DATA SCIENCE & ARTIFICIAL INTELLIGENCE

Probability

Lecture No. 01

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic PERMUTATIONS & COMBINATIONS

Topics to be Covered

Topic

BASICS of PROBABILITY (Part 01)

Q. In how many ways B'days of 6 different persons will fall in (i) 1 Calendar Months (ii) In exactly 2 calendar Months? (ii) Ans: (66×62)

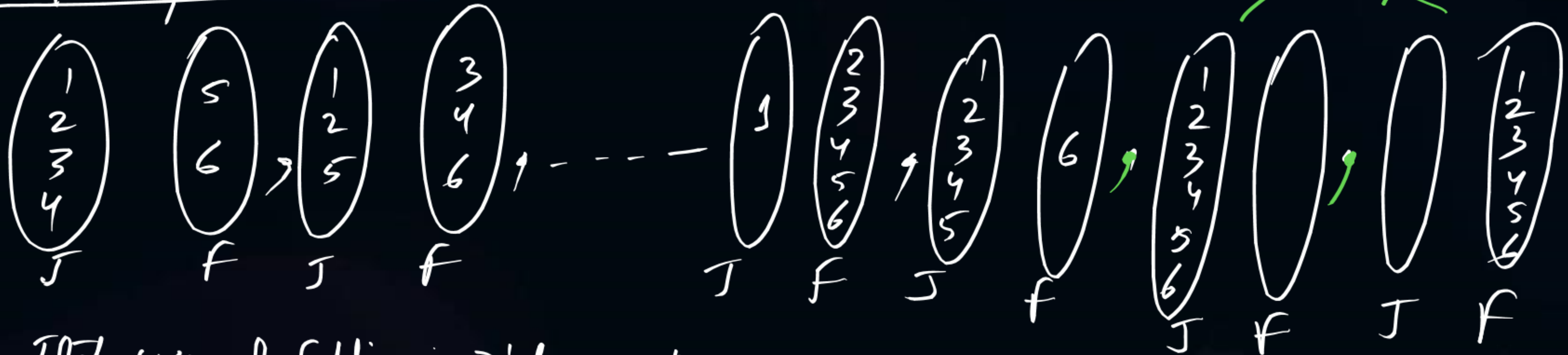
Sol: (1) Month = 12, Persons = 6 so $\underbrace{\text{Total ways of falling B'day}}_{\text{(No Restriction)}} = \underset{P_1}{12} \times \underset{P_2}{12} \times \underset{P_3}{12} \times \underset{P_4}{12} \times \underset{P_5}{12} \times \underset{P_6}{12} = (12^6)$ ways

(2) No. of ways to select 2 Months $= {}^{12}C_2 = 66$ ways

for eg

$\underbrace{\text{J} \quad \text{F}}_{1^{st}\text{ Pair}}, \underbrace{\text{J} \quad \text{M}}_{2^{nd}\text{ Pair}}, \underbrace{\text{J} \quad \text{A}}_{3^{rd}\text{ Pair}}, \underbrace{\text{J} \quad \text{May}}_{4^{th}\text{ Pair}}, \ldots \ldots \underbrace{\text{Oct} \quad \text{Nov}}_{64^{th}\text{ Pair}}, \underbrace{\text{Oct} \quad \text{Dec}}_{65^{th}\text{ pair}}, \underbrace{\text{Nov} \quad \text{Dec}}_{66^{th}\text{ Pair}}$

let us take 1^{st} pair :



Total ways of falling B'day in these months $= \underset{P_1}{2} \times \underset{P_2}{2} \times \underset{P_3}{2} \times \underset{P_4}{2} \times \underset{P_5}{2} \times \underset{P_6}{2} = 2^6 = 64$

So for Exactly 2 month fav cases $= 64 - 2 = 62$ ways.

Hence Req Ans $=$ $1^{st} \longrightarrow 62$ ways
$2^{nd} \longrightarrow 62$;
$3 \longrightarrow 62$;
⋮
$66^{th} \longrightarrow 62$;

is Ans $= 66 \times 62 = {}^{12}C_2 \times (2^6 - 2)$ ways

Note- $P(\text{exactly 2 month}) = \dfrac{66 \times 62}{12^6}$

PROBABILITY (Possibility)

(1) % = Base is of 100 units (2) Prob = Base is of 1 unit

RANDOM EXPERIMENT → whenever we are not sure about the outcome of an experiment the such types of experiments are called R. Exp.

Sample space (S) Total possible outcomes associated with Random Experiment, when written in a set form, is called S. space. for eg

$D = \{1, 2, 3, 4, 5, 6\}$, $C = \{H, T\}$, $F = \{B, G, T\}$

Event (E) Any subset of S is called Event for eg

$D = \{1, 2, 3, 4, 5, 6\}$, $E_1 = \{1, 3, 5\}$, $E_2 = \{2, 4, 6\}$, $E_3 = \{1, 2, 3, 4\}$, ... etc

Note Total No. of events associated with Sample space S having n elements $= 2^n$

Note: if $A=\{a,b,c\}$ then Total Subsets of A are $= 2^3 = 8$

eg: $\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}, \phi$ ie Total $= 8$

Impossible Event → $\because \phi \subset S$ so ϕ is also an Event & it is Called Impossible Event.

& $P(\phi) = 0$

Sure Event/Certain Event → $\because S \subseteq S$ so S is also an Event & it is Called Sure Event &

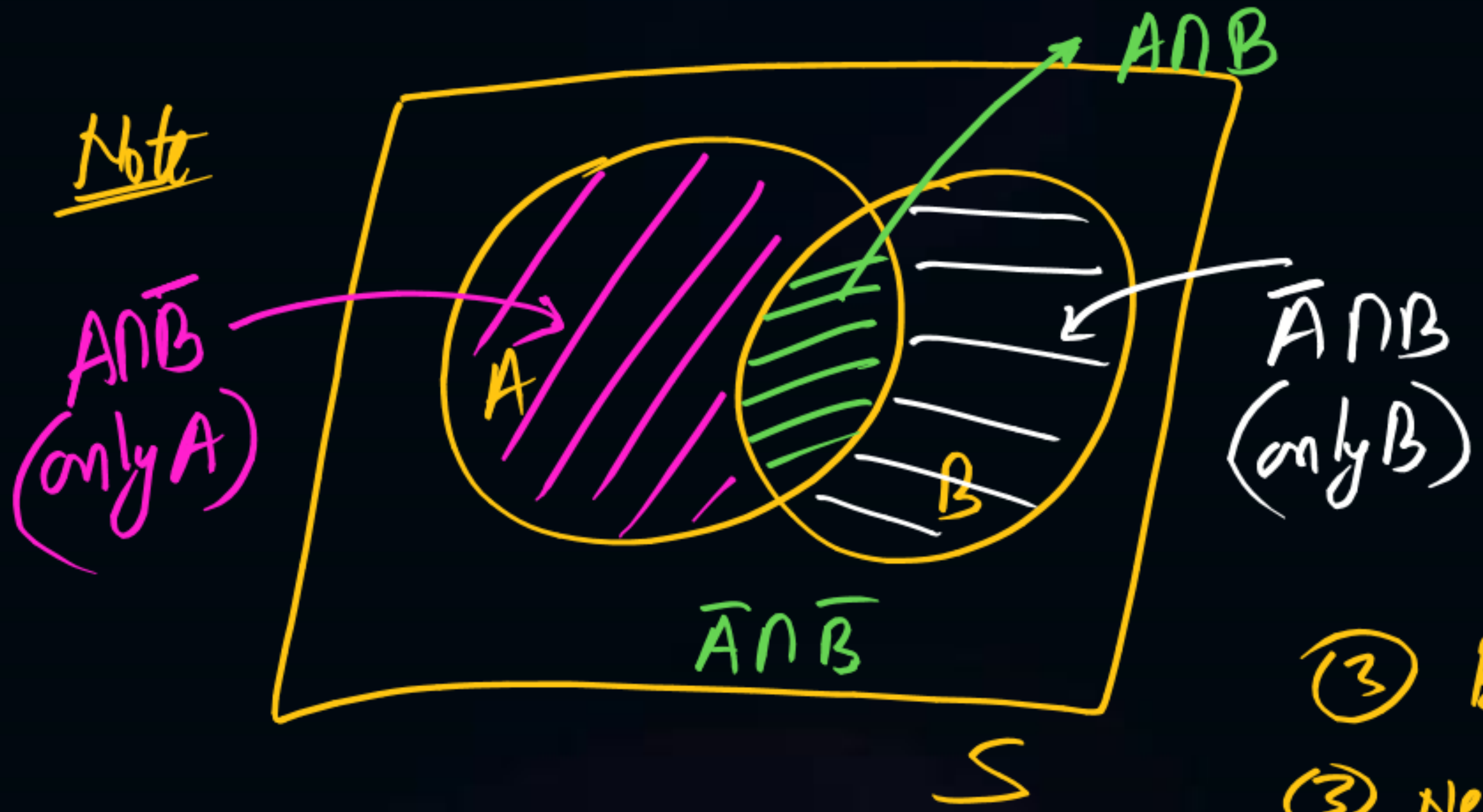
$P(S) = 1$

Note → ① P(Nothing occurs) = 0, ② P(something occurs) = 1

③ P(given that ....) = 1 ④ P(Death) = 1, ⑤ P(GOD) = 1

⑥ $0 \leq P(E) \leq 1$

Note



① $(A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B) = A \cup B$

(*) $A \cap \bar{B} = A - A \cap B$ & $\bar{A} \cap B = B - A \cap B$

② Either A or B or Both = Atleast A or B = $A \cup B$

③ Both A & B = Simultaneous occurrence of A & B = $A \cap B$

③ Neither A Nor B = None of A & B = $\bar{A} \cap \bar{B}$

④ Addition Theorem of Prob → $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$ (w/o proof)

& $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

⑤ Multiplication theorem of Prob → $\boxed{P(A \cap B) = P(A/B) \cdot P(B)}$

⑥ P(Neither A Nor B) = 1 − P(either A or B or Both) ⇒ $\boxed{P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)}$

⑦ $\boxed{\text{P(atleast one)} = 1 - \text{P(None)}}$

Note (Neither A Nor B) or (A But not B) or (B but not A) or (Both A & B) = Total possibilities

$$\underset{\text{(None)}}{(\bar{A}\cap\bar{B})} + \underbrace{(A\cap\bar{B}) + (\bar{A}\cap B) + (A\cap B)}_{\text{At least one of A or B}=(A\cup B)} = S$$

ie At least one of A or B = S − (None of A & B)

$$\boxed{P(\text{at least one}) = 1 - P(\text{None})}$$

Mutually Exclusive Events →

Two Events A & B are called Mutually Exclusive if "they can't occur simultaneously" i.e

[A & B are ME] if $\Longleftrightarrow$ [$A \cap B = \phi$]

→ $P(A \cap B) = 0$

→ $P(A \cup B) = P(A) + P(B) - 0$

eg $S_{Dice} = \{1, 2, 3, 4, 5, 6\}$ & let $E_1 = \{1, 3, 5\}$, $E_2 = \{2, 4, 6\}$, $E_3 = \{1, 2, 3, 4\}$

$\because E_1 \cap E_2 = \phi$ so E_1 & E_2 are ME

$\because E_1 \cap E_3 = \{1, 3\} \neq \phi$ so E_1 & E_3 are not ME

$\because E_2 \cap E_3 = \{2, 4\} \neq \phi$ so E_2 & E_3 are not ME

*Special Note → ME Events are associated with same sample space

while Ind Events " " " with Different sample spaces.

Independent Events → "if occurrence or Non occurrence of one event do not alter the occurrence or Non occurrence of other events then events are Called Ind Events"

$$\boxed{A \,\&\, B \text{ are Ind}} \iff \boxed{P(A\cap B) = P(A)\cdot P(B)}$$

Note: In Case of Independency, we can multiply individual probabilities in order to find their simultaneous probability i.e if A, B, C are Ind then $P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C)$

Eg: $S_{coin} = \{H, T\}$, $S_{Dice} = \{1,2,3,4,5,6\}$

$A = \{H\}$, $B = \{No \le 4\} = \{1,2,3,4\}$

$P(A) = 1/2$, $P(B) = \frac{4}{6}$

$\because$ A & B are Ind then

$P(A\cap B) = ? = P(A)\cdot P(B) = \frac{1}{2}\times\frac{4}{6}$

Doubt: Here in above Question, $A\cap B = \phi$ then why $P(A\cap B)$ is not Coming Zero in this Quest?

Senseless Discussion.

Nature of Elements in Sample Space → if we are repeating our R Exp n times then elements of sample space are in the form of ordered n-tuples

(i) if Die is thrown once, then $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

(ii) if Die is thrown twice, then
$$S = \left\{\begin{matrix} (11) & (12) & (13) & (14) & (15) & (16) \\ (21) & (22) & (23) & \text{---} & & (26) \\ (31) & (32) & \cdots & & & (36) \\ (41) & \cdots & (51) & \cdots & & (66) \end{matrix}\right\} \Rightarrow n(S) = \underset{D_1}{6} \times \underset{D_2}{6} = 36 \text{ pair} = 36 \text{ two tuples}$$

(iii) A Coin is tossed 5 times then $S = \{(HHHHH), (HHHHT), (HHHTT), \ldots, (TTTTT)\} \Rightarrow n(S) = 2^5 = 32$ five tuples

(iv) A Couple has 3 children. then
$$S = \left\{\begin{matrix} (BBB), & (BBG) & (BGB) & (BGG) \\ (GBB), & (GBG), & (GGB), & (GGG) \end{matrix}\right\} \Rightarrow n(S) = 2^3 = 8 \text{ Triplets}$$

(*) Various Approaches to Solve Question →

Approach 1 :- By listing all the elements of Sample space (S) & Favourable Event (E)

Req prob $P(E) = \dfrac{n(E)}{n(S)}$

App 2 :- if it is not easy to write sample space and Fav. Event (E) then directly find Fav No. of Cases and Total No of Cases by using the Concept of P&C

& then Req Prob = $\dfrac{\text{Fav Cases}}{\text{Total Cases}}$

App (3) :- By using Some Standard Definitions & Standard theorems.

if given information is in the form of Probability then we can use App (3)

Note :- Favourable → which is Required should be assumed as favourable.

Q: Four Dices are thrown simultaneously then find the prob that sum of the outcomes on upper faces will be 22 ?

