Benchmark Methods & Forecast Accuracy

In this tutorial, you will learn general tools that are useful for many different forecasting situations. It will describe some methods for benchmark forecasting, methods for checking whether a forecasting method has adequately utilized the available information, and methods for measuring forecast accuracy. Each of the tools discussed in this tutorial will be used repeatedly in subsequent tutorials as you develop and explore a range of forecasting methods.

tl;df

This tutorial serves as an introduction to basic benchmarking approaches for time series data and covers:

1. Replication requirements: What you'll need to reproduce the analysis.
2. Naive Forecasting Methods: A simple but useful benchmark approach.
3. Fitted Values and Residuals: Always check the residuals.
4. Training and Test Sets: How to partition time series data.
5. Evaluating forecast Accuracy: How to evaluate accuracy of non-seasonal and non-season forecast methods.
6. Exercises: Practice what you've learned.

Replication Requirements

This tutorial leverages a variety of data sets to illustrate unique time series features. The data sets are all provided by the `forecast` and `fpp2` packages. Furthermore, these packages provide various functions for computing and visualizing basic time series components.

```
library(forecast)
library(fpp2)
```

Naive Forecasting Methods

Although it is tempting to apply "sophisticated" forecasting methods, one must remember to consider *naive forecasts*. A naive forecast is simply the last observed value in the series. In other words, at time t, the k-step-ahead naive forecast is given by

$$F_{t+k} = y_t \tag{1}$$

Sometimes, this is the best that can be done for many time series including most stock price data (for reasons illustrated in the previous tutorial's exercise). Even if it is not a good forecasting method, it provides a useful benchmark for other forecasting methods. Here, we use `naive` to forecast the next 10 values.

The resulting output is an object of class `forecast`. This is the core class of objects in the forecast package, and there are many functions for dealing with them. We can print off the model summary with `summary`, which provides us with the residual standard deviation, some error measures (which we will cover later), and the actual forecasts.

```
fc_goog <- naive(goog, 10)
summary(fc_goog)
##
## Forecast method: Naive method
##
```

```
## Model Information:
## Call: naive(y = goog, h = 10)
##
## Residual sd: 8.9145
##
## Error measures:
##                     ME     RMSE      MAE        MPE      MAPE MASE
## Training set 0.4436236 8.921089 6.008889 0.06493981 0.9815741    1
##                    ACF1
## Training set 0.04680557
##
## Forecasts:
##      Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 1001         838.96 827.5272 850.3928 821.4750 856.4450
## 1002         838.96 822.7915 855.1285 814.2325 863.6875
## 1003         838.96 819.1577 858.7623 808.6751 869.2449
## 1004         838.96 816.0943 861.8257 803.9900 873.9300
## 1005         838.96 813.3954 864.5246 799.8623 878.0577
## 1006         838.96 810.9554 866.9646 796.1306 881.7894
## 1007         838.96 808.7116 869.2084 792.6990 885.2210
## 1008         838.96 806.6231 871.2969 789.5049 888.4151
## 1009         838.96 804.6615 873.2585 786.5050 891.4150
## 1010         838.96 802.8062 875.1138 783.6675 894.2525
```

You will notice the forecast output provides a point forecast (the last observed value in the `goog` data set) and prediction confidence levels at the 80% and 95% level. A prediction interval gives an interval within which we expect y_i to lie with a specified probability. For example, assuming the forecast errors are uncorrelated and normally distributed, then a simple 95% prediction interval for the next observation in a time series is

$$\hat{y}_t \pm 1.96\hat{\sigma} \tag{2}$$

where $\hat{\sigma}$^ is an estimate of the standard deviation of the forecast distribution. In forecasting, it is common to calculate 80% intervals and 95% intervals, although any percentage may be used.

When forecasting one-step ahead, the standard deviation of the forecast distribution is almost the same as the standard deviation of the residuals. (In fact, the two standard deviations are identical if there are no parameters to be estimated such as with the naive method. For forecasting methods involving parameters to be estimated, the standard deviation of the forecast distribution is slightly larger than the residual standard deviation, although this difference is often ignored.)

