An Improved Mayr-Type Arc Model Based on Current-Zero Measurements

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***Abstract*—In this paper it is shown that the Mayr-type arc models can be used to describe the arc behavior before current zero exactly. Based on this analysis the different types of 'modified' Mayr arc models can be explained. As a result of the theory, an improved Mayr-type arc model - with a constant time parameter and a cooling power which is dependent on the electrical power input - is introduced and used to reproduce current zero measurements successfully.**

***Index Terms*—circuit breaker, current zero measurement, arc modeling, Mayr-type differential equation, transients.**

I. Introduction

HIGH-VOLTAGE circuit breakers play an important role in transmission and distribution systems. They must clear faults and isolate faulted sections rapidly and reliably. In order to be sure of its capabilities, the breaker must be tested in a High-Power Laboratory.

Arc models give the possibility to extend the information obtained during the tests in a High-Power Laboratory. When a model has been developed that shows good correspondence with the circuit breaker behavior as observed during the tests, additional information such as limiting curves can be obtained from computations.

Therefore, arc modeling has a rich history. From the moment that O. Mayr and A.M. Cassie introduced their differential equations for describing the dynamic arc behavior [1]–[3] a lot has been published about the application of arc models. Even more publications are devoted to modifying arc models in order to fit measured data. A summary on the different type of arc models and its applications can be found in [4].

II. Theoretical Investigation of the Mayr Arc Model

The Mayr arc model is described by the following equation:

$$\frac{1}{g}\frac{dg}{dt} = \frac{d\ln g}{dt} = \frac{1}{\tau}\left(\frac{ui}{P} - 1\right) \tag{1}$$

g	the arc conductance
u	the arc voltage
i	the arc current
τ	the are 'time constant'
P	the cooling power

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The 'time constant' τ and cooling power P can be either constant or can be assumed to be functions of the electrical quantities conductance, current or voltage. A lot of varieties can be found in literature and a comparison of these variants is given by Haupt [5]. In order to find the possible relations between the parameters of the arc model and the electrical quantities, assumptions are made to acquire a system with as many equations as there are parameters to be determined:

- Amsinck [6] assumes the parameters to be equal at points of equal conductance (therefore, a reignition is required to obtain the parameters)
- Hochrainer [7] assumes the parameters to be equal at points of equal current (therefore, a reignition is required to obtain the parameters)
- Zückler [8] assumes the parameters to be equal in two following time steps (therefore, no reignition is required)
- Sporckmann [9] and other authors seek to the parameters as a function of the time by using polynomials (note that the higher the degree of the polynomial, the more equations are needed)

In fact, all of the above mentioned methods can lead to good results. However, it will become clear from the following analysis that there is not one unique relation between electrical quantities and arc parameters.

Equation 1 can be rewritten at a certain time instant k as:

$$\begin{bmatrix} -\left.\dfrac{d\ln g}{dt}\right|_k & ui|_k \end{bmatrix} \begin{bmatrix} \tau|_k \\ 1/P|_k \end{bmatrix} = 1 \quad \text{or} \quad A\underline{x} = b \tag{2}$$

Equation 2 is an underdetermined system [10]: it consists of one equation and two unknowns. There is not a unique solution vector x for this equation. A particular solution to the system (2) can be found easily: $\underline{x}_p^T [0 \quad ui|_k]$. The solution to the homogeneous system $A\underline{x}_h = 0$ has at least one nontrivial solution (i.e. $\underline{x}_h \neq 0$) because the number of equations is smaller than the number of unknowns. Since the matrix A has one row, there is only one nontrivial solution which is called the nullspace of A. The solutions of equation 2 can now be written as a sum of the particular solution and a solution to the homogeneous system:

$$\underline{x} = \underline{x}_p + c \cdot \underline{x}_h \tag{3}$$

c arbitrary number

It is easy to verify that, for every c, this is a solution of the underdetermined system (2):

$$A\underline{x} = A(\underline{x}_p + c \cdot \underline{x}_h) = A\underline{x}_p + c \cdot A\underline{x}_h = b + c \cdot 0 = b \tag{4}$$

