An Improved Arc Model Before Current Zero Based on the Combined Mayr and Cassie Arc Models

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***Abstract*—In this paper, a computer model that describes the dynamic arc behavior in the high- and low-current regions before current zero is proposed. The model divides the current and voltage waveform in two regions. A differential equation for both regions which unifies current and voltage time derivatives is obtained by means of a generalized function method. The computer waveforms reproduced with the model show good agreement with measured results published in [1] in the low and high current regions, but further comparison with other test measurements are required to know if the model has any feature of predictability.**

***Index Terms*—Arc model, Cassie and Mayr type differential equations, circuit breaker, electric arc modeling.**

I. Introduction

THE performance of high-voltage circuit breakers during current interruption has been a topic of interest for many years. Along this time a great number of publications have been made regarding circuit breaker testing and modeling [1]–[8].

Current interruption is closely associated with the electric arc physical phenomena. During circuit breaker operation, the gap between electrodes changes rapidly from a conductive plasma to an insulating gas. The electrical arc performance during this short period of time is dependant on the energy balance in the plasma column. To operate successfully, the circuit breaker must be capable of interrupting this current in a short period of time, around current zero.

In order to assess circuit breaker capability during current interruption a test program is required. These tests provide the opportunity to study the dynamic arc behavior in detail. Based on this experimental work several arc models have been proposed, many of them focused on fitting measured data and model results. The usefulness of these arc models is that they provide additional information about circuit breaker performance under varied conditions. A general description about arc models can be found in [3], [4].

Actually, black box modeling is commonly used to describe the arc interaction with the electrical network in a current interruption processes. The aim of black box models is to use voltage and current traces from a circuit breaker test, together with a given differential equation, to deduce a mathematical model for the arc under study [4].

Most of the published work on black box models is based on the well known Cassie and Mayr models [1], [2], which provide a qualitative description of the arc in the high- and low-current regions, respectively. The fitting of model results to measured data is achieved by means of a proper selection of arc parameters like the time constant and the cooling power, which is normally taken as a function of arc current and voltage [4]–[7].

Since the Cassie and Mayr models, several approaches to arc modeling have been proposed. Some authors have combined Cassie and Mayr models in order to represent the arc dynamic in a wide current region [6]. More recently, by using the fact that the cooling power constant is closely related to the electric power input, a new arc model for high-voltage circuit breakers has been proposed [5]. This model reproduces the voltage and current traces accurately in the low current region. However, the arc model has some voltage inaccuracy in the high current region.

The aim of this paper is to develop an arc model that describes the dynamic arc behavior in the high- and low-current regions. At the same time, a generalized method of analysis for the development of arc models is proposed. This methodology facilitates the development of arc models from current and voltage traces before current zero.

The paper has been organized as follows. Section II presents a general form for arc models. In Section III, a general description of the theory related to the arc model developed is presented. In Section IV, some model results are presented and analyzed. Finally, conclusions and recommendations are presented.

II. General Form for Arc Models

Most of the published arc models have the form of ordinary differential equations of first order that usually have the following form [2], [4], [5]:

$$\frac{d\ln g}{dt} = \frac{1}{\tau(u,i)}\left[\frac{ui}{P(u,i)} - 1\right] \tag{1}$$

where g, u, and i are the arc conductance, voltage, and current, respectively. $P(u,i)$ and $\tau(u,i)$ are the cooling power and the time constant, respectively. A general description about the techniques for calculating these model parameters can be found in [4]. However, some cases exist where the arc model is not represented in this form. For example, the model developed by

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Rieder and Urbanek [7], which takes into account nonequilibrium effects in the arc plasma caused by high electric fields around current zero, is described by the following equation:

$$\frac{d\ln g}{dt} = \beta\frac{du}{dt} + \frac{1}{\tau_0}\left(\frac{ui}{P_0} - 1\right). \tag{2}$$

Sometimes in the literature, more sophisticated forms of black box arc models appear based on considering different arc characteristics or by the combinations of well-known models [4]. In order to facilitate the analysis, in this paper the following form for arc models is proposed, which can be obtained directly from (1)

$$\frac{du}{dt} = \Phi(u,i)\frac{di}{dt} - u\cdot F(u,i). \tag{3}$$

$\Phi(u,i)$ and $F(u,i)$ are different for each model. For the Mayr and Cassie arc models, these functions are

$$\begin{aligned}\Phi_{M,C}(u,i) &= \frac{u}{i}\\ F_M(u,i) &= \frac{1}{\tau_M}\left(\frac{ui}{P_0} - 1\right)\\ F_C(u,i) &= \frac{1}{\tau_C}\left(\frac{u^2}{E_0^2} - 1\right).\end{aligned} \tag{4}$$

This can be verified by direct substitution of (4) in (3). On the other hand, for the Reider and Urbanek model, the expressions for these functions are given by

$$\Phi(u,i) = \frac{u}{i(1+\beta u)} \quad F(u,i) = \frac{1}{\tau(1+\beta u)}\left(\frac{ui}{P_0} - 1\right).$$

The proposal of a general form for arc models provides a general frame for the analysis.

III. Improved Arc Model

The arc model to develop has to satisfy the generalized (3), where the functions $\Phi(u,i)$ and $F(u,i)$ are to be deduced.

A. Arc Model Background

Recently, in [5], Schavemaker *et al.* have proposed a Mayr type arc model based on current zero measurements. The arc model equation for the high- and low-current regions is

$$\frac{d\ln g}{dt} = \frac{1}{\tau}\left(\frac{ui}{\max(U_{\text{arc}}|i|, P_0 + P_1 ui)}\right) \tag{5}$$

where P_0 and P_1 are parameters related to the arc cooling and τ is the time constant. For this particular expression, the function $\Phi(u,i)$ is the same that for the Mayr model, u/i. On the other hand, $F(u,i)$ is obtained in [5]

$$F(u,i) = \frac{1}{\tau_M}\left(\frac{ui}{P_0 + P_1 ui} - 1\right). \tag{6}$$

Fig. 1 shows current and voltage measurements in a 245 kV/50 kA/50 Hz/single-unit SF6 puffer circuit breaker, used for validation purposes during a normal interruption [5]. This figure also show the calculated results using a Mayr type arc model, (5). On the other hand, Fig. 2 shows measured and computed results during reignition for the same model. From Figs. 1 and 2, it is evident that the Mayr type arc model reproduces reasonably well current and voltage traces in the low-current region.

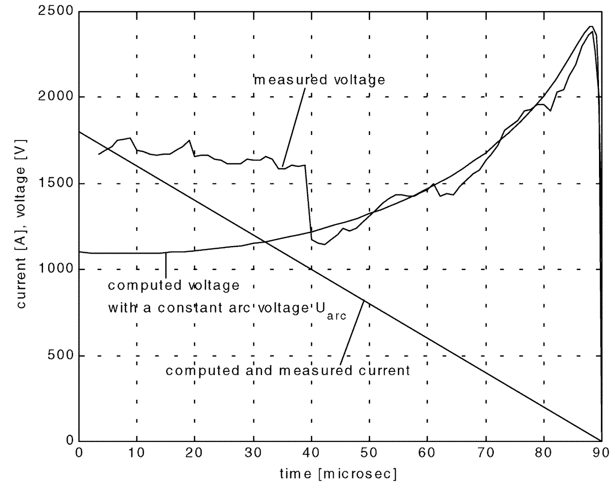

Fig. 1. Measured and computed voltage and current traces during a normal interruption.

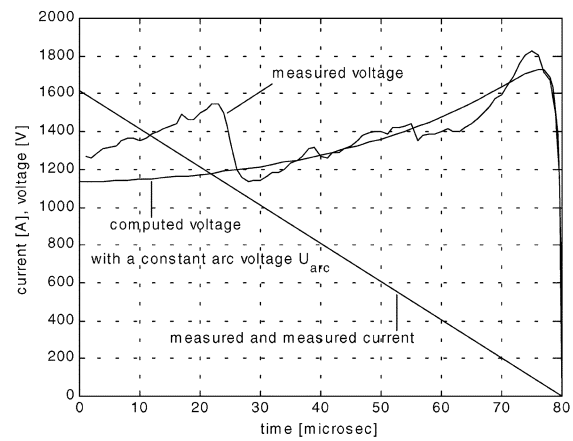

Fig. 2. Measured and computed voltage and current traces during a reignition.

