Draft version August 29, 2017
Typeset using LaTeX **twocolumn** style in AASTeX61

A JOINT DECONVOLUTION ALGORITHM TO COMBINE SINGLE DISH AND INTERFEROMETER DATA FOR WIDEBAND MULTI-TERM IMAGING

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(Received TBD; Revised August 29, 2017; Accepted TBD)

Submitted to ApJ

ABSTRACT

Imaging in Radio Astronomy is done by the means of either Single-Dish Radio Telescopes or Interferometric Arrays. Image Formation with a radio interferometer offers excellent angular resolutions but suffers from the short-spacing problem, where sources with large angular size and low surface brightness are not seen in the image. On the other hand, single-dish radio telescopes offer very coarse resolution, but respond very well to large angular scales. This paper describes an algorithm to combine data from these two types of telescopes in order to bring about a good reconstruction of the true sky image at a wide range of frequencies. Contrary to traditional methods that only combine interferometer and single dish images after their respective reconstructions, our method combines the image and the point spread function before deconvolution in a scheme that can be interpreted simply as a choice of image weighting. This allows the algorithm to be relatively immune to scale factors traditionally used to align single dish and interferometer data in the spatial frequency domain. This algorithm also naturally extends to wide-band multi-term imaging and can be used to reconstruct the spectrum of the sky at spatial scales probed primarily by the single dish while also preserving high resolution information from the interferometer data. The algorithm can also be run on only the wideband single dish data to produce a reconstruction of spectral structure at an angular resolution better than that offered by the lowest frequency single dish image. Results for all of the above are demonstrated via simulations between 1-2 GHz for the VLA and GBT.

Keywords: Radio Astronomy — Imaging — Deconvolution Algorithms — Wideband Imaging

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1. INTRODUCTION

1.1. *Imaging with single-dish radio telescopes*

Single-Dish radio telescopes, such as the GBT Radio telescope are designed to respond linearly to the intensity of the radiation received from a point in the sky. The strategy followed to create an image is as follows : The antenna beam sweeps across the region of interest. The response of each point on the sky as the beam sweeps across it is summed and put into a pixel. This is carried out for the whole Field-of-View (FOV) till the region has been fully sampled. This strategy is known colloquially as "basket-weaving". Mathematically, the basket-weaving technique is equivalent to a convolution, where the antenna beam is convolved with the true sky image in order to form the image :

$$I^{Image} = I^{sky} * P^{antenna} \tag{1}$$

Properties of Single-dish images: From the theory of diffraction, it is known that the highest possible resolution in an image that is created from an aperture of diameter D for radiation of wavelength λ is given by

$$\theta \sim \frac{\lambda}{D} \tag{2}$$

From this equation, it is immediately seen that the resolution for radio frequencies is rather coarse. For the GBT dish with a diameter of 100m, the resolution at $\nu \sim 1.5$GHz is still ~ 5 arcmin. Hence, single dish radio-telescopes are not useful for obtaining high-resolution radio images. This logically leads to the technique of interferometry for conducting high-resolution imaging.

1.2. *Imaging with Interferometers*

Interferometric arrays are telescopes that have multiple elements or antennas. The radiation received at each antenna is combined using signal-processing software/hardware for creating the image - this technique is called 'Interferometry'. Each pair of antennas measures an *interference fringe*, and n antennas give $\frac{n(n-1)}{2}$ such fringes. Measurement of these fringes leads to the mathematical reconstruction of the *Visibility Function V(u,v)*. By the Van Cittert - Zernicke theorem of optics, it is known that the Intensity of radiation in the plane[1] of the sky and the visibility function are related by a two-dimensional Fourier transform. Mathematically, we write

$$V(u,v) = \int\int \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i[lu+mv]} dudv \tag{3}$$

[1] If the FOV is too large, the assumption of the sky being a plane does break down

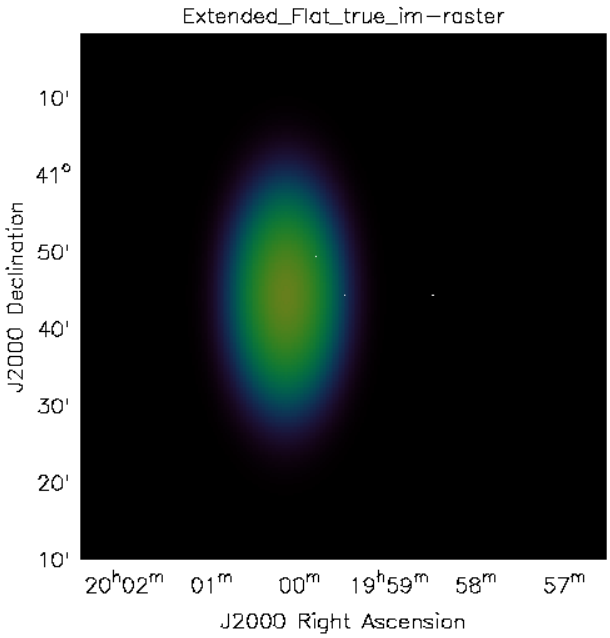

Figure 1. Simulated Image of the True Sky

Where $V(u,v)$ is the visibility function, $I(l,m)$ is the intensity in space, l,m are the *direction cosines* and (u,v) are the *Fourier-domain* coordinates.

Properties of interferometer images From Equation (2), we know that the finest resolution that can be measured by an aperture of diameter D is λ/D. In case of an interferometer, D is the *longest baseline*, defined as the maximum separation between a pair of antennas, which is unconstrained. This makes the technique of interferometry extremely powerful and it leads to very high-resolution images (arcsecond or sub-arcsecond) of the radio sky. As can be seen in Figure 2 the image of the same sky as seen by a single-dish radio telescope and an interferometer can look very different in terms of resolution.

1.3. *Motivation Behind the development of the WMCA algorithm*

As was seen in Figure 2, a single-dish instrument provides images with very coarse resolution. An interferometer provides images with excellent resolution, however, it does not faithfully reproduce objects that have very large angular sizes and low surface brightness. Scientific goals of various kinds can be achieved by combining best of what these two classes of instruments have to offer. Techniques such as *feathering* exist to combine the two kinds of images. However, these suffer from many problems, rendering them insufficient as a reliable technique for image combination. These include :

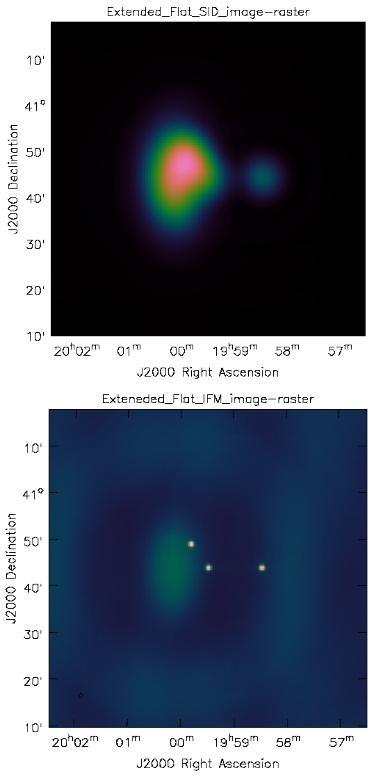

Figure 2. Simulated Image of the sky, from a single dish and the sky image from an interferometer

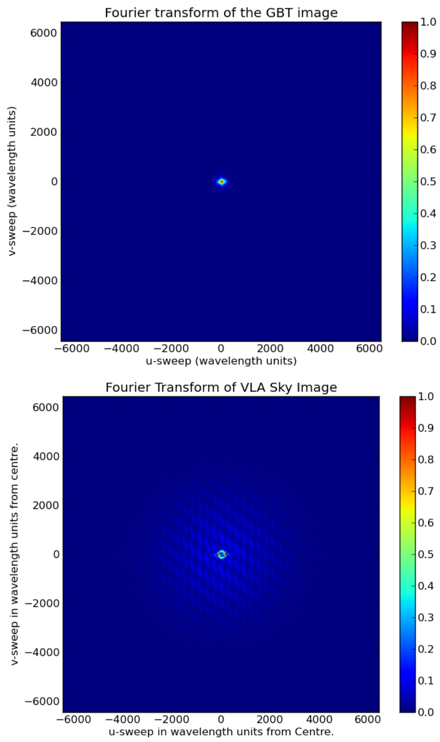

Figure 3. The short spacing-problem. Note the hole in the Fourier Transform of the interferometer image (VLA), which is filled by the single-dish image (GBT)

1. It depends heavily on the arbitrary scaling of single-dish data, which may not always lead to physically sensible results.

2. It works only for data taken at exactly one frequency. It fails to utilise the vast potential of multi-frequency data taken with modern telescopes such as the VLA or the GMRT.

3. Wideband Imaging algorithms exist (Rau and Cornwell 2011)(Rau 2011, PhD thesis) for pure interferometer data, but due to the inherent limitations of imaging with an interferometer (called the *short spacing problem*), these are not useful for sources which have large angular extent.

The Short Spacing Problem As any two antennas must always be separated by a finite nonzero distance, the values u and v can never be simultaneously zero. Hence, the smallest spacings always remain un-sampled. As sources with large angular extent have visibility values clustered around the origin, they are always undersampled and missed by the interferometer. This is clearly seen in Figure 2.

The short spacing problem is illustrated in Figure 3. In this figure, it can be seen that the Fourier transform of the interferometer image insufficiently samples the region around $(0, 0)$. This is called the *central uv-hole*. On the other hand, the single-dish data samples this precise region very well (bottom half), but lacks the data at large values of (u, v). This is overcome using the feathering technique, but it suffers from a few problems. The technique and its shortcomings are discussed in Section 2.

2. PREVIOUS WORK AND DEVELOPMENT OF THE WMCA ALGORITHM

2.1. *Feathering*

Feathering is the technique that is conventionally used to combine images from single-dish instruments and interferometers. The composite image formed by feathering is obtained by computing a weighted sum in the Fourier domain and inverting the modified data to create the image. The weight given to the interferometer image is $(1 - \mathcal{F}(P^{antenna}))$. $\mathcal{F}$ is the Fourier transform and $P^{antenna}$ is the antenna pattern from Section 1. The Fourier transform of the single-dish image is multiplied by the volume ratio of the interferometer restoring beam to the single dish antenna pattern. Mathematically, this can be written as :

$$\mathcal{F}(\mathbf{I}^{feathered}) = \frac{w_s V_s + w_I V_I}{w_s + w_I} \quad (4)$$

here, V_s and V_I are the Fourier transforms of the single-dish and interferometer data respectively. Generally, the single-dish data is also pre-scaled by a gain parameter.

Limitations of the feathering algorithm The feathering algorithm suffers from a few limitations, including:

1. It depends strongly on the single-dish gain parameter which is used to pre-scale the single-dish data before beginning the calculation. This may lead to situations where the data is scaled too much and does not reflect the physical reality.
2. It does not allow manipulation of the visibility data from the raw data file, as it takes the final images as its input which have already been deconvolved and processed. Hence one cannot handle any potential artifacts which may crop up.
3. It is highly sensitive to noise levels in the data.

2.2. *Multiterm, Multi-Frequency Synthesis (MTMFS) Algorithm*

Any radio image containing information on multiple angular scales can be written as a linear sum of a *sky model* comprising of δ functions, and a *smoothing kernel* which is usually a tapered, inverted, truncated paraboloid (Cornwell 2008). Mathematically, this amounts to

$$\mathbf{I}^m = \sum_{s=0}^{N_s-1} \mathbf{I}_s^{shp} * \mathbf{I}_s^{sky,\delta} \quad (5)$$

$\mathbf{I}^m$ is the model sky image, $\mathbf{I}_s^{shp}$ is the kernel function and $\mathbf{I}_s^{sky,delta}$ is the δ function model of the sky. '*' denotes the convolution operator. The sky changes with the observing frequency. To model this frequency dependence, the sky is modeled as a Taylor series about a centre frequency :

$$\mathbf{I}_\nu^m = \sum_{t=0}^{N_t-1} w_\nu^t \mathbf{I}_t^{sky} \quad (6)$$

Moreover, the dependence of the sky on the frequency is modeled as a power-law with a curvature term :

$$\mathbf{I}_\nu^{sky} = \mathbf{I}_{\nu_0}^{sky} (\frac{\nu}{\nu_0})^{I_\alpha^{sky} + I_\beta^{sky} log(\frac{\nu}{\nu_0})} \quad (7)$$

Next, putting together equations (4) and (5) leads to :

$$\mathbf{I}_\nu^m = \sum_{s=0}^{N_s} \sum_{t=0}^{N_t} w_\nu^t [\mathbf{I}_s^{shp} * \mathbf{I}_{s_t}^{sky}] \quad (8)$$

In all these equations, $w_t = (\frac{\nu-\nu_0}{\nu_0})^t$
In Equation (7), $\mathbf{I}_s$ represents a collection of δ-functions that describe the sky locations and amplitudes of flux components corresponding to angular scale s in the image formed by the t^{th} Taylor series coefficient. The list of visibilities $\mathbf{V}$ can be expressed in Matrix notation as,

$$\mathbf{V}_{n\times 1}^{obs} = [\mathbf{S}_{n\times m}][\mathbf{F}_{m\times m}]\mathbf{I}_{m\times 1}^{sky} \quad (9)$$

This is just the Van Cittert-Zernicke theorem (Equation (3)) restated, in case of discrete values and adding the presence of a sampling function (matrix) $\mathbf{S}$. $\mathbf{I}$ is the intensity matrix and $\mathbf{F}$ is the discrete Fourier transform matrix. To account for the frequency dependency of the sky intensity, we re-write (9) as

$$\mathbf{V}_\nu^{obs} = \sum_{s=0}^{N_s} \sum_{t=0}^{N_t} w_\nu^t [\mathbf{S}_\nu][\mathbf{T}_s][\mathbf{F}]\mathbf{I}_{s_t}^{sky} \quad (10)$$

This has an additional *taper function* $\mathbf{T}$ for the frequency variation. In case there are multiple frequencies, the weight function will also turn into a matrix $\mathbf{W}$. Computing intensities $\mathbf{I}$ is now reduced to an inversion problem, because $\mathbf{S}$ and $\mathbf{V}$ are known. Further details may be found in (Rau and Cornwell 2011) and (Rau 2011, PhD Thesis).

Limitations of the MTMFS algorithm Due to the short-spacing problem (Section 2.1), it does not work for sources that have low surface brightness and large angular extents. However, there are important science goals that can be achieved by extending to this technique for such sources. This can be clearly seen in Figure 5, where the image looks very similar to that of the original sky in Figure 1, which has a flat spectrum. However, the spectrum is not reproduced properly at any point on the extended cloud source.

3. METHODOLOGY AND ALGORITHM

3.1. *MTMFS and WMCA : Points of Divergence*

The Wideband Multiterm Combination Algorithm (WMCA) is an extension of the MTMFS algorithm (Rau 2011, PhD thesis). The major conceptual differences are notes as below.

1. The Multiterm Wideband Combination Algorithm developed by us does a joint deconvolution, namely, that it feathers both the single-dish data as well as the PSF with the corresponding interferometer ones. In case of a single-dish telescope, the PSF is simply the antenna beam or the power pattern.

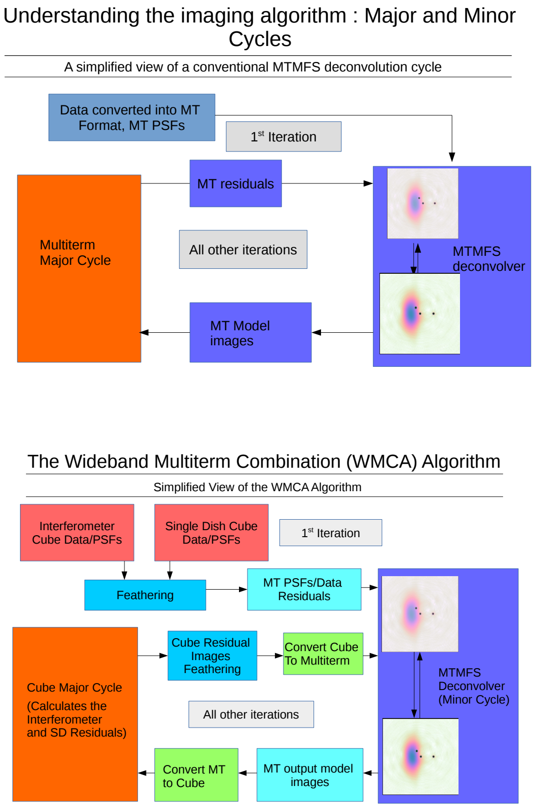

Figure 4. Illustration of the conceptual differences between the MTMFS algorithm and the WMCA algorithm

2. In the first run of the setup, the gridded interferometer residual is first feathered with the single-dish data, then passed on to carry out the deconvolution.

3. After the run of each minor cycle, the multiterm images are converted back to the images at all the observing frequencies as a model (hereafter called a "cube" of images) .Thereafter, the residual contribution of the single dish image is calculated by smoothing these "model" images with their respective beams at their respective frequencies.

4. After the run of each major cycle, the single-dish residual images are recalculated. These are then again feathered with the interferometer residuals and written out.

5. These composite residual images are converted back into the multiterm residuals and the cycle is begun afresh.

The major differences between an MTMFS algorithm and the WMCA are shown in Figure 4.

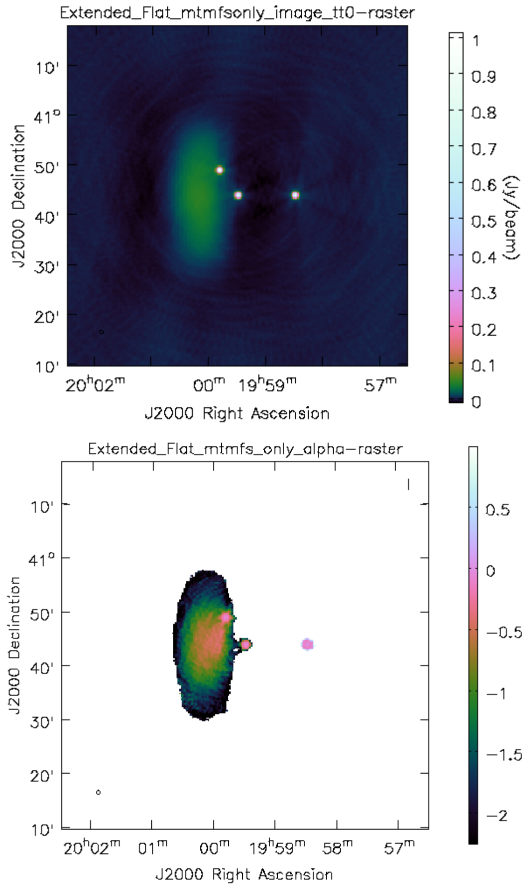

Figure 5. The reconstructed multiterm images using pure MTMFS for the flat-spectrum sky shown in Figure 1

3.2. *Detailed Description of the WMCA Algorithm*

1. **Data Input** : Interferometer visibility values, stored in cube format $\mathbf{V}^{obs}$

2. **Data Input** : Processed single-dish raw image values $\mathbf{I}^{SD}$, stored pixel-wise in a cube format

3. **Data input** : Beam information of the Interferometer restoring beam and the single-dish antenna pattern $P^{antenna}$

4. **Data input** : *uv* - sampling function, choice of weighting scheme and the input reference frequency for the deconvolver to compute the weight terms w

5. further work TBD.............................. I don't have the "Algorithm" command

3.3. *Outputs from this Algorithm*

A run of this algorithm produces the following outputs.

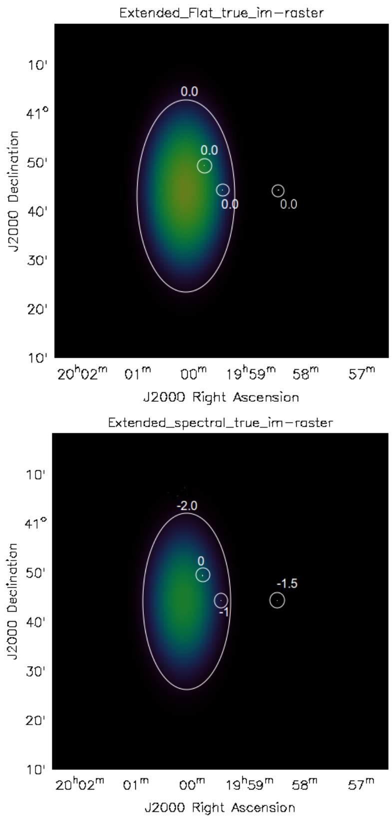

Figure 6. Two different simulated "sky images" with different spectral indices. Both derive from Figure 1.

- A number of images equal to the number of terms in the Taylor series is produced.

- Residual images and model images equaling the number of terms in the Taylor series are produced by the multiterm algorithm, plus a cube residual and model produced by the cube portion of the major cycle.

- A spectral index map is produced by the multiterm portion of the algorithm, which shows the spectral index of the sky for each pixel in the image. Another image that holds the errors for the index estimates per pixel is also created.

- For n terms, this algorithm creates $2n - 1$ multiterm PSFs and one cube PSF.

4. TESTS AND RESULTS

4.1. *Simulation of the Trial Data*

This new algorithm was tested on two different test cases, both of which derive from Figure 1 :

1. A *flat-spectrum* sky, where the sky does not change with frequency. In this case, the plot of the spectrum must show a value of zero for every pixel

2. A *steep-spectrum* sky, where each object shown in Figure 1 has a nonzero spectral index. The spectral indices are indicated on the true sky image.

Both these "true skies" are shown in Figure 6.

4.2. *Results from the WMCA Algorithm*

4.2.1. *The Flat-Spectrum sky*

The results produced by the WMCA algorithm on the flat-spectrum sky are shown in the subsequent figures. The images show the zero-order Taylor image of the sky (*i.e.*- the reconstructed image of the sky at the centre frequency), the spectral map and its corresponding error map and the residual error on the sky image.

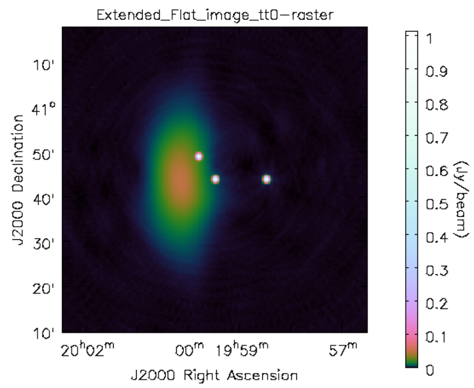

Figure 7. The Reconstructed Sky Image at the reference frequency

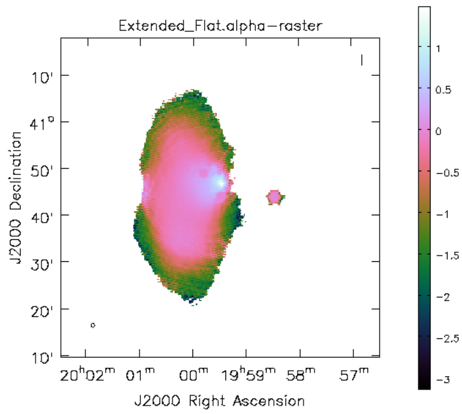

Figure 8. The Reconstructed Spectral map of the sky using WMCA. Comapre this to Figure 5 where the spectrum of the cloud was incorrectly measured by the MTMFS deconvolver

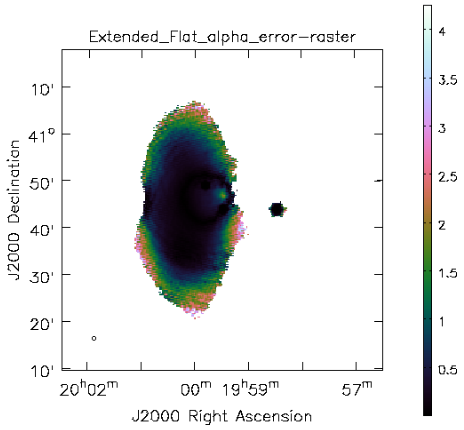

Figure 9. The error on the spectral estimate α in Figure 8

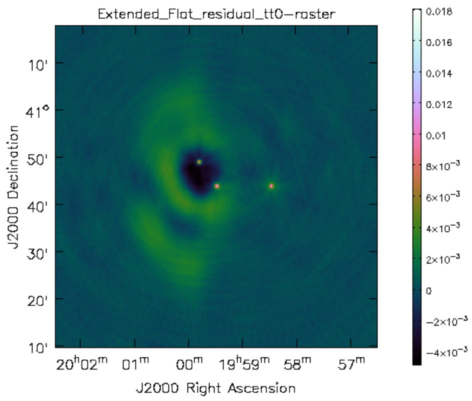

Figure 10. The residual error on Figure 7

4.2.2. *The Steep-Spectrum sky*

The results produced for the steep-spectrum sky shown in Figure 6 (bottom panel) are shown. The outputs are ordered like the previous section.

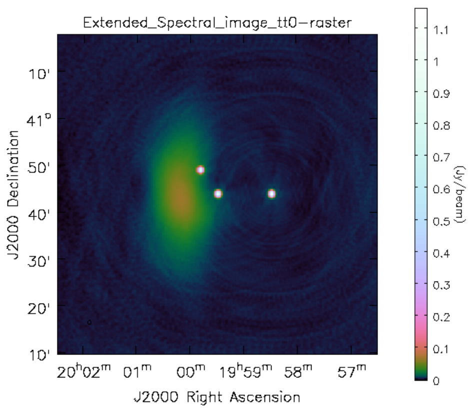

Figure 11. The Reconstructed Sky Image at the reference frequency

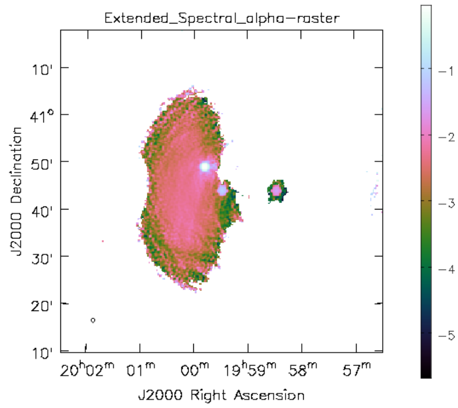

Figure 12. The Reconstructed Spectral map using WMCA. It is seen that this reasonably matches Figure 6.

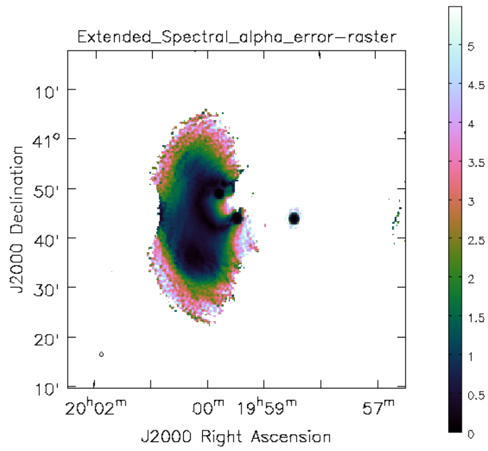

Figure 13. The error on the spectral estