

# TYPESETTING EXAMS

Dyaga Nikhil Reddy  
B191258

October 6, 2022

Maths

Assignment

IITB #

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**problem 1.** Show that there exists no non trivial unramified extension of  $\mathbb{Q}$

**solution :** If  $K/\mathbb{Q}$  is a non-trivial number field then  $|disc K| > 1$ . But then  $disc K$  has a prime factor so that so same prime ramifies in  $K$

**Problem 2.** complete the following :

(a) how does one prove a cot theorem ?

(b) compute  $\int \cos x dx$

(c) how does one square  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ ?

**solutions :**

(a) use rollaries

(b) we have

$$\int \cos x dx = \sin x + c \quad (1)$$

we can check (1)

$$\frac{d}{dx}(\sin x + c) = \cos x$$

(c) This is routine. □

**Problem 3.** Prove that  $\sqrt{2}$  is irrational.

**Proof :** . Assume that  $\sqrt{2} = \frac{a}{b}$ , Where  $a, b \in \mathbb{Z}$  . Without loss of generality , we may assume  $\gcd(a, b) = 1$  . Then we have

$$\sqrt{2} = \frac{a}{b}$$

$$\sqrt{2}^2 = \left(\frac{a}{b}\right)^2 \quad (2)$$

$$2 = \frac{a^2}{b^2}$$

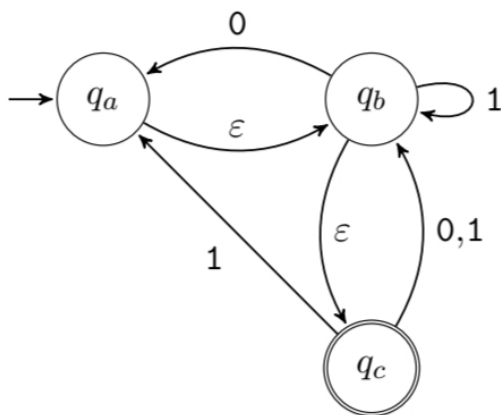
$$a^2 = 2b^2 \quad (3)$$

But then from (3), we know that  $a^2$  is even so that  $a$  is even . But then we must have .

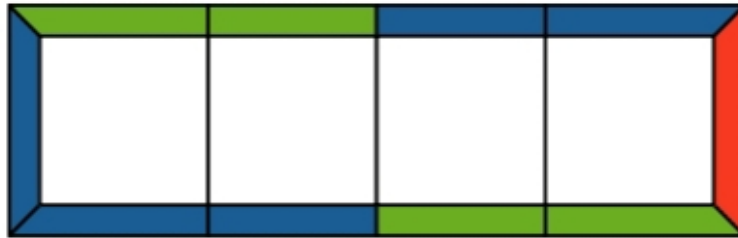
$$2a^2 = b^2$$

so that  $b^2$  is even , implying  $b$  is even. But then  $\gcd(a, b) \geq 2$  , a contradiction . □

(b)



**4.Solving Puzzles # 1** IN clas we did three puzzles, the first one which is equivalent to finite automata. In general , a puzzle of this type has a frame like (but possibly with more/fewer squares and different colors) :



and a finite set of tiles like this (but possibly with more/fewer tiles and different colors): The tiles must be arranged so that adjustment areas have



matching colors. there is an unlimited number of copies of each tile

(a) show how every puzzle of this type can be converted in to a finite automaton  $M$  and a string  $w$  that  $M$  accepts  $w$  if and only if the puzzle has a solution.

(b) Apply your construction to the above instance.

(c) Briefly describe how this gives an  $O(n)$  algorithm for solving puzzles of this type