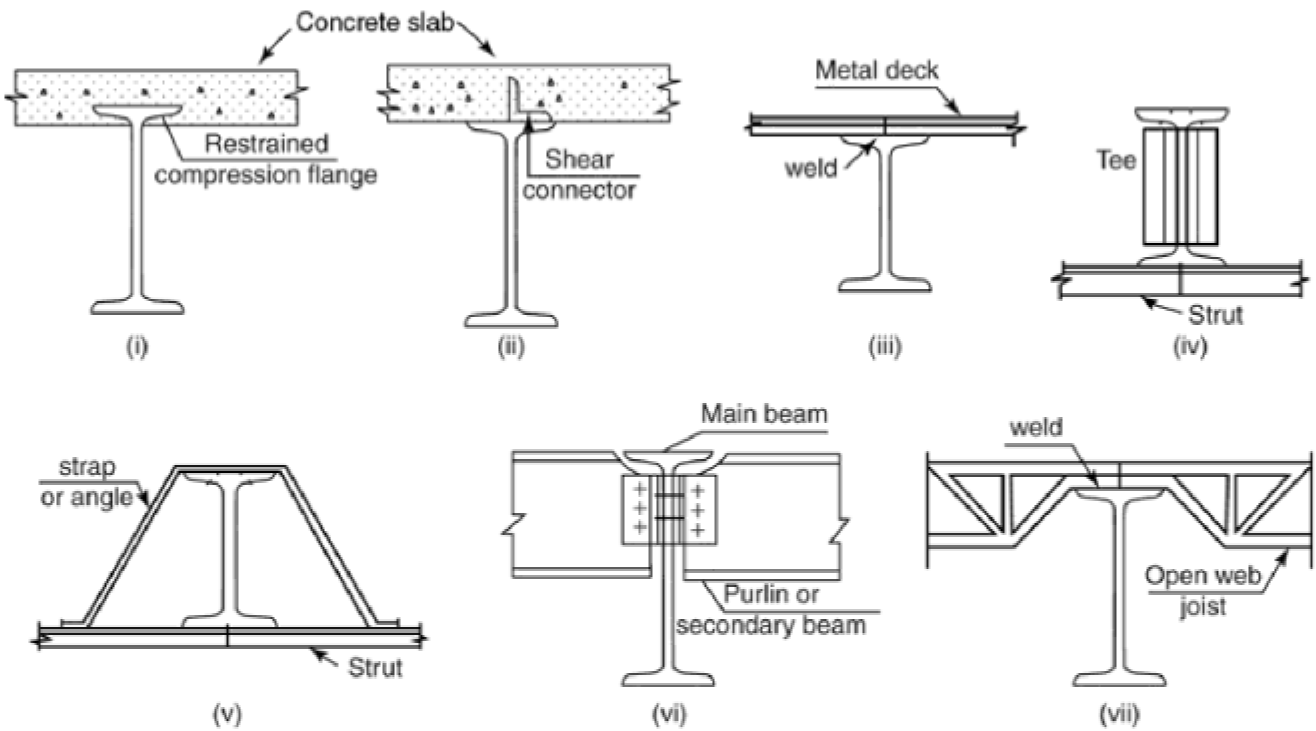
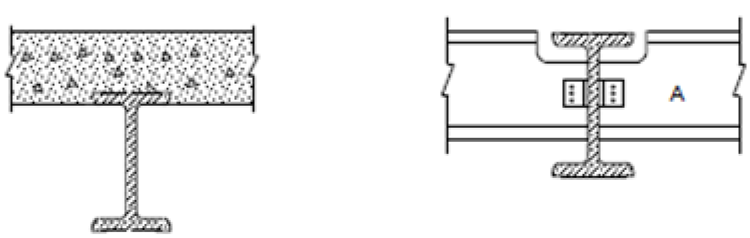
SUPPORTED AND UNSUPPORTED BEAMS (Restrained and Unrestrained)

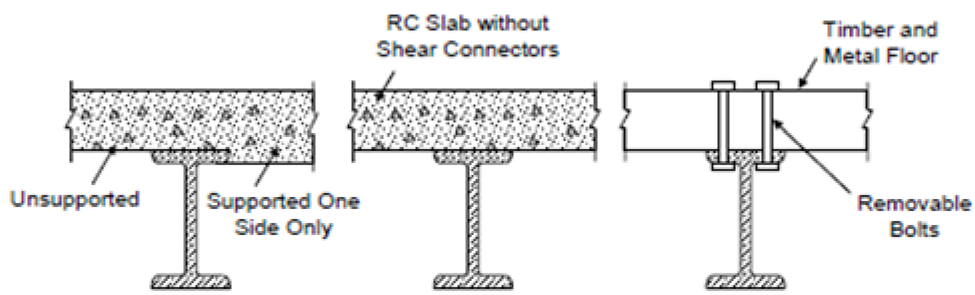
Figure Showing *restrained or Lateraly Supported Beams.*





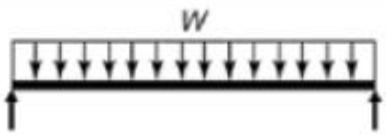
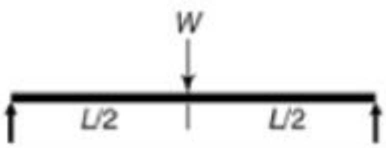
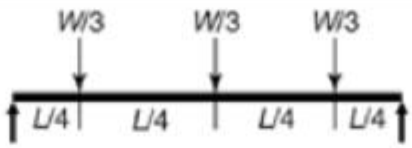
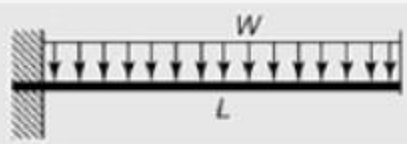
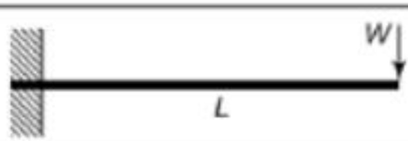
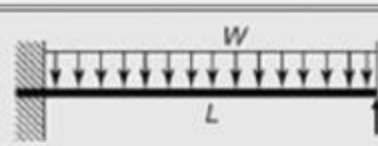
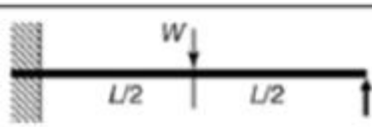
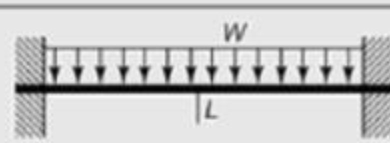
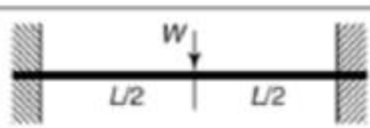
(a) Continuous **(b) Local**

