Series EF1GH/5

SET~3

रोल नं.
Roll No.

प्रश्न-पत्र कोड
Q.P. Code **65/5/3**

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*
Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।*
Please check that this question paper contains 23 printed pages.

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।*
Please check that this question paper contains 38 questions.

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period..

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This question paper contains 38 questions. All questions are compulsory.*

(ii) *Question paper is divided into FIVE Sections – Section A, B, C, D and E.*

(iii) *In Section A – Question Number 1 to 18 are Multiple Choice Questions (MCQ) type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.*

(iv) *In Section B – Question Number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.*

(v) *In Section C – Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.*

(vi) *In Section D – Question Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.*

(vii) *In Section E – Question Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section – B, 3 questions in Section – C, 2 questions in Section – D and 2 questions in Section – E.*

(ix) *Use of calculators is NOT allowed.*

SECTION – A

(Multiple Choice Questions)

Each question carries 1 mark.

Select the correct option out of the four given options :

1. Let R be a relation in the set N given by

$$R = \{(a, b) : a = b - 2, b > 6\}.$$

Then

(a) $(8, 7) \in R$

(b) $(6, 8) \in R$

(c) $(3, 8) \in R$

(d) $(2, 4) \in R$

2. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, where A^T is the transpose of the matrix A, then

(a) $x = 0, y = 5$

(b) $x = y$

(c) $x + y = 5$

(d) $x = 5, y = 0$

3. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

4. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals

(a) A

(b) $A + I$

(c) $I - A$

(d) $A - I$

5. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

(a) 1

(b) 2

(c) 3

(d) 4

6. If $f(x) = |\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is

(a) 1
(b) -1
(c) $\frac{-1}{\sqrt{2}}$
(d) $\frac{1}{\sqrt{2}}$

7. If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

(a) x
(b) $-x$
(c) $16x$
(d) $-16x$

8. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$
(b) $x = 1.5$
(c) $x = -2$
(d) $x = 4$

9. The function $f(x) = x^3 + 3x$ is increasing in interval

(a) $(-\infty, 0)$
(b) $(0, \infty)$
(c) $\mathbb{R}$
(d) $(0, 1)$

10. $\int_{-1}^{1} \frac{|x-2|}{x-2}\, dx$, $x \neq 2$ is equal to

(a) 1
(b) -1
(c) 2
(d) -2

11. $\int \frac{\sec x}{\sec x - \tan x}\, dx$ equals

(a) $\sec x - \tan x + c$
(b) $\sec x + \tan x + c$
(c) $\tan x - \sec x + c$
(d) $-(\sec x + \tan x) + c$

12. The order and the degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$ respectively are :

(a) $1, \frac{2}{3}$ (b) 3, 1

(c) 3, 3 (d) 1, 2

13. If $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$, then $\vec{a}$ is

(a) $\hat{k}$ (b) $\hat{i}$

(c) $\hat{j}$ (d) $\hat{i} + \hat{j} + \hat{k}$

14. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{32}$

15. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

16. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 0° (b) 30°

(c) 45° (d) 90°

17. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$

(d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

18. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is

(a) 1

(b) 5

(c) 7

(d) 12

Assertion – Reason Based Questions

In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. **Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$

Reason (R) : $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

20. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

SECTION – B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Find the value of k for which the function f given as

$$f(x) = \begin{cases} \dfrac{1-\cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$ is continuous at $x = 0$.

OR

(b) If $x = a \cos t$ and $y = b \sin t$, then find $\dfrac{d^2y}{dx^2}$.

22. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1} 1$.

23. Find the vector and the cartesian equations of a line that passes throug the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

24. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

25. (a) If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

SECTION – C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. Show that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

27. Using integration, find the area of the region bounded by $y = mx$ $(m > 0)$, $x = 1$, $x = 2$ and the x-axis.

28. (a) Find the coordinates of the foot of the perpendicular drawn from point (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

(b) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

29. Find the distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k});$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

30. (a) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

31. (a) Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$

OR

(b) Find : $\int \frac{x^4}{(x-1)(x^2+1)}\, dx$

SECTION – D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. Solve the following Linear Programming Problem graphically :

Minimise : $Z = 60x + 80y$

subject to constraints :

$3x + 4y \geq 8$

$5x + 2y \geq 11$

$x, y \geq 0$

33. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

34. Evaluate : $\int_{0}^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x)\, dx$

35. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

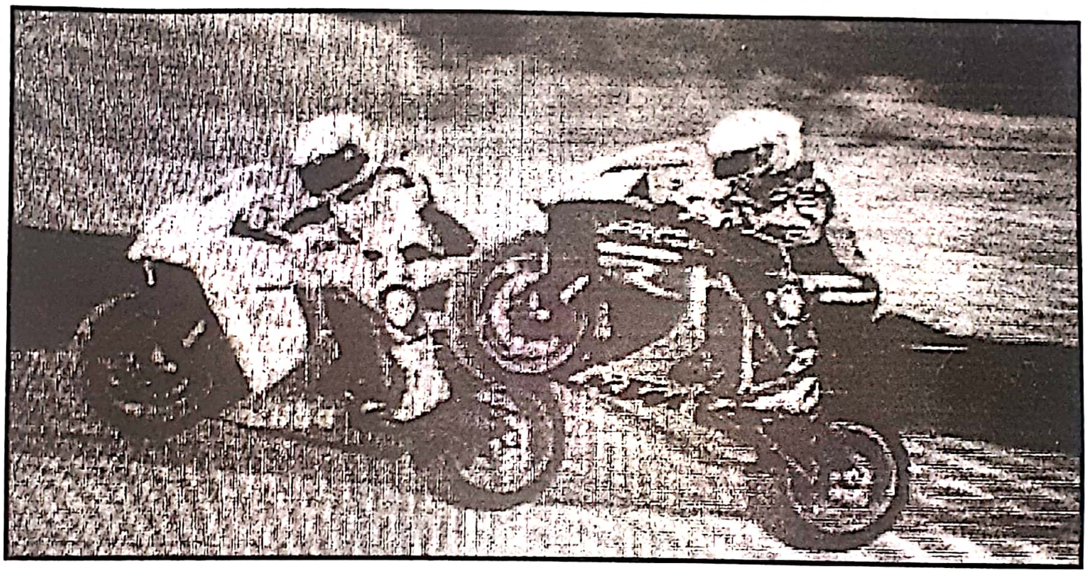
SECTION – E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub – parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub – parts (I) and (II) of marks 2 each.

Case Study-I

36. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective. Justify your answer.

Case Study-II

37. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions :

(I) Convert the given above situation into a matrix equation of the form $AX = B$.

(II) Find $|A|$.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)\, dx + 2xy\, dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.