For example, consider our naive forecast for the `goog` data. The last value of the observed series is 838.96, so the forecast of the next value is 838.96 and the standard deviation of the residuals from the naive method is 8.91. Hence, a 95% prediction interval for the next value of `goog` is

$$838.96 \pm 1.96(8.91) = [856, 821].$$

Similarly, an 80% prediction interval is given by

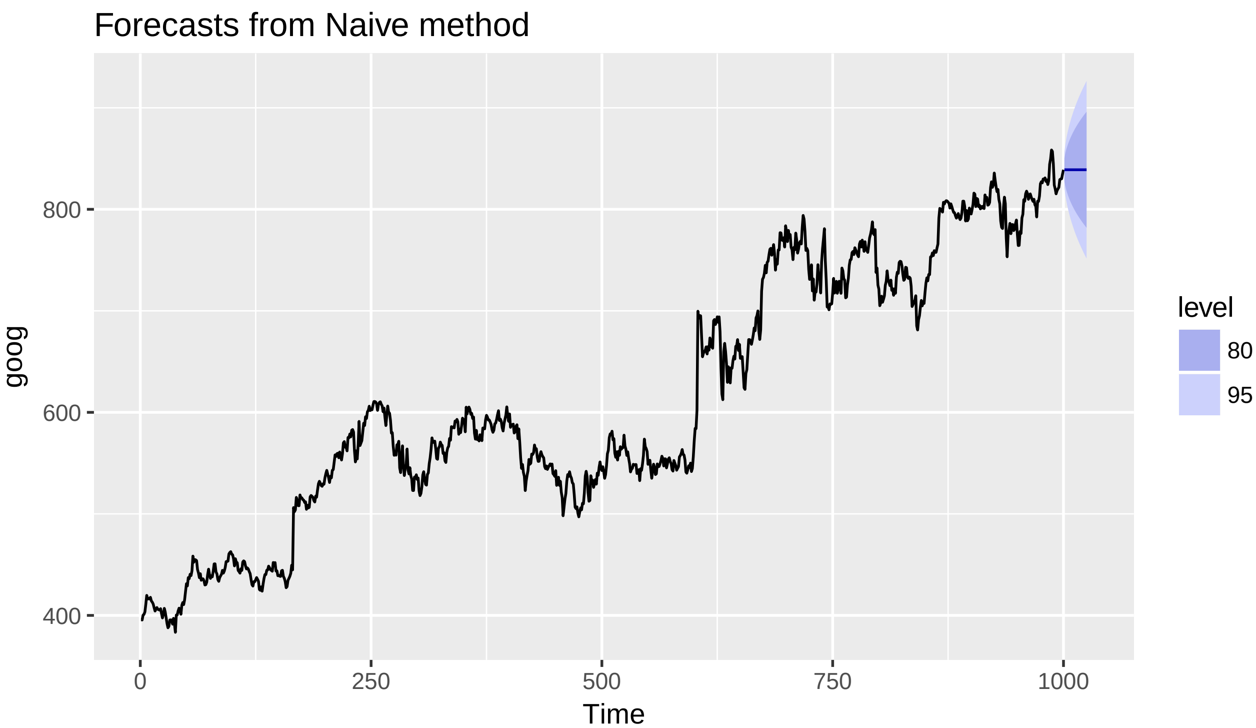
$$838.96 \pm 1.28(8.91) = [850, 828].$$

The value of prediction intervals is that they express the uncertainty in the forecasts. If we only produce point forecasts, there is no way of telling how accurate the forecasts are. But if we also produce prediction intervals, then it is clear how much uncertainty is associated with each forecast. Thus, with the naive forecast on the next goog value, we can be 80% confident that the next value will be in the range of 828-850 and 95% confident that the the value will be between 821-856.

We can illustrate this prediction interval by plotting the naive model (fc_goog). Here, we see the black point estimate line flat-line (equal to the last observed value) and the colored bands represent our 80% and 95% prediction confidence interval.

```
# forecast next 25 values
fc_goog <- naive(goog, 25)

autoplot(fc_goog)
```



For seasonal data, a related idea is to use the corresponding season from the last year of data. For example, if you want to forecast the sales volume for next March, you would use the sales volume from the previous March. For a series with M seasons, we can write this as

$$F_{t+k} = y_{t-M+k} \tag{3}$$

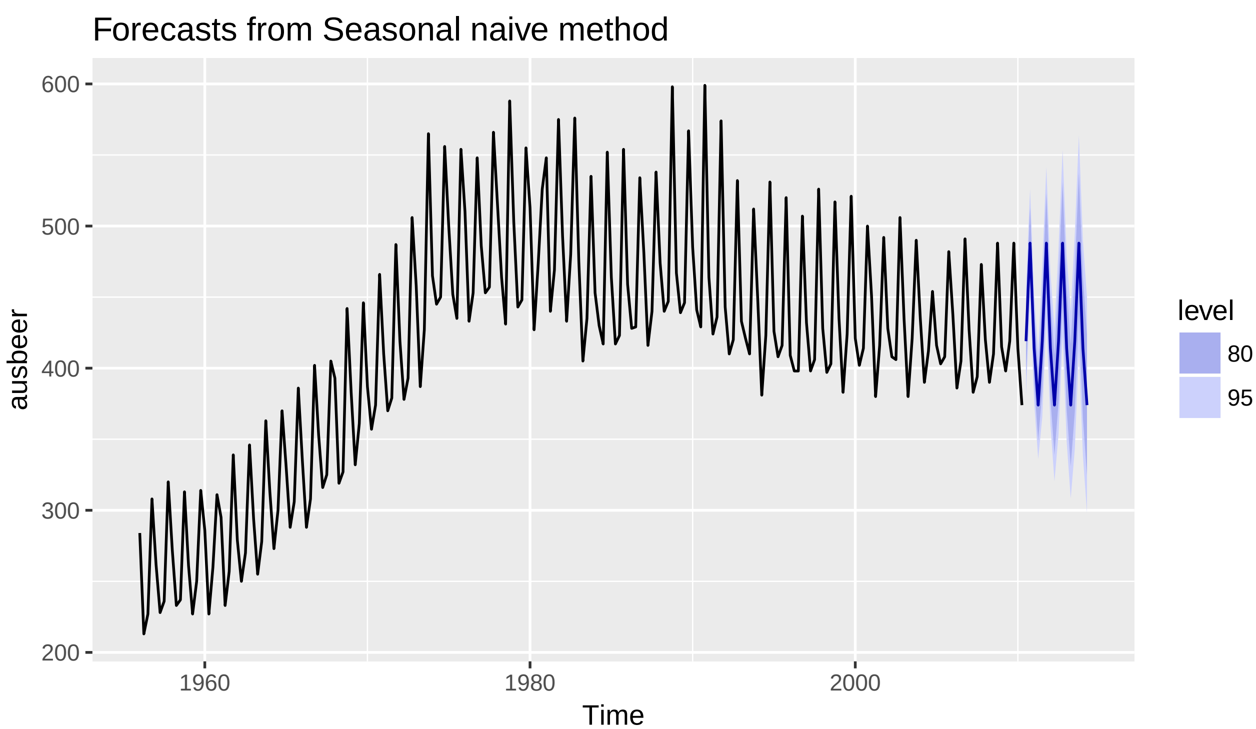
This is implemented in the snaive function, meaning, *seasonal naive*. Here I use snaive to forecast the next 16 values for the ausbeer series. Here we see that the 4th quarter for each future year is 488 which is the last observed 4th quarter value in 2009.

```
fc_beer <- snaive(ausbeer, 16)
summary(fc_beer)
##
## Forecast method: Seasonal naive method
##
## Model Information:
## Call: snaive(y = ausbeer, h = 16)
##
## Residual sd: 19.1207
```

```
##
## Error measures:
##                    ME     RMSE      MAE      MPE    MAPE MASE       ACF1
## Training set 3.098131 19.32591 15.50935 0.838741 3.69567    1 0.01093868
##
## Forecasts:
##         Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 2010 Q3            419 394.2329 443.7671 381.1219 456.8781
## 2010 Q4            488 463.2329 512.7671 450.1219 525.8781
## 2011 Q1            414 389.2329 438.7671 376.1219 451.8781
## 2011 Q2            374 349.2329 398.7671 336.1219 411.8781
## 2011 Q3            419 383.9740 454.0260 365.4323 472.5677
## 2011 Q4            488 452.9740 523.0260 434.4323 541.5677
## 2012 Q1            414 378.9740 449.0260 360.4323 467.5677
## 2012 Q2            374 338.9740 409.0260 320.4323 427.5677
## 2012 Q3            419 376.1020 461.8980 353.3932 484.6068
## 2012 Q4            488 445.1020 530.8980 422.3932 553.6068
## 2013 Q1            414 371.1020 456.8980 348.3932 479.6068
## 2013 Q2            374 331.1020 416.8980 308.3932 439.6068
## 2013 Q3            419 369.4657 468.5343 343.2438 494.7562
## 2013 Q4            488 438.4657 537.5343 412.2438 563.7562
## 2014 Q1            414 364.4657 463.5343 338.2438 489.7562
## 2014 Q2            374 324.4657 423.5343 298.2438 449.7562
```

Similar to `naive`, we can plot the `snaive` model with autoplot.

```r
autoplot(fc_beer)
```



Fitted Values and Residuals

When applying a forecasting method, it is important to always check that the residuals are well-behaved (i.e., no outliers or patterns) and resemble white noise. Essential assumptions for an appropriate forecasting model include residuals being:

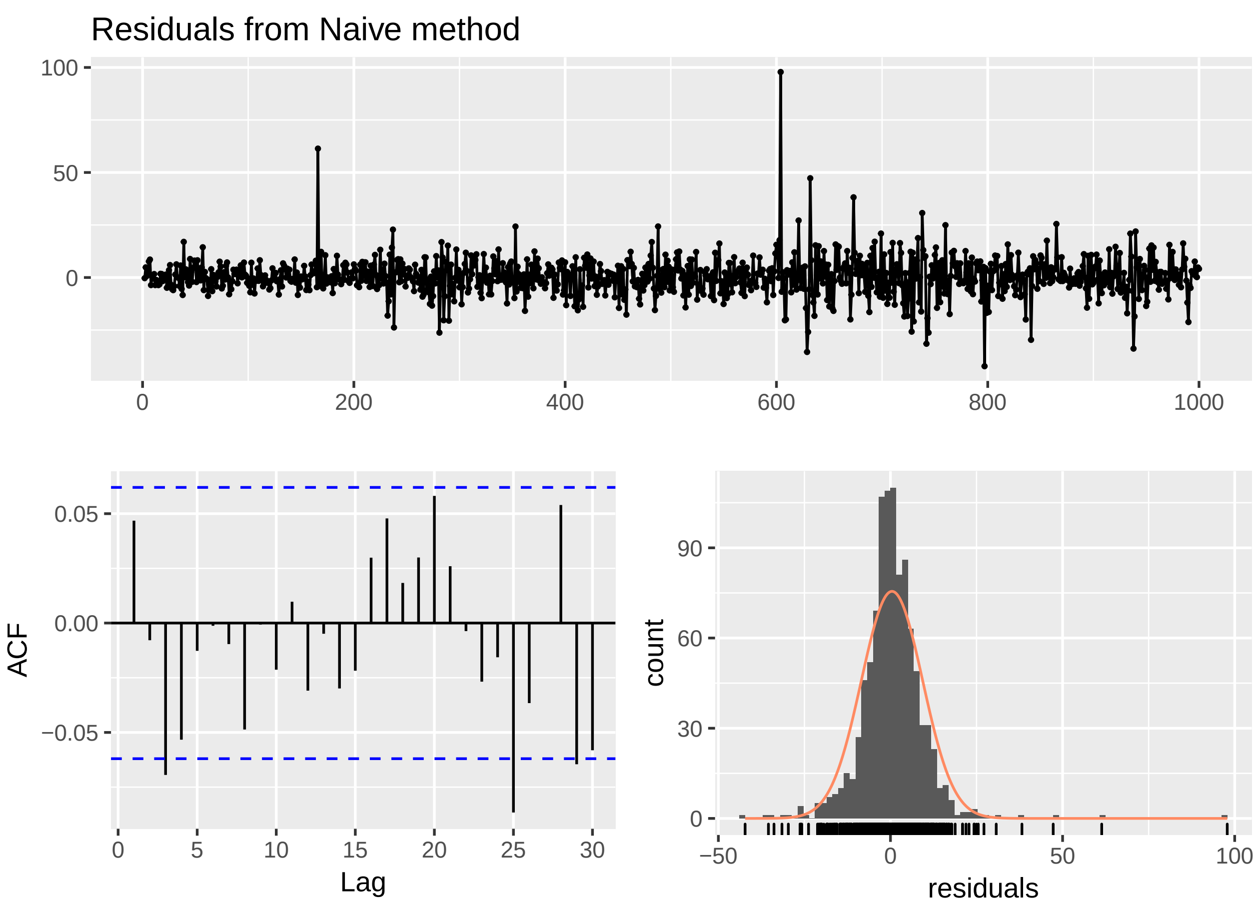
- uncorrelated
- have mean zero

Furthermore, the prediction intervals are computed assuming that the residuals:

- have constant variance
- are normally distributed

A convenient function to use to check these assumptions is the `checkresiduals` function. This function produces a time plot, ACF plot, histogram, and a Ljung-Box test on the residuals. Here, I use `checkresiduals` for the `fc_goog` naive model. We see that the top plot shows residuals that appear to be white noise (no discernable pattern), the bottom left plot shows only a couple lags that exceed the 95% confidence interval, bottom right plot shows the residuals to be approximately normally distributed, and the Ljung-Box test results shows a p-value of 0.22 suggesting the residuals are white noise.

```r
checkresiduals(fc_goog)
```

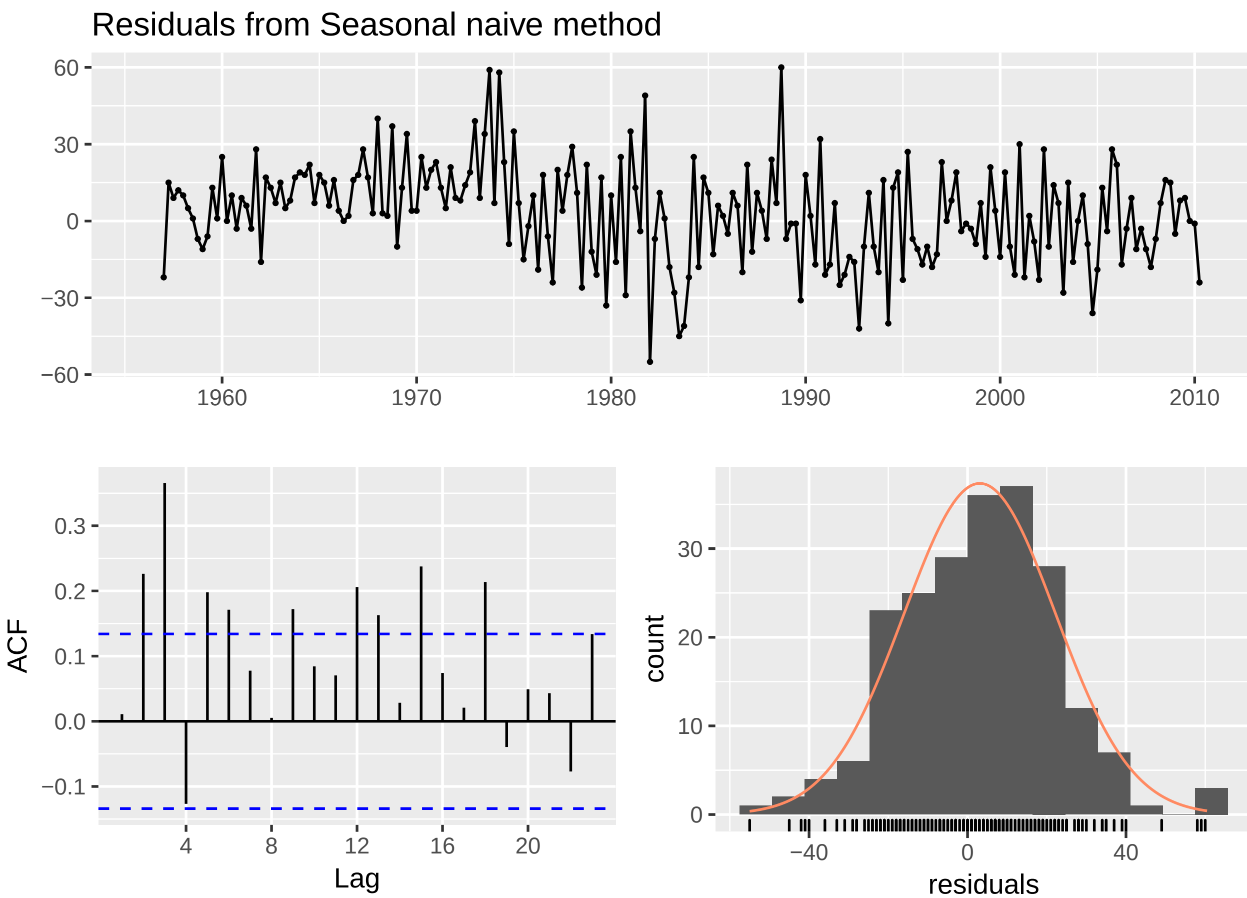



```
##
##  Ljung-Box test
##
## data:  residuals
```

```
## Q* = 13.061, df = 10, p-value = 0.2203
##
## Model df: 0.   Total lags used: 10
```

If we compare that to the `fc_beer` seasonal naive model we see that there is an apparent pattern in the residual time series plot, the ACF plot shows several lags exceeding the 95% confidence interval, and the Ljung-Box test has a statistically significant p-value suggesting the residuals are not purely white noise.

```
checkresiduals(fc_beer)
```



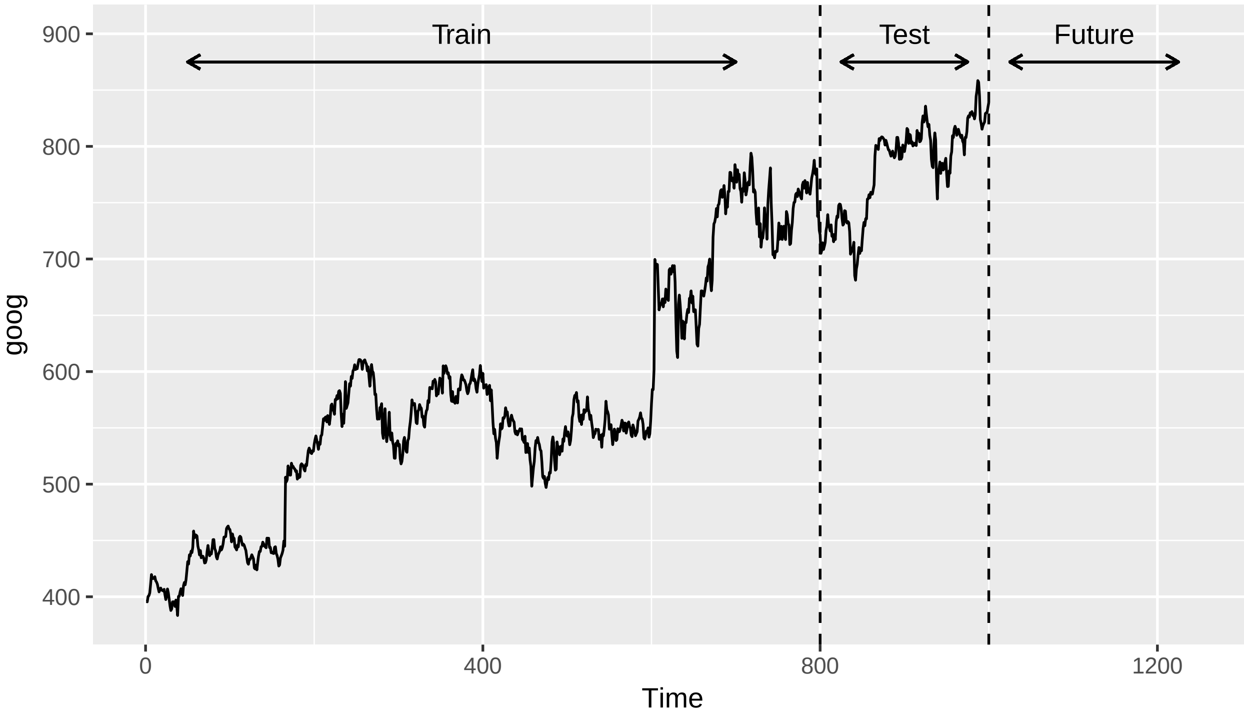

```
##
##  Ljung-Box test
##
## data:  residuals
## Q* = 60.535, df = 8, p-value = 3.661e-10
##
## Model df: 0.   Total lags used: 8
```

Training & Test Sets {training}

A training set is a data set that is used to discover possible relationships. A test set is a data set that is used to verify the strength of these potential relationships. When you separate a data set into these parts, you generally allocate more of the data for training, and less for testing. There is one important difference between data partitioning in cross-sectional and time series data. In cross-sectional data the partitioning is usually done randomly, with a random set of observations designated as training data and the remainder as test data. However, in time series, a random partition creates two problems:

1. It does not mimic the temporal uncertainty where we use the past and present to forecast the future.
2. It creates two time series with "holes", whereas many standard forecasting methods cannot handle time series with missing values.

Therefore, time series partitioning into training and test sets is done by taking a training partition from earlier observations and then using a later partition for the test set.



One function that can be used to create training and test sets is `subset.ts()`, which returns a subset of a time series where the start and end of the subset are specified using index values. The `gold` time series comprises daily gold prices over 1108 trading days. Let's use the first 1000 days as a training set. We can also create a test data set of the remaining data.

```
train <- subset.ts(gold, end = 1000)
test <- subset.ts(gold, start = 1001, end = length(gold))
```

Evaluating Forecast Accuracy

For evaluating predictive performance, several measures are commonly used to assess the predictive accuracy of a forecasting method. In all cases, *the measures are based on the test data set*, which serves as a more objective basis than the training period to assess predictive accuracy. Given a forecast and it's given errors (e_t), the commonly used accuracy measures are listed below:

Note that each measure has its strengths and weaknesses. For example, if you want to compare forecast accuracy between two series on very different scales you can't compare the MAE or MSE for these forecast as they measures depend on the scale of the time series data. MAPE is often better for comparisons but only if our data are all positive and have no zeros or small values. It also assumes a natural zero so it can't be used for temperature forecasts as these are based on arbitrary zero scales. MASE is similar to MAE but is scaled so that it can be compared across different data series. You can read more about each of these measures here; however, for now just keep in mind that for all these measures a smaller value signifies a better forecast.

Definitions	Observation y_t	Forecast $\hat{y}_t$	Forecast e $e_t = y_t$ -

Accuracy measure	Calculation
Mean Absolute Error	$MAE = average(\lvert e_t \rvert)$
Mean Squared Error	$MSE = average(e_t^2)$
Mean Absolute Percentage Error	$MAPE = 100 \times average(\lvert \frac{e_t}{y_t}$
Mean Absolute Scaled Error	$MASE = MAE/Q$

* Where Q is a scaling c

Figure 1: Forecast Accuracy Measures

We can compute all of these measures by using the `accuracy` function. The accuracy measures provided include *root mean squared error* (RMSE) which is the square root of the mean squared error (MSE). Minimizing RMSE, which corresponds with increasing accuracy, is the same as minimizing MSE. In addition, other accuracy measures not illustrated above are also provided (i.e. ACF1, Theil's U). The output of `accuracy` allows us to compare these accuracy measures for the residuals of the training data set against the forecast errors of the test data. However, our main concern is how well different forecasting methods improve the predictive accuracy on the test data.

Using the training data we created from the `gold`