Therefore, there is not one unique combination of $(\tau,\ P)$-values to describe the measured current and voltage traces. This

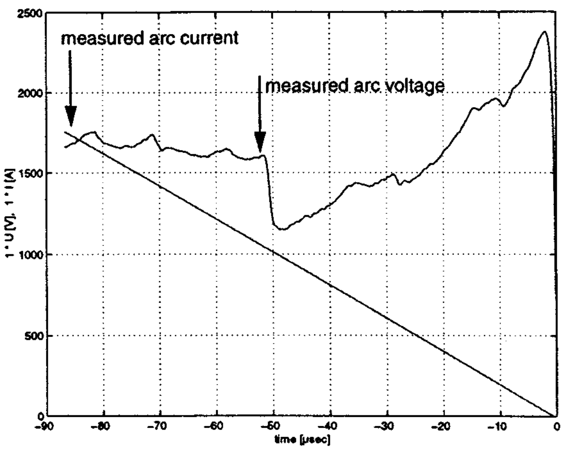

Fig. 1. Measured current and voltage traces before current zero.

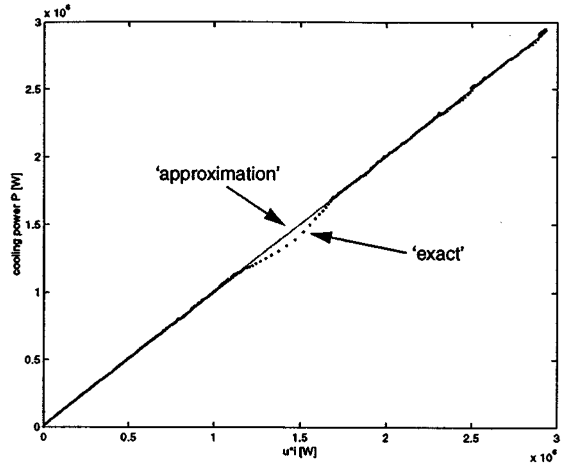

Fig. 2. Cooling power P versus the electrical power input.

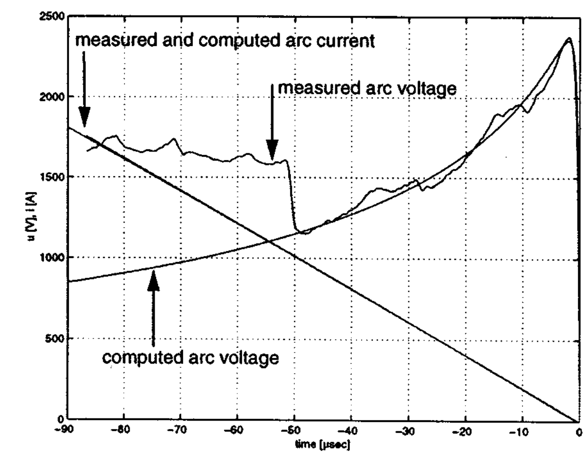

Fig. 3. Computed (equation 6) and measured current and voltage traces.

explains why there are so many variations on the Mayr-type arc model that can be used successfully in simulations.

III. Application of the Theory

The theory as described in section II will be applied to Short Line Fault (SLF) measurements performed on a 245kV/50kA/50Hz/single unit SF6 puffer circuit breaker. The voltages and currents were recorded with a 10MHz/12 bit measuring system at the KEMA High Power Laboratory in the Netherlands. A sample of a recorded voltage and current trace for a 90% SLF is shown in Fig. 1.

From the measurements, the particular and homogeneous solution can be determined for each sample point. Therefore, by choosing a certain relation for the time parameter τ, the arbitrary number c in equation 3 is prescribed at each sample point and so is the cooling power P. When the time constant τ is chosen to be constant ($\tau = 0.27\ \mu$sec), the cooling power P is computed at each sample point such that equation 3 is fulfilled. With these parameters, the measured voltage trace as shown in Fig. 1 can be recomputed exactly when the following equation is solved:

$$\frac{1}{g}\frac{dg}{dt} = \frac{d\ln g}{dt} = \frac{1}{\tau}\left(\frac{{i_{\text{measured}}}^{2/g}}{P} - 1\right) \tag{5}$$

In Fig. 2, the 'exact' computed cooling power P is shown versus the electrical power input of the are. From this figure it is evident that the cooling power shows a very strong linear relation with the electrical power input. Therefore, a new arc model is proposed with a constant time parameter r and the cooling power a function of the electrical power input:

$$\frac{1}{g}\frac{dg}{dt} = \frac{d\ln g}{dt} = \frac{1}{\tau}\left(\frac{ui}{P_0 + P_1 ui} - 1\right) \tag{6}$$

P_0 cooling constant in Watt
P_1 cooling constant

The arc model with the cooling power dependent on the electrical power input is a mathematical arc model but it does have a relation with arc physics as the cooling power is built up of two components. The cooling constant P_0 represents the cooling power originating from the design of the circuit breaker (i.e. nozzle layout, pressure etc.). The cooling constant P_1 regulates the influence of the electrical power input on the cooling power and therefore embodies the pressure built-up in the breaker caused by ohmic heating of the extinguishing medium by the arc.

The parameters are determined from a least squares fit: $\tau = 0.27\ \mu$sec, $P_0 = 15\,917$ W and $P_1 = 0.9943$. The approximate cooling power $P_0 + P_1 ui$, is shown in Fig. 2 for the above mentioned parameters as well. The measured and computed (see section V) arc currents and voltages using the differential equation (6) with the above mentioned parameters, are shown in Fig. 3. It is evident from Fig. 3 that in the high current area the arc voltage is lower than the measured one. This can be improved by adapting the arc model in the following way.

$$\frac{1}{g}\frac{dg}{dt} = \frac{d\ln g}{dt} = \frac{1}{\tau}\left(\frac{ui}{\max(U_{arc}|i|,\ P_0 + P_1 ui)} - 1\right) \tag{7}$$

U_{arc} the constant arc voltage in the high current area; when this value is set to zero equation 6 results

In the high current area, equation 7 reduces to the following differential equation.

$$\frac{1}{g}\frac{dg}{dt} = \frac{d\ln g}{dt} = \frac{1}{\tau}\left(\frac{ui}{U_{arc}|i|} - 1\right) = \frac{1}{\tau}\left(\frac{u}{U_{arc}} - 1\right) \tag{8}$$

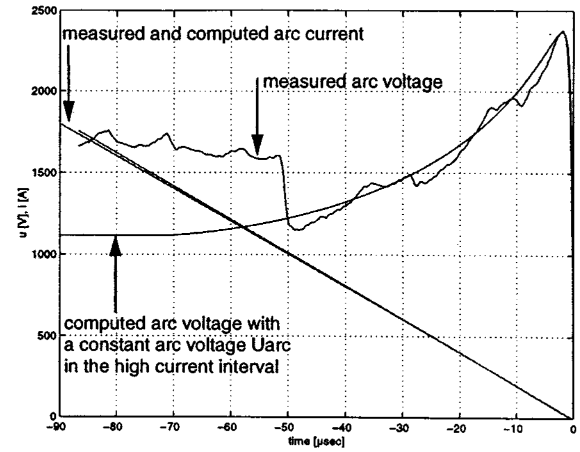

Fig. 4. Computed (equation 7) and measured current and voltage traces.

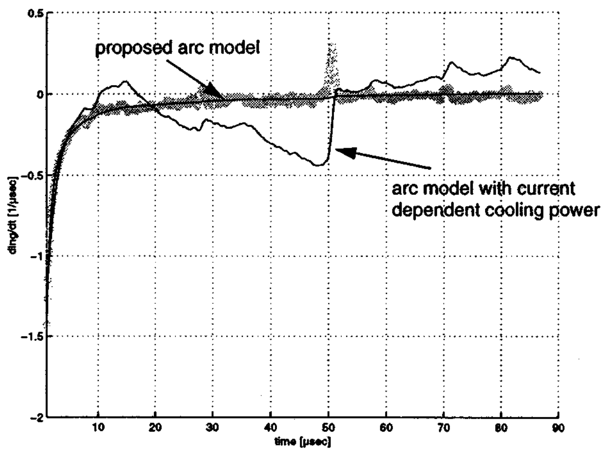

Fig. 5. 'Measured' (grey o's) and computed $d \ln g/dt$-curves.

This equation shows a clear conformity with the Cassie arc model [1], which has proven its validity in the high current area. At current zero, equation 7 (and equation 6) reduces to the following differential equation.

$$\frac{1}{g}\frac{dg}{dt} = \frac{d \ln g}{dt} = \frac{1}{\tau}\left(\frac{ui}{P_0} - 1\right) \tag{9}$$

This is exactly the Mayr arc model [2], [3], which has proven its validity in the current zero region. After current zero the constant P_1 is set to zero (the arc has been extinguished) and equation 9, i.e. the Mayr arc model, is used.

The measured and computed (see section V) arc currents and voltages using the differential equation (7) with the same parameters as mentioned earlier and the constant $U_{arc} = 1100$ V, is shown in Fig. 4.

IV. Advantages Over Existing (Mayr-Type) Arc Models

What are the advantages of the proposed arc model over other existing Mayr-type arc models? To answer this question, we shall compare the proposed arc model with the two following widely applied arc models [5]:

$$\frac{1}{g}\frac{dg}{dt} = \frac{d \ln g}{dt} = \frac{1}{\tau_0 g^{\alpha}}\left(\frac{ui}{P_0 g^{\beta}} - 1\right) \tag{10}$$

$$\frac{1}{g}\frac{dg}{dt} = \frac{d \ln g}{dt} = \frac{1}{\tau}\left(\frac{ui}{P_0 + P_1|i|} - 1\right) \tag{11}$$

Advantages compared to the model in equation 10:

- the proposed arc model only has three free parameters to be determined (i.e. τ, P_0, P_1; using U_{arc} is optional and not necessary) while for the model in equation 10 four parameters must be determined. This results in serious faster arc parameter determination, because there are no exponents involved.
- After current zero, the proposed arc model has two constant parameters in the denominators of the differential equation (i.e. τ and P_0 in equation 9). The arc model in equation 10 has power functions of the conductance in the denominators. After current zero, this conductance becomes very small (in case of a successful interruption) and therefore must be limited in its value to avoid numerical errors.

Advantages compared to the model in equation 11:

- the deviation from the measured curves is much larger than in the case of the proposed arc model as shown in Fig. 5. Both curves have been computed by applying curve fitting techniques. It is evident from Fig. 5 that the arc model with current dependent cooling power (equation 11) 'fits' the $d \ln g/dt$, as computed from the measured data, much worse than the proposed arc model. Although the predictive value (successful/unsuccessful interruption) of the arc model is the most important property, acceptance of an arc model is very much based on the conformity between measured and computed curves.

V. Numerical Computations With the Proposed Arc Model

The proposed arc model is incorporated into the new transient program XTrans, which is developed at the Delft University of Technology [11], [12]. The program was developed especially for analysis of arc-circuit interaction involving nonlinear elements (such as arc models) in relation to stiff differential equations. Therefore, the program is based on the use of Differential Algebraic Equations (DAE's). The calculations are performed with a variable stepsize and adjustable accuracy of the computed currents, voltages and conductances. The program has been verified by comparing computations with the well-known EMTP.

The program runs on a Windows computer and uses dynamic link libraries (dll's) that contain the compiled code of the elements. Therefore the models are separate from the main program, which made it easy to create the new arc model and use it in the main program. The program structure is shown in Fig. 6.

The synthetic test circuit used in the High-Power Laboratory consists of a current supply circuit with generators and a parallel current injection circuit to supply the transient recovery voltage after current interruption. The parallel current injection circuit and the artificial line used for the 90% SLF-test are shown in Fig. 7 (the current supply circuit is not shown). A successful interruption has been recomputed with the transient program and the arc model with the previously mentioned parameters

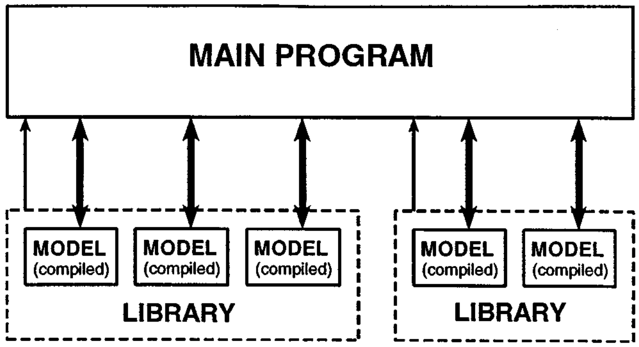

Fig. 6. Transient program structure.

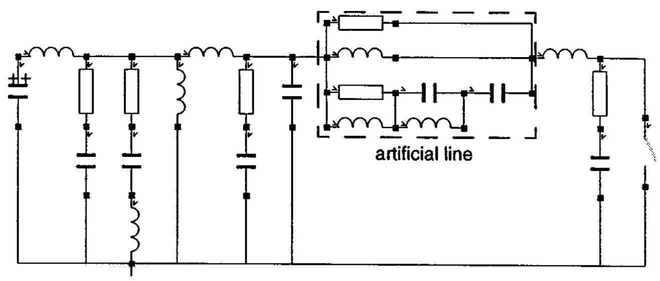

Fig. 7. Parallel current injection circuit.

($\tau = 0.27$ μsec, $P_0 = 15\,917$ W and $P_1 = 0.9943$; see Fig. 3). The arc model predicts the successful interruption and the calculated post-arc current is shown in Fig. 8. In the measurement data, however, no physical post-arc current can be detected and therefore, the computed post-arc current can not be compared to the 'real' one. Furthermore, an unsuccessful interruption has been recomputed with the following parameters obtained from the corresponding measurement: $\tau = 0.57$ μs, $P_0 = 24\,281$ W, $P_1 = 0.9942$ and $U_{arc} = 1135$ V. The arc model predicts the reignition. Both measured and computed voltage and current traces around current zero are shown in Fig. 9.

Both computations show good correspondence between the measured current and voltage traces and the ones resulting from the transient program. Furthermore, the model is able to reproduce both successful interruptions and reignitions.

VI. Conclusions

In this paper an improved Mayr-type arc model—with a constant time parameter and a cooling power which depends on the electrical power input—is proposed with three free parameters. The parameters of the arc model are obtained from current-zero measurements. Numerical computations in the XTrans transient program show that the proposed arc model gives good correspondence with the measured voltage and current traces and is able to reproduce the measurements successfully.

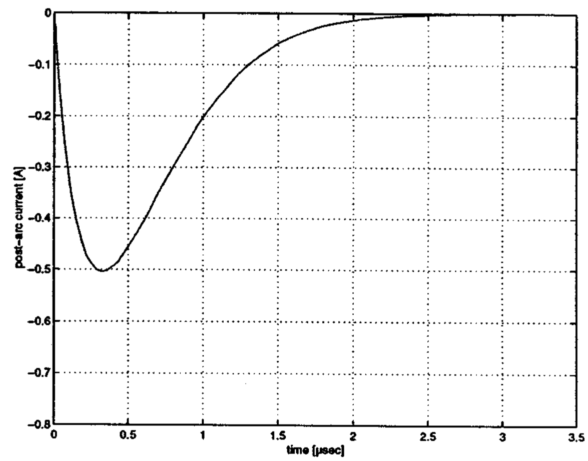

Fig. 8. Computed post-arc current.