In the high-current region, the Mayr type arc model is not as accurate as in the low current region. In Fig. 1, the differences are notably high, up to 600 V at 1.6 kA. Another important phenomena is the sudden voltage fall ($\approx$425 V) at 1 kA, which has also been found in other measurements [4]. Hence, a mathematical model that describes the arc dynamic in the high- and low-current regions is needed, which must also be capable of representing the arc voltage fall at around 1 kA.

B. Arc Model Development

From Fig. 1, it is evident that for currents greater than 1 kA the arc current and voltage behaves approximately linear. Thus, beginning at 1.75 kV, the arc voltage decreases up to $u_{\text{cr}} \approx$ 1.6 kV. Along this time, the arc current decrease from 1.75 kA up to $i_{\text{cr}} = 1$ kA. This fact can be described by the following first-order differential equation:

$$\frac{du}{dt} = \frac{u - u_{\text{cr}}}{i - i_{\text{cr}}}\frac{di}{dt} - u.F(u,i) \tag{7}$$

where the second term on the right-hand side of (7) must be very small, in the neighborhood of the transition point $u = u_{\text{cr}}, i =$

$i_{\rm cr}$. This can be achieved by considering a Cassie's form for the region $i > i_{\rm cr}$ with $E_0 = u_{\rm cr}$ and τ_C great enough in order to neglect the last term in (7) near the transition point. Hence, two equations have been obtained which describe the electric arc dynamic in the low- and high-current regions. The next step is to modify these equations in order to obtain the voltage fall at $i = i_{\rm cr}$. Mathematically, this can be represented by the following equation:

$$\frac{du}{dt} = \Delta u \cdot \delta\left(i - i_{\rm cr}\right)\frac{di}{dt} \tag{8}$$

where $\Delta u = 425$ V, for this particular case, and $\delta(i - i_{\rm cr})$ is the δ-function of Dirac [9]. Finally, (6)–(8) are combined into a generalized model where the functions $\Phi(u,i)$ and $F(u,i)$ are defined by

$$\begin{cases} \Phi(u,i) = \frac{u-u_{\rm cr}}{i-i_{\rm cr}}, \\ F(u,i) = \frac{1}{\tau_C}\left(\frac{u^2}{u_{\rm cr}^2} - 1\right) \end{cases}, \quad \text{if } i > i_{\rm cr},$$
$$\begin{cases} \Phi(u,i) = \Delta u \delta(i - i_{\rm cr}), \\ F(u,i) = 0, \end{cases} \quad \text{if } i = i_{\rm cr},$$
$$\begin{cases} \Phi(u,i) = \frac{u}{i}, \\ F(u,i) = \frac{1}{\tau_M}\left(\frac{ui}{P_0+P_1 ui} - 1\right) \end{cases}, \quad \text{if } i < i_{\rm cr}. \tag{9}$$

This can be rewritten in a single formulation using the Heavyside function $\theta(x)$ with the following properties:

$$\theta(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x > 0 \end{cases}. \tag{11}$$

Then

$$\begin{aligned}\Phi(u,i) &= \frac{u - u_{\rm cr}\cdot\theta\left(i - i_{\rm cr}\right)}{i - i_{\rm cr}\cdot\theta\left(i - i_{\rm cr}\right)} + \Delta u \cdot \delta(i - i_{\rm cr}) \\ F(u,i) &= \frac{1}{\tau_c}\left(\frac{u^2}{u_{\rm cr}^2} - 1\right)\theta\left(i - i_{\rm cr}\right) \\ &\quad + \frac{1}{\tau_m}\left(\frac{ui}{P_0 + P_1 ui} - 1\right)\theta\left(i_{\rm cr} - i\right).\end{aligned} \tag{12}$$

However, in expression (9), the lack of a defined derivative at the transition point can give rise to convergence problems. On the other hand, expression (12) contains a generalized function which cannot be used directly into circuit transient simulations. Hence, we have to replace θ by a smooth function like $T_\xi(i - i_{\rm cr})$, with the parameter ξ such that

$$\lim_{\xi\to 0} T_\xi(i - i_{\rm cr}) = \theta(i - i_{\rm cr}). \tag{13}$$

This function is similar to the well-known Fermi-Dirac distribution function, expression (14), and the parameter ξ must be chosen small enough to provide a rapid arc voltage fall in the transition point, $i = i_{\rm cr}$. $T_\xi(i - i_{\rm cr})$ for two different parameters $\xi = 0.2 i_{\rm cr}$ (curve I) and $\xi = 0.4 i_{\rm cr}$ (curve II), and θ-function are shown graphically in Fig. 3

$$T_\xi(i - i_{\rm cr}) = \frac{1}{1 + \exp\left(-\frac{i - i_{\rm cr}}{\xi}\right)}. \tag{14}$$

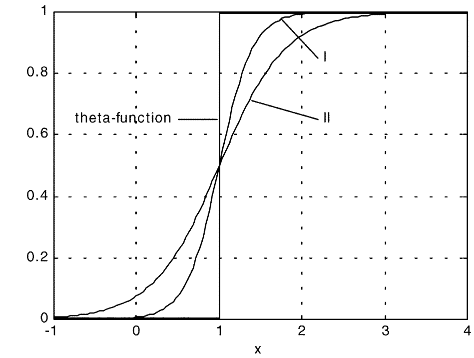

Fig. 3. Fermi-Dirac like distribution function, $x = i/i_{\rm cr}$.

From expressions (9) and (14), $D_\xi(i - i_{\rm cr})$ is defined as the derivative of $T_\xi(i - i_{\rm cr})$

$$D_\xi(i - i_{\rm cr}) = \frac{d}{di}T_\xi(i - i_{\rm cr}). \tag{15}$$

Again, ξ must be selected in (15) with the parameter ξ such that

$$\lim_{\xi\to 0} D_\xi(i - i_{\rm cr}) = \delta(i - i_{\rm cr}). \tag{16}$$

Finally, the following generalized hybrid model for the electric arc before current zero is obtained:

$$\begin{aligned}\frac{du}{dt} = \Phi(u,i)\frac{di}{dt} - \frac{u}{\tau_c(i)}\left(\frac{u^2}{u_{\rm cr}^2} - 1\right) \\ - \frac{u}{\tau_m(i)}\left(\frac{ui}{P_0 + P_1 ui} - 1\right)\end{aligned} \tag{17}$$

where

$$\begin{aligned}\Phi(u,i) &= \frac{u\left(1 + e^{\left(-\frac{i - i_{\rm cr}}{\xi}\right)}\right) - u_{\rm cr}}{i\left(1 + e^{\left(-\frac{i - i_{\rm cr}}{\xi}\right)}\right) - i_{\rm cr}} \\ &\quad + \frac{\Delta u}{\xi}\frac{e^{\left(-\frac{i - i_{\rm cr}}{\xi}\right)}}{\left(1 + e^{\left(-\frac{i - i_{c{\rm cr}}}{\xi}\right)}\right)^2} \\ \tau_m(i) &= \tau_{\rm mo}\cdot\left(1 + e^{\left(\frac{i - i_{\rm cr}}{\xi}\right)}\right) \\ \tau_c(i) &= \tau_{\rm co}\cdot\left(1 + e^{\left(-\frac{i - i_{\rm cr}}{\xi}\right)}\right).\end{aligned}$$

This model contains four additional parameters: $u_{\rm cr}, i_{\rm cr}, \Delta u$, and ξ. All these parameters can be obtained directly from Figs. 1 and 2.

IV. Numerical Results and Discussion

The arc model developed in the previous section must be validated. Current and voltages measurements presented in [5] were used for validation purposes. Computer programs were developed in order to solve expression (17) for varied simulation conditions, assuming that the arc current decreases in a linear form, i.e., di/dt is constant.

Figs. 4 and 5 show both experimental and computed voltage and current curves. The magnitude for the simulation parameters can be obtained directly from Fig. 1 and they are $i_0 = 1800$ A and $u_0 = 1731$ V, $i_{\rm cr} = 1010$ A, $u_{\rm cr} = 1600$ V,

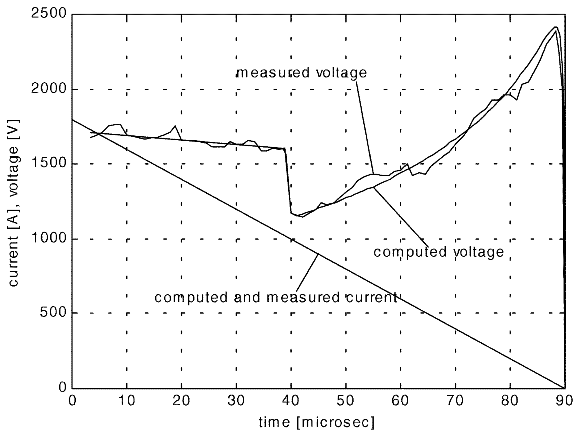

Fig. 4. Computed and measured voltage and current traces during normal interruption.

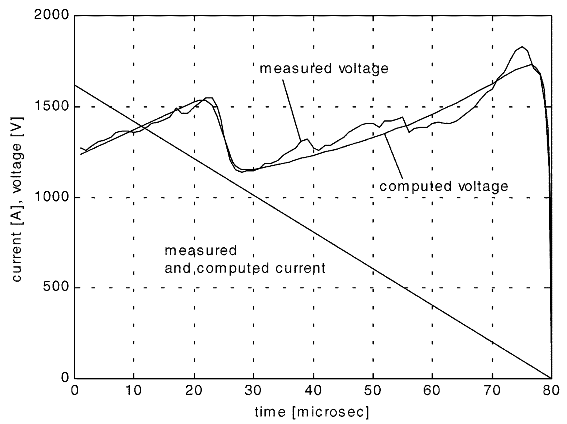

Fig. 5. Computed and measured voltage and current traces during a reignition.

$di/dt = -20$ A/μs, $\Delta u = 450$ V. On the other hand, the arc parameters considered in this simulation are $\tau_M = 0.27$ μs, $P_0 = 15\,917$ W, and $P_1 = 0.9943$. The magnitude of ξ is equal to 4 A, which is obtained from experimental data as the magnitude of the derivative du/dt at the point $i = i_{\rm cr}$

$$\xi = \frac{\Delta u}{4(du/dt)_{i=i_{\rm cr}}} \frac{di}{dt}. \tag{18}$$

For Fig. 5, the simulation parameters are as follows: $i_0 = 1615$ A, $u_0 = 1240$ V, $i_{\rm cr} = 1110$ A, $u_{\rm cr} = 1596$ V, $di/dt = -20.1875$ A/μs, $\tau_{M0} = 0.57$ μs, $P_0 = 24\,281$ W, $P_1 = 0.9942$ W, $\xi = 18$ A, $\Delta u = 420$ V. It should be kept in mind that these computer model results and simulation parameters are valid only for these measurements, and cannot be used in other designs, tests, or voltage and current conditions.

The comparison between measured and calculated results, Figs. 4 and 5, shows that the proposed arc model is capable of reproducing the measured results in the high- and low-current region. It should be mentioned that the approach used in the development of this arc model can be extended in order to include different patterns of current and voltages waveforms, typical of circuit breaker testing. For example, the Fermi-Dirac like function can be used to simulate several voltage falls or sudden increases in voltage magnitudes by adjusting its parameters. In this sense, the approach used in the development of this model is generalized.

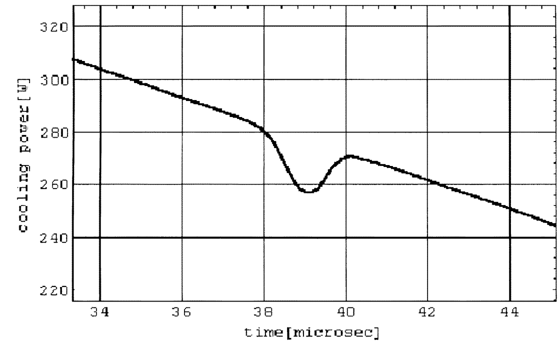

Fig. 6. Cooling power time dependence for normal interruption.

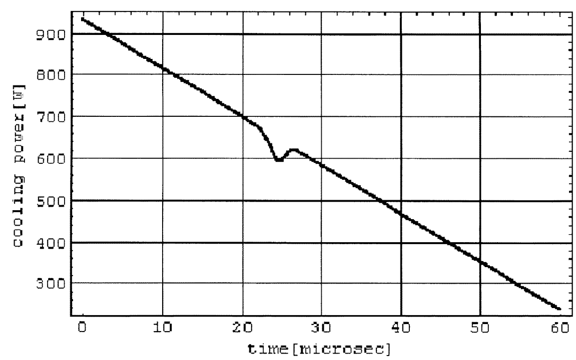

Fig. 7. Cooling power time dependence during reignition.

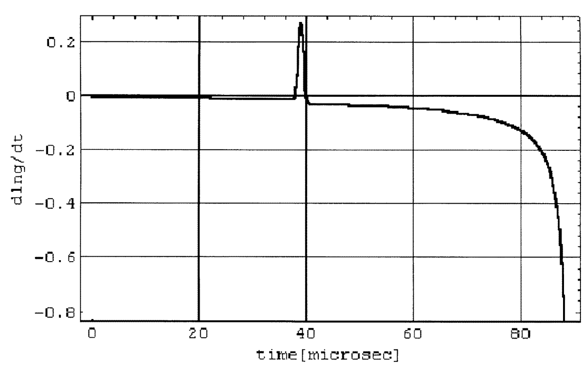

Fig. 8. Computed $d\ln g/dt$ curve for normal interruption.

Figs. 6 and 7 show the computed cooling power performance for Figs. 4 and 5, respectively. In both cases, the cooling power time dependence is linear, except in an area around $i_{\rm cr}$, where the cooling power decrease. This cooling power performance is in agreement with measured results published in the literature, which were the basis for the arc model proposed in [5]. This phenomena can be explained by the fact that the thermal energy produced by the arc does not disappears but supply some internal physical processes, which is incorporated externally in the arc model developed.

For validation purposes, Figs. 8 and 9 show the performance of the term $d\ln g/dt$ From these figures it is evident that this parameter behaves in a smooth manner except nearby the transition point $i = i_{\rm cr}$. Around this point, the $d\ln g/dt$ curves exhibit a sharp increase and decrease in magnitude. This performance is related to the arc voltage fall at the transition point, as shown by measurements in [5].

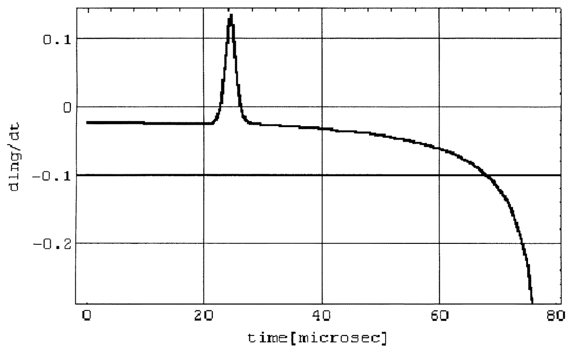

Fig. 9. Computed $d \ln g/dt$ curve during reignition.

V. Conclusion

A computer model that describes the arc dynamic performance in the high- and low-current regions has been presented. A comparison between measurements presented in [1] and the arc model results shows good agreement in the high- and low-current regions before current zero. However, comparisons with other test measurements are needed to determine if the model has any predictability for tests of different circuit breaker designs with varying voltage and current conditions.

The approach used in the arc model development can be effectively applied to varied patterns of current and voltage waveforms, typical of circuit breaker tests. In this sense, the method proposed is quite general. For example, in order to achieve a better precision in a highly distorted waveform, current and voltage measurements can be divided in several sections, where the techniques proposed in this paper can be applied independently.

In general, measurements and computer results show that the cooling power performance is linear. However, in an area close to the transition point, $i = i_{\mathrm{cr.}}$, the cooling power decrease significantly, which may be interpreted as some internal physical processes, phase change for example, which must be investigated in future works and incorporated into the modeling.

The arc model developed in this paper, (17), can be rewritten in the form of (2) as follows:

$$\frac{d \ln g}{dt} = \beta(u,i) + \frac{1}{\tau_c(u,i)}\left(\frac{u^2}{u_{\mathrm{cr}}^2} - 1\right) + \frac{1}{\tau_m(u,i)}\left(\frac{ui}{P_0 + P_1 ui} - 1\right) \quad (19)$$

where

$$\begin{aligned} \beta(u,i) &= \frac{1}{u} - \frac{1}{i\Phi(u,i)} \\ \tau_c(u,i) &= \frac{i}{u}\Phi(u,i)\tau_c(i) \\ \tau_m(u,i) &= \frac{i}{u}\Phi(u,i)\tau_m(i) \end{aligned} \quad (20)$$

and the $\Phi(u,i)$ is the same function that in (17). In this sense, the arc model (19) can be considered as an extended model (2) of the proposed by Rieder and Urbanek.

Finally, it should be mentioned that the arc model proposed in this paper is useful to simulate the arc behavior before current zero and does not preclude anything happening after current zero. Further developments are needed in the arc model to be capable of determining the circuit breaker interrupting capability.

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