Shear strength

Note : while solving numericals, always take same scale on X – axis and Y – axis.

Problems on Direct shear test (DST)

Problem 1. A specimen of clean dry cohesionless soil is tsted in shear box and the soil is failed at shear stress of 40 KN/m^2 When the normal load on the specimen was 50 KN/m^2 . Determine

a) The angle of shearing resistance
b) Principal stresses during the failure
c) The direction of the principal planes.

OR

A specimen clean dry sand tested in shear box and the soil failed at a shear stress of 40 KN/m^2

When the normal load on specimen is 50 KN/m^2. Determine

a) Angle of shearing resistance
b) Magnitude and direction of principal stresses

OR

In DST , on dry sand the sample failed at shear stress of 40 and the normal stress of 50

Determine

1) Shear strength parameters
2) Principal stresses and their orientation of these planes.

OR

For DST on the soil failed at shear stress of 40 KN/m^2 when normal stress is 50 KN/m^2

Determine C , $\emptyset$ and principal stresses.

ANSWER : following are steps (procedure) to solve above problem .

1) On X axis : 1 cm = 10 KN/m^2 , on Y Axis : 1 cm = **10** KN/m^2

2) Mark the ordinate C (50,40) .as the specimen fails at shear stress of 40 KN/m^2

3) Draw the line passing through C and intersect to Y – Axis. Here in this case, line connects at origin. Because soil is cohesionless or sandy soil (Cohesion = C = Zero)

4) Draw perpendicular line from point C , we get point E .

5) With origine E and distance EC , draw semicircle . we get points B and D

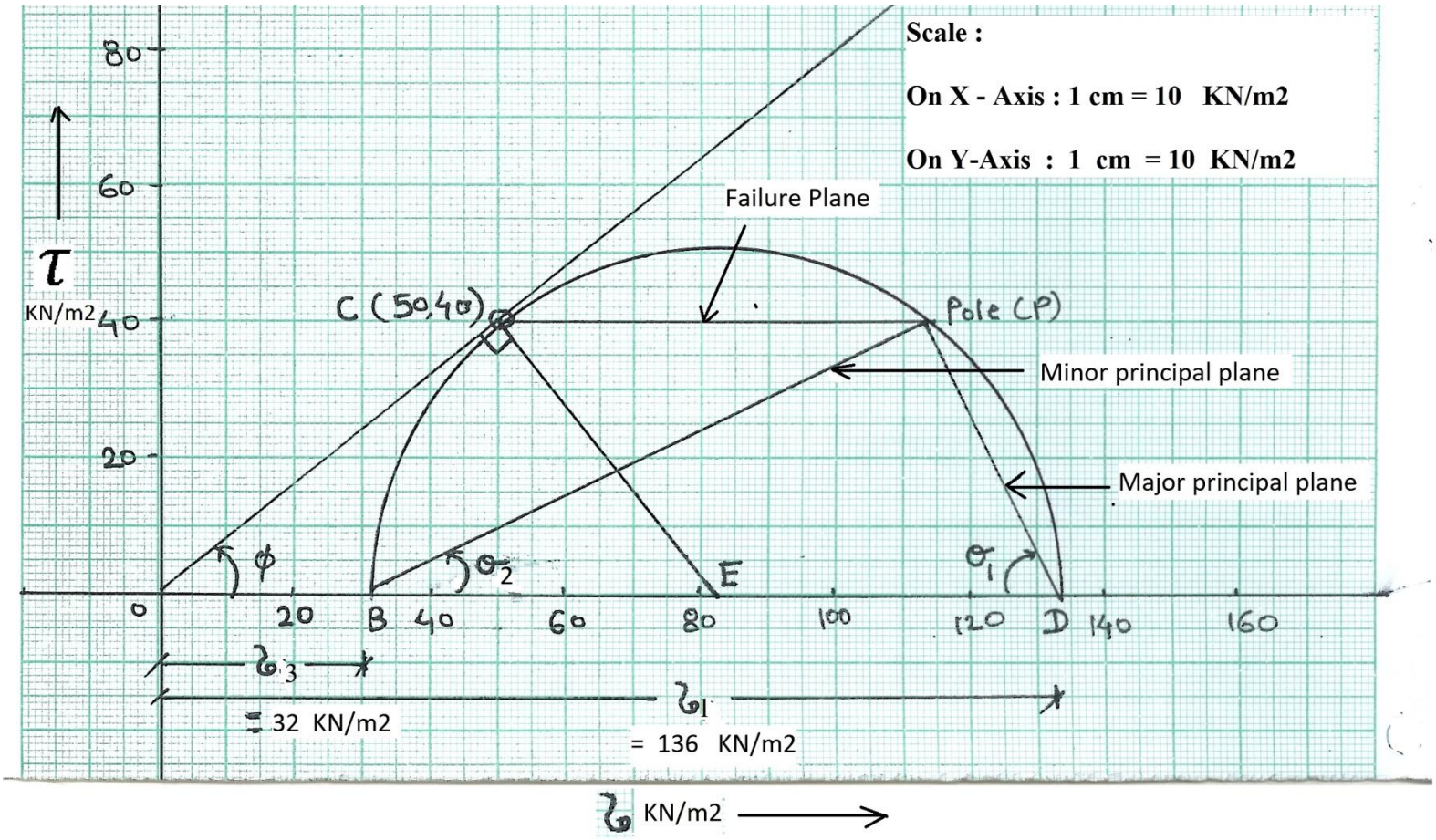
6) Draw horizontal straight line from Pt. C , we get point P (Pole)

7) Connect Points B and D to the pole (p) .

8) Now, PD = major principal plane with orientation θ_1 and
9) PB = minor principal plane with orientation θ_2

Cohesion = C = 0 ; major principal stresss = σ_1 = 136 KN/m2

minor principal stresss = σ_3 = 32 KN/m2 : $\theta_1 = 65^0$ and $\theta_2 = 25^0$



Problem 2 following are the observations for direct shear test of size 6 cm x 6 cm .

Normal load (N)	100	200	300	400
Shear load (N)	90	181	270	362

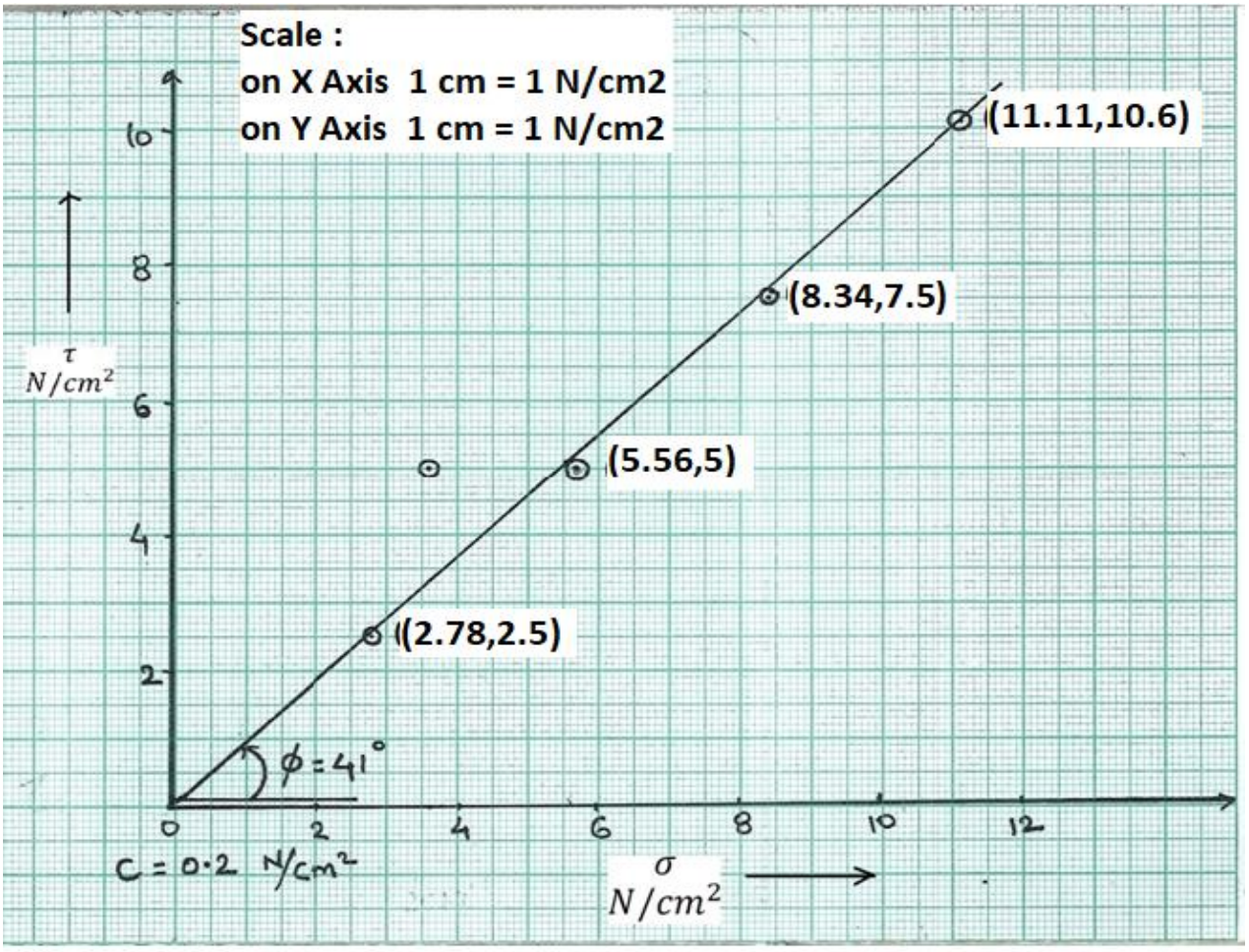
Determine shear strength parameters :