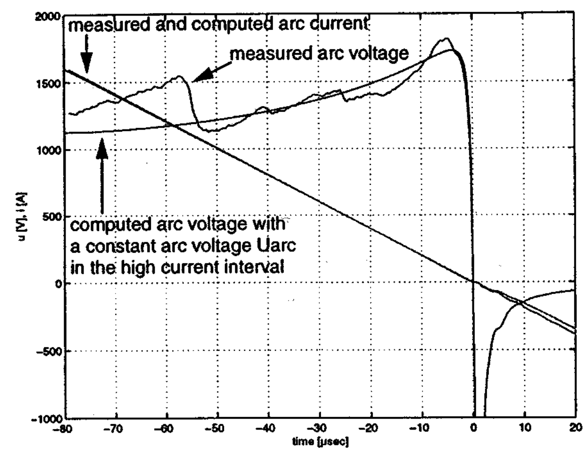

Fig. 9. Measured and computed currents and voltages for a reignition.

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References

[1] A. M. Cassie, "Theorie Nouvelle des Arcs de Rupture et de la Rigidité des Circuits," Cigre, Report 102, 1939.

[2] O. Mayr, "Beitrage zur Theorie des Statischen und des Dynamischen Lichthogens," *Archiv für Elektrotechnik*, vol. Band 37, no. Heft 12, pp. 588–608, 1943.

[3] O. Mayr, "Über die Theorie des Lichtbogens und seiner Löschung," in *Elektrotechnische Zeitschrift*, vol. Jahrgang 64, Dezember 16, 1943, pp. 645–652.

[4] WG 13.01, "Applications of Black Box Modeling to Circuit Breakers," *Electra*, no. 149, pp. 41–71, August 1993.

[5] M. Haupt, "Untersuchung der Anwendungsmöglichkeiten von Lichtbogenzweipolmodellen zur Beschreibung des Thermischen Schaltverhaltens von SF6-Leistungsschaltem," Dissertation RWTH, Aachen, 1988.

[6] R. Amsinck, “Verfahren zur Ermittlung der das Ausschaltverhalten Bestimmenden Lichtbogenkenngröβen,” *ETZ-A*, vol. Bd. 98, no. H. 8, pp. 566–567, 1977.
[7] A. Hochrainer, “Study of Arcs in Breakers with the Help of a Cybernetic Model and Under the Influence of Turbulence,” Cigre, Report 13-10, 1972.
[8] K. Zückler, “Untersuchungen zum Dynarnischen Verhatten von Schalterlichtbögen,” *ETZ-A*, vol. Bd. 99, no. H. 9, pp. 546–548, 1978.
[9] B. Sporckmann, “Auswerteverfahren zur Bestimmung der Kenngröβen phänomenologischer Lichtbogenmodelle,” *Archiv für Elektrotechnik*, vol. 66, pp. 135–141, 1983.
[10] G. Strang, *Linear Algebra and its Applications*, Third ed: Harcourt Brace Jovanovich.
[11] N. D. H. BijI and L. van der Sluis, “New Approach to the Calculation of Electrical Transients, Part 1: Theory,” *European Transactions on Electrical Power Engineering (ETEP)*, vol. 8, no. 3, pp. 175–179, May/June 1998.
[12] N. D. H. BijI and L. van der Sluis, “New Approach to the Calculation of Electrical Transients, Part 11: Applications,” *European Transactions on Electrical Power Engineering (ETEP)*, vol. 8, no. 3, pp. 181–186, May/June 1998.

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Lou van der Sluis was born in Geervliet, the Netherlands on July 10, 1950. He obtained his M.Sc. in electrical engineering from the Delft University of Technology in 1974. He joined the KEMA High Power Laboratory in 1977 as a test engineer and was involved in the development of a data acquisition system for the High Power Laboratory, computer calculations of test circuits and the analysis of test data by digital computer. In 1990 he became a part-time professor and since 1992 he has been employed as a full-time professor at the Delft University of Technology in the Power Systems Department. Prof. van der Sluis is a senior member of IEEE and convener of CC-03 of Cigre and Cired to study the transient recovery voltages in medium and high voltage networks.