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Roll No. : ☐☐☐

HALF YEARLY EXAMINATION (2022-23)

SUBJECT : MATHEMATICS

CLASS : XII

Time : 3 hrs   MM:80

General Instructions:

1.   This question paper contains five sections–A, B,C,D and E. Each part is compulsory.

2.   Section-A has 14 MCQs and carry 1 marks each.

3.   Section-B has 2 questions based on case study of 5 marks each.

4.   Section-C has 5 very short answer questions and carry 2 marks each.

5.   Section - D has 10 short answer questions and carry 3 marks each.

6.   Section - E has 4 long answer questions and carry 4 marks each.

7.   There is an internal choice in some of the Questions.

SECTION-A

1.   If $A = \begin{bmatrix} Cos\,x & -Sin\,x \\ Sin\,x & Cos\,x \end{bmatrix}$, then $A + A' = I$, if the value of x is :

   (a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{3}$

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(c) π

(d) $\dfrac{3\pi}{2}$

2. $Sin\left[\dfrac{\pi}{3} - Sin^{-1}\left(\dfrac{-1}{2}\right)\right]$ is equal to :

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{3}$

(c)   1

(d)   – 1

3.   If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, then x.y is equal to :

(a)   9

(b)   14

(c)   7

(d)   18

4.   If $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of |adj A| is :

(a)   64

(b)   16

(c)   0

(d)   – 8

5.   If $Cos\left(Sin^{-1}\dfrac{3}{5} + Cos^{-1}x\right) = 0$, then x is equal to :

(a) $\dfrac{1}{5}$

(b) $\dfrac{3}{5}$

(c)   0

(d)   1

6.   For what value of k, points (1, 3), (3, 9) and (0, k) are collinear?

(a)   3

(b)   – 3

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(c) 1

(d) 0

OR

Let A be a square matrix of order 3 × 3, the value of |2A|, Where |A| = 4 is :

(a) 8

(b) 16

(c) 4

(d) 32

7. The function is $f(x) = \dfrac{x(4-x^2)}{4-x^2}$ is :

(a) Discontinuous at only one point

(b) Discontinuous at exactly two points

(c) Discontinuous at exactly three points

(d) None of these

OR

The function $f(x) = e^{|x|}$ is :

(a) Continuous everywhere but not differentiable at x = 0

(b) Continuous and differentiable everywhere

(c) Not continuous at x = 0

(d) Differentiable everywhere

8. If $y = \log(\sec e^{x^2})$, then $\dfrac{dy}{dx} =$

(a) $x^2 e^{x^2} \tan e^{x^2}$

(b) $e^{x^2} \tan e^{x^2}$

(c) $2x e^{x^2} \tan e^{x^2}$

(d) None of these

9.   If $x = t^2$ and $y = t^3$, then $\dfrac{d^2y}{dx^2}$ is equal to :

     (a) $\dfrac{3}{2}$ (b) $\dfrac{3}{2}t$

     (c) $\dfrac{3}{2t}$ (d) $\dfrac{3}{4t}$

10.   If the volume of a sphere is changing at the same rate as its radius, then the surface area of the sphere is :

     (a)   2 sq units   (b)   1 sq units

     (c)   ½ sq units   (d)   ¼ sq units

11.   The function $f(x) = \tan x - x$:

     a)   Always increases

     b)   Always decreases

     c)   Never increases

     d)   Sometimes increases and sometimes decreases

<div align="center">OR</div>

     Which of the following functions is decreasing on $\left(0, \dfrac{\pi}{2}\right)$

     (a)   sin2x   (b)   tan x

     (c)   cos x   (d)   cos 3x

12. $\int \dfrac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ is equal to :

     (a) $2\cos\sqrt{x} + c$ (b) $2\sin\sqrt{x} + c$

     (c) $-2\sin\sqrt{x} + c$ (d) $\sin\sqrt{x} + c$

13. $\int \log x \, dx$ is equal to :

(a) $x \log x + x + c$

(b) $x \log x - x + c$

(c) $- x \log x + x + c$

(d) $- x \log x - x + c$

OR

$\int_{-\pi/2}^{\pi/2} \cos x \, dx$ is equal to :

(a)   0   (b)   1

(c)   2   (d)   4

14. The order and degree of the differential equation $x - Sin\left(\dfrac{dy}{dx}\right)$

= 0 respectively are :

a)   1, Not defined   b)   1, 1

c)   Not defined, 1   d)   Both not defined

SECTION B

In this section, each question is of 1 mark weightage. Questions 15-16 are based on Case Study.

15. Read the following and answer questions from (i) to (v).

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote. Voter turnout was about 67%, the highest ever.

XII-MATHEMATICS   [P.T.O.]
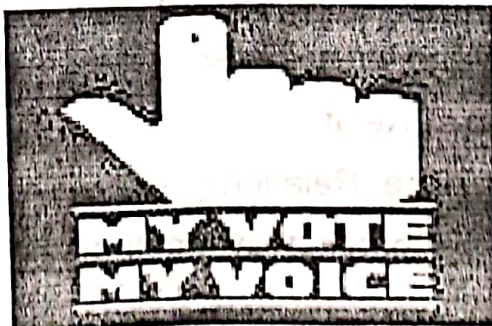
Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation R is defined on I as follows :

R = {(V1, V2) : V1, V2 ∈ I and both use their voting right in general election-2019}

1. Two neighbors X and Y ∈ I, X exercised his voting right and Y did not cast her vote in general election-2019. Which of the following is true :

    (a)   (X, Y) ∈ R

    (b)   (Y, X) ∈ R

    (c)   (Y, X) ∉ R

    (d)   (X, Y) ∉ R

2. Mr. X and his wife W both exercised their voting right in general election-2019. Which of the following is true :

    (a)   Both (X, W) and (W, X) ∈ R

    (b)   (X, W) ∈ R and (W, X) ∉ R

    (c)   Both (X, W) and (W, X) ∉ R

    (d)   (W, X) ∈ R but (W, X) ∉ R

3. Three friends F_1, F_2 and F_3 exercised their voting right in general election -2019, then which of the following is true

    (a) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

    (b) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \notin R$

    (c) $(F_1, F_2) \in R$, $(F_2, F_2) \in R$ and $(F_3, F_3) \notin R$

4. The above defined relation R is :

    (a)   Symmetric and Transitive but not reflexive

    (b)   Universal Relation

    (c)   Equivalence Relation

    (d)   Reflexive but not symmetric and Transitive

5.   Mr Shyam exercised his voting right in General Election -2019 then Mr. Shyam is related to which of the following :

   ✓(a)   All those eligible voters who cast their votes

   (b)   Family members of Mr. Shyam

   (c)   All citizens of India

   (d)   Eligible voters of India

16.   Deepak residing in Jaipur went to see an apartment of 3 BHK house in Delhi. The window of the house in the form of a rectangle surmounted by a semicircular opening and perimeter of the window is 10 m as shown in the figure.
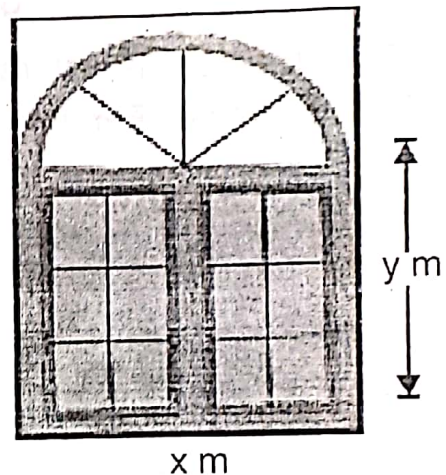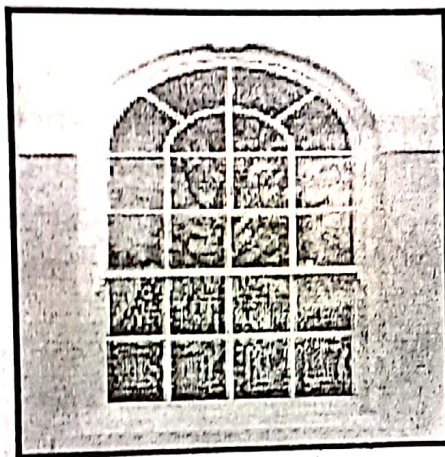
1.   If x and y represent the length and breadth of the rectangular region, then the relation between the variables is :

   (a) $x + y + \dfrac{x}{2} = 10$

   (b) $x + 2y + \dfrac{x}{2} = 10$

   (c) $x + 2y + \pi\dfrac{x}{2} = 10$.

   (d) $2x + 2y = 10$

2. The area of the Window (A) expressed as a function of x is :

(a) $A = x - \dfrac{\pi x^3}{8} - \dfrac{x^3}{2}$

(b) $A = 5x - \dfrac{\pi x^2}{8} - \dfrac{x^2}{2}$

(c) $A = 5x + \dfrac{3x^2}{8} - \dfrac{x^2}{2}$

(d) $A = 5x + \dfrac{\pi x^2}{8} + \dfrac{x^3}{2}$

3. Deepak is interested in maximizing the area of the whole window. For this to happen the value of x should be :

(a) $\dfrac{20}{\pi}$

(b) $\dfrac{20}{4 - \pi}$

(c) $\dfrac{20}{2 + \pi}$

(d) $\dfrac{20}{4 + \pi}$

4. For maximum value of A, the breadth of rectangular part of window is :

(a) $\dfrac{20}{4 + \pi}$ 　　　　(b) $\dfrac{20}{\pi}$

(c) $\dfrac{10}{4 + \pi}$ 　　　　(d) $\dfrac{205}{2 + \pi}$

5.   The maximum area of window is :

(a) $\dfrac{100}{(4+x)^2}$ sq.m

(b) $\dfrac{10\pi}{(4+x)^2}$ sq.m

(c) $\dfrac{800}{(4+x)^2}$ sq.m

(d) $\dfrac{200+50\pi}{(4+x)^2}$ sq.m

SECTION-C

17.   Evaluate : $\displaystyle\int \dfrac{\cos x}{\sqrt{1+\sin x}}\, dx.$

OR

Evaluate : $\displaystyle\int \dfrac{(x-3)e^x}{(x-1)^3}\, dx.$

18.   If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = kA$, then write the value of k.

19.   If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

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20. Find $\dfrac{dy}{dx}$: $\sin^2 x + \cos xy = 1$.

21. Find the intervals in which the following function is strictly increasing or decreasing : $f(x) = -2x^3 - 9x^2 - 12x + 1$.

SECTION-D

22. Evaluate : $\int_0^1 x(1-x)^n \, dx$.

23. Evaluate : $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{dx}{1 + \sqrt{\tan x}}$.

24. Evaluate : $\displaystyle\int \dfrac{1}{\cos(x-a)\sin(x-b)} \, dx$.

OR

Evaluate : $\displaystyle\int \dfrac{\cos x}{(1 - \sin x)(2 - \sin x)} \, dx$.

25. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

26. Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3} y$ and the circle $x^2 + y^2 = 4$.

27. Find the general solution of the differential equation $y \log y \, dx - x \, dy = 0$.

28. Show that the function $f : R \to R$ defined by $f(x) = \dfrac{x}{x^2 + 1}$, $\forall x \in R$ is neither one-one nor onto.

XII-MATHEMATICS   [P.T.O.]

29. Express the following matrix as the sum of a symmetric and skew symmetric matrix :

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

30. Differentiate the function $x^x - 2^{\sin x}$ with respect to x.

31. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then Prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

OR

If $f(x) = \begin{cases} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} & , \text{ if } x \neq 0 \\ k & , \text{ if } x = 0 \end{cases}$

is Continuous at $x = 0$, then find k.

SECTION-E

32. Evaluate : $\displaystyle\int_0^\pi \dfrac{x\tan x}{\sec x + \tan x}\,dx.$

OR

Evaluate : $\displaystyle\int_1^4 (|x-1| + |x-2| + |x-3|)\,dx.$

33. Solve the following system of linear equations using matrix method

x - y + 2z = 7

3x + 4y - 5z = - 5

2x - y + 3z = 12

XII-MATHEMATICS   [P.T.O.]

34. Sand is pouring from a pipe at the rate of 12 cm³/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm ?

35. Evaluate : $\int \dfrac{6x+5}{\sqrt{(x-5)(x-4)}}\,dx.$

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