ANSWER : Area = 6 x 6 = 36 cm^2

Normal load (N)	Shear load (N)	$\sigma = \frac{Normal\ load}{A}$ N/cm^2	$\tau = \frac{shear\ load}{A}$ N/cm^2
100	90	2.78	2.5
200	181	5.56	5.03
300	270	8.34	7.5
400	362	11.11	10.06

Cohesion = C = 0.2 N/cm2

Angle of shearing resistance or angle of internal friction = Ø = 41^0



Problem 3. A shear box test carried out on the sample gave :

Test No.	$Stress \quad {}^{KN}/_{m^2}$	Shear $Stress \quad {}^{KN}/_{m^2}$
1	100	80
2	200	144
3	300	216

Determine the magnitude of the major and minor principal stresses at failure when the normal (horizontal) stresses on the sample was 200 ${}^{KN}/_{m^2}$. determine also the inclination to the horizontal of these stresses.

ANSWER : following are steps (procedure) to solve above problem .

1) On X axis : 1 cm = 25 ${}^{KN}/_{m^2}$, on Y Axis : 1 cm = **25** ${}^{KN}/_{m^2}$

2) Mark the all three ordinates on graph i.e. (100,80) , (200,144) , (300,216) respectively.

3) Draw the line passing through these all three points and further it will intersect to Y – Axis.

4) major and minor principal stresses at failure when the normal (horizontal) stresses on the sample was 200 ${}^{KN}/_{m^2}$.

5) Draw perpendicular line from point C , we get point E .

6) With origine E and distance EC , draw semicircle . we get points B and D

7) Draw horizontal straight line from Pt. C , we get point P (Pole)

8) Connect Points B and D to the pole (p) .

9) Now, PD = major principal plane with orientation θ_1 and

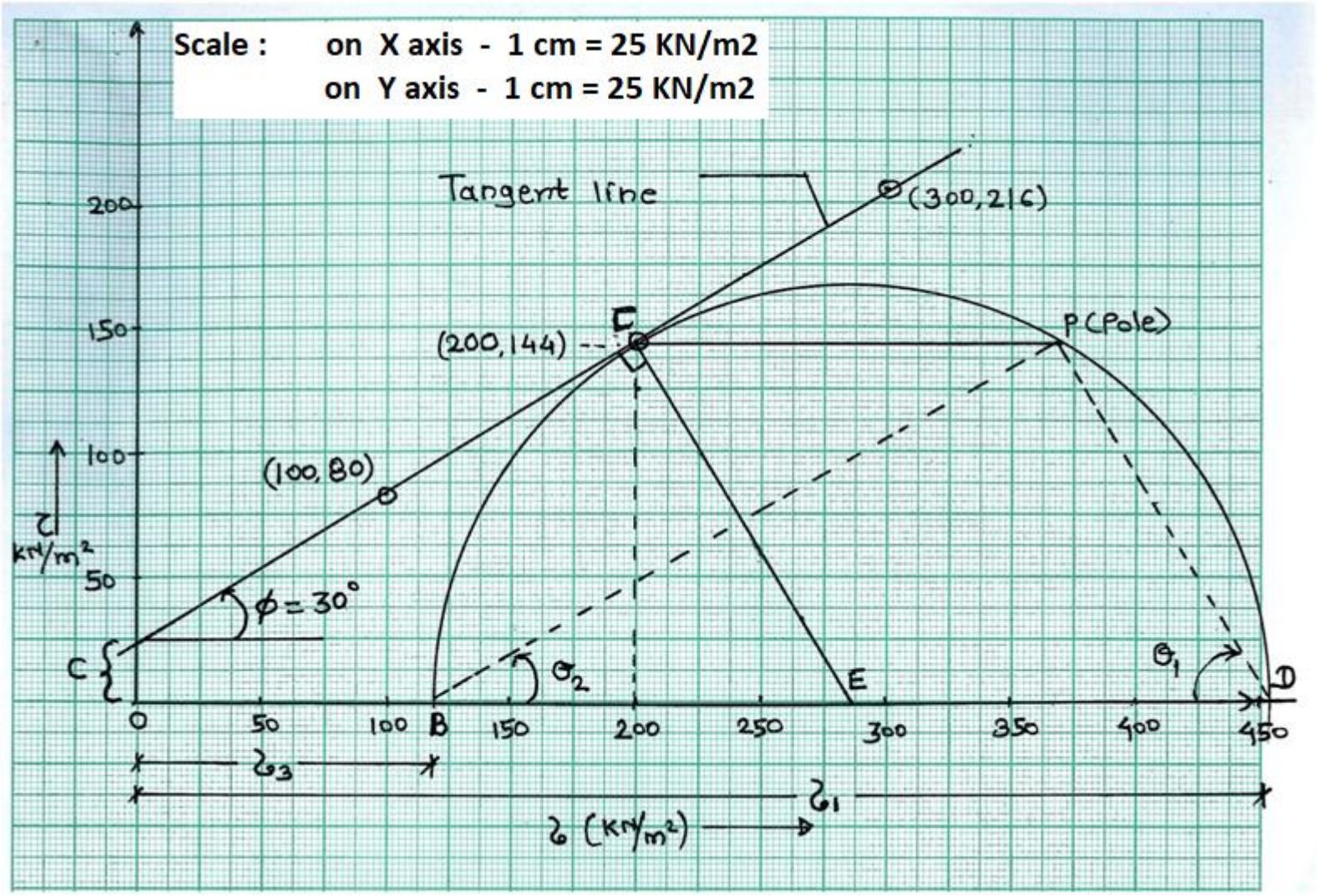
10) PB = minor principal plane with orientation θ_2

Cohesion = C = 0 ;

major principal stresss = σ_1 = 425 KN/m2 with orientation $\theta_1 = 58^0$

minor principal stresss = σ_3 = 125 KN/m2 with orientation $\theta_2 = 32^0$

Angle of shearing resistance or internal friction = $\emptyset = 34^0$



Problem 4. Following results were obtained when loose sand was tested in shear box test.:

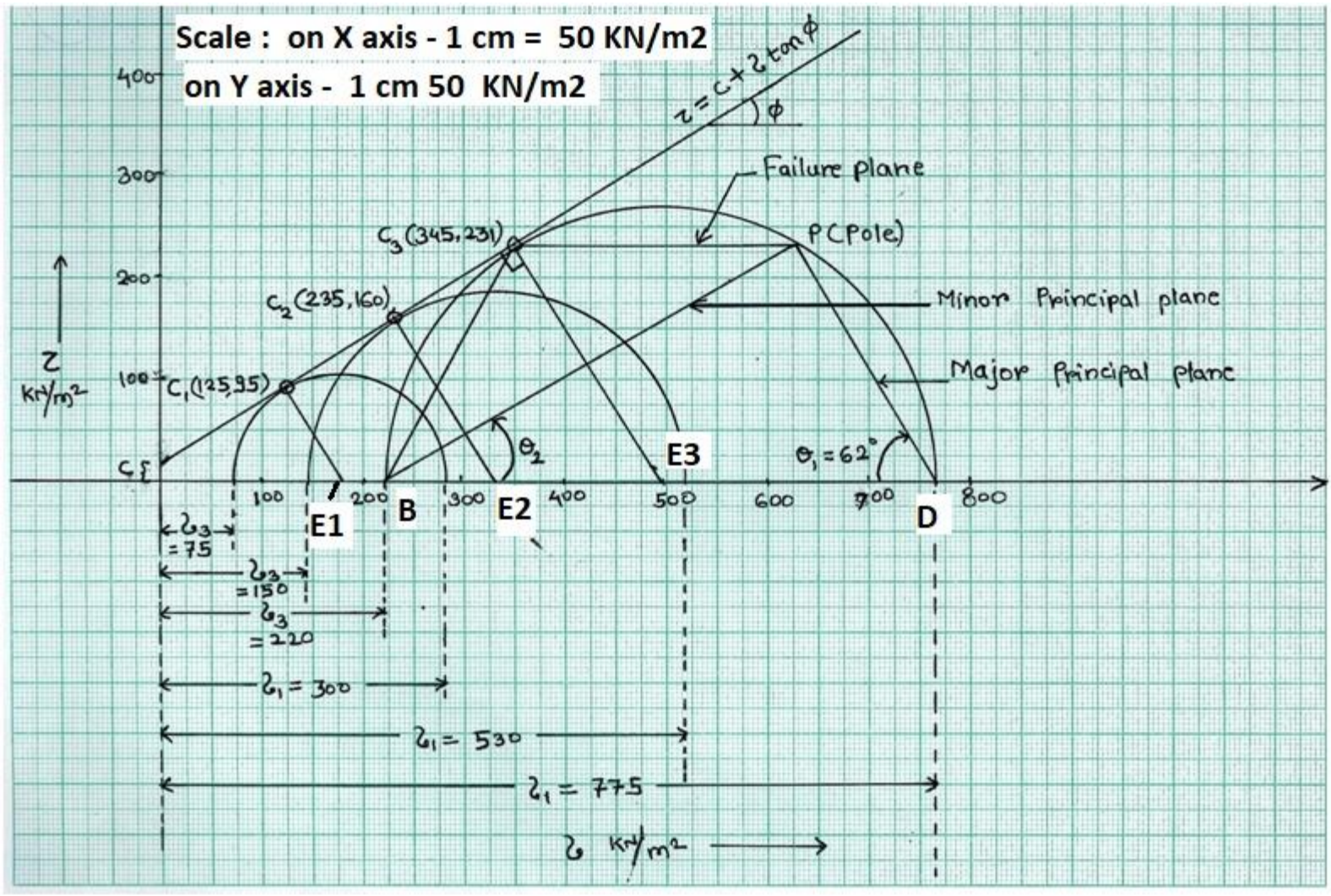
Normal stress $^{kn}/_{m^2}$	125	235	345
Shear stress $^{kn}/_{m^2}$	95	160	231

Draw mohr's circle and strength envelope

Determine : 1) shear strength parameters

2) Principal stresses

3) orientation of principal streses.



ANSWER : following are steps (procedure) to solve above problem .

1) On X axis : 1 cm = 50 $^{KN}/_{m^2}$, on Y Axis : 1 cm = **50** $^{KN}/_{m^2}$

2) Mark the all three ordinates on graph i.e. C1, C2, C3 respectively.

3) Draw the line passing through these all three points and further it will intersect to Y – Axis.

4) Draw perpendicular line from points C1, C2, C3 , we get point E1, E2, E3 .

5) With origine E1, E2, E3 and distance E1C1, E2C2, E3C3, , draw semicircle . we get points B and D finaly.

6) Draw straight line from C3 and it will intersect somewhere on semicircle , we get point P (Pole).

7) Connect Points B and D to the pole (p) .

8) Now, PD = major principal plane with orientation θ_1 and

9) PB = minor principal plane with orientation θ_2

Since the soil is sandy soil , hence Cohesion = C = 0

Angle of shearing resistance or internal friction = $\emptyset = 32^0$

Average major principal stresss $= \sigma_1 = \frac{(300+530+775)}{3} = 535$ KN/m2

Average minor principal stresss $= \sigma_3 = \frac{(75+150+220)}{3} = 148.33$ KN/m2

orientation of major principal plane with horizontal $\theta_1 = 62^0$

orientation of minor principal plane with horizontal $\theta_2 = 28^0$

Problem :5 :

From direct shear test of soil sample . the following data have been obtained ; determine shear strength parameters. If the same soil sample is subjected to normal stress of 100 $^{kn}/_{m^2}$.. determine shear strength , major and minor principal stresses.

Normal stress $^{kn}/_{m^2}$	70	96	114
Shear stress $^{kn}/_{m^2}$	138	156	170

Answer :

From graph, we get shear strength parameters as

Cohesion = C = 87 KN/m2 and ` Angle of shearing resistance or internal friction = $\emptyset = 35^0$

Part II -

At normal stress, $\sigma = 100\ kN/m^2$

we've to find shear strength (s)

we know Equation of tangent / shear strength

$$S = \tau = c + \sigma \tan\phi$$

Shear strength

$$S = 87 + (100 \times \tan 35)$$

$$= 157\ kN/m^2$$

From Mohr's Coulom's failure envelope,

$$\sigma_1 = \sigma_3 \tan^2\theta + 2c \tan\theta$$

where;

$$\theta = 45 + \phi/2 = 45 + 35/2 = 62.5^\circ$$

$$\sigma_1 = \sigma_3 \tan^2(62.5) + 2 \times 87 \times \tan(62.5)$$

$$\therefore \sigma_1 = 3.69\,\sigma_3 + 334.08 \quad \text{---------} (1)$$

Again we've Mohr's stress Circle,

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy} \sin 2\theta$$

$$\therefore 100 = \sigma_3 \cos^2\theta + \sigma_1 \sin^2\theta$$

$$100 = \sigma_3 \cos^2(62.5) + \sigma_1 \sin^2(62.5)$$

$$100 = 0.213\,\sigma_3 + 0.786\,\sigma_1$$

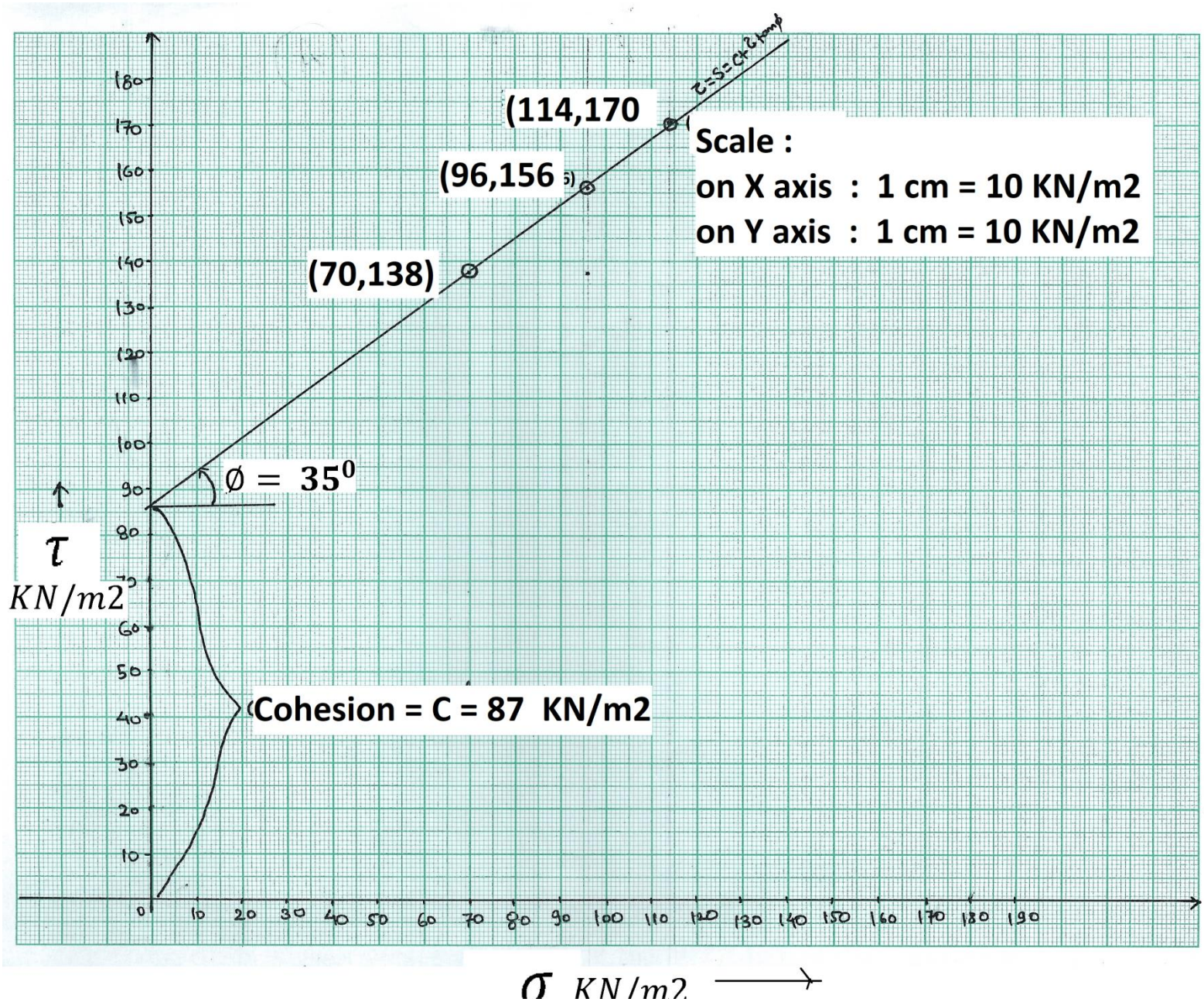
Eq^n (1) →

$$\therefore 100 = 0.2132\,\sigma_3 + 0.786\,(3.69\,\sigma_3 + 334.8)$$

$$\sigma_3 = -52.21\ kN/m^2$$

Put this value in Eq^n (1)

$$\therefore \sigma_1 = 142.34\ kN/m^2$$


(114,170
(96,156
(70,138)
Scale :
on X axis : 1 cm = 10 KN/m2
on Y axis : 1 cm = 10 KN/m2
Ø = 35⁰
τ
KN/m2
Cohesion = C = 87 KN/m2
σ KN/m2

Problems on Unconfined Compression test

Problem :6 :

An unconfined compression test was conducted on saturated clay specimen 38 mm Ø and 76 mm in length . the vertical deformation is measured to be 10 mm at failure load of 0.25 KN . compute Cohesion (C) ?

Answer :

$$A_f = \frac{A_o}{(1-\epsilon)} = \frac{\pi/4 \times 38^2}{1-(10/76)}$$

$$= 1306 \text{ mm}^2 = 0.0013 \text{ m}^2$$

$$q_u = \frac{0.25}{0.0013} = 192.31 \text{ KN/m}^2$$

$$\text{Cohesion} = C = \frac{192.31}{2} = 96.15 \text{ KN/m}^2$$

Problem :7 :

An unconfined compression test on saturated clay, yhe unconfined compression strength was found to be 160 $^{kn}/_{m^2}$. It is known that the same soil showed an angle of shearing resistance f 10^0 . in consolidated undrained test . what is the % error and is conservative or not . conservative to use Cu = $^{q_u}/_2$

Answer :

case 1]

when $\sigma_1 = q_u = 160 \text{ KN/m}^2$; $\phi = 0^\circ$

cohesion $C_u = \frac{q_u}{2} = \frac{160}{2} = 80 \text{ KN/m}^2$

$\theta = 45^\circ$

There is no lateral / confining Stress (σ_3)

$\therefore \sigma_3 = 0$

Case II] $\phi = 10^\circ$

we've $\theta = 45 + \phi/2 = 45 + 10/2 = 50^\circ$

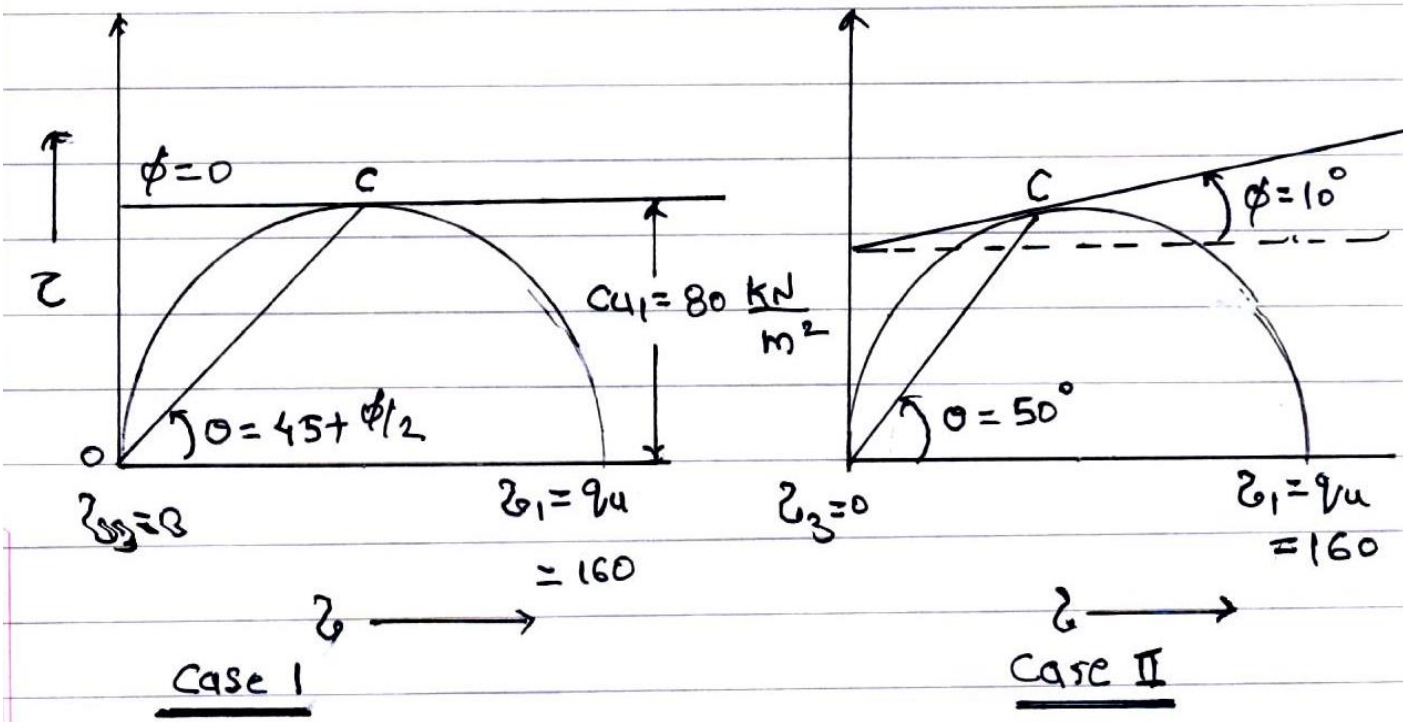
Again;

$$\sigma_1 = \sigma_3 \tan^2\theta + 2C \tan\theta$$

$$\therefore 160 = 0 \times \tan^2\theta + 2C \tan 50^\circ$$

$$\therefore 160 = 2C \tan 50^\circ$$

Cohesion $C_{u2} = 67.11 \ KN/m^2$



$$\% \text{ error} = \left(\frac{C_{u1} - C_{u2}}{C_{u1}}\right) \times 100 = \left(\frac{80 - 67.11}{80}\right) \times 100 = 16.11\ \%$$

Problems on Triaxial shear test

Important Notes :

Minor principal stress (σ_3) = radial stresss =cell pressure=confining pressure.

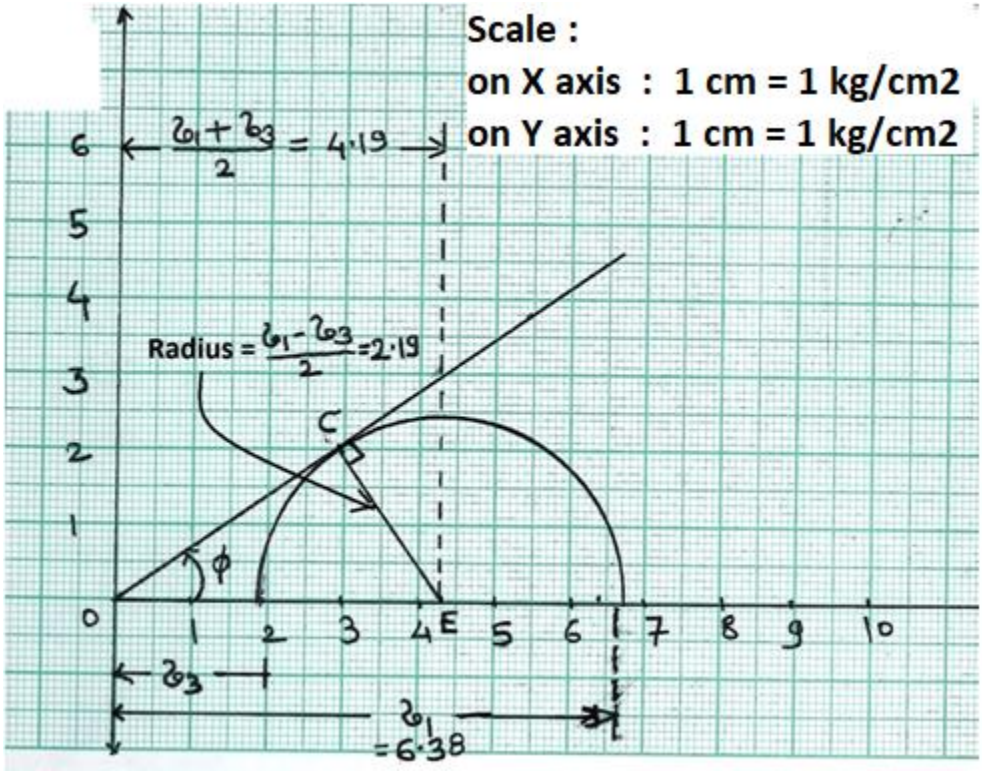
Major principal stress (σ_1) = axial stress.= $\sigma_3 + \sigma_d$

where, σ_d - deviator stress.

Problem 8. A soil sample of dry coarse sand is tested in triaxial apparatus in undrained condition under a cell pressure of 2 $^{kg}/_{cm^2}$. The sample fails when deviator stress reached at 4.38 $^{kg}/_{cm^2}$.

Determine :

1) Shear strength parameters
2) At what deviator stress , will be soil fail if the cell pressure be 3 $^{kg}/_{cm^2}$.



As we know, Minor principal stress = cell pressure

$\therefore \sigma_3 = 2$ Kg/cm^2

Again, Major principal Stress (σ_1)

$\sigma_1 = \sigma_3 + \sigma_d$

$= 2 + 4.38 = 6.38$ Kg/cm^2

Since the soil sample is coarse sand, hence cohesion = C = 0

$$OE = \frac{\sigma_1 + \sigma_3}{2} = 4.19 \text{ kg/cm}^2$$

$$CE = \text{Radius} = \frac{\sigma_1 - \sigma_3}{2} = 2.19 \text{ kg/cm}^2$$

Shear strength parameters:

1) Angle of shearing Resistance = $\phi = 31^\circ$

2) As the soil is cohesionless (sand);

cohesion $C = 0$.

Now, $\sigma_1 = \sigma_3 \tan^2\theta + 2C \tan\theta$

$\therefore \sigma_1 = \sigma_3 \tan^2\theta = \sigma_3 \tan^2(45 + \phi/2)$

Here, $\sigma_3 = 3 \text{ kg/cm}^2$; $\phi = 31^\circ$

$$\therefore \sigma_1 = 3 \tan^2\left(45 + \frac{31}{2}\right) = 9.37 \text{ kg/cm}^2$$

$\therefore$ deviator stress $= (\sigma_1 - \sigma_3) = (9.37 - 3)$

$= 6.37 \text{ kg/cm}^2$

Problem 9. The shear strength parameters of the given soil are C = 0.26 kg/cm2 and $\emptyset = 21^0$. Undrained triaxial shear test can be carried out on specimen.

Determine :

1) Daviator stress at which failure will occur if the cell pressure be 2.5 kg/cm2
2) The cell pressure during this test , if sample fails when the deviator stress reaches 1.68 kg/cm2

Answer :

We know that, $\sigma_1 = \sigma_3 \tan^2\theta + 2c\tan\theta$

for Given Soil, $C = 0.26\ kg/cm^2$; $\emptyset = 21^\circ$

$$\theta = 45 + \emptyset/2 = 45 + \frac{21}{2} = 55.5^\circ$$

$$\therefore \tan^2\theta = \tan^2 55.5 = 2.117 \quad \&$$

$$\tan\theta = \tan 55.5 = 1.455$$

$$\therefore \sigma_1 = 2.117\,\sigma_3 + 0.757 \quad \text{—— (1)}$$

When $\sigma_3 = 2.5\ kg/cm^2$

Eq^n (1) →

$$\sigma_1 = (2.117 \times 2.5) + 0.757 = 6.05\ kg/cm^2$$

Now,

deviator stress $\sigma_d = (\sigma_1 - \sigma_3)$

$= (6.05 - 2.5) = 3.55\ kg/cm^2$

let, the required cell pressure be $x\ kg/cm^2$

$$\therefore \sigma_1 = \sigma_3 + \sigma_d = 1.68 + x$$

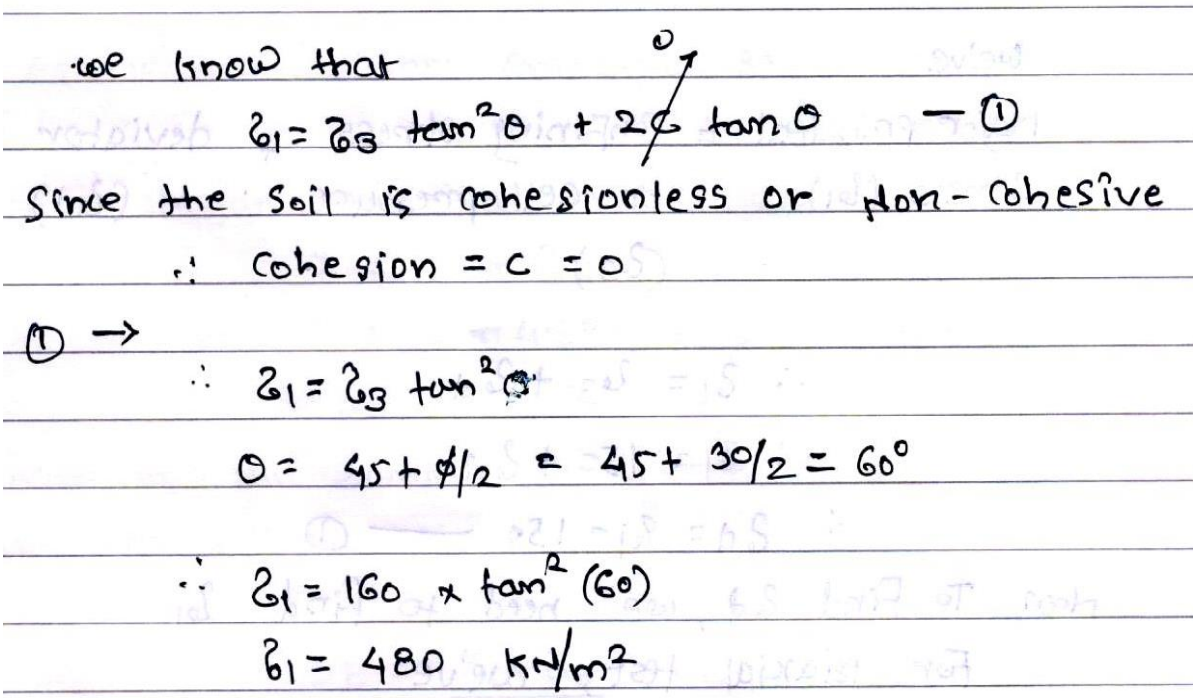
(1) →

$$(1.68 + x) = 2.117\,x + 0.757$$

$$\therefore x = \sigma_3 = 0.83\ kg/cm^2$$

Problem 10. A specimen of non cohesive soil are to be tested . angle of shearing resistance of the soil is 30^0 If the cell pressure in test is 160 KN/m2 , what would be the axial stress at failure.

Answer :



Problem 11.

Determine shear strength parameters of soil from triaxial test data given

Test	1	2
Cell pressure (KN/m2)	140	280
Axial stress (KN/m2)	320	640

Answer :

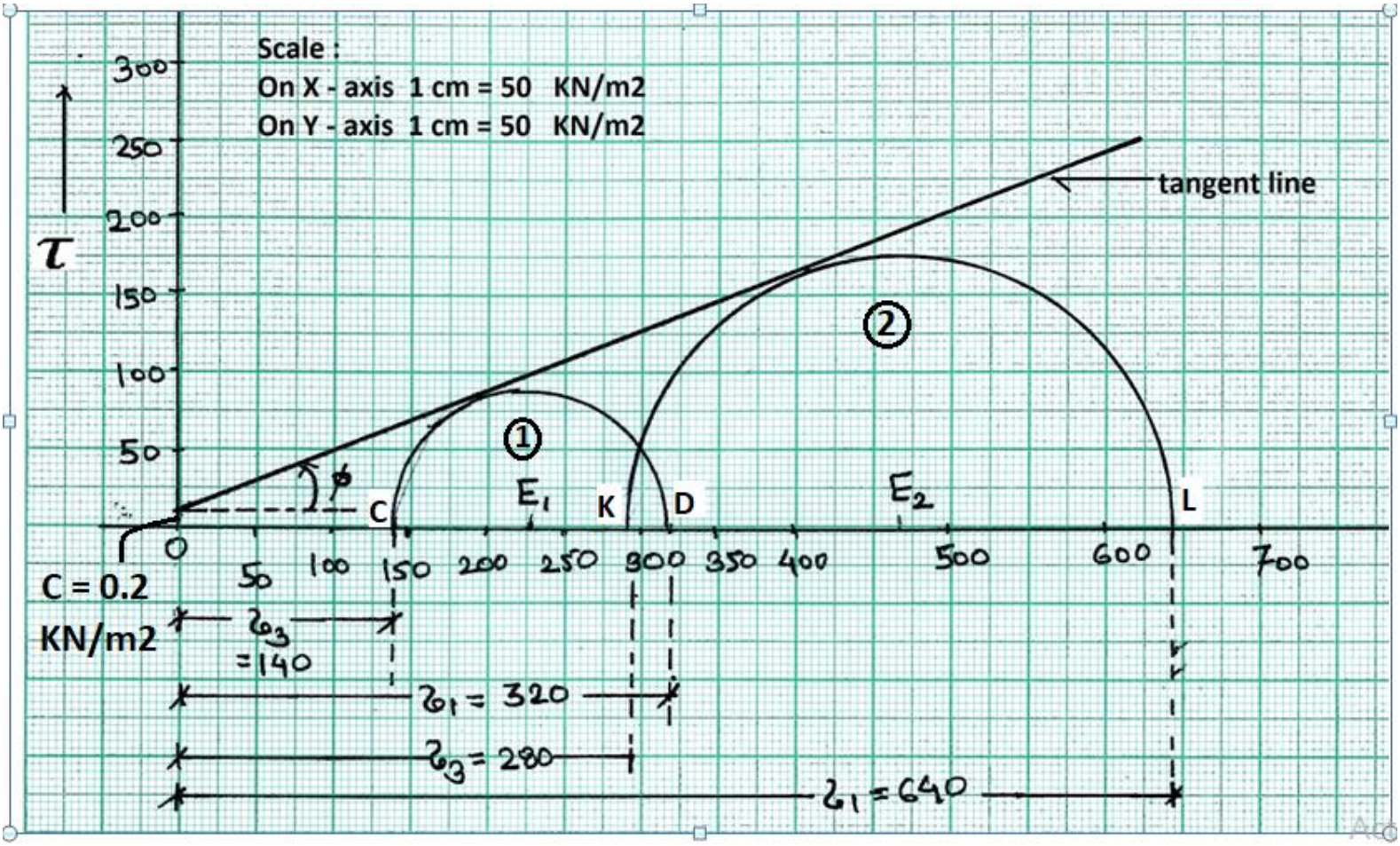
Test	Cell pressure (KN/m2)/minor principal stress $(\boldsymbol{\sigma_3})$	Axial stress (KN/m2))/major principal stress $\boldsymbol{\sigma_1}$)
1	140	280
2	320	640

Procedure :

1) Take suitable scale on x and y axis .
2) Let us consider that how to draw 1st mohr circle.

We already know that for 1st circle, $\sigma_3 = 140$ as point C and $\sigma_1 = 280$ as point D. Mark these points on X axis.

3) Take ruler or simple scale , measure distance CD and find out its center . say E1.
4) Draw Mohr circle with origine E1 and radius CE1.
5) For 2nd mohr circle, revise and repeat above procedure . we get points K,L, and E2. Draw semicircle.
6) Now, draw the tangent line which connects both circles and further extended to Y axis.
7) Cohesion = C = 0.2 KN/m2 and ` Angle of shearing resistance or internal friction = $\emptyset = 23^0$



Problem 12

Following results were obtained . from a series of consolidated undrained test of soil. In which a pore water pressure was not determined. Determine shear strength parameters?

Test	Confining pressure (KN/m2)	Deviator stress (KN/m2) At failure
1	100	600
2	200	750
3	300	870

Answer : **Important Notes :**

Minor principal stress (σ_3) = radial stresss =cell pressure=confining pressure.

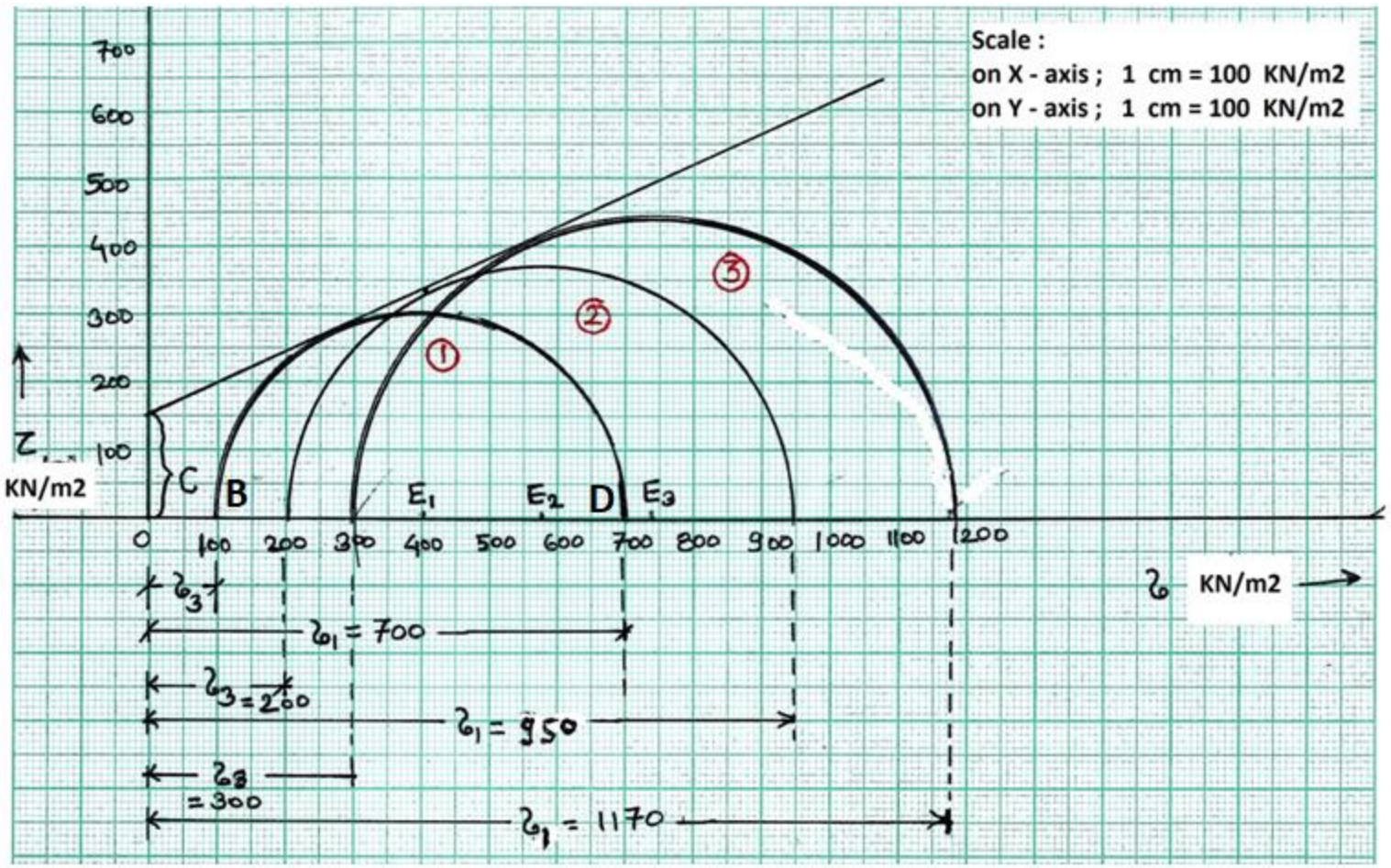
Major principal stress (σ_1) = axial stress.= $\sigma_3 + \sigma_d$

where, σ_d - deviator stress.

Test	Minor principal stress = Confining pressure (KN/m2)= σ_3	Deviator stress (KN/m2) At failure	Major principal stress = (KN/m2)= σ_1
1	100	600	700
2	200	750	950
3	300	870	1170

Procedure :

8) Take suitable scale on x and y axis .
9) Let us consider that how to draw 1st mohr circle.
   We already know that for 1st circle, $\sigma_3 = 100$ as point B and $\sigma_1 = 700$ as point D. Mark these points on X axis.
10) Take ruler or simple scale , measure distance BD and find out its center . say E1.
11) Draw Mohr circle with origine E1 and radius BE1.
12) For 2nd and 3rd mohr circle, revise and repeat above procedure to get E2 and E3 . Draw semicircle.
13) Now, draw the tangent line which connects all three circles and further extended to Y axis.
14) Cohesion = C = 150 KN/m2 and ` Angle of shearing resistance or internal friction = $\emptyset = 24^0$



Problem 13: A direct shear test was carried out on cohesive soil and following results were obtained.

Normal stress KN/m2	150	250
Shear test at failure KN/m2	110	120

What would be the deviator stress at failure if a triaxial test is carried out on same soil with cell pressure of 150 KN/m2

Answer :

We've

Major principal stress (σ_1) = Confining stress or cell pressure (σ_3) + deviator press. (σ_d)

$$\therefore \sigma_1 = \sigma_3 + \sigma_d$$

$$\therefore \sigma_1 = 150 + \sigma_d$$

$$\therefore \sigma_d = \sigma_1 - 150 \quad \text{——} \ (1)$$

Now, To Find σ_d, we need to find σ_1

For triaxial test, we've

$$\sigma_1 = \sigma_3 \tan^2\theta + 2c \tan\theta \quad \text{——} \ (2)$$

From Graph;

Cohesion $= c = 98\ kN/m^2$

Angle of internal friction $= \phi = 6°$.

(2) $\rightarrow$ $\sigma_1 = 150 \times \tan^2(45+\phi/2) + 2\times 98 \times \tan(45+\phi/2)$

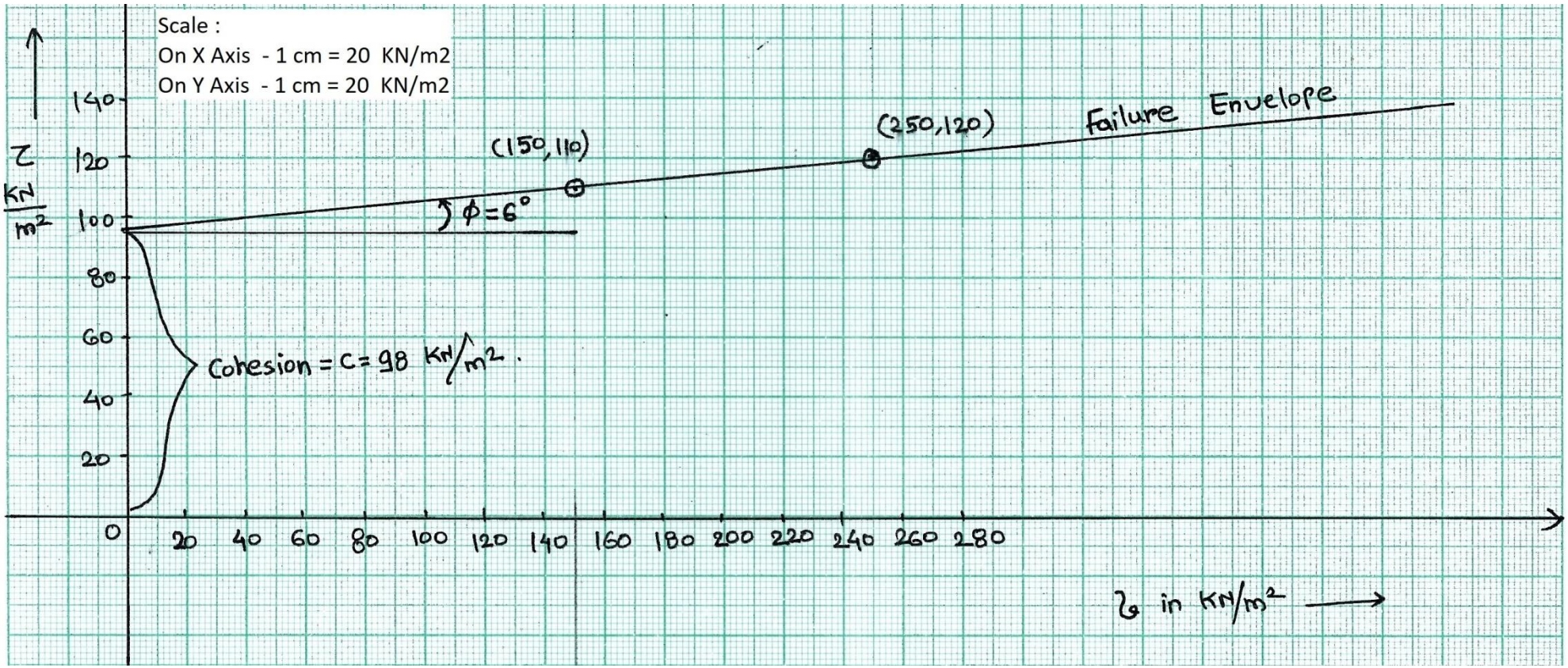
$= 150 \times \tan^2(45+6/2) + 2\times 98 \times \tan(45+6/2)$

$= 393\ kN/m^2$

(1) $\rightarrow$ deviator stress $(\sigma_d) = \sigma_1 - \sigma_3$

$= 393 - 150$

$= 243\ kN/m^2$


Scale :
On X Axis - 1 cm = 20 KN/m2
On Y Axis - 1 cm = 20 KN/m2
τ KN/m2
140
120
100
80
60
40
20
0
20 40 60 80 100 120 140 160 180 200 220 240 260 280
(150, 110)
(250, 120)
Failure Envelope
φ = 6°
Cohesion = C = 98 KN/m2.
σ in KN/m2

Problem 14. A series of consolidated undrained triaxial test was conducted on over consolidated clay and following results were obtained

sample	Cell pressure (KN/m2)	Deviator stress (KN/m2)	Pore water pressure (μ) (KN/m2)
1	125	510	-70
2	250	620	-10
3	500	850	120