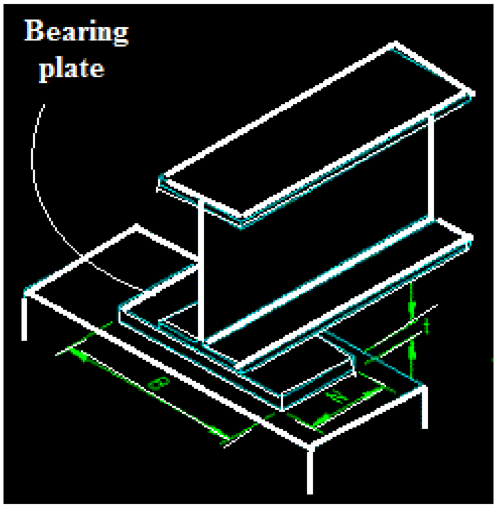
(A) Adequate Lateral Support



(c) **(d)** **(e)**

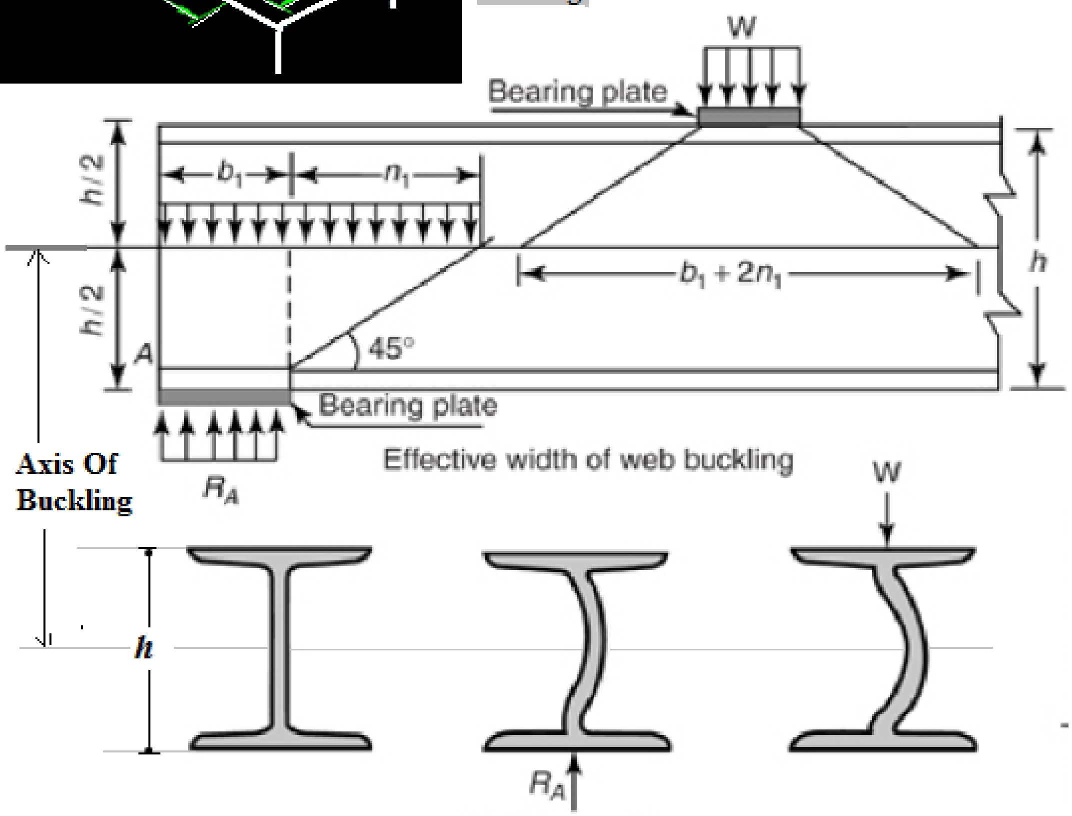
(B) Inadequate Lateral Support

Type of beam, the maximum deflection and the maximum bending moment		*L/h* ratio
	$\Leftarrow \quad \Delta_{max} = \frac{5WL^3}{384\,EI}; M_{max} = \frac{WL}{8}$	19.39
$\Delta_{max} = \frac{WL^3}{60\,EI}; M_{max} = \frac{WL}{6} \quad \Rightarrow$		20.20
	$\Leftarrow \quad \Delta_{max} = \frac{WL^3}{48\,EI}; M_{max} = \frac{WL}{4}$	24.24
$\Delta_{max} = \frac{23WL^3}{1296\,EI}; M_{max} = \frac{WL}{6} \quad \Rightarrow$		18.97
	$\Leftarrow \quad \Delta_{max} = \frac{19WL^3}{1152\,EI}; M_{max} = \frac{WL}{6}$	20.41
$\Delta_{max} = \frac{WL^3}{8\,EI}; M_{max} = -\frac{WL}{2} \quad \Rightarrow$		8.08
	$\Leftarrow \quad \Delta_{max} = \frac{WL^3}{3\,EI}; M_{max} = -WL$	6.06
$\Delta_{max} = \frac{WL^3}{185\,EI}; M_{max} = -\frac{WL}{8} \quad \Rightarrow$		46.72
	$\Leftarrow \quad \Delta_{max} = \frac{WL^3}{48\sqrt{5}\,EI}; M_{max} = \frac{3WL}{16}$	40.66
$\Delta_{max} = \frac{WL^3}{384\,EI}; M_{max} = -\frac{WL}{12} \quad \Rightarrow$		64.65
	$\Leftarrow \quad \Delta_{max} = \frac{WL^3}{192\,EI}; M_{max} = \frac{WL}{8}$	48.48

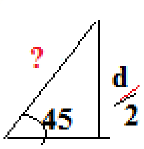


bearing plate takes the reaction of a steel beam and distributes that force over a larger area that is made of a material that is not as strong as steel. The action is essentially the same as a footing that distributes a large load to softer soil. The bearing plate is necessary to avoid crushing of the weaker material to prevent web yielding or crippling. Or Buckling.

W E B B U C K L I N G



The Dipersion at 45° to the level of neutral axis. The diagonal compression is considered to be inclined at 45°. Consider a diagonal column of unit thickness and of length equal to $d\sqrt{2}$ (as inclined at 45° with NA) with fixed ends, the effective length is



$$\text{Sin } 45 = \frac{d}{l}$$

$$\therefore\ l = \frac{d\sqrt{2}}{2} = 0.70\,(d)$$

and minimum radius of gyration is

$$r_{min} = \sqrt{\frac{I}{A_{net}}} = \sqrt{\frac{\frac{b\ x\ t^3}{12}}{b\ x\ t}} = \frac{t_w}{\sqrt{12}}$$

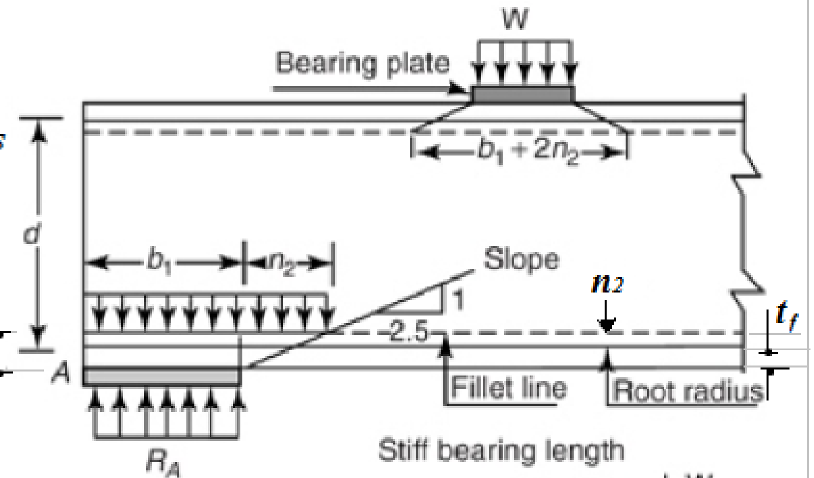
$$\therefore\ (\lambda) = \frac{l_{eff}}{r_{min}} = \frac{0.7\ d}{t_w/\sqrt{12}} = 2.45\ \frac{d}{t_w}$$

Web Buckling strength at the support and Web Buckling strength at the Center

$P_{wb} = (b_1 + n_1)\ t\ f_c$ | $P_{wb} = (b_1 + 2\,n_1)\ t\ f_c$

Where , f_c Design compressive stress curresponding to (λ)

Web Crippling :