Sol: **App ①** $S = \begin{Bmatrix} (1111),(1112),(1113),(1114),(1115),(1116) \\ (2111),(2112),(2113)\ \ldots\ldots\ldots\ (6666) \end{Bmatrix} \Rightarrow n(S) = \underset{D_1}{6} \times \underset{D_2}{6} \times \underset{D_3}{6} \times \underset{D_4}{6} = 6^4$

$= 1296$ Quadruples

fav Cases = fav Quadruples = $\{$sum is 22$\}$ = $\begin{Bmatrix} (6664),(6646),(6466),(4666) \\ (6655),(6565),(6556) \\ (5566),(5656),(5665) \end{Bmatrix} = 10$

Hence Req Prob $= \frac{\text{fav}}{\text{Total}} = \frac{10}{1296}$

App II → fav Cases $= \begin{Bmatrix} \text{eg. } (6664)\ldots\ldots \\ \text{eg. } (6655)\ldots\ldots \end{Bmatrix}$

$\frac{4!}{3!} = 4$

$\frac{4!}{2!2!} = 6$

i.e fav outcomes = 10

Req Prob $= \frac{f}{T} = \frac{10}{6^4} = \frac{10}{1296}$

Qs 7 Surgical Strikes occurred in a week from INDIA on PAKISTAN then find the prob that all will occur on a same day ?

Sol: **App II** Total ways of occurring S Strikes $= \underset{S_1}{7} \times \underset{S_2}{7} \times \underset{S_3}{7} \times \underset{S_4}{7} \times \underset{S_5}{7} \times \underset{S_6}{7} \times \underset{S_7}{7} = 7^7$ ways.

(No Restriction / RA)

Fav ways of occurring S Strikes = (All will occur on a same day)

$= \underset{S_1}{7} \times \underset{S_2}{1} \times \underset{S_3}{1} \times \underset{S_4}{1} \times \underset{S_5}{1} \times \underset{S_6}{1} \times \underset{S_7}{1} = 7$ ways

Explanation:

S_1	S_2	S_3	S_4	S_5	S_6	S_7
(M	M	M	M	M	M	M)
or (T	T	T	T	T	T	T)
(W	W	W	W	W	W	W)
⋮						
or (S	S	S	S	S	S	S)

Hence Req Prob $= \frac{f}{T} = \frac{7}{7^7} = \frac{1}{7^6} = \frac{{}^7C_1}{7^7}$

THANK - YOU

DATA SCIENCE & ARTIFICIAL INTELLIGENCE

Probability

Lecture No. 02

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

Theoretical Concepts of Probability

Topics to be Covered

Topic

BASICS of PROBABILITY (QUESTIONS)

(Part 2)

Note :- Learn By →

Operation	P&C	Prob	ME	Ind
Either-or	Add	union (use Addition Th)	$P(A \cup B) = P(A) + P(B)$	$P(A \cup B) = P(A) + P(B) - P(A)P(B)$
AND	Multiply	Intersection (use Multi Theorem)	$P(A \cap B) = 0$	$P(A \cap B) = P(A) \cdot P(B)$

Fundamental Quest 1 → Q: Two Dice are thrown simultaneously then write it's Sample Space.

$$S = \left\{ \begin{matrix} (11)(12)(13)\ldots(16),(21),(22)\ldots(26) \\ (31)(32)\ldots(36),(41),(42),\ldots(46) \\ (51),(52)\ldots(56),(61),(62)\ldots(66) \end{matrix} \right\} \Rightarrow n(S) = \underset{D_1}{\underline{6}} \times \underset{D_2}{\underline{6}} = 36 \text{ pair}$$

App 1

① find the prob that sum of the outcomes will be 8 = ? $\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$ Ans

$A = \{\text{sum is } 8\} = \{ \cancel{(17)}\ (26),(62),(35),(53),(44) \} \Rightarrow n(A) = 5$

App 2

② find the prob that sum is 9 = ? $\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{4}{36}$ Ans

$B = \{\text{sum is } 9\} = \{(36),(63),(45),(54)\} \Rightarrow n(B) = 4$

App 3

③ find the prob that sum is Both 8 & 9 ?

$\because A \cap B = \phi \Rightarrow A \,\&\, B$ are ME so $P(A \cap B) = 0$

App 3

④ find the Prob that sum is either 8 or 9 ? $\Rightarrow P(A \cup B) = P(A) + P(B) - 0 = \frac{5}{36} + \frac{4}{36} = \frac{9}{36}$

App 3 Gate

⑤ " " " " sum is Neither 8 Nor 9 ? $\Rightarrow P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - \frac{9}{36} = \frac{3}{4}$

PYQ

(6) find the prob that all the outcomes are identical = ?

App 1

(a) 1/6 (b) 8/36

(c) 0 (d) 1

$C = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\} \Rightarrow n(C) = 6$

$P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

PYQ

(7) find the prob that, product on the upper faces will be perfect square

App 1

(a) 1/6 (b) 8/36

(c) 0 (d) 1

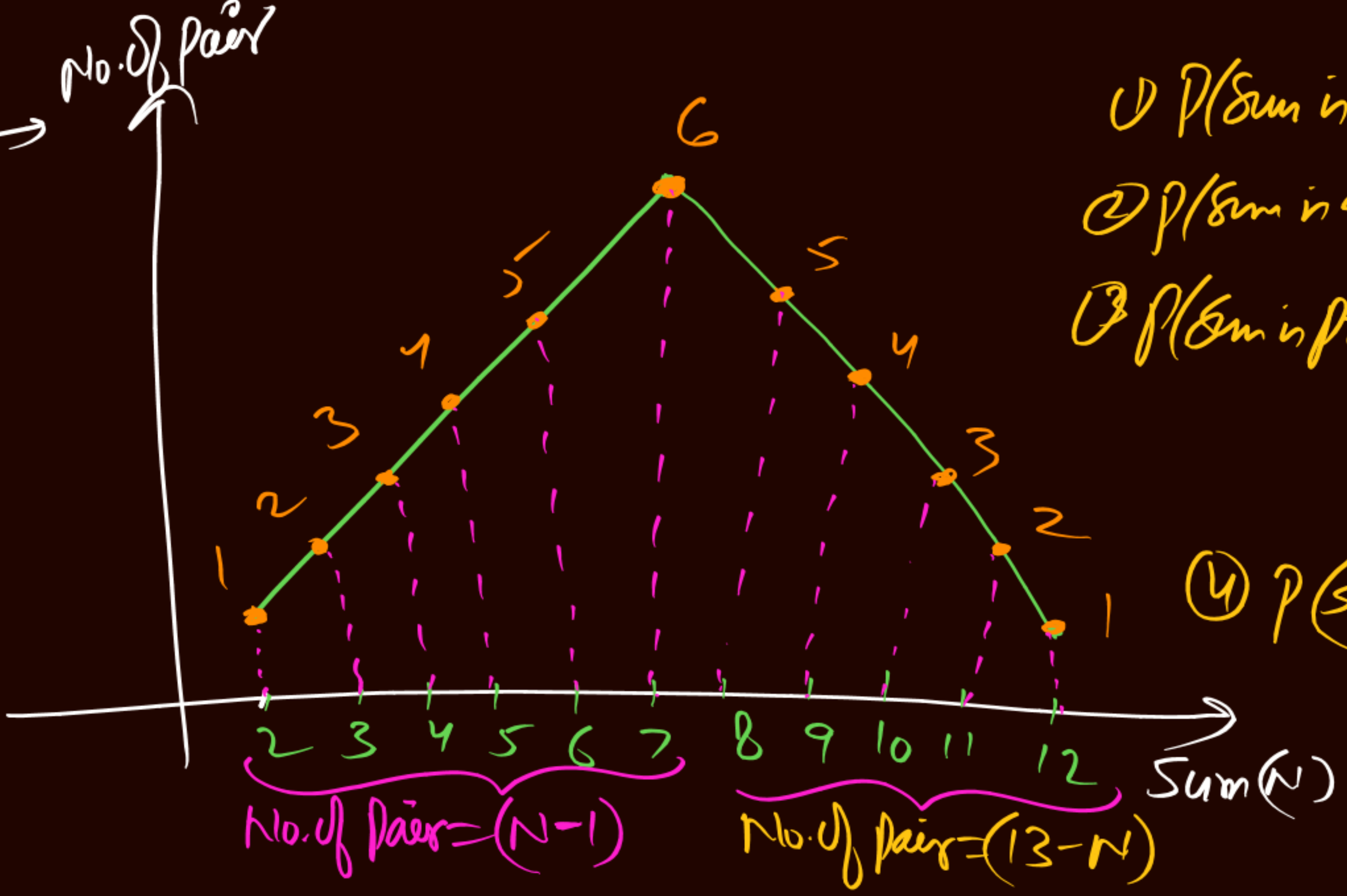
$D = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,4),(4,1)\} \Rightarrow n(D) = 8$

$P = \frac{8}{36}$

(8) find the Prob that sum of the Nos on upper faces will be Prime Number = ?

$E = \{\text{Sum is Prime No}\} = \{\text{Sum} = 2,3,5,7,11\} = \frac{15}{36}$ (will be cleared by Shortcut)

Shortcut →



1) $P(\text{Sum is } 8) = \frac{5}{36}$

2) $P(\text{Sum is } 9) = 4/36$

3) $P(\text{Sum is prime No}) = P(\text{Sum} = 2,3,5,7,11)$

$= \frac{1+2+4+6+2}{36} = \frac{15}{36}$

4) $P(\text{Sum is divisible by } 4) = ?$

$= P(\text{Sum} = 4 \text{ or } 8 \text{ or } 12)$

$= \frac{3+5+1}{36} = \frac{9}{36}$

5) $P(\text{Sum is atleast } 9) = ?$

$= P(\text{Sum} = 9,10,11,12) = \frac{4+3+2+1}{36} = \frac{10}{36}$

Fundamental Quest 2 :- A Coin is tossed 6 times then write it's sample space ?

$$S = \{(HHHHHH), (HHHHHT), (HHHHTT), (HHHTTT), (HHTTTT), (HTTTTT), (TTTTTT)\}$$

$${}^6C_6 = 1,\quad {}^6C_5 = 6,\quad {}^6C_4 = 15,\quad {}^6C_3 = 20,\quad {}^6C_2 = 15,\quad {}^6C_1 = 6,\quad {}^6C_0 = 1$$

$\Rightarrow n(S) = \underset{C_1}{2} \times \underset{C_2}{2} \times \underset{C_3}{2} \times \underset{C_4}{2} \times \underset{C_5}{2} \times \underset{C_6}{2} = 2^6 = 64$ Six tuples, $P(H) = P(T) = \frac{1}{2}$, All tosses are Ind

① Find the Prob that all the outcomes are identical?

App 1 :- $A = \{(HHHHHH), (TTTTTT)\} \Rightarrow n(A) = 2$

So $P(A) = \frac{2}{64} = \frac{1}{32}$

App ③ Req Prob $= P[\underbrace{(HHHHHH)}_{ME} \text{ or } \underbrace{(TTTTTT)}_{ME}]$

$$= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^5$$

② Find the Prob that H & T appears alternately

App 1 :- fav outcomes $= \{(HTHTHT), (THTHTH)\} = 2$

So Prob $= \frac{\text{fav}}{\text{Total}} = \frac{2}{64}$

App 3 :- Req Prob $= P[(HTHTHT) \text{ or } (THTHTH)]$ ME

$$= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^5$$

(3) find the prob that Both H & T appears atleast once?

App(1). unfav outcomes $= \{(HHHHHH), (TTTTTT)\} = 2$

So fav. outcomes $= 64 - 2 = 62$

i.e Req prob $= \frac{fav}{Total} = \frac{62}{64}$

(4) find the prob that H appears at least once?

App(1) unfav $= \{(TTTTTT)\} = 1$ so fav $= 64 - 1 = 63$

Hence Req Prob $= \frac{fav}{Total} = \frac{63}{64}$

App(3) → $P(\text{at least one } H) = 1 - P(\text{No Head})$

$= 1 - P(\text{all } T)$

$= 1 - P(TTTTTT) = 1 - \left(\frac{1}{2}\right)^6$

(5) find the prob that H and T appears equal number of times?

Sol: $P(H \,\&\, T \text{ appears equal no. of times})$

$= P(\text{exactly } 3H \text{ appears})$

$= \frac{fav}{Total} = \frac{{}^6C_3}{2^6} = \frac{20}{64}$

(6) If 1^{st} 3 outcomes are H, H, H then find the prob of getting T when coin is tossed again?

condition

Sol: M-I Req Prob = $P(T \text{ in } 4^{th} \text{ toss}) = \frac{1}{2}$

App 3

M-II Req Prob = $P[\underbrace{HHH}_{\text{given}} T \underbrace{\text{Something occurs}}]$

$= (1\times1\times1)\times\frac{1}{2}\times(1\times1) = \frac{1}{2} \neq \frac{32}{64}$

whenever our Question is Based on Conditional Prob then we can't generalize the final Ans

App(1) will be discussed in the chapter of Conditional Prob

(7) If 1^{st} three outcomes are all heads then find the prob of occurring Tail in Remaining tosses?

condition.

Sol: App III

Req Prob $= P[\underbrace{HHH}_{\text{given}} TTT] = 1^3 \times \left(\frac{1}{2}\right)^3$

$= \frac{1}{8} \neq \frac{8}{64}$

(8) Find the prob that 1st two tosses produces H ?

Sol: App(3):

$P[1^{st}$ two tosses produces $H] = P[H\,H\,\underbrace{\text{something occurs}}]$

$= \left(\frac{1}{2}\right)^2 \times 1^4 = \frac{1}{4}$

App(1): fav elements $= \{(H\,H\,\_\_\_\_\_\_\_\_)\ldots\} = ?$

= Tough to count.

App(2) → fav elements $= \{$eg $(\underline{H}\ \underline{H}\ \underline{H/T}\ \underline{H/T}\ \underline{H/T}\ \underline{H/T})$ ...... $\}$

$= 1 \times 1 \times 2 \times 2 \times 2 \times 2 = 16$

Hence Req Prob $= \frac{fav}{Tol} = \frac{16}{64} = \frac{1}{4}$

(9) Find the prob that only 1st two tosses produces H ?

Sol App(2)

fav outcomes $= \{(H\,H\,T\,T\,T\,T)\} = 1$

Req. Prob $= \frac{fav}{Total} = \frac{1}{64}$

App(3)

$P(\text{only } 1^{st}$ two tosses produces $H) = P[(H\,H\,\underline{T\,T\,T\,T})]$

$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^6$

(11) Find the prob that exactly 2 H produces ?

$= \frac{{}^6C_2}{2^6} = \frac{\frac{6!}{2!\,4!}}{64} = \frac{15}{64}$

(10) find the prob that exactly 3 tosses produces Head ?

App(1): fav tuples = { eg (HTTHTH) ...... } = ? Tough to Count

App(2): fav tuples = { eg (HTTHTH) ...... } = $\frac{6!}{3!\,3!} = {}^6C_3 = 20$

So Req Prob $= \frac{fav}{Total} = \frac{20}{64} = \frac{{}^6C_3}{2^6}$ Ans

App 3: use Binomial Dist.

