PAGE NO.
DATE / /

Assignment 0

Q-1 Find out the z-transform and ROC of the following function

1) $x(n) = \{1, 2, 1, 2, 3\}$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= 1 \cdot z^{-0} + 2z^{-1} + 1z^{-2} + 2z^{-3} + 3z^{-4}$$

$$= 1 + 2z^{-1} + z^{-2} + 2z^{-3} + 3z^{-4}$$

ROC $= z - \{0, \infty\}$

stability: system is stable because Roc includes unit circle

causal: system is causal because it is defined for $n \geq 0$

2) $x[n] = (n+1)^2 u(n)$

$$\rightarrow x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (n+1)^2 z^{-n}$$

ROC: $|z| > 1$

stability: system is not stable because ROC does not include the unit circle.

consality: The system is causal because it is defines only for $n \geq 0$.

PAGE NO.
DATE / /

3) $x(n) = \log n$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=\infty}^{\infty} \log n \, z^{-n}$$

- The function $\log(n)$ is not typically defined for $n=0$ and is not causal function. it is also not well defined for $n<1$. therefore z-transform isn't usually unstable for this function.

ROC: NA because standard z-transform is not exist.

Stability: NA

Causality: The system is not causal since $\log(A)$ is not defined for $n<1$.

4) $x(n) = 2^n u(n) - 0.5^n u(-n-1)$

$$\rightarrow x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 2^n u(n) z^{-n} - 0.5^n u(-n-1) z^{-n}$$

$$= \sum_{n=0}^{\infty} 2^n z^{-n} - \sum_{n=-\infty}^{-1} 0.5^n z^{-n}$$

$$= \frac{1}{1-2z^{-1}} - \frac{1}{1-0.5z}$$

$$= \frac{1}{z-2} - \frac{1}{1-0.5z}$$

PAGE NO.
DATE / /

ROC : i) for $2^n u(n) \rightarrow |z| > 2$
ii) for $0.5^n u(n) \rightarrow |z| < 0.5$

The final ROC will be the intersection of both sequence which is empty so the z-transform does not exist in this combined form.

- Stability : NA because no overlapping ROC.

- Causality : The first part is causal but the second part is anti causal.

5) $x(n) = u(-n-6)$

$$x(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} u(-n-6) z^{-n}$$

$$= \sum_{n=-\infty}^{-6} z^{-n} = z^6 \sum_{n=\infty}^{-6} z^n = \frac{z^6}{1-z}$$

ROC : The ROC is $|z| < 1$ where series is converges.

Stability : NA because ROC does not include the unit circle.

Causality : The system is non causal because $x[n]$ is defined $n \leq 6$.

PAGE NO.
DATE / /

6) $x(n) = \delta(n+5)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= z^{-(-5)} = z^{5}$$

ROC: Z - {0}

stability: The system is stable as the ROC includes the unit circle.

consaity: The system is non causal because the impulse is at $n=-5$

Q-2 Find out the inverse z-transform of the following H(z) if the system is causal.

1) $H(z) = 1 + z + z^{-1}$

- 1 is for $\delta[n]$
- z is for $\delta[n+1]$
- z^{-1} is for $\delta[n-1]$

Here the $\delta[n+1]$ is defined for $n<0$, so the system is non causal.

2) $H(z) = \log(1+z^{-1})$

Power series of $\log(1+z^{-1})$ is

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^{-k}}{k}$$

So, the Inverse z transform is

$$h[n] = \begin{cases} -\frac{1}{k}(-1)^{k+1}, & \text{for } n=k \ (k \geq 1) \\ 0, & \text{otherwise} \end{cases}$$

3) $H(z) = \dfrac{z^2+2z-3}{z^3+z^2-4z+3}$

- Here, $z^3+z^2-4z+3 = (z-1)^2(z+3)$

Now, $H(z) = \dfrac{A}{z-1} + \dfrac{B}{(z-1)^2} + \dfrac{C}{(z+3)}$

Now multiply by demoninator

$A(z-1)(z+3) + B(z+3) + C(z-1)^2$
$= z^2+2z-3$

$\therefore A(z^2+2z-3) + B(z+3) + C(z^2-2z+1) = z^2+2z-3$

$\therefore Az^2 + 2Az - 3A + Bz + 3B + Cz^2 - 2Cz + C = z^2+2z-3$

$\therefore z^2(A+C) + z(2A+B-2C) + (C+3B-3A) = z^2+2z-3$

So, $A+C=1$
$2A+B-2C=2$
$-3A+3B+C=-3$

So, $A=1,\ B=2,\ C=-1$

Thus, $H(z) = \dfrac{1}{z-1} + \dfrac{2}{(z-1)^2} + \dfrac{1}{z+3}$

So, the inverse z transform are

PAGE NO.
DATE / /

$H[n] = u[n] + 2(n+1)u[n] - (-3)^n u[n]$

4) $H(z) = \dfrac{(z^2+1)(2z-3)}{z^2-4z+3}$

Here, $(z^2+1)(2z-3) = 2z^3 - 3z^2 + 2z - 3$

so, $H(z) = \dfrac{2z^3-3z^2+2z-3}{z^2-4z+3}$

and, $z^2 - 4z + 3 = (z-1)(z-3)$

so, $H(z) = \dfrac{2z^3-3z^2+2z-3}{(z-1)(z-3)}$

now, $H(z) = \dfrac{A}{z-1} + \dfrac{B}{z-3}$

so, $2z^3 - 3z^2 + 2z - 3 = A(z-3) + B(z-1)$

now $z = 3 > B(3-1) = 2(27) - 3(9) + 2(3) - 3$

$2B = 54 - 27 + 6 - 3$

$2B = 27 + 3$

$B = 15$

now $z = 1 > 2(1) - 3(1) + 2(1) - 3 = -2A$

$2 - 3 + 2 - 3 = -2A$

$-2 = -2A$

$A = 1$

so, $H(z) = \dfrac{1}{z-1} + \dfrac{15}{z-3}$

PAGE NO.
DATE / /

Now, Inverse z-transform

$$H(n) = u[n] + 15 \cdot 3^n u[n]$$
$$= (1 + 15 \cdot 3^n) u[n]$$

5) $H(z) = \dfrac{5}{(z-3)^2 (z-1)}$

Here $H(z) = \dfrac{A}{z-1} + \dfrac{B}{z-3} + \dfrac{C}{(z-3)^2}$

So, $5 = A(z-3)^2 + B(z-1)(z-3) + C(z-1)$

$5 = A[z^2 - 3z + 9] + B(z^2 - 4z + 3)\ C[z-1)$

$5 = Az^2 - 3Az + 9A + Bz^2 - 4Bz + 3B + Cz - C$

$5 = z^3 [A] + z^2 (A+B) + z[C - 4B - 3A] + (9A + 3B - C)$

So, $A = 5/4$, $C = 5/2$

So, $H(z) = \dfrac{5/4}{z-1} + \dfrac{B}{z-3} + \dfrac{5/2}{(z-3)^2}$

So, Inverse Transform

$$h[n] = 5/4\, u[n] + B \cdot 3^n u[n] + 5/2\, n \cdot 3^n \cdot u[n]$$

Q-3) LCCD equation of LTI system is as following:

$y(n) = y(n-1) + 5x(n) + 2x(n-1)$

Find the impulse response frequency response magnitude response and phase response of system. If input is $2^n u[n]$. what will be the output.

PAGE NO.

DATE / /

- for impulse response, $x(n) = \delta(n)$

so, $h(n) = h(n-1) + 5\delta(n) + 2\delta(n-1)$

now $n=0 > h(0) = h(-1) + 5\delta(0) + 2\delta(-1)$

assuming system is causal

$h(0) = 0 + 5 + 2(0) = 5$

and $n=1 > h(1) = h(0) + 5\delta(1) + 2\delta(0)$

$= 5 + 5(0) + 2(1)$

$= 7$

for $n=2 > h(2) = h(2) + 5\delta(2) + 2\delta(2)$

$= 7 + 2(0) + 5(0)$

$= 7$

so, $h(n) = \begin{cases} 5, & n=0 \\ 7, & n>0 \end{cases}$

- Now, frequency responce $H(e^{j\omega})$

$$H(e^{j\omega}) = 5 + 7\sum_{n=1}^{\infty} e^{-j\omega n}$$

$$H(e^{j\omega}) = 5 + \frac{1}{e^{j\omega}-1}$$

- magnitude response

$$|H(e^{j\omega})| = \left| 5 + \frac{7}{e^{j\omega}-1} \right|$$

- Phase Response

$$H(e^{j\omega}) = \arg\left(5 + \frac{7}{e^{j\omega}-1}\right)$$

output for input $x(n) = 2^n(n)$

$$y(n) = \sum_{K0}^{n} h(K)\,x(n-K)$$

$$y(n) = 5(2^n) + 7\sum_{K=1}^{n} 2^{n-K}$$

$$= 5(2^n) + 7(2^n - 1)$$

$$= 12.2^n - 7$$

Q-4) If transfer function of LTI system is $\frac{z^2+2z-3}{z^3+z^2-4z+3}$. Find out relationship between input and output. also find output unit step response of following system.

$$H(z) = \frac{z^2+2z-3}{z^3+z^2-4z+3}$$

$$\frac{y(z)}{x(z)} = \frac{z^2+2z-3}{z^3+z^2-4z+3}$$

$$(z^3+z^2-4z+3)\,y(z).\,(z^2+2z-3)\,x(z)$$

$$z^3y(z) + z^2y(z) - 4zy(z) + 3y(z) = z^2x(z) + 2zx(z) - 3x(z)$$

using difference equation

$y(n+3) + y(n+2) - 4y(n+1) + 3y(n) = x(n+2) + 2x(n+1)$
$3x(n)$

- Now, unit step response

$$X(z) = \frac{1}{1-z^{-1}} \quad \text{for } |z|>1$$

The output $y(z)$ is given by

$$y(z) = H(z) \cdot X(z)$$

$$= \frac{z^2+2z-3}{z^3+z^2-4z+3} \cdot \frac{1}{1-1/z}$$

Q-5) Find out convolution of $2^n u(n)$ and $5^n(-n-1)$ using z transforms Q-4)

Let $x_1[n] = 2^n u(n)$
$x_2[n] = 5^n u(-n-1)$

so, $$X_1(z) = \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (2/z)^n$$

$$= \frac{1}{1-2/z} = \frac{z}{z-2}$$

so, $$X_2(z) = \sum_{n=-\infty}^{-1} 5^n z^{-n}$$

PAGE NO.
DATE / /

putting $m = -n \rightarrow x_2(z) = \sum_{m=1}^{m=\infty} 5^{-m} z^m$

so, $x_2(z) = \frac{(z/5)^2}{1 - z/5}$

$$= \frac{z^2}{25(1 - z/5)}$$

$$= \frac{z^2}{25(z-5)}$$

now, $x(z) = x_1(z) \cdot x_2(z)$

$$= \frac{z}{z-2} \cdot \frac{z^2}{25(z-5)}$$

$$= \frac{z^3}{25(z-5)(z-2)}$$

now, Inverse transform is

$$\frac{z^3}{25(z-5)(z-2)} = \frac{A}{z-5} + \frac{B}{z-2}$$

$$z^3 = A(z-2) + B(z-5)$$
$$z^3 = Az - 2A + Bz - 5B$$
$$z^3 = z(A+B) - (2A + 5B)$$

$A + B = 0$ & $5B + 2A = 0$

solving $A = 0, B = 0$

so, $y(n) = x_1(n) * x_2(n)$

$$= \frac{1}{25}\left[2^n u(n) - 5^n u(n)\right]$$

PAGE NO.
DATE / /

Q-6) Find DTFT of the following sequence.

a) $(n+1)a^n u(n)$

$$x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (n+1)a^n u(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (n+1)a^n e^{-j\omega n}$$

$$= \frac{1}{(1-ae^{-j\omega})^2}$$

b) $x(n) = \frac{\sin(\omega_0 n)}{\pi n}$

$$x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \sin(\omega_0 n)/\pi n \cdot e^{-j\omega n}$$

$$= \begin{cases} 1, & |\omega-\omega_0| \leq \omega_0 \\ 0, & \text{otherwise} \end{cases}$$

c) $x(n) = \cos(\omega_0 n + \beta)$

$$x(e^{j\omega}) = \frac{1}{2}\left[e^{j\beta}\,\delta(\omega-\omega_0) + e^{-j\omega\beta}\,\delta(\omega+\omega_n)\right]$$

d) $x(n) = e^{j\omega_0 n} u[n]$

$$x(e^{-j\omega}) = \sum_{n=0}^{\infty} e^{j\omega n} e^{-j\omega n}$$

$$= \frac{1}{1-e^{j(\omega_0-\omega)}}, \quad |\omega_0-\omega|<1$$

PAGE NO.

DATE / /

g) $x(n) = 1$ For $-m < n < m$

$$x(e^{j\omega}) = \sum_{n=-m}^{m} e^{-j\omega n}$$

$$= \frac{\sin(2m+1)\omega/2}{\sin \omega/2} e^{-j\omega\omega}$$

Q-7) Determine IDTFT of the following sequence.

$$x(n) = \frac{1}{2\pi} \sin(\pi/4)$$

$$H_{bs}(e^{j\omega}) = \begin{cases} 1, & \text{if } |\omega| < \omega_0 \\ 0, & \text{if } |\omega| > \omega_0 \end{cases}$$

- The IDTFT of the above sequences

$$x(n) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} H_{bs}(e^{j\omega k}) e^{j\omega k n}$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{j\omega k n}$$

$$= \frac{1}{2\pi} \times \frac{1-e^{-j\omega k n}}{1-e^{-j\omega a n}}$$

$$= \frac{\sin \pi/4}{\pi n}$$

Q-8) Find out linear and circular convolution of the following sequences.

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

PAGE NO.
DATE / /

$= \{0,0,1,2,3,4,5,6,5,4,3,2,1,0,0\}$

circular convolution

$$y(n) = \sum_{k=0}^{n-1} x[k]\,h(n-k) \bmod N$$

$$= \{5,4,4,4,4,4,5,6\}$$

Q-9) consider an LTI system with $|H(e^{j\omega})|$ and arg $|H(e^{j\omega})|$ is shown in figure Find impulse response of the system

$$H(e^{j\omega}) = e^{ja(\omega)}$$

$$h(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega t}\, e^{ja(\omega)}\, d\omega$$

$$= \frac{1}{2\pi j}\left(e^{j5\pi t/6} - e^{j\pi t/2}\right)$$

$$+ \frac{1}{2\pi j}\left(e^{j\pi t/2} - e^{-j5\pi t/6}\right)$$

$$= \frac{2}{\pi}\sin\frac{5\pi t}{6}$$

for $x(t) = \cos(\pi n)u(n)$

$$y(t) = x(t) * h(t)$$

$$= \frac{2}{\pi}\int_{-\infty}^{\infty} \cos(\pi n)\sin 5\pi t/6 \; u(\tau)\, dt$$

$$= \frac{2}{\pi}\int_{0}^{\infty} \cos(\pi n)\sin 5\pi t/6 \; dt$$

$$= \frac{1}{\pi}\left[\frac{\sin(5\pi t/6 - \pi m)}{5\pi/6} - \frac{\sin(5\pi/6 + \pi n)}{5\pi/6}\right]$$

Q-8)

PAGE NO.

DATE / /

for unit step input $x(t) = u(t)$

$$y(t) = \frac{2}{\pi}\int_0^t \sin \frac{5\pi t}{6} \, dt$$

$$= \frac{2}{\pi}\left[\frac{-6}{5\pi} \cos \frac{5\pi t}{6}\right]_0^t$$

$$= \frac{-12}{\pi} \cos \frac{5\pi t}{6} + \frac{12}{\pi}$$

Q-10) Prove the following properties of z-transform

1) Differentiation of $X(z)$

if $X(z) = \{x[n]\}$, then the z-transform of $n.x[n]$ is given by

$$Z\{n.x[n]\} = -z \frac{d\,X(z)}{dz}$$

Proof :

$$Z\{n.x[n]\} = \sum_{n=-\infty}^{\infty} n.x[n] z^{-n}$$

$$\text{so, } \frac{dX(z)}{dz} = \frac{d}{dz}\left(\sum_{n=-\infty}^{\infty} x[n] z^{-n}\right)$$

$$\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} x[n] \frac{d}{dz} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \cdot (-n) z^{-n-1}$$

now multiplying $-z$ both side

$$-z \frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} n . x[n] z^{-n}$$

hence, $Z \{n . x[n]\} = -z \frac{dX(z)}{dz}$

4) Time shifting

if $X(z) = Z\{x[n]\}$, then the z-transform of $x[n-k]$ is given by

$$Z\{x[n-k]\} = z^{-k} X(z)$$

Proof:

cosider the z transform of $x[n-k]$

$$Z\{x[n-k]\} = \sum_{n=-\infty}^{\infty} x[n-k] z^{-n}$$

now putting $m = n-k$, hence $n = m+k$

$$Z\{x[n-k]\} = \sum_{m=\infty}^{6} x[m] z^{-(m+k)}$$

$$= z^{-k} \sum_{m=-\infty}^{\infty} x[m] z^{-m}$$

this simplifies to:

$$Z\{x(n-k]\} = z^{-k} X(z)$$

PAGE NO.
DATE / /

Q-11) State and Prove symmetry property of DTFT

Statement: The DTFT of a real valued sequence is Hermitian symmetric, meaning its magnitude is even and its Phase is odd. mathematically.

$$x^* (-e^{jw}) = x(e^{jw})$$

where :-

$x(-e^{jw})$ is the dfdt of the real valued sequence $x(n)$
$x(e^{-jw})$ is the complex conjugate of the DTFT evalted at e^{jw}

Proof :

Let $x(n)$ be a real valued sequence then its DTFT is

$$x(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}$$

$$x^*(e^{jw}) = \sum_{n=-\infty}^{\infty} x^*(n) e^{jwn}$$

since $x(n)$ is real, $x^*(n) = x(n)$ therefore

$$x^*(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) e^{jwn} = x(e^{jw})$$

Now let's evalute $x^*(e^{jw})$ at $-e^{jw}$

$x^*(-e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n}$

Q-12) $x(n) = \{1, 1, 2, 2\}$, $h(n) = \{1, 2, 3, 4\}$

a)

	1	2	3	4
1	1	2	3	4
1	1	2	3	4
2	2	4	6	8
2	2	4	6	8

linear = $\{1, 3, 7, 13, 14, 8\}$
circular = $\{15, 17, 15, 13\}$

Q-13) Find 8 point DFT of $x(n) = \cos(n\pi/4)$. then find amplitude and Phase spectrum.

Here $x[n] = \cos^{\pi n/4}$

$x[n] = \{1, 1/\sqrt{2}, 0, -1/\sqrt{2}, -1, -1/\sqrt{2}, 0, 1/\sqrt{2}\}$

now $X(K) = \sum_{n=0}^{7} x(n) \cdot e^{-j\frac{\pi}{2}Kn}$

$X(0) = \sum_{n=0}^{7} x(n) = 0$

$X(1) = \sum_{n=0}^{7} x(n) \cdot e^{-j\pi/4 n}$

$= 1 + 1/\sqrt{2}\, e^{-j\pi/4} + 0 - 1/\sqrt{2}\, e^{-j3\pi/4} - 1 e^{-j\pi/2} +$
$1/\sqrt{2} + 1/\sqrt{2}\, e$

Similarly for $X(2), X(3) \dots X(7)$ the non-zero values will occur at $X(2)$ and $X(6)$

PAGE NO.

DATE / /

4) For each of the following system, determine whether the system is i) stable ii) causal iii) linear iv) time Invariant memoryless

1) $y(n) = x[n-n_0]$

- Here output is shifted version of input. So it is stable

- Here, if $n_0 \geq 0$ it is causal else non causal

- Here, the system is line shifted so it linear

2) $y(n) = x[-n]$

the system is time reversal so it is stable

- the system is defined for $x<0$, so it is non causal.

- The system depends on a different time index so, it is not memoryless.

3) $y(n) = \log x(n)$

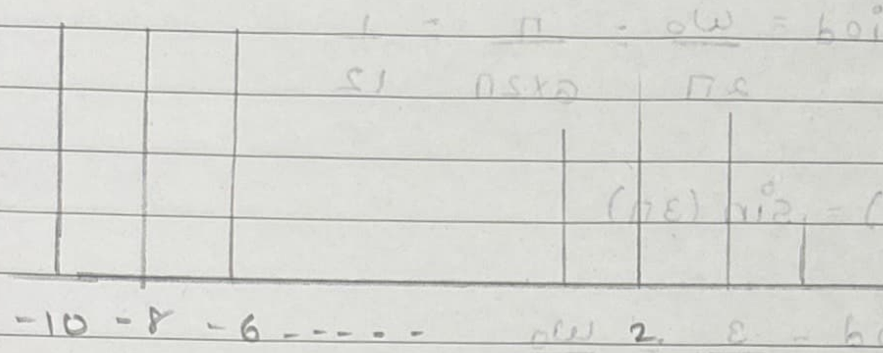
if $x \geq 0$ then it is stable

- the system is defined for $n \geq 0$ only so it is causal

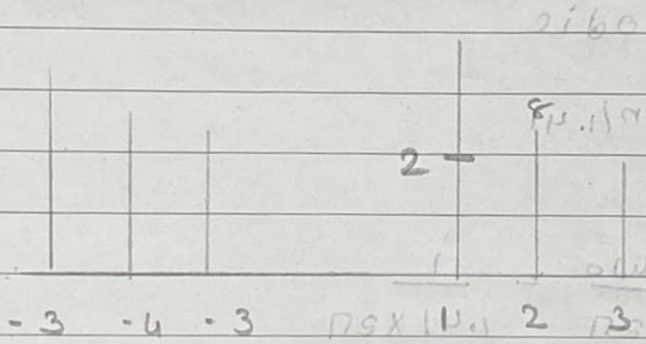
- the system is non-linear

Q-15] Draw following signal $x(n) = 4(n) + 3*$ delta $(n-1) + 44(n-3)$

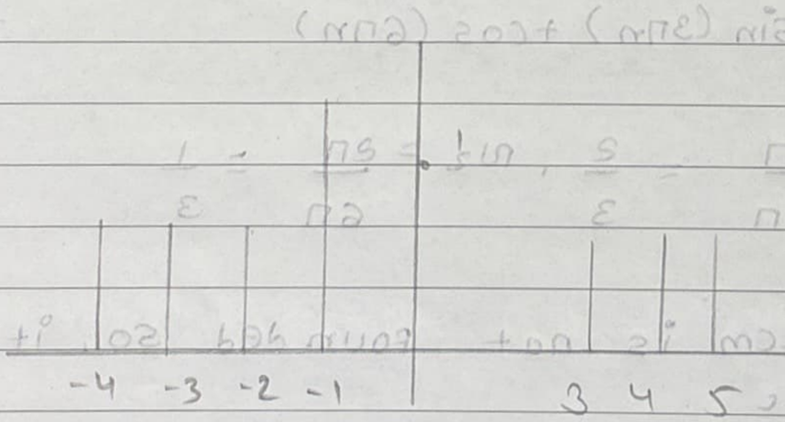
$x(n/2)$



$2x(n)$



$x(-n)$



PAGE NO.
DATE / /

Q.16) find out whether the signal is periodical aperiodic. find the period of following signals.

1) $x(n) = e^{j\pi n/6}$

$$\text{period} = \frac{\omega_0}{2\pi} = \frac{\pi}{6\times 2\pi} = \frac{1}{12}$$

2) $x(n) = \sin(3n)$

$$\text{Period} = \frac{3}{2\pi} = \frac{\omega_0}{2\pi}$$

$3/2\pi$ is not rational number so the signal is aperiodic

3) $x(n) = e^{jn/1.41}$

$$\text{Period} = \frac{\omega_0}{2\pi} = \frac{1}{1.41\times 2\pi}$$

signal is aperiodic

4) $x(n) = \sin(3\pi n) + \cos(6\pi n)$

$$N_1 = \frac{2\pi}{3\pi} = \frac{2}{3}, \quad N_2 = \frac{2\pi}{6\pi} = \frac{1}{3}$$

Here, LCM is not founded so, it is aperodic

PAGE NO.

DATE / /

g) $x(n) = \frac{\sin \pi}{\pi n}$

this is a sin c function which is aperodic.