PROJECT ADVANCE STATISTICS - NIKHIL



A research laboratory was developing a new compound for the relief of severe cases of hay fever. In an experiment with 36 volunteers, the amounts of the two active ingredients (A & B) in the compound were varied at three levels each. Randomization was used in assigning four volunteers to each of the nine treatments. The data on hours of relief can be found in the following .csv file: Fever.csv

[Assume all of the ANOVA assumptions are satisfied]

- 1.1) State the Null and Alternate Hypothesis for conducting one-way ANOVA for both the variables 'A' and 'B' individually. [both statement and statistical form like Ho=mu, Ha>mu]
- 1.2) Perform one-way ANOVA for variable 'A' with respect to the variable 'Relief'. State whether the Null Hypothesis is accepted or rejected based on the ANOVA results.
- 1.3) Perform one-way ANOVA for variable 'B' with respect to the variable 'Relief'. State whether the Null Hypothesis is accepted or rejected based on the ANOVA results.
- 1.4) Analyse the effects of one variable on another with the help of an interaction plot. What is the interaction between the two treatments? [hint: use the 'pointplot' function from the 'seaborn' function]
- 1.5) Perform a two-way ANOVA based on the different ingredients (variable 'A' & 'B' along with their interaction 'A*B') with the variable 'Relief' and state your results.
- 1.6) Mention the business implications of performing ANOVA for this particular case study.

Sample data

	A	В	Volunteer	Relief
0	1	1	1	2.4
1	1	1	2	2.7
2	1	1	3	2.3
3	1	1	4	2.5
4	1	2	1	4.6

Shape:

(36,4)

Information of the dataframe

#	Column	Non-Null Count	Dtype
0	Α	36 non-null	int64
1	В	36 non-null	int64
2	Volunteer	36 non-null	int64
3	Relief	36 non-null	float64
dtyp	es: float64	(1), int64(3)	

Summary of the dataframe

	A	В	Volunteer	Relief
count	36.000000	36.000000	36.000000	36.000000
mean	2.000000	2.000000	2.500000	7.183333
std	0.828079	0.828079	1.133893	3.272090
min	1.000000	1.000000	1.000000	2.300000
25%	1.000000	1.000000	1.750000	4.675000
50%	2.000000	2.000000	2.500000	6.000000
75%	3.000000	3.000000	3.250000	9.325000
max	3.000000	3.000000	4.000000	13.500000

Checking for missing value

 $\begin{array}{ccc} A & & 0 \\ B & & 0 \\ Volunteer & & 0 \\ Relief & & 0 \end{array}$

There are no missing values present

1.1) State the Null and Alternate Hypothesis for conducting one-way ANOVA for both the variables 'A' and 'B' individually. [both statement and statistical form like Ho=mu, Ha>mu]

For variable A

Ho :The mean hours of relief for different levels of active ingredient A are equal H0: u1A=u2A=U3A

Ha:At least at one of the levels, the mean hours of relief is different from the other. Ha:u1A!=u2A=U3A

For variable B

Ho :The mean hours of relief for different levels of active ingredient B are equal H0: u1B=u2B=U3B

Ha:At least at one of the levels, the mean hours of relief is different from the other Ha:u1B!=u2B=U3B

1.2) Perform one-way ANOVA for variable 'A' with respect to the variable 'Relief'. State whether the Null Hypothesis is accepted or rejected based on the ANOVA results.

One Way Anova:

Ho:The mean hours of relief for different levels of active ingredients A are equal .

Ha:At least at one of the levels, the mean hours of relief is different from the other

```
df sum_sq mean_sq F PR(>F)
C(A) 2.0 220.02 110.010000 23.465387 4.578242e-07
Residual 33.0 154.71 4.688182 NaN NaN
```

Since the p value is less than the significance level, we can reject the null hupothesis and conclude that the mean number of hours of relief is different for the levels in compound A

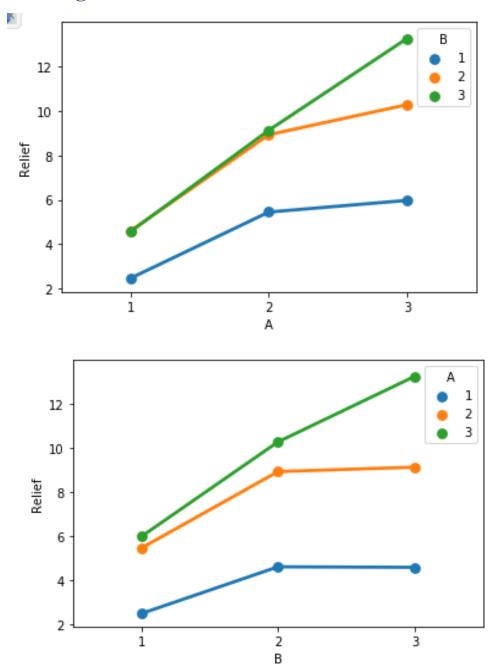
1.3) Perform one-way ANOVA for variable 'B' with respect to the variable 'Relief'. State whether the Null Hypothesis is accepted or rejected based on the ANOVA results.

Ho:The mean hours of relief for different levels of active ingredients B are equal Ha:At least at one of the levels the mean hours of relief is different from the other

Since the p value is less than the significance level, we can reject the null hupothesis and conclude that the mean number of hours of relief is different for the levels in compound B

1.4) Analyse the effects of one variable on another with the help of an interaction plot. What is the interaction between the two treatments? [hint: use the 'pointplot' function from the 'seaborn' function]

Drawing a Point Plot



By looking at the interaction plot between Variable A and Variable B

We can say that there is an interaction between Variable A and Variable B based on response variable Relief.

The responses significantly differs across the levels of both active ingredients the A and B. Relief hours are affected by interaction of A and B and relief number of hours increases significantly by level 3 of A and B.

Number of relief hours in not much impacted by level 2 and 3 of A combining with level 1 and 2 of B.

But it is not the case with level 3 of B. Relief hours change when level 3 of B interacts with A.

1.5) Perform a two-way ANOVA based on the different ingredients (variable 'A' & 'B' along with their interaction 'A*B') with the variable 'Relief' and state your results.

	df	sum_sq	mean_sq	F	PR(>F)
C(A)	2.0	220.020	110.010000	1827.858462	1.514043e-29
C(B)	2.0	123.660	61.830000	1027.329231	3.348751e-26
C(A):C(B)	4.0	29.425	7.356250	122.226923	6.972083e-17
Residual	27.0	1.625	0.060185	NaN	NaN

Two-way ANOVA -

At least at one of the levels, the mean hours of relief is different from the other

Ho: The mean hours of relief for different levels of active ingredients A and B are equal

H1: At least at one of the levels, the mean hours of relief of active ingredients A and B is different from the other .

By performing Two way anova p value is 6.972083e-17 which is way less than 0.05.

Hence null hypothesis Ho is rejected that means At least at one of the levels the mean hours of relief of active ingredients A and B is different from the other. There is interaction between the levels of ingredient A and ingredient B. Considering both the factors(A and B) both are significant factor as the p value is <0.05.

1.6) Mention the business implications of performing ANOVA for this particular case study.

The one-way ANOVA compares the means between the groups you are interested in and determines whether any of those means are statistically significantly different from each other.

In this case study after performing ANOVA, we can make the inference and conclude that both the active ingredients A and B have a significant interaction between the levels and both active ingredients A and B plays a significant and vital role in the cure of severe cases of hay fever.

When we performed Two way anova and checked the interaction between both variable A and B we found that interaction between the two ingredients is not significant while if we can see that A individually is the contributing factor in relief and B individually is a contributing factor in relief.

Variance between each level of ingredient A is 1827 times the variance within each level of ingredient A

Variance between each level of ingredient B is 1027 times the variance within each level of ingredient B

But when variance between of both A and B is taken together is 122 times the variation within which is very less as compared to when A and B are contributing individually and thus we can conclude that for preparing the cure for the relief of severe cases of hay fever we can consider that both the ingredients have significant impact if used individually but using both together is not that significant.

2

The dataset Education - Post 12th Standard.csv is a dataset that contains the names of various colleges. This particular case study is based on various parameters of various institutions. You are expected to do Principal Component Analysis for this case study according to the instructions given in the following rubric. The data dictionary of the 'Education - Post 12th Standard.csv' can be found in the following file: Data Dictionary.xlsx.

- 2.1) Perform Exploratory Data Analysis [both univariate and multivariate analysis to be performed]. The inferences drawn from this should be properly documented.
- 2.2) Scale the variables and write the inference for using the type of scaling function for this case study.
- 2.3) Comment on the comparison between covariance and the correlation matrix.
- 2.4) Check the dataset for outliers before and after scaling. Draw your inferences from this exercise.
- 2.5) Build the covariance matrix, eigenvalues, and eigenvector
- 2.6) Write the explicit form of the first PC (in terms of Eigen Vectors).
- 2.7) Discuss the cumulative values of the eigenvalues. How does it help you to decide on the optimum number of principal components? What do the eigenvectors indicate? Perform PCA and export the data of the Principal Component scores into a data frame.
- 2.8) Mention the business implication of using the Principal Component Analysis for this case study. [Hint: Write Interpretations of the Principal Components Obtained]

2.1) Perform Exploratory Data Analysis [both univariate and multivariate analysis to be performed]. The inferences drawn from this should be properly documented.¶

Shape:

(777,18)

Datatype of the dataframe

#	Column	Non-	-Null Cour	nt Dtype
0	Names	777	non-null	object
1	Apps	777	non-null	int64
2	Accept	777	non-null	int64
3	Enroll	777	non-null	int64
4	Top10perc	777	non-null	int64
5	Top25perc	777	non-null	int64
6	F. Undergrad	777	non-null	int64
7	P.Undergrad	777	non-null	int64
8	Outstate	777	non-null	int64
9	Room.Board	777	non-null	int64
10	Books	777	non-null	int64
11	Personal	777	non-null	int64
12	PhD	777	non-null	int64
13	Terminal	777	non-null	int64
14	S.F.Ratio	777	non-null	float64
15	perc.alumni	777	non-null	int64
16	Expend	777	non-null	int64
17	Grad.Rate		non-null	int64
dtyp	es: float64(1	nt64(16),	object(1)	

	count	mean	std	min	25%	50%	75%	max
Apps	777.0	3001.638353	3870.201484	81.0	776.0	1558.0	3624.0	48094.0
Accept	777.0	2018.804376	2451.113971	72.0	604.0	1110.0	2424.0	26330.0
Enroll	777.0	779.972973	929.176190	35.0	242.0	434.0	902.0	6392.0
Top10perc	777.0	27.558559	17.640364	1.0	15.0	23.0	35.0	96.0
Top25perc	777.0	55.796654	19.804778	9.0	41.0	54.0	69.0	100.0
F.Undergrad	777.0	3699.907336	4850.420531	139.0	992.0	1707.0	4005.0	31643.0
P.Undergrad	d 777.0 855.298584	855.298584	1522.431887	1.0	95.0	353.0	967.0	21836.0
Outstate	777.0	10440.669241	4023.016484	2340.0	7320.0	9990.0	12925.0	21700.0
Room.Board	777.0	4357.526384	1096.696416	1780.0	3597.0	4200.0	5050.0	8124.0
Books	777.0	549.380952	165.105360	96.0	470.0	500.0	600.0	2340.0
Personal	777.0	1340.642214	677.071454	250.0	850.0	1200.0	1700.0	6800.0
PhD	777.0	72.660232	16.328155	8.0	62.0	75.0	85.0	103.0
Terminal	777.0	79.702703	14.722359	24.0	71.0	82.0	92.0	100.0
S.F.Ratio	777.0	14.089704	3.958349	2.5	11.5	13.6	16.5	39.8
perc.alumni		12.391801	0.0	13.0	21.0	31.0	64.0	
Expend		5221.768440	3186.0	6751.0	8377.0	10830.0	56233.0	
Grad.Rate	777.0	65.463320	17.177710	10.0	53.0	65.0	78.0	118.0

Checking for duplicates

0 duplicates are present

Checking for Missing values

Names	0
Apps	0
Accept	0
Enroll	0
Top10perc	0
Top25perc	0
F.Undergrad	0
P.Undergrad	0
Outstate	0
Room.Board	0

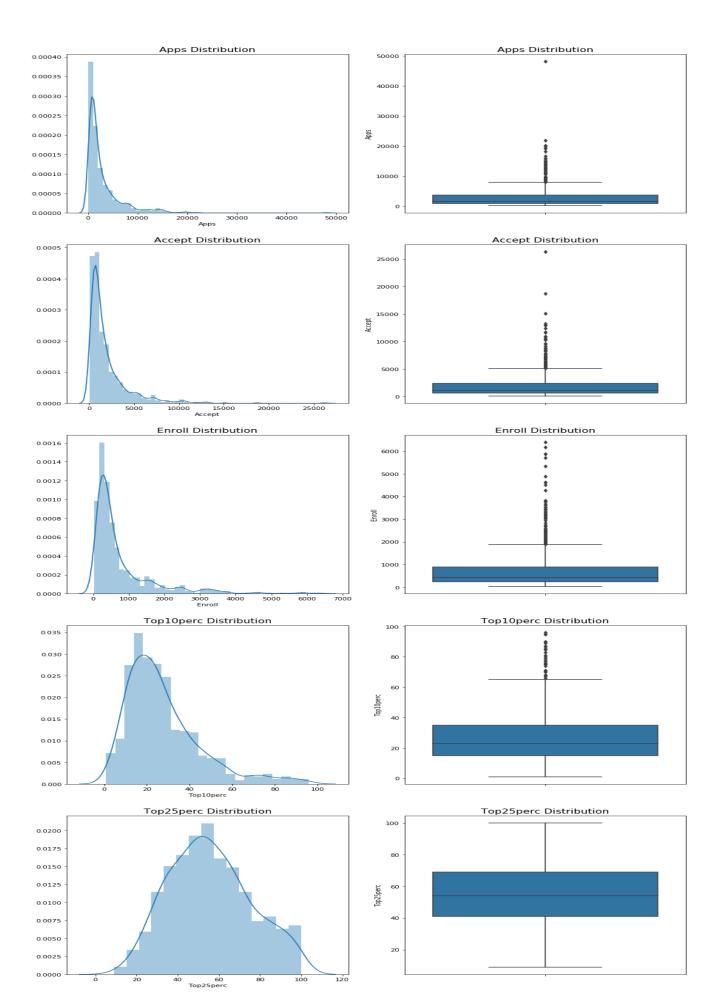
Books 0
Personal 0
PhD 0
Terminal 0
S.F.Ratio 0
perc.alumni 0
Expend 0
Grad.Rate 0
dtype: int64

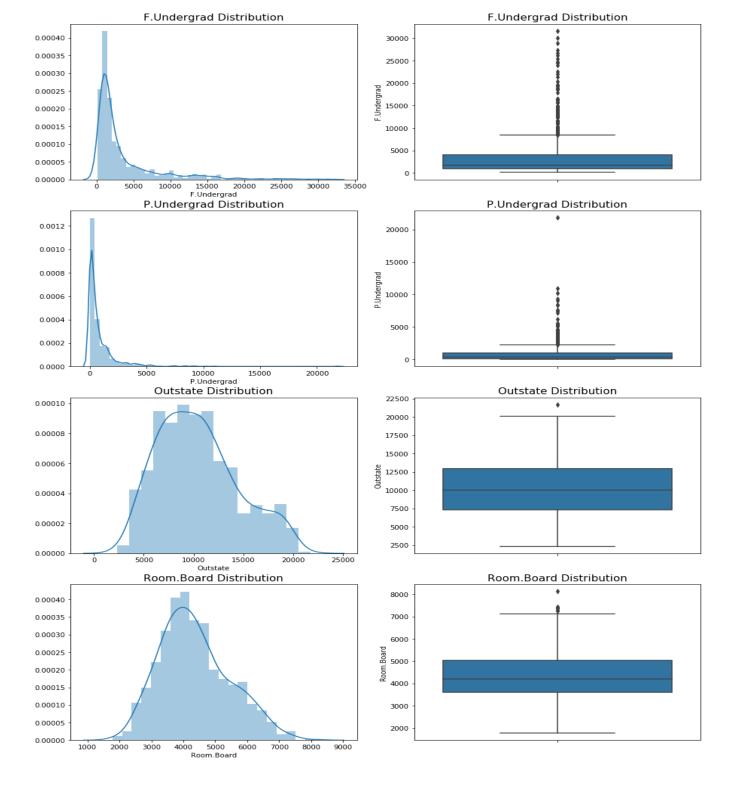
No missing values are present.

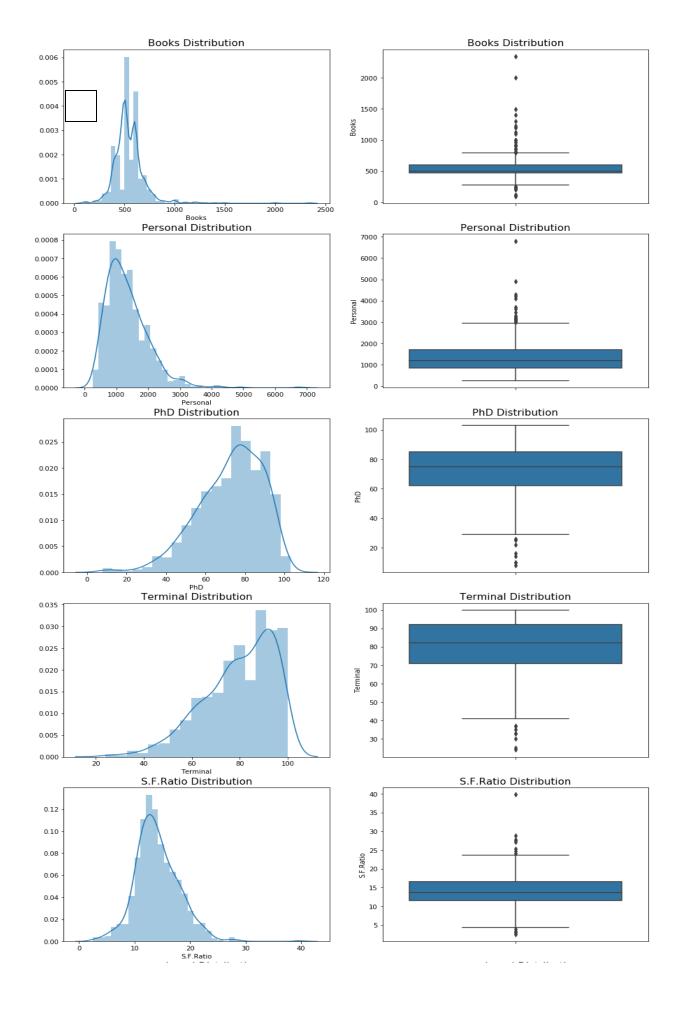
BOX PLOT AND DIST PLOT

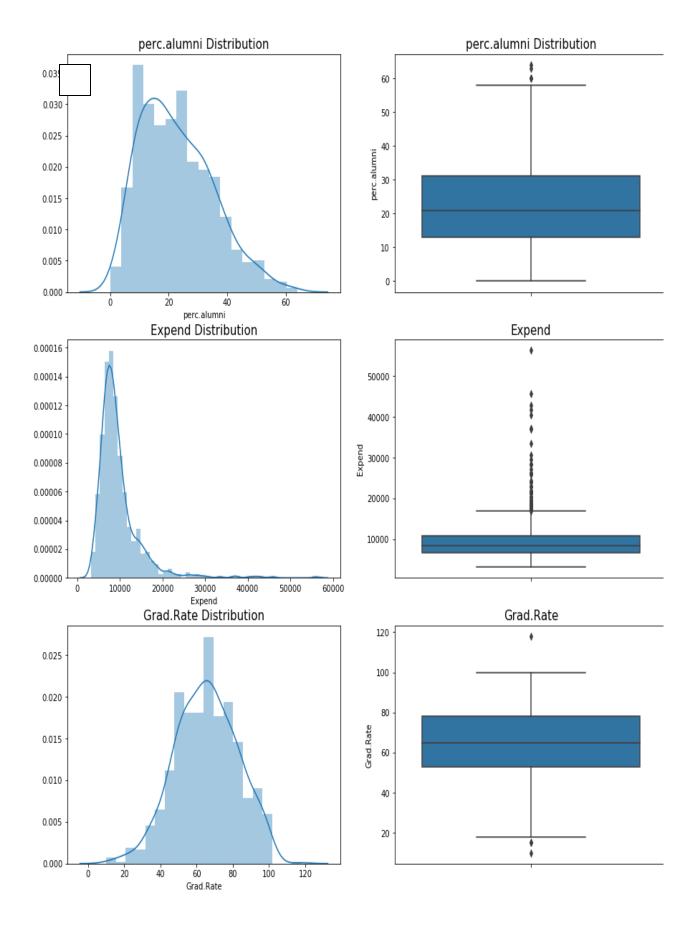
Boxplot gives us a good indication of how the values in the data are spread out and also tells us if any outlier is present.

Displot shows us univariant set of observations .

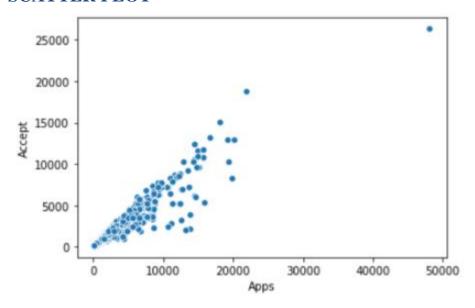








SCATTER PLOT



SKEWNESS

Apps	3.723750
Accept	3.417727
Enroll	2.690465
Top10perc	1.413217
Top25perc	0.259340
F.Undergrad	2.610458
P.Undergrad	5.692353
Outstate	0.509278
Room.Board	0.477356
Books	3.485025
Personal	1.742497
PhD	-0.768170
Terminal	-0.816542
S.F.Ratio	0.667435
perc.alumni	0.606891
Expend	3.459322
Grad.Rate	-0.113777

Univariate analysis refer to the analysis of a single variable.

The purpose of univariate analysis is to summarize and find patterns in the data.

No null value found

No missing value found

Mean of Apps is 3001.6, Median is 1558 and S.D is 3870.2 whereas Max value is 48094.0

Mean of Accept is 2018.8, Median is 1110 and S.D is 2451.1 whereas Max value is 26330.0

By seeing at the distplot for Apps we can see that most of the colleges have received applications in the range 0-5000

By seeing at the distplot for Accept we can see that most of the colleges have received applications in the range 0-2500

Highest mean is of outstate 10440.66 and lowest mean is of S.F ratio 14.08

Highest median is of outstate 9990 and lowest is of S.F ratio 13.6

Max value is of expand 56233

If Mode< Median< Mean then the distribution is positively skewed.

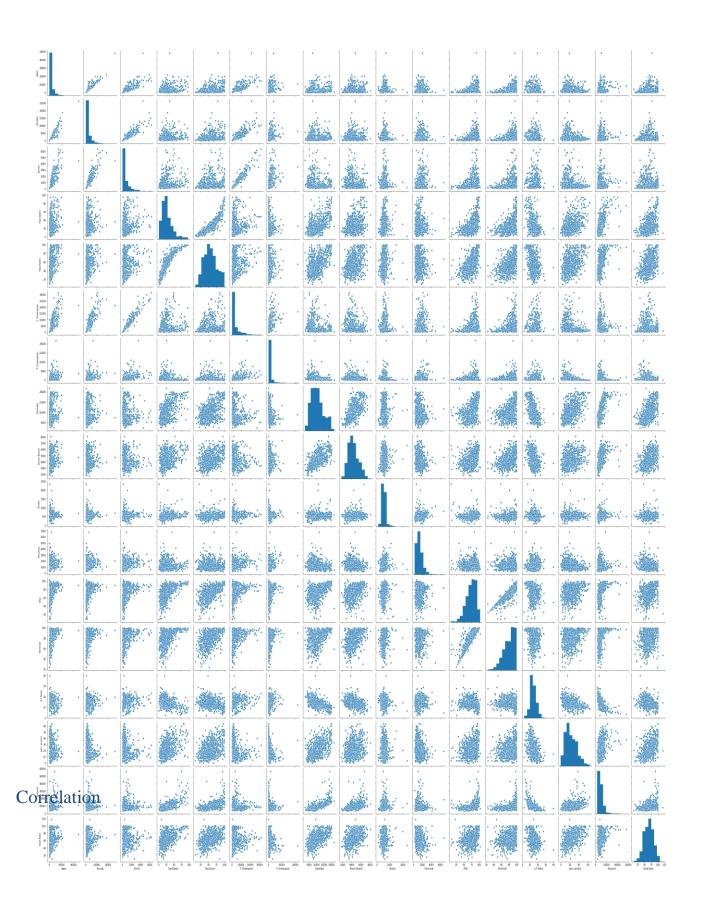
If Mode> Median> Mean then the distribution is negatively skewed.

Positively skewed: Most frequent values are low and tail is towards high values. Negatively skewed: Most frequent values are high and tail is towards low values.

Presence of skew in data with long thick tail can impact the effective of PCA which hurts the notion of variance and therefore hurts the notion of variance and therefore hurts the notion of PCA.

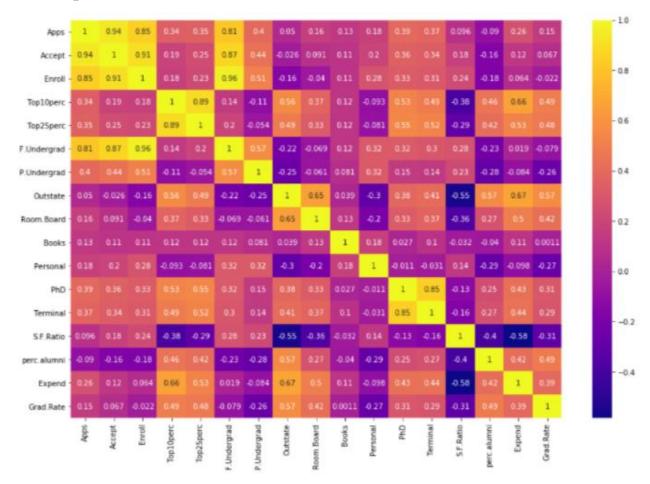
We observe that all the variables have an outliers except Top25perc

Pair plot



	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board	Books	Personal	PhD
Apps	1.000000	0.943451	0.846822	0.338834	0.351640	0.814491	0.398264	0.050159	0.164939	0.132559	0.178731	0.390697
Accept	0.943451	1.000000	0.911637	0.192447	0.247476	0.874223	0.441271	-0.025755	0.090899	0.113525	0.200989	0.355758
Enroll	0.846822	0.911637	1.000000	0.181294	0.226745	0.964640	0.513069	-0.155477	-0.040232	0.112711	0.280929	0.331469
Top10perc	0.338834	0.192447	0.181294	1.000000	0.891995	0.141289	-0.105356	0.562331	0.371480	0.118858	-0.093316	0.531828
Top25perc	0.351640	0.247476	0.226745	0.891995	1.000000	0.199445	-0.053577	0.489394	0.331490	0.115527	-0.080810	0.545862
F.Undergrad	0.814491	0.874223	0.964640	0.141289	0.199445	1.000000	0.570512	-0.215742	-0.068890	0.115550	0.317200	0.318337
P.Undergrad	0.398264	0.441271	0.513069	-0.105356	-0.053577	0.570512	1.000000	-0.253512	-0.061326	0.081200	0.319882	0.149114
Outstate	0.050159	-0.025755	-0.155477	0.562331	0.489394	-0.215742	-0.253512	1.000000	0.654256	0.038855	-0.299087	0.382982
Room.Board	0.164939	0.090899	-0.040232	0.371480	0.331490	-0.068890	-0.061326	0.654256	1.000000	0.127963	-0.199428	0.329202
Books	0.132559	0.113525	0.112711	0.118858	0.115527	0.115550	0.081200	0.038855	0.127963	1.000000	0.179295	0.026906
Personal	0.178731	0.200989	0.280929	-0.093316	-0.080810	0.317200	0.319882	-0.299087	-0.199428	0.179295		-0.010936
PhD	0.390697	0.355758	0.331469	0.531828	0.545862	0.318337	0.149114	0.382982	0.329202	0.026906	-0.010936	1.000000
Terminal	0.369491	0.337583	0.308274	0.491135	0.524749	0.300019	0.141904	0.407983	0.374540	0.099955	-0.030613	0.849587
S.F.Ratio	0.095633	0.176229	0.237271	-0.384875	-0.294629	0.279703	0.232531	-0.554821	-0.362628	-0.031929	0.136345	-0.130530
perc.alumni	-0.090226	-0.159990	-0.180794	0.455485	0.417864	-0.229462	-0.280792	0.566262	0.272363	-0.040208		0.249009
	PhD	Term	ninai	5.F.	Ratio	perc.	alumni	_	×pend		I.Rate	
0.390	697	0.369	9491	0.09	5633	-0.0	90226	0.2	59592	0.1	46755	•
0.355	758	0.337	7583	0.17	6229	-O.1	59990	0.1	24717	0.0	67313	:
0.331	469	0.308	3274	0.23	7271	-0.1	80794	0.0	64169	-0.0	22341	L
0.531	828	0.49	1135	-0.38	4875	0.4	55485	0.6	60913	0.4	94989	,
0.545	862	0.524	1749	-0.29	4629	0.4	17864	0.5	27447	0.4	77281	L
0.318	337	0.300	0019	0.27	9703	-0.2	29462	0.0	18652	-0.0	78773	
0.149	114	0.141	1904	0.23	2531	-0.2	80792	-0.08	83568	-0.2	57001	1
0.382	982	0.407	7983	-0.55	4821	0.5	66262	0.6	72779	0.5	71290	
0.329	202	0.374	1540	-0.36	2628	0.2	72363	0.5	01739	0.4	24942	2
0.026	906	0.099	955	-0.03	1929	-0.0	40208	0.1	12409	0.0	01061	L
-0.010	936	-0.030	0613	0.13	6345	-0.2	85968	-0.09	97892	-0.2	69344	+
1.000	000	0.849	587	-0.13	0530	0.2	49009	0.4	32762	0.3	05038	
0.849	349587 1.000000 -0.160104		0.2	67130	0.4	38799	0.289527		-			
-0.130	0530 -0.160104 1.000000 -0.402929		02929	-0.5	83832	-0.3	06710					
0.249	009	0.267	7130	-0.40	2929	1.0	00000	0.417712		0.490898		
0.432	762	0.438	3799	-0.58	3832	0.4	17712	1.00	00000	0.3	90343	
0.305038		0.289	527	-0.30	6710	0.4	90898	0.39	90343	1.00	00000	

Heatmap



We can see that many columns are co-related to each other

There is strong correlation exists between the applications received and application accepted.

There is strong correlation exists between the applications received and applications enrolled.

Scatter plot gave us an idea of the association between different variables. It gives visual representation of the degree of correlation between any two columns.

To obtain more precise information we have used the .corr() and heatmap to see that many columns are co-related to each other

So we can perform PCA.

2.2) Scale the variables and write the inference for using the type of scaling function for this case study.

First we can drop the variable 'Names' than we can use standard scaling or z score for scaling the data

	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board	Books	Personal	PhD	Terminal
0	-0.346882	-0.321205	-0.063509	-0.258583	-0.191827	-0.168116	-0.209207	-0.746356	-0.964905	-0.602312	1.270045	-0.163028	-0.115729
1	-0.210884	-0.038703	-0.288584	-0.655656	-1.353911	-0.209788	0.244307	0.457496	1.909208	1.215880	0.235515	-2.675646	-3.378176
2	-0.406866	-0.376318	-0.478121	-0.315307	-0.292878	-0.549565	-0.497090	0.201305	-0.554317	-0.905344	-0.259582	-1.204845	-0.931341
3	-0.668261	-0.681682	-0.692427	1.840231	1.677612	-0.658079	-0.520752	0.626633	0.996791	-0.602312	-0.688173	1.185206	1.175657
4	-0.726176	-0.764555	-0.780735	-0.655656	-0.596031	-0.711924	0.009005	-0.716508	-0.216723	1.518912	0.235515	0.204672	-0.523535

5.F.Ratio	perc.alumni	Expend	Grad.Rate
1.013776	-0.867574	-0.501910	-0.318252
-0.477704	-0.544572	0.166110	-0.551262
-0.300749	0.585935	-0.177290	-0.667767
-1.615274	1.151188	1.792851	-0.376504
-0.553542	-1.675079	0.241803	-2.939613

StandardScaler removes the mean and scales the data to unit variance.

Standard Scaler is the number of standard deviations by which the value of a raw score is above or below the mean value of what is being observed or measured.

Raw scores above the mean have positive standard scores, while those below the mean have negative standard scores.

StandardScaler removes the mean and scales the data to unit variance.

Often the variables of the data set are of different scales In this data set all variables are integers except SF Ratio is a float It gets difficult to compare the data when it is in different scales.

StandardScaler score the method used to standardize the range of features of data. Since, the range of values of data may vary widely, so we have choose Standard Scaler In this method, we convert variables with different scales of measurements into a single scale. StandardScaler normalizes the data using the

Formula (x-mean)/standard deviation.

We do this only for the numerical variables.

2.3) Comment on the comparison between covariance and the correlation matrix.

"Covariance" indicates the direction of the linear relationship between variables.

"Correlation" on the other hand measures both the strength and direction of the linear relationship between two variables.

Correlation is a function of the covariance.

You can obtain the correlation coefficient of two variables by dividing the covariance of these variables by the product of the standard deviations of the same values.

We can say that after scaling - the covariance and the correlation have the same values correlation value itself is scaled covariance value.

The covariance or correlation matrix captures relationships between different variables in their original dimensions. When the sign is positive, the variables are said to be positively correlated When the sign is negative, the variables are said to be negatively correlated When the sign is 0, the variables are said to be uncorrelated.

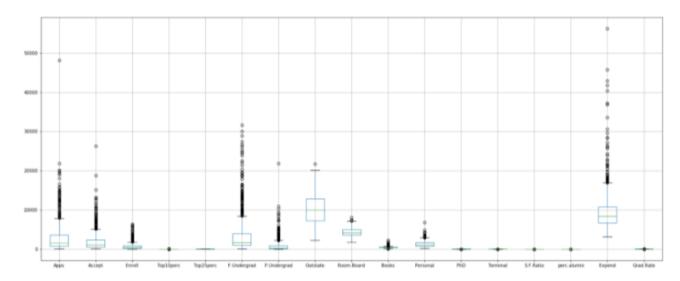
We can state that above three approaches yield the same eigenvectors and eigenvalue pairs:

- 1. Eigen decomposition of the covariance matrix after standardizing the data.
- 2. Eigen decomposition of the correlation matrix.
- 3. Eigen decomposition of the correlation matrix after standardizing the data.

Finally we can conclude that after scaling - the covariance and the correlation have the same values.

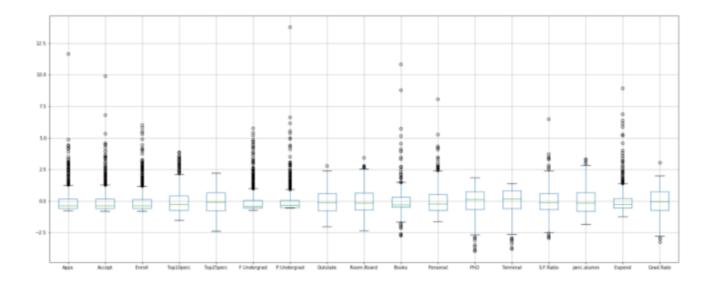
2.4) Check the dataset for outliers before and after scaling. Draw your inferences from this exercise.

BEFORE SCALING



All have outliers except Top25perc

AFTER SCALING



In both cases before scaling and after scaling

We observed that almost all variables have outliers except Top25perc

So scaling doesn't remove outliers it just helps in standardising the values.

Before scaling the boxplot shows the amount of total variance explained in the different principal components where we have not normalized the data.

After scaling we have normalized the data. Here it is clear that PCA seeks to maximize the variance of each component.

There is no significant change in the outlier pattern.

Covariance matrix must be built on the scaled data and the same should be given as an input to calculate the Eigen values and vectors.

Outliers are valid and need to be retained inmodel in this case and should not be treated

2.5) Build the covariance matrix, eigenvalues, and eigenvector.

COVARIANCE MATRIX

```
array([[ 1.001e+00,
                     9.450e-01,
                                  8.480e-01,
                                               3.390e-01,
                                                           3.520e-01,
                                  5.000e-02,
         8.160e-01,
                     3.990e-01,
                                               1.650e-01,
                                                          1.330e-01,
         1.790e-01,
                     3.910e-01,
                                  3.700e-01,
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         2.600e-01,
                     1.470e-01],
                                              1.930e-01,
       [ 9.450e-01,
                     1.001e+00,
                                  9.130e-01,
                                                           2.480e-01,
         8.750e-01,
                     4.420e-01, -2.600e-02,
                                              9.100e-02,
                                                          1.140e-01,
                     3.560e-01,
                                              1.760e-01, -1.600e-01,
         2.010e-01,
                                  3.380e-01,
         1.250e-01,
                     6.700e-02],
       [ 8.480e-01,
                     9.130e-01,
                                  1.001e+00,
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         9.660e-01,
                                                          1.130e-01,
         2.810e-01,
                     3.320e-01,
                                  3.090e-01,
                                              2.380e-01, -1.810e-01,
         6.400e-02, -2.200e-02],
       [ 3.390e-01,
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                                              3.720e-01,
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        -9.300e-02,
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         2.000e-01, -5.400e-02,
                                  4.900e-01,
                                              3.320e-01,
                                                           1.160e-01,
        -8.100e-02,
                     5.470e-01,
                                  5.250e-01, -2.950e-01,
                                                           4.180e-01,
         5.280e-01,
                     4.780e-01],
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                                  9.660e-01,
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                     5.710e-01, -2.160e-01, -6.900e-02,
         1.001e+00,
                                                           1.160e-01,
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                     3.190e-01,
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        -2.990e-01,
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                     1.000e-03],
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                     1.490e-01,
                                  3.830e-01,
                                               3.300e-01,
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                     3.050e-01],
       [ 3.700e-01,
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                                  3.090e-01,
                                              4.920e-01,
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-2.700e-01, 3.050e-01, 2.900e-01, -3.070e-01, 4.920e-01,
 3.910e-01, 1.001e+00]])
```

Eigen Vectors

```
% [[-2.48765602e-01 3.31598227e-01 6.30921033e-02 -2.81310530e-01
   5.74140964e-03 1.62374420e-02 4.24863486e-02 1.03090398e-01
   9.02270802e-02 -5.25098025e-02 3.58970400e-01 -4.59139498e-01
   4.30462074e-02 -1.33405806e-01 8.06328039e-02 -5.95830975e-01
  2.40709086e-021
 [-2.07601502e-01 3.72116750e-01 1.01249056e-01 -2.67817346e-01
   5.57860920e-02 -7.53468452e-03 1.29497196e-02 5.62709623e-02
  1.77864814e-01 -4.11400844e-02 -5.43427250e-01 5.18568789e-01
  -5.84055850e-02 1.45497511e-01 3.34674281e-02 -2.92642398e-01
  -1.45102446e-01]
 [-1.76303592e-01 4.03724252e-01 8.29855709e-02 -1.61826771e-01
  -5.56936353e-02 4.25579803e-02 2.76928937e-02 -5.86623552e-02
  1.28560713e-01 -3.44879147e-02 6.09651110e-01 4.04318439e-01
  -6.93988831e-02 -2.95896092e-02 -8.56967180e-02 4.44638207e-01
  1.11431545e-02
 [-3.54273947e-01 -8.24118211e-02 -3.50555339e-02 5.15472524e-02
  -3.95434345e-01 5.26927980e-02 1.61332069e-01 1.22678028e-01
  -3.41099863e-01 -6.40257785e-02 -1.44986329e-01 1.48738723e-01
  -8.10481404e-03 -6.97722522e-01 -1.07828189e-01 -1.02303616e-03
   3.85543001e-021
 [-3.44001279e-01 -4.47786551e-02 2.41479376e-02 1.09766541e-01
  -4.26533594e-01 -3.30915896e-02 1.18485556e-01 1.02491967e-01
 -4.03711989e-01 -1.45492289e-02 8.03478445e-02 -5.18683400e-02
 -2.73128469e-01 6.17274818e-01 1.51742110e-01 -2.18838802e-02
```

```
-8.93515563e-021
[-1.54640962e-01 4.17673774e-01 6.13929764e-02 -1.00412335e-01
 -4.34543659e-02 4.34542349e-02 2.50763629e-02 -7.88896442e-02
 5.94419181e-02 -2.08471834e-02 -4.14705279e-01 -5.60363054e-01
-8.11578181e-02 -9.91640992e-03 -5.63728817e-02 5.23622267e-01
 5.61767721e-02]
[-2.64425045e-02 3.15087830e-01 -1.39681716e-01 1.58558487e-01
 3.02385408e-01 1.91198583e-01 -6.10423460e-02 -5.70783816e-01
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-6.35360730e-02]
[-2.94736419e-01 -2.49643522e-01 -4.65988731e-02 -1.31291364e-01
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 4.57332880e-03 -1.86675363e-01 5.08995918e-02 -1.01594830e-01
 1.43220673e-01 -3.83544794e-02 -3.40115407e-02 1.41856014e-01
-8.23443779e-011
[-2.49030449e-01 -1.37808883e-01 -1.48967389e-01 -1.84995991e-01
 5.60919470e-01 -1.62755446e-01 -2.09744235e-01 2.21453442e-01
-2.75022548e-01 -2.98324237e-01 1.14639620e-03 2.59293381e-02
-3.59321731e-01 -3.40197083e-03 -5.84289756e-02 6.97485854e-02
 3.54559731e-01]
[-6.47575181e-02 5.63418434e-02 -6.77411649e-01 -8.70892205e-02
-1.27288825e-01 -6.41054950e-01 1.49692034e-01 -2.13293009e-01
 1.33663353e-01 8.20292186e-02 7.72631963e-04 -2.88282896e-03
 3.19400370e-02 9.43887925e-03 -6.68494643e-02 -1.14379958e-02
-2.81593679e-02]
[ 4.25285386e-02 2.19929218e-01 -4.99721120e-01 2.30710568e-01
-2.22311021e-01 3.31398003e-01 -6.33790064e-01 2.32660840e-01
 9.44688900e-02 -1.36027616e-01 -1.11433396e-03 1.28904022e-02
-1.85784733e-02 3.09001353e-03 2.75286207e-02 -3.94547417e-02
-3.92640266e-02]
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 1.40166326e-01 -9.12555212e-02 1.09641298e-03 7.70400002e-02
 1.85181525e-01 1.23452200e-01 1.38133366e-02 -2.98075465e-02
 4.03723253e-02 1.12055599e-01 -6.91126145e-01 -1.27696382e-01
 2.32224316e-021
[-3.17056016e-01 4.64294477e-02 6.60375454e-02 5.19443019e-01
 2.04719730e-01 -1.54927646e-01 2.84770105e-02 1.21613297e-02
 2.54938198e-01 8.85784627e-02 6.20932749e-03 2.70759809e-02
-5.89734026e-02 -1.58909651e-01 6.71008607e-01 5.83134662e-02
 1.64850420e-02]
[ 1.76957895e-01 2.46665277e-01 2.89848401e-01 1.61189487e-01
-7.93882496e-02 -4.87045875e-01 -2.19259358e-01 8.36048735e-02
-2.74544380e-01 -4.72045249e-01 -2.22215182e-03 2.12476294e-02
 4.45000727e-01 2.08991284e-02 4.13740967e-02 1.77152700e-02
-1.10262122e-02]
[-2.05082369e-01 -2.46595274e-01 1.46989274e-01 -1.73142230e-02
-2.16297411e-01 4.73400144e-02 -2.43321156e-01 -6.78523654e-01
 2.55334907e-01 -4.22999706e-01 -1.91869743e-02 -3.33406243e-03
-1.30727978e-01 8.41789410e-03 -2.71542091e-02 -1.04088088e-01
```

```
1.82660654e-01]
[-3.18908750e-01 -1.31689865e-01 -2.26743985e-01 -7.92734946e-02 7.59581203e-02 2.98118619e-01 2.26584481e-01 5.41593771e-02 4.91388809e-02 -1.32286331e-01 -3.53098218e-02 4.38803230e-02 6.92088870e-01 2.27742017e-01 7.31225166e-02 9.37464497e-02 3.25982295e-01]
[-2.52315654e-01 -1.69240532e-01 2.08064649e-01 -2.69129066e-01 -1.09267913e-01 -2.16163313e-01 -5.59943937e-01 5.33553891e-03 -4.19043052e-02 5.90271067e-01 -1.30710024e-02 5.00844705e-03 2.19839000e-01 3.39433604e-03 3.64767385e-02 6.91969778e-02 1.22106697e-01]]
```

Eigen Values

```
% [5.45052162 4.48360686 1.17466761 1.00820573 0.93423123 0.84849117 0.6057878 0.58787222 0.53061262 0.4043029 0.02302787 0.03672545 0.31344588 0.08802464 0.1439785 0.16779415 0.22061096]
```

2.6) Write the explicit form of the first PC (in terms of Eigen Vectors).

Explicit form of the FIRST PRINCIPAL COMPONENT

```
PC1 = (2.48765602e-01)*Apps + (2.07601502e-01)*Accept + (1.76303592e-01)*Enroll + (3.54273947e-01)*Top10perc + (3.44001279e-01)*Top25perc + (1.54640962e-01)*F.Undergrad + (2.64425045e-02)*P.Undergrad + (2.94736419e-01)*Outstate + (2.49030449e-01)*Room.Board + (6.47575181e-02)*Books + (-4.25285386e-02)*Personal + (3.18312875e-01)*PhD + (3.17056016e-01)*Terminal + (-1.76957895e-01)*S.F.Ratio + (2.05082369e-01)*perc.alumni + (3.18908750e-01)*Expend + (2.52315654e-01)*Grad.rate
```

2.7) Discuss the cumulative values of the eigenvalues. How does it help you to decide on the optimum number of principal components? What do the eigen

vectors indicate? Perform PCA and export the data of the Principal Component scores into a data frame.

TOTAL(sum of eigen values)=17.021907216494846

Cumulative Variance Explained

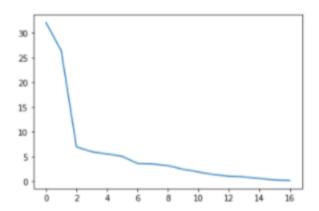
```
    [ 32.0206282
    58.36084263
    65.26175919
    71.18474841
    76.67315352

    81.65785448
    85.21672597
    88.67034731
    91.78758099
    94.16277251

    96.00419883
    97.30024023
    98.28599436
    99.13183669
    99.64896227

    99.86471628
    100.
    ]
```

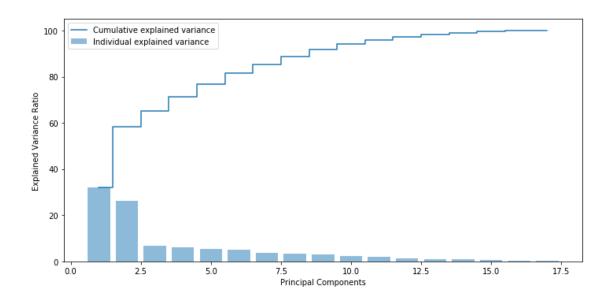
First Principal component explains 32.02 percent variance
Second Principal component explains 58.36 percent variance
Third Principal component explains 65.26 percent variance
Forth Principal component explains 71.18 percent variance
Fifth Principal component explains 76.67 percent variance
Sixth Principal component explains 81.65 percent variance
Seventh Principal component explains 85.21 percent variance
Eight Principal component explains 88.67 percent variance
Nineth Principal component explains 91.78 percent variance



Screeplot is also called Elbow curve

Based on this graph we will take a decision on how many PC can be taken where ever there is a sudden change in the graph that is taken as the Principal Component.

Visually we can observe that their is steep drop in variance explained with increase in number of PC's.



The Cumulative % gives the percentage of variance accounted for by the n components.

For example, the cumulative percentage for the second component is the sum of the percentage of variance for the first and second components.

It helps in deciding the number of components by selecting the components which explained the high variance .

For about 60percent variance we need 2PC

For about 80percent variance we need 5PC

For about 85percent variance we need 7PC

For about 90percent variance we need 9PC

We will proceed with 7 components here which explains ~ 85% of the variance within the dataset.

Eigen vector indicate

Eigen vector points in the direction where the maximum variance is explained.

Eigen vector determine the directions of the new feature space, and the eigenvalues determine their magnitude.

Eigenvectors are the coefficients of new feature components which is obtained by multiplying the eigen vector values by the features.

Eigenvectors can help us in calculating an approximation of a large matrix as a smaller vector.

Eigenvectors are used to make linear transformation understandable.

Eigenvectors and eigenvalues are used to reduce noise in data. They can help us improve efficiency in intensive tasks.

pca.components_

```
[-6.30921033e-02, -1.01249056e-01, -8.29855709e-02,
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 -2.26584481e-01, 5.59943937e-01],
[-1.03090398e-01, -5.62709623e-02, 5.86623552e-02,
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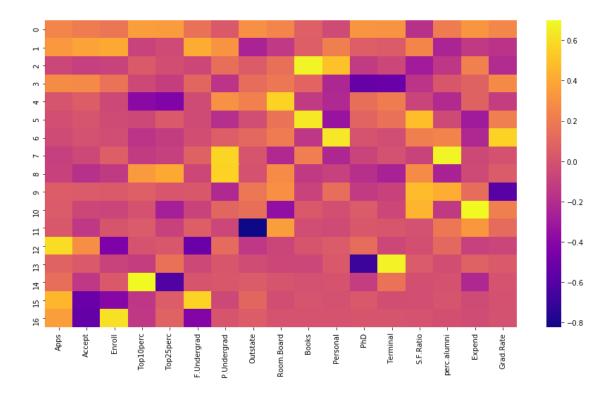
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Principal Component scores into a data frame

	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board
0	0.248766	0.207602	0.176304	0.354274	0.344001	0.154641	0.026443	0.294736	0.249030
1	0.331598	0.372117	0.403724	-0.082412	-0.044779	0.417674	0.315088	-0.249644	-0.137809
2	-0.063092	-0.101249	-0.082986	0.035056	-0.024148	-0.061393	0.139682	0.046599	0.148967
3	0.281311	0.267817	0.161827	-0.051547	-0.109767	0.100412	-0.158558	0.131291	0.184996
4	0.005741	0.055786	-0.055694	-0.395434	-0.426534	-0.043454	0.302385	0.222532	0.560919
5	-0.016237	0.007535	-0.042558	-0.052693	0.033092	-0.043454	-0.191199	-0.030000	0.162755
6	-0.042486	-0.012950	-0.027693	-0.161332	-0.118486	-0.025076	0.061042	0.108529	0.209744

Books	Personal	PhD	Terminal	5.F.Ratio	perc.alumni	Expend	Grad.Rate
0.064758	-0.042529	0.318313	0.317056	-0.176958	0.205082	0.318909	0.252316
0.056342	0.219929	0.058311	0.046429	0.246665	-0.246595	-0.131690	-0.169241
0.677412	0.499721	-0.127028	-0.066038	-0.289848	-0.146989	0.226744	-0.208065
0.087089	-0.230711	-0.534725	-0.519443	-0.161189	0.017314	0.079273	0.269129
-0.127289	-0.222311	0.140166	0.204720	-0.079388	-0.216297	0.075958	-0.109268
0.641055	-0.331398	0.091256	0.154928	0.487046	-0.047340	-0.298119	0.216163
-0.149692	0.633790	-0.001096	-0.028477	0.219259	0.243321	-0.226584	0.559944

HEATMAP



2.8) Mention the business implication of using the Principal Component Analysis for this case study. [Hint: Write Interpretations of the Principal Components Obtained]

This case study has 17 variables and to analyse we need reduction of variables and to do that we need Principal Component Analysis

After doing analysis we can consider 7 PCAs to get about 85% variability.

PCA is a statistical technique and uses orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables.

PCA also is a tool to reduce multidimensional data to lower dimensions while retaining most of the information.

Principal Component Analysis (PCA) is a well-established mathematical technique for reducing the dimensionality of data, while keeping as much variation as possible.

PCA can only be done on continous variables. Large datasets are increasingly common and are often difficult to interpret.

Principal component analysis (PCA) is a technique for reducing the dimensionality of such datasets, increasing interpretability but at the same time minimizing information loss. It does so by creating new uncorrelated variables that successively maximize variance.

After scaling we have normalized the data. Here it is clear that PCA seeks to maximize the variance of each component.

There is no significant change in the outlier pattern.

Covariance matrix is built on the scaled data and the same has been given as an input to calculate the Eigen values and vectors.

Outliers are valid and need to be retained inmodel in this case and should not be treated