Numerical Algorithm for Overhead Lines Arcing Faults Detection and Distance and Directional Protection

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***Abstract*—In this paper an overhead lines protection numerical algorithm, based on one terminal data and derived in the time domain, is presented. The fault location, direction and its nature (arcing or arcless fault) are estimated using Least Error Squares Technique. The faulted phase voltage is modeled as a serial connection of fault resistance and arc voltage, offering by this the more sophisticated line protection. The algorithm can be applied for both the ordinary and the high impedance faults detection, distance protection, intelligent autoreclosure, as well as for the purpose of directional relaying. The approach presented does not require the line zero sequence resistance as an input datum. The algorithm is derived for the case of most frequent single-phase to ground unsymmetrical faults. The results of algorithm testing through computer simulation are given. The influence of remote infeed, fault resistance, higher order harmonies, power system frequency, network topology, line parameters and other factors are investigated and systematically presented. Finally, an example of real life data processing is given.**

***Index Terms*—Autoreclosure, computer relaying, directional relaying, distance protection, fault location.**

I. Introduction

THE FAST AND accurate determination of fault location on electric power transmission lines is utilized as an aid in the fault analysis and power restoration. At the same time, the fault determination adversely impacts service reliability, operation costs and the quality of power delivery. Various numerical algorithms for fault location were developed in the past. A class of algorithms using one terminal information (one-terminal algorithms) [1]–[6] has a major advantage: it does not need data communication channels between line terminals. When communication channels are available, two terminal fault location numerical algorithms [7], [8] may be used following a fault.

Fault location numerical algorithms are predominantly developed in the complex/spectral domain, i.e., they use voltage and current phasors/spectral as input data. Traveling wave algorithms are theoretically the fastest, but rather complicated, putting high demands on the dynamics of current and voltage sensors. Time domain solutions [9]–[12], based on the fundamental differential equations describing the transients on the transmission line are not sensitive to the decaying DC component as well as to the changes of power system frequency, improving thus the algorithm quality. The algorithm properties depend on the physical model adopted. Too many model parameters make the algorithm not suitable for real-time application, or even cause ill-condition situations when the estimation theory is applied.

The existing numerical algorithms can be classified into the class of algorithms in which the result is calculated and the class of algorithms in which the result is estimated. In the nonsinusoidal nonlinear power systems and in the circumstances of frequent measurement errors it is recommended to apply estimation theory approaches, capable of filtering out bad data and minimizing the influence of random errors and noise (the so called *robust estimation algorithms*).

In this paper a fault location numerical algorithm, derived in the time domain, will be presented. Both the continuous-time and discrete-time equations will be given. It is based on voltages and currents measured at one terminal of protected line and on the application of Least Error Squares Technique. The algorithm presented in this paper is an important extension of the algorithm presented in [11]. The key improvements are: a) the model order has been reduced from $n = 4$ to $n = 3$, so the CPU requirements are reduced, what is very important for real-time applications, b) the mathematical model of the problem is so derived, that the undesired ill-condition cases never occur, so the additional post-filtering is now not necessary to get useful results, c) the directional properties of the algorithm are presented and demonstrated through computer simulated examples, and d) the paper reports further algorithm testing through computer simulation and processing input data recorded in a real network. The solutions presented in [9] and [10] do not consider the fault resistance and its influence to the fault location, so the algorithm presented in this paper, as well as the algorithm presented in [11], are an improvement in comparison to them.

Time domain algorithms require input data describing the line, such as per unit length impedance. The algorithm sensitivity to these, unavoidable, input data will be investigated in the paper. The approach presented does not require the line zero sequence resistance as an input datum, thus providing more flexibility, from the user point of view. Also, it will be demonstrated that the algorithm presented can be utilized for the design of directional relays.

The results of algorithm testing through computer simulation will be given, too. In the last section the results of processing real life data recorded in an 110 kV network will be presented.

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II. Algorithm Derivation

Let us consider the most frequent single-phase to ground fault on a transmission line fed from both line terminals. The new approach of modeling the fault voltage u_F presented in [13], models it as a serial connection of the fault resistance R_F and arc voltage u_a, so $u_F = u_a + R_F i_F$, where i_F is the fault (arc) current. There, the arc voltage is assumed to be of square wave shape, in phase with the arc current and corrupted by the random noise. Thus, the faulted phase voltage becomes:

$$u_F = U_a \operatorname{sgn}(i_F) + R_F i_F + \xi \tag{1}$$

where U_a is the arc voltage amplitude and ξ is a random noise.

For the known line per unit length resistance r and reactance x, the phase voltage measured by the relay can be and expressed as follows:

$$u = \left[ri + \frac{x}{\omega_0}\frac{d}{dt}(i + K_L i_0)\right]\ell + \operatorname{sgn}(i_0)U_a + i_0 R_e + \varepsilon \tag{2}$$

where: $K_L = (x_0 - x)/x$ (here x_0 is line per unit length zero sequence reactance) is the coefficient which can be precalculated, $R_e = (r_0 - r)\ell + k_a R_F$ (here r_0 is line per unit length zero sequence resistance) is an equivalent resistance, ω_0 is the fundamental angular velocity and ε is the error modeling all measuring errors and errors in modeling the transmission line and long arc in free air, existing at the fault point. Equation (2) is derived under the assumption that there exists a proportionality between the fault and zero sequence currents, i.e., $i_F = k_a i_0$, where k_a is a proportional coefficient. The value of k_a is not necessary to be known in advance.

The line zero sequence parameters r_0 and x_0 are frequency dependent. The actual variation depends on a number of factors, such as tower configuration, soil resistivity, etc. That means that the algorithm will not give accurate estimates if the line parameters are not exact. Through the algorithm development, r_0 was excluded from the list of input parameters which means that the algorithm accuracy is now not dependent on this input parameter.

Equation (2) requires the numerical calculation of the derivative of current (di/dt). The following approximate formula may be used for this calculation:

$$\frac{di(t)}{dt} \approx \frac{i_{n+1} - i_{n-1}}{2T}. \tag{3}$$

At one line terminal, e.g., the left one, line voltages and currents can be uniformly sampled with the preselected sampling frequency f_s, and used as an input to the algorithm. A set of N voltage and $N + 2$ current samples can be obtained.

For the kth sample ($k = 1, \cdots, N$), the following holds:

$$\begin{aligned} u_k = \Big\{ & ri_k + \frac{x}{2T\omega_0}[i_{k+1} - i_{k-1} \\ & + K_L(i_{0(k+l)} - i_{0(k-1)})]\Big\}\ell \\ & + \operatorname{sgn}(i_{0k})U_a + i_{0k}R_e + \varepsilon_k. \end{aligned} \tag{4}$$

Discrete-time (4) can be rewritten in the following abbreviated matrix form:

$$u_k = [\,a_{k1} \quad a_{k2} \quad a_{k3}\,]\boldsymbol{x} + \varepsilon_k \tag{5}$$

where

$$a_{k1} = ri_k + \frac{x}{2T\omega_0}\left[i_{k+1} - i_{k-1} + K_L(i_{0(k+I)} - i_{0(k-I)})\right]$$

$$a_{k2} = \operatorname{sgn}(i_{0k})$$

and

$$a_{k3} = i_{0k}$$

are timely dependent coefficients and $\boldsymbol{x}^T = [\ell U_a R_e]$ is the vector of unknown model parameters, to be estimated.

By writing (5) for all k samples, the following matrix equation is obtained:

$$\boldsymbol{u} = \boldsymbol{A}\boldsymbol{x} + \varepsilon \tag{6}$$

where $\boldsymbol{u} = [u_1, \cdots, u_N]^T$, $\boldsymbol{A}$ is an $N \times 3$ coefficient matrix and $\varepsilon = [\varepsilon_1, \cdots, \varepsilon_N]^T$.

Now, the vector of unknown model parameters may be estimated by using Least Error Squares Technique, i.e., by minimizing the error vector ε:

$$\boldsymbol{x} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{u}. \tag{7}$$

The algorithm presented in this paper estimates both the fault distance (at the same time its direction) and the arc voltage amplitude. It provides the solution for the most frequent unsymmetrical single-phase to ground faults. Based on the same methodology, the solutions for other fault types can be derived, too. The algorithm is not sensitive to decaying DC component appearing in fault current, as well as to the changes of power system frequency, which is a drawback of algorithms developed in the spectral domain and on the application of discrete Fourier technique. Due to this important feature, the convergence (speed) and accuracy of the algorithm are essentially improved.

The precise fault location is essentially improved by introducing the arc voltage into the problem formulation, modeling and development. In particular it is significant for close in faults, when the voltage, measured by relays, has very small values (e.g., 0.01 p.u.). Additionally it is badly distorted and the voltage drop depends on die arc voltage, or length (know that arc voltage is directly proportional to its length).

Since the fault resistance has been also taken into account, the algorithm covers the cases of high impedance faults, as well, and delivers precise fault distances for a wide range of fault resistances existing at the fault point.

The existence of a positive arc voltage amplitude value indicates the existence of arc at the location of the fault, so such faults can be considered as transient faults. In these cases auto-reclosure should operate. On the other hand, for arcless (permanent) faults this value is zero (or near zero) and in these cases auto-reclosure should be blocked. Such *intelligent auto-reclosure technique* can avoid reclosure onto the permanent faults and thus reduce damages on the elements of power system [14].

The estimated fault distance carry the information of the fault direction, so it can be used for the purpose of directional relaying. By taking the sign of the estimated distance, the logical blocking/operating signal can be obtained. If it is 1 (the positive distance, in front of the relay), the directional relay should operate. Contrary, if it is -1 (the negative distance, behind the relay), it should be blocked.

III. Computer Simulated Tests

The tests have been done using the Electromagnetic Transient Program (EMTP) [15]. The schematic diagram of the 400 kV power system on which the tests are based is shown in Fig. 1. Here $u(t)$ and $i(t)$ are digitized voltages and currents, and D is the line length. The line parameters, calculated via line constants program were $D = 100$ km, $r = 0.065$ Ω/km, $x = 0.3$ Ω/km, $r_0 = 0.195$ Ω/km and $x_0 = 0.9$ Ω/km. Data for network A were: $R_A = 1$ Ω, $L_A = 0.064$ H, $R_{A0} = 2$ Ω and $L_A = 0.128$ H. Data for network B were: $R_B = 0.5$ Ω, $L_B = 0.032$ H, $R_{B0} = 1$ Ω and $L_B = 0.064$ H. The equivalent electromotive forces of networks A and B were $E_A = 400$ kV and $E_B = 395$ kV, respectively. The phase angle between them was 20°.

Single-phase to ground faults are simulated at different points on the transmission line. The prefault load was present on the line. The left line terminal voltages and currents are sampled with the sampling frequency $f_s = 6400$ Hz. The duration of data window was $T_{dw} = 20$ ms.

In all tests network A was active. The nature of network B was changed: 1) Network B is a pure passive load, 2) network B is an active network; transformer B has "delta-wye" connection, and 3) Network B is an active network; transformer B has "wye-wye" connection and it is solidly (effectively) grounded.

In the first case, the algorithm did not introduce any error. During these and all further tests, the capacitive nature of the simulated test power system was neglected.

In the second case, the maximal relative errors for fault distances and arc voltage amplitudes, for the high impedance faults, were less than 0.01 and 0.0001%, respectively. From the practical point of view, these errors are negligible. They are mainly cause due to random noise modeled in the arc voltage. The estimation in the presence of harmonics will be presented in the next test example. The fault distance, fault resistance and arc voltage were $\ell = 10$ km, $R_F = 8$ Ω and $U_a = 3.5$. The 3rd, 5th, and 7th harmonics were generated from network A. Their content was 5, 2.5, and 1% of the fundamental harmonic, respectively. The distorted input voltages and currents are plotted in Figs. 2 and 3.

The estimated fault distance and arc voltage are depicted in Figs. 4 and 5. The exact unknown model parameters ($\ell = 10$ km and $U_a = 3.5$ kV) are obtained very fast, after 20 ms. From the algorithm speed and accuracy point of view, the results obtained confirm that the algorithm is useful for the application to real overhead lines protection.

In the third, from the point of view of the conditions under which the algorithm should operate, the worst case, the fault is fed with the zero sequence currents from both active networks, so the undesired errors appeared. Their appearance was mainly caused by the additional voltage drop on the fault resistance R_F

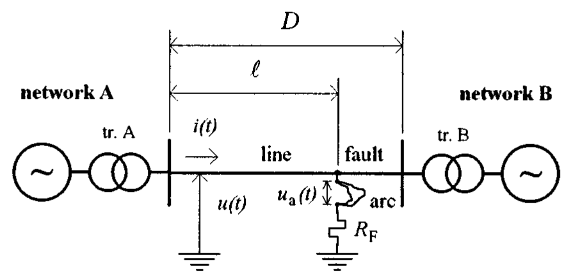

Fig. 1. Test power system.

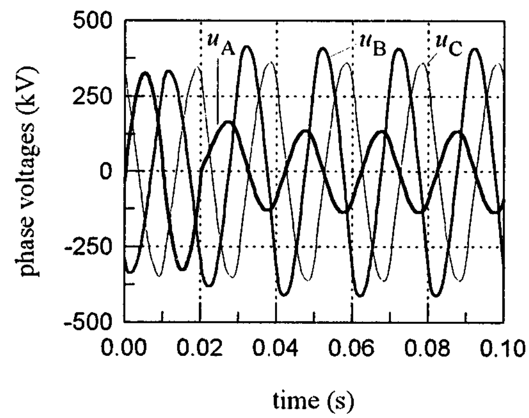

Fig. 2. Distorted input voltages.

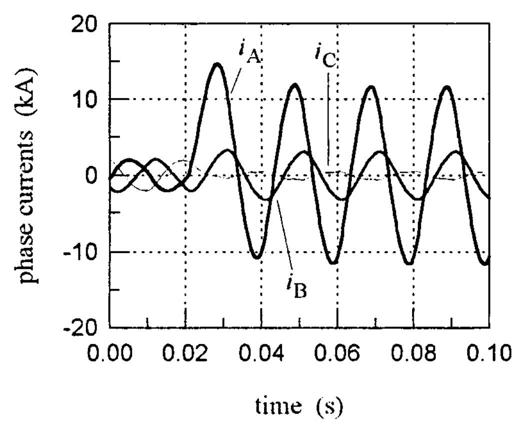

Fig. 3. Distorted input currents.

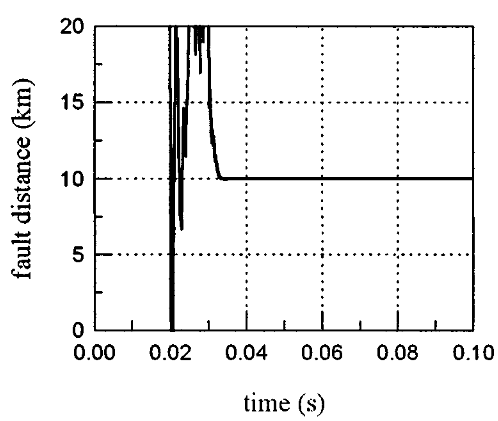

Fig. 4. Estimated fault distance.

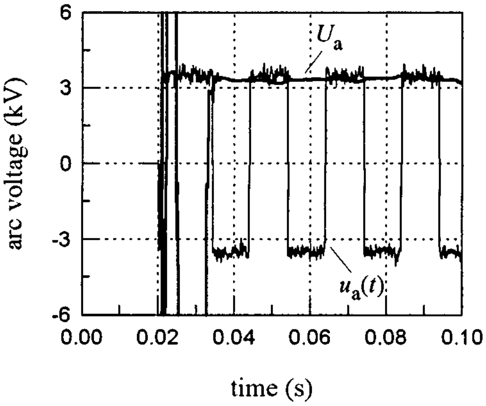

Fig. 5. Are voltage wave form and its estimated amplitude.

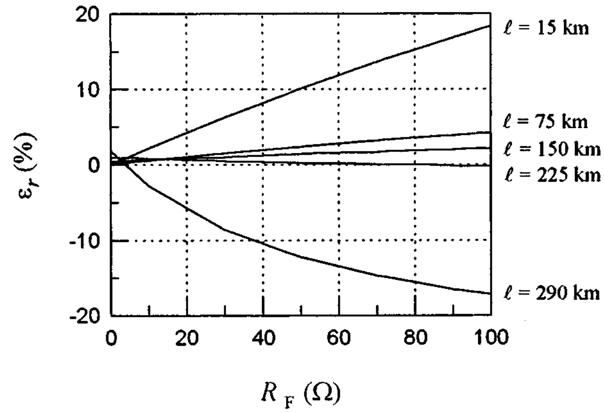

Fig. 6. Relative errors for various fault distances and resistances.

initiated by the remote end zero sequence current infeed. Note that in the second case transformer B blocked the flow of the zero sequence currents from network B to the fault, due to its delta-wye connection. From the arc voltage amplitude estimation point of view, the remote end infeed did not affect the results essentially. On the other hand, it affected the fault distance estimates. These errors are minimized with the fault resistance taken into account into the fault model. In Fig. 6 relative errors for the faults over resistance on a 300 km long line are depicted. The fault resistance R_F was changed in the range 0–100 Ω. The family of curves, for various fault distances marked in the right hand side of the graph, is presented. The errors due to huge values of the fault resistance and remote and infeed could be eliminated by using two terminal numerical algorithms approaches [7], [8].

The line zero sequence reactance x_0 determines the value of K_L and if not correctly selected, the errors occur. The algorithm sensitivity to K_L is investigated and these results are presented in Fig. 7. In Fig. 7 the relative errors for the faults on a 100 km long line are depicted. In the test the fault resistance was $R_F = 0$ Ω. Value of K_L was changed in the range 1.5–2.5 (±25% error in K_L; its exact value was $K_L = 2$), for various fault distances ℓ (5, 25, 50, 75, and 100% of the whole line length D).

As mentioned in the first section of the paper, the results are more accurate if the estimation approach, instead of simple calculation, is used. This investigation is provided for the model not including the arc voltage, when the calculation is possible. The model order is 2, the unknowns are line resistance and inductance, and, as shown in [12], they can be calculated. Through the analysis of the determinant of the normal matrix $\boldsymbol{A}^T\boldsymbol{A}$, inverted during estimation, for data windows lasting 0.02 s and containing 128 samples, it was concluded that it is high enough (approximately 10^6) and the singularity never occurs. For shorter data windows and smaller number of samples belonging to data window the value of determinant decreases. If the number of input data N exceeds the number of unknown model parameters ($n = 3$), i.e., if $N > 3$, (7) delivers us an estimate of the unknown vector x. If $N = 3$, through (7) the unknown vector x is calculated. The superiority of estimation with respect to calculation is shown in Fig. 8, in which two curves for an arcing fault at the middle of the line are depicted. The first is obtained

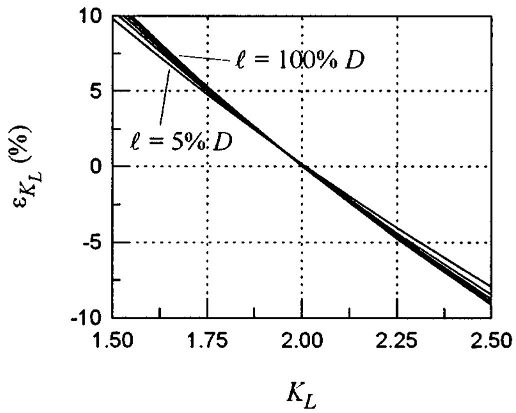

Fig. 7. Relative errors for various fault distances and values of K_L.

through estimation and the second (less accurate) through calculation. Arc, as a nonlinear phenomenon, injects higher order harmonics into the network and distorts other voltages and currents, thus introducing significant errors if the estimation is not used. Through the estimation, i.e., through the minimization of random errors, the more accurate result is obtained.

One of the good features of the algorithm presented is the fact that it is not sensitive to the variation of system frequency. The same algorithm will deliver precise estimates without regard if the signal frequency is 50 or 60 Hz, or any other, off-nominal, value. This important feature lies in the fact that the algorithm is developed in the time domain by using the differential equations as the problem model. As it can be observed, power system frequency does not appear in the model as a variable [do not mention it with the ω_0 in (4), which is a parameter equal to 2 π 50 (or 60) rad/s].

The line capacitance is not modeled, so the algorithm should introduce errors when applied on long lines. Fortunately, these errors are acceptable from the protection point of view. For example, for 300 km long line with the same parameters as for the line used in the algorithm testing, per unit length positive and zero sequence line capacitances $c = 0.1105 \times 10^7$ F/km and $c_0 = 0.1105 \times 10^7$ F/km, and the fault distance 200 km, the algorithm estimates distance 202 k. The relative error for this example was 0.0099%.

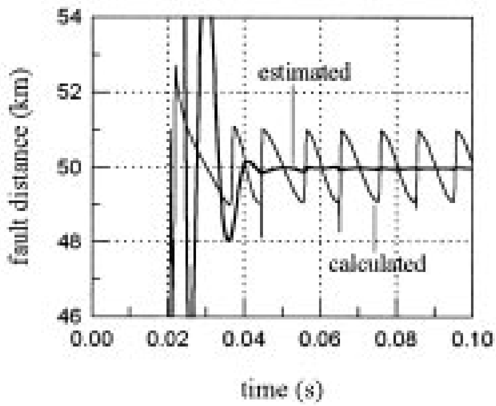

Fig. 8. Estimated and calculated distances.

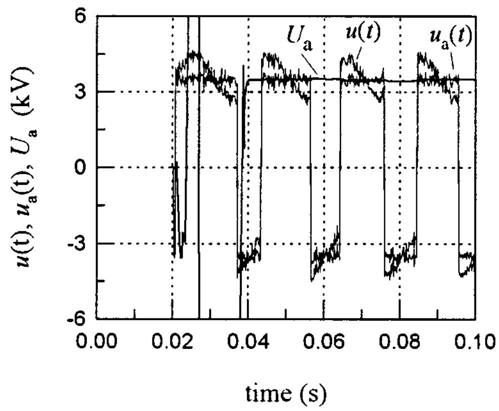

Fig. 10. Distorted input relay voltage, arc voltage and its estimated amplitude.

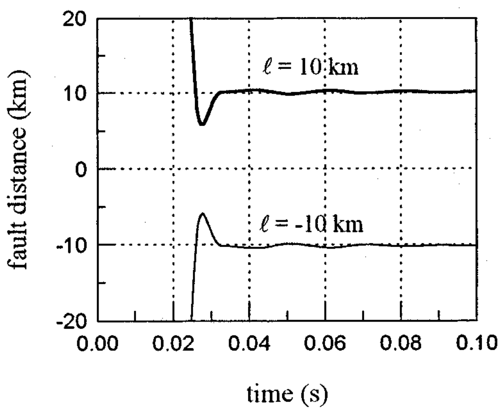

Fig. 9. Estimated fault distances.

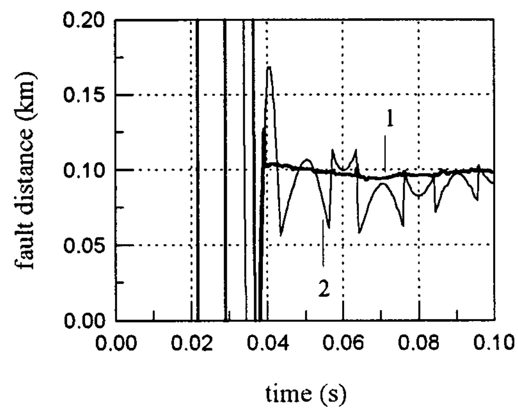

Fig. 11. Estimated distances for the close in fault.

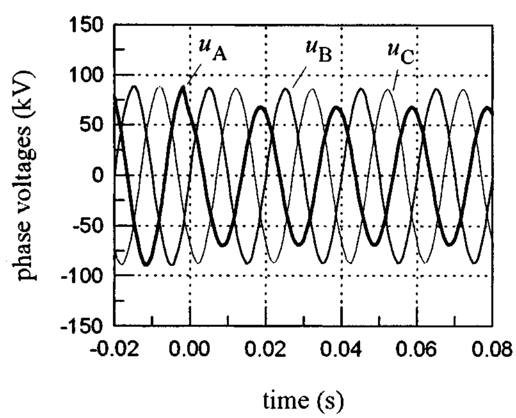

Fig. 12. Input voltages measured by the relay.

In the last test example the directional properties of the algorithm presented will be demonstrated. Two single-phase line to ground faults, the first behind and the second in front of relay, are simulated. The fault distances are ±10 km. Their estimates are presented in Fig. 9.

A very serious problem in operation of directional relays are faults which are very close to relay. The relay input voltage is very small, near zero. For directional relays it is difficult to distinguish such a close faults. If the arc voltage appears at the fault point, it can be helpful for the fault direction determination. On the other hand, under these circumstances, the input voltage, processed by the relay, is badly distorted and this distortion introduces errors. In Fig. 10 distorted relay input voltage $u(t)$, rectangular arc voltage $u_a(t)$ and its estimated amplitude U_a, for a close fault ($\ell = 0.1$ km) are presented.

The estimated fault distance is presented in Fig. 11. Two curves are depicted. The more accurate (curve 1) presents the estimated distance by using the model including arc voltage. The second (curve 2) is obtained by the model excluding the arc voltage term. It is obvious how important is the enclosure of arc voltage term in the problem modeling.

IV. Field Testing

In order to check the validity of the algorithm presented, voltages and currents, recorded during faults on a 110 kV network, are processed. Over 20 faults are analyzed. Here, a typical example of an arcing fault will be demonstrated. In Figs. 12 and 13 voltages and currents measured by the relay before and during a single-phase line to ground fault over arc are, respectively presented.

All signals are sampled with the sampling frequency $f_s 1600$ Hz. The duration of data window was $T_{dw} = 20$ ms. After data processing, the results of the application of the algorithm presented are depicted in Figs. 14 and 15. The exact fault location (see Fig. 14) $\ell = 12.8$ km was estimated. Additionally, it is determined that the fault was over an arc with the estimated amplitude plotted in Fig. 15. As it can be observed, the arc voltage am-

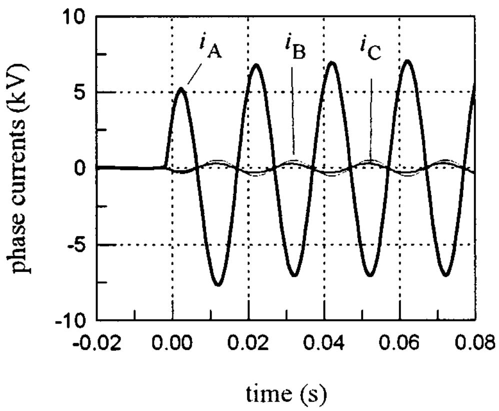

Fig. 13. Input currents measured by the relay.

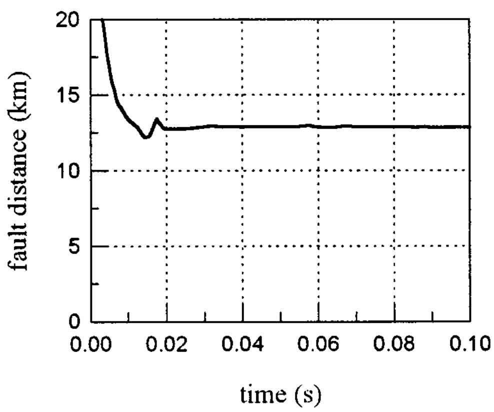

Fig. 14. Estimated fault location.

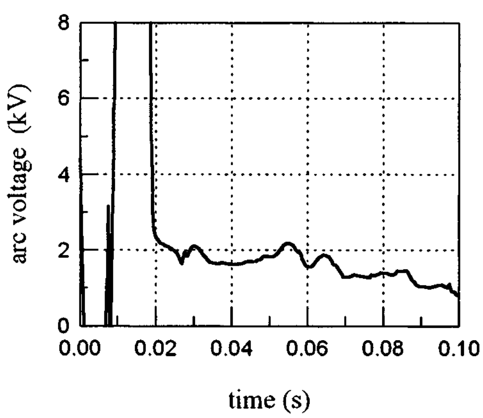

Fig. 15. Estimated arc voltage amplitude.

plitude was approximately 1–2 kV. This result can be checked by the following analysis: the minimal length of the arc path, determined by the arcing horns distance was 0.85 m and the minimal arc voltage amplitude expected was 1.1 kV (obtained as a product of the minimal arc length and the electric field inside the arc, which is practically constant along the arc and has the average value of 1.3 kV/m). The estimated arc voltage amplitude indicates that the arc was prolonged with regard to its minimal length, equal to the distance between arcing horns.

By the inspection of the fault analyzed, it was concluded that the estimated distance was the exact one. The exact result is obtained because the fault was fed only from one active network. Algorithm testing with field data in the cases when the fault is fed from both sides is the topic of authors current research project.

V. Conclusion

An efficient numerical algorithm for simultaneous estimation of the fault distance, fault direction and arc voltage amplitude, as well as for blocking auto-reclosure during permanent faults, is presented. It does not require zero-sequence line resistance as an input parameter, offering by this more flexibility to users. The errors caused by the remote end infeed are reduced to the minimum. The immunity to the higher order harmonics, generated from other nonlinear sources is shown. The advantages with respect to the algorithms based on simple calculation are presented. The influence of transformer connection, i.e., zero sequence currents feeding the fault, is investigated, too. It is concluded that the algorithm is not sensitive to frequency changes and to decaying DC component existing in the fault current. By this, the speed of the algorithm convergence and accuracy are essentially improved with respect to the algorithms developed in a spectral domain and based on the application of discrete Fourier technique. The excellent algorithm performances during close in faults are demonstrated, too. The processing of real life data confirmed the results obtained through computer simulations and showed that the algorithm can be applied for the protection of overhead lines.

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