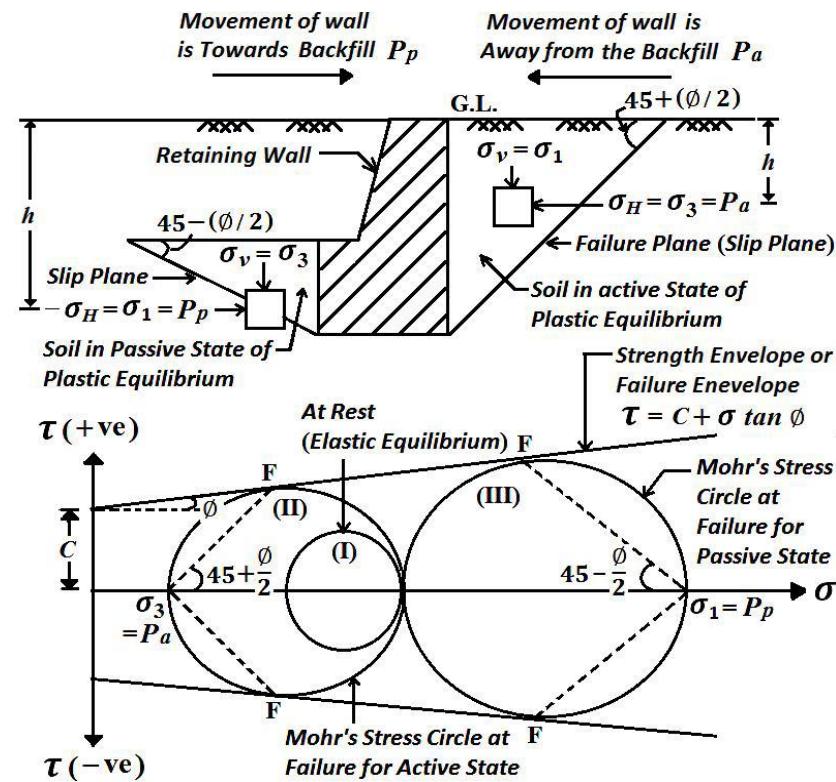


0 Assumptions in Rankine's Theory of Earth Pressure :

- i) The Soil is homogeneous, isotropic and semi infinite.
- ii) Rankine's theory is applicable to incompressible soil.
- iii) The failure wedge acts as rigid body.
- iv) The soil mass is in the state of plastic equilibrium i.e. on the verge of failure.
- v) The ground surface is considered as plane which may be horizontal or inclined.
- vi) The Retaining wall is yields about its base.
- vii) Rankine's theory is satisfactory for brittle materials , and not applicable to ductile material.
- viii) The back of retaining wall is smooth and vertical. i.e. completely Frictionless.



Rankine (1857) considered the equilibrium of a soil element at any depth in the backfill below the ground level behind a retaining wall . Figure shows Retaining wall of height H with Cohesionless backfill having bulk unit weight

(γ) and angle of shearing resistance (ϕ). Consider an element of the soil mass at depth h below ground level. Rankine assumed that the soil element is subjected to only two types of stresses:

- a) Vertical stress i.e. $\sigma_v = (\gamma \times h)$ due to the weight of the soil.

At depth $h = H$; $\sigma_v = (\gamma \times H)$

- b) Lateral earth pressure ($\sigma_H = P_a$ or P_p).

The impact of movement of the retaining wall can be very well studied and explain by drawing Mohr's Circle which is follows.

- 1] When the soil is at rest i.e. soil is in the state of elastic equilibrium , stress circle does not touch failure envelope. $\therefore \sigma_1 = \sigma_v$ and $\sigma_3 = \sigma_H = P_{Zero}$
- 2] If the wall is allowed to move laterally, $\sigma_3 = \sigma_H$ reduces as the wall moves away from the backfill . this decreasing intensity of the stress (σ_3) attains the Minimum peak value where the stress circle (II) touches the failure envelope. So, general state of plastic equilibrium will be developed at all points within the soil mass A . here, two sets of failure planes will developed in the soil mass making and angle $45 + (\phi/2)$ with horizontal. Major principal stress, $\sigma_1 = \sigma_v = (\gamma \times H)$ and minor principal stress, $\sigma_3 = \sigma_H = P_a$ (Active earth Pressure)

- 3] With the wall moves towards the backfill, σ_H in the soil B will increase and hence,

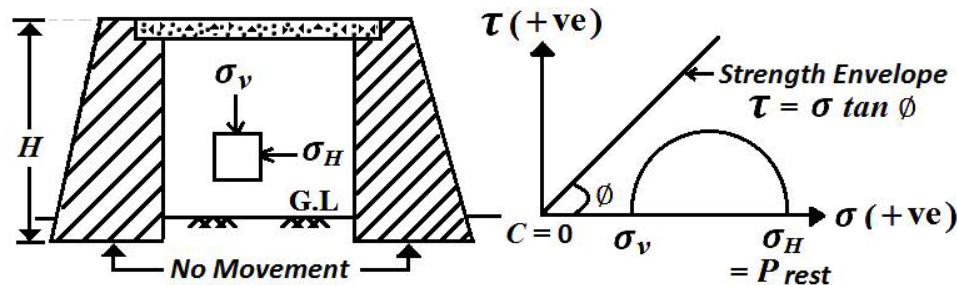
σ_H may become larger than σ_v . Major principal stress $\sigma_1 = \sigma_H = P_p$ (Passive earth Pressure), and minor principal stress, $\sigma_3 = \sigma_v = (\gamma \times H)$. this increasing intensity of the stress (σ_1) attains the Maximum peak value where the stress circle (III) touches the failure envelope. So, general state of plastic equilibrium will be developed at all points within the soil mass B . here, two sets of failure

planes will develop in the soil mass making an angle $45 - (\phi/2)$ with horizontal.

Earth Pressure at Rest Condition :

The lateral pressure corresponding to no deformation is called as earth pressure at rest Condition. In other words, when the wall is at rest and the soil material (Backfill) which is retained by the wall remains in natural state then the pressure applied by the backfill is known as earth pressure at rest Condition. Here, there is no movement of retaining wall as the pressure intensity is Zero. This can be explained by Mohr's Circle as the stress circle does not touch failure envelope. $\therefore \sigma_1 = \sigma_v$ and $\sigma_3 = \sigma_H = P_{zero}$. Consider bridge abutments connected by RCC Slab as shown in figure

$$\text{Coefficient of earth Pressure at Rest} \therefore K_{zero} = \frac{\sigma_H(\text{rest})}{\sigma_v} = \frac{P_{zero}}{\sigma_v}$$

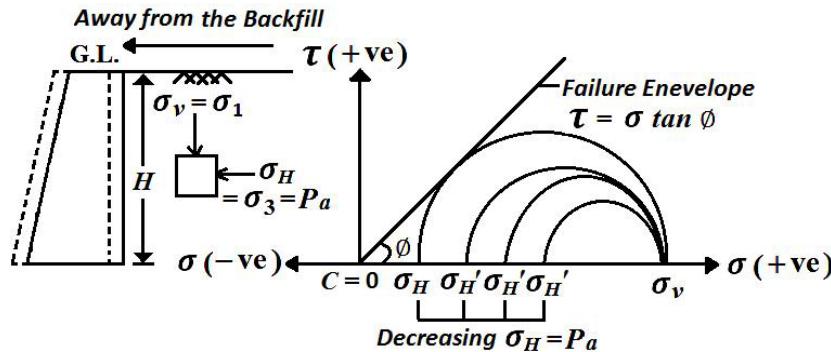


Active Earth Pressure :

When the soil allows to expand or if the Retaining wall moves away from the Backfill , the Horizontal Pressure (σ_H) goes on reducing till it reaches to minimum value or till Failure occurs. In the Active case, the vertical stress (σ_v) is more than the horizontal stress (σ_H). hence, Major principal stress, $\sigma_1 = \sigma_v = (\gamma \times H)$ and minor principal stress, $\sigma_3 = \sigma_H = Pa$ (Active earth Pressure). Figure shows the Mohr's circle of stresses and the failure envelope for the active case. The Horizontal Pressure (σ_H) goes on reducing from $\sigma_{H'}$ to peak Minimum value of σ_H where the Mohr Circle touches the failure

envelope. This Minimum peak value of Earth Pressure is called as active Earth Pressure (Pa) and the corresponding Coefficient is known as Coefficient of active Earth Pressure (K_a).

$$\therefore K_a = \frac{\sigma_H(\text{active})}{\sigma_v} = \frac{Pa}{\sigma_v}$$

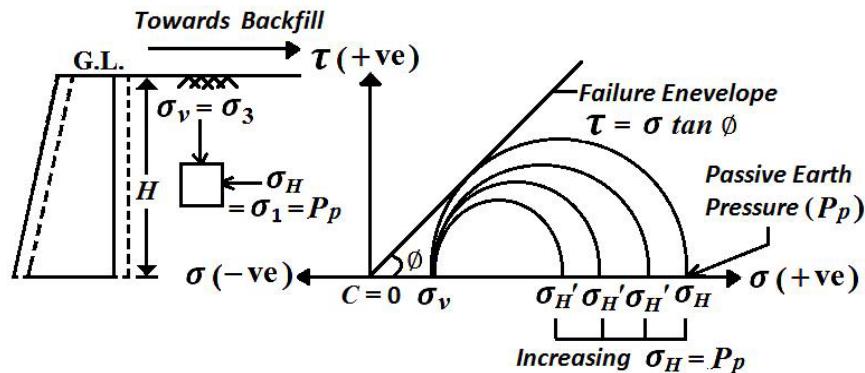


Passive Earth Pressure :

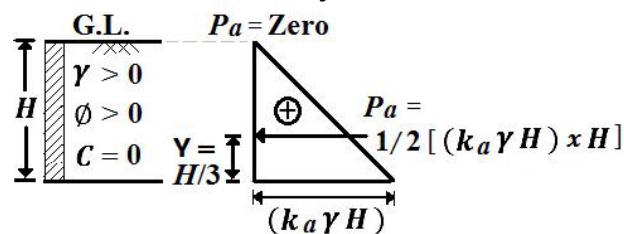
If the soil is compressed or the Retaining wall is pushed into the Backfill, the Lateral earth pressure (σ_H) goes on increasing until it reaches to maximum value or till Failure occurs. In the Passive case, the horizontal stress (σ_H) is more than the vertical stress (σ_v). hence, Major principal stress $\sigma_1 = \sigma_H = Pp$ (Passive earth Pressure), and minor principal stress, $\sigma_3 = \sigma_v = (\gamma \times H)$.

Figure shows the Mohr's circle of stresses and the failure envelope for the Passive case. The Horizontal Pressure (σ_H) goes on increasing from $\sigma_{H'}$ to peak Maximum value of σ_H where the Mohr Circle touches the failure envelope. This Maximum peak value of Earth Pressure is called as Passive Earth Pressure (Pp) and the corresponding Coefficient is known as Coefficient of Passive Earth Pressure (K_p).

$$\therefore K_p = \frac{\sigma_H(\text{passive})}{\sigma_v} = \frac{Pp}{\sigma_v}$$



Rankine's Active Earth Pressure for Dry Cohesionless Backfill ($C = 0$) :



It is known that the principal stresses are related to the shear strength parameters of the backfill material (C & ϕ) by the Bell's equation which is as follows :

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2C \tan \alpha \quad \dots \dots \dots (i)$$

For dry Cohesionless Backfill, Cohesion (C) = 0

For active state, Major Principal Stress, $\sigma_1 = \sigma_v = (\gamma x H)$ and

Minor Principal Stress ; $\sigma_3 = \sigma_H = Pa$ (Active earth Pressure)

Substituting in Equation (i)

$$\therefore (\gamma x H) = Pa \tan^2 \alpha + 0 ; \quad \therefore Pa = (\gamma x H) \frac{1}{\tan^2 \alpha} = (K_a \gamma H)$$

Hence, determination of active earth pressure (Pa) at any level in Rankine's theory is to multiply the vertical stress $\sigma_v = (\gamma x H)$ at that depth with the Rankine's coefficient of active earth pressure (K_a) and it is given by,

$$K_a = \frac{1}{\tan^2 \alpha} = \cot^2 \alpha = \frac{\cos^2 \alpha}{\sin^2 \alpha} = \frac{(1 + \cos 2\alpha)/2}{(1 - \cos 2\alpha)/2} = \frac{(1 + \cos 2\alpha)}{(1 - \cos 2\alpha)}$$

We have, $\alpha = 45 + (\phi/2)$;

$$\therefore K_a = \frac{1 + \cos(90 + \phi)}{1 - \cos(90 + \phi)} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

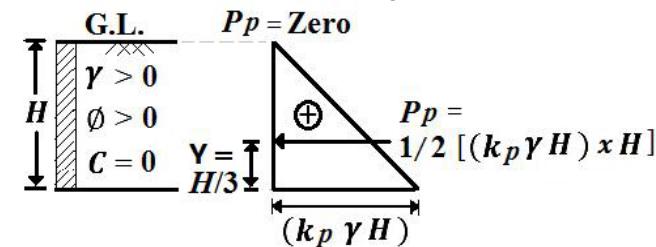
Where, $\cos(90 + \phi) = -\sin \phi$ and $\cos(90 - \phi) = +\sin \phi$

the active earth pressure is zero at the top surface of the backfill ($H = 0$) and increases linearly with depth below the surface. The distribution of active earth pressure is shown in figure. The total or resultant active earth pressure (Pa) exerted on the wall is obtained by computing the area of the pressure diagram.

$$\therefore Pa = 1/2 [(K_a \gamma H) x H]$$

The total active earth pressure acts horizontally through the centroid of the pressure diagram and it is acting at distance of $Y = (H/3)$ from the base of Retaining wall.

Rankine's Passive Earth Pressure for Dry Cohesionless Backfill ($C = 0$) :



We have the Bell's equation which is as follows :

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2C \tan \alpha \quad \dots \dots \dots (i)$$

For dry Cohesionless Backfill, Cohesion (C) = 0

For Passive state, Major Principal Stress, $\sigma_1 = \sigma_H = Pp$ (passive earth Pressure)

and Minor Principal Stress ; $\sigma_3 = \sigma_v = (\gamma x H)$

Substituting in Equation (i)

$$Pp = (\gamma x H) \tan^2 \alpha + 0 ; \quad \therefore Pp = (\gamma x H) \tan^2 \alpha = K_p (\gamma x H)$$

Hence, determination of passive earth pressure (P_p) at any level in Rankine's theory is to multiply the vertical stress $\sigma_v = (\gamma \times H)$ at that depth with the Rankine's coefficient of passive earth pressure (K_p) and it is given by,

$$K_p = \tan^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \frac{(1 - \cos 2\alpha)/2}{(1 + \cos 2\alpha)/2} = \frac{(1 - \cos 2\alpha)}{(1 + \cos 2\alpha)}$$

We have, $\alpha = 45 + (\phi/2)$;

$$\therefore K_p = \frac{1 - \cos(90 + \phi)}{1 + \cos(90 + \phi)} = \frac{1 + \sin \phi}{1 - \sin \phi}$$

Where, $\cos(90 + \phi) = -\sin \phi$ and $\cos(90 - \phi) = +\sin \phi$

In other words,

$$K_p =$$

$$1/K_a = 1/(1 - \sin \phi/1 + \sin \phi) = (1 + \sin \phi/1 - \sin \phi)$$

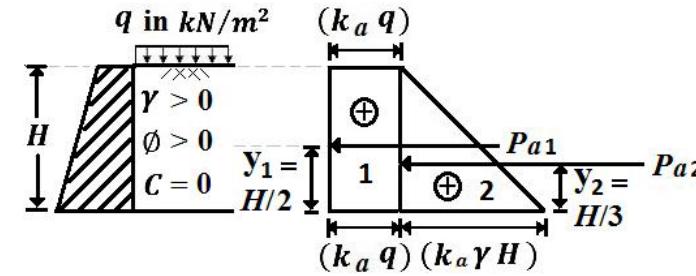
the Passive earth pressure is zero at the top surface of the backfill ($H = 0$). The total or resultant Passive earth pressure (P_p) exerted on the wall is obtained by computing the area of the pressure diagram.

$$\therefore P_p = 1/2 [(K_p \gamma H) \times H]$$

The total Passive earth pressure acts horizontally through the centroid of the pressure diagram and it is acting at distance of $Y = (H/3)$ from the base of Retaining wall.

Rankine's Active Earth Pressure for Cohesionless Backfill with Surcharge :

Figure shows a retaining wall with a horizontal backfill subjected to additional pressure (surcharge) of intensity q (kN/m^2) on the backfill surface. The surcharge applied at the top may be assumed to be uniform throughout the depth of the wall.



vertical stress σ_v = Due to weight of the soil ($\gamma \times H$) + Surcharge (q)

As we have discussed earlier, active earth pressure (Pa) at any level in Rankine's theory can be determined by multiplying the vertical stress σ_v at that depth with the Rankine's coefficient (K_a).

At depth $Z = 0$; $\sigma_v = Pa = K_a [(\gamma \times H) + q] = [0 + K_a q] = (K_a \times q)$

\therefore Vertical stress at (σ_v) at depth ($Z = H$) is given by,

$$\sigma_v = Pa = K_a [(\gamma \times H) + q] = [K_a \gamma H + K_a q]$$

Where, K_a = coefficient of active earth pressure = $\frac{1 - \sin \phi}{1 + \sin \phi}$

Now, Due to Surcharge Loading (q), rectangle of Intensity ($K_a \times q$) is added in the Pressure diagram (Figure 1) with triangle of intensity ($K_a \gamma H$) for Dry Cohesionless Backfill i.e. due to weight of the soil (Figure 2)

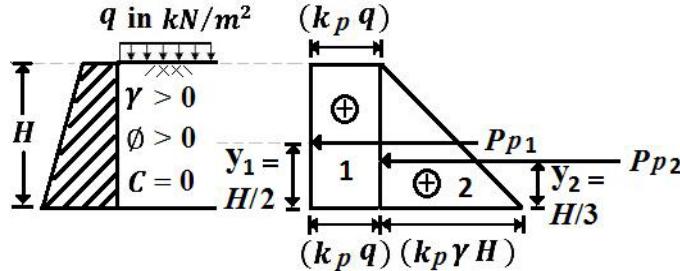
Active earth pressure due to Surcharge (q) having intensity ($K_a \times q$) = Area of Figure 1. $\therefore Pa_1 = [(K_a q) \times H]$ which acts at horizontally through the centroid of the pressure diagram 1 i.e. $Y_1 = (H/2)$ from the base of Retaining wall.

Active earth pressure due to weight of the soil having intensity ($K_a \gamma H$) = Area of Figure 2. $\therefore Pa_2 = 1/2 [(K_a \gamma H) \times H]$ which acts at horizontally through the centroid of the pressure diagram 2 i.e. $Y_2 = (H/3)$ from the base of Retaining wall.

\therefore Total Resultant Active earth pressure (Pa) = $Pa_1 + Pa_2$

Which is acting at distance of $P_a x Y = (P_{a1} Y_1 + P_{a2} Y_2)$ From the principles of mechanics. $\therefore Y = (P_{a1} Y_1 + P_{a2} Y_2) / P_a$ from the base of Retaining wall.

Rankine's Passive Earth Pressure for Cohesionless Backfill with Surcharge :



vertical stress σ_v = Due to weight of the soil ($\gamma \times H$) + Surcharge (q)

As we have discussed earlier, passive earth pressure (P_p) at any level in Rankine's theory can be determined by multiplying the vertical stress σ_v at that depth with the Rankine's coefficient (K_p) .

At depth $Z = 0$; $\sigma_v = P_p = K_p [(\gamma \times H) + q] = [0 + K_p \cdot q] = (K_p \times q)$

\therefore Vertical stress at (σ_v) at depth ($Z = H$) is given by,

$$\sigma_v = P_p = K_p [(\gamma \times H) + q] = [K_p \gamma H + K_p q]$$

Where, K_p = coefficient of Passive earth pressure = $\frac{1+\sin\phi}{1-\sin\phi}$

Now, Due to Surcharge Loading (q), rectangle of Intensity ($K_p \times q$) is added in the Pressure diagram (Figure 1) with triangle of intensity ($K_p \gamma H$) for Dry Cohesionless Backfill i.e. due to weight of the soil (Figure 2)

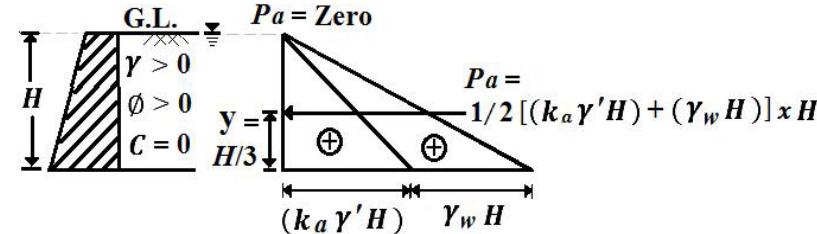
passive earth pressure due to Surcharge (q) having intensity ($K_p \times q$) = Area of Figure 1. $\therefore P_p 1 = [(K_p q) \times H]$ which acts at horizontally through the centroid of the pressure diagram 1 i.e. $Y_1 = (H/2)$ from the base of Retaining wall.

passive earth pressure due to weight of the soil having intensity ($K_a \gamma H$) = Area of Figure 2. $\therefore P_p 2 = 1/2 [(K_p \gamma H) \times H]$ which acts at horizontally through the centroid of the pressure diagram 2 i.e. $Y_2 = (H/3)$ from the base of Retaining wall.

\therefore Total Resultant passive earth pressure (P_p) = $P_p 1 + P_p 2$

Which is acting at distance of $P_p x Y = (P_{p1} Y_1 + P_{p2} Y_2)$ From the principles of mechanics. $\therefore Y = (P_{p1} Y_1 + P_{p2} Y_2) / P_p$ from the base of Retaining wall.

Rankine's Active Earth Pressure for Fully Submerged Cohesionless Backfill :



At depth H , vertical stress (σ_v) = Due to Submerged weight of the soil ($\gamma' \times H$) + Pore water Pressure or hydrostatic pressure ($\gamma_w \times H$)

Figure shows a retaining wall with a fully submerged backfill, with the groundwater table at the surface of the backfill. The principle of determination of active earth pressure is to multiply the effective vertical stress with the lateral pressure coefficient (K_a) and then add the hydrostatic pressure due to water table, if any. This is because the hydrostatic pressure is equal in all directions as per Pascal's law, and hence, the lateral pressure should not be applied to the hydrostatic pressure.

\therefore Vertical stress at (σ_v) at depth $Z = 0$;

$$\sigma_v = Pa = K_a (\gamma' \times H) + (\gamma_w \times H) = (0 + 0) = Zero$$

Hence, the active earth pressure is zero at the top surface of the backfill ($Z = 0$)

At depth ($Z = H$) ; $\sigma_v = Pa = K_a (\gamma' x H) + (\gamma_w x H)$

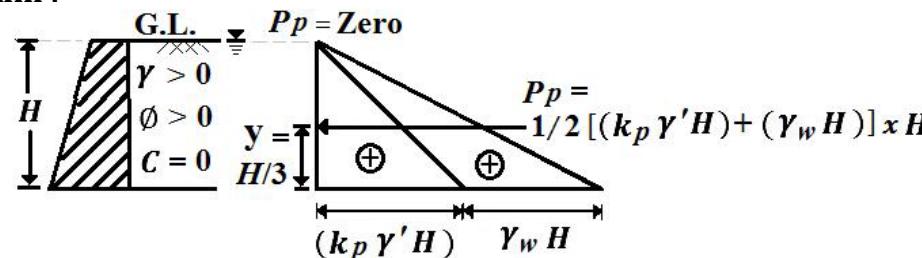
Where, K_a = coefficient of active earth pressure = $\frac{1-\sin\phi}{1+\sin\phi}$

The total or resultant active earth pressure (Pa) exerted on the wall is obtained by computing the area of the pressure diagram.

$$\therefore Pa = 1/2 [(K_a \gamma' H) + (\gamma_w H)] x H$$

Which acts horizontally through the centroid of the pressure diagram at distance of $Y = (H/3)$ from the base of Retaining wall.

Rankine's Passive Earth Pressure for Fully Submerged Cohesionless Backfill :



At depth ($Z = H$), vertical stress (σ_v) = Due to Submerged weight of the soil ($\gamma' x H$) + Pore water Pressure or hydrostatic pressure ($\gamma_w x H$)

Figure shows a retaining wall with a fully submerged backfill, with the groundwater table at the surface of the backfill. The principle of determination of active earth pressure is to multiply the effective vertical stress with the lateral pressure coefficient (K_p) and then add the hydrostatic pressure due to water table, if any. This is because the hydrostatic pressure is equal in all directions as per Pascal's law, and hence, the lateral pressure should not be applied to the hydrostatic pressure.

\therefore Vertical stress at (σ_v) at depth $Z = 0$;

$$\sigma_v = Pp = K_p (\gamma' x H) + (\gamma_w x H) = (0 + 0) = Zero$$

Hence, the active earth pressure is zero at the top surface of the backfill ($Z = 0$)

At depth ($Z = H$) ; $\sigma_v = Pp = K_p (\gamma' x H) + (\gamma_w x H)$

Where, K_p = coefficient of active earth pressure = $\frac{1+\sin\phi}{1-\sin\phi}$

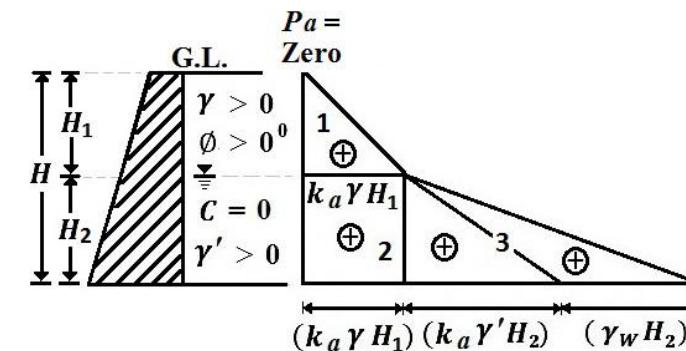
The total or resultant active earth pressure (Pa) exerted on the wall is obtained by computing the area of the pressure diagram.

$$\therefore Pp = 1/2 [(K_p \gamma' H) + (\gamma_w H)] x H$$

Which acts horizontally through the centroid of the pressure diagram at distance of $Y = (H/3)$ from the base of Retaining wall.

Rankine's Active Earth Pressure for Partially Submerged Cohesionless Backfill:

Figure shows a retaining wall of height H , with a partially submerged backfill, with the groundwater table at a depth (H_1) below the surface of the backfill. The soil above the water table may be either partially or fully saturated. The bulk density (γ) of the soil is to be used for computation of vertical stress for soil above the water table.



At depth H , vertical stress (σ_v) can be Computed as

Due to dry weight of Backfill for ht. $Z = H_1$ is $(K_a x \gamma x H_1) +$ Submerged weight of the backfill for ht. $Z = H_2$ is $(K_a x \gamma' x H_2) +$ Pore water Pressure or hydrostatic pressure for ht. $Z = H_2$ is $(\gamma_w x H_2)$

At depth $Z = 0$; Vertical stress (σ_v) is given by,

$$\sigma_v = Pa = [(K_a x \gamma x H_1) + (K_a x \gamma' x H_2) + (\gamma_w x H_2)] = Zero$$

At depth $Z = H$; Vertical stress (σ_v) is given by,

$$(\sigma_v) = Pa = [(Ka x \gamma x H_1) + (Ka x \gamma' x H_2) + (\gamma_w x H_2)]$$

Where, Ka = coefficient of active earth pressure $= \frac{1-\sin\phi}{1+\sin\phi}$

Active earth pressure Due to dry weight of Backfill for ht. H_1 having intensity $(Ka x \gamma x H_1)$ = Area of Figure 1. $\therefore Pa_1 = 1/2 [(Ka x \gamma x H_1) x H_1]$ which acts at horizontally through the centroid of the pressure diagram 1 i.e. $Y_1 = H_2 + (H_1/3)$ from the base of Retaining wall.

Active earth pressure Due to dry weight of Backfill for ht. H_2 having intensity $(Ka x \gamma x H_1)$ = Area of Figure 2. $\therefore Pa_2 = (Ka x \gamma x H_1) x H_2$ acting at $Y_2 = (H_2/2)$ from the base of Retaining wall.

Active earth pressure due to Submerged weight of the backfill having intensity $(Ka x \gamma' x H_2)$ and hydrostatic pressure having intensity $(\gamma_w x H_2)$ for ht. H_2

= Area of Figure 3. $\therefore Pa_3 = 1/2 [(Ka \gamma' H_2) + (\gamma_w H_2)] x H_2$ acting at $Y_3 = (H_2/3)$ from the base of Retaining wall.

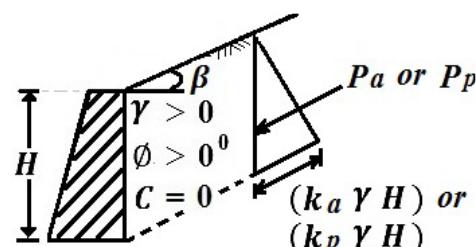
\therefore Total Resultant Active earth pressure (Pa) = $Pa_1 + Pa_2 + Pa_3$

Which is acting at distance of $Pa x Y = (P_{a1} Y_1 + P_{a2} Y_2 + P_{a3} Y_3)$ From the principles of mechanics.

$\therefore Y = (P_{a1} Y_1 + P_{a2} Y_2 + P_{a3} Y_3) / Pa$ from the base of Retaining wall.

Cohesionless Backfill with Sloping Surface :

shows a retaining wall with a cohesionless backfill having its surface sloping at an angle β with the horizontal.



Active Case :

The Total Resultant Active earth Pressure (Pa) = Area of Pressure diagram

$$\therefore Pa = 1/2 [(Ka x \gamma x H) x H] = 1/2 (Ka x \gamma x H^2)$$

Where, Ka = coefficient of active earth pressure

$$Ka = \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

Passive Case :

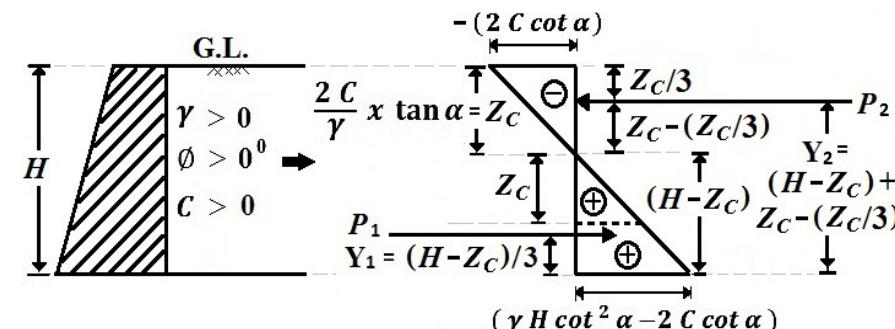
The Total Resultant Active earth Pressure (Pp) = Area of Pressure diagram

$$\therefore Pp = 1/2 [(Kp x \gamma x H) x H] = 1/2 (Kp x \gamma x H^2)$$

Where, Kp = coefficient of Passive earth pressure

$$Kp = \cos \beta \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

Rankine's Active Earth Pressure for Cohesive Backfill ($C > 0$):



We know Bell's equation which is as follows :

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2C \tan \alpha$$

.....(i)

At depth $Z = H$, Vertical stress i.e. Major Principal Stress $\sigma_v = \sigma_1 = (\gamma x H)$ will act due to the weight of the soil . and Minor Principal Stress $\sigma_H = \sigma_3 = Pa$

Substituting these in equation (i) $\therefore (\gamma H) = Pa \tan^2 \alpha + 2C \tan \alpha$

$$Pa \tan^2 \alpha = (\gamma H) - 2C \tan \alpha$$

$$\therefore Pa = \left[\frac{(\gamma H)}{\tan^2 \alpha} - \frac{2C \tan \alpha}{\tan^2 \alpha} \right] = \left[\frac{(\gamma H)}{\tan^2 \alpha} - \frac{2C}{\tan \alpha} \right] = Ka \gamma H - 2C \sqrt{Ka}$$

$$\text{Or } Pa = (\gamma H \cot^2 \alpha - 2C \cot \alpha)$$

$$\text{At depth } Z = 0 ; Pa = (\gamma H \cot^2 \alpha - 2C \cot \alpha) = (0 - 2C \cot \alpha) = -2C \cot \alpha$$

where, Ka = Coefficient of active earth pressure

$$Ka = 1/\tan^2 \alpha = \cot^2 \alpha = \tan^2 \alpha = \frac{1 - \sin \phi}{1 + \sin \phi} \text{ and } \alpha = 45 + (\phi/2)$$

Thus, active earth pressure is negative at the top of the wall and increases linearly with the increase in depth. As the soil is weak in tension, tension cracks will develop in the negative active earth pressure zone of the backfill. The depth of a tension crack or potential crack (Z_c) can be obtained by substituting $Pa = 0$

$$Pa = (\gamma Z_c \cot^2 \alpha - 2C \cot \alpha) = 0 ; (\gamma Z_c \cot^2 \alpha) = 2C \cot \alpha ;$$

$$\therefore Z_c = \frac{2C \cot \alpha}{\gamma \cot^2 \alpha} = \frac{2C}{\gamma \cot \alpha} = \frac{2C}{\gamma} \tan \alpha \quad \text{or}$$

$$Pa = (Ka \gamma Z_c - 2C \sqrt{Ka}) = 0 ; (Ka \gamma Z_c) = (2C \sqrt{Ka})$$

$$\therefore Z_c = \frac{2C \sqrt{Ka}}{\gamma Ka} = \left[\frac{2C \sqrt{Ka}}{\gamma Ka} \times \frac{\sqrt{Ka}}{\sqrt{Ka}} \right] = \frac{2C \sqrt{Ka}^2}{\gamma Ka \sqrt{Ka}} = \frac{2C}{\gamma \sqrt{Ka}}$$

If the soil is able to withstand the negative active earth pressure, the negative pressure over the depth (Z_c) is balanced by a positive pressure over the same depth (Z_c) below. Hence, the resultant active earth pressure is zero over the depth ($H_c = 2Z_c$) known as critical height or Caving depth. Thus, excavations in cohesive soils can stand with vertical sides without any lateral support over the critical height, (H_c). hence it also called as depth of unsupported Excavation.

$$\therefore H_c = 2Z_c = 2 \left[\frac{2C}{\gamma} \tan \alpha \right] = \frac{4C}{\gamma} \tan \alpha \quad \text{or}$$

$$H_c = 2Z_c = 2 \left[\frac{2C}{\gamma \sqrt{Ka}} \right] = \frac{4C}{\gamma \sqrt{Ka}}$$

The Active Earth Pressure Can be Computed by following two Cases :

- i) Active Earth Pressure after occurrence of Tension Cracks or assuming tension cracks developed or after the formation of an tension crack :**

Figure shows a retaining wall of height H with a cohesive backfill. the active earth pressure diagram when a tension crack (Z_c) is formed. If a tension crack is developed, it indicates that the soil has failed in the negative pressure zone and hence the negative pressure will no longer be acting on the wall. The total active earth pressure (Pa) is obtained by computing the area of the positive pressure diagram over the depth ($H - Z_c$) only , ignoring the negative active earth pressure

Active Earth Pressure after occurrence of tension Cracks (Pa) = Area of +ve Portion only. $\therefore Pa = P_1 = 1/2 [(\gamma H \cot^2 \alpha - 2C \cot \alpha) \times (H - Z_c)]$
Acting at distance of $Y = (H - Z_c)/3$ From the Base of the Retaining wall.

- ii) Active Earth Pressure before occurrence of Tension Crack. or Assuming No Tension cracks developed or Before the formation of an tension crack :**

If a tension crack is not developed, the negative pressure over the depth (Z_c) is balanced by a positive pressure over the depth ($H - Z_c$) below. The wall will be therefore subjected to the net active earth pressure. i.e. The total Resultant active earth pressure (Pa) is obtained by computing the net difference in between positive Pressure area over the depth ($H - Z_c$) say P_1 and negative Pressure area over the depth (Z_c) say P_2 .

$$\therefore Pa = P_1 - P_2$$

$$\text{Where, } P_1 = 1/2 [(\gamma H \cot^2 \alpha - 2C \cot \alpha) x (H - Z_c)]$$

Acting at distance of $Y_1 = (H - Z_c)/3$ From the Base of the Retaining wall.
and $P_2 = 1/2 [(2C \cot \alpha) x (Z_c)]$ = Area of negative pressure area

$$\text{Acting at distance of } Y_2 = (H - Z_c) + [Z_c - (Z_c/3)]$$

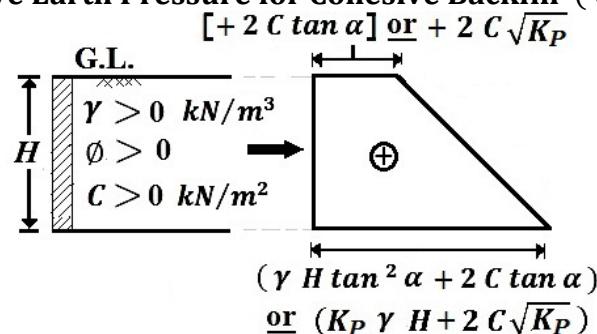
From the Base of the Retaining wall.

The total Resultant active earth pressure (Pa) acting horizontally at distance of Y

$$Pa \times Y = (P_1 x Y_1 - P_2 x Y_2); \quad \therefore Y = (P_1 x Y_1 - P_2 x Y_2) / Pa$$

From the Base of the Retaining wall.

Rankine's Passive Earth Pressure for Cohesive Backfill ($C > 0$):



We have the Bell's equation which is as follows :

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2C \tan \alpha$$

.....(i)

For Passive state, At depth $Z = H$, Major Principal Stress, $\sigma_1 = \sigma_H = P_p$
and vertical stress or Minor Principal Stress ; $\sigma_3 = \sigma_V = (\gamma x H)$

Substituting in Equation (i)

$$\therefore P_p = (\gamma x H) \tan^2 \alpha + 2C \tan \alpha = (K_p \gamma H + 2C \sqrt{K_p})$$

where , K_p = Coefficient of Passive earth pressure = $\tan^2 \alpha = \frac{1+\sin \phi}{1-\sin \phi}$

and $\alpha = 45 + (\phi/2)$

$$\text{At depth } Z = H = 0; P_p = (\gamma x H) \tan^2 \alpha + 0 = (0 + 2C \sqrt{K_p})$$

$$\therefore P_p = 2C \tan \alpha = 2C \sqrt{K_p}$$

The total or resultant Passive earth pressure (P_p) exerted on the wall is obtained by computing the area of the pressure diagram.

$$P_p =$$

$$[(a+b)/2] x H = [(2C \tan \alpha) + (\gamma H \tan^2 \alpha + 2C \tan \alpha)/2] x H$$

Or

$$[(a+b)/2] x H = [(2C \sqrt{K_p}) + (K_p \gamma H + 2C \sqrt{K_p})/2] x H$$

Acting at distance of y at the centroid of the pressure diagram from the base of retaining wall.

$$\therefore y = \left[\frac{(2a+b)}{(a+b)} \right] x \frac{H}{3}$$

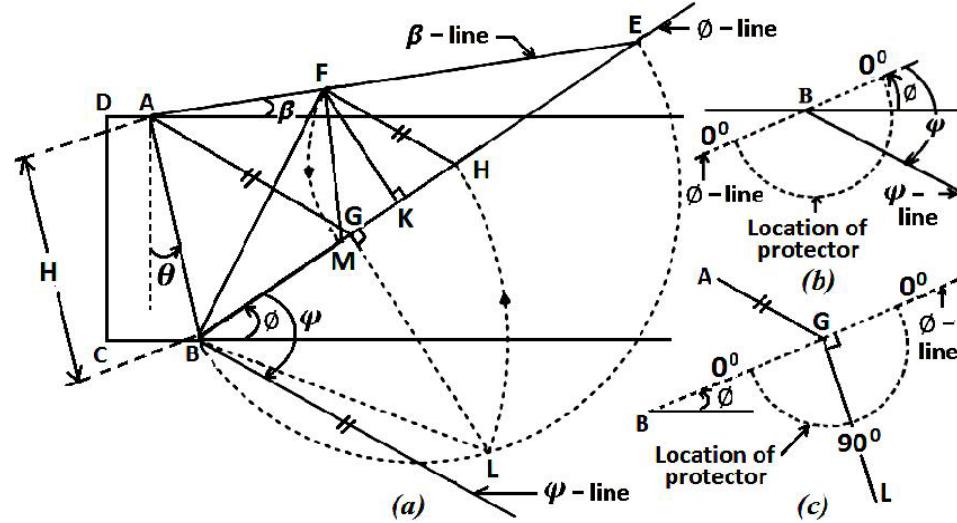
$$= \left[\frac{(2 \times 2C \tan \alpha) + (\gamma H \tan^2 \alpha + 2C \tan \alpha)}{(2C \tan \alpha) + (\gamma H \tan^2 \alpha + 2C \tan \alpha)} \right] x \frac{H}{3}$$

Or

$$y = \left[\frac{(2a+b)}{(a+b)} \right] x \frac{H}{3} = \left[\frac{(2 \times 2C \sqrt{K_p}) + (K_p \gamma H + 2C \sqrt{K_p})}{(2C \sqrt{K_p}) + (K_p \gamma H + 2C \sqrt{K_p})} \right] x \frac{H}{3}$$

Rehban's Graphical Method :

Case 1) When , $\phi > \beta$. i.e. When the Intersection of ϕ – line and β – line lies in the borders of graph Paper.



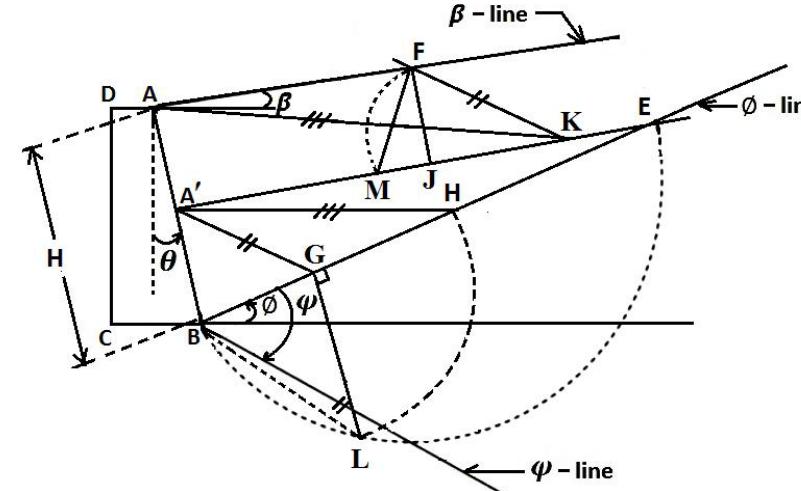
Construction :

- Draw the cross section $ABCD$ of the retaining wall to suitable scale. Let the wall face AB Retains the soil.
- Draw a line which makes an angle of ϕ (Angle of Internal friction or shearing resistance or angle of repose) with the horizontal at B called as ϕ – line . and the line at an angle of β (angle of surcharge) as β – line or Surcharge line at A with Horizontal. Let them intersect at E . So, here, we found point E .
- Calculate value of angle $\varphi = 90 - \theta - \delta$
Where θ – Angle of inclination of wall face AB with the vertical ,
 δ – Angle of wall friction and θ – positive battened angle.
- Draw the line making angle φ w.r.t. ϕ – line as shown in figure which is called as earth pressure line (φ – line) .
- Draw the semicircle on the line BE by measuring a distance of BE by ruler scale.
- Draw parallel line to φ – line from point A such that it will further intersects ϕ – line at point G . So, here, we found point G .
- Erect perpendicular GL to BE (ϕ – line) , cutting the semicircle at L . So, here, we found point L .

- With B as centre and BL as radius, draw an arc which cuts ϕ – line at H . So, here, we found point H .
- Draw HF parallel to φ – line such that it will further intersects β – line at point F . So, here, we found point F .
- With H as centre and radius HF , draw an arc to cut ϕ – line at M . Join FM .
- Active Earth Pressure ; $P_a = \gamma x (\text{Area of } \Delta HFM)$

$$\therefore P_a = \gamma x [1/2 x l(MH) x l(FK)]$$

Case 2) When , $\phi \gg \beta$. i.e. When the Intersection of ϕ – line and β – line not lies within borders of graph Paper.

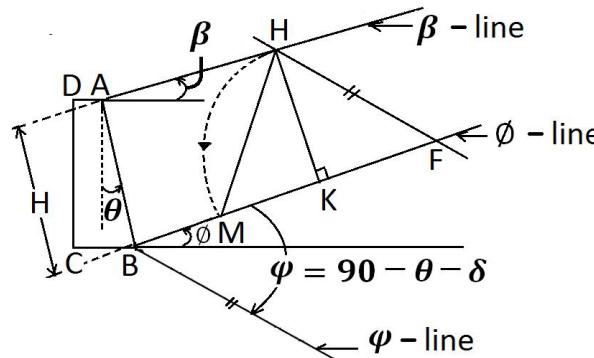


Construction :

- Refer procedure from points (a) to (d) from Case 1. Then ,
- As the intersection not lies within borders of graph Paper, we have to shift A to A' (assumed or arbitrary point on the face AB)
- Draw parallel line to β – line from point A' such that it will further intersects ϕ – line at point E . So, here, we found point E .
- Draw the semicircle on the line BE by measuring a distance of BE by ruler scale.
- Draw parallel line to φ – line from point A' such that it will further intersects ϕ – line at point G . So, here, we found point G .
- Erect perpendicular GL to ϕ – line , cutting the semicircle at L . So, here, we found point L .

- g) With B as centre and BL as radius, draw an arc which cuts ϕ – line at H . So, here, we found point H . Join $A'H$.
- h) Draw the line parallel to $A'H$ from A such that it will further intersects $A'E$ at point K . So, here, we found point k .
- i) Draw KF parallel to φ – line such that it will further intersects β – line at point F . So, here, we found point F .
- j) With K as centre and radius KF , draw an arc to cut ϕ – line at M . Join FM .
- k) Active Earth Pressure ; $P_a = \gamma x (\text{Area of } \Delta KFM)$
 $\therefore P_a = \gamma x [1/2 x l(MK) x l(FJ)]$

Case 3) When, $\phi = \beta$.



Construction :

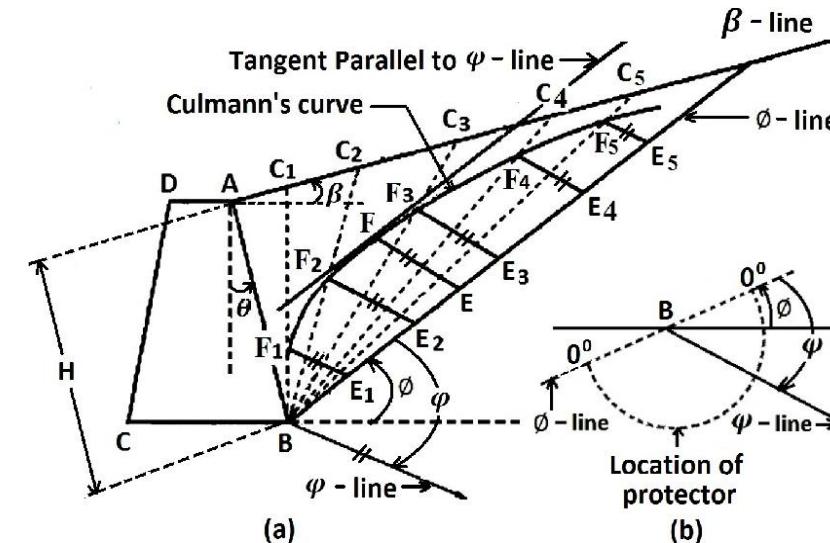
- a) Refer procedure from points (a) to (C) from Case 1. Then,
- b) Draw a line AB at an angle of ϕ (Angle of Internal friction or shearing resistance or angle of repose) called as ϕ – line and line AB at an angle of β (angle of surcharge) with Horizontal called as β – line or Surcharge line .

These lines will never intersect because $\phi = \beta$

- i) Draw the line parallel to φ – line such that it will intersects ϕ – line and β – line at points H and F respectively. So, here, we directly found HF .
- c) With H as centre and radius HF , draw an arc to cut ϕ – line at M . Join FM .
- d) Active Earth Pressure ; $P_a = \gamma x (\text{Area of } \Delta HFM)$
 $\therefore P_a = \gamma x [1/2 x l(MF) x l(KH)]$

Culmann's Graphical Method :

Case 1) Active Earth Pressure :



- a) Draw the cross section $ABCD$ of the retaining wall to suitable scale. Let the wall face AB Retains the soil.
- b) Draw the ϕ – line , β – line or Surcharge line and φ – line as usual which is earlier discussed earlier in Rehban's method (Case 1).
- c) Take minimum five assumed (arbitrary) points say C_1, C_2, C_3, C_4, C_5 at an distance of 2 to 3 mt. on β – line or Surcharge line . Join all these points with B .
- d) Now, Soil wedges $ABC_1, ABC_2, ABC_3, ABC_4$ and ABC_5 will be form with base width of AC_1, AC_2, AC_3, AC_4 and AC_5 respectively. Find out weight of all soil wedges say W_1, W_2, W_3, W_4 and W_5 resp. by using table as below.

Wedge s	Base (B) in mt.	Height (H) in mt.	Area (A) = $1/2 x B x H$	Bulk unit wt. (γ) in kN/m^3	Weight (W) = ($A x \gamma$) in kN/m
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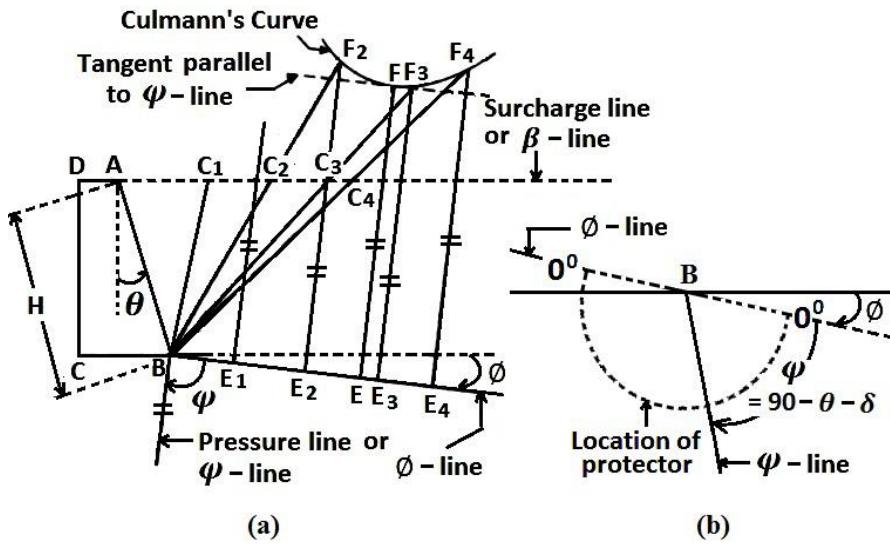
- e) If the Surcharge load (q) in kN/m exist, then the above table can be modified as below.

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Wedge s	Bas e (B)	Heig ht (H)	Are a (A)	γ in kN/m	$w = (A \times \gamma)$ in kN/m	q in kN/m	Weight (W) = (w + q)
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- f) Plot weight of all these wedges as $W_1 = BE_1$, $W_2 = BE_2$, $W_3 = BE_3$, $W_4 = BE_4$ and $W_5 = BE_5$ by converting it in selected scale on \emptyset – line. So, here, we found E_1, E_2, E_3, E_4 and E_5 on \emptyset – line .
- g) Draw parallel line to φ – line from E_1, E_2, E_3, E_4 and E_5 such that it will further intersects slip planes BC_1, BC_2, BC_3, BC_4 and BC_5 . We get points F_1, F_2, F_3, F_4 and F_5 respectively.
- h) Draw smooth Curve through points F_1, F_2, F_3, F_4 and F_5 which is called as Culmann's curve .
- i) Draw a tangent to this culmann's line which is parallel to \emptyset – line . the point where this tangent line touches the Culmann's curve , say F.
- j) Again draw parallel; line to φ – line from point F to \emptyset line, we get point E.
- k) Join EF
- ∴ Active earth pressure :
- ∴ $P_a = \gamma x l(EF)$

Case 2) Passive Earth Pressure :



- 1) Draw the cross section $ABCD$ of the retaining wall to suitable scale. Let the wall face AB Retains the soil.
Draw the ϕ – line , β – line or Surcharge line and φ – line . Note : This time ϕ – line and φ – line , both angles are taken below the horizontal plane. See the protector image.
 - 2) Take minimum five assumed (arbitrary) points say C_1, C_2, C_3, C_4, C_5 at an distance of 2 to 3 mt. on β – line or Surcharge line . Join all these points with B .
 - 3) Now, Soil wedges $ABC_1, ABC_2, ABC_3, ABC_4$ and ABC_5 will be form with base width of AC_1, AC_2, AC_3, AC_4 and AC_5 respectively. Find out weight of all soil wedges say W_1, W_2, W_3, W_4 and W_5 resp. by using table as below.

Wedge s	Base (B) in mt.	Height (H) in mt.	Area (A) $= 1/2 x B x H$	Bulk unit wt. (γ) in kN/m	Weight (W) $= (A \times \gamma)$ in kN/m
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- 4) If the Surcharge load (q) in kN/m exist, then the above table can be modified as below.

Wedge s	Bas e (B)	Heig ht (H)	Are a (A)	γ in kN/m^3	$= \frac{w}{\text{in } kN/m}$	q in kN/m	Weight (W) = (w + q)
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- 5)** Plot weight of all these wedges as $W_1 = BE_1$, $W_2 = BE_2$, $W_3 = BE_3$, $W_4 = BE_4$ and $W_5 = BE_5$ by converting it in selected scale on \emptyset – line. So, here, we found E_1, E_2, E_3, E_4 and E_5 on \emptyset – line .

6) Draw parallel line to φ – line from E_1, E_2, E_3, E_4 and E_5 such that it will further intersects slip planes BC_1, BC_2, BC_3, BC_4 and BC_5 . We get points F_1, F_2, F_3, F_4 and F_5 respectively.

7) Draw smooth Curve through points F_1, F_2, F_3, F_4 and F_5 which is called as Culmann's curve .

8) Draw a tangent to this culmann's line which is parallel to \emptyset – line .
the point where this tangent line touches the Culmann's curve , say F .

9) Again draw parallel; line to φ – line from point F to \emptyset line, we get point E .

10) Join EF

\therefore Passive earth pressure :

Important Note : If Surcharge angle (β) and Battered angle (θ) is not given in the problem then always take values as zero.

Problem 1. Following data pertains the retaining wall :

Height of wall - 7 mt.

Battened angle = 10^0

Angle of wall friction = 20^0

Unit weight of soil = 16.5 kN/m^3

Angle of internal friction or angle of shearing resistance = 30^0

Surcharge angle = 10^0

Compute the total active thrust on wall . use Rehban's method

Answer : Here, $\phi > \beta$. Also ϕ - line and β - line intersects each other at point E hence, Refer Rehban's graphical method Case 1 , page no. 10 . Refer graph on page no. 16.

Given :

ϕ – Angle of internal friction or shearing resistance = 30^0

δ – Angle of wall friction = 20^0 ; θ – Positive battened angle = 10^0

γ – Unit weight of soil = 16.5 kN/m^3 β – Surcharge angle = 10^0

$$\varphi = 90 - \theta - \delta$$

$$= 90 - 10 - 20 = 60^0$$

Active earth pressure :

$$P_a = \gamma x \text{ Area of } \Delta \text{ MHF}$$

$$= \gamma x \frac{1}{2} x \text{ MH } x \text{ FK}$$

$$= 16.5 x \frac{1}{2} x 5 x 4.3 = 177.37 \text{ KN/m}$$

Problem 2.

The retaining wall achieved vertical backfill of 7 m high supports cohesionless backfill of unit weight 19.6 kN/m^3 . The upper surface of the backfill rises at an angle of 10^0 with horizontal from crest of the wall . The angle of internal friction for the soil is 35^0 and the angle of wall friction is 20^0 . find total active earth pressure per meter length of the wall. Use Rehban's method.

Answer :

: Here, $\phi > \beta$. Also ϕ - line and β - line intersects each other at point E hence, Refer Rehban's graphical method Case 1 , page no. 10 . Refer graph on page no. 17.

Given :

ϕ – Angle of internal friction or shearing resistance = 35^0

δ – Angle of wall friction = 20^0

θ – Positive battened angle = 0^0

γ – Unit weight of soil = 19.6 kN/m^3

β – Surcharge angle = 10^0

$$\varphi = 90 - \theta - \delta$$

$$= 90 - 0 - 20 = 70^0$$

Active earth pressure :

$$P_a = \gamma x \text{ Area of } \Delta \text{ MHF}$$

$$= \gamma x \frac{1}{2} x \text{ MH } x \text{ FK}$$

$$= 19.6 x \frac{1}{2} x 2 x 2 = 40 \text{ KN/m}$$

Problem 3.

Determine the active earth pressure by Rehban's graphical method for retaining wall of 6 mt. Achieved at an angle of 15^0 With vertical . the backfill inclined at an angle of 20^0 Upward from the top of retaining wall . The material has an unit weight of 18.2 KN/m^3 and the angle of internal friction as 20^0 and wall friction angle of 20^0 .

Answer :

Here, $\phi = \beta$. Also $\phi - line$ and $\beta - line$ intersects each other at point E hence, Refer Rehban's graphical method Case 1 , page no. 11 .

Refer graph on page no. 18.

Given :

ϕ – Angle of internal friction or shearing resistance = 20^0

δ – Angle of wall friction = 20^0

θ – Positive battened angle = 15^0

γ – Unit weight of soil = 18.2 kN/m^3

β – Surcharge angle = 20^0

$$\varphi = 90 - \theta - \delta$$

$$= 90 - 15 - 20 = 55^0$$

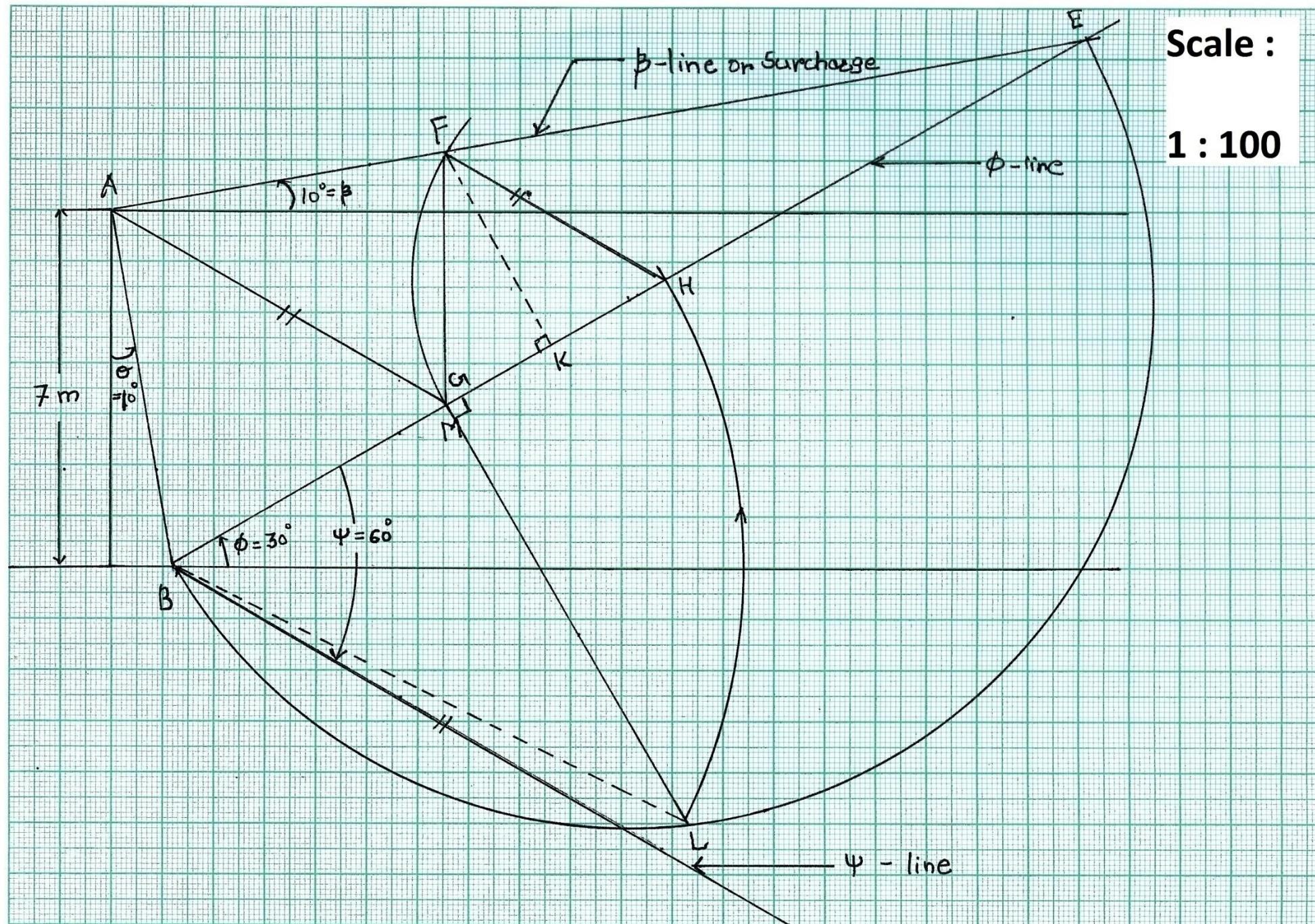
Active earth pressure :

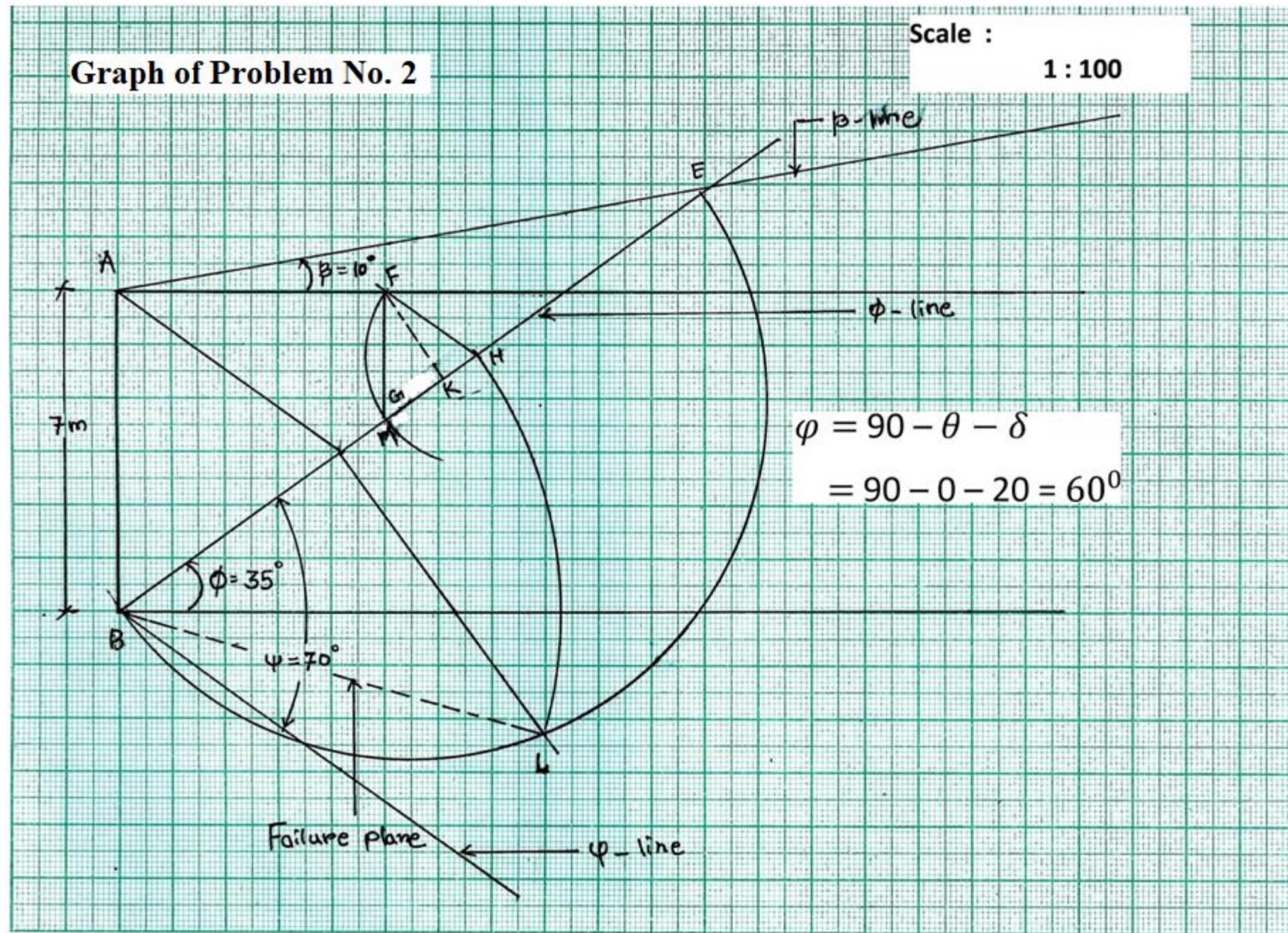
$$P_a = \gamma \times \text{Area of } \Delta \text{ MHF}$$

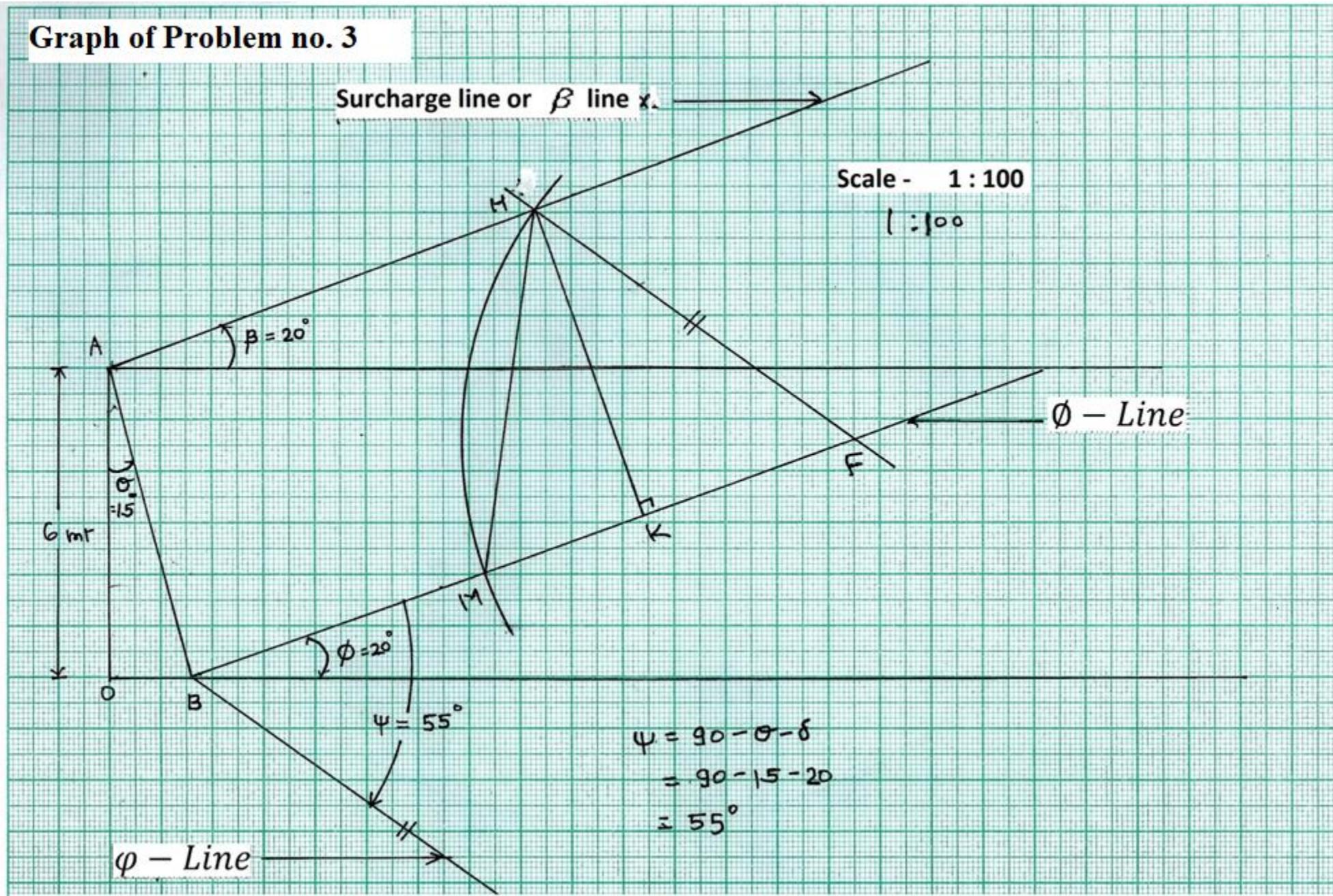
$$= \gamma \times \frac{1}{2} \times MF \times HK$$

$$= 18.2 \times \frac{1}{2} \times 7.6 \times 6.2 = 430 \text{ KN/m}$$

$$= 430 \text{ KN/m}$$





Graph of Problem no. 3

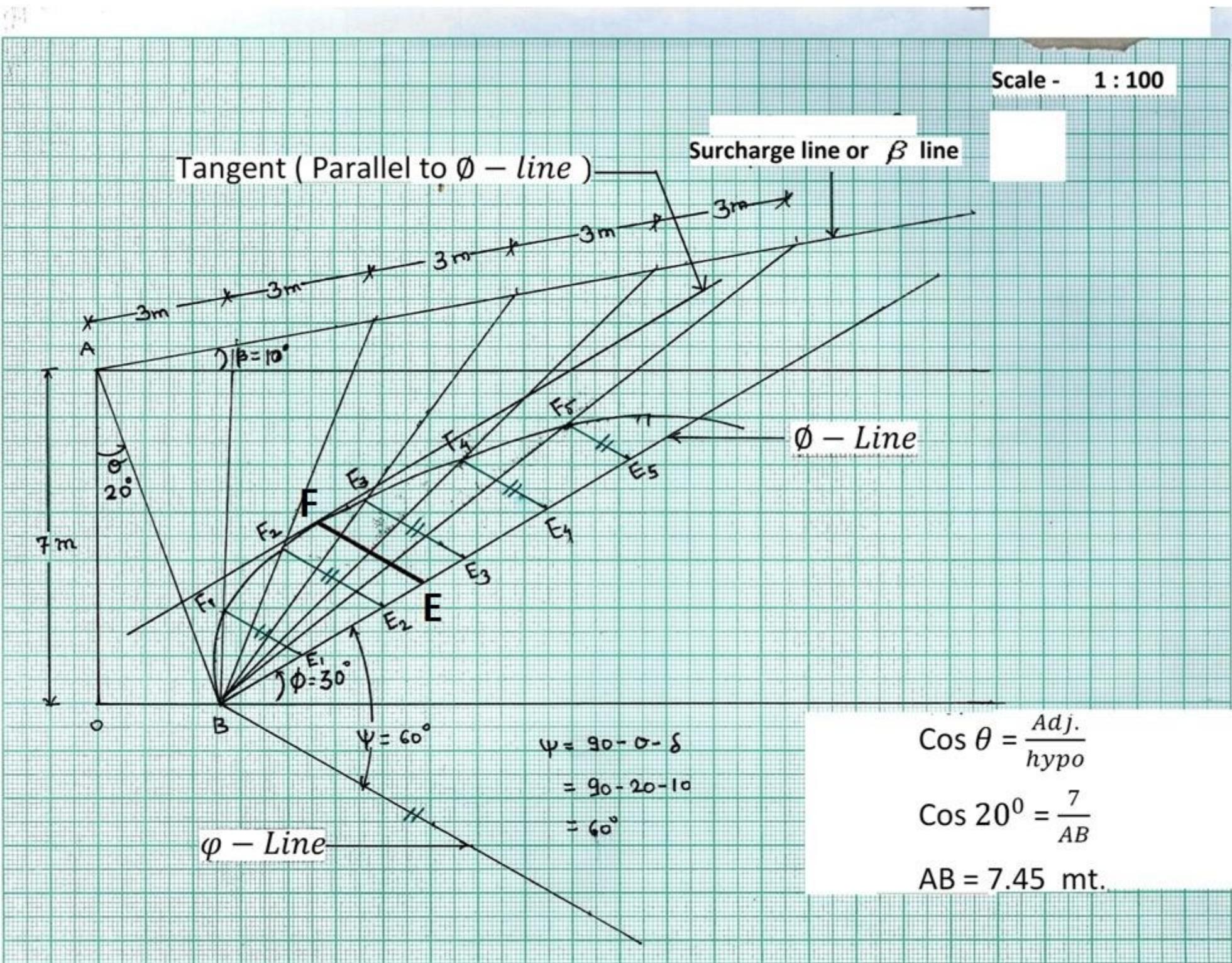
Problem 4.

Determine the active earth pressure by Culman's graphical method for retaining wall of 7 mt. Achieved at an angle of 20^0 With vertical . the backfill inclined at an angle of 10^0 Upward from the top of retaining wall . The material has an unit weight of 18.2 KN/m^3 and the angle of internal friction as 30^0 and wall friction angle of 10^0 .

Answer : Refer Culmann's graphical method for procedure , page no. 11.

Wedges	Base (B) in mt.	Height (H) in mt.	Area (A) $= 1/2 \times B \times H$	Bulk unit wt. (γ) in kN/m^3	Weight (W) $= (A \times \gamma)$ in kN/m	WEIGHT IN SCALE 1:100
ABC1	3	7.45	11.175	18	201.15	$201.15/100 = 2.01 \text{ cm}$ $= BE1$
ABC2	6	7.45	22.35	18	402.3	BE 2 = 4.023 cm
ABC3	9	7.45	33.525	18	603.45	BE 3 = 6.03 cm
ABC4	12	7.45	44.70	18	804.6	BE 4 = 8.046 cm
ABC5	15	7.45	55.87	18	1005.66	BE 5 = 10.05 cm

ANSWER - Total active earth pressure $P_a = \text{length FE} \times \text{Scale}$ i.e. $P_a = 2.5 \times 100 = 250 \text{ KN/mt}$



Problem 5. A retaining wall of 7 m height achieved an angle of 20^0 with vertical. the footing runs parallel to retaining wall.

Properties of the backfill as follows .

Angle of internal friction or shearing resistance = 30^0

Unit weight of the backfill = $18 \text{ KN}/\text{m}^3$

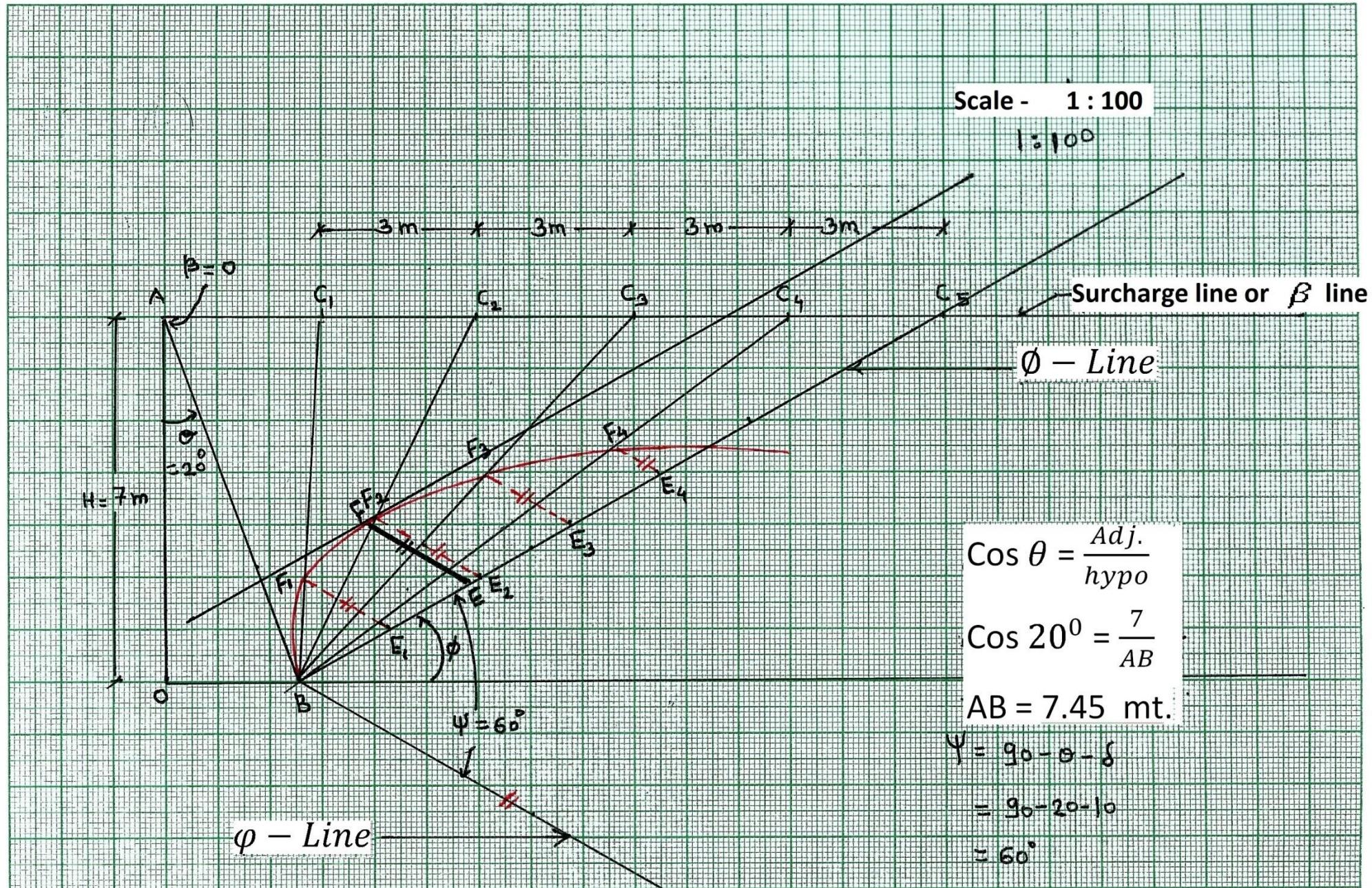
Wall friction angle = 20^0

Find the total active earth pressure by Culmann's graphical method.

Answer : Refer Culmann's graphical method procedure , page no. 11.

Wedges	Base (B) in mt.	Height (H) in mt.	Area (A) $= 1/2 \times B \times H$	Bulk unit wt. (γ) in kN/m^3	Weight (W) $= (A \times \gamma)$ in kN/m	WEIGHT IN SCALE 1:100
ABC1	3	7.45	11.175	18	201.15	$201.15/100 = 2.01 \text{ cm}$ $= BE1$
ABC2	6	7.45	22.35	18	402.3	BE 2 = 4.023 cm
ABC3	9	7.45	33.525	18	603.45	BE 3 = 6.03 cm
ABC4	12	7.45	44.70	18	804.6	BE 4 = 8.046 cm
ABC5	15	7.45	55.87	18	1005.66	BE 5 = 10.05 cm

ANSWER - Total active earth pressure $\text{Pa} = \text{length FE} \times \text{Scale}$ i.e. $\text{Pa} = 2.2 \times 100 = 220 \text{ KN/mt}$



Problem 6. Determine the active earth pressure by culmann's graphical method for retaining all of 6 m hight with vertical back . the backfill is inclined at an angle of 10^0 upward from the top of retaining wall . the backfill has an unit weight 18.2 KN/m^3 Angle of internal friction 33^0 and wall friction of 20^0 carrying line load of 33.34 KN/m .

Answer : Refer Culmann's graphical method , page no. 10.

Wedges	Base (B) in mt.	Height (H) in mt.	Area (A) $= 1/2 \times B \times H$	Bulk unit wt. (γ) in kN/m	Weight (W) $= (A \times γ)$ in kN/m	Line load q	W + q	(W + q) in scale 1 : 100
ABC1	3	6	9	18.2	163.8	33.34	197	$197/100 = 1.97 \text{ cm} = BE1$
ABC2	6	6	18	18.2	327.6	33.34	360.84	BE2 = 3.6 cm
ABC3	9	6	27	18.2	491.4	33.34	524.74	BE3 = 5.247 cm
ABC4	12	6	36	18.2	655.2	33.34	688.54	BE4 = 6.88 cm
ABC5	15	6	45	18.2	819	33.34	852.35	BE5 = 8.5 cm

ANSWER - Total active earth pressure $P_a = \text{length FE} \times \text{Scale}$ i.e. $P_a = 1.1 \times 100 = 110 \text{ KN/mt}$

Scale - 1 : 100

1 : 100

Tangent (Parallel to \emptyset – line)

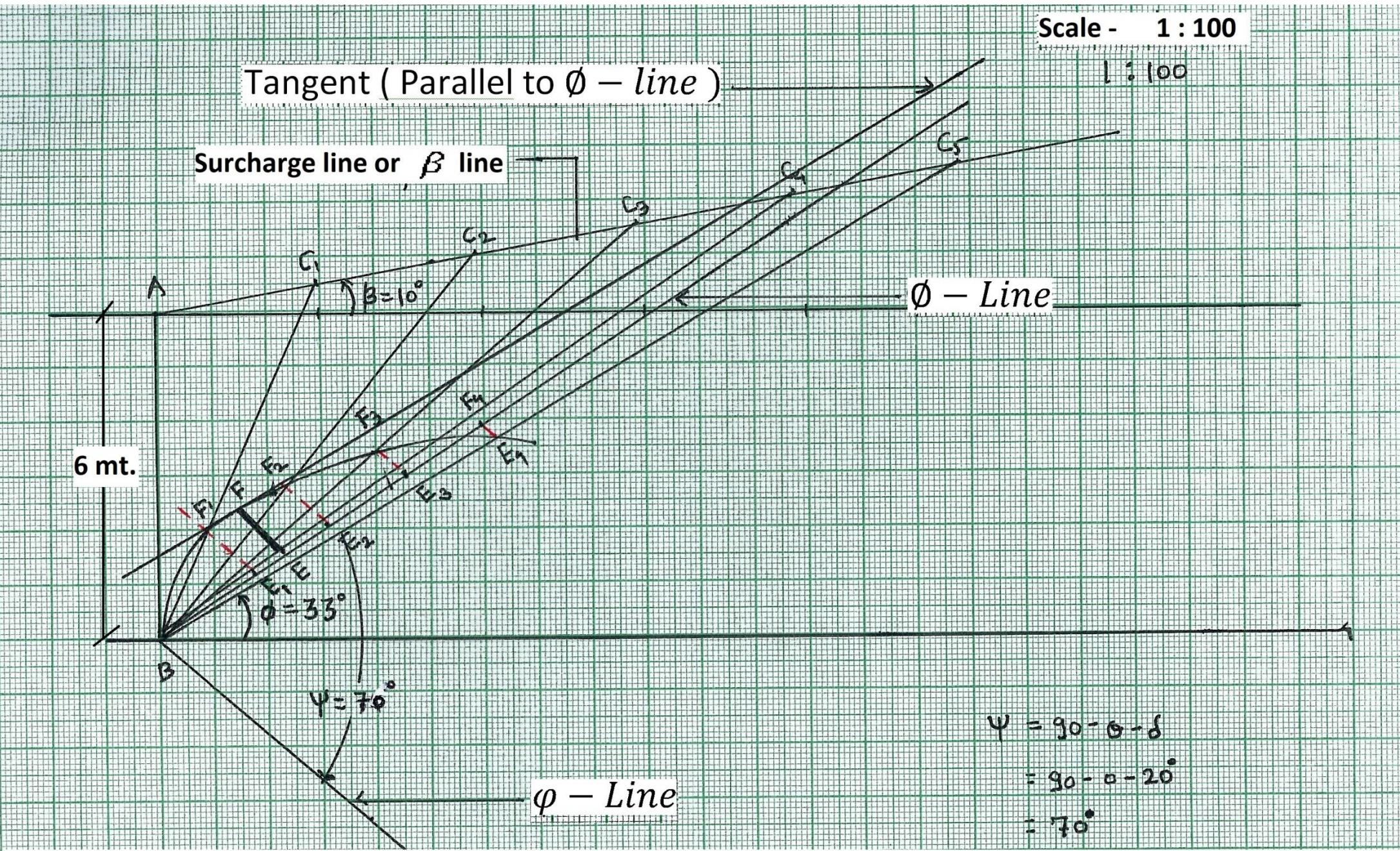
Surcharge line or β line

\emptyset – Line

6 mt.

$$\psi = 70^\circ$$

φ – Line



Problem 7. A retaining wall of 7 m ht. With batten angle 20^0 supports backfill . the backfill inclined at an angle of 10^0 Upward and carrying line load of 100 KN at a distance of 3 mt. Find the safe distance of footing from the face of the wall so that there is no increase in lateral pressure on wall due to construction of footing . properties of backfill is

$$\gamma = 18 \text{ } KN/m^3 \quad \phi = 30^0 \quad \delta = 10^0$$

Answer :

- 1) Draw the cross section $ABCD$ of the retaining wall to suitable scale. Let the wall face AB Retains the soil.
- 2) Draw the ϕ – line , β – line or Surcharge line and φ – line as usual which is earlier discussed earlier in Rehban's method (*Case 1*).
- 3) Take minimum five assumed (arbitrary) points say C_1, C_2, C_3, C_4, C_5 at an distance of 2 to 3 mt. on β – line or Surcharge line . Join all these points with B .
- 4) Now, Soil wedges $ABC_1, ABC_2, ABC_3, ABC_4$ and ABC_5 will be form with base width of AC_1, AC_2, AC_3, AC_4 and AC_5 respectively. Find out weight of all soil wedges say W_1, W_2, W_3, W_4 and W_5 resp. by using table as below.

Wedge s	Base (B) in mt.	Height (H) in mt.	Area (A) $= 1/2 \times B \times H$	Bulk unit wt. (γ) in kN/m^3	Weight (W) $= (A \times \gamma)$ in kN/m
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- 5) Here, the Surcharge load $q = 33.34 \text{ } kN/m$ exist, then the above table can be modified as below.

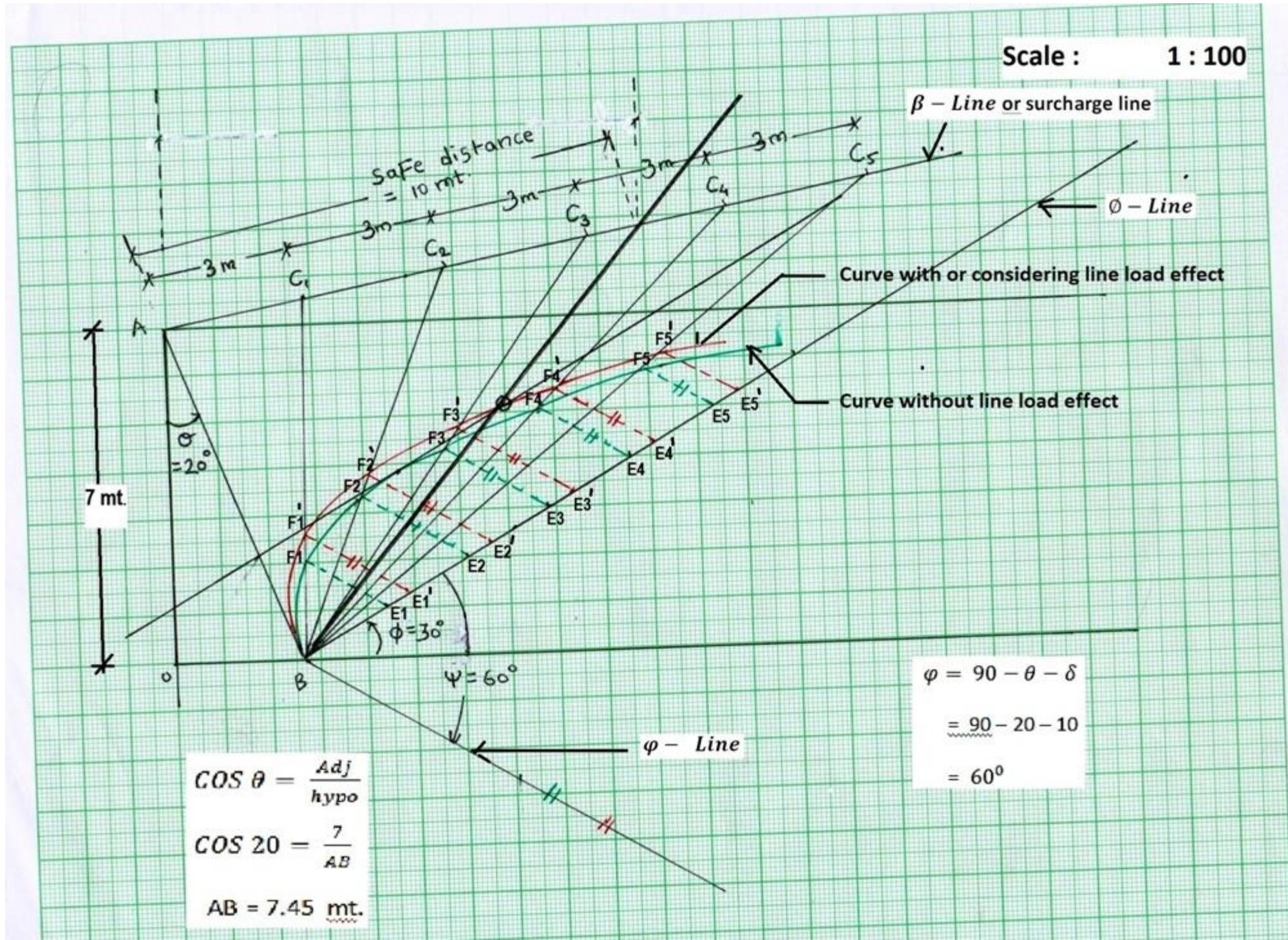
Wedges	Base (B)	Heig ht (H)	Area (A)	γ in kN/m^3	$w = (A \times \gamma)$ in kN/m	q in kN/m	Weight (W) $= (w + q)$ in kN/m
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- 6) Plot weight of all these wedges as $W_1 = BE_1, W_2 = BE_2, W_3 = BE_3, W_4 = BE_4$ and $W_5 = BE_5$ by converting it in selected scale on ϕ – line. So, here, we found E_1, E_2, E_3, E_4 and E_5 on ϕ – line .
- 7) Draw parallel line to φ – line from E_1, E_2, E_3, E_4 and E_5 such that it will further intersects slip planes BC_1, BC_2, BC_3, BC_4 and BC_5 at F_1, F_2, F_3, F_4 and F_5 respectively. So, here, we found F_1, F_2, F_3, F_4 and F_5 .

- 8) Draw smooth Curve through points B, F_1, F_2, F_3, F_4 and F_5 which is called as Culmann's curve without line load effect .
- 9) Here the line load effect 33.34 KN/m . for this, follow table which we have already seen in point 5 and find out weight.
- 10) Follow same above procedure for this line load effect value such that we found $E1', E2' E3' E4' E5'$ and $F1', F2' F3' F4' F5'$
- 11) Draw smooth Curve through points $B, F1', F2' F3' F4' F5'$ which is with line load effect .
- 12) Draw a tangent (parallel to \emptyset – line) to culman's curve (without line load effect.). and it will further intersect to culman's curve (with line load effect.) Say point **S**.
- 13) From B , Draw the line passes through point S such that it intersect β – Line or surcharge line.
- 14) Now, Find out the safe distance from face of the wall = 10 mt. - **ANSWER**

Wedges	Base (B) in mt.	Height (H) in mt.	Area (A) $= 1/2 \times B \times H$	Bulk unit wt. (γ) in kN/m^3	Weight (W) $= (A \times \gamma)$ in kN/m	WEIGHT IN SCALE 1:100	q Line load	$W + q$	WEIGHT WITH LINE LOAD IN SCALE 1:100
ABC1	3	7.45	11.175	18.2	203.38	$201.15 / 100$ $= 2.03 \text{ cm} = BE1$	$100 / 3$ $= 33.34$	236.72	$BE 1' = 2.36$
ABC2	6	7.45	22.35	18.2	406.77	$BE 2 = 4.06 \text{ cm}$	33.34	440.11	$BE 2' = 4.40$
ABC3	9	7.45	33.525	18.2	610.15	$BE 3 = 6.10 \text{ cm}$	33.34	643.50	$BE 3' = 6.43$
ABC4	12	7.45	44.70	18.2	813.54	$BE 4 = 8.13 \text{ cm}$	33.34	846.88	$BE 4' = 8.46$
ABC5	15	7.45	55.87	18.2	1016.83	$BE 5 = 10.16 \text{ cm}$	33.34	1050.17	$BE 5' = 10.50$

Answer : Safe distance of footing from the face of the wall so that there is no increase in lateral pressure on wall due to construction of footing = 10 mt.

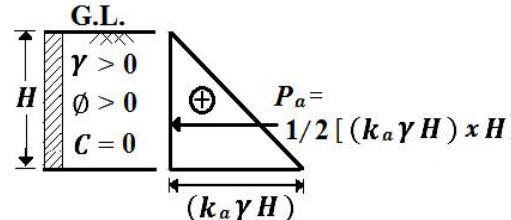


Rankine's Earth Pressure Theory (Analytical Approach) :

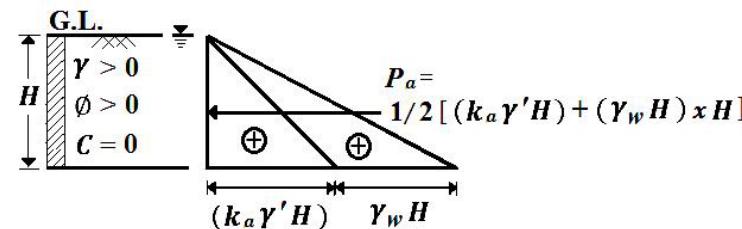
Case 1] Active Earth Pressure (P_a) and Passive Earth Pressure (P_p) for Cohesion less soil ($C = 0$) :

a) When there is no Interfaces (Soil Layers) :

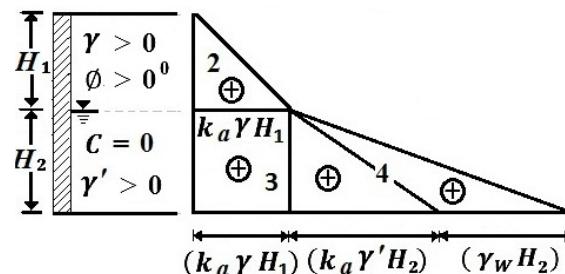
i) Dry Backfill :



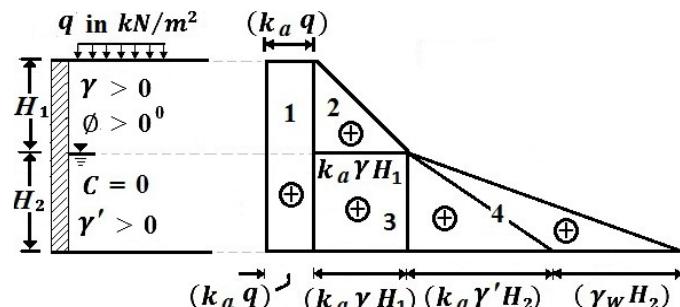
ii) Water Table is at Ground Surface (Submerged Backfill) :



iii) Water Table is in between (Partially Submerged Backfill) :

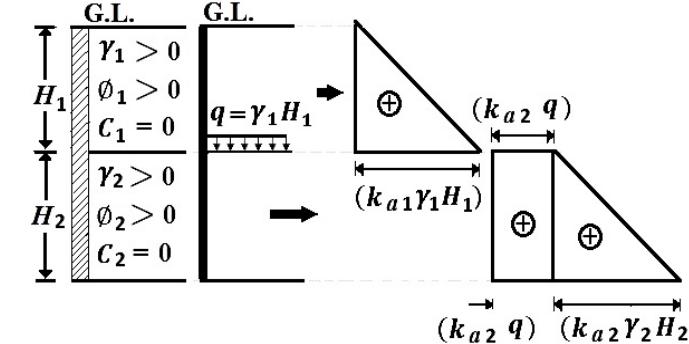


iv) Surcharge Load (q) in any Case : Due to Surcharge Loading, Extra rectangle of Intensity ($k_a x q$) is added in following way.

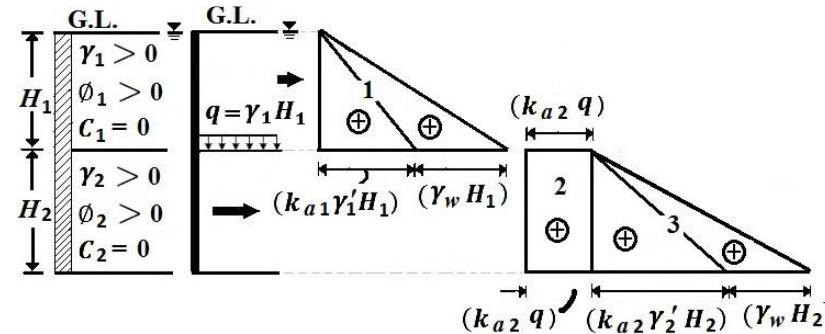


b) When there is soil Interfaces (Soil Layers) :

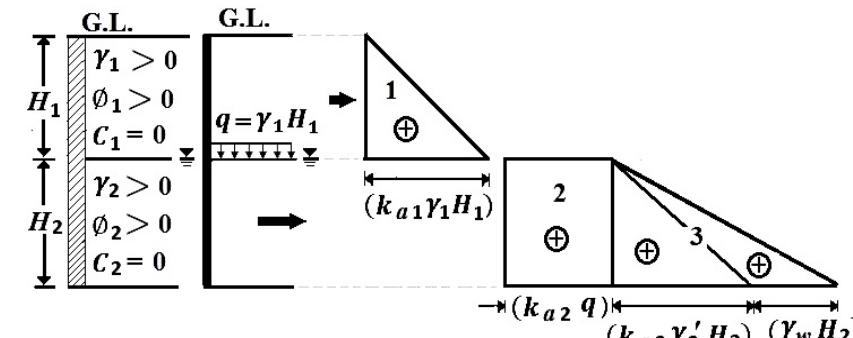
i) Dry Backfill :



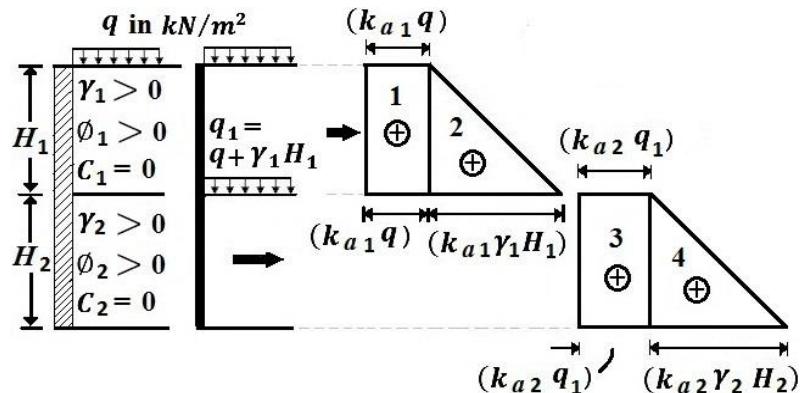
ii) Water Table is at Ground Surface (Submerged Backfill) :



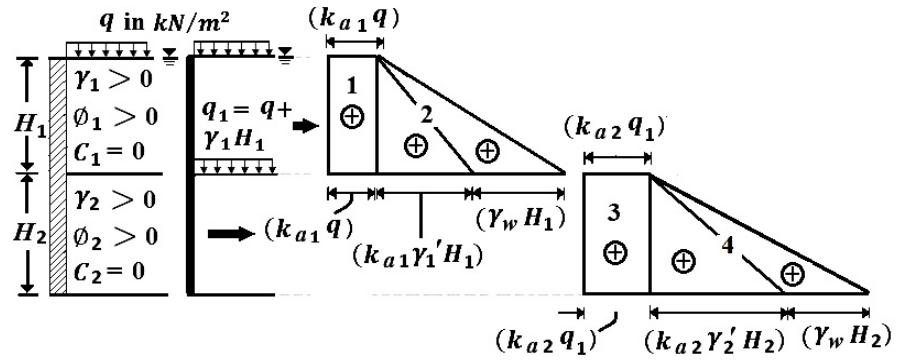
iii) Water Table is located at Interfaces between two layers :



iv) Surcharge Load (q) at Ground Surface :



v) Water Table is located at Ground Surface with Surcharge Load (q):



Problem 1.1 Determine Rankine's Active Earth Pressure (P_a) of 6 m High Retaining wall for Cohesion less Backfill having unit weight as 18 kN/m^3 and Angle of Internal Friction (Shearing Resistance) as $\phi = 30^\circ$. Also determine the Change in Active earth pressure when there is surcharge load of 24 kN/m^2 at Ground Level. ?

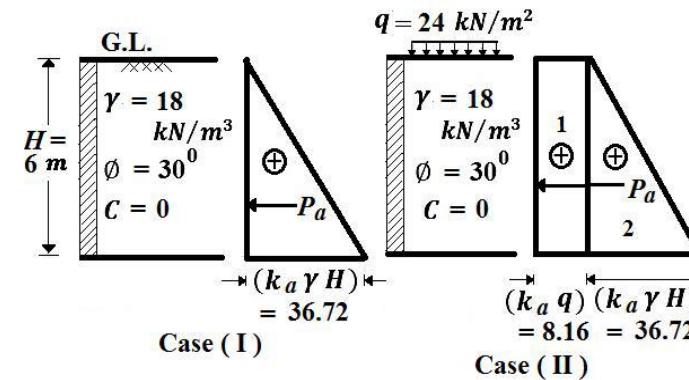
Solution :

Given ; $\phi = 30^\circ$; $\gamma = 18 \text{ kN/m}^3$; $H = 6 \text{ m}$

Coefficient of Active earth pressure

$$(K_a) = 1 - \sin(\phi) / 1 + \sin(\phi)$$

$$= 1 - \sin(30^\circ) / 1 + \sin(30^\circ) = 0.34$$



Case 1) Dry Backfill :

$$(K_a \times \gamma \times H) = (0.34 \times 18 \times 6) = 36.72 \text{ kN/m}$$

Active Earth Pressure P_{p1} (considering +ve portion only)

$$P_{p1} = 1/2 \times \text{Base} \times \text{height} = [1/2 \times 36.72 \times 6] = 110.16 \text{ kN/m}$$

Case 2) Dry Backfill with Surcharge at GL :

Due to Surcharge Loading ($q = 24 \text{ kN/m}^2$), Extra rectangle of Intensity ($k_a q$) is added in the Pressure diagram as shown in Figure.

$$\therefore (K_a \times q) = (0.34 \times 24) = 8.16; \quad (K_a \times \gamma \times H) = (0.34 \times 18 \times 6) = 36.72$$

$$\text{Active Earth Pressure for Figure 1 } (P_1) = (8.16 \times 6) = 48 \text{ kN/m}$$

$$\text{Active Earth Pressure for Figure 2 } (P_2) = (1/2 \times 36.72 \times 6) = 110.16 \text{ kN/m}$$

$$\text{Total Active Earth Pressure } (P_{a2}) = (110.16 + 48) = 158 \text{ kN/m}$$

Change in Active Earth Pressure between case 1 and case 2

$$P_{a\text{change}} = 158 - 110.16 = 48 \text{ kN/m}$$

Problem 1.2 Determine the Change in Passive earth pressure in Problem 1.1

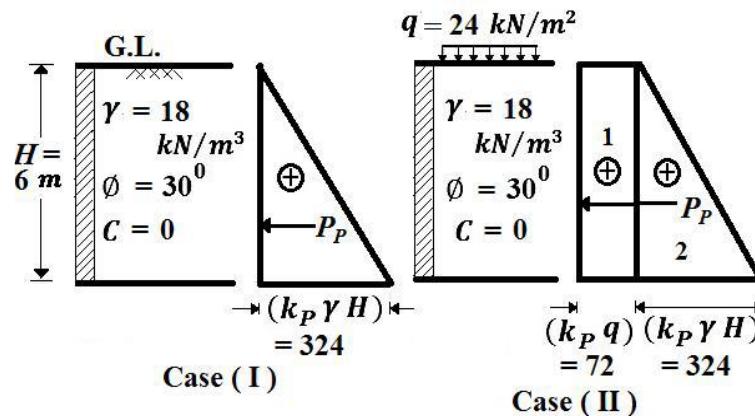
Solution :

Given ; $\phi = 30^\circ$; $\gamma = 18 \text{ kN/m}^3$; $H = 6 \text{ m}$

Coefficient of Passive earth pressure

$$(K_p) = 1 + \sin(\phi) / 1 - \sin(\phi)$$

$$= 1 + \sin(30^\circ) / 1 - \sin(30^\circ) = 3.0$$



Case 1) Dry Backfill :

$$(K_p \times \gamma \times H) = (3 \times 18 \times 6) = 324 \text{ kN/m}$$

Passive Earth Pressure (P_{p1}) = Area of Pressure Figure = $1/2 \times \text{Base} \times \text{height}$
 $= 1/2 \times 324 \times 6 = 972 \text{ kN/m}$

Case 2) Dry Backfill with Surcharge at GL :

Due to Surcharge Loading ($q = 24 \text{ kN/m}^2$), Extra rectangle of Intensity ($K_p \times q$) is added in the Pressure diagram as shown in Figure.

$$\therefore (K_p \times q) = (3 \times 24) = 72; (K_p \times \gamma \times H) = (3 \times 18 \times 6) = 324$$

$$\text{Passive Earth Pressure for Figure 1 } (P_1) = (72 \times 6) = 432 \text{ kN/m}$$

$$\text{Passive Earth Pressure for Figure 2 } (P_2) = (1/2 \times 324 \times 6) = 972 \text{ kN/m}$$

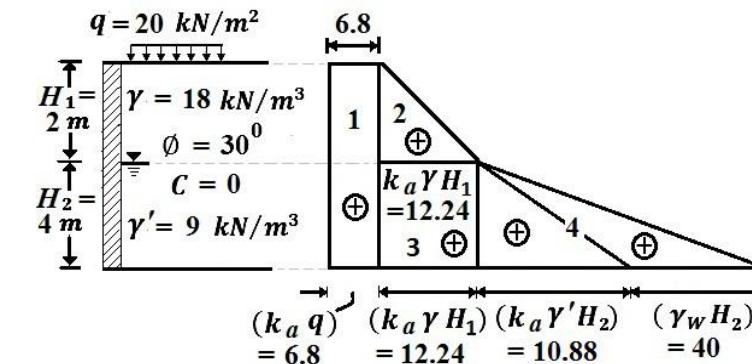
$$\text{Total Passive Earth Pressure } (P_p) = (432 + 972) = 1404 \text{ kN/m}$$

Change in Passive Earth Pressure in between case 1 and case 2

$$P_{p_change} = (1404 - 972) = 432 \text{ kN/m}$$

Problem 1.3 A retaining wall 6 m height pertains cohesion less Backfill with unit weight of 18 kN/m^3 and the angle of shearing Resistance as 30° . The water table is at 2 m from ground surface (4 m from base of the wall) with Surcharge load of 20 kN/m^2 . Determine Rankine's Active Earth Pressure (P_a) per meter length of the wall . Also determine location of the line of action of Resultant.?

Solution :



Coefficient of Active earth pressure (K_a) = $1 - \sin(\phi)/1 + \sin \phi$

$$K_a = 1 - \sin(30^\circ)/1 + \sin(30^\circ) = 0.34$$

Due to Surcharge Loading ($q = 20 \text{ kN/m}^2$),

Extra rectangle of Intensity ($K_a \times q$) = $(0.34 \times 20) = 6.8 \text{ kN/m}$
 is added in the Pressure diagram as shown in Figure.

$$(K_a \times \gamma \times H_1) = (0.34 \times 18 \times 2) = 12.24 \text{ kN/m}$$

$$(K_a \times \gamma' \times H_2) = (0.34 \times 8 \times 4) = 10.88 \text{ kN/m}$$

$$\text{Submerged unit weight } (\gamma') = (\gamma - \gamma_w) = (18 - 10) = 8 \text{ kN/m}^3$$

$$\text{Where, } \gamma_w - \text{Unit weight of water} = 9.81 \approx 10 \text{ kN/m}^3$$

$$\text{Pore Water Pressure } (\gamma_w \times H_2) = (10 \times 4) = 40 \text{ kN/m}$$

$$\text{Active Earth Pressure for Figure 1 } (P_1) = (6.8 \times 6) = 40.8 \text{ kN/m}$$

$$\text{acting at distance of } (Y_1) = (H/2) = (6/2) = 3 \text{ m from base.}$$

$$\text{Active Earth Pressure for Figure 2 } (P_2) = (1/2 \times 12.24 \times 2) = 12.24 \text{ kN/m}$$

$$\text{acting at distance of } (Y_2) = 4 + (H_1/3) = 4 + (2/3) = 0.67 \text{ m from base.}$$

$$\text{Active Earth Pressure for Figure 3 } (P_3) = [12.24 \times 4] = 49 \text{ kN/m}$$

$$\text{acting at distance of } (Y_3) = (H_2/2) = (4/2) = 2 \text{ m from base}$$

$$\text{Active Earth Pressure for Figure 4 } (P_4) = [1/2 \times (10.88 + 40) \times 4] = 101.76 \text{ kN/m}$$

$$\text{acting at distance of } (Y_4) = (H_2/3) = (4/3) = 1.34 \text{ m from base}$$

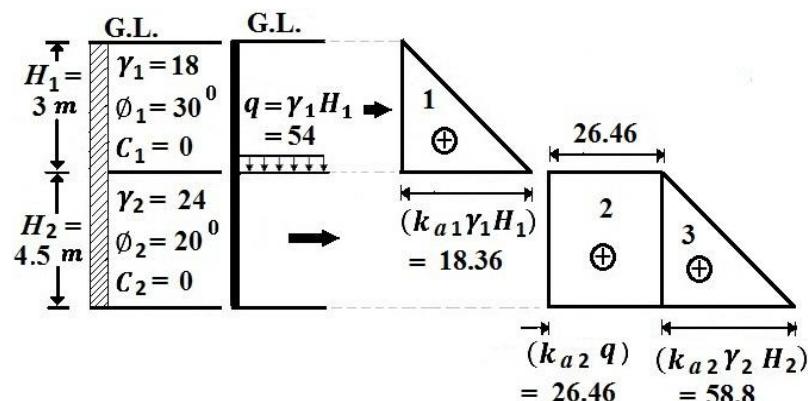
$$\text{Total Active Earth Pressure } (P_a) = (P_1 + P_2 + P_3 + P_4) = 203.8 \text{ kN/m}$$

$$\begin{aligned} \text{Acting at } Y &= (P_1 Y_1 + P_2 Y_2 + P_3 Y_3 + P_4 Y_4) / P_a = \\ &[(40.8 \times 3) + (12.24 \times 0.67) + (49 \times 2) + (101.76 \times 1.34)] / 203.8 = \\ &= (365 / 203.8) = 1.8 \text{ m} \end{aligned}$$

Hence The total Active Earth Pressure (Resultant) $P_a = 203.8 \text{ kN/m}$ will be act at distance of 1.8 m from the base of the Retaining wall.

Problem 1.4. Determine the active earth pressure and point of application of pressure on 7.5 m high retaining wall which retains a cohesion less backfill. The top 3 m of the fill has a unit weight of 18 kN/m^3 and $\phi = 30^\circ$ and rest has unit weight 24 kN/m^3 and $\phi = 20^\circ$. [SGBAU,W-15/7 m]

Solution :



Calculations for Top layer :

Coefficient of Active earth pressure

$$\begin{aligned} (K_a)_1 &= 1 - \sin(\phi_1) / 1 + \sin(\phi_1) \\ &= 1 - \sin(30^\circ) / 1 + \sin(30^\circ) = 0.34 \end{aligned}$$

$$(K_a \gamma_1 H_1) = (0.34 \times 18 \times 3) = 18.36 \text{ kN/m}$$

Calculations for Bottom layer :

$$K_a_2 = 1 - \sin(\phi_2) / 1 + \sin(\phi_2)$$

$$= 1 - \sin(20^\circ) / 1 + \sin(20^\circ) = 0.49$$

Due to top layer, The Surcharge Pressure will be developed on the Bottom layer in the form of UDL (q).

$$\text{Hence, } q = \gamma_1 H_1 = (18 \times 3) = 54 \text{ kN/m}^2$$

$$\text{Due to this Surcharge Loading (} q\text{), Extra rectangle of Intensity (} K_a_2 \times q\text{) }= (0.49 \times 54) = 26.46 \text{ kN/m}$$

is added in the Pressure diagram as shown in Figure.

$$(K_a_2 \gamma_2 H_2) = (0.49 \times 24 \times 4.5) = 58.8 \text{ kN/m}$$

$$\begin{aligned} \text{Active Earth Pressure for Figure 1 } (P_1) &= 1/2 (18.36 \times 3) = 27.54 \text{ kN/m} \\ \text{acting at distance of } (Y_1) &= 4.5 + (H_1/3) = 4.5 + (3/2) = 6 \text{ m from base.} \end{aligned}$$

$$\begin{aligned} \text{Active Earth Pressure for Figure 2 } (P_2) &= (26.46 \times 4.5) = 119.07 \text{ kN/m} \\ \text{acting at distance of } (Y_2) &= (H_2/2) = (4.5/2) = 2.25 \text{ m from base} \end{aligned}$$

$$\begin{aligned} \text{Active Earth Pressure for Figure 3 } (P_3) &= [1/2 \times 58.8 \times 4.5] = 132.3 \text{ kN/m} \\ \text{acting at distance of } (Y_3) &= (H_2/3) = (4.5/3) = 1.5 \text{ m from base} \end{aligned}$$

$$\text{Total Active Earth Pressure } (P_a) = (P_1 + P_2 + P_3) = 278.9 \text{ kN/m}$$

$$\begin{aligned} \text{Acting at } Y &= (P_1 Y_1 + P_2 Y_2 + P_3 Y_3) / P_a \\ &= [(27.54 \times 6) + (119.07 \times 2.25) + (132.3 \times 1.5)] / 278.9 = 2.464 \text{ m} \end{aligned}$$

Hence The total Active Earth Pressure (Resultant) $P_a = 278.9 \text{ kN/m}$ will be act at distance of 2.464 m from the base of the Retaining wall.

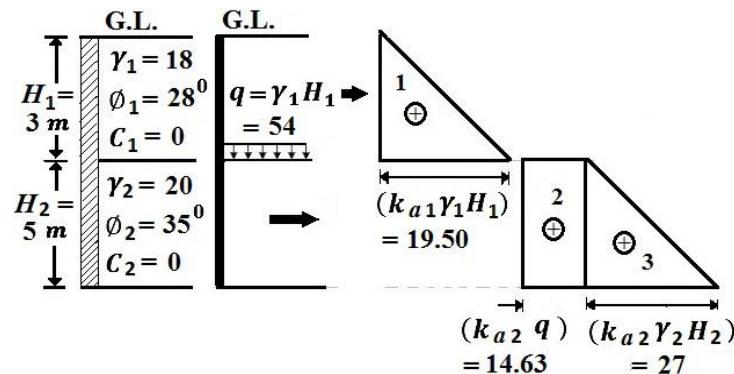
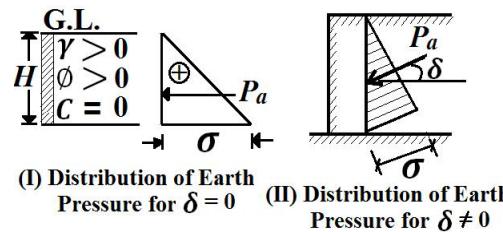
Problem 1.5. A vertical retaining wall retains backfill up to top level. The backfill is cohesion less and having the properties as :

- The top 3 m backfill is sand having $C_1 = 0$, $\phi_1 = 28^\circ$, $\gamma_1 = 18 \text{ kN/m}^3$
- The rest 5.0 m backfill having $C_2 = 0$, $\phi_2 = 35^\circ$, $\gamma_2 = 20 \text{ kN/m}^3$

Compute the lateral thrust per unit run of wall neglecting frictional force on back of wall. [SGBAU,W-18/7 m]

Solution : The magnitude of active or passive earth pressure, respectively depends upon friction between the soil and the back of wall and by the

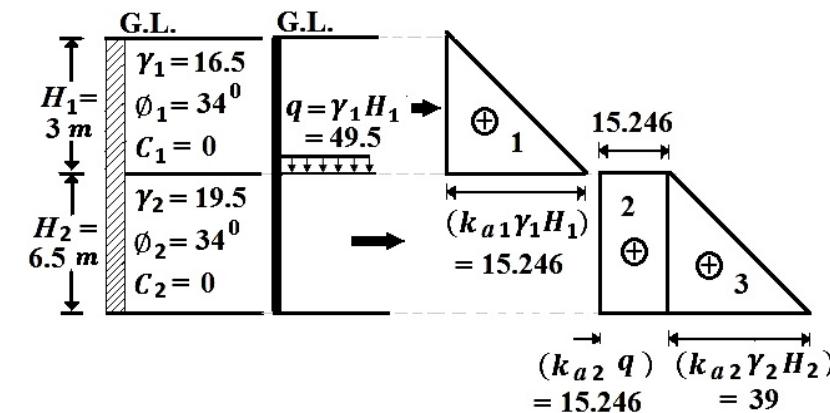
adhesion of soil to the structure face represented by the angle δ . If $\delta = 0$ then the pressure σ acts in the direction normal to the back of wall. With increasing value of δ , Active earth pressure decreases.



Similar to Problem 1.4. For further Calculations, Refer the Problem 1.4.

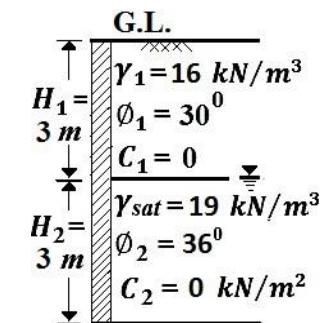
Problem 1.6. A retaining wall 9.5 m high retains a cohesion less soil with an angle of internal friction 34° . The surface of level with the top of the wall. The unit weight of the top 3 m of the fill is 16.5 kN/m^3 and that of the rest is 19.5 kN/m^3 . Calculate the magnitude and application of the resultant active thrust.
[SGBAU,S-14/7 m]

Solution :

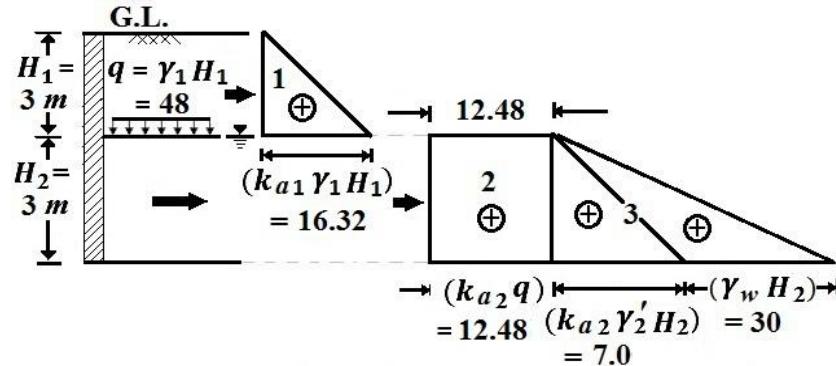


Similar to Problem 1.4. For further Calculations, Refer the Problem 1.4.

Problem 1.7. Refer the Figure assume that the wall can yield sufficiently and Determine Rankine's Active Earth Pressure per meter length of the wall . Also determine location of the line of action of Resultant ?
[SGBAU; W-16/7 m]



Solution :



Calculations for Top layer :

Coefficient of Active earth pressure

$$(K_a)_1 = 1 - \sin(\phi_1)/1 + \sin(\phi_1) \\ = 1 - \sin(30^\circ)/1 + \sin(30^\circ) = 0.34$$

$$(K_a_1 \times \gamma_1 \times H_1) = (0.34 \times 16 \times 3) = 16.32 \text{ kN/m}$$

Calculations for Bottom layer :

$$K_a_2 = 1 - \sin(\phi_2)/1 + \sin(\phi_2) = 1 - \sin(36^\circ)/1 + \sin(36^\circ) = 0.26$$

Due to top layer, The Surcharge Pressure will be developed on the Bottom layer in the form of UDL (q).

$$\text{Hence, } q = \gamma_1 H_1 = (16 \times 3) = 48 \text{ kN/m}^2$$

$$\text{Due to this Surcharge Loading (}q\text{), Extra rectangle of Intensity (}K_a_2 \times q\text{)} = (0.26 \times 48) = 12.48 \text{ kN/m}$$

is added in the Pressure diagram as shown in Figure . ;

$$(K_a_2 \times \gamma_2' \times H_2) = (0.26 \times 9 \times 3) = 7 \text{ kN/m}$$

$$\text{Submerged unit weight (} \gamma_2' \text{)} = (\gamma_2 - \gamma_w) = (19 - 10) = 9 \text{ kN/m}^3$$

$$\text{Where, } \gamma_w - \text{Unit weight of water} = 9.81 \approx 10 \text{ kN/m}^3$$

$$\text{Pore Water Pressure; (} \gamma_w \times H_2 \text{)} = (10 \times 3) = 30 \text{ kN/m}$$

$$\text{Active Earth Pressure for Figure 1 (}P_1\text{)} = (1/2 \times 16.32 \times 3) = 24.50 \text{ kN/m} \\ \text{acting at distance of (}Y_1\text{)} = 3 + (H_1/3) = 3 + (3/3) = 4 \text{ m from base.}$$

Active Earth Pressure for Figure 2 (P_2) = (12.48×3) = 37.44 kN/m

acting at distance of (Y_2) = ($H_2/2$) = ($3/2$) = 1.5 m from base

Active Earth Pressure for Figure 3 (P_3) = [$1/2 \times (7 + 30) \times 3$] = 55.5 kN/m

acting at distance of (Y_3) = ($H_2/3$) = ($3/3$) = 1 m from base

Total Active Earth Pressure (P_a) = ($P_1 + P_2 + P_3$) = 117.44 kN/m

$$\text{Acting at } Y = (P_1 Y_1 + P_2 Y_2 + P_3 Y_3) / P_a$$

$$= [(24.50 \times 4) + (37.44 \times 1.5) + (55.5 \times 1)] / 117.44 = 1.785 \text{ m}$$

Hence The total Active Earth Pressure (Resultant) $P_a = 117.44 \text{ kN/m}$

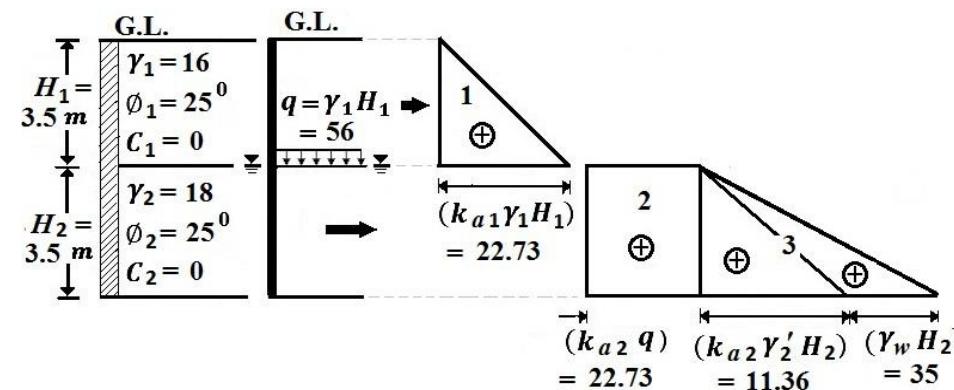
will be act at distance of 1.785 m from the base of the Retaining wall.

Problem 1.8. Determine the magnitude and point of application of active earth pressure for a 7.0 m high retaining wall. The back fill is considering of two layers of equal height. The upper layer is filled with soil of 16.0 kN/m^3 unit weight and angle of internal friction 25° with saturated density of 18.0 kN/m^3 . Ground water table is 3.5 m from top of the wall .

[SGBAU,W-18/7m]

Solution : As We Know basic Properties of Soil as :

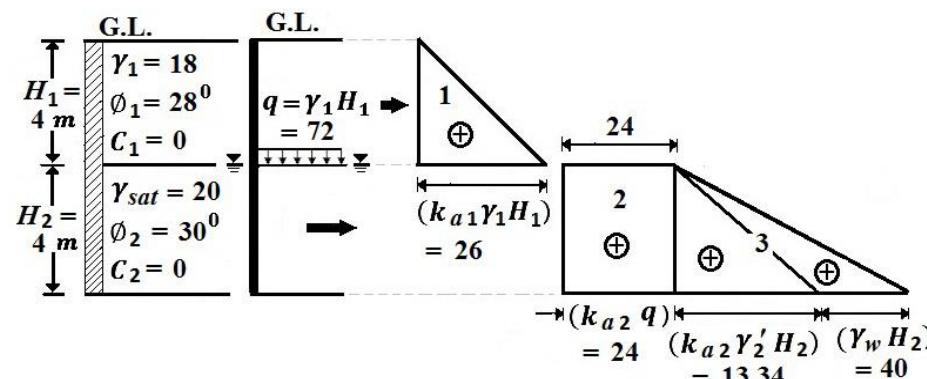
For Cohesive soil, $\phi = 0^\circ$, $C > 0$ and For Cohesion less soil, $\phi > 0$, $C = 0$. In this Problem, for Both top and bottom layer, We can observe that Values of Cohesion (C) is not Given , But $\phi > 0$. Hence the Backfill must be Cohesion less i.e. $C = 0$.



Similar to Problem 1.7. For further Calculations, Refer the Problem 1.7.

Problem 1.9. Determine the magnitude and point of application of active earth pressure for 8 m high retaining wall. The back fill consist of two layers of equal height. The upper layer is filled with soil of 18 kN/m³ unit weight and 28° angle of internal friction. The lower layer has soil with 30° angle of internal friction with 20 kN/m² saturated density. Ground water table is at 4 m from top of wall. [SGBAU,W-19/7 m]

Solution :



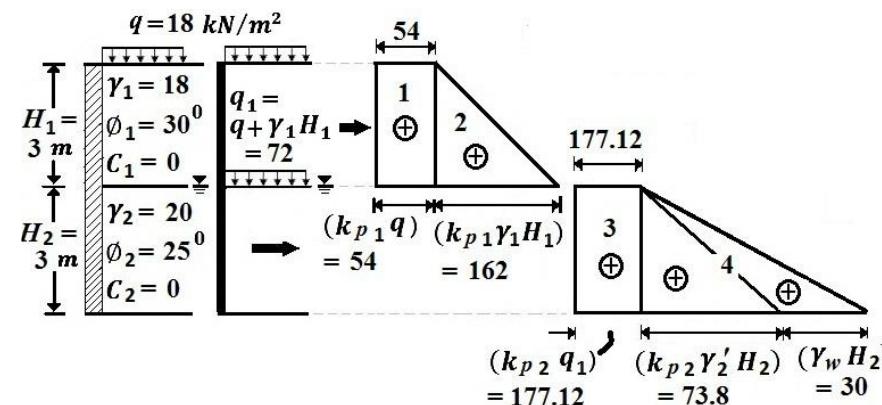
Similar to Problem 1.7. For further Calculations, Refer the Problem 1.7.

Problem 2.0. Determine Rankine's Passive Earth Pressure per meter length for 2 layered Backfill behind 6 m high Retaining wall with smooth vertical Back

. Also determine location of the line of action of Resultant if the Ground Carries surcharge load of 18 kN/m². The water table is at 3 m below Ground Surface.

Layer	Height in mt	Unit Wt (γ) kN/m ³	Cohesion (C) kN/m ²	ϕ
I (Top)	3	18	0	30°
II (Bottom)	3	20	0	25°

Solution :



Calculations for Top layer :

$$\text{Coefficient of Passive earth pressure } (K_p)_1 = 1 + \sin(\phi_1)/1 - \sin(\phi_1) \\ = 1 + \sin(30^\circ)/1 - \sin(30^\circ) = 3.0$$

Due to this Surcharge Loading (q), Extra rectangle of Intensity ($K_p_1 \times q$) is added in the Pressure diagram as shown in Figure.

$$\therefore (K_p_1 \times q) = (3 \times 18) = 54 \text{ kN/m} \\ (K_p_1 \gamma_1 H_1) = (3 \times 18 \times 3) = 162 \text{ kN/m}$$

Passive Earth Pressure for Figure 1 (P_1) = (54×3) = 162 kN/m
acting at distance of (H_1) = $3 + (H_1/2) = 3 + (3/2) = 4.5$ m from base.

Passive Earth Pressure for Figure 2 (P_2) = ($1/2 \times 162 \times 3$) = 243 kN/m

acting at distance of (Y_2) = $3 + (H_1/3) = 3 + (3/3) = 4.0$ m from base.

Calculations for Bottom layer :

$$Kp_2 = 1 + \sin(\phi)/1 - \sin(\phi) = 1 + \sin(25^\circ)/1 - \sin(25^\circ) = 2.46$$

Pressure Exerted by Top Layer ($\gamma_1 \times H_1$) + Surcharge (q), will be act on the Bottom layer in the form of UDL (q_1)

$$\therefore q_1 = q + (\gamma_1 \times H_1) = 18 + (18 \times 3) = 72 \text{ kN/m}^2$$

$$\text{Submerged unit weight } (\gamma'_2) = (\gamma_2 - \gamma_w) = (20 - 10) = 10 \text{ kN/m}^3$$

$$\text{Where, } \gamma_w - \text{Unit weight of water} = 9.81 = 10 \text{ kN/m}^3$$

$$(Kp_2 \times \gamma'_2 \times H_2) = (2.46 \times 10 \times 3) = 73.8 \text{ kN/m}$$

$$(Kp_2 \times q_1) = (2.46 \times 72) = 177.12 \text{ kN/m}$$

$$\text{Pore Water Pressure, } (\gamma_w H_2) = (10 \times 3) = 30 \text{ kN/m}$$

$$\text{Passive Earth Pressure for Figure 3 } (P_3) = (177.12 \times 3) = 531.36 \text{ kN/m}$$

$$\text{acting at distance of } (Y_3) = (H_2/2) = (3/2) = 1.5 \text{ m from base.}$$

$$\begin{aligned} \text{Passive Earth Pressure for Figure 4 } (P_4) &= [1/2 \times (73.8 + 30) \times 3] \\ &= 155.70 \text{ kN/m} \end{aligned}$$

$$\text{acting at distance of } (Y_4) = (H_2/3) = (3/3) = 1.0 \text{ m from base.}$$

$$\text{Total Passive Earth Pressure } (P_p) = (P_1 + P_2 + P_3 + P_4) = 1092 \text{ kN/m}$$

$$\text{Acting at } Y = (P_1 Y_1 + P_2 Y_2 + P_3 Y_3 + P_4 Y_4) / P_p$$

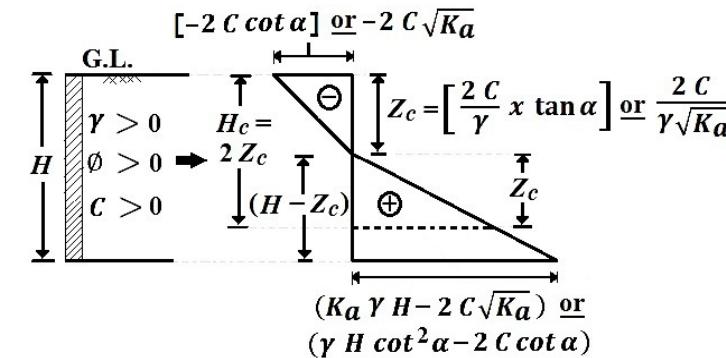
$$\begin{aligned} &= [(162 \times 4.5) + (243 \times 4) + (531.36 \times 1.5) + (155.7 \times 1)] / 1092 \\ &= [2653.74 / 1092] = 2.43 \text{ m} \end{aligned}$$

Hence The total Passive Earth Pressure (Resultant) $P_p = 1092 \text{ kN/m}$ will be act at distance of 2.43 m from the base of the Retaining wall

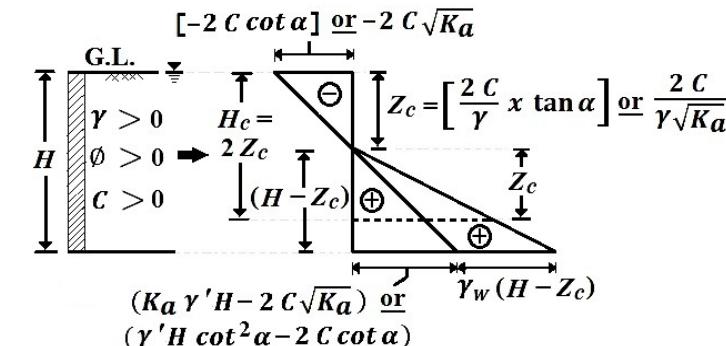
Rankin's Earth Pressure Theory :

Case 2] Active Earth Pressure for Cohesive soil ($C > 0$)

i) Dry Backfill :

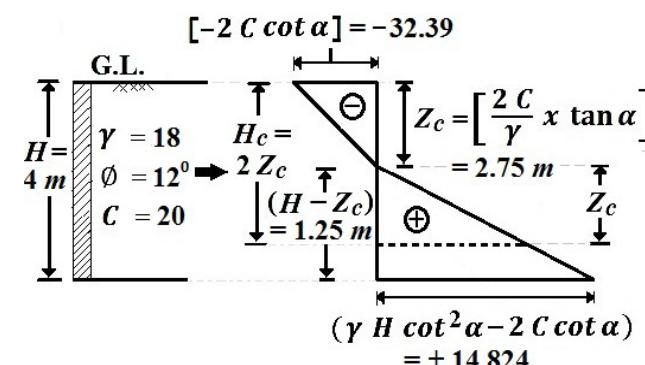


ii) Water table is located at at Ground Surface (Submerged Backfill) :



Problem 2.1. A vertical cut of 4 m depth has to be made In the soil having properties , $C = 20 \text{ kN/m}^2$, $\phi = 12^\circ$ and $\gamma = 18 \text{ kN/m}^3$. Determine lateral stresses in soil at top and Bottom of the Cut. Also determine maximum depth of Potential Cracks and the maximum depth of supported Excavation ? [RTMNU, W-14, W-08/7 m]

Solution :



$$\text{Coefficient of Active earth pressure } (K_a) = 1 - \sin (\phi)/1 + \sin (\phi) \\ = 1 - \sin (12^\circ)/1 + \sin (12^\circ) = 0.655$$

$C = 20 \text{ kN/m}^2 > 0$; hence the soil is Cohesive in nature.

$$\alpha = 45 + (\phi/2) = 45 + (12^\circ/2) = 51^\circ$$

Depth of Tension Crack or Potential Crack or Supported Excavation (Z_C):

$$Z_C = (2C/\gamma) \times \tan \alpha = [(2 \times 20)/18] \times \tan 51^\circ = 2.75 \text{ m}$$

$$\text{Lateral Stress at Top ; } -2C \cot \alpha = (-2 \times 20 \times 1/\tan 51^\circ) = \\ -32.39 \text{ kN/m}$$

$$\text{Lateral Stresses at base (Bottom Intensity) : } (\gamma H \cot^2 \alpha - 2C \cot \alpha) \\ = [(18 \times 12 \times (1/\tan 51^\circ)^2)] - 32.39 = +14.824 \text{ kN/m}$$

Problem 2.2. A Cohesive soil having , Internal angle of friction as 20° and Unit weight of backfill is 18 kN/m^3 caved after depth of digging reach 4 m . Determine Cohesion of soil which is used as Backfill against retaining wall of height 12 m ?

Solution :

$$\alpha = 45 + (\phi/2) = 45 + (20/2) = 55^\circ$$

Here , $H_C = 4 \text{ m}$ (given in problem)

where , H_C - Unsupported length or Caving Depth or Critical Height or Cut or

Depth up to

which Excavation can be done without any lateral Support .

$$\text{Again we know that, } H_C = (2 Z_C) = [(4C/\gamma) \times \tan \alpha]$$

$$\text{Equating ; } 4 = [(4 \times C)/18] \times \tan 55^\circ$$

$$4 = [(4 \times C)/18] \times \tan 55^\circ$$

$$\therefore \text{Cohesion } (C) = 12.60 \text{ kN/m}^2$$

Problem 2.3. A vertical bank was formed during the excavation of a soil having

$\phi = 15^\circ$ and unit weight of 18 kN/m^3 . When the depth of excavation reached 5.5 m , the bank failed. What was the approximate value of the cohesion of the clay ?

[SGBAU,S-15,S-19/7 m]

$$\text{Solution : } \alpha = 45 + (\phi/2) = 45 + (15^\circ/2) = 52.5^\circ$$

Here , $H_C = 5.5 \text{ m}$ (given in problem)

$$\text{We know that, } H_C = (2 Z_C) = [(4C/\gamma) \times \tan \alpha]$$

$$\text{Equating ; } 5.5 = [(4 \times C)/18] \times \tan 52.5^\circ$$

$$5.5 = [(4 \times C)/18] \times \tan 52.5^\circ$$

$$\therefore \text{Cohesion } (C) = 19 \text{ kN/m}^2$$

Problem 2.4. Calculate the depth of tension crack , unsupported height of backfill and total active Rankine thrust with point of application of 12 m high retaining wall with a smooth vertical back. The back fill is sandy 100 m with cohesion as 10 kN/m^2 , angle of internal friction 20° and unit weight 18 kN/m^3 .

[SGBAU,S-15,S-19/7 m]

Solution :

$$\alpha = 45 + (\phi/2) = 45 + (20/2) = 55^\circ$$

$$\text{Coefficient of Active earth pressure } (K_a) = 1 - \sin (\phi)/1 + \sin (\phi)$$

$$= 1 - \sin (20^\circ)/1 + \sin (20^\circ) = 0.49$$

$C = 10 \text{ kN/m}^2 > 0$; hence the soil is Cohesive in nature.

$$\text{Depth of Tension Crack or Potential Crack } (Z_C) = (2C/\gamma) \times \tan \alpha \\ = [(2 \times 10)/18] \times \tan 55^\circ = 1.6 \text{ m}$$

$$\text{Maximum depth of unsupported Excavation } (H_C) = (2 Z_C) = (2 \times 1.6) = 3.2 \text{ m}$$

$$\text{Top Intensity ; } -2C \cot \alpha = (-2 \times 10 \times 1/\tan 55^\circ) = -14 \text{ kN/m}$$

$$\text{Bottom Intensity ; } (\gamma H \cot^2 \alpha - 2C \cot \alpha)$$

$$= [(18 \times 12 \times (1/\tan 55^\circ)^2)] - 14 = +91.90 \text{ kN/m}$$

To Find Active Earth Pressure (P_a), Consider +ve Portion only

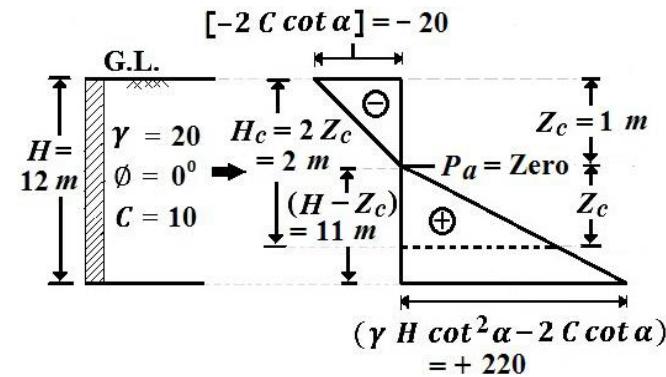
$$P_a = [1/2 \times \text{base} \times (H - Z_C)] = [1/2 \times 91.90 \times 10.4] = 477.88 \text{ kN/m}$$

Point of Application of Resultant force $P_a = 477.88 \text{ kN/m}$ acting at distance of

$$Y = (H - Z_c) / 3 = (10.4 / 3) = 3.47 \text{ m} \text{ from base of Retaining wall.}$$

Problem 2.6. The backfill of a 5 m high retaining wall is having following properties : Angle of internal friction = 30° , Cohesion = 10 kN/m^2 , unit weight = 17.5 kN/m^3 Determine the unsupported length & calculate active earth pressure (Resultant Force) on the wall and Its Point of Application ? [RTMNU, S-16/7 m]

Solution : Refer above Problem for further Calculations as the Case is same.



Problem 2.7. Determine the active earth pressure on a retaining wall with smooth vertical back, retaining cohesive soil. The height of wall 12.0 m and unit Weight of backfill is 20 kN/m^3 and cohesive is 10 kN/m^2 , also determine the depth of point of application and depth where pressure intensity is zero.

[SGBAU,W-14/7 m]

Solution :

As We Know basic Properties of Soil as :

For Cohesive soil , $\phi = 0^\circ$, $C > 0$ and For Cohesion less soil , $\phi > 0$, $C = 0$

$C = 10 \text{ kN/m}^2 > 0$; hence the soil is Cohesive in nature , $\therefore \phi = 0^\circ$

Coefficient of Active earth pressure (K_a) = $1 - \sin (0^\circ) / 1 + \sin (0^\circ) = 1$
 $\alpha = 45 + (0^\circ/2) = 45^\circ$

Depth where the Pressure Intensity is zero ($P_a = \text{zero}$) or Depth of Tension Crack or Depth Potential Crack (Z_c) = $(2C/\gamma) \times \tan \alpha$

$$= [(2 \times 10) / 20] \times \tan 45^\circ = 1 \text{ m}$$

Maximum depth of unsupported Excavation (H_c) = $(2 Z_c) = (2 \times 1) = 2 \text{ m}$

Top Intensity ; $-2 C \cot \alpha = (-2 \times 10 \times 1/\tan 45^\circ) = -20 \text{ kN/m}$

Bottom Intensity ; $(\gamma H \cot^2 \alpha - 2 C \cot \alpha)$

$$= [(20 \times 12 \times (1/\tan 45^\circ)^2)] - 20 = +220 \text{ kN/m}$$

To Find Active Earth Pressure (P_a), Consider +ve Portion only

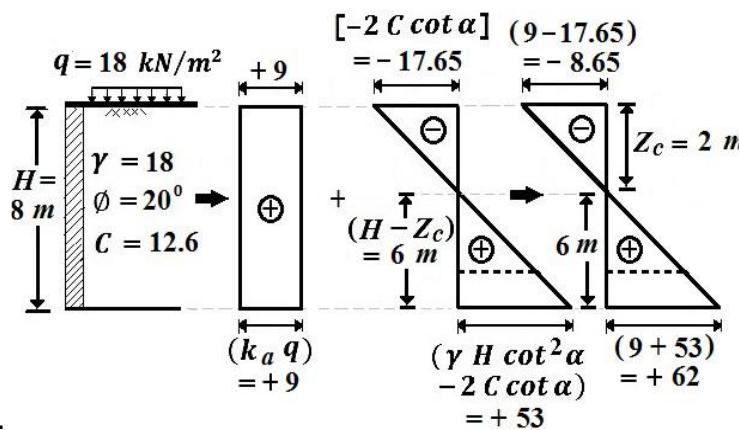
$$P_a = [1/2 \times \text{base} \times (H - Z_c)] = [1/2 \times 220 \times 11] = 1210 \text{ kN/m}$$

Point of Application of Resultant force $P_a = 1210 \text{ kN/m}$ acting at distance of

$$Y = (H - Z_c) / 3 = (11 / 3) = 3.67 \text{ m} \text{ from base of Retaining wall.}$$

Problem 2.8. A retaining wall 8 mt. High, with a smooth vertical back retains a clay backfill with Cohesion 12.6 kN/m^2 , Internal angle of friction as 20° and Unit weight of backfill is 18 kN/m^3 . Calculate the total active thrust per meter Run if the Ground Carries surcharge load of 18 kN/m^2 ?

Solution :



Coefficient of Active earth pressure

$$(K_a) = 1 - \sin(\phi)/1 + \sin(\phi) = 0.49 \text{ and } \alpha = 45 + (\phi/2) = 45 + (20/2) = 55^{\circ}$$

Due to Surcharge Loading ($q = 18 \text{ kN/m}^2$), Extra rectangle of Intensity

$$(K_a \times q) = (0.49 \times 18) = 9.0 \text{ kN/m} \text{ is added as shown in Figure.}$$

Depth of Tension Crack (Z_c) = $(2C/\gamma) \times \tan \alpha$

$$= [(2 \times 12.6) / 18] \times \tan 55^{\circ} = 2 \text{ m}$$

$$\text{Top Intensity; } -2C \cot \alpha = (-2 \times 12.6 \times 1/\tan 55^{\circ}) = -17.65 \text{ kN/m}$$

Bottom Intensity; $(\gamma H \cot^2 \alpha - 2C \cot \alpha)$

$$= [(18 \times 8 \times (1/\tan 55^{\circ})^2] - 17.65 = +53 \text{ kN/m}$$

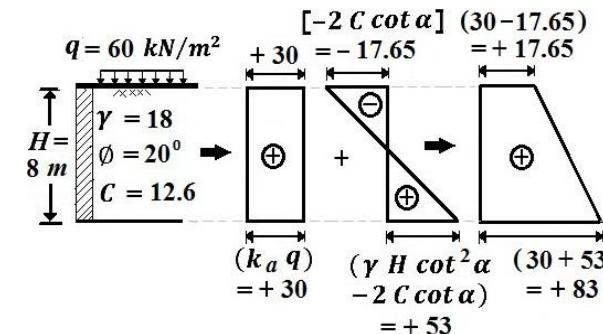
Active Earth Pressure (P_a) = Area of +ve Portion only

$$(P_a) = [1/2 \times 62 \times 6] = 186 \text{ kN/m}$$

acting at distance of (Y) = $(H - Z_c) / 3 = (6/3) = 2 \text{ m}$ from base of Retaining wall.

Problem 2.9. Solve above Problem For surcharge Load of 60 kN/m^2 ?

Solution :



Coefficient of Active earth pressure

$$(K_a) = 1 - \sin(\phi)/1 + \sin(\phi) = 0.49$$

$$\alpha = 45 + (\phi/2) = 45 + (20/2) = 55^{\circ}$$

Due to Surcharge Loading ($q = 60 \text{ kN/m}^2$),

Extra rectangle of Intensity

$$(K_a \times q) = (0.49 \times 60) = 30 \text{ kN/m}$$

is added as shown in Figure.

Top Intensity; $-2C \cot \alpha =$

$$= (-2 \times 12.6 \times 1/\tan 55^{\circ}) = -17.65 \text{ kN/m}$$

Bottom Intensity; $(\gamma H \cot^2 \alpha - 2C \cot \alpha)$

$$= [(18 \times 8 \times (1/\tan 55^{\circ})^2] - 17.65 = +53 \text{ kN/m}$$

Active Earth Pressure (P_a) = Area of +ve Portion (Trapezoid)

$$= [(a + b)/2] \times \text{height} = [(17.65 + 83)/2] \times 8 = 402.56 \text{ kN/m}$$

Acting at distance of Y

$$Y = \left[\frac{(2a + b)}{(a + b)} \right] \times \frac{H}{3} = \left[\frac{(2 \times 17.65) + 83}{(17.65 + 83)} \times (8/3) \right] = 3.14 \text{ m}$$

Hence The total Active Earth Pressure (Resultant) $P_a = 402.56 \text{ kN/m}$ will be act at distance of 3.14 m from the base of the Retaining wall.

Problem 3.0. A Retaining wall of 6 m high with Soft Saturated clay is having $C = 5 \text{ kN/m}^2$, $\phi = 0^\circ$ and $\gamma = 15.2 \text{ kN/m}^3$.

Determine :

- Maximum Depth of tension Crack.
- Active Earth Pressure before occurrence of Tension Crack. **or**
Assuming No Tension cracks developed **or** Before the formation of an tension crack.
- Active Earth Pressure after occurrence of Tension Crack. **or**
Assuming Full Tension cracks developed **or** after the formation of an tension crack. [SGBAU,W-17 /8 m]

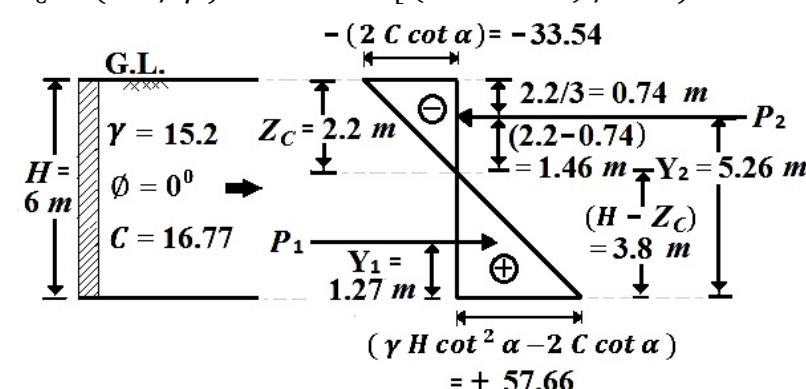
Solution :

$$\alpha = 45 + (\phi/2) = 45 + (0^\circ/2) = 45^\circ$$

$C = 16.77 \text{ kN/m}^2 > 0$; hence the soil is Cohesive in nature.

Case i) Depth of Tension Crack or Potential Crack :

$$Z_C = (2C/\gamma) \times \tan \alpha = [(2 \times 16.77)/15.2] \times \tan 45^\circ = 2.2 \text{ m}$$



Case ii) Active Earth Pressure Before occurrence of Tension Cracks

Active Pressure for +ve Portion (P_1) = $[1/2 \times 57.66 \times 3.8] = 109.55 \text{ kN/m}$

Acting at distance of (Y_1) = $(3.8)/3 = 1.27 \text{ m}$ from base of the wall.

Active Pressure for -ve Portion (P_2) = $[1/2 \times 33.54 \times 2.2] = 36.90 \text{ kN/m}$

Acting at (Y_2) = $(3.8 + 1.46) = 5.26 \text{ m}$ from base of the wall.

\therefore Active Earth Pressure before occurrence of Tension Crack (P_a)
 $= (P_1 - P_2)$

$$= (109.55 - 36.90) = 72.66 \text{ kN/m} \text{ acting at distance of } Y$$

$$(P_1 Y_1 - P_2 Y_2) / P_a = [(109.55 \times 1.27) - (36.90 \times 5.26)] / 72.66 = 0.75 \text{ m.}$$

Hence The total Active Earth Pressure (Resultant) $P_a = 72.66 \approx 73 \text{ kN/m}$ before occurrence of Tension Cracks will be act at distance of 0.75 m from the base of the Retaining wall.

Case iii) Active Earth Pressure after occurrence of Tension Cracks :

Active Earth Pressure after occurrence of Tension Cracks (P_a) = Area of +ve Portion only.

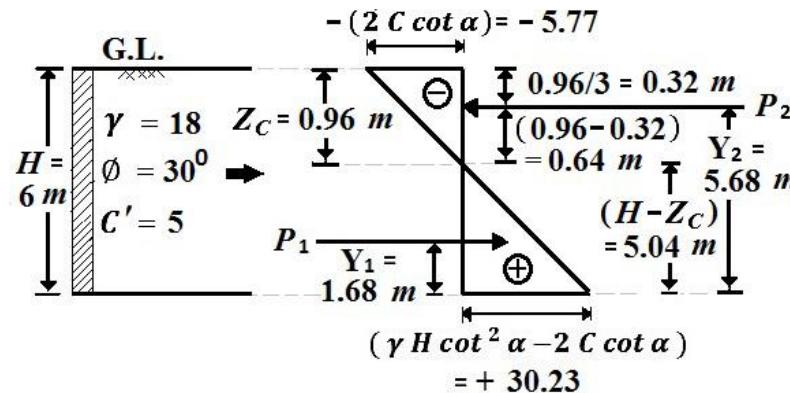
$$\therefore P_a = [1/2 \times 57.66 \times (H - Z_C)] = [1/2 \times 57.66 \times 3.8] = 109.55 \text{ kN/m} \text{ acting at distance of } (Y) = (3.8/3) = 1.27 \text{ m} \text{ from base of Retaining wall.}$$

Problem 3.1. A 6 m high Retaining wall pertains following properties ; $C' = 5 \text{ kN/m}^2$, $\phi = 30^\circ$ and $\gamma = 18 \text{ kN/m}^3$. Determine the Rankine's active earth pressure on the wall at

- Before the formation of the crack.
- After the formation of an crack.

[RTMNU, S-17/7
m]

Solution :



Problem 3.2. A Retaining wall of 10 m high with smooth vertical back retains $C - \phi$ soil with $C = 30 \text{ kN/m}^2$

and Angle of Internal Friction = 18° , Unit weight of soil is 20 kN/m^3 . Calculate :

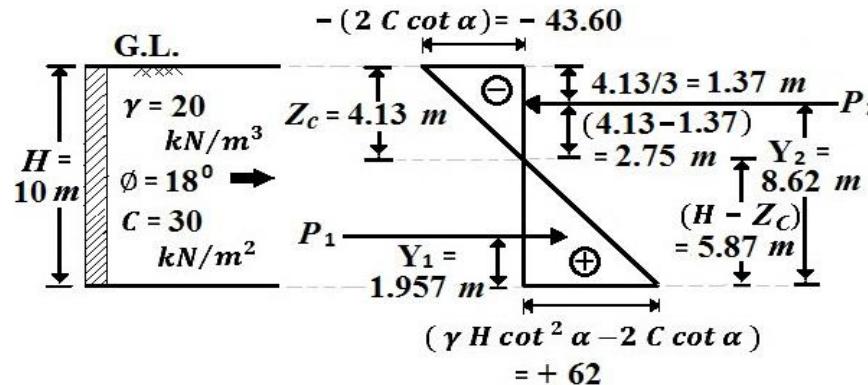
- Active Earth Pressure assuming no tension cracks developed .
- Active Earth Pressure assuming Full tension cracks developed and is Filled with water. [RTMNU, S-08/7 m]

Solution : $C = 30 \text{ kN/m}^2 > 0$; hence the soil is Cohesive in nature.

$$\alpha = 45 + (\phi/2) = 45 + (18^\circ/2) = 54^\circ$$

$$\text{Depth of Tension Crack or Potential Crack } (Z_c) = (2C/\gamma) \times \tan \alpha \\ = [(2 \times 30)/20] \times \tan 54^\circ = 4.13 \text{ m}$$

Case i) Active Earth Pressure assuming no tension cracks developed :



$$\text{Top Intensity ; } -2C \cot \alpha = (-2 \times 30 \times 1/\tan 54^\circ) = -43.60 \text{ kN/m}$$

$$\text{Bottom Intensity ; } (\gamma H \cot^2 \alpha - 2C \cot \alpha) =$$

$$= [(20 \times 10 \times (1/\tan 54^\circ)^2) - (2 \times 30 \times 1/\tan 54^\circ)] = + 62 \text{ kN/m}$$

$$\text{Active Pressure for +ve Portion } (P_1) = [1/2 \times 62 \times 5.87] \\ = 182 \text{ kN/m}$$

Acting at distance of $(Y_1) = (5.87/3) = 1.957 \text{ m}$ from base of the wall.

$$\text{Active Pressure for -ve Portion } (P_2) = [1/2 \times 43.60 \times 4.13] = \\ 90 \text{ kN/m}$$

Acting at $(Y_2) = (5.87 + 2.753) = 8.623 \text{ m}$ from base of the wall.

$$\therefore \text{Active Earth Pressure before occurrence of Tension Crack } (P_a) \\ = (P_1 - P_2)$$

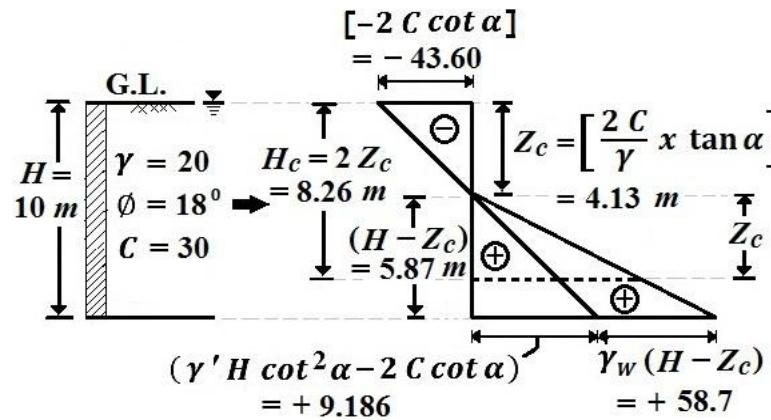
$$= (182 - 90) = 92 \text{ kN/m}$$

Acting at distance of $Y =$

$$(P_1 Y_1 - P_2 Y_2) / P_a = [(182 \times 1.957) - (90 \times 8.623)] / 92 = 4.56 \text{ m}.$$

Hence The total Active Earth Pressure (Resultant) $P_a = 72.66 \text{ kN/m}$ will be act at distance of 4.56 m from the base of the Retaining wall.

Case ii) Active Earth Pressure assuming Full tension cracks developed and is Filled with water. i.e. Water Table at Ground surface or Submerged Backfill :



$$\text{Top Intensity; } -2C \cot \alpha = (-2 \times 30 \times 1/\tan 54^\circ) = -43.60 \text{ kN/m}$$

$$\text{Bottom Intensity; } (\gamma' H \cot^2 \alpha - 2C \cot \alpha) = [(10 \times 10 \times 1/\tan 54^\circ)^2 - (2 \times 30 \times 1/\tan 54^\circ)] = +9.186 \text{ kN/m}$$

$$\text{Submerged unit weight } (\gamma') = (\gamma - \gamma_w) = (20 - 10) = 10 \text{ kN/m}^3$$

$$\text{Where, } \gamma_w - \text{Unit weight of water} = 9.81 \approx 10 \text{ kN/m}^3$$

$$\text{Pore Water Pressure; } \gamma_w (H - Z_c) = (10 \times 5.87) = 58.7 \text{ kN/m}$$

Active Earth Pressure after occurrence of tension crack (P_a) = Area of +ve Portion only. (P_a) = $1/2 \times [9.186 + 58.7] \times 5.87 = 199.245 \text{ kN/m}$

acting at distance of (Y) = $(H - Z_c)/3 = (5.87/3) = 1.957 \text{ m}$ from base of Retaining wall.

Problem 3.3. A two layer cohesive horizontal backfill is supported by a 10 m high vertical smooth wall. Determine the Rankine's active force per unit length of the wall before and after tension crack develop in the top layer. Also determine the line of action of the resultant in both cases. The details of the soil properties are given below. [RTMNU, S-19, S-16/8 m]

Properties	Top Layer	Bottom Layer
Depth	0 - 5 m	5 - 10 m
Cohesion	12 kN/m ²	35 kN/m ²
Angle of internal friction	0°	10°

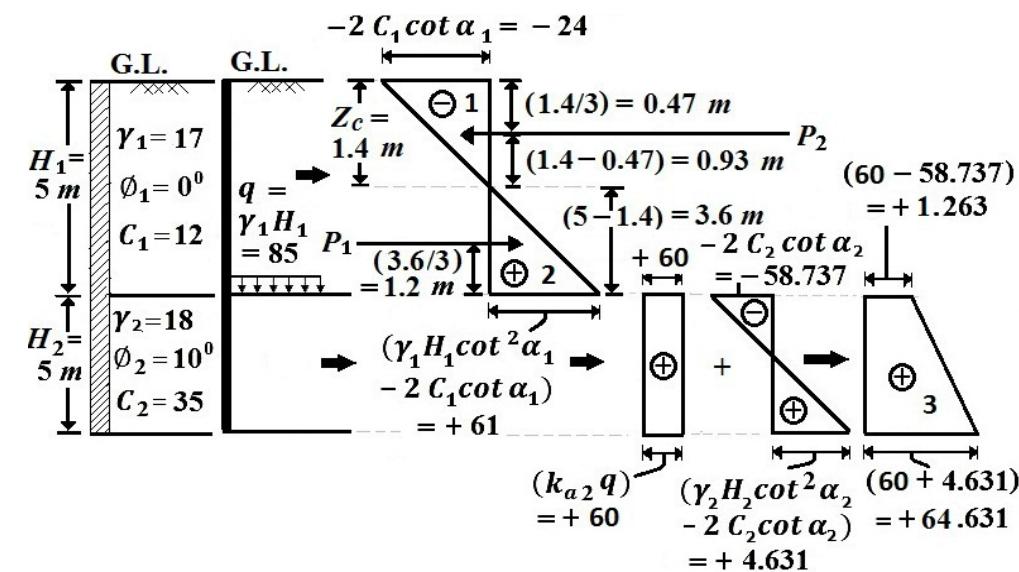
Unit weight

17 kN/m³

18 kN/m³

Solution :

Case i) Active Earth Pressure Before occurrence of Tension Cracks :



$$\alpha_1 = 45 + (\phi_1/2) = 45 + (0^\circ/2) = 45^\circ$$

$$\text{Coefficient of Active Earth Pressure } (K_a) = 1 - \sin(0^\circ)/1 + \sin(0^\circ) = 1$$

$$\text{Top Intensity; } -2C_1 \cot \alpha_1 = (-2 \times 12 \times 1/\tan 45^\circ) = -24 \text{ kN/m}$$

$$\text{Bottom Intensity; } (\gamma_1 H_1 \cot^2 \alpha_1 - 2C_1 \cot \alpha_1) =$$

$$= [(17 \times 5 \times 1/\tan 45^\circ)^2] - 24 = +61 \text{ kN/m}$$

$$\text{Active Pressure for +ve Portion } (P_1) = [1/2 \times 61 \times 3.6] = 109.8 \text{ kN/m}$$

Acting at distance of (Y_1) = $5 + (3.6/3) = 6.2 \text{ m}$ from base of the wall.

$$\text{Active Pressure for -ve Portion } (P_2) = [1/2 \times 24 \times 1.4] = 16.8 \text{ kN/m}$$

$$\text{Acting at } (Y_2) = (5 + 3.6 + 0.93) = 9.53 \text{ m from base of the wall.}$$

$$\therefore \text{Hence The total Active Earth Pressure for top layer } (P_a) = (P_1 - P_2) = (109.8 - 16.8) = 93 \text{ kN/m}$$

$$\text{Acting at distance of } Z_1 = (P_1 Y_1 - P_2 Y_2) / P_a =$$

$$= [(109.8 \times 6.2) - (16.8 \times 9.53)] / 93 = 5.60 \text{ m.}$$

Hence The Active Earth Pressure for Top Layer (Pa_1) = 93 kN/m will be act at distance of $Z_1 = 5.6$ m from the base of the Retaining wall.

Calculations for Bottom layer :

$$\text{Coefficient of Active earth pressure } (Ka_2) = 1 - \sin(\phi_2)/1 + \sin(\phi_2) \\ 1 - \sin(10^\circ)/1 + \sin(10^\circ) = 0.70$$

$$\alpha_2 = 45 + (\phi_2/2) = 45 + (10^\circ/2) = 50^\circ$$

Due to top layer, The Surcharge Pressure will be developed on the Bottom layer in the form of UDL (q). Hence, $q = \gamma_1 H_1 = (17 \times 5) = 85$ kN/m²
 \therefore The extra Rectangle of Intensity ($Ka_2 \times q$) = (0.7 \times 85) = 60 kN/m is added due to surcharge load (q) as shown in Figure.

$$\text{Top Intensity} ; -2 C_2 \cot \alpha_2 = (-2 \times 35 \times 1/\tan 50^\circ) = -58.737 \text{ kN/m}$$

$$\text{Bottom Intensity} ; (\gamma_2 H_2 \cot^2 \alpha_2 - 2 C_2 \cot \alpha_2) =$$

$$= [(18 \times 5 \times (1/\tan 50^\circ)^2) - 58.737] = +4.631 \text{ kN/m}$$

$$\text{Active Earth Pressure } (Pa_2) \text{ for Bottom Layer (Considering Figure 3) :}$$

$$= [(a+b)/2] \times H_2 = [(1.263 + 64.631)/2] \times 5 = +164.735 \text{ kN/m}$$

Acting at distance of Z_2

$$Z_2 = \left[\frac{(2a+b)}{(a+b)} \right] \times \frac{H_2}{3} = \left[\frac{(2 \times 1.263) + 64.631}{(1.263 + 64.631)} \times (5/3) \right] = 1.7 \text{ m}$$

\therefore The total Active Earth Pressure for bottom layer Pa_2 = 164.735 kN/m will be act at $Z_2 = 1.7$ m from the base of the Retaining wall.

Hence, The Total resultant Active Earth Pressure Before occurrence of Tension Cracks :

$$Pa = (Pa_1 + Pa_2) = (93 + 164.735) = 257.735 \text{ kN/m}$$

Acting at distance of

$$Z = (Pa_1 Z_1 + Pa_2 Z_2) / Pa = [(93 \times 5.6) + (164.735 \times 1.7)] / 257.735$$

= 3.107 m . Hence The total Active Earth Pressure (Resultant) Pa = 257.735 kN/m will be act at distance of 3.107 m from the base of the Retaining wall.

Case ii) Active Earth Pressure after occurrence of Tension Cracks : Calculations for top layer :

$$\text{Active Earth Pressure } (Pa_1) \text{ for top Layer (Considering Figure 2 i.e. +ve Portion only) : } \therefore Pa_1 = [1/2 \times 61 \times 3.6] = 109.8 \text{ kN/m}$$

Acting at distance of (Z_1) = $5 + (3.6/3) = 6.2$ m from base of the wall.

Calculations for Bottom layer :

$$\text{Active Earth Pressure } (Pa_2) \text{ for Bottom Layer (Considering Figure 3 i.e. + ve Portion only. } \therefore Pa_2 = [(a+b)/2] \times H_2$$

$$= [(1.263 + 64.631)/2] \times 5 = +164.735 \text{ kN/m acting at distance of } Z_2$$

$$Z_2 = \left[\frac{(2a+b)}{(a+b)} \right] \times \frac{H_2}{3} = \left[\frac{(2 \times 1.263) + 64.631}{(1.263 + 64.631)} \times (5/3) \right] = 1.7 \text{ m}$$

\therefore Total Active Earth Pressure after occurrence of Tension Cracks :

$$Pa = (Pa_1 + Pa_2) = (109.8 + 164.735) = 274.535 \text{ kN/m}$$

Acting at distance of

$$Z = (Pa_1 Z_1 + Pa_2 Z_2) / Pa = [(109.8 \times 6.2) + (164.735 \times 1.7)] / 274.535$$

= 3.5 m . Hence The total Active Earth Pressure (Resultant) Pa = 274.535 kN/m after occurrence of tension crack will be act at distance of 3.5 m from the base of the Retaining wall.

Problem 3.4. A Two layered Cohesive horizontal backfill is supported by 10 m high vertical smooth wall . Determine Rankine's active force per meter length of the wall before and after tensile cracks occurs in the top layer. Also determine the line of action of the resultant in both cases. The Details of the layer is as follows : **[RTMNU, W-15/8 m]**

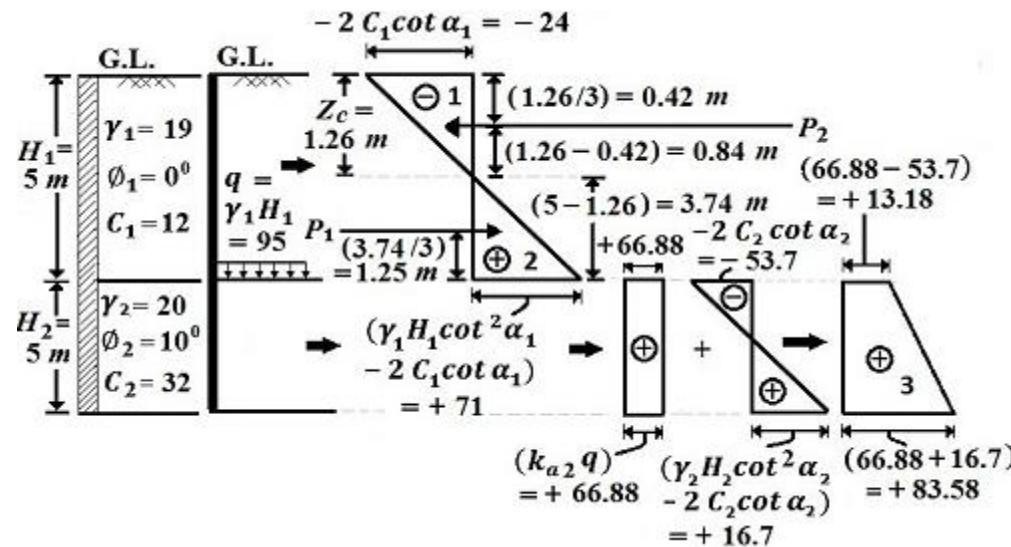
Depth	0 – 5 m	5 – 10 m
Cohesion	12 kN/m ²	32 kN/m ²
Angle of internal friction	0 ⁰	10 ⁰

Unit weight

19 kN/m³

20 kN/m³

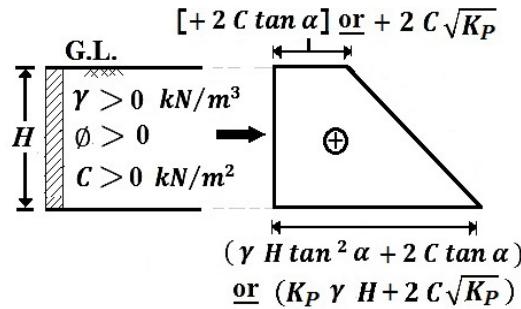
Solution :



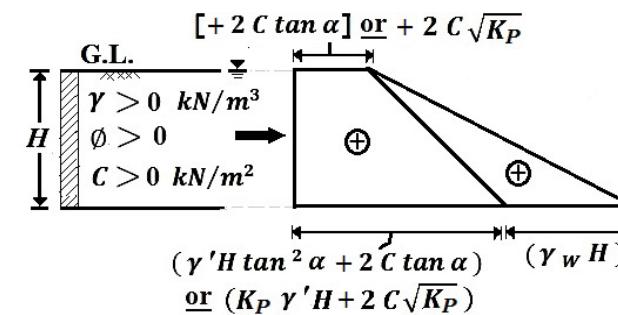
Rankin's Earth Pressure Theory :

Case 3] Passive Earth Pressure for Cohesive soil ($C > 0$)

i) Dry Backfill :



ii) Water Table is Located at Ground Surface (Submerged Backfill) :



Problem 3.5. A vertical excavation in cohesive soil caved after a depth of 2 m. If the unit weight of soil is 16 kN/m³ and $\phi = 0^\circ$. Determine the cohesion of soil. If the same soil was used as a backfill for 6 m high Vertical retaining wall. Determine the total passive pressure and point of application. [SGBAU,W-19/7 m]

Solution :

Case i) Active Earth Pressure : $\alpha = 45 + (\phi/2) = 45 + (16^\circ/2) = 53^\circ$

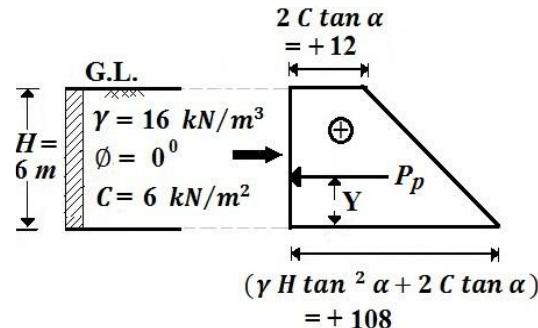
Here, $H_C = 2 \text{ m}$ (given in problem)

where, H_C - Unsupported length or Caving Depth or Critical Height or Cut or Depth up to which Excavation can be done without any lateral Support.

We know that, $H_C = (2 Z_C) = [(4 C / \gamma) \times \tan \alpha]$

Equating; $2 = [(4 \times C) / 16] \times \tan 53^\circ$

$$2 = [(4 \times C) / 16] \times \tan 53^\circ ; \therefore \text{Cohesion } (C) = 6 \text{ kN/m}^2$$

Case i) Passive Earth Pressure :

$$\text{Top Intensity} ; + 2 C \tan \alpha = (2 \times 6 \times \tan 45^\circ) = + 12 \text{ kN/m}$$

$$\text{Bottom Intensity} ; (\gamma H \tan^2 \alpha + 2 C \tan \alpha)$$

$$= [(16 \times 6 \times \tan^2 45^\circ)] + 12 = + 108 \text{ kN/m}$$

Passive Earth Pressure (P_p) = Area of +ve Portion (Trapezoid)

$$= [(a + b)/2] \times H = [(12 + 108)/2] \times 6 = 360 \text{ kN/m}$$

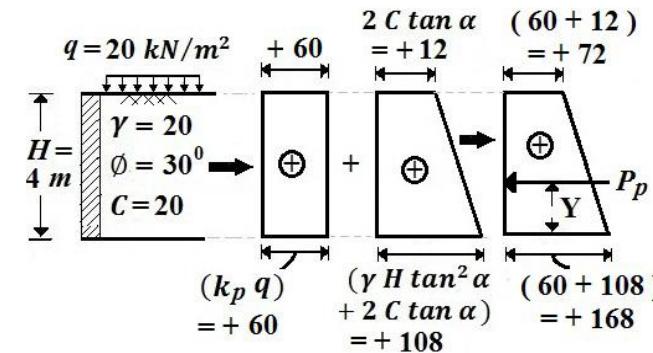
Acting at distance of (Y)

$$Y = \left[\frac{(2a + b)}{(a + b)} \right] \times \frac{H}{3} = \left[\frac{(2 \times 12) + 108}{(12 + 108)} \times (6/3) \right] = 2.2 \text{ m}$$

Resultant Force (Passive Earth Pressure) $P_p = 360 \text{ kN/m}$ is acting at a Distance of 2.2 m from base of retaining wall

Problem 3.6. A Retaining wall 4 m High supports Backfill having Properties as $C = 20 \text{ kN/m}^2$, Internal angle of Friction = 30° , Unit weight of soil is 20 kN/m^3 with Horizontal top, Flush with the top of the wall. The Backfill Carries A Surcharge of 20 kN/m^2 . If the wall is flushed towards the Backfill, Compute the Total Passive Pressure on the wall, and its point of Application ? [RTMNU,S-11/7 m]

Solution : $\alpha = 45 + (\phi/2) = 45 + (30^\circ/2) = 60^\circ$



$$\begin{aligned} \text{Coefficient of Passive earth pressure } (K_p) &= 1 + \sin(\phi)/1 - \sin(\phi) \\ &= 1 + \sin(30^\circ)/1 - \sin(30^\circ) = 3 \end{aligned}$$

The extra Rectangle of Intensity ($K_p \times q$) = $(3 \times 20) = 60 \text{ kN/m}$ is added due to surcharge load (q) as shown in Figure.

$$\text{Top Intensity} ; + 2 C \tan \alpha = (2 \times 20 \times \tan 60^\circ) = + 12 \text{ kN/m}$$

$$\text{Bottom Intensity} ; (\gamma H \tan^2 \alpha + 2 C \tan \alpha)$$

$$= [(20 \times 4 \times \tan^2 60^\circ)] + 12 = + 108 \text{ kN/m}$$

Passive Earth Pressure (P_p) = Area of +ve Portion (Trapezoid)

$$= [(a + b)/2] \times \text{height} = [(72 + 168)/2] \times 4 = 480 \text{ kN/m}$$

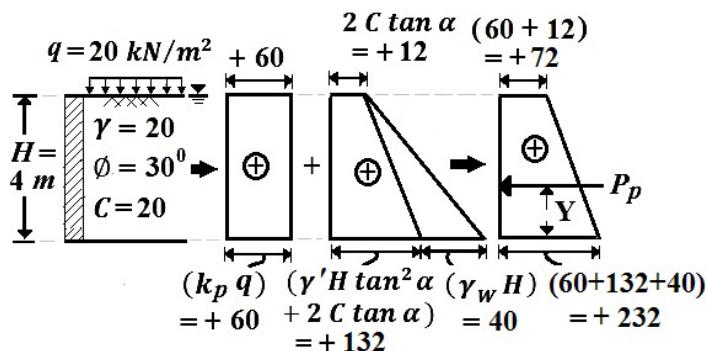
Acting at distance of (Y)

$$Y = 5 + \left[\frac{(2a + b)}{(a + b)} \right] \times \frac{H}{3} = \left[\frac{(2 \times 72) + 168}{(72 + 168)} \times (4/3) \right] = 1.74 \text{ m}$$

Resultant Force (Passive Earth Pressure) $P_p = 480 \text{ kN/m}$ is acting at a Distance of 1.74 m from base of retaining wall .

Problem 3.7. Solve the Problem 3.6 if the water table is located at ground Surface.

Solution :



Top Intensity ; $+ 2 C \tan \alpha = (2 \times 20 \times \tan 60^\circ) = + 12 \text{ kN/m}$

Bottom Intensity ; $(\gamma' H \tan^2 \alpha + 2 C \tan \alpha) =$

$$= [(10 \times 4 \times \tan^2 60^\circ) + (2 \times 20 \times \tan 60^\circ)] = + 132 \text{ kN/m}$$

Submerged unit weight (γ') = $(\gamma - \gamma_w) = (20 - 10) = 10 \text{ kN/m}^3$

Where, γ_w - Unit weight of water = $9.81 \approx 10 \text{ kN/m}^3$

Pore Water Pressure ; $(\gamma_w \times H) = (10 \times 4) = 40 \text{ kN/m}$

Passive Earth Pressure (P_p) = Area of +ve Portion only (Trapezoid)

$$P_p = [(a + b)/2] \times H = [(72 + 232)/2] \times 4 = 608 \text{ kN/m}$$

Acting at distance of (Y)

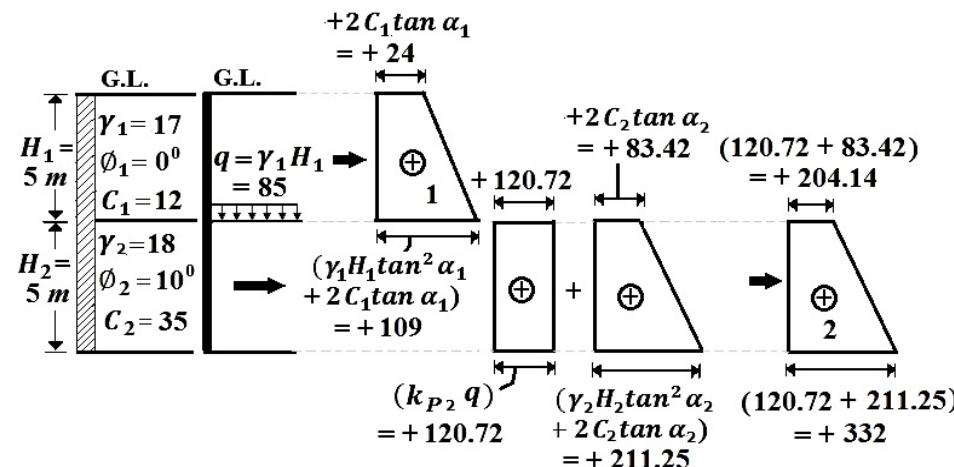
$$Y = \left[\frac{(2a + b)}{(a + b)} \right] \times \frac{H}{3} = \left[\frac{(2 \times 72) + 232}{(72 + 232)} \times (4/3) \right] = 1.65 \text{ m}$$

\therefore Passive Earth Pressure (P_p) = 608 kN/m will be act at distance of $Y = 1.65 \text{ m}$ from the base of the Retaining wall.

Problem 3.8. Determine Rankine's Passive Earth Pressure per meter length for 2 layered Backfill behind 10 m high Retaining wall . Also determine location of the line of action of Resultant?

Properties	Top Layer	Bottom Layer
Depth	0 - 5 m	5 - 10 m
Cohesion	12 kN/m²	35 kN/m²
Angle of internal friction	0°	10°
Unit weight	17 kN/m^3	18 kN/m^3

Solution :



Calculations for top layer : $\alpha_1 = 45 + (\phi_1/2) = 45 + (0^\circ/2) = 45^\circ$

$$\text{Coefficient of Passive Earth Pressure } (K_{P_1}) = 1 + \sin(0^\circ)/1 - \sin(0^\circ) = 1$$

Top Intensity ; $+ 2 C_1 \tan \alpha_1 = (2 \times 12 \times \tan 45^\circ) = + 24 \text{ kN/m}$

Bottom Intensity ; $(\gamma_1 H_1 \tan^2 \alpha_1 + 2 C_1 \tan \alpha_1)$

$$= [(17 \times 5 \times (\tan 45^\circ)^2) + 24] = + 109 \text{ kN/m}$$

Passive Earth Pressure P_1 (considering +ve portion only, i.e. figure 1)

$$P_{p1} = [(a + b)/2] \times H_1 = [(24 + 109)/2] \times 5 = 332.5 \text{ kN/m}$$

Acting at distance of (Y_1)

$$Y_1 = 5 + \left[\frac{(2a + b)}{(a + b)} \right] \times \frac{H_1}{3} = 5 + \left[\frac{(2 \times 24) + 109}{(24 + 109)} \times (5/3) \right] = 6.97 \text{ m}$$

\therefore Passive Earth Pressure for Top Layer (P_{p1}) = 332.5 kN/m will be act at distance of $Y_1 = 6.97 \text{ m}$ from the base of the Retaining wall.

Calculations for Bottom layer : $\alpha_2 = 45 + (\phi_2/2) = 45 + (10^0/2) = 50^0$

$$\begin{aligned} Kp_2 &= 1 + \sin(\phi_2)/1 - \sin(\phi_2) = 1 + \sin(10^0)/1 - \sin(10^0) \\ &= 1.42 \end{aligned}$$

Due to top layer, The Surcharge Pressure will be developed on the Bottom layer in the form of UDL (q). Hence, $q = \gamma_1 H_1 = (17 \times 5) = 85 \text{ kN/m}^2$
 \therefore The extra Rectangle of Intensity ($Kp_2 \times q$) = $(1.42 \times 85) = 120.72 \text{ kN/m}$ is added due to surcharge load (q) as shown in Figure.

Top Intensity ; $+2C_2 \tan \alpha_2 = (2 \times 35 \times \tan 50^0) = +83.42 \text{ kN/m}$

$$\begin{aligned} \text{Bottom Intensity} ; & (\gamma_2 H_2 \tan^2 \alpha_2 + 2C_2 \tan \alpha_2) = \\ & = [(18 \times 5 \times (\tan 50^0)^2) + 83.42] = +211.25 \text{ kN/m} \end{aligned}$$

Passive Earth Pressure (Pp_2) for Bottom Layer (Considering Figure 2) :

$$= [(a+b)/2] \times H_2 = [(204.14 + 332)/2] \times 5 = 1340.35 \text{ kN/m}$$

Acting at distance of Y_2

$$Y_2 = \left[\frac{(2a+b)}{(a+b)} \right] \times \frac{H_2}{3} = \left[\frac{(2 \times 204.14) + 332}{(204.14 + 332)} \times (5/3) \right] = 2.30 \text{ m}$$

\therefore Active Earth Pressure for bottom layer $Pp_2 = 1340.35 \text{ kN/m}$ will be act at $Y_2 = 2.30 \text{ m}$ from the base of the Retaining wall.

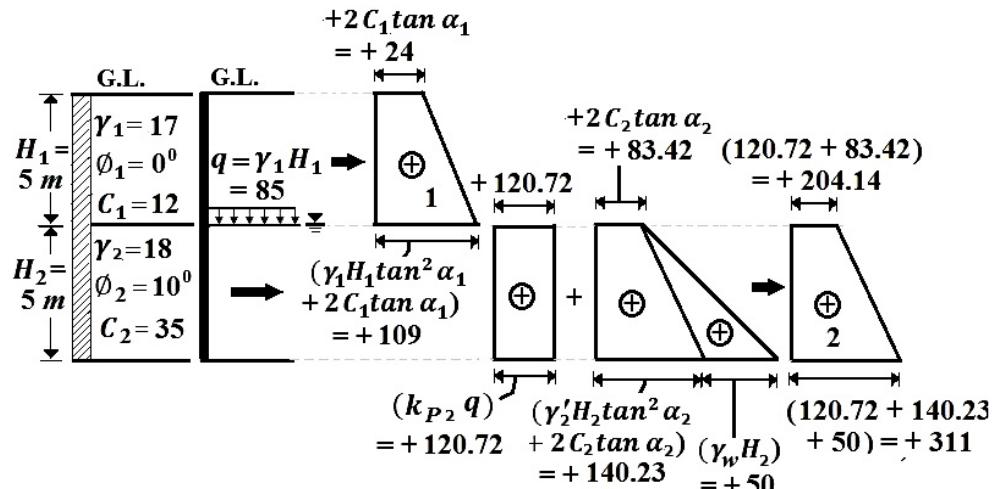
Hence, The Total Resultant Passive Earth Pressure (Pa) :

$$Pp = (Pp_1 + Pp_2) = (332.5 + 1340.35) = 1672.85 \text{ kN/m}$$

Acting at distance of

$$\begin{aligned} Y &= (Pp_1 Y_1 + Pp_2 Y_2) / Pp = [(332.5 \times 6.97) + (1340.35 \times 2.3)] / 1672.85 \\ &= 3.23 \text{ m} \text{ . Hence The total Passive Earth Pressure (Resultant) } Pp = \\ & 1672.85 \text{ kN/m will be act at distance of } 3.23 \text{ m from the base of the} \\ & \text{Retaining wall.} \end{aligned}$$

Problem 3.9. Solve Above Problem if the Ground water table is located at 5 m from base of wall.



Calculations for top layer :

Refer the above problem as we have already calculated.

\therefore Passive Earth Pressure for Top Layer (Pp_1) = 332.5 kN/m will be act at distance of $Y_1 = 6.97 \text{ m}$ from the base of the Retaining wall.

Calculations for Bottom layer : $\alpha_2 = 45 + (\phi_2/2) = 45 + (10^0/2) = 50^0$

$$\begin{aligned} Kp_2 &= 1 + \sin(\phi_2)/1 - \sin(\phi_2) = 1 + \sin(10^0)/1 - \sin(10^0) \\ &= 1.42 \end{aligned}$$

Due to top layer, The Surcharge Pressure will be developed on the Bottom layer in the form of UDL (q). Hence, $q = \gamma_1 H_1 = (17 \times 5) = 85 \text{ kN/m}^2$
 \therefore The extra Rectangle of Intensity ($Kp_2 \times q$) = $(1.42 \times 85) = 120.72 \text{ kN/m}$ is added due to surcharge load (q) as shown in Figure.

Top Intensity ; $+ 2 C_2 \tan \alpha_2 = (2 \times 35 \times \tan 50^\circ) = + 83.42 \text{ kN/m}$

Bottom Intensity ; $(\gamma_2' H_2 \tan^2 \alpha_2 + 2 C_2 \tan \alpha_2) =$

$$= [(8 \times 5 \times (\tan 50^\circ)^2) + 83.42] = + 140.23 \text{ kN/m}$$

Submerged unit weight (γ_2') = $(\gamma_2 - \gamma_w) = (18 - 10) = 10 \text{ kN/m}^3$

Where, γ_w - Unit weight of water = $9.81 \approx 10 \text{ kN/m}^3$

Pore Water Pressure ; $(\gamma_w \times H_2) = (10 \times 5) = 50 \text{ kN/m}$

Passive Earth Pressure (Pp_2) for Bottom Layer (Considering Figure 2) :

$$= [(a+b)/2] \times H_2 = [(204.14 + 311)/2] \times 5 = + 1287.85 \text{ kN/m}$$

Acting at distance of Y_2

$$Y_2 = \left[\frac{(2a+b)}{(a+b)} \right] \times \frac{H_2}{3} = \frac{(2 \times 204.14) + 311}{(204.14 + 311)} \times (5/3) = 2.33 \text{ m}$$

\therefore Active Earth Pressure for bottom layer $Pp_2 = 1287.85 \text{ kN/m}$ will be act at $Y_2 = 2.33 \text{ m}$ from the base of the Retaining wall.

Hence, The Total Resultant Passive Earth Pressure (Pp) :

$$Pp = (Pp_1 + Pp_2) = (332.5 + 1287.85) = 1620.35 \text{ kN/m}$$

Acting at distance of

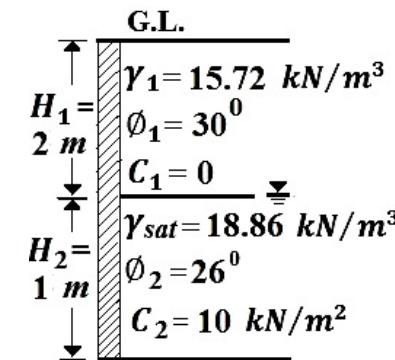
$$Y = (Pp_1 Y_1 + Pp_2 Y_2) / Pp = [(332.5 \times 6.97) + (1287.85 \times 2.33)] / 1620.35$$

= 3.28 m . Hence The total Passive Earth Pressure (Resultant) $Pp =$

1620.35 kN/m will be act at distance of 3.28 m from the base of the Retaining wall.

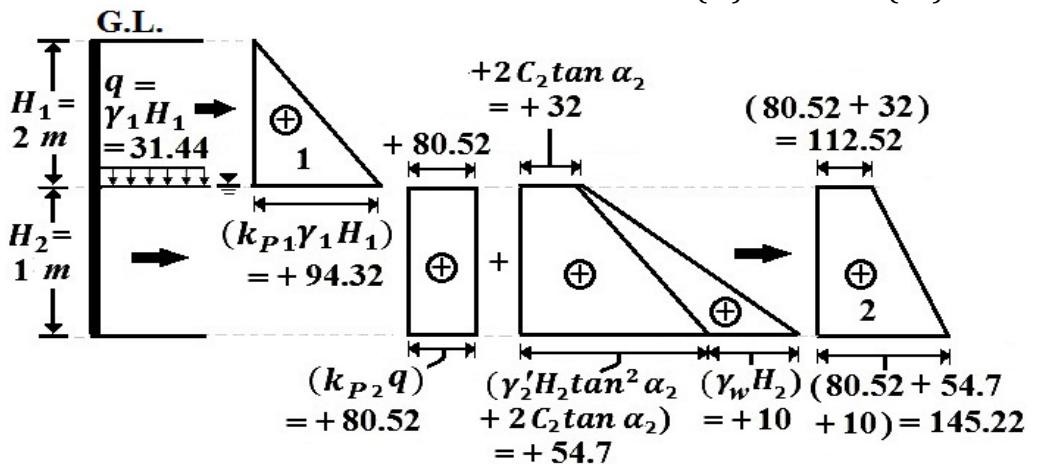
Problem 4.0 A 3 m high wall is shown in figure below. Determine Rankine's Passive Earth Pressure per unit length of wall ?

[SGBAU,S-18/8 m]



Since, for top layer, $C_1 = 0$, hence the backfill is Cohesion less in nature and for bottom layer, $C_2 = 10 \text{ kN/m}^2 > 0$, hence the backfill is Cohesive in nature.

\therefore This Numerical is Combination of Rankine's Case (I) and Case (iii)



Calculations for top layer :

$$\begin{aligned} \text{Coefficient of Passive Earth Pressure } (Kp_1) &= 1 + \sin(\phi_1) / 1 - \sin(\phi_1) \\ &= 1 + \sin(30^\circ) / 1 - \sin(30^\circ) = 3.0 \end{aligned}$$

$$(Kp_1 \gamma_1 H_1) = (3 \times 15.72 \times 2) = 94.32 \text{ kN/m}$$

Passive Earth Pressure Pp_1 (considering +ve portion only, i.e. figure 1)

$$Pp_1 = 1/2 \times \text{Base} \times \text{height} = [1/2 \times 94.32 \times 2] = 94.32 \text{ kN/m}$$

acting at distance of (Y_1) = $1 + (H_1/3) = 1 + (2/3) = 1.67 \text{ m}$ from base of wall.

Calculations for Bottom layer : $\alpha_2 = 45 + (\emptyset_2/2) = 45 + (26^0/2) = 58^0$

$$Kp_2 = 1 + \sin(\emptyset_2)/1 - \sin(\emptyset_2) = 1 + \sin(26^0)/1 - \sin(26^0) = 2.56$$

Due to top layer, The Surcharge Pressure will be developed on the Bottom layer in the form of UDL (q). Hence, $q = \gamma_1 H_1 = (15.72 \times 2) = 31.44 \text{ kN/m}^2$

∴ The extra Rectangle of Intensity ($Kp_2 \times q$) = $(2.56 \times 31.44) = 80.52 \text{ kN/m}$ is added due to surcharge load (q) as shown in Figure.

$$\text{Top Intensity; } + 2 C_2 \tan \alpha_2 = (2 \times 10 \times \tan 58^0) = + 32 \text{ kN/m}$$

$$\text{Bottom Intensity; } (\gamma_2' H_2 \tan^2 \alpha_2 + 2 C_2 \tan \alpha_2)$$

$$= [8.86 \times 1 \times (\tan 58^0)^2] + 32 = + 54.7 \text{ kN/m}$$

$$\text{Submerged unit weight } (\gamma_2') = (\gamma_2 - \gamma_w) = (18.86 - 10) = 8.86 \text{ kN/m}^3$$

$$\text{Where, } \gamma_w - \text{Unit weight of water} = 9.81 \approx 10 \text{ kN/m}^3$$

$$\text{Pore Water Pressure; } (\gamma_w \times H_2) = (10 \times 1) = 10 \text{ kN/m}$$

Passive Earth Pressure (Pp_2) for Bottom Layer (Considering Figure 2) :

$$= [(a+b)/2] \times H_2 = [(112.52 + 145.22)/2] \times 1 = + 128.87 \text{ kN/m}$$

Acting at distance of Y_2

$$Y_2 = \left[\frac{(2a+b)}{(a+b)} \right] \times \frac{H_2}{3} = \frac{(2 \times 112.52) + 145.22}{(112.52 + 145.22)} \times (1/3) = 0.48 \text{ m}$$

∴ Active Earth Pressure for bottom layer $Pp_2 = 128.87 \text{ kN/m}$ will be act at $Y_2 = 0.48 \text{ m}$ from the base of the Retaining wall.

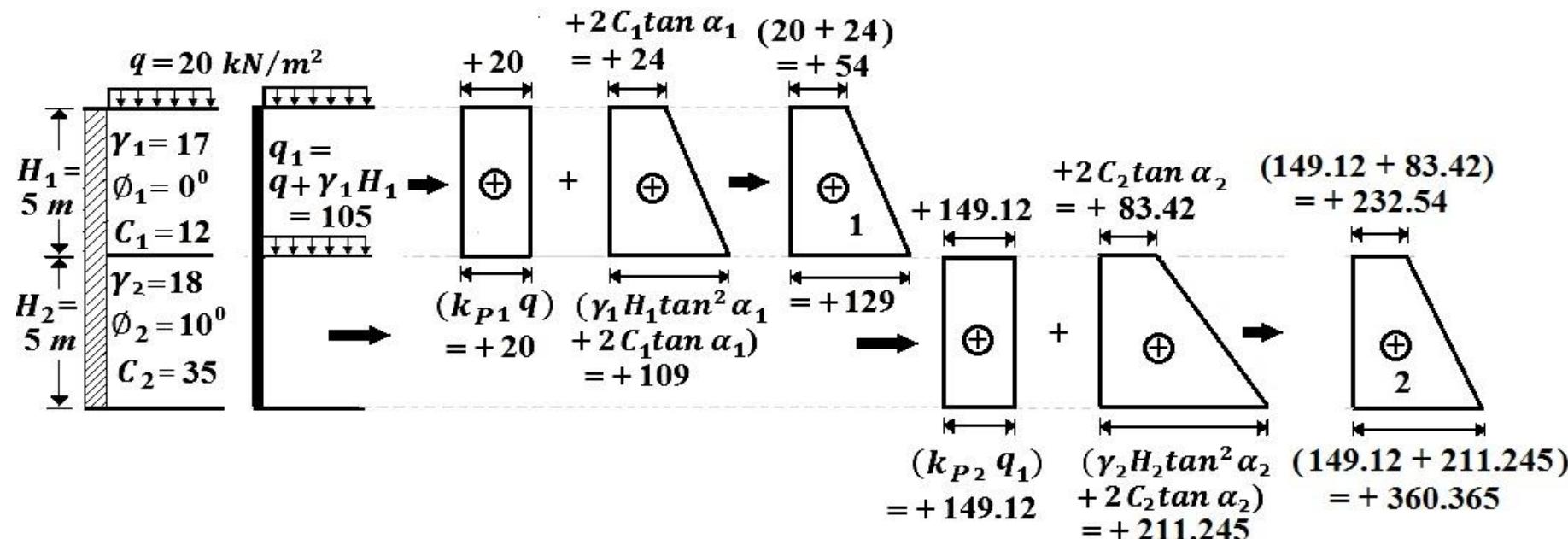
Hence, The Total Resultant Passive Earth Pressure (Pp) :

$$Pp = (Pp_1 + Pp_2) = (94.32 + 128.87) = 223.20 \text{ kN/m}$$

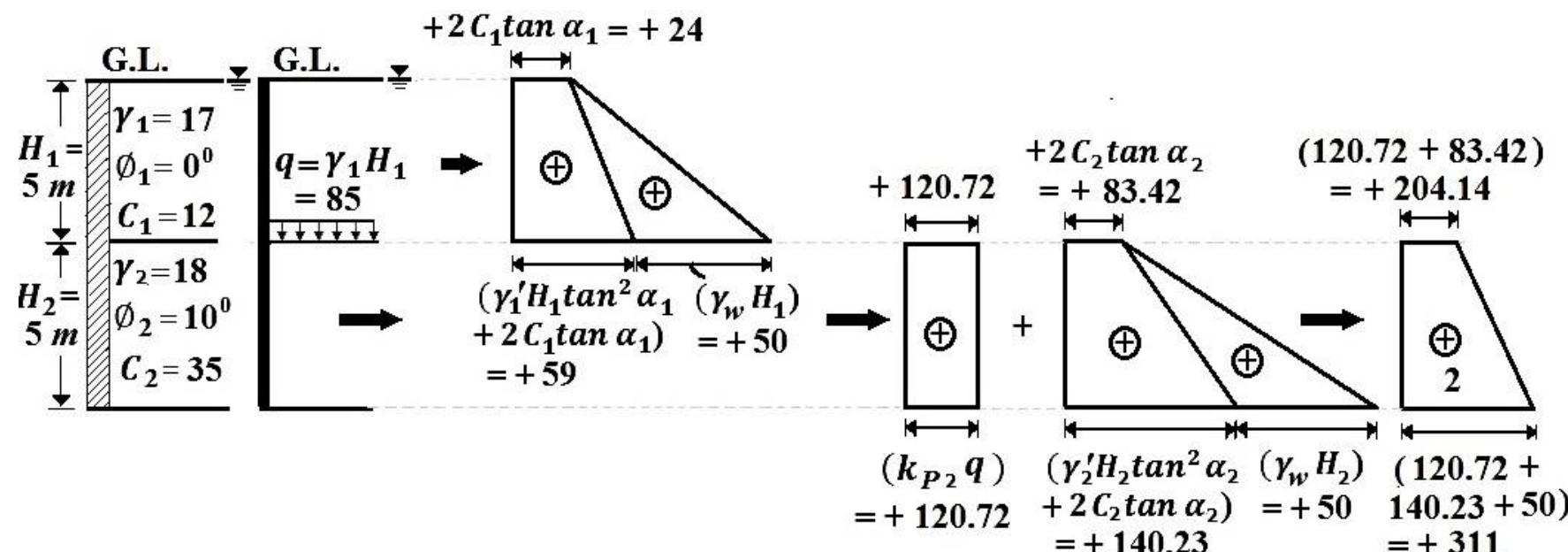
Acting at distance of

$$Y = (Pp_1 Y_1 + Pp_2 Y_2) / Pp = [(94.32 \times 1.67) + (128.87 \times 0.48)] / 223.20 = 0.73 \text{ m.}$$

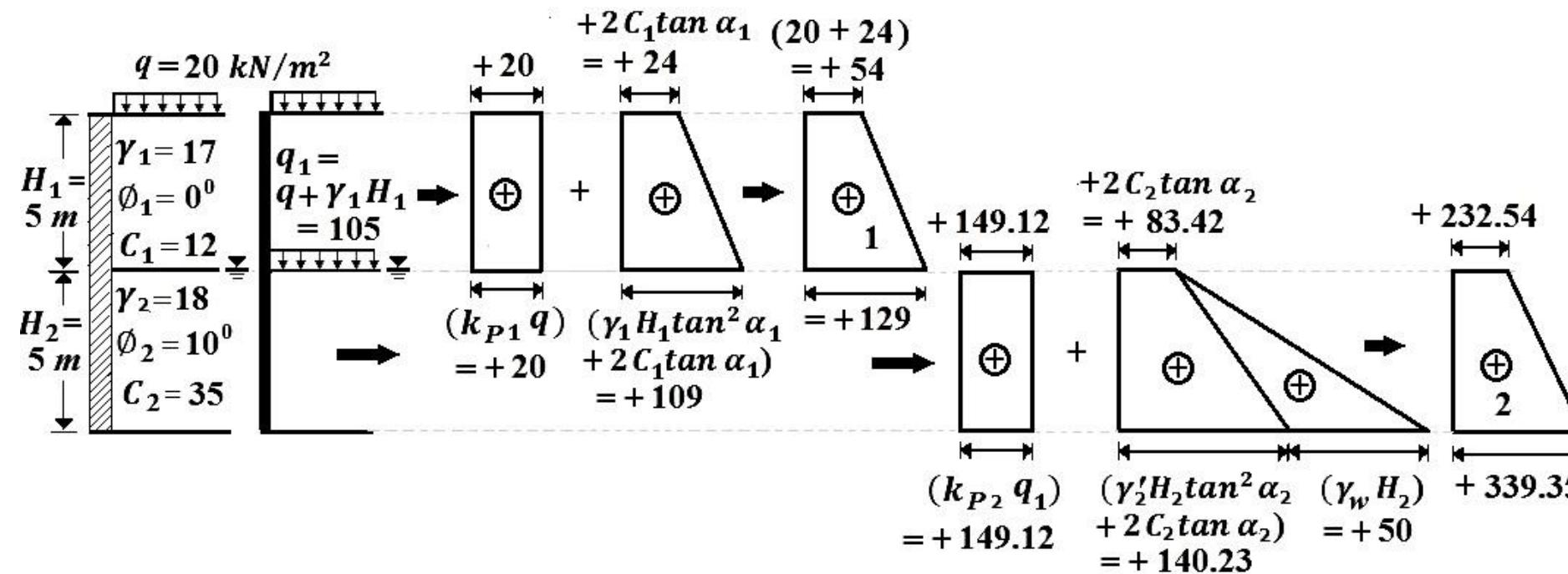
Problem 4.1. Draw an Passive Earth Pressure Distribution diagramm for the Problem 3.8, if there is surcharge load of 20 KN/m².



Problem 4.2. Draw an Passive Earth Pressure Distribution diagramm for the Problem 3.8, if the Water table is located at ground level.



Problem 4.3. Draw an Passive Earth Pressure Distribution diagram for the Problem 3.8, if the Water table is located at 5 mt. below Ground level (at the interface of two layers)..



Problem 4.4. Draw an **Active Earth Pressure** distribution for 3 layered backfill behind a 15 m high retaining wall with smooth vertical backfill.
[RTMNU,S-12/13 m]

Layer	Height in mt	Unit Wt (γ) kN/m ³	Cohesion (C) kN/m ²	ϕ
I	5	20	0	35^0
II	5	18	20	25^0
III	5	16	35	0^0

