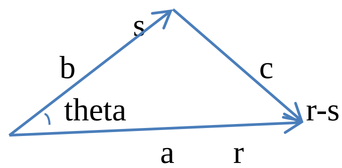
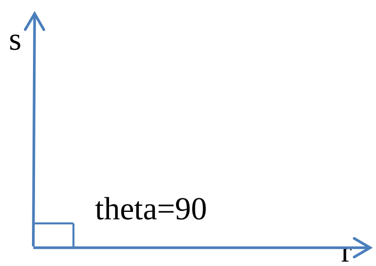
Vectors

1, Cosine rule:



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$
$$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$$
$$(r-s).(r-s) = r.r - r.s - s.r + s.s = |r|^2 - 2s.r + |s|^2$$
$$\rightarrow -2s.r = -2|r||s|\cos\theta$$
$$\rightarrow s.r = |r||s|\cos\theta$$



$$r.s = |r||s|\cos 90 = 0$$

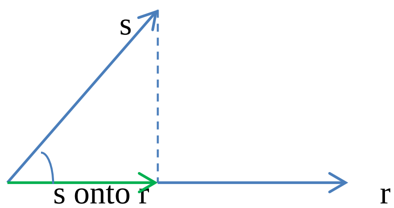
$$r.s = -|r||s|$$

2, The *size* of a vector is the square root of the sum of the squares of its components.

3, For two *n* component vectors,

$$a.b = a_1b_1 + a_2b_2 + \cdots + a_nb_n$$

4,



The *scalor projection* is the size of the green vector. If the angle between s and r is greater than $\pi/2$, the projection will also have a minus sign.

$$r.s = |r||s|\cos\theta = |r| * projection$$

The *vector projection* is the green vector.

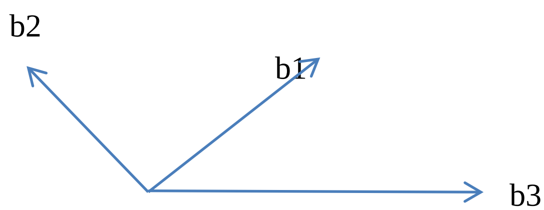
$$proj_{\vec{v}}\vec{u} = \left(\frac{\vec{u}.\vec{v}}{||\vec{v}||^2}\right)\vec{v}$$

5, Basis is a set of *n* vectors that:

(i) are not linear combinations of each other (linear independent);

(ii) span the space;

- The space is then n-dimensional.



$$b_3 \neq a_1 b_1 + a_2 b_2 \ (linear\ independent)$$

If b_3 is linear independent to b_1 and b_2:

1) $b_3 \neq a_1 b_1 + a_2 b_2$, for any a_1 or a_2;

2) b_3 does not lie in the plane spanned by b_1 and b_2.