Nikhil Aswal - Sec-F - 17 - 2016877

Tutorial - 4

Ans. 1. $T(n) = 3T(n/2) + n^2$

$a \geq 1, \quad b \geq 1$

Or Comparing

$a = 3, \quad b = 2, \quad f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$T(n) = O(n^2)$

Ans. 2 $T(n) = 4T(n/2) + n^2$

$a \geq 1, \quad b > 1$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 \Rightarrow f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

Ans. 3. $T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 0 = 0$

$n^c = n^0 = 1$

$f(n) \geq n^c$

$T(n) = \theta(2^n)$

Ans. 4. $T(n) = 2^n T(n/2) + n^2$

$a = 2^n$

$b = 2, \quad f(n) = n^2$

$c = \log_n a = \log_2 2^n$

$= n$

$n^c \Rightarrow n^n$

$f(n) = n^c$

$f(n) = \theta(n^2 \log_2 n)$

Ans. 5. $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2$

$= 2 \log_4 4$

$= 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

Ans 6. $T(n) = 2T(n/2) + n \log n$

$a = 2, \quad b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n \geq n$

$f(n) \geq n^c$

$T(n) = \theta(n \log n)$

Ans.7. $T(n) = 2T(n/2) + n(\log n)$

$a = 2,\ b = 2,\ f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore \frac{n}{\log n} < n$

$f(n) \leq n^c$

$T(n) = \theta(n)$

Ans.8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.31}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.31}$

$f(n) \geq n^c$

$\therefore T(n) = \theta(n^{0.51})$

Ans.9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b = 2$

$a \geq 1$ but here a is 0.5

So we cannot apply master's Theorem

Ans.10. $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

Ans.11. $4T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\because \log n < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$= \theta(n^2)$

Ans.12. $T(n) = 8\text{sqrt}(n)T(n/2) +$

$a = \sqrt{n},\ b = 2$

$c = \log_b a = \log_2 \sqrt{n} =$

$\frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) \leq n^c$

$T(n) = \theta(f(n))$

$= \theta(\log(n))$

Ans.13. $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.5439}$

$n < n^{1.3843}$

$f(n) < n^c$

$T(n) = \theta(n^{1.5841})$

Ans 14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3,\ b = 3,\ c = \log_b a = 1$

$n^c = n^1 = n$

As sqrt $(n) < n$

$= \theta(n)$

Ans.19. $T(n) = 4T(n/2) + n$

$a = 4,\ b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$f(n) = \theta(n^2)$

Ans 16. $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4, f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \theta(n\log n)$

Ans.17. $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3$

$c = \log_b a \rightarrow \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \theta(n)$

Ans.18. $T(n) = 6T(n/3) + n^2\log n)$

$a = 6,\ b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2\log n$

$T(n) = \theta(n^2\log n)$

Ans.19. $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \theta(n^2)$

Ans.20 $T(n) = 64T(n/3) - n^2\log n$

$a = 64,\ b = 3$

$c = \log_b a = \log_8 64 = \log_{\frac{1}{8}}$

$c = 2$

$n^c = n^2$

$n^2\log n > n^2$

$T(n) = \theta(n^2\log n)$

Ans.21 $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ if(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \theta(n^2)$

Ans.22. $T(n) = T(n/2) + n(2 - \cos x)$

$a = 1,\ b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos x) > n^c$

$T(n) = \theta(n(2 - \cos x))$