

# Chapter 3: Fundamentals of Structure of Crystalline Solids

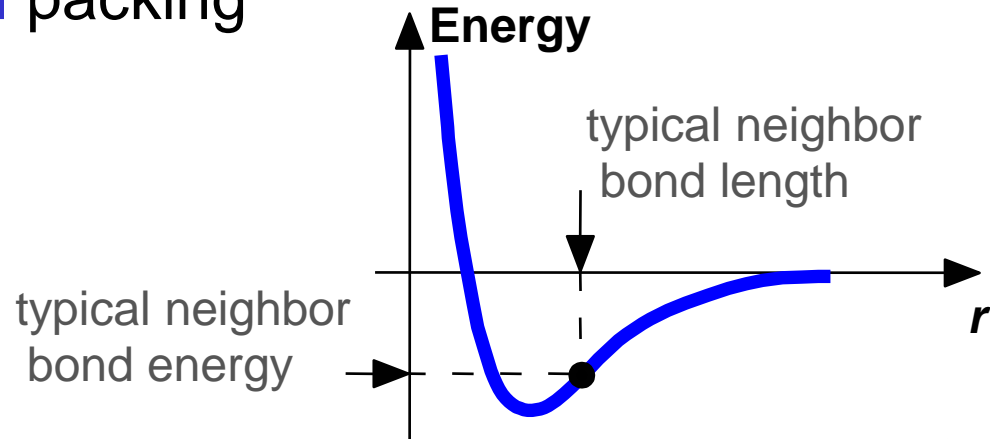
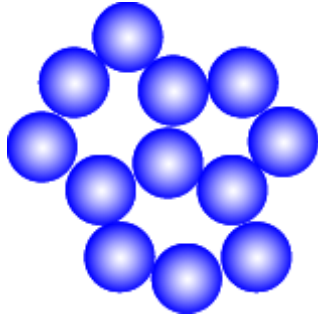
## ISSUES TO ADDRESS...

- How do atoms assemble into solid structures in crystalline and non-crystalline materials?
- How do we specify directions and planes in unit cells?
- How do we distinguish between single crystals and polycrystalline materials?
- When do material properties vary with the sample (i.e., part) orientation?

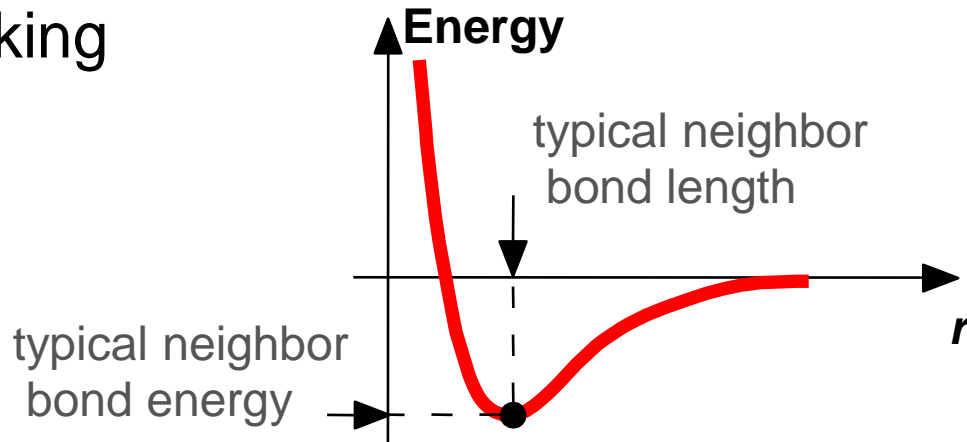
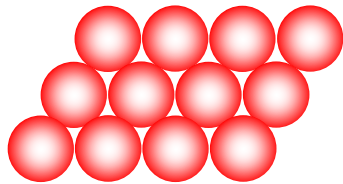


# Energy and Packing

- Non dense, **random** packing



- Dense, **ordered** packing

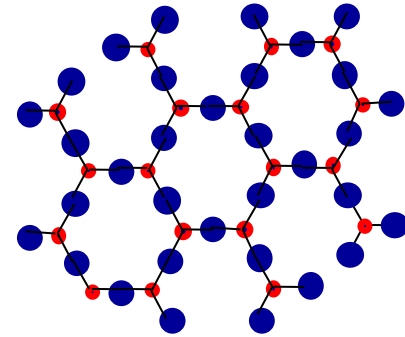


Dense, ordered packed structures tend to have lower energies.