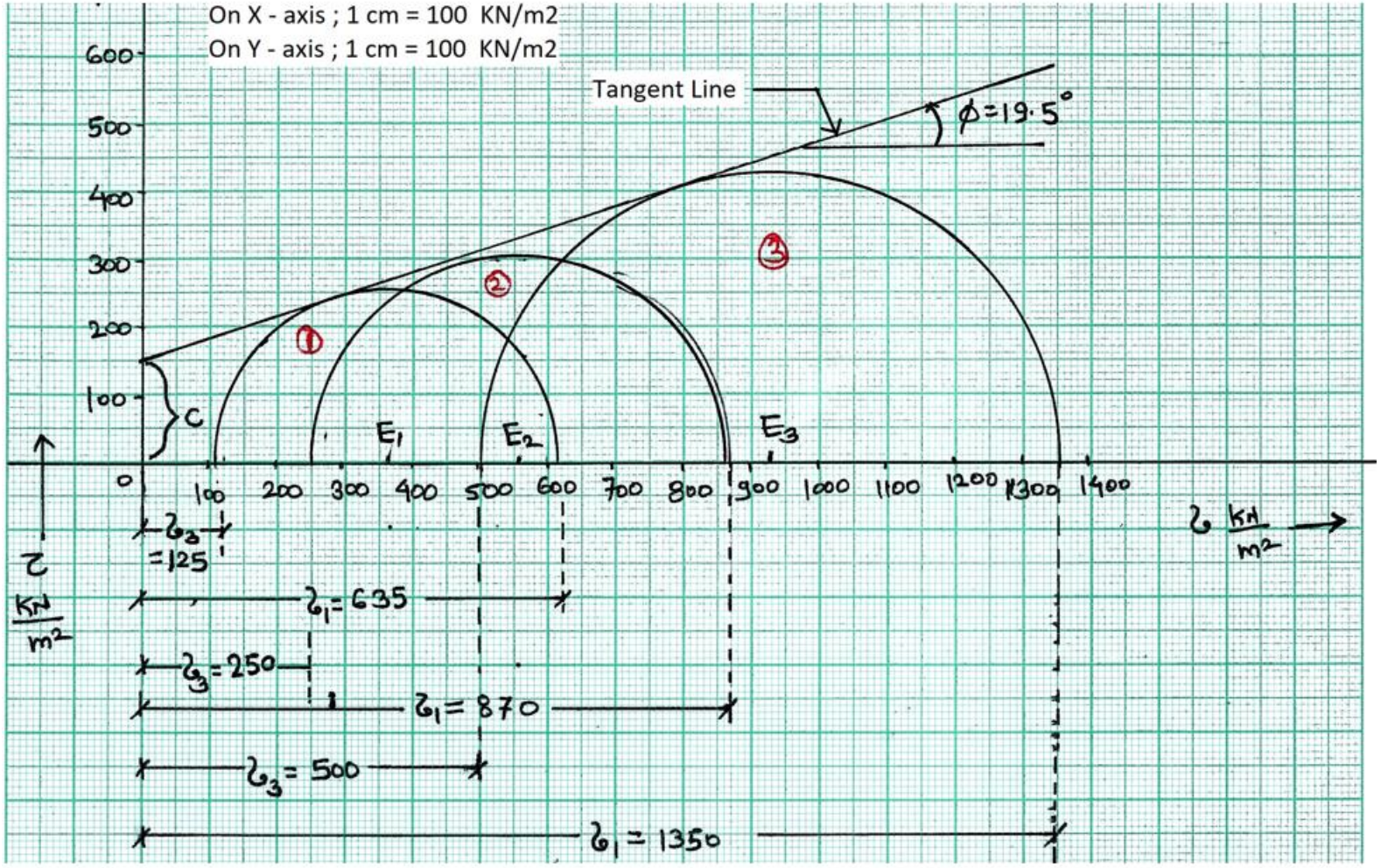
Plot the strength envelope in terms of total stresses and effective stresses . and hence determine shear strength parameters.

Answer :

sample	Total stress parameters		μ	Effective stress parameters	
	Minor principal stress = Confining or cell pressure (KN/m2)= $\boldsymbol{\sigma_3}$	Major principal stress ($\boldsymbol{\sigma_1}$) $\boldsymbol{\sigma_1 = \sigma_3 + \sigma_d}$		$\overline{\sigma_3} = \sigma_3 - \mu$	$\overline{\sigma_1} = \sigma_1 - \mu$
1	100	600	-70	125-(-70)= 195	705
2	200	750	-10	260	880
3	300	870	+120	380	1230

Procedure : Refer problem No. 11 and 12 . the procedure is same.

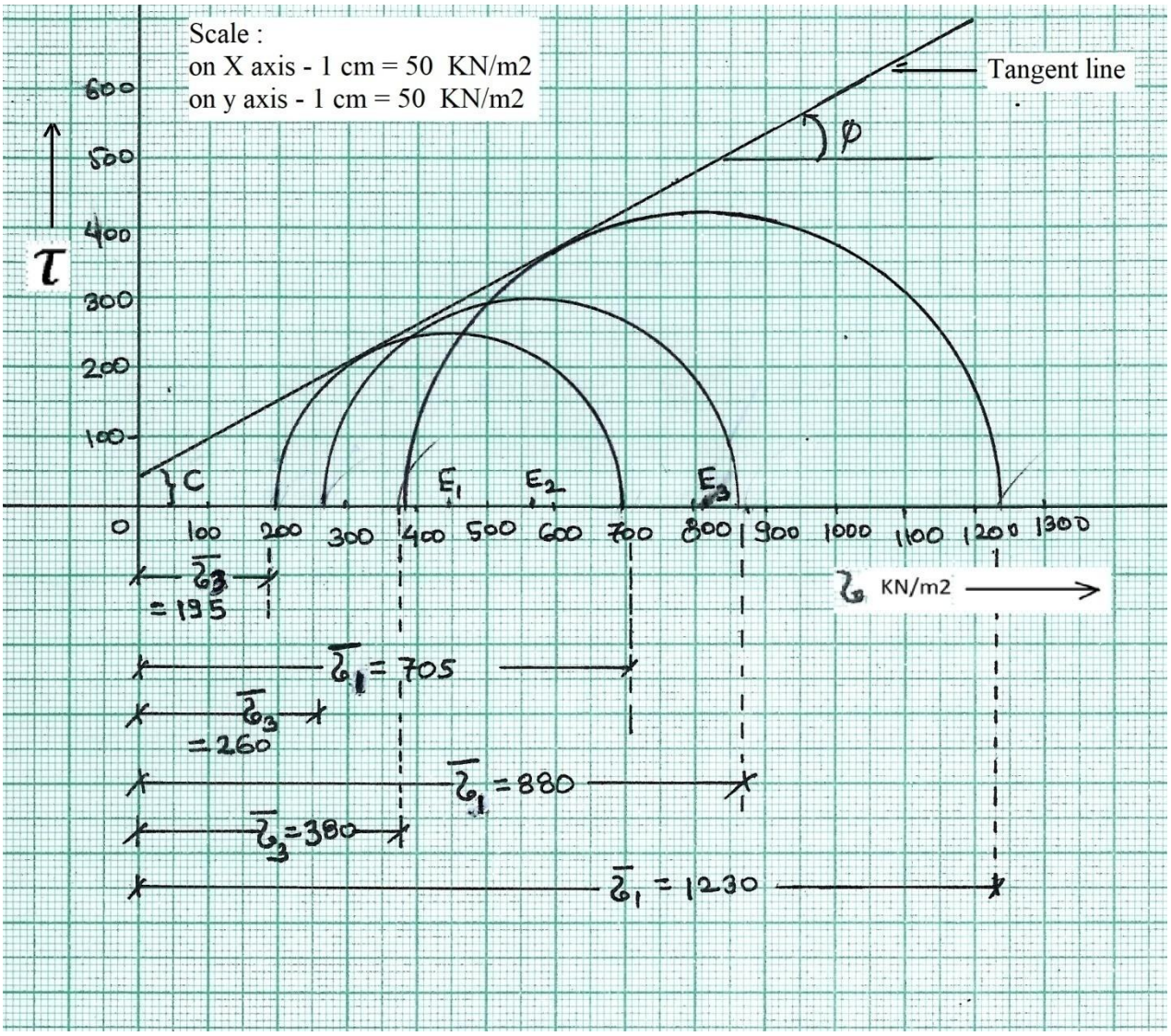
Graph of Total Stresses



Shear strength parameters are :

1) Cohesion = C = 150 KN/m2
2) Angle of shearing resistance or internal friction = $\emptyset = 20^0$

Graph of effective Stresses



Shear strength parameters are :

1) Cohesion = C = 50 KN/m2
2) Angle of shearing resistance or internal friction = ∅ = 28.5^0