(11) find the prob that exactly 2 H appears ? $= \frac{fav}{Total} = \frac{{}^6C_2}{2^6} = \frac{15}{64}$

(12) " " of getting at least 4 Heads = ? $= P(\text{exactly } 4H) \text{ or } P(\text{exactly } 5H) \text{ or } P(\text{exactly } 6H)$

$= \frac{{}^6C_4}{2^6} + \frac{{}^6C_5}{2^6} + \frac{{}^6C_6}{2^6} = \frac{15+6+1}{64}$

HW8 A coin is tossed 10 times then find the prob that

① exactly 3 H appears $= ?$ $= \frac{fav}{Total} = \frac{^{10}C_3}{2^{10}} = \frac{\left(\frac{10!}{3!\,7!}\right)}{120} = \frac{120}{1024}$

② 4^{th} head appears in 9^{th} toss $= ?$ $(7/64)$

App II

App III

THANK - YOU

DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE

Probability

Lecture No. 03

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

THEORETICAL CONCEPTS

Topics to be Covered

Topic

BASICS of PROBABILITY (QUESTIONS)
(Part 3)

& Tree diagram Concept

Note : Learn By ♡ →

Operation	P&C	Prob	ME	Ind
Either-or	Add	union (use Addition Th)	$P(A\cup B)=P(A)+P(B)$	$P(A\cup B)=P(A)+P(B)-P(A)P(B)$
AND	Multiply	Intersection (use Multi Theorem)	$P(A\cap B)=0$	$P(A\cap B)=P(A)\cdot P(B)$

HW8 A coin is tossed 10 times then find the prob that

(1) exactly 3 H appears $= ? \quad = \dfrac{fav}{Total} = \dfrac{{}^{10}C_3}{2^{10}} = \dfrac{\frac{10!}{3!\,7!}}{120} = \dfrac{120}{1024}$

(2) 4^{th} head appears in 9^{th} toss $= ?$ $(7/64)$

Sol(2) App I: Total Cases $= 2^{10} = (1024)$ Ten tuples

fav Cases $= \{ \text{eg} \left(\underbrace{\overset{\text{exactly } 3H}{\text{\_ \_ \_ \_ \_ \_ \_ \_}}}_{\text{In } 1^{st}\ 8 \text{ tosses}} \ \underset{9^{th}}{\underline{H}} \ \underset{10^{th}}{\underline{H/T}} \right) \ldots\ldots \} = {}^{8}C_3 \times 1 \times 2$ So Req Prob $= \dfrac{{}^{8}C_3 \times 1 \times 2}{2^{10}}$

$= \dfrac{{}^{8}C_3}{2^8} \times \left(\dfrac{1}{2}\right) \times \left(\dfrac{2}{2}\right) = \dfrac{7}{64}$

App II Req Prob $= P[4^{th}\ H \text{ in } 9^{th} \text{ toss}]$

$= P(\text{exactly } 3H \text{ in } 1^{st}\ 8 \text{ tosses}) \times P(H \text{ in } 9^{th} \text{ toss}) \times P(\text{something occurs in } 10^{th} \text{ toss})$

$= \left(\dfrac{{}^{8}C_3}{2^8}\right) \times \dfrac{1}{2} \times 1 = \dfrac{7}{64}$

Q. 6 Dice are thrown simultaneously then find the prob that

(1) All will show the same face = ? $= \frac{f}{T} = \frac{{}^6C_1}{6^6} = \frac{6}{6^6} = \frac{1}{6^5}$

$S = \left\{ \begin{matrix} (111111), (111112), \ldots (111116) \\ (211111), (211112), \ldots (666666) \end{matrix} \right\} \Rightarrow n(S) = 6^6$, fav tuples $= \left\{ \begin{matrix} (111111), (222222), (333333) \\ (444444), (555555), (666666) \end{matrix} \right\} = 6$

(2) All will show the different faces = ? $= \frac{f}{T} = \frac{6!}{6^6}$

where fav cases = $\{$ eg $(\underset{D_1}{1}\,\underset{D_2}{2}\,\underset{D_3}{3}\,\underset{D_4}{4}\,\underset{D_5}{5}\,\underset{D_6}{6}), \ldots\ldots \} = 6!$

(3) Exactly 3 will show the same face and Rest three different faces = ? $= \frac{f}{T} = \frac{{}^6C_1 \times {}^5C_3 \times \frac{6!}{3!}}{6^6}$

eg (414435) ......

(4) At least 4 will show the same face = ? = 4 same face + 5 same face + 6 same faces

$= \frac{{}^6C_1 \times {}^5C_2 \times \frac{6!}{4!}}{6^6} + \frac{{}^6C_1 \times {}^5C_1 \times \frac{6!}{5!}}{6^6} + \frac{{}^6C_1 \times \frac{6!}{6!}}{6^6}$ = Do yourself

eg (515255) (333332)

Q. Aishwarya studies either CS or Maths on each day.

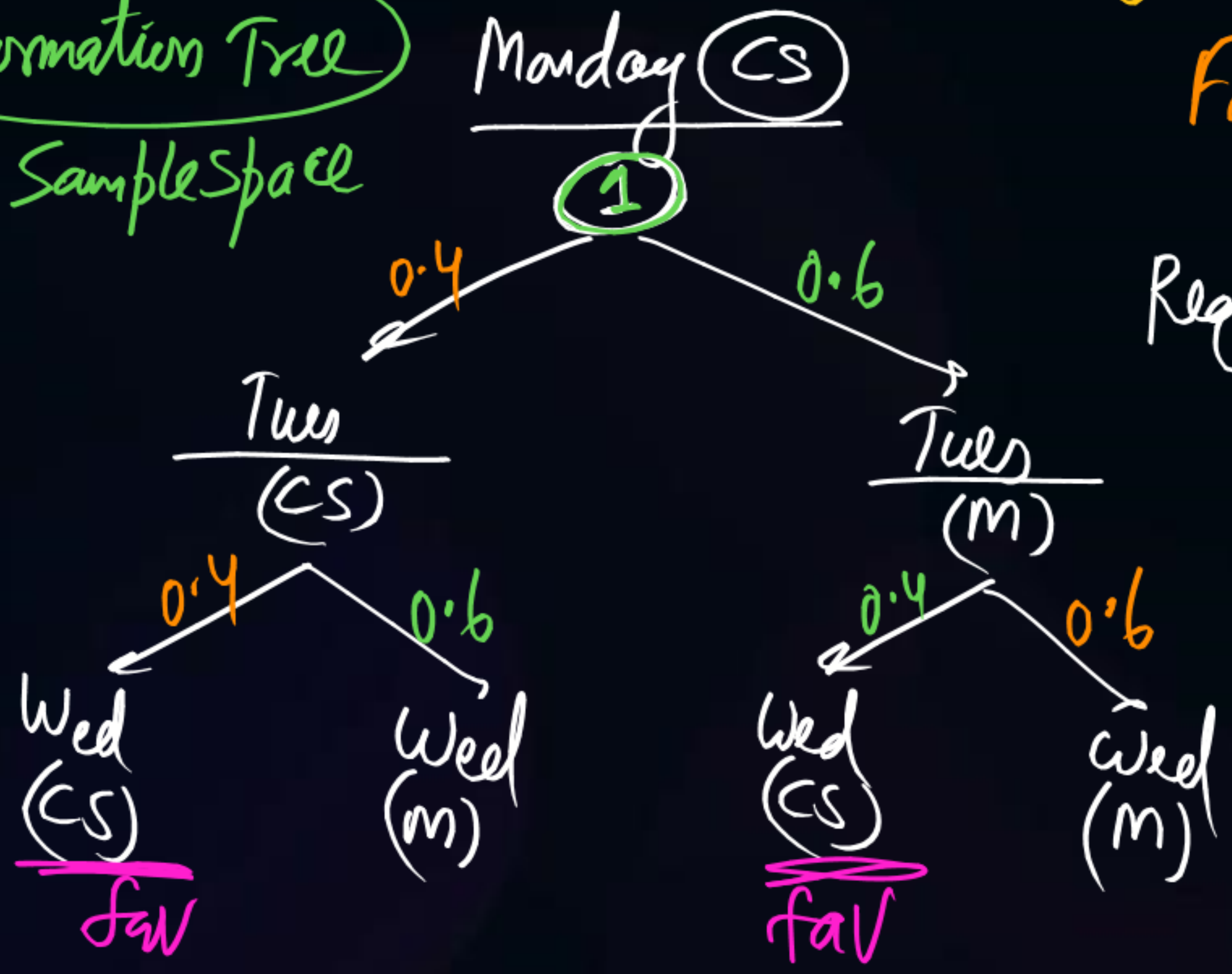
If she studies CS on a day then prob of studying M on next day is 0.6 &

" " M " " " " " " CS " " " is 0.4.

Given that Aish studies CS on Monday then find the prob that she will also study CS on Wed.

Information Tree

≈ Sample Space



Fav Path $= \begin{pmatrix} \frac{M}{CS} & \frac{T}{CS} & \frac{W}{CS} \end{pmatrix}$ or $\begin{pmatrix} \frac{M}{CS} & \frac{T}{M} & \frac{W}{CS} \end{pmatrix}$

Req Prob $= P[CS\ CS\ CS] \cup P[CS\ M\ CS]$

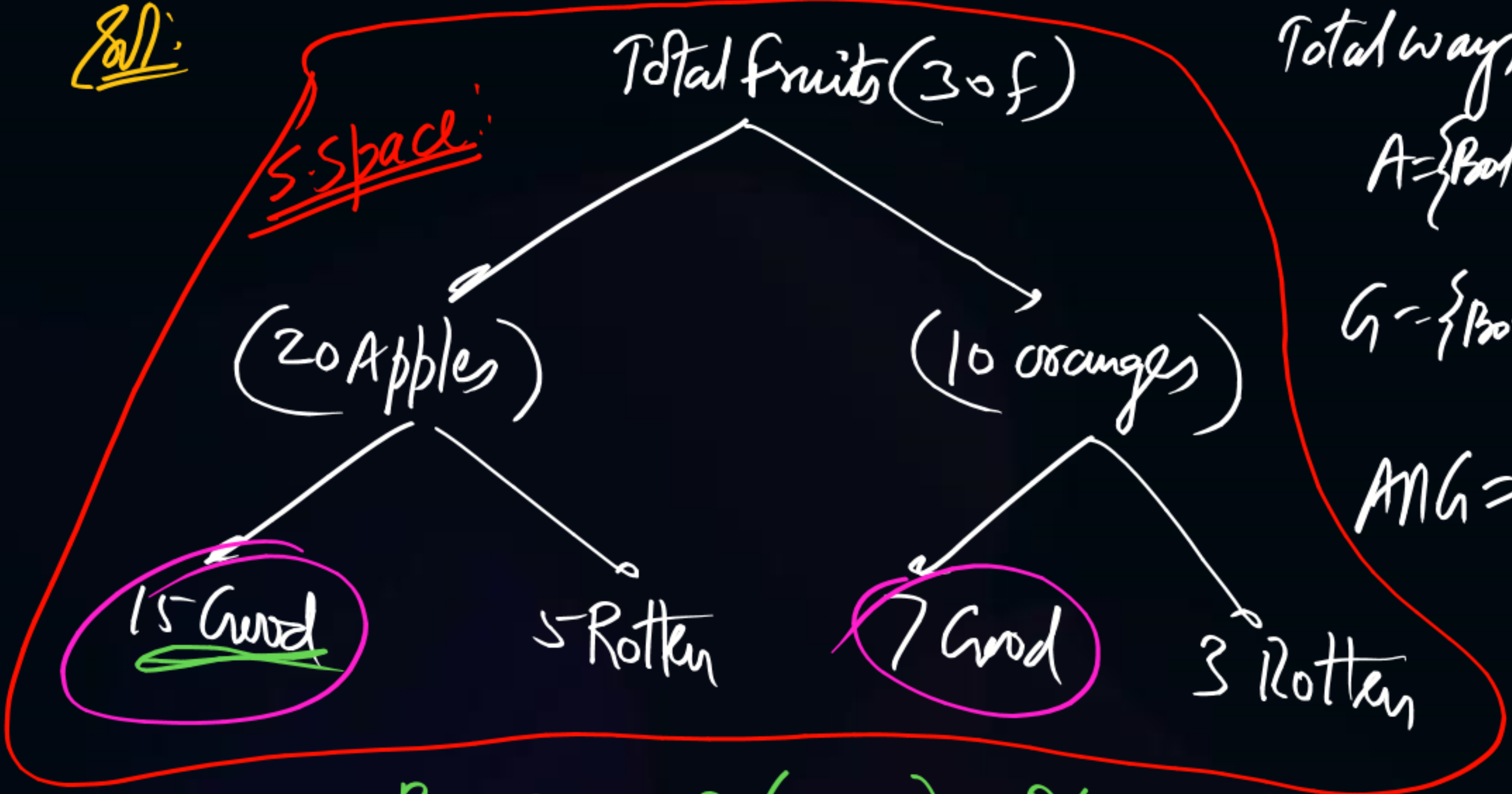
ME Paths

$= (1 \times 0.4 \times 0.4) + (1 \times 0.6 \times 0.4)$

$= 0.4 \approx 40\%$ chance that she will also study CS on Wed.

Q. A Basket (20) Apples and (10) oranges in which 5 Apples and 3 oranges are Rotten.
If two fruits are choosen at Random then find the prob that,
Either both are apples or both are Good? (min Question of App 2 & App 3)

Sol:



Total ways of selecting 2f $= {}^{30}C_2 = 435$ pair

$A = \{\text{Both fruit are Apples}\} \Rightarrow P(A) = \dfrac{{}^{20}C_2}{{}^{30}C_2} = \dfrac{190}{435}$

$G = \{\text{Both fruits are Good}\} \Rightarrow P(G) = \dfrac{{}^{22}C_2}{{}^{30}C_2} = \dfrac{231}{435}$

$A \cap G = \{\text{Both Fruits are Good Apples}\}$

is $P(A \cap G) = \dfrac{{}^{15}C_2}{{}^{30}C_2} = \dfrac{105}{435}$

Req Prob $= P(A \cup G) = P(A) + P(G) - P(A \cap G) = \dfrac{190 + 231 - 105}{435} = \dfrac{316}{435}$

Q. A Coin is tossed until Head appears then find the prob that Req No. of tosses will be odd?

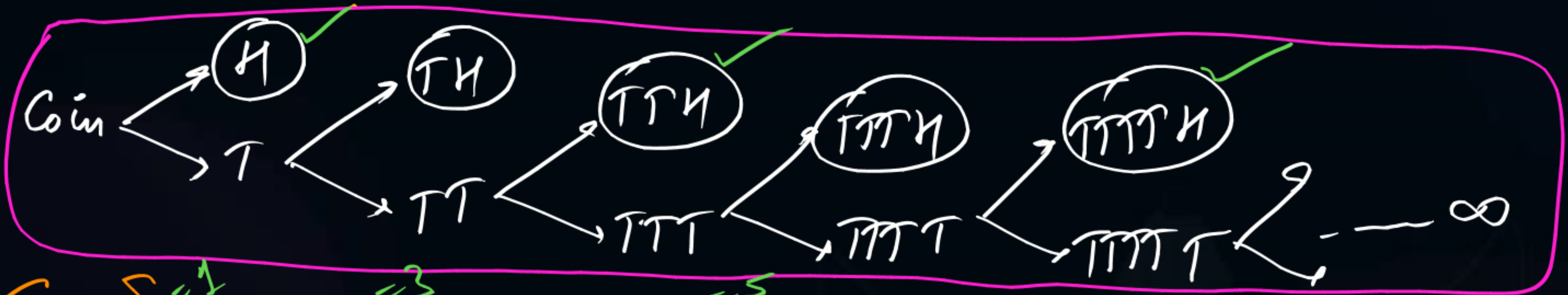
or A Coin is tossed " " " " " " " " Such type of Game will end in odd number of tosses ?

(a) 1/2

(b) 2/3

(c) 1/3

(d) 0



$S = \{\underset{=1}{H}, TH, \underset{=3}{TTH}, TTTH, \underset{=5}{TTTTH}, \ldots\ldots\}$

Fav Cases = {odd No. of Tosses are Required} = $\{H, TTH, TTTTH, \ldots\}$

Here $P(H) = \frac{1}{2} = P(T)$ And all tosses are Independent

Req Prob $= P[H \text{ or } TTH \text{ or } TTTTH \text{ or } \ldots]$

$= P(H) + P(TTH) + P(TTTTH) + \ldots$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$

$P_{\text{odd tosses}} = \dfrac{1/2}{1-\left(1/2\right)^2} = \dfrac{2}{3}$

$$\boxed{S_\infty = a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r} \quad \because -1 < r < 1}$$

Similarly $P_{(\text{even tosses})} = 1 - \frac{2}{3} = \frac{1}{3}$

HW Two Persons A & B play a Game of Dice alternately in which any one can win if 6 appears 1^{st} time. then find their Respective chances of winning? if A begins.

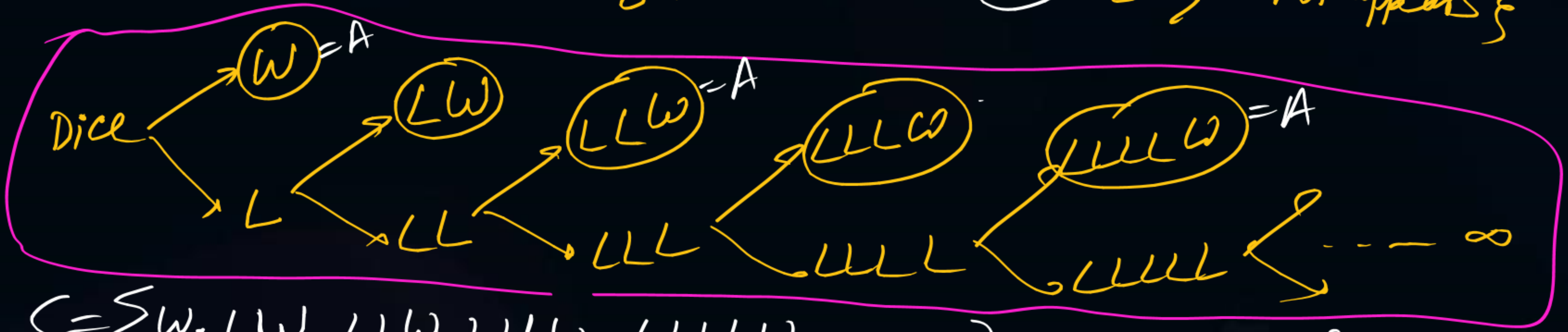
$P(A\text{ win}) = 6/11$

$P(B\text{ win}) = 5/11$

Soln: $P(\text{win}) = \frac{1}{6}$, $P(\text{Loose}) = \frac{5}{6}$, A & B are Ind,

$W = \{\text{six appears}\}$

$L = \{\text{six Not appears}\}$



$S = \{W, LW, LLW, LLLW, LLLLW, \ldots\}$

fav cases for A $= \{W, LLW, LLLLW, \ldots\}$

$P(A) = P(W) + P(LLW) + P(LLLLW) + \ldots$

$= \frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \ldots$

$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\right]$

$= \frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^2}\right] = \frac{6}{11}$

$P(B\,\text{win}) = 1 - P(A\,\text{win}) = \frac{5}{11}$

A, B, C

one after another

HWQ Three persons throw die in succession till one gets six and wins the game find their respective chances of winning? if A starts

Probability

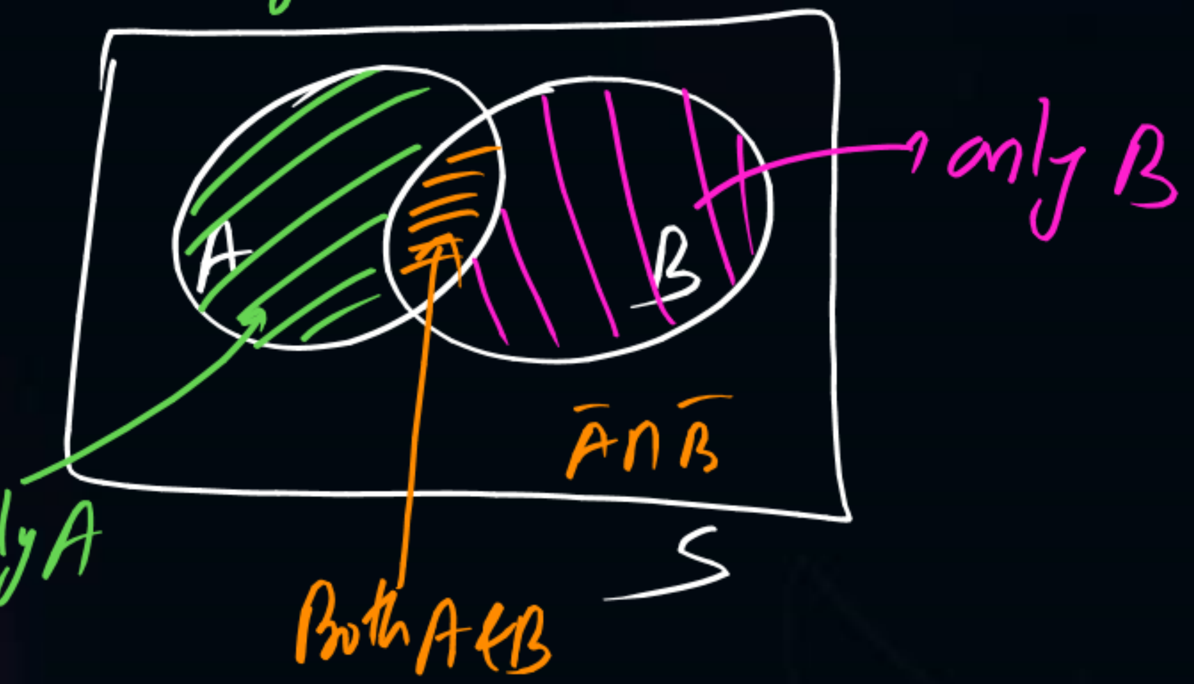
Ans: A → 36/91

B → 30/91

C → 25/91

Concept of M.E and Independency in a Single Questions →

eg Two persons A & B fire at the target once then write the various possible cases



Here A & B are Ind.

(None will hit) or (A hit & B missed) or (A missed & B hit) or (Both will hit) = Total cases

$$\underbrace{(\bar{A}\cap\bar{B})}_{E_1} \cup \underbrace{(A\cap\bar{B})}_{E_2} \cup \underbrace{(\bar{A}\cap B)}_{E_3} \cup \underbrace{(A\cap B)}_{E_4} = S$$

i.e $E_1 \cup E_2 \cup E_3 \cup E_4 = S$ Here E_1, E_2, E_3, E_4 are ME events & these are associated with same S. space S

Analysis:

(1) $\underbrace{(\bar{A}\cap\bar{B})}_{}^{=E_1} \cup \underbrace{\underbrace{(A\cap\bar{B})^{=E_2} \cup (\bar{A}\cap B)^{=E_3}}_{\text{exactly one will hit}} \cup (A\cap B)^{=E_4}}_{\text{At least one will hit}} = \text{S.Space}$

(2) A & B are Ind

$\Rightarrow \left\{\begin{array}{lll} A \,\&\, \bar{B} & \text{''} & \text{''} \\ \bar{A} \,\&\, B & \text{''} & \text{''} \\ \bar{A} \,\&\, \bar{B} & \text{''} & \text{''} \end{array}\right\}$

Q: There are two Gangsters Munna Mobile $= A$ & Pappu Pazer $= B$. they both fire at the target once with prob of their hitting is $\frac{4}{5}$ & $\frac{3}{4}$ respt then find the prob that,

Sol: $P(A)=\frac{4}{5}$, $P(\bar{A})=\frac{1}{5}$, $P(B)=\frac{3}{4}$, $P(\bar{B})=\frac{1}{4}$, A & B are Ind

(1) Both will hit $= ?$ $\Rightarrow P(A\cap B)=P(A)\cdot P(B)=\frac{4}{5}\cdot\frac{3}{4}=\frac{3}{5}$

(2) None will hit $= ?$ $\Rightarrow P(\bar{A}\cap\bar{B})=P(\bar{A})\cdot P(\bar{B})=\frac{1}{5}\cdot\frac{1}{4}=\frac{1}{20}$

(3) At least one will hit $= ?$ $\Rightarrow$ P(at least one will hit) $= 1-$ P(None will hit) $=1-\frac{1}{20}=\frac{19}{20}$

(4) Either of them will hit $= ?$ $\Rightarrow P(A\cup B)= ? = P(A)+P(B)-P(A\cap B)$

$= \frac{4}{5}+\frac{3}{4}-\left(\frac{3}{5}\right)=\frac{19}{20}$ i.e Same as (3)

(5) Target will be hit $= ?$ = At least one will hit = Same as (3,4)

(6) Find the prob that A hit & B missed = ? $\Rightarrow P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})$

$$= \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$$

(7) Find the prob that only one will hit = ?

$$= P(A \cap \bar{B} \text{ or } \bar{A} \cap B)$$

$$= P(E_2 \cup E_3)$$

$$= P(E_2) + P(E_3) \quad (\because E_2 \,\&\, E_3 \text{ are ME})$$

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= \left(\frac{4}{5} \times \frac{1}{4}\right) + \left(\frac{1}{5} \times \frac{3}{4}\right) \quad (\because \text{Events are Ind})$$

$$= \frac{7}{20}$$

(8) HW if exactly one person hit (Condition) then find the prob that A hit & B missed = ?

Ans = 4/7

(a) Speed is OK, (c) Speed is Fast

(b) " " Slow, (d) Can't say anything

THANK - YOU

DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE

Probability

Lecture No. 04

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

THEORETICAL CONCEPTS

Topics to be Covered

Topic CONDITIONAL PROBABILITY

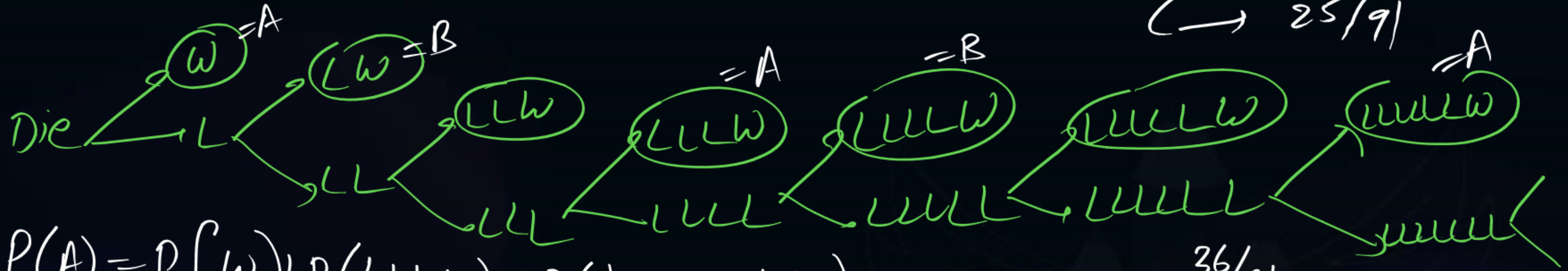
HWQ Three persons (A, B, C) throw die in succession (one after another) till one gets six and wins the game. Find their respective chances (Probability) of winning? if A starts

Ans: $A \rightarrow 36/91$

$B \rightarrow 30/91$

$C \rightarrow 25/91$

Sol: $P(W) = \frac{1}{6}$, $P(L) = \frac{5}{6}$, And all A, B, C are Ind.



$$P(A) = P(W) + P(LLLW) + P(LLLLLLW) + \ldots = 36/91$$

$$P(B) = P(LW) + P(LLLLW) + P(LLLLLLLW) + \ldots = 30/91$$

$$P(C) = 1 - (P(A) + P(B)) = 1 - \left(\frac{36}{91} + \frac{30}{91}\right) = \frac{25}{91}$$

Q: There are two Gangsters Munna Mobile $= A$ & Pappu Pazer $= B$. they both fire at the target once with prob of their hitting is $\frac{4}{5}$ & $\frac{3}{4}$ resp. then find the prob that,

Sol: $P(A) = \frac{4}{5}$, $P(\bar{A}) = \frac{1}{5}$, $P(B) = \frac{3}{4}$, $P(\bar{B}) = \frac{1}{4}$, A & B are Ind

(1) Both will hit $= ?$ $\Rightarrow P(A \cap B) = P(A) \cdot P(B) = \frac{4}{5} \cdot \frac{3}{4} = \frac{3}{5}$

(2) None will hit $= ?$ $\Rightarrow P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B}) = \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$

(3) At least one will hit $= ?$ $\Rightarrow$ P(at least one will hit) $= 1 -$ P(None will hit) $= 1 - \frac{1}{20} = \frac{19}{20}$

(4) Either of them will hit $= ?$ $\Rightarrow P(A \cup B) = ? = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{5} + \frac{3}{4} - \left(\frac{3}{5}\right) = \frac{19}{20}$ i.e. Same as (3)

(5) Target will be hit $= ?$ $=$ At least one will hit $=$ Same as (3/4)

(6) Find the prob that A hit & B missed = ? $\Rightarrow P(A\cap\bar{B}) = P(A)\cdot P(\bar{B})$

$= \frac{4}{5}\times\frac{1}{4} = \frac{1}{5}$

(7) Find the prob that only one will hit = ?

$= P\left((A\cap\bar{B}) \text{ or } (\bar{A}\cap B)\right)$

$= P(E_2 \cup E_3)$

$= P(E_2) + P(E_3)$ ($\because E_2$ & E_3 are ME)

$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$= \left(\frac{4}{5}\times\frac{1}{4}\right) + \left(\frac{1}{5}\times\frac{3}{4}\right)$ ($\because$ Events are Ind)

$= \frac{7}{20}$

(8) HW: if exactly one person hit (Condition) then Find the prob that A hit & B missed = ?

Sol: Original Prob $= P(S) = 1$

Reduced Prob $= P(\text{Condition}) = \frac{7}{20}$

fav Prob $= P(A\cap\bar{B}) = 1/5 = \frac{1/5}{1}$

So Conditional Prob $= \frac{\text{fav Prob}}{\text{R-Prob}} = \frac{1/5}{7/20} = \frac{4}{7}$

Conceptual Questions of Conditional Prob

(we can Reduce the sample space according to condition)

Q. A couple has 2 children find the prob that both are boys?

$S = \{(BB), (BG), (GB), (GG)\} \cong 4$ & $fav = \{(BB)\} \cong 1 \Rightarrow$ Req Prob $= \frac{f}{T} = \frac{1}{4}$

Q. A couple has 2 children, find the prob that both are boys if

(i) $\underbrace{\text{one of the child is Boy}}_{\text{Condition}}$ (App I) : Reduced S Space $= \{(BB), (BG), (GB)\} \cong 3$

Conditional Prob $= \frac{\text{fav Cases}}{\text{R. SSpace}} = \frac{1}{3}$ Here fav $= \{(BB)\} = 1$

(ii) $\underbrace{\text{Elder child is Boy}}_{\text{Condition}}$ (App I) : Reduced SSp $= \{(BB), (BG)\} \cong 2$

Conditional Prob $= \frac{f}{R} = \frac{1}{2}$

Q: A Coin is tossed thrice & 1st outcome is known to be Head then find the prob of getting exactly 2 Heads?

$\underbrace{\text{1st outcome is known to be Head}}_{\text{Condition}}$

Sol: original S space $= \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\} = 2^3 = 8$ Triplets

App I

Reduced S space (According to Condition) $= \{\text{1st outcome is Head}\} = \{(HHH), (HHT), (HTH), (HTT)\} = 4$

fav Cases $= \{\text{Exactly } 2H\} = \{(HHT), (HTH)\} = 2$

Conditional Prob $= \dfrac{\text{fav Cases}}{\text{R-S Space}} = \dfrac{2}{4} = \dfrac{1}{2} \neq \dfrac{4}{8}$

Note: Had the Condition was not there then ans would have been $= ? = \dfrac{f}{T} = \dfrac{3}{8}$

M-II Req Prob $= P[(H\,HT) \text{ or } (H\,TH)] = 1 \times \frac{1}{2} \times \frac{1}{2} + 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

ME ✓

Q. A coin is tossed twice. Events A & B are defined as follows;

$A = \{\text{Both H \& T appears}\}$, $B = \{\text{At most one T appears}\}$

then find $P(A)$, $P(B)$, $P(A/B)$, $P(B/A) = ?$

Sol: $S = \{(HH), (HT), (TH), (TT)\} = 4$

① $A = \{(HT), (TH)\} = 2 \Rightarrow P(A) = \frac{f}{T} = \frac{2}{4}$

② $B = \{(HH), (HT), (TH)\} = 3 \Rightarrow P(B) = \frac{f}{T} = \frac{3}{4}$

③ Reduced SSp (B) $= \{(HH), (HT), (TH)\} = 3$

App I

fav Case (A) $= \{(HT), (TH)\} = 2 \Rightarrow P(A/B) = \frac{2}{3}$

App I

④ $P(B/A) = ?$ Condition is $= A$

Reduced SSp $(A) = \{(HT), (TH)\} = 2$

fav Cases $(B) = \{\text{at most one T}\}$

$= \{(HT), (TH)\} = 2$

Conditional Prob $P(B/A) = \frac{f}{R} = \frac{2}{2} = 1$

Q: Two integers are to be selected from integers $\{1, 2, 3, 4 \ldots\ldots, 10, 11\}$ ≃ Togs ≃ RNA

If their sum is Even then Find the prob that Both the selected integers are odd ?

Condition

Sol: $\underbrace{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}_{^{11}C_2 = 55 \text{ pair}}$

$2, 4, 6, 8, 10 = {}^5C_2 = 10$ pair

$1, 3, 5, 7, 9, 11 = {}^6C_2 = 15$ pair

App II Total ways of selecting two integers $= {}^{11}C_2 = 55$ pair

Reduced ways (Acc to condition) = {Sum is Even}

= Either (Both even) or (Both odd) $= {}^5C_2 + {}^6C_2 = 25$ pair

Addition Principle

fav ways = {Both should be odd} $= {}^6C_2 = 15$ pair

Hence Conditional Prob $= \frac{f}{T} = \frac{{}^6C_2}{{}^6C_2 + {}^5C_2} = \frac{15}{25} = \frac{3}{5}$

App I : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 → 2, 4, 6, 8, 10 ; 1, 3, 5, 7, 9, 11

original sspace = $\left\{\begin{matrix} \cancel{(11)}, (12), (13), (14) \ldots (1\,10), (1\,11) \\ \cancel{(21)}, \cancel{(22)}, (23), (24) \ldots (2\,10), (2\,11) \\ (34), (35) \ldots (3\,10), (3\,11) \\ (45), (46) \ldots (10, 11) \end{matrix}\right\}$ = 55 pair

Reduced ssp = {Sum is Even} = {Both even or Both odd}

$= \left\{\begin{matrix} (24), (26), (28), (2\,10), (46), (48), (4\,10), (68), (6\,10), (8\,10) \\ (13), (15) \ldots (1, 11), (35), (37) \ldots (9\,11) \end{matrix}\right\}$ = 25 pair

fav cases = {Both odd} = {(13), (15), ... (9 11)} = 15 pair

So Conditional Prob $= \dfrac{f}{R} = \dfrac{15}{25}$

Note Had the Condition was not there then answer would have been = ? $= \dfrac{f}{T} = \dfrac{15}{55}$

Qs (Gate) Parcels are sending from Senders S to Receiver R sequentially through two Post offices. The prob of loosing an incoming parcel is $\frac{1}{5}$ by each Post office, independently of all other parcels. Given that parcel is lost then Find the prob that it was lost by 2^{nd} Post office?

Condition

Sol: M-I (App III) $\rightarrow$ original Prob $= P(S) = 1$

Reduced Prob $=$ P(Condition) $=$ P(Parcel is Lost)

$= P\{\text{Either Lost by } 1^{st} \text{ or Lost by } 2^{nd}\}$

$= P(\text{Lost by } 1^{st}) + P(\text{Lost by } 2^{nd})$

$= P(\text{Lost by } 1^{st}) + P\{NL \text{ by } 1^{st} \,\&\, L \text{ by } 2^{nd}\}$

$= \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{9}{25}$

$P(L) = \frac{1}{5}, \quad P(NL) = \frac{4}{5}$

Fav Prob $= P(\text{Lost by } 2^{nd}) = \frac{4}{5} \times \frac{1}{5} = \frac{4/25}{1}$

Conditional Prob $= \frac{f}{R} = \frac{4/25}{9/25} = \frac{4}{9}$

M-II Using Baye's Theorem →

Q: A Die is thrown twice and sum of the numbers on upper faces is observed to be 7 then find the prob that Number 2 has appeared at least once ?

(a) 2/5

(b) 1/3

(c) 1/6

(d) $\frac{1}{2}$

App. I original SSp $= \left\{ \begin{matrix} (11)(12) \ldots (16) \\ (21)(22) \ldots (66) \end{matrix} \right\} \Rightarrow n(S) = 36$

Reduced SSp $= \{\text{Sum is } 7\}$

$= \{(16)(61)(25)(52)(34)(43)\} = 6$

Fav Cases $= \{\text{'2' should occur at least once}\}$

$= \{(25),(52)\} = 2$

So Cond^n Prob $= \frac{f}{R} = \frac{2}{6} = \frac{1}{3}$

App. II $P(S) = 1$

Reduced Prob $= P(\text{Sum} = 7) = \frac{6}{36} = \frac{1}{6}$

Fav Prob $= P(2 \text{ occurs at least once})$

$= \frac{2}{36} = \frac{1}{18}$

Cond^n Prob $= \frac{\text{f Prob}}{\text{R Prob}} = \frac{1/18}{6/36} = \frac{1}{3}$

Standard Results of Conditional Prob $\rightarrow$

① $P(A/B) = \dfrac{P(A\cap B)}{P(B)} = P\{$A when B has already occured$\}$

② $P(B/A) = \dfrac{P(B\cap A)}{P(A)} = P\{$B when A " " " $\}$

③ $P(A\cap B/C) = \dfrac{P(A\cap B\cap C)}{P(C)} = P\{$Simultaneous occurrence of A & B when C has already occurred$\}$

Note: if A & B are Independent Events then Cond^n has No Significance.

$\because P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{P(A)\cdot P(B)}{P(B)} = P(A)$ Similarly $P(B/A) = P(B)$

(2) To check Independency of Events we have following 3 Methods;

(M-I) By using Def^n

(M-II) if $\left(P(A\cap B)=P(A)\cdot P(B)\right) \xLeftrightarrow{\text{then}} \left(A \ \& \ B \text{ are Ind.}\right)$

(M-III) if $\left[P(A/B)=P(A)\right] \xLeftrightarrow{\text{then}} \left[A \ \& \ B \text{ are Ind.}\right]$

eg Coin $=\{H,T\}$, Die $=\{1,2,3,4,5,6\}$

$A=\{H\}$

$P(A)=\frac{1}{2}$

$B=\{\text{No}\le 4\}=\{1,2,3,4\}$

$P(B)=\frac{4}{6}=\frac{2}{3}$

So $P(A/B)=?=P(A)=\frac{1}{2}$

$P(B/A)=?=P(B)=\frac{2}{3}$

Q. (PYQ) if $\underbrace{P(A)=1}$ & $P(B)=\frac{1}{2}$ then

$\rightarrow P(A/B) = ? = P(A) = 1$

$\rightarrow P(B/A) = ? = P(B) = \frac{1}{2}$

$\Downarrow$

A is Sure Event

$\Downarrow$

A will definitely occur

$\Downarrow$

A is Ind from B (By def^n)

$\Downarrow$

B is also Ind from A

Qs if $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A/B) = \frac{1}{6}$ then find the Prob of their simultaneous occurrence ? $P(A \cap B) = ?$

PYQ

(a) 1/12

(b) 1/24

(c) 1/18

(d) 0

$\because P(A/B) \neq P(A) \Longrightarrow$ A & B are Not Ind (By M-III)

ie $P(A \cap B) \neq P(A) \cdot P(B)$

So $\boxed{P(A \cap B) = P(A/B) \cdot P(B)}$

$= \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

HWQ Two persons p & q are considering to apply for job.

The prob that p apply for job is $1/4$.

The prob that p apply for job given that q apply for job is $\frac{1}{2}$

& " " " q " " " " p " " " is $\frac{1}{3}$ then

find the prob that p does not apply for job given that q does not apply for job ?

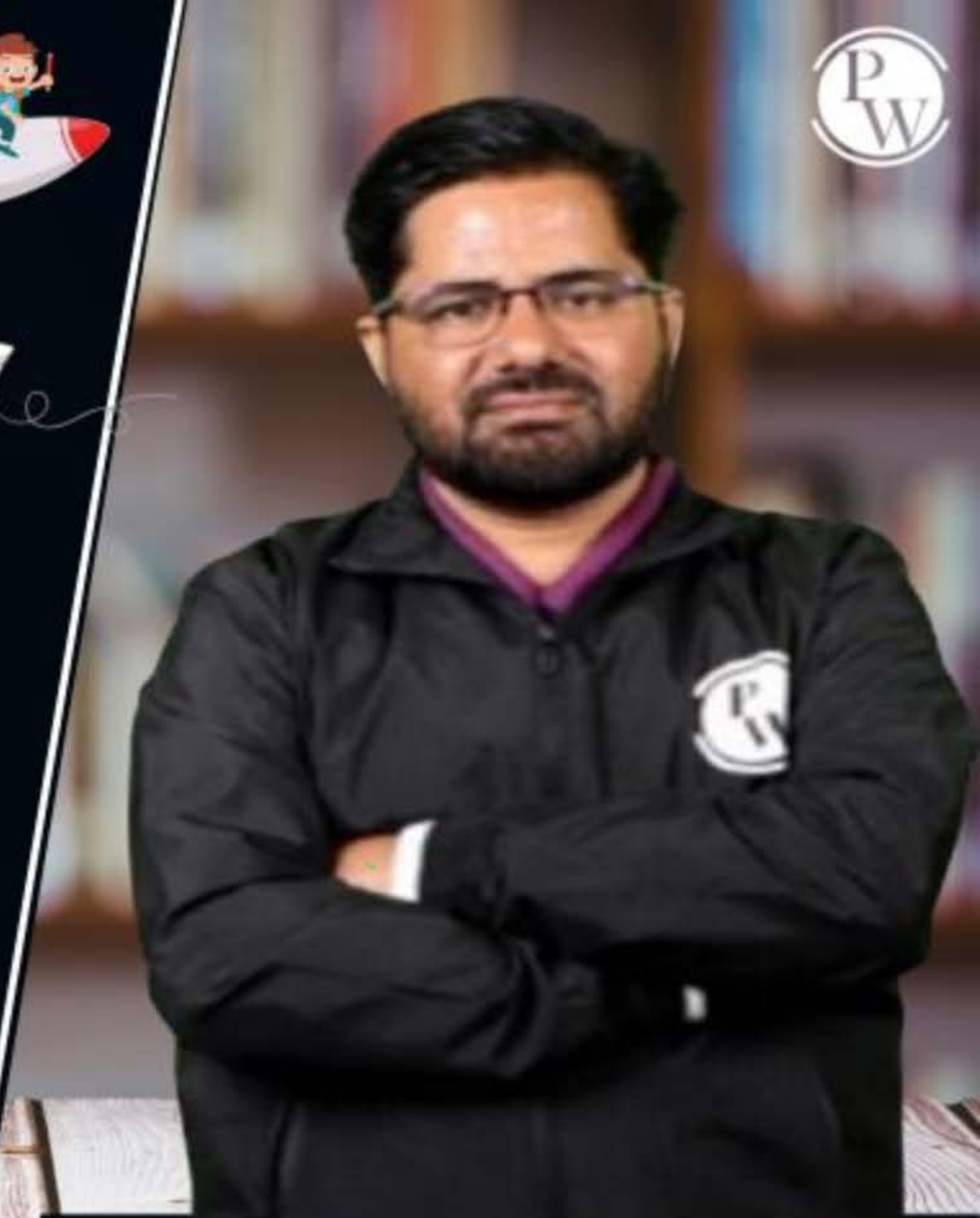
Sol: $P(\bar{p}/\bar{q}) = ?$ Ans $= 4/5$

THANK - YOU

DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE

Probability

Lecture No. 05



By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

CONDITIONAL PROBABILITY

Topics to be Covered

Topic

BAYE'S THEOREM

Qs (Gate) Parcels are sending from Senders S to Receiver R sequentially through two Post offices. The prob of loosing an incoming parcel is $\frac{1}{5}$ by each Post office, independently of all other parcels. Given that parcel is lost then Find the prob that it was lost by 2^{nd} Post office?

Condition.

Sol: M-I / App III → original Prob $= P(S) = 1$

Reduced Prob $= P(\text{Condition}) = P(\text{Parcel is Lost})$

$= P\{\text{Either Lost by } 1^{st} \text{ or Lost by } 2^{nd}\}$

$= P(\text{Lost by } 1^{st}) + P(\text{Lost by } 2^{nd})$

$= P(\text{Lost by } 1^{st}) + P\{\text{NL by } 1^{st} \text{ \& L by } 2^{nd}\}$

$= \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{9}{25}$

$P(L) = \frac{1}{5}, \; P(NL) = \frac{4}{5}$

Fav Prob $= P(\text{Lost by } 2^{nd}) = \frac{4}{5} \times \frac{1}{5} = \frac{4/25}{1}$

Conditional Prob $= \frac{f}{R} = \frac{4/25}{9/25} = \frac{4}{9}$

M-II Using Baye's Theorem → $A = \{\text{Parcel is Lost}\}$, $P(E_1) = P(\text{L by } 1^{st}) = \frac{1}{5}$

$P(E_2) = P(\text{NL by } 1^{st} \text{ \& L by } 2^{nd})$

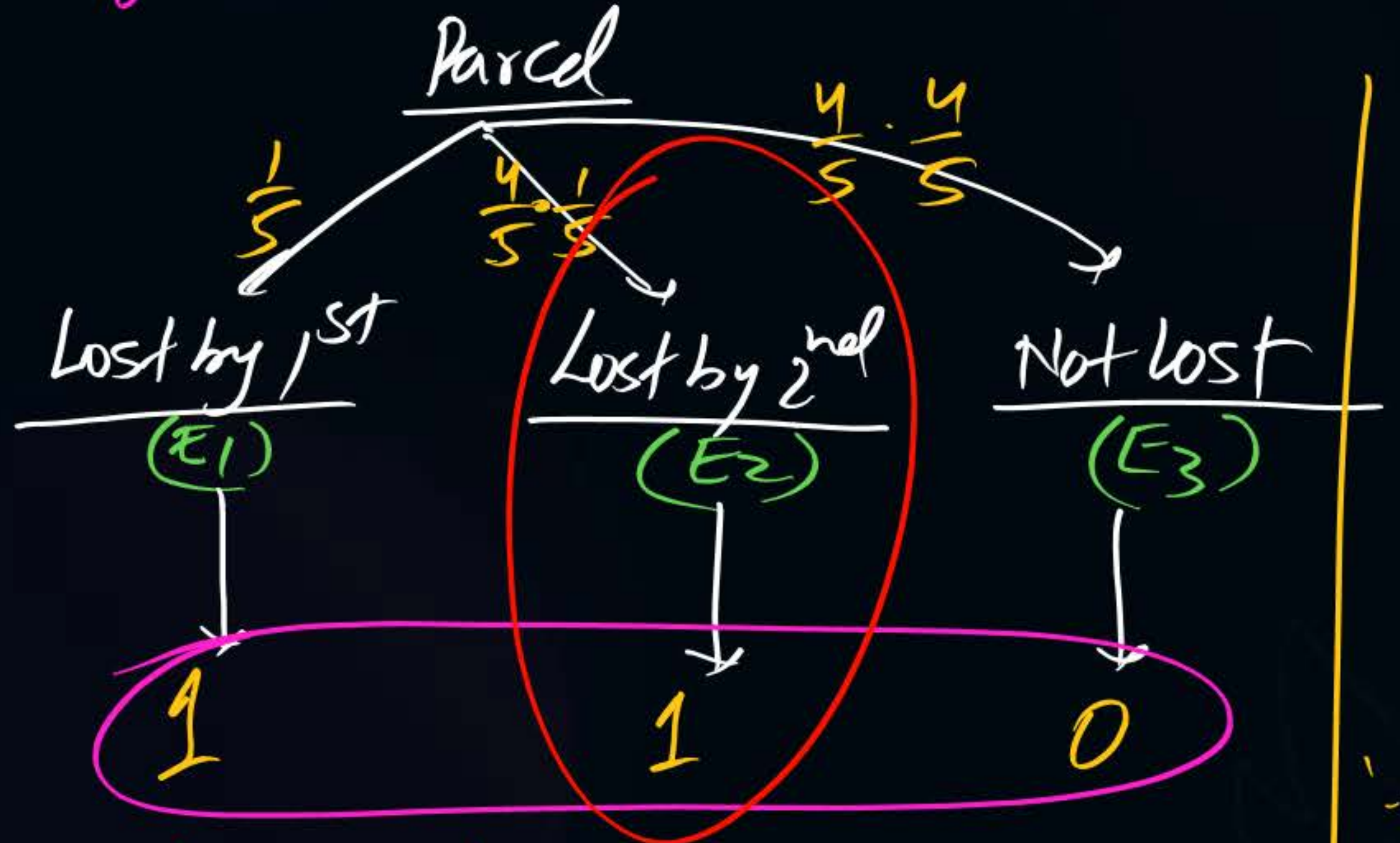
$= \frac{4}{5} \cdot \frac{1}{5} = \frac{4}{25}$

$P(E_3) = P(\text{NL by } 1^{st} \text{ \& NL by } 2^{nd})$

$= \frac{4}{5} \cdot \frac{4}{5} = \frac{16}{25}$

$\because P(E_1) + P(E_2) + P(E_3) = 1$

ie Events are ME & Exhaustive



Parcel is Lost (A):

$P(A) = \frac{1}{5} \times 1 + \frac{4}{25} \times 1 + \frac{16}{25} \times 0 = \frac{9}{25}$

$P(E_2/A) \leftarrow \frac{\text{f. Path}}{\text{H. Path}} = \frac{\text{fav Path}}{\text{R. Prob}} = \frac{4/25}{9/25} = \frac{4}{9}$

RECAP

(1) To check Independency of Events we have following 3 Methods;

(M-I) By using Def^n

(M-II) if $\left(P(A\cap B)=P(A)\cdot P(B)\right) \overset{\text{then}}{\Longleftrightarrow} \left(A \,\&\, B \text{ are Ind.}\right)$

(M-III) if $\left[P(A/B)=P(A)\right] \overset{\text{then}}{\Longleftrightarrow} \left[A \,\&\, B \text{ are Ind.}\right]$

(2) In case of Independency Condition has No Significance i.e $P(A/B)=P(A)$, $P(B/A)=P(B)$

(3) $P(A/B)=\dfrac{P(A\cap B)}{P(B)}$, $P(B/A)=\dfrac{P(B\cap A)}{P(A)}$

HWQ Two persons p & q are considering to apply for job.

The prob that p apply for job is $1/4$.

The prob that p apply for job given that q apply for job is $\frac{1}{2}$

& " " " q " " " " p " " " is $\frac{1}{3}$ then

find the prob that p does not apply for job given that q does not apply for job ?

Sol: $P\{p\} = \frac{1}{4}$, $P(p/q) = \frac{1}{2}$, $P(q/p) = \frac{1}{3}$, $P(\bar{p}/\bar{q}) = ?$

$\because P(q/p) = \frac{1}{3} \Rightarrow \frac{P(q \cap p)}{P(p)} = \frac{1}{3} \Rightarrow P(q \cap p) = \frac{1}{3}P(p) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$P(p/q) = \frac{1}{2} \Rightarrow \frac{P(p \cap q)}{P(q)} = \frac{1}{2} \Rightarrow P(q) = 2P(p \cap q) = 2 \times \frac{1}{12} = \frac{1}{6}$

$$P(\bar{p}/\bar{q}) = \frac{P(\bar{p}\cap\bar{q})}{P(\bar{q})} = \frac{1-P(p\cup q)}{1-P(q)}$$

$$= \frac{1-\{P(p)+P(q)-P(p\cap q)\}}{1-P(q)}$$

$$= \frac{1-\left\{\frac{1}{4}+\frac{1}{6}-\frac{1}{12}\right\}}{1-\frac{1}{6}} = \frac{4}{5}$$

PYQ

Q. An hydraulic structure has four Gates, which operates Independently. The Prob of failure of each Gate is 0.2. Given that Gate 1 has failed then find the Prob that Gate 2 & Gate 3 will also fail

Sol: $P(G_1) = P(G_2) = P(G_3) = P(G_4) = 0.2$

$$P\left\{\frac{G_2\cap G_3}{G_1}\right\} = P(G_2\cap G_3) = 0.2\times 0.2 = 0.04$$

due to Independency

M-II

Req Prob $= P[\underset{\text{given}}{G_1} \cap G_2 \cap G_3] = 1\times 0.2\times 0.2$

M.E Events → (E_1, E_2, E_3 are called ME) $\Longleftrightarrow$ if ($E_i \cap E_j = \phi \ \forall i \neq j$)

& $P(E_i \cap E_j) = 0$ & $P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$

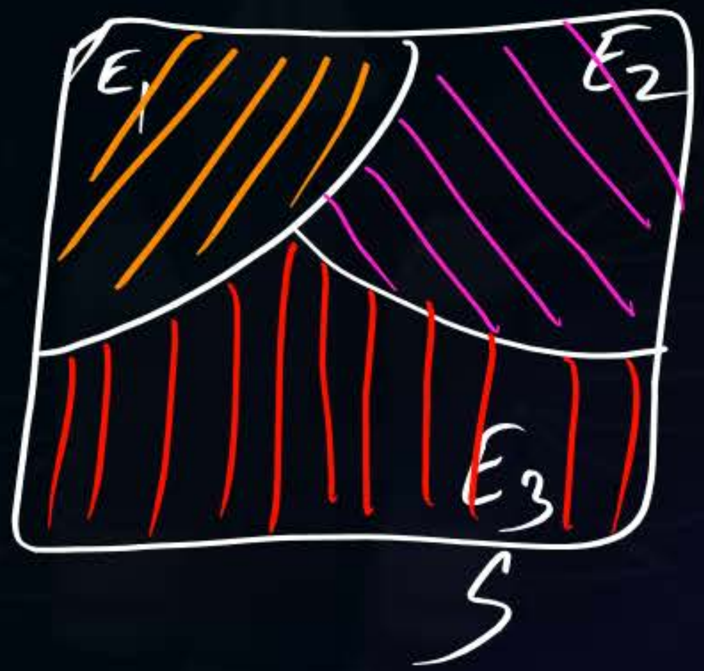
Exhaustive Events → (E_1, E_2, E_3 are called Exhaustive) $\Longleftrightarrow$ if ($E_1 \cup E_2 \cup E_3 = S$)

ME & Exhaustive Events → E_1, E_2, E_3 are called ME & Exhaustive if

$E_i \cap E_j = \phi$ & $E_1 \cup E_2 \cup E_3 = S$

ie $P(E_1 \cup E_2 \cup E_3) = P(S)$

$$\boxed{P(E_1) + P(E_2) + P(E_3) = 1}$$



if ($P(E_1) + P(E_2) + P(E_3) = 1$) then $\Longleftrightarrow$ (E_1, E_2, E_3 are ME & Exhaustive) eg: $S_{Dice} = \{1, 2, 3, 4, 5, 6\}$, $E_1 = \{1, 3, 5\}$, $E_2 = \{2, 4, 6\}$

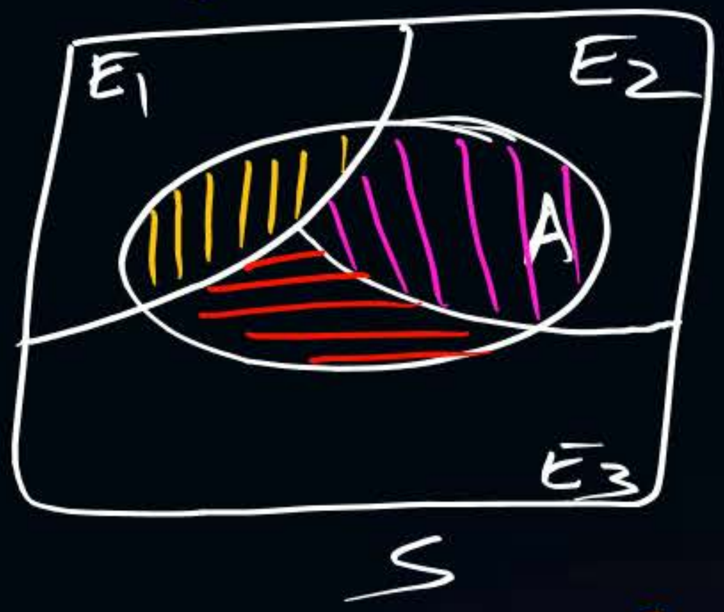
$$\left.\begin{array}{l} \because E_1 \cap E_2 = \phi \Rightarrow \text{ME} \\ \& \; E_1 \cup E_2 = S \Rightarrow \text{Exhaustive} \end{array}\right\} \Rightarrow P(E_1) + P(E_2) = \frac{3}{6} + \frac{3}{6} = 1$$

MAG

eg: $S_{Dice} = \{1, 2, 3, 4, 5, 6\}$, $E_1 = \{1, 3, 5\}$, $E_2 = \{\text{No} \geq 2\} = \{2, 3, 4, 5, 6\}$

$$\left.\begin{array}{l} \because E_1 \cap E_2 \neq \phi \text{ so these are not ME} \\ \& \; E_1 \cup E_2 = S \text{ so these are Exhaustive} \end{array}\right\} P(E_1) + P(E_2) = \frac{3}{6} + \frac{5}{6} \neq 1$$

Law of Total Probability → Let E_1, E_2, E_3 are ME & Exhaustive events & A is an event which can occur with all E_1, E_2, E_3



then $$\boxed{P(A) = P(E_1)P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)} \quad \text{--- (1)}$$

Baye's Theorem (Inverse Prob theorem) → (To solve complex problems of conditional prob we will use this theorem)

theory same as above then

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(A)}, \quad P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(A)}, \quad P(E_3/A) = \frac{P(E_3)P(A/E_3)}{P(A)}$$

Note (1) Necessary Conditions for B.Th → Associated Events must be ME & Exhaustive

(2) In Baye's Th, $A = \{$Assume that event as A which is given as condition$\}$

(3) while in Law of Total Prob, $A = \{$" " " " " which is Required$\}$

(4) whenever in a Question there is a feeling of Cross check then use B.Th.

(5) write A, at the Bottom of a Tree

Q. Computers are supplied to an organisation according to the following

Company	% of Computers Supplied	Probability of Being Defective
X	60 %	0.01
Y	30 %	0.02
Z	10 %	0.03

(1) Find the prob that supplied computer is Defective ?

(2) Given that supplied Comp is Def. then find the prob that it was supplied from Y

Cond^n.

$A = \{\text{Supplied Comp is Def}\}$

Computers

$60\% = 0.6$ | $30\% = 0.3$ | $10\% = 0.1$

Supplied from X	Supplied from Y	Supplied from Z
(E_1)	(E_2)	(E_3)
0.01	0.02	0.03

Supplied Comp is Defective (A)

$\because P(E_1) + P(E_2) + P(E_3) = 1$

$\Rightarrow E_1, E_2, E_3$ are ME & Exhaustive

$P(A/E_1) = 0.01$

$P(A/E_2) = 0.02$

$P(A/E_3) = 0.03$

$$P(A) = (0.6 \times 0.01) + (0.3 \times 0.02) + (0.1 \times 0.03)$$

$$= 0.015 = \frac{15}{1000}$$

(ii) $$P\left(E_2/A\right) = \frac{P(E_2) \cdot P(A/E_2)}{P(A)}$$

$$= \frac{\text{V-Path}}{\text{H-Path}} = \frac{0.3 \times 0.02}{0.015} = \frac{6}{15}$$

(iii) Similarly $P(E_1/A) = \frac{\text{V. Path}}{\text{H. Path}} = \frac{0.6 \times 0.01}{0.015} = \frac{6}{15}$

& $P(E_3/A) = \frac{\text{V. Path}}{\text{H. Path}} = \frac{0.1 \times 0.03}{0.015} = \frac{3}{15}$

Analysis :→ $P(A) = \frac{0.015}{1} = \frac{15}{1000}$ & $P(E_1/A) = \frac{6}{15}$, $P(E_2/A) = \frac{6}{15}$, $P(E_3/A) = \frac{3}{15}$

ie out of 1000 Supplied computers 15 are Defective

& out of 15 Defective computers 6 are supplied from X, 6 from Y & 3 from Z

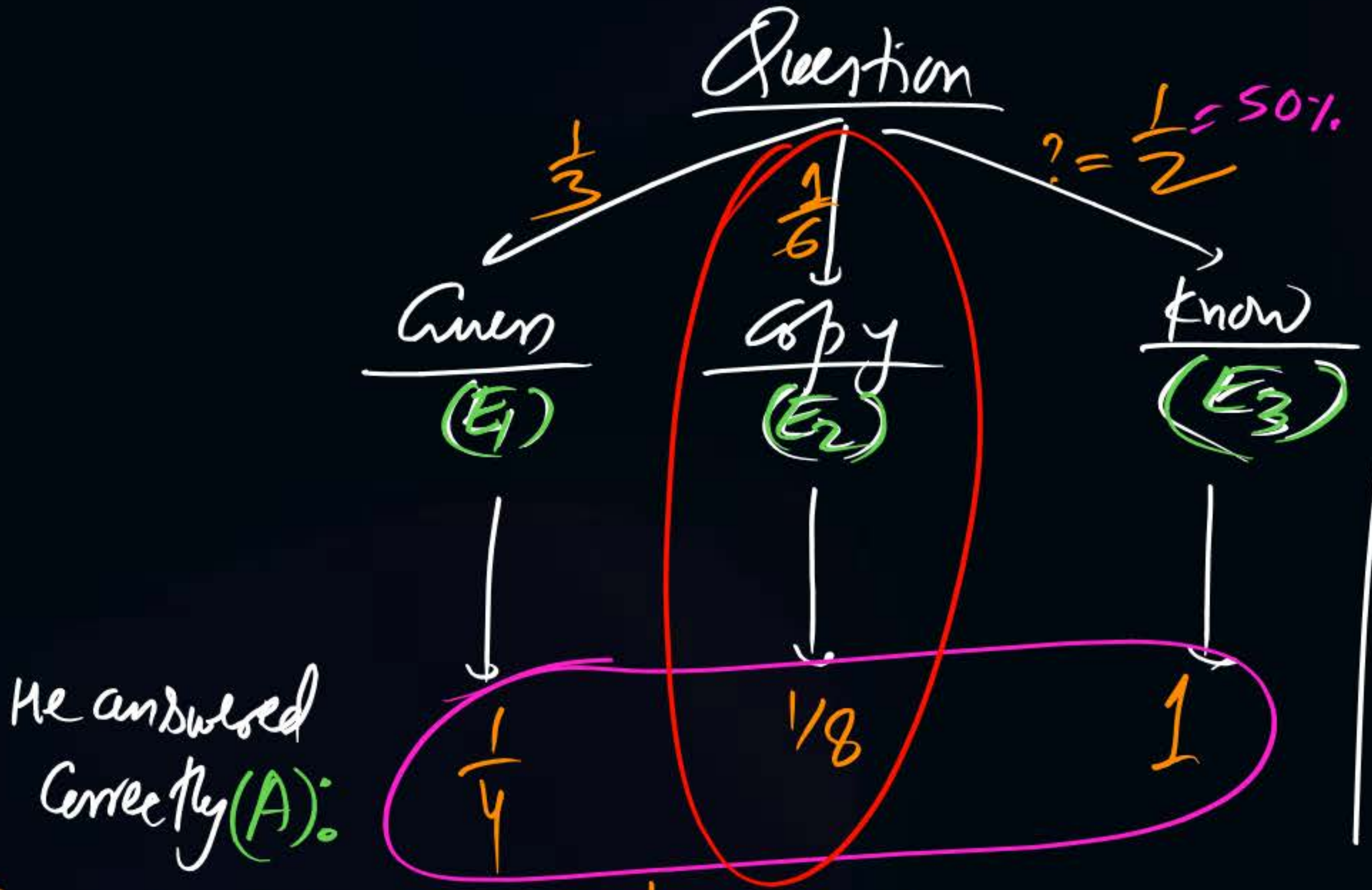
Q. In an examination, an Examinee either Guess or Copy or Know the answer of an objective type Question having 4 choices (in which only one choice is correct). The prob of making Guess is $\frac{1}{3}$ & Copying is $\frac{1}{6}$. the prob that his answer is correct given that he copied it is $\frac{1}{8}$ then find the prob that he copy the answer if he answered correctly?

Condition

Sol: ∵ there is a feeling of cross checking of correct Ans so we will use B-Th

$A = \{\text{He answered correctly}\}$, $E_1 = \{\text{Guess}\}$, $E_2 = \{\text{Copy}\}$, $E_3 = \{\text{Know}\}$

$P(A/E_1) = P(\text{Correct Ans}/\text{Guess}) = \frac{^1C_1}{^4C_1} = \frac{1}{4}$, $P(A/E_2) = \frac{1}{8}$, $P(A/E_3) = 1$



$\because E_1, E_2, E_3$ are ME & Exhaustive

so $P(E_1)+P(E_2)+P(E_3)=1$

$\frac{1}{3}+\frac{1}{6}+P(E_3)=1 \Rightarrow P(E_3)=\frac{1}{2}$

$P(A)=\frac{1}{3}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{8}+\frac{1}{2}\times 1=\frac{29}{48}$

$P(E_2/A)=\frac{\text{fav Path}}{\text{T. Path}}=\frac{\frac{1}{6}\times\frac{1}{8}}{29/48}=\frac{1}{29}$

Similarly $P(E_1/A)=\frac{\frac{1}{3}\times\frac{1}{4}}{29/48}=\frac{4}{29}$ & $P(E_3/A)=\frac{24}{29}$

Note: out of 48 attempted Question, only 29 are correct

& out of 29 correct answers, he copied only 1, guessed 4 & he knew 24

Q. A person is known to speak truth 3 out of 4 times.
HW He throw a die & Reports that it is six then find the Prob that it is actually six?

Exp

Condition

Ans $= 3/8$

$A = \{$Man Reports that it is six$\} \Rightarrow P(A) = \frac{8}{24}$

$E_1 = \{$six occur$\}$

$E_2 = \{$six Not occur$\}$

& $P(E_1/A) = ? = \frac{3}{8}$

------.

THANK - YOU

DATA SCIENCE & ARTIFICIAL INTELLIGENCE

Probability

Lecture No. 06

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

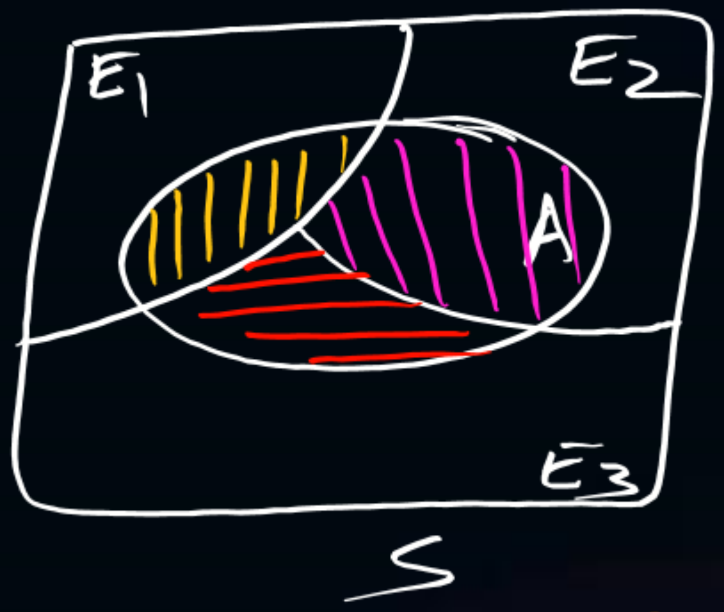
BAYE'S THEOREM

Topics to be Covered

Topic

MISCELLANEOUS CONCEPTS

Law of Total Probability →



Let E_1, E_2, E_3 are ME & Exhaustive events & A is an event which can occur with all E_1, E_2, E_3

then $$\boxed{P(A) = P(E_1)P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)} \quad \text{— (1)}$$

Baye's Theorem (Inverse Prob theorem) →

(To solve complex problems of conditional prob we will use this theorem)

theory same as above then

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(A)}, \quad P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(A)}, \quad P(E_3/A) = \frac{P(E_3)P(A/E_3)}{P(A)}$$

Note (1) Necessary Conditions for B.Th → Associated Events must be ME & Exhaustive

(2) In Baye's Th, $A = \{$Assume that event as A which is given as condition$\}$

(3) while in Law of Total Prob, $A = \{$" " " " " which is Required$\}$

(4) whenever in a Question there is a feeling of Cross check then use B.Th.

(5) write A, at the Bottom of a Tree

Q: A person is known to speak truth 3 out of 4 times.

(HW) He throw a die & Reports that it is six then Find the Prob that it is actually six?

Exp $S = \{1, 2, 3, 4, 5, 6\}$ Condition

Sol: $E_1 = \{\text{six occurs}\} = \{6\}$, $E_2 = \{\text{six Not occurs}\} = \{1, 2, 3, 4, 5\}$ & $P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{5}{6}$

$\because E_1 \cap E_2 = \phi$ ie ME & $\because E_1 \cup E_2 = S$ ie Exhaustive

$A = \{\text{Man Reports that it is six}\}$,

$P(A/E_1) = P(\text{Man is telling truth}) = \frac{3}{4}$

$P(A/E_2) = P(\text{" " " lie}) = \frac{1}{4}$

Now $P(\text{actually six})$

$P(E_1/A) = \frac{\text{Fav Prob}}{\text{R. Prob}} = \frac{\frac{1}{6} \times \frac{3}{4}}{8/24}$

DIE

$\frac{1}{6}$ → Six occurs (E_1) → Telling Truth $3/4$

$5/6$ → Six Not occurs (E_2) → Telling Lie $1/4$

Man Reports that it is six A:

$$P(A) = \frac{1}{6}\times\frac{3}{4} + \frac{5}{6}\times\frac{1}{4} = \frac{8}{24}$$

$$P(\text{Actually six}) = P(E_1/A) = \frac{\text{V. Path}}{\text{H. Path}}$$

$$= \frac{\frac{1}{6}\times\frac{3}{4}}{8/24} = \frac{3}{8}$$

Note: out of 24 Reports given by him, 8 reports are representing that "It is six" & out of 8 six Representing Reports, only 3 are Correct.

Th II

DIE

$\frac{3}{4}$ Telling Truth (E_1) → Six occurs ($1/6$)

$\frac{1}{4}$ Telling Lie (E_2) → Six Not occurs ($5/6$)

Man Reports that it is Six A:

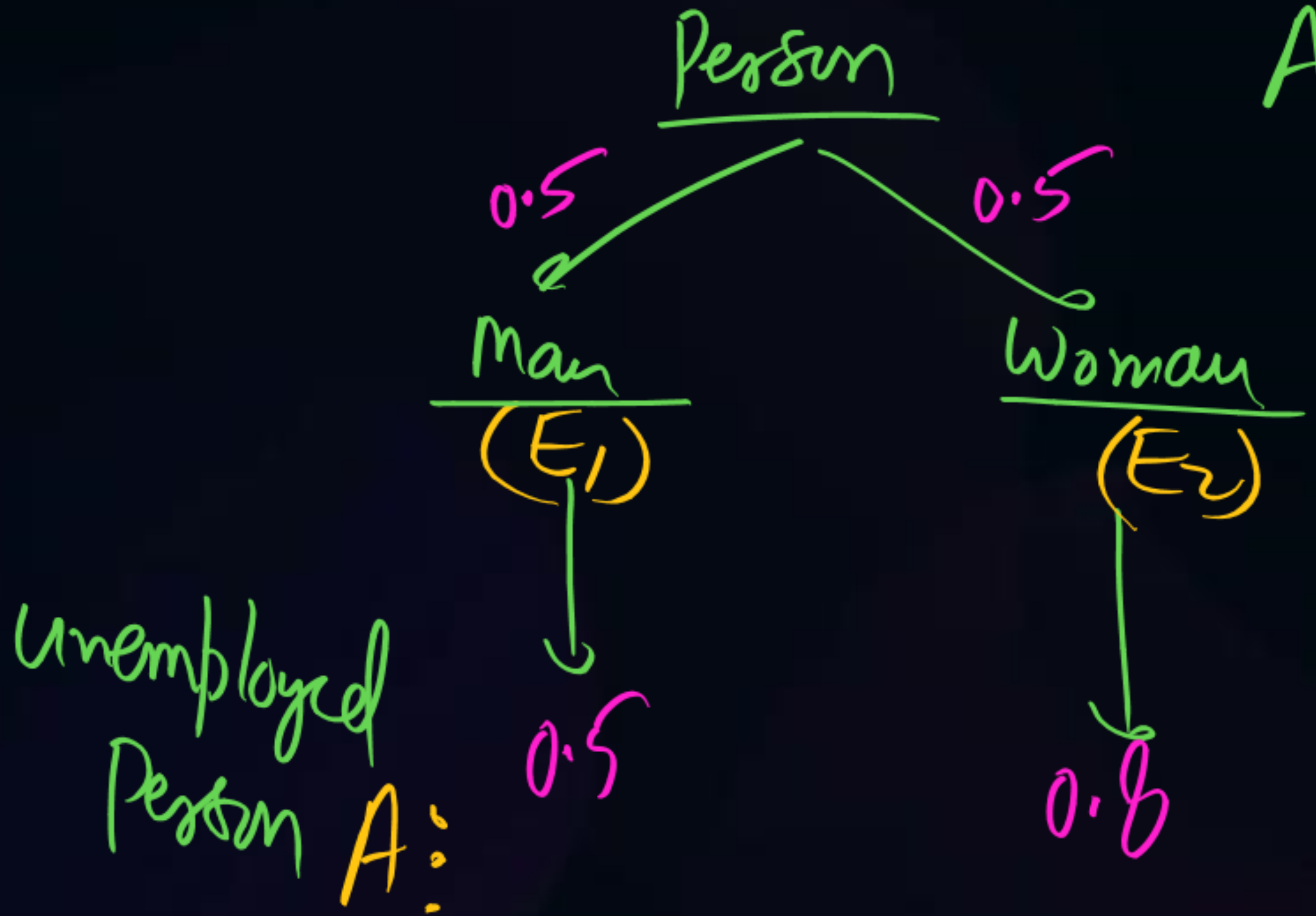
$$P(A) = \frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6} = \frac{8}{24}$$

$$P(E_1/A) = P(\text{actually Six})$$

$$= \frac{\frac{3}{4} \times \frac{1}{6}}{8/24} = \frac{3}{8}$$

Q: (PYQ) In a town there are equal number of Men & Women in which 50% M and 20% W are employed. If a person is selected at Random then Find the prob that person is an unemployed person? Exp

Sol: ∵ there is No feeling of cross check anything so we will Not use Baye's Th.



A = { Selected person is an unemployed person }

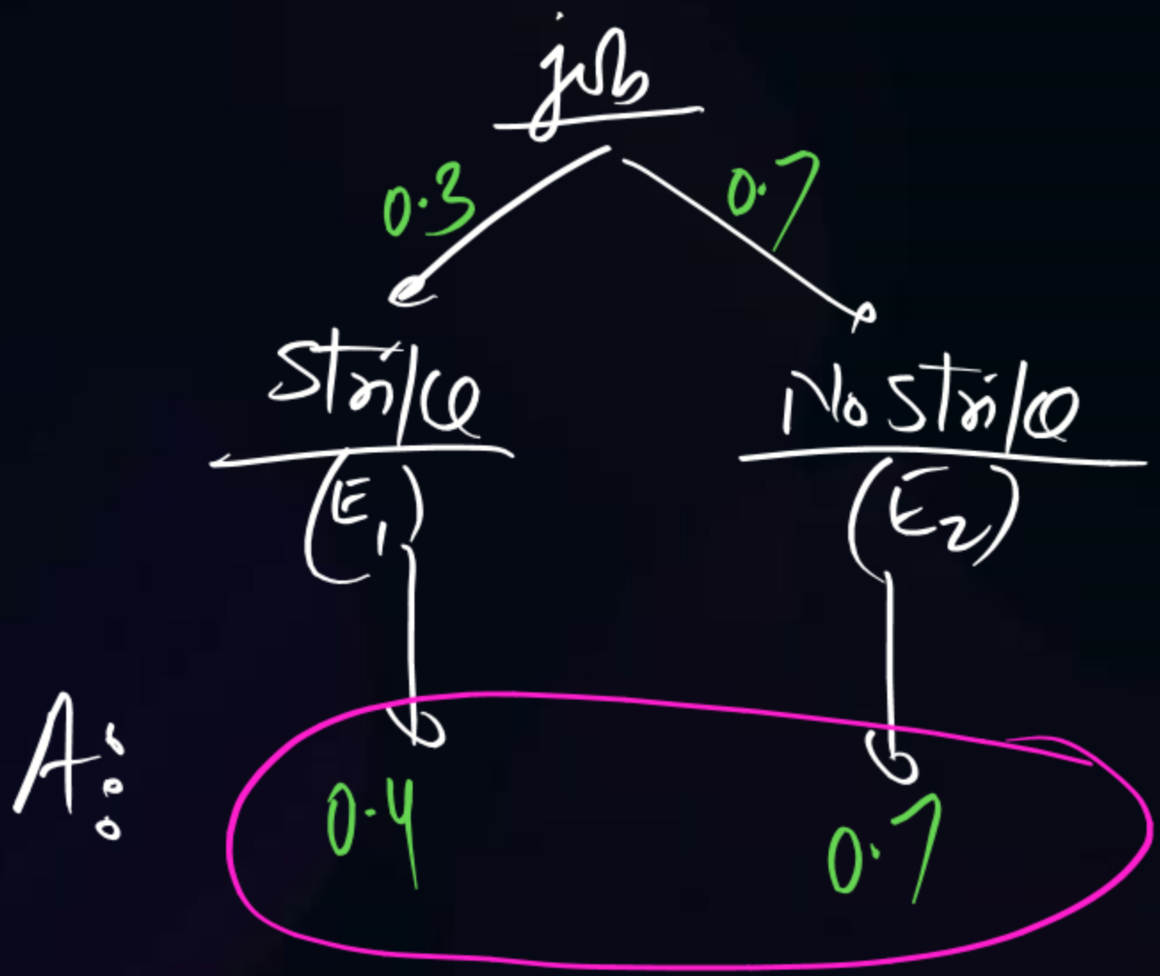
By Law of Total Prob

$$P(A) = 0.5 \times 0.5 + 0.5 \times 0.8 = 0.65 = 65\%$$

Q. A person has undertaken a construction job. The prob that there will be strike is 0.3. The prob that construction job will be completed on time if there is strike is 0.4 and if there is no strike is 0.7 then Find the prob that construction job will be completed on time?

R. Exp.

Sol: $A = \{\text{Const. job will be completed on time}\}$



$$P(A) = 0.3 \times 0.4 + 0.7 \times 0.7$$

$$= 0.12 + 0.49 = 0.61$$

$$= 61\% \text{ chance}$$

Q. In a town 10% of the population is Covid +ve. A new diagonostic kit arrives in the market. this kit correctly identifies Covid +ve idividual 95% of time and Covid -ve individual 89% of time.

A person is tested by this kit and is found to be +ve then Find the prob that person is Actually +ve ?

$\underbrace{\text{A person is tested by this kit}}_{\text{R. Exp}}$ and is $\underbrace{\text{found to be +ve}}_{\text{Condition}}$

$$\text{Ans} = \frac{95}{194} = 0.48$$

Concept of Playing Cards →

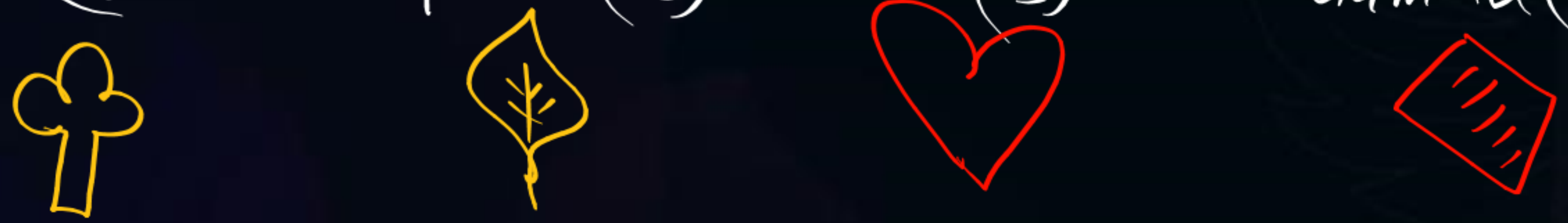
Face Cards (12) → K, Q, J

Honours Cards (16) → K, Q, J, A

K	Q	J	A
↓	↓	↓	↓
4	4	4	4

Total Cards (52)

- Black Suit (26)
  - Club (13)
  - Spade (13)
- RED Suit (26)
  - Heart (13)
  - Diamond (13)



Q: from a pack of 52 cards, while shuffling, four cards are accidently dropped. then find the prob that missing cards will be one from each suits?

Sol: Total ways of missing cards $= {}^{52}C_4$

fav ways " " " = one from each suits $= S \times H \times C \times D$

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

Req Prob $= \frac{f}{T} = \frac{({}^{13}C_1)^4}{{}^{52}C_4}$

Q. From a pack of 52 cards, while shuffling, one card is dropped & then two cards are drawn at Random then Find the Prob that both the selected cards will be spade?

$\underbrace{\text{one card is dropped}}_{Q\text{-}1}$ $\underbrace{\text{two cards are drawn at Random}}_{Q\text{-}2}$

(M-I) Req Prob = (Spade Missing) × (Two Cards drawn are spade)
OR
(Spade Not Missing) × (Two cards drawn are spade)

$$= \frac{{}^{13}C_1}{{}^{52}C_1} \times \frac{{}^{12}C_2}{{}^{51}C_2} + \frac{{}^{39}C_1}{{}^{52}C_1} \times \frac{{}^{13}C_2}{{}^{51}C_2} = 0.058$$

M-II

$$
\begin{tikzcd}
& \text{Cards (shuffling) } (52) \text{ (13 sp \& 39 others)} \arrow[dl, "\frac{13}{52}"'] \arrow[dr, "\frac{39}{52}"] & \\
\text{Spade Missing } (E_1) \text{ (12 sp \& 39 oth)} & & \text{Spade Not Missing } (E_2) \text{ (13 sp \& 38 oth)}
\end{tikzcd}
$$

Both selected cards Spade A : $\dfrac{{}^{12}C_2}{{}^{51}C_2}$ $\qquad$ $\dfrac{{}^{13}C_2}{{}^{51}C_2}$

$$P(A) = \frac{13}{52} \times \frac{{}^{12}C_2}{{}^{51}C_2} + \frac{39}{52} \times \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$= \frac{1}{4} \times \frac{12 \times 11}{51 \times 50} + \frac{3}{4} \times \frac{13 \times 12}{51 \times 50}$$

$$= \frac{132 + 468}{10200} = \frac{600}{10200}$$

$$= 0.058$$

Concept of with & w/o Replacement →

Q. Three Cards are drawn from a well shuffled pack of 52 Cards then find the Number of ways if Cards are drawn

(1) At Random $= {}^{52}C_3$

(2) one by one with Replacement $= {}^{52}C_1 \times {}^{52}C_1 \times {}^{52}C_1$

(3) " " w/o " $= {}^{52}C_1 \times {}^{51}C_1 \times {}^{50}C_1$

Eg: From a pack of Regular playing Cards, Two Cards are drawn then Find the Prob that both will be King if 1^{st} Card is Not Replaced.

Sol: it is a Case of one by one w/o Replacement.

M-I

$$\text{Req Prob} = P(K \cap K) = \frac{f}{T}$$

$$= \frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^3C_1}{{}^{51}C_1} = \frac{4}{52} \times \frac{3}{51}$$

$$= \frac{1}{221}$$

M-II

Hyp Dist

52

4 Kings | 2 | 48 others

2K | 0 others

$$\text{Req Prob} = \frac{f}{T} = \frac{{}^4C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

Hypergeometric Dist: it is applicable in that type of Quest where you are drawing anything one by one w/o Replacement

Q2) A Box Contains 4R & 3B Marbles & we want to draw 3 Marbles one by one w/o Replacement then find the Prob of drawing 1R & 2B Marbles ?

Sol: M-I Appb III

Req Prob = Case I or Case II or Case III

$= (RBB) \text{ or } (BRB) \text{ or } (BBR)$

$= \left(\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\right) + \left(\frac{3}{7}\times\frac{4}{6}\times\frac{2}{5}\right) + \left(\frac{3}{7}\times\frac{2}{6}\times\frac{4}{5}\right)$

$= \left(\frac{4\times3\times2}{7\times6\times5}\right)\times 3$

$= 0.34$

M-II Using Hypergeometric Distribution:

7 Marbles

$\swarrow \quad \downarrow \quad \searrow$

4 Red $\quad$ 3M $\quad$ 3 Black

$\downarrow \qquad\qquad\qquad \downarrow$

1R $\qquad\qquad\qquad$ 2B

$\text{Req Prob} = \frac{f}{T} = \frac{{}^4C_1 \times {}^3C_2}{{}^7C_3} = \left(\frac{4\times3\times2}{7\times6\times5}\right)\times 3$

$= 0.34$

Explanation:-

$$\text{Req Prob} = \frac{f}{T} = \frac{{}^4C_1 \times {}^3C_2}{{}^7C_3} = \frac{4\times\left(\frac{3\times2}{2\times1}\right)}{\left(\frac{7\times6\times5}{3\times2\times1}\right)} = \left(\frac{4\times3\times2}{7\times6\times5}\right)\times 3 = 0.34$$

(ii) Also find the Ans if Marbles are drawn one by one with Replacement?

$$7\text{ Marbles} \begin{cases} 4R \\ 3B \end{cases} \longrightarrow 3M$$

(M-I) Using Binomial Distr

will be discussed later

(M-II) Req Ans $= P[RBB \text{ or } BRB \text{ or } BBR]$

$$= \frac{4}{7}\times\frac{3}{7}\times\frac{3}{7} + \frac{3}{7}\times\frac{4}{7}\times\frac{3}{7} + \frac{3}{7}\times\frac{3}{7}\times\frac{4}{7} = \left(\frac{4}{7}\right)\left(\frac{3}{7}\right)^2\times 3$$

Q. There are 10 Calci on a Table in which 6 are Defective & 4 are Non Def. & we want to draw three Calci one by one w/o Replacement then find the prob that there will be exactly one Defective ?

Sol: M-I

Hypergeometric

10 Calci → 6 Def, 3, 4 Non Def

6 Def → 1 Def

4 Non Def → 2 N.D.

$$\text{Req Prob} = \frac{F}{T} = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3} = \frac{6 \times \left(\frac{4\times3}{2\times1}\right)}{\frac{10\times9\times8}{3\times2\times1}} = \left(\frac{6\times4\times3}{10\times9\times8}\right)\times 3 = 0.3$$

M-II

$$\text{Req Prob} = P[DNN \text{ or } NDN \text{ or } NND]$$

$$= \frac{6}{10}\times\frac{4}{9}\times\frac{3}{8} + \frac{4}{10}\times\frac{6}{9}\times\frac{3}{8} + \frac{4}{10}\times\frac{3}{9}\times\frac{6}{8}$$

$$= \left(\frac{6\times4\times3}{10\times9\times8}\right)\times 3 = 0.3$$

(ii) also find the answer if we are drawing Calci one by one with Replacement?

M-I

$$10 \text{ Calci} \begin{cases} \to 6 \text{ Def} \\ \to 4 \text{ Non Def} \end{cases}$$

Req Prob $= P[DNN \text{ or } NDN \text{ or } NND]$

$$= \frac{6}{10} \times \frac{4}{10} \times \frac{4}{10} + \frac{4}{10} \times \frac{6}{10} \times \frac{4}{10} + \frac{4}{10} \times \frac{4}{10} \times \frac{6}{10}$$

$$= \frac{6 \times 4 \times 4}{10 \times 10 \times 10} \times 3 = \left(\frac{6}{10}\right)\left(\frac{4}{10}\right)^2 \times 3$$

M-II Shortcut

(Using Binomial Dist)

will be discussed later

THANK - YOU