# Materials and Packing

## Crystalline materials...

- atoms pack in periodic, 3D arrays
- typical of:
  - metals
  - many ceramics
  - some polymers



crystalline SiO<sub>2</sub>

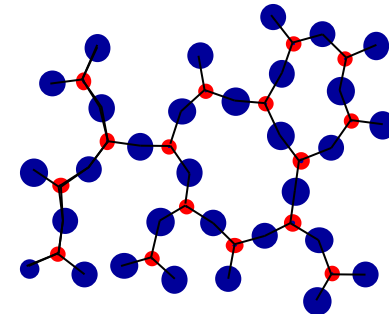
From Fig. 3.10(a)

*Callister's Materials Science and Engineering,  
Adapted Version.*

• **Si** • **Oxygen**

## Noncrystalline materials...

- atoms have no periodic packing
- occurs for:
  - complex structures
  - rapid cooling



noncrystalline SiO<sub>2</sub>

From Fig. 3.10(b)

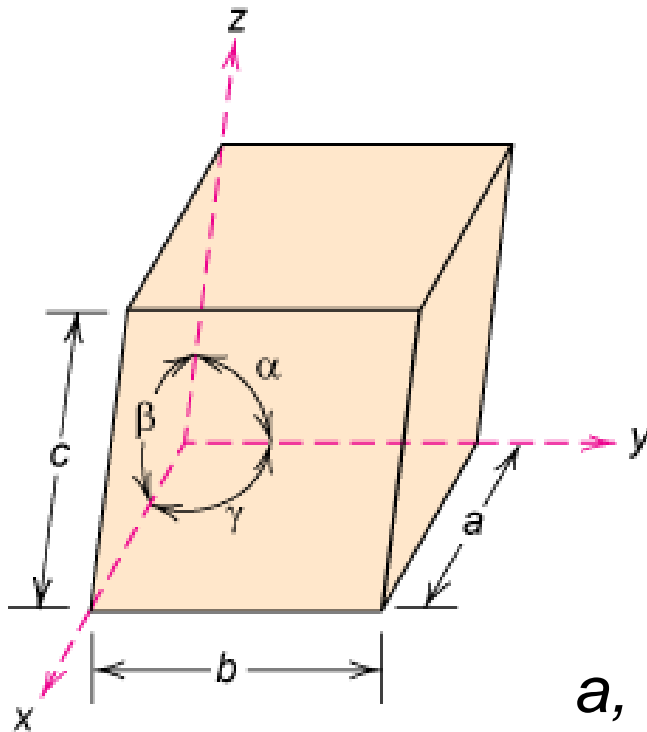
*Callister's Materials Science and Engineering,  
Adapted Version.*