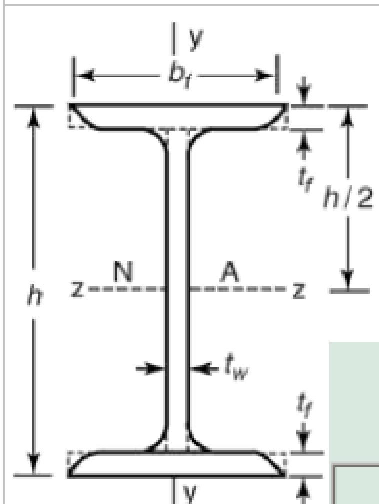
d_1 = *Thickness of flange + root radius*

$= t_f + n_1$

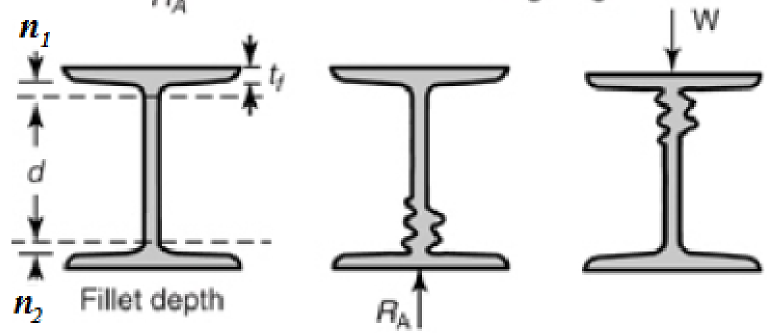
Dispersion length (1:2.5) $n_2 = 2.5.\ d_1$

∴ Design crippling strength Of Web =

$$P_{crip} = \frac{(b_1 + n_2)\ t\ f_y}{\gamma_{mo}}$$

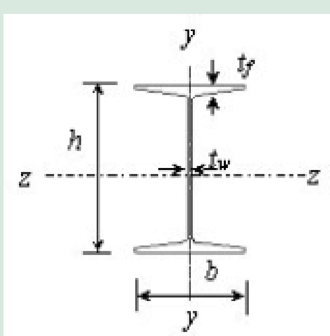


I-section



(b) Web crippling

***IS 800 - 2007; Pg. 44;* Table5.3 Buckling class of cross sections (Section 7.1.2.2)**

Cross Section	Limits		Buckling about axis	Buckling Class
Rolled I-Sections 	h/b > 1.2 :	$t_f \le 40$ mm	z-z	a
			y-y	b
		40 mm< $t_f \le 100$ mm	z-z	b
			y-y	c
	h/b ≤ 1.2 :	$t_f \le 100$ mm	z-z	b
			y-y	c
		t_f >100 mm	z-z	d
			y-y	d

Design *1) A simply supported beam of span 5 m is subjected to a factored maximum bending moment of 180 kNm and a factored maximum shear force of 225 kN. The compression flange of the beam is laterally supported. Design the beam.*

Solution.

Factored moment $M = 180\ kNm$, Factored shear $V = 225\ kN$

Plastic modulus required $= Z_p = \dfrac{180\times10^6}{250}\times1.1 = 792\times10^3\ cm^3$; $M_{pz} = Z_p\, f_y / \gamma_{m0}$

Let us select ISLB 350 @ 485.6 N/m ....*Cl. 9.2.2 ; Pg. 70*

Properties of this section are,

$h = 350\ mm$, $b_f = 165\ mm$, $t_f = 11.4\ mm$, $t_w = 7.4\ mm$, Radius at root, $r_1 = 16\ mm$

Plastic modulus $Z_p = 851.11 \times 10^3\ mm^3$,

Section modulus $Z_e = 751.9 \times 10^3\ mm^3$

Table 2 Limiting Width to Thickness Ratio

(*Clauses* 3.7.2 *and* 3.7.4 *; Pg. 18*)

Compression Element			Ratio	Class of Section: Class 1 Plastic	Class 2 Compact	Class 3 Semi-compact
(1)			(2)	(3)	(4)	(5)
Outstanding element of compression flange	Rolled section		b/t_f	9.4ε	10.5ε	15.7ε
	Welded section		b/t_f	8.4ε	9.4ε	13.6ε
Internal element of compression flange	Compression due to bending		b/t_f	29.3ε	$33.5\ \varepsilon$	42ε
	Axial compression		b/t_f	Not applicable		
Web of an I, H or box section	Neutral axis at mid-depth		d/t_w	84ε	105ε	126ε
	Generally	If r_1 is negative:	d/t_w	$\dfrac{84\varepsilon}{1+r_1}$ but $\le 42\varepsilon$	$\dfrac{105.0\,\varepsilon}{1+r_1}$	$\dfrac{126.0\,\varepsilon}{1+2r_2}$ but $\le 42\varepsilon$
		If r_1 is positive :	d/t_w		$\dfrac{105.0\,\varepsilon}{1+1.5r_1}$ but $\le 42\varepsilon$	
	Axial compression		d/t_w	Not applicable		42ε

Section classification

Outstand of flange $= b = \dfrac{b_f}{2} = \dfrac{165}{2} = 82.5\ mm$,

depth of web

$= d = h - 2\,(t_f + r_1)$

$= 350 - 2\,(11.4 + 16)\ mm$

$= 295.2\ mm$

$$\frac{b}{t_f} = \frac{82.5}{11.4} = 7.24 < 9.4$$

$$\frac{d}{t_w} = \frac{295.2}{7.4} = 39.89 < 84$$

$\therefore$ The section is plastic....*(see table 2 ; IS 800 -2007)*

Since $\dfrac{d}{t_w} < 67$ shear buckling check will not be necessary...........*(see bottom notes in table 2 ; Pg. 18)*

Check for shear strength

Design shear strength of the beam section ....*(Cl. 8.4.1 & 8.4.1.1 ;Pg. 59 ; IS 800 - 2007)*

$$= V_d = \frac{f_y}{\sqrt{3}\,\gamma_{mo}} h t_w = \frac{250}{\sqrt{3}\times 1.1}(350\times 7.4)\times 10^{-3}\,kN = 339.86\,kN > 240\,kN$$

$0.6\ V_d = 0.6 \times 339.86 = 203.92\ kN$ But $V = 225\ kN$, $\therefore V > 0.6\ V_d$

∴ The beam is in a state of high shear.

The design bending strength for a beam in a state of high shear *(Cl. 8.2.1.2 ; Pg. 53)*

$$= M_{dv} = M_d - \beta[M_d - M_{fd}] \le 1.2 Z_e \frac{f_y}{\gamma_{mo}} \text{ ....(Cl. 9.2.2 ; Pg. 70)}$$

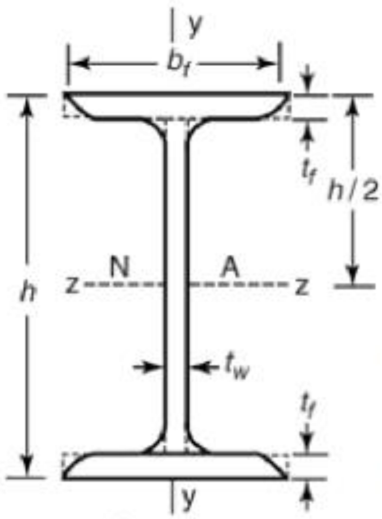
where,

$$M_d = Z_p \cdot \frac{f_y}{\gamma_{mo}} = 851.11\times 10^3 \times \frac{250}{1.1}\times 10^{-6}\,kNm$$

$$= 193.43\ kNm$$

$$\beta = \left[\frac{2V}{V_d} - 1\right]^2 = \left[\frac{2\times 225}{339.86} - 1\right]^2 = 0.105$$

Z_{fd} = Plastic modulus of flanges = Plastic modulus of the whole section − Plastic modulus of the web



I-section

$$= 851.11 \times 10^3 - \frac{t_w h^2}{4} = 851.11\times 10^3 - \frac{7.4\times 350^2}{4}$$

$$= 851.11 \times 10^3 - 226.63 \times 10^3 = 624.48 \times 10^3\ mm^3$$

Now, M_{fd} = Plastic Design Moment Of The Section is ;

$$= Z_{fd} \frac{f_y}{\gamma_{mo}} = 624.48 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}\ kNm$$

$$= 141.93\ kNm$$

$$\therefore M_{dv} = 193.43 - 0.105[193.43 - 141.93]$$

$$= 193.43 - 5.41 = 188.02\ kNm > 180\ kNm \text{ ; Safe}$$

Also, $$1.2\frac{Z_e f_y}{\gamma_{mo}} = 1.2\times 751.9\times 10^3 \times \frac{250}{1.1}\times 10^{-6}$$

$$= 205.06\ kNm > 180\ kNm \text{ ; Safe}$$

Check for web crippling at support

$b_1 = 100\ mm$, $n_2 = 2.5(t_f + r_1) = 2.5(11.4 + 16) = 68.5\ mm$

Design crippling strength $$= \frac{(b_1 + n_2)t_w\ f_y}{1.1}$$

$$= \frac{(100 + 68.5)7.4\times 250}{1.1}\times 10^{-3} kN$$

$$= 283.39\ kN > 225\ kN \text{ ; Safe}$$

Check for web buckling at support

We have already stated that in this case, since $\frac{d}{t_w} < 67$ shear buckling check will not be necessary.

However the check has been made to demonstrate the method of checking.

Provide a stiff bearing length of 100 *mm* *i.e.,* $b_1 = 100\ mm$

Depth of the web $= d = h - 2(t_f + r_1) = 350 - 2(11.4 + 16) = 295.2\ mm$

Least radius of gyration of the web section $= \frac{t_w}{\sqrt{12}} = \frac{7.4}{\sqrt{12}} = 2.136$

$\therefore$ *Slenderness ratio* ; $(\lambda) = \frac{d}{t_w} \times 2.5 = 96.74$

We have ; $\frac{h}{b} = \frac{350}{165} = 2.121 > 1.2$ & $t_f = 11.4\ mm \leq 40$

IS 800 - 2007; Pg. 44; ***Table5.3 Buckling class of cross sections***
(Section 7.1.2.2)

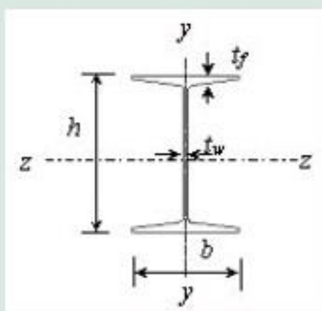
Cross Section	Limits		Buckling about axis	Buckling Class
Rolled I-Sections	$h/b > 1.2$:	$t_f \leq 40$ mm	z-z	a
			y-y	b
		40 mm< $t_f \leq 100$ mm	z-z	b
			y-y	c
	$h/b \leq 1.2$:	$t_f \leq 100$ mm	z-z	b
			y-y	c
		$t_f > 100$ mm	z-z	d
			y-y	d

Table 9(c) Design Compressive Stress, f_{cd} (MPa) for Column Buckling Class a
IS 800-2007 ; Pg. 40; (*Clause* 7.1.2.1)

$(\lambda) = KL/r$	Yield Stress, f_y (MPa) = 250 Mpa
90	149
96.74	??
100	132

Interpolation :

$\left\{\frac{(149-132)}{(100-90)} \times (100-96.74)\right\} - 149$

$\therefore (f_{cd}) = 143.45$

$\therefore$ Compressive strength =

$P_{Buckling} = (b_1 + n_2)\ t_w f_{cd}$

$= (275) \times 7.4 \times 143.45 \times 10^{-3}$

$= 291.92\ KN > 225$; **Safe**

Design 2) *: A simply supported beam of span 5 m has its compression flange laterally supported. It has to support the following loads. Design the Beam ;*

Dead load excluding the self weight of the beam = 80 *kN*
Point load at mid span = 65 *kN*
Live load = 120 *kN*

Answer :

Step i) Maximum Bending Moment &S. f. Calculation :

a) Self Wt. Of Beam $= \frac{W \times l}{300} = \frac{80 \times 5}{300} = 1.34$ say $2 \ \frac{KN}{m}$

Or **you can take directly as 2 to 3** KN/m

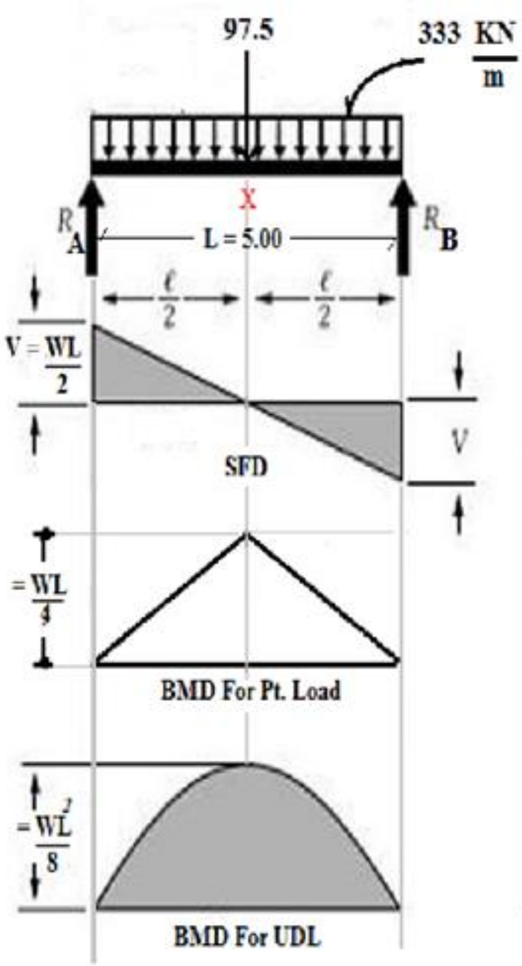
b) D.L Excluding Self Wt......................... $= 80 \ KN/m$

C) Live Load... $= 120 \ KN/m$

$\therefore$ **Total Udl** $= 120 + 80 + 2 = 222$ **KN**

Total Factored UDL $= 1.5 \times 222 = 333$ **KN**

d) Factored point load $= 1.5 \times 65 = 97.5$ **KN**



$$\text{Maximum S. F. (V)} = \frac{WL}{2} = \frac{\{(97.5) + (333 \times 5)\}}{2}$$

$$= 881.25 \ KN$$

$$\text{Maximum B.M.} = \frac{WL}{4} + \frac{WL}{8}$$

$$= \frac{(97.5 \times 5)}{4} + \frac{(333 \times 5)}{8}$$

$$= 333 \ KN$$

Plastic modulus required $= Z_p = \dfrac{333 \times 10^6}{250} \times 1.1 = 1465.25 \times 10^3 \; mm^3$

Let us select ISMB 450 @ 724 N/m

Properties of the section

Plastic modulus $= Z_p = 1533.36 \times 10^3 \; mm^3$, Section modulus $= Z_e = 1350.7 \times 10^3 \; mm^3$,
$h = 450 \; mm$, b_f 150 mm, $t_f = 17.4 \; mm$, $t_w = 9.4 \; mm$, Radius at root $= r_1 = 18 \; mm$

$$d = h - 2(t_f + r_1) = 450 - 2(17.4 + 1 \;) = 414 \; mm, \quad b = \frac{b_f}{2} = \frac{150}{2} = 75 \, mm$$

Carry steps from Design 01

Check For Deflection:

Therefore, the vertical deflection is given by

$$\delta_v = \frac{1}{EI_z}\left(\frac{5w_{(udl)}L^4}{384} + \frac{W_{(pt)}\;L^3}{48}\right)$$ **Where I_{zz} = M. I. @ major axis**

$$= \frac{1}{(2 \times 10^5 \times \qquad \times 10^4)}\left(\frac{5 \text{ x } 333 \times 5^4}{384} + \frac{97.5 \times 5^3}{48}\right) \times 10^{12} = \qquad \text{mm}$$

The vertical deflection limit is $\delta_{\lim} = \dfrac{L}{300} = \dfrac{5000}{300} = 16.67 \text{ mm} > \delta \;\; (O.K.)$

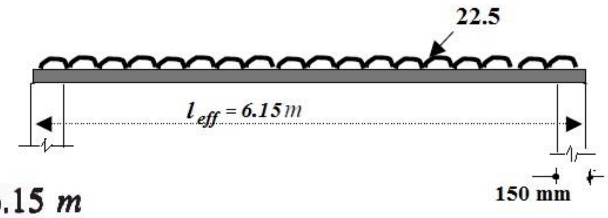
.... Cl. 5.6.1 ; Pg. 31 ; IS 800 - 2007

Deflection Limits ***(IS 800 - 2007 ; Pg. 31)***

Type of beam	Design service load	Supporting	Limiting deflection
1. Industrial buildings			
Beams supporting floors and false ceiling: (a) Simply supported (b) Cantilever	Live load	Elastic Cladding Brittle Cladding Elastic Cladding Brittle Cladding	$L/240$ $L/300$ $L/120$ $L/150$
Purlins and grits	Live/wind load	Elastic Cladding Brittle Cladding	$L/150$ $L/180$
Rafter supporting	Live/wind load	Profiled Metal Sheeting Plastered Sheets	$L/180$ $L/240$
Crane girders supporting manually operated cranes	Crane load	Crane	$L/500$
Crane girders with E.O.T. crane of capacity up to 500 kN	Crane load	Crane	$L/750$
Crane girders with E.O.T. crane of capacity more than 500 kN	Crane load	Crane	$L/1000$
2. Other buildings			
Beams supporting floors and roofs: (a) Simply supported (b) Cantilever	Live load	*Type-1-:Element Type-2: Element Type-1: Element Type-2: Element	$L/300$ $L/360$ $L/150$ $L/180$

Design *3) Steel beams have to be provided in a hall at a spacing of 2.75 m. The beams have a clear span of 6 meters with end bearings of 150 mm at each end. The loading consists of a dead load of 3 kN/m² (including the weight of beams) and a live load of 12 kN/m². The compression flanges of the beams are laterally supported. Design the beam. Beam depth shall not exceed 375 mm.*

Solution.



Factored dead load $1.5 \times 3 = 4.5\ kN/m^2$

Factored live load $1.5 \times 12 = 18\ kN/m^2$

Total load $= 22.5\ kN/m^2$

Effective span of the beam $= l = 6 + 0.15 = 6.15\ m$

Total load on one beam $= 22.5 \times 6.15 \times 2.75 = 380.53\ kN$

Maximum shear force $= V = \dfrac{380.53}{2} = 190.26\ kN$

Maximum bending moment $= M = \dfrac{380.53 \times 6.15}{8} = 292.53\ kNm = \dfrac{W \times L}{8}$

Plastic modulus required $= \dfrac{M}{f_y} \cdot \gamma_{mo} = \dfrac{292.53 \times 10^6 \times 1.1}{250}\ mm^3$

$= 1287.132 \times 10^3\ mm^3$

Let us provide ISMB 350 @ 514 N/m with cover plates.

Plastic modulus of ISMB 350 $= 889.57 \times 10^3\ mm^3$

Plastic modulus to be provided by cover plates

$$= 1287.132 \times 10^3 - 889.57 \times 10^3 = 397.562 \times 10^3\ mm^3$$

Let b = width of cover plate and t = thickness of cover plate

Plastic modulus of cover plates $= bt(d+t) = 397.562 \times 10^3\ mm^3$

Let $t = 8\ mm$, $b = \dfrac{397.562 \times 10^3}{8(350+8)} = 139\ mm$

Provide 140 mm × 8 mm cover plates

Actual plastic modulus provided by cover plates

$$= bt(d+t) = 140 \times 8(350+8) = 400.96 \times 10^3\ mm^3$$

Actual total plastic modulus provided

$$= 889.57 \times 10^3 + 400.96 \times 10^3 = 1290.53 \times 10^3\ mm^3$$

Total depth of the beam $= 350 + 2(8) = 366\ mm < 375\ mm$

Properties of ISMB 350 @ 514 N/m

$h = 350\ mm,\ b_f = 140\ mm,\ t_f = 14.2\ mm,\ t_w = 8.1\ mm,\ r_1 = 14\ mm$

Depth of web $= d = h - 2(t_f + r_1)$

$= 350 - 2(14.2 + 14) = 293.6\ mm$

Flange outstand $= b = \dfrac{b_f}{2} = \dfrac{140}{2} = 70\ mm$

$Z_e = 778.9 \times 10^3\ mm^3,\ Z_p = 889.57 \times 10^3\ mm^3$

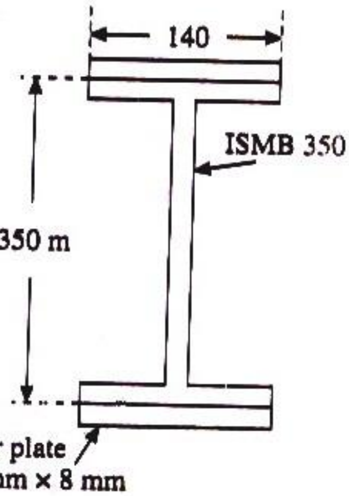


Fig. 9.21

Section classification

$$\left.\begin{aligned} \frac{b}{t_f} &= \frac{70}{14.2} = 4.93 < 9.4 \\ \frac{d}{t_w} &= \frac{293.6}{8.1} = 36.25 < 84 \end{aligned}\right\} \therefore \text{This is a plastic beam section.}$$

Design shear strength $= V_d = \dfrac{f_y}{\sqrt{3}\gamma_{mo}} h t_w$

$$= \frac{250 \times 350 \times 8.1}{\sqrt{3} \times 1.1} \times 10^{-3}\ kN = 372\ kN$$

$V = 190.26\ kN$

$0.6V_d = 0.6 \times 372 = 223.2\ kN$ $\quad \therefore V < 0.6V_d \quad \therefore$ This is a case of low shear.

Check for web buckling at support

Provide a stiff bearing length of 150 mm, *i.e.*, $b_1 = 150\ mm$

Depth of the web $= d = h - 2(t_f + r_1) = 350 - 2(14.2 + 14) = 293.6\ mm$

Least radius of gyration of the web section

$$= r = \frac{t_w}{\sqrt{12}} = 2.34\ \text{mm}$$

Slenderness ratio $(\lambda) = \dfrac{l_{eff}}{r_{min}} = 2.45\ \dfrac{d}{t_w} = 87.83$

Design compressive stress corresponding to this slenderness ratio (buckling class c)

$$f_{cd} = 124.25\ N/mm^2$$

$$n_1 = \frac{h}{2} = \frac{350}{2} = 175\ mm, \quad b_1 + n_1 = 150 + 175 = 325\ mm$$

Design buckling strength $= (b_1 + n_1) t_w f_{cd}$

$= 325 \times 8.1 \times 124.25 \times 10^{-3}\ kN$

$= 327.09\ kN > 190.26\ kN$

Check for crippling at support

$$b_1 = 150\ mm, \quad n_2 = 2.5(t_f + r_1) = 2.5(14.2 + 14) = 70.5\ mm$$

Design crippling strength $= \dfrac{(b_1 + n_2)t_w f_y}{\gamma_{mo}}$

$$= \frac{(150 + 70.5)8.1 \times 250}{1.1} \times 10^{-3}\ kN$$

$$= 405.92\ kN > 190.26\ kN$$

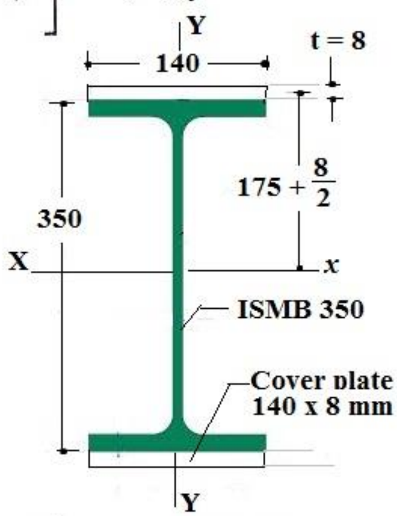
Check for deflection

M.I. of ISMB 350 : $I_1 = 13630.3 \times 10^4\ mm^4$

Since, S/c not symetrical on XX Axis

M.I. of cover plates $I_{xx} = 2\left[\dfrac{140 \times 8^3}{12} + 140 \times 8(175 + 4)^2\right] = 2\{ I_{xx\,self} + A\ (\text{dist})^2 \}$

$$= 7178.4 \times 10^4\ \ mm^4$$



Total M.I. $= I = (13630.3 + 7178.4)10^4$

$$= 20868.7 \times 10^4\ mm^4$$

Total factored load on the beam $= 380.53\ kN$

Total service load on the beam $= \dfrac{380.53}{1.5} = 253.69\ kN$

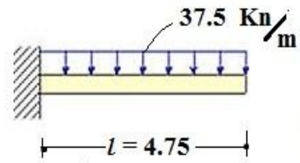
Deflection $= \delta = \dfrac{5}{384} \cdot \dfrac{Wl^3}{EI}$

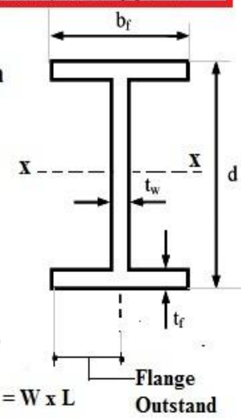
$$= \frac{5}{384} \cdot \frac{253.69 \times 6.15^3}{200 \times 20808.7 \times 10^4} \times 10^9\ mm = 18.46\ mm$$

Allowable deflection $= \dfrac{l}{300} = \dfrac{6150}{300} = 20.5\ mm$

Note: Problems may asked in prescribed loading conditions of beam cases which is on pg. no. 02

Design *4) : Design a 4.75 m long cantilever built firmly into a concrete wall at the fixed end. It is subjected to a dead load of 15 kN/m and a live load of 10 kN/m. The cantilever is laterally unsupported.*





Solution.

Dead load $= 15\ kN/m$

live load $= 10\ kN/m$

Total load $= 25\ kN/m$

Total factored load $= 1.5 \times 25 = 37.5\ kN/m$

Maximum bending moment $= M = \dfrac{37.5 \times 4.75^2}{2} = 423.045\ kNm = \dfrac{W \times L^2}{2}$

.... see pg. 02

Maximum shear force $= V = 37.5 \times 4.75 = 178.125\ kN = W \times L$

The cantilever is given to be laterally unsupported.

Considering this condition, the approximate plastic modulus required **as Follows**

Is 800 - 2007 ; Cl. 8.2.1.2 ; Pg. 53 { To avoid irreversible deformation under serviceability loads, M_d shall be less than $1.2\ Z_e f_y / \gamma_{m0}$ incase of simply supported and $1.5\ Z_e f_y / \gamma_{m0}$ in cantilever beams;

$$\therefore \quad Z_p = 1.5 \times \frac{M_u}{(f_y / \gamma_{m0})} = 1.5 \times \frac{(423 \times 10^6) \times 1.1}{250} = \mathbf{2792.1} \times 10^3\ mm^3$$

Table 05 ; Pg. 30; IS Code

Let us select ISWB 550 @ 1103.6 *N/m* having $Z_p = 3066.29 \times 10^3\ mm^3$

Properties of ISWB 550 area,

$h = 550\ mm$, $b_f = 250\ mm$, $t_f = 17.6\ mm$, $t_w = 10.5\ mm$, $r_1 = 16\ mm$,

$I_y = 3740.6 \times 10^4 mm^4$, $\quad Z_e = 2723.9 \times 10^3 mm^3$, $\quad Z_p = 3066.29 \times 10^3 mm^3$

Depth of the web $= d = h - 2\,(t_f + r_1) = 550 - 2(17.6 + 16) = 482.8\ mm$

Flange outstand $= b = \dfrac{b_f}{2} = \dfrac{250}{2} = 125\ mm$

Section Classification :

$\dfrac{b_f}{T_f} = \dfrac{250}{17.6} = 7.4 < 9.4\varepsilon$

$\dfrac{d}{t_w} = \dfrac{550}{10.5} = 52.38 < 84\varepsilon$

where ; $\varepsilon = \sqrt{\dfrac{250}{f_y}}$

$\varepsilon = \sqrt{\dfrac{250}{250}} = 1$

$\therefore$ **The Section is classified as Plastic**

Table 2 Limiting Width to Thickness Ratio

(Clauses 3.7.2 and 3.7.4 ; Pg. 18 ; IS 800 - 2007))

Compression Element (1)			Ratio (2)	Class 1 Plastic (3)	Class 2 Compact (4)	Class 3 Semi-compact (5)
Outstanding element of compression flange	Rolled section		b/t_f	9.4ε	10.5ε	15.7ε
	Welded section		b/t_f	8.4ε	9.4ε	13.6ε
Internal element of compression flange	Compression due to bending		b/t_f	29.3ε	$33.5\ \varepsilon$	42ε
	Axial compression		b/t_f	Not applicable		
Web of an I, H or box section	Neutral axis at mid-depth		d/t_w	84ε	105ε	126ε
	Generally	If r_1 is negative:	d/t_w	$\dfrac{84\varepsilon}{1+r_1}$	$\dfrac{105.0\,\varepsilon}{1+r_1}$	$\dfrac{126.0\,\varepsilon}{1+2r_2}$ but $\le 42\varepsilon$
		If r_1 is positive:	d/t_w	but $\le 42\varepsilon$	$\dfrac{105.0\,\varepsilon}{1+1.5r_1}$ but $\le 42\varepsilon$	
	Axial compression		d/t_w	Not applicable		42ε

Design shear strength of the section.....*(Cl. 8.4.1 & 8.4.1.1 ; Pg. 59 ; IS 800 - 2007)*

$$V_d = \frac{f_y}{\sqrt{3}\gamma_{mo}} h t_w = \frac{250 \times 550 \times 10.5}{\sqrt{3} \times 1.1} \times 10^{-3}\, kN = 757.8\, kN$$

$0.6V_d = 0.6 \times 757.8 = 454.68\, kN$, $\quad V = 178.125\, kN \quad \therefore V < 0.6\, V_d$; Safe

$\therefore$ The section is in a state of low shear. *(Cl. 8.2.1.2 ; Pg. 53)*

Finding Elastic lateral Torsional Buckling moment :

Table 15 Effective Length for Simply Supported Beams, L_{LT}

(*Clause* 8.3.1 ; Pg. 58 ; IS 800 - 2007)

Sl No.	Conditions of Restraint at Supports		Loading Condition	
	Torsional Restraint	Warping Restraint	Normal	Destabilizing
(1)	(2)	(3)	(4)	(5)
i)	Fully restrained	Both flanges fully restrained	0.70 L	0.85 L
ii)	Fully restrained	Compression flange fully restrained	0.75 L	0.90 L
iii)	Fully restrained	Both flanges fully restrained	0.80 L	0.95 L
iv)	Fully restrained	Compression flange partially restrained	0.85 L	1.00 L
v)	Fully restrained	Warping not restrained in both flanges	1.00 L	1.20 L
vi)	Partially restrained by bottom flange support connection	Warping not restrained in both flanges	1.0 L + 2 D	1.2 L + 2 D
vii)	Partially restrained by bottom flange bearing support	Warping not restrained in both flanges	1.2 L + 2 D	1.4 L + 2 D

NOTES

1 Torsional restraint prevents rotation about the longitudinal axis.

2 Warping restraint prevents rotation of the flange in its plane.

3 D is the overall depth of the beam.

This is a laterally unsupported member. $L_{LT} = 0.85L = 0.85 \times 4.75 = 4.0375\, m$

Let us determine the elastic lateral torsional buckling moment given by,

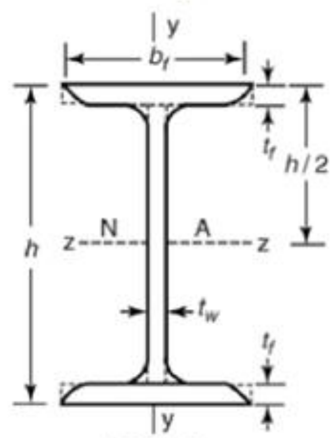
$$M_{cr} = \sqrt{\left\{\left(\frac{\pi^2 EI_y}{(L_{LT})^2}\right)\left[GI_t + \frac{\pi^2 EI_w}{(L_{LT})^2}\right]\right\}} = \beta_b Z_p f_{cr,b} \text{ .... } \textit{Cl. 8.2.2.1 ; Pg. 54}$$

Where; G = (Modulus Of Rigidity) $= \frac{E}{2(1+\nu)} = 7.692 \times 10^4 \, \frac{N}{mm^2}$ & $E = 2 \times 10^5$

$$I_t = \Sigma \frac{b_i t_i^3}{3} = 2\frac{b_f t_f^3}{3} + (h - t_f)\frac{t_w^3}{3} = \frac{2 \times 250 \times 17.6^3}{3} + \frac{(550 - 17.6)10.5^3}{3}$$

$$= 9.086293 \times 10^5 + 2.054399 \times 10^5$$

$$= 11.140692 \times 10^5\, mm^4$$



I-section

Distance between centres of flanges

$$= h_f = h - t_f = 550 - 17.6 = 532.4\, mm$$

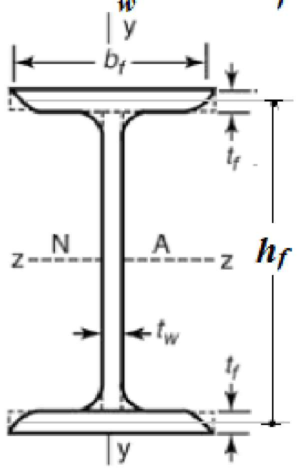
Now, Warping Constant $(I_w) = (1 - \beta_f)\beta_f I_y h_f^2$

Where $\quad \beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5$

I_{fc} & I_{ft} = M. I. Of compression and Tension Flanges @ minor axis of Entire S/c.

Note that For Equal Flange Beams; β_f = 0.5

$$\therefore \; I_w = (1-\beta_f)\beta_f I_y h_f^2 = (1-0.5)0.5\times3740.6\times10^4\times532.4^2 = 2.650680\times10^{12}$$



I-section

$$\frac{\pi^2 EI_y}{L_{LT}^2} = \frac{\pi^2\times2\times10^5\times3740.6\times10^4}{4037.5^2} = 4.529455\times10^6$$

$$GI_t = 7.6923\times10^4\times11.140692\times10^5 = 8.569755\times10^{10}$$

$$\frac{\pi^2 EI_w}{L_{LT}^2} = \frac{\pi^2\times2\times10^5\times2.650680\times10^{12}}{4037.5^2} = 3.209682\times10^{11}$$

$$\therefore \; M_{cr} = \sqrt{4.529455\times10^6(8.569755\times10^{10}+3.209682\times10^{11}}$$

$$= 1357.194\times10^6 \; Nmm = 1357.194 \; kNm$$

Finding Design bending strength = $M_d = \beta_b Z_p f_{bd}$ **....Cl. 8.2.2 ; Pg. 54**

where β_b = 1.0 for plastic and compact sections.
= Z_e/Z_p for semi-compact sections.

Non-dimensional slenderness ratio = $\lambda_{LT} = \sqrt{\dfrac{\beta_b Z_p f_y}{M_{cr}}} = \sqrt{\dfrac{1\times3066.29\times10^3\times250}{1357.194\times10^6}}$

$$= 0.7515$$

$$\phi_{LT} = 0.5[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2], \text{ Note } \alpha_{LT} = 0.21$$

$$= 0.5[1+0.21(0.7515-0.2)+0.7515^2]$$

$$= 0.5[1+0.1158+0.5647) = 0.8403$$

$$X_{LT} = \frac{1}{\phi_{LT}+[\phi_{LT}^2-\lambda_{LT}^2]^{0.5}}$$

$$= \frac{1}{0.8403+[0.8403^3-0.7515^2]^{0.5}} = 0.8222$$

Design bending compressive stress

$$= f_{bd} = \frac{X_{LT} f_y}{\gamma_{mo}} = \frac{0.8222\times250}{1.1} = 186.86 \; N/mm^2$$

$\therefore$ Design bending strength $= M_d = \beta_b Z_p f_{bd}$

$$= 1 \times 3066.29 \times 10^3 \times 186.86 \times 10^{-6}\ kNm$$

$$= 572.97\ kNm > 423.045\ kNm$$

Safe.

Check for web buckling

Provide a bearing length of 150 mm *i.e.,* $b_1 = 150\ mm$ ...see Pg. 03 for reference

Depth of the web $= d = h - 2(t_f + r_1) = 550 - 2(17.6 + 16) = 482.8\ mm$

Least radius of gyration of the web section $= r = \dfrac{t_w}{\sqrt{12}} = 3.031$

Slenderness ratio $(\lambda) = \dfrac{l_{eff}}{r_{min}} = 2.45\,\dfrac{d}{t_w} = 111.5$

We have ; $\dfrac{h}{b} = \dfrac{550}{250} = 2.2 > 1.2$ & $t_f = 17.6\ mm \leq 40$

IS 800 - 2007; Pg. 44; Table5.3 Buckling class of cross sections (Section 7.1.2.2)

Cross Section	Limits	Buckling about axis	Buckling Class
Rolled I-Sections	$h/b > 1.2$: $t_f \leq 40$ mm	z-z	a
		y-y	b
	40 mm< $t_f \leq$ 100 mm	z-z	b
		y-y	c
	$h/b \leq 1.2$: $t_f \leq$100 mm	z-z	b
		y-y	c
	t_f >100 mm	z-z	d
		y-y	d

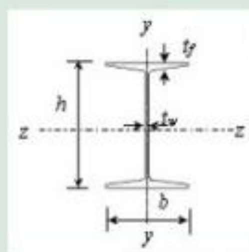


Table 9(c) Design Compressive Stress, f_{cd} (MPa) for Column Buckling Class
IS 800-2007 ; Pg. 40 ; (*Clause* 7.1.2.1)

$(\lambda) = \dfrac{l_{eff}}{r_{min}} = KL/r$ $= 111.5$	Yield Stress, f_y (MPa) $= 250$ Mpa
110	115
111.5	???
120	101

Interpolation :

$$\left\{\frac{(115 - 101)}{(120 - 110)} \times (115.5 - 110)\right\} - 115$$

$\therefore (f_{cd}) = 107.3$

Design buckling strength $= (b_1 + n_1)t_w f_{cd}$

$$= (150 + 275)10.5 \times 107.3 \times 10^{-3}\ kN$$

$$= 478.82\ kN > 178.125\ kN$$

Check for web crippling at support ....*see Pg. no. 04 for reference*

$$b_1 = 150\ mm, \quad n_2 = 2.5(t_f + r_1) = 2.5(17.5 + 16) = 83.75\ mm$$

Design crippling strength $= \dfrac{(b_1 + n_2)t_w f_y}{\gamma_{mo}}$

$$= \frac{(150 + 83.75)10.5 \times 250}{1.1} \times 10^{-3}\ kN$$

$$= 557.8\ kN > 178.125\ kN$$

= 557.8 KN > v = 178.125 KN ; Safe

Note : Check For Deflection is only applicable in Supported (Restrained Beams)

Problem 5) Design economical built up column to carry 2000 KN using Four angles. The length of column is 6.4 mt. with both ends are held in position and one end is restrained against rotation. Design a suitable **Lacing / Battening also** ? Take $f_y = 250 \ \ Mpa$ *[Summer 2013]*

Answer : Given ;

a) Length of member (clear) = 6.4 mt = 6.4 $\times$ 10^3 mm

Given case is **One End Fix And Other Is Hinge (case 02)**

$\therefore$ l_{eff} = KL = 0.8$\times$ 5$\times$ 10^3 = 5.12 $\times$ 10^3 mm ……... *IS 800-2007; pg. 45*

Let. λ (slenderness ratio) = 70

For f_y=250; (compressive stress) $\sigma_{ac} = 152 \ {}^{N}/_{mm^2}$ ……… *IS 800-2007; pg. 42 (See table)*

Area Reqd. For the strut (A_{Req}) $= \frac{2000\times 10^3}{152}$ = 13.57 X 10^3 mm^2

Area reqd. of Each angle = $\frac{13.57 \text{ X } 10^3}{4}$ = 3289.47 mm^2

Note : Always select Equal angle s/c for easy calculations

Refer the steel table and select rolled steel equal angle such that $A_{prov} > A_{req}$ thus selecting ISA 150 x 150 x 12

For this Angle ; Area = 3459 mm^2; width (b) = 150 mm ;

Center of gravity ; C_{yy}= C_{XX} = 41.4 mm

Here, Area = 3459 > 3289.47 mm^2 ; M. I. = I_{xx} = I_{yy} = 735.4 $\times$ 10^4 mm^4;

Radius Of Gyration ; r_{xx} = r_{yy} = 46.1 mm

IS 800 - 2007; Pg. 44;* *Table5.3 Buckling class of cross sections
(Section 7.1.2.2)

Cross Section	Limits	Buckling about axis	Buckling Class
Channel, Angle, T and Solid Sections		Any	c

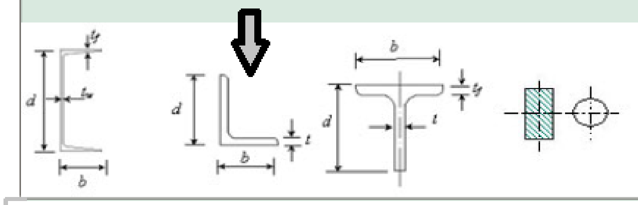


Table 9(c) Design Compressive Stress, f_{cd} (MPa) for Column Buckling Class c
(*Clause* 7.1.2.1) **; Pg. 42 ; IS 800 - 2007**

(λ)

KL/r ↓	Yield Stress, f_y (MPa) = 250													
	200	210	220	230	240	250	260	280	300	320	340	360	380	400
10	182	191	200	209	218		236	255	273	291	309	327	345	364
20	182	190	199	207	216		233	250	266	283	299	316	332	348
30	172	180	188	196	204		219	234	249	264	278	293	307	321
40	163	170	177	184	191		205	218	231	244	256	268	280	292
50	153	159	165	172	178		189	201	212	222	232	242	252	261
60	142	148	153	158	163		173	182	191	199	207	215	222	228
70						152	156	163	170	176	182	187	192	197
80	120	123	127	130	133	136	139	145	149	154	158	162	165	169

We have ; $\lambda = \frac{l_{eff}}{r_{min}}$; $70 = \frac{5.12 \times 10^3}{r_{min}}$; $r_{min} = 73.142$ mm

Again We have ; $r_{min} = \sqrt{\frac{I_{min}}{A}}$ $\therefore$ $I_{min} = A \, x \, r_{min}{}^2$

$\therefore$ I_{min} = (4 x 3459) x 73.142 ^{2} = 74.01 x 10^6 mm^4

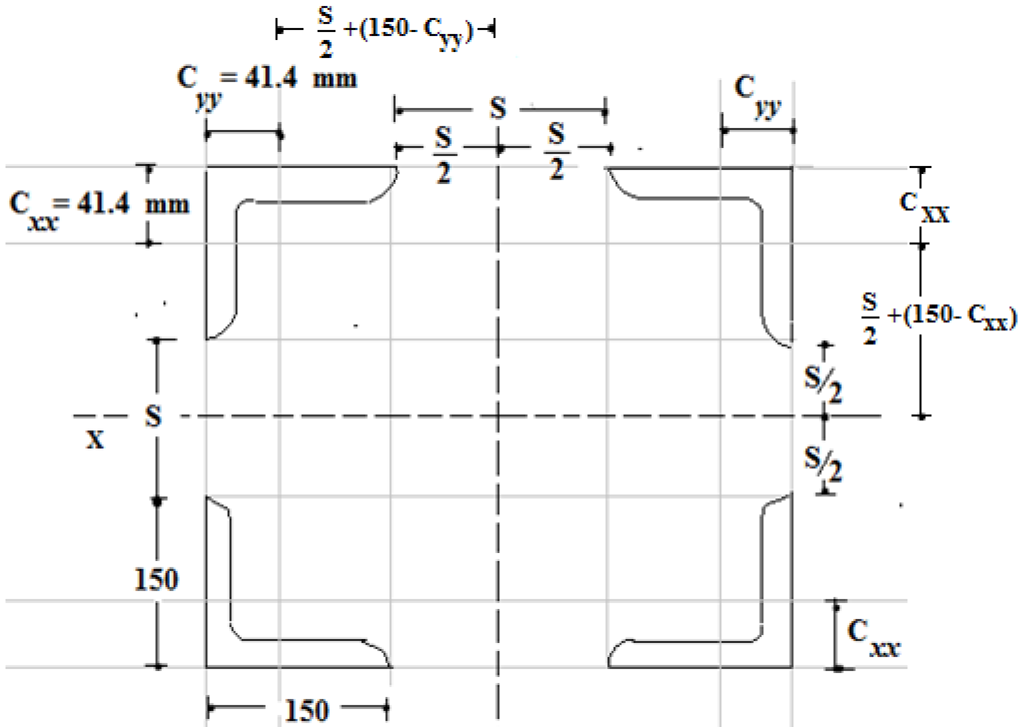
Now, Finding actual M. I. of the S/C And Comparing With

I_{min} = 74.01 x 10^6 mm^4 ; We will get spacing (S)

$$\therefore \; 74.01 \times 10^6 = 4 \; [\; I_{xx(self)} + \text{Area} \, (distance^2) \,]$$

$$74.01 \times 10^6 = 4 \, [\, 735.4 \times 10^4 + 3459 \, (\tfrac{S}{2} + (150 - 41.4 \,) \,)^2]$$

Find S = spacing and carry steps in lacing and battening



$\frac{S}{2}+(150-C_{yy})$
$C_{yy} = 41.4$ mm
S
$\frac{S}{2}$
$\frac{S}{2}$
C_{yy}
$C_{xx} = 41.4$ mm
C_{XX}
$\frac{S}{2}+(150-C_{xx})$
$S/2$
S
X
$S/2$
150
C_{xx}
150