"Amorphous" = Noncrystalline



# Section 3.3 – Crystal Systems

**Unit cell:** smallest repetitive volume which contains the complete lattice pattern of a crystal.



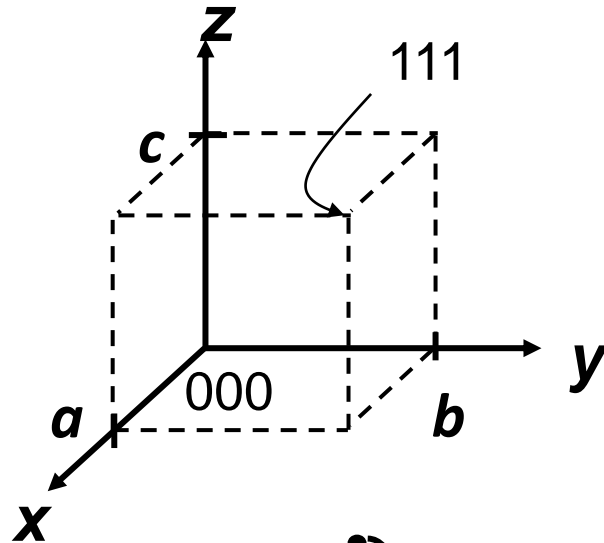
7 crystal systems

14 crystal lattices

$a$ ,  $b$ , and  $c$  are the lattice constants

Fig. 3.2  
Callister's Materials  
Science and  
Engineering,  
Adapted Version.

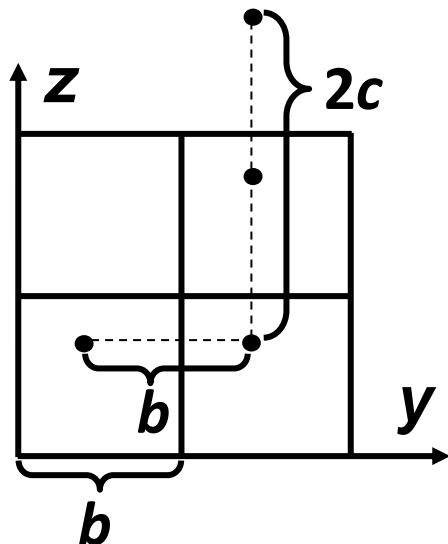
# Section 3.5 Point Coordinates



Point coordinates for unit cell center are

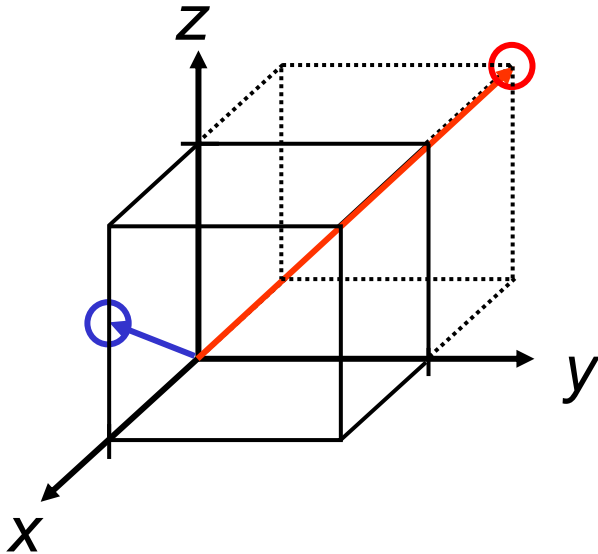
$$a/2, b/2, c/2 \quad \frac{1}{2} \frac{1}{2} \frac{1}{2}$$

Point coordinates for unit cell corner are 111



Translation: integer multiple of lattice constants  $\rightarrow$  identical position in another unit cell

# Crystallographic Directions



## Algorithm

1. Vector repositioned (if necessary) to pass through origin.
2. Read off projections in terms of unit cell dimensions  $a$ ,  $b$ , and  $c$
3. Adjust to smallest integer values
4. Enclose in square brackets, no commas

$[uvw]$

ex:  $1, 0, \frac{1}{2} \Rightarrow 2, 0, 1 \Rightarrow [201]$

$-1, 1, 1 \Rightarrow [\bar{1}11]$  where overbar represents a negative index

families of directions  $\langle uvw \rangle$

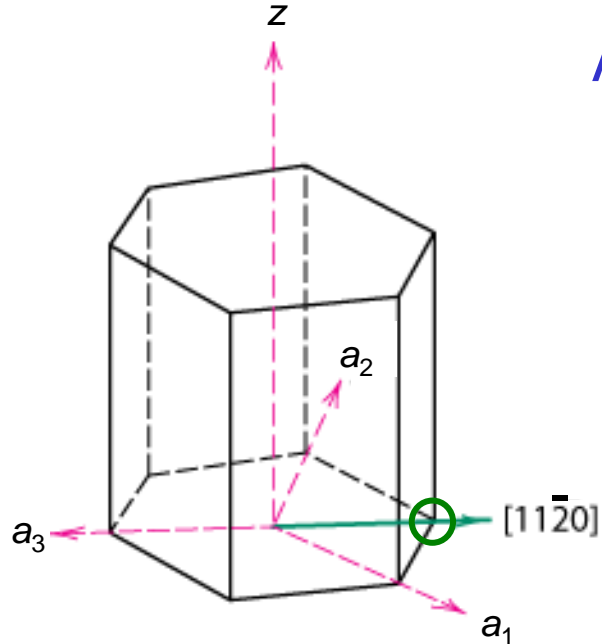


# HCP Crystallographic Directions

## Algorithm

1. Vector repositioned (if necessary) to pass through origin.
2. Read off projections in terms of unit cell dimensions  $a_1$ ,  $a_2$ ,  $a_3$ , or  $c$
3. Adjust to smallest integer values
4. Enclose in square brackets, no commas

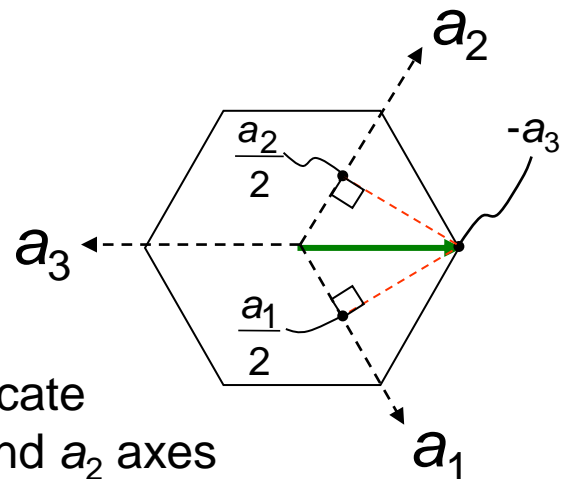
$[uvw]$



From Fig. 3.6(a)  
Callister's Materials Science and Engineering, Adapted Version.

ex:  $\frac{1}{2}, \frac{1}{2}, -1, 0 \Rightarrow [11\bar{2}0]$

dashed red lines indicate  
projections onto  $a_1$  and  $a_2$  axes



# HCP Crystallographic Directions

- Hexagonal Crystals

- 4 parameter Miller-Bravais lattice coordinates are related to the direction indices (i.e.,  $u'v'w'$ ) as follows.

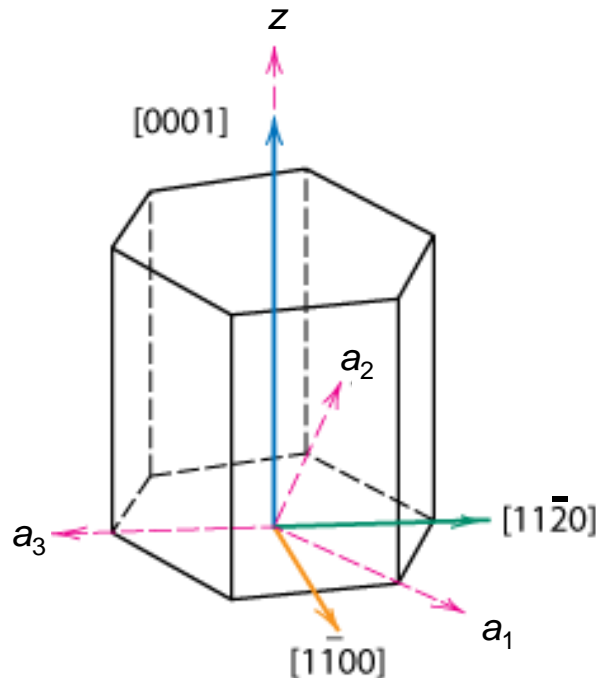


Fig. 3.6(a)  
Callister's Materials Science and Engineering,  
Adapted Version.

$$[u'v'w'] \rightarrow [uvtw]$$

$$u = \frac{1}{3}(2u' - v')$$

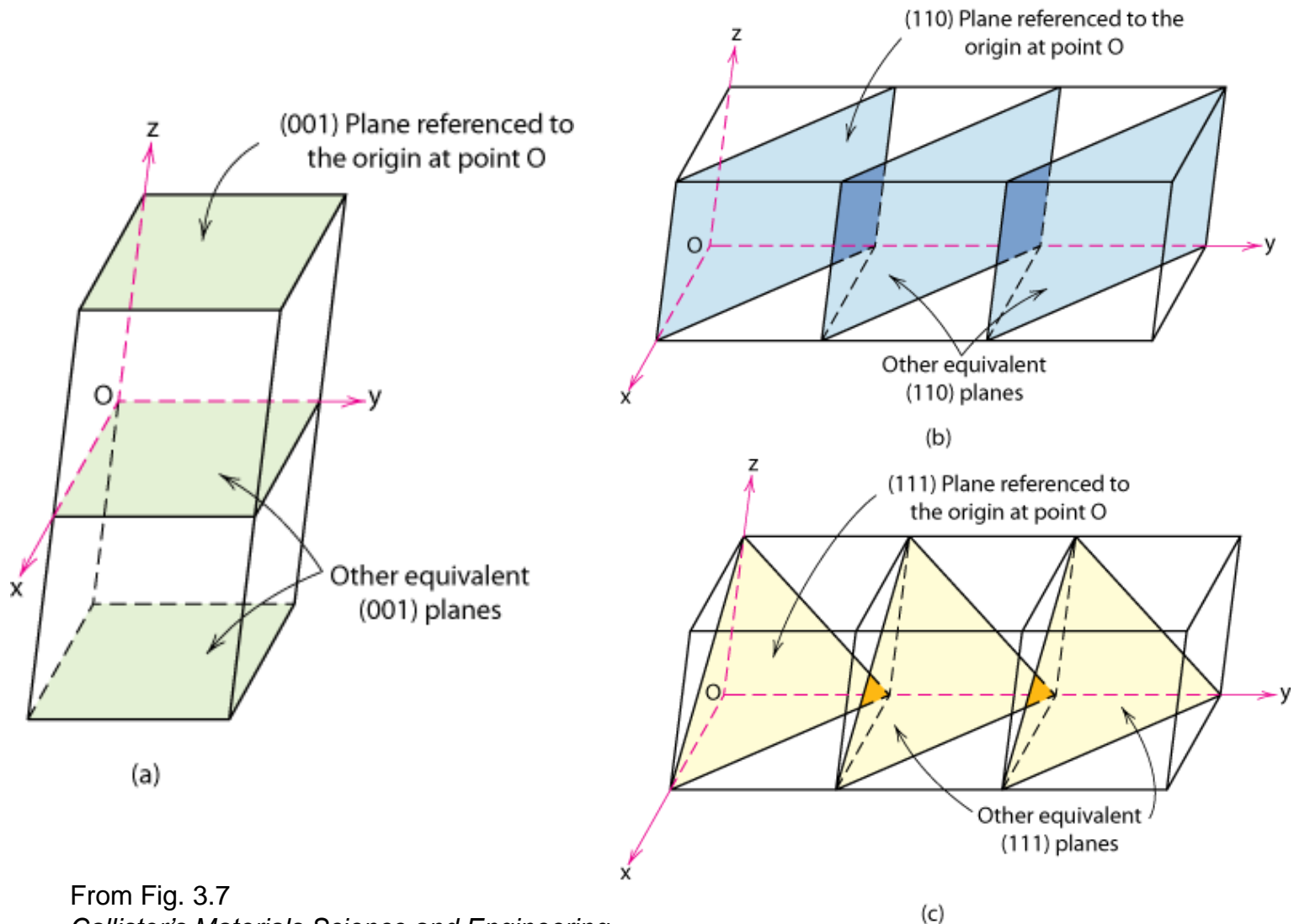
$$v = \frac{1}{3}(2v' - u')$$

$$t = -(u + v)$$

$$w = w'$$



# Crystallographic Planes



From Fig. 3.7  
*Callister's Materials Science and Engineering,*  
*Adapted Version.*

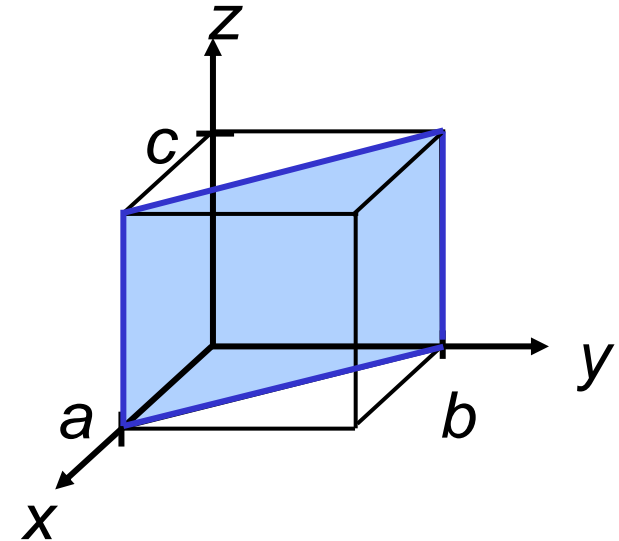
# Crystallographic Planes

- Miller Indices: Reciprocals of the (three) axial intercepts for a plane, cleared of fractions & common multiples. All parallel planes have same Miller indices.
- Algorithm
  1. Read off intercepts of plane with axes in terms of  $a$ ,  $b$ ,  $c$
  2. Take reciprocals of intercepts
  3. Reduce to smallest integer values
  4. Enclose in parentheses, no commas i.e.,  $(hkl)$

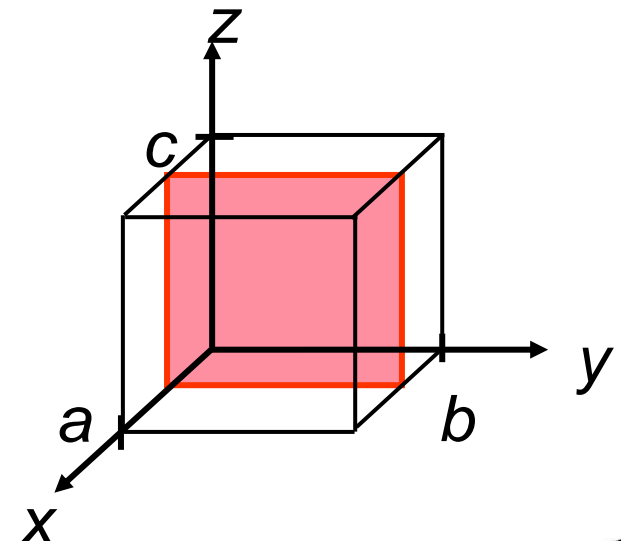


# Crystallographic Planes

<u>example</u>	<i>a</i>	<i>b</i>	<i>c</i>
1. Intercepts	1	1	$\infty$
2. Reciprocals	1/1	1/1	1/ $\infty$
	1	1	0
3. Reduction	1	1	0
4. Miller Indices	(110)		

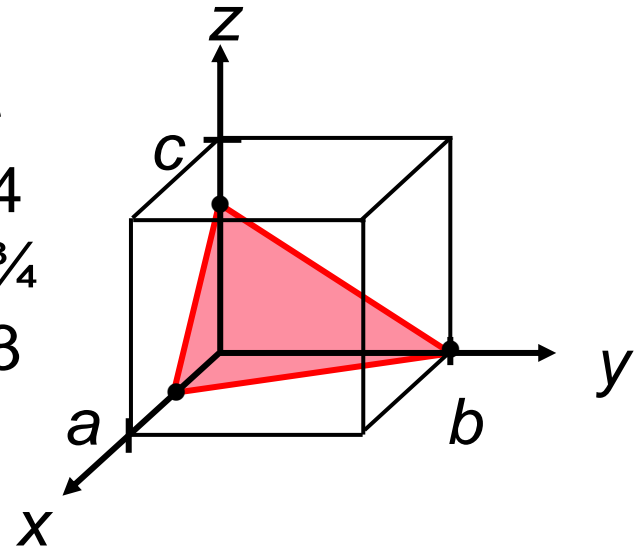


<u>example</u>	<i>a</i>	<i>b</i>	<i>c</i>
1. Intercepts	1/2	$\infty$	$\infty$
2. Reciprocals	1/1/2	1/ $\infty$	1/ $\infty$
	2	0	0
3. Reduction	2	0	0
4. Miller Indices	(100)		



# Crystallographic Planes

<u>example</u>	<i>a</i>	<i>b</i>	<i>c</i>
1. Intercepts	1/2	1	3/4
2. Reciprocals	1/1/2	1/1	1/3/4
	2	1	4/3
3. Reduction	6	3	4
4. Miller Indices	(634)		



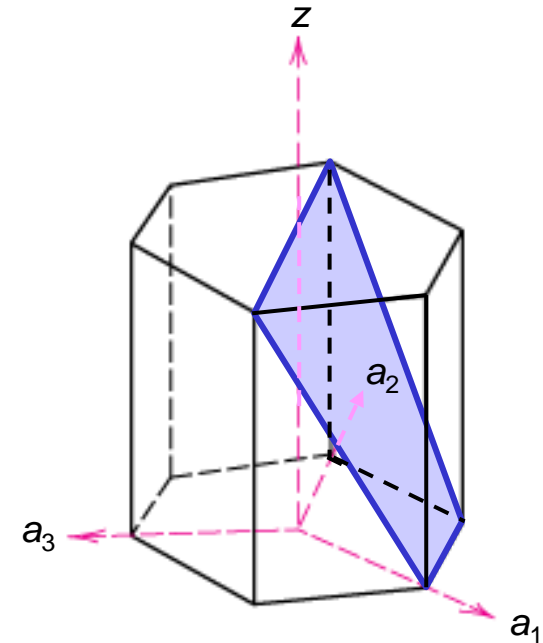
Family of Planes  $\{hkl\}$

Ex:  $\{100\} = (100), (010), (001), (\bar{1}00), (0\bar{1}0), (00\bar{1})$

# Crystallographic Planes (HCP)

- In hexagonal unit cells the same idea is used

<u>example</u>	$a_1$	$a_2$	$a_3$	$c$
1. Intercepts	1	$\infty$	-1	1
2. Reciprocals	1	$1/\infty$	-1	1
	1	0	-1	1
3. Reduction	1	0	-1	1
4. Miller-Bravais Indices	$(10\bar{1}1)$			



From Fig. 3.6(b)  
*Callister's Materials Science and Engineering,*  
 Adapted Version.

# Crystals as Building Blocks

- Some engineering applications require single crystals:
  - diamond single
  - turbine blades

crystals for abrasives



(Courtesy Martin Deakins, GE Superabrasives, Worthington, OH. Used with permission.)

- Properties of crystalline materials often related to crystal structure.

--Ex: Quartz fractures more easily along some crystal planes than others.

Fig. 11.33(c)  
*Callister's Materials Science and Engineering, Adapted Version.*  
(Fig.11.33(c) courtesy of Pratt and Whitney).



(Courtesy P.M. Anderson)

# Polycrystals

- Most engineering materials are polycrystals.



Anisotropic

From Fig. K, color inset pages of *Callister 5e*.  
(Fig. K is courtesy of Paul E. Danielson, Teledyne Wah Chang Albany)

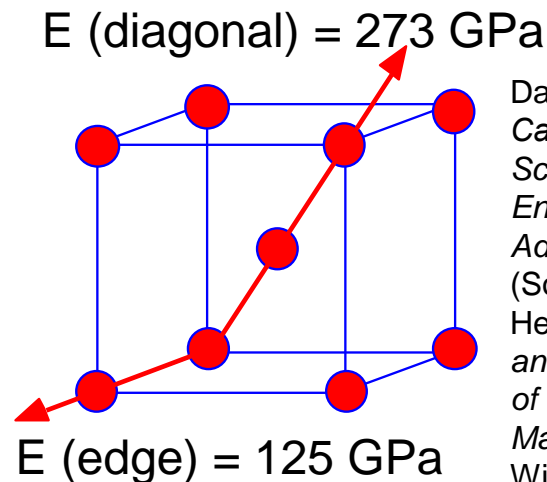
- Nb-Hf-W plate with an electron beam weld.
- Each "grain" is a single crystal.
- If grains are randomly oriented, overall component properties are not directional.
- Grain sizes typ. range from 1 nm to 2 cm (i.e., from a few to millions of atomic layers).

Isotropic

# Single vs Polycrystals

- Single Crystals

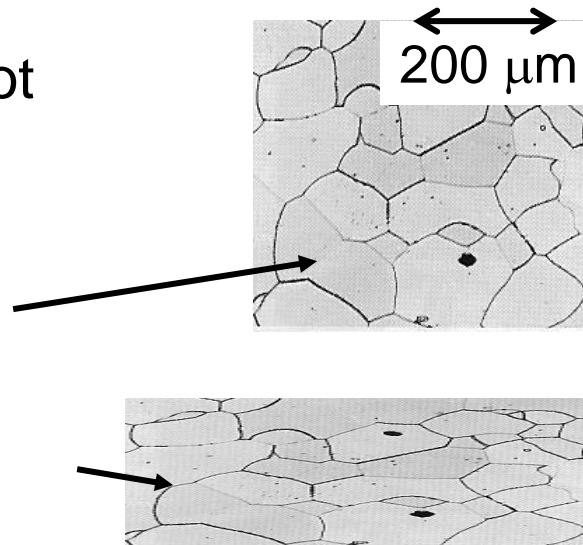
- Properties vary with direction: **anisotropic**.
- Example: the modulus of elasticity ( $E$ ) in BCC iron:



Data from Table 3.2, *Callister's Materials Science and Engineering, Adapted Version*. (Source of data is R.W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 3rd ed., John Wiley and Sons, 1989.)

- Polycrystals

- Properties may/may not vary with direction.
- If grains are randomly oriented: **isotropic**. ( $E_{\text{poly iron}} = 210 \text{ GPa}$ )
- If grains are **textured**, anisotropic.



From Fig. 5.19(b), *Callister's Materials Science and Engineering, Adapted Version*. (Fig. 5.19(b) is courtesy of L.C. Smith and C. Brady, the National Bureau of Standards, Washington, DC [now the National Institute of Standards and Technology, Gaithersburg, MD].)



# SUMMARY

- Atoms may assemble into **crystalline** or **amorphous** structures.
- **Crystallographic points**, **directions** and **planes** are specified in terms of indexing schemes.
- Materials can be **single crystals** or **polycrystalline**.  
Material properties generally vary with single crystal orientation (i.e., they are **anisotropic**), but are generally non-directional (i.e., they are **isotropic**) in polycrystals with randomly oriented grains.



# ANNOUNCEMENTS

Reading:

Core Problems:

Self-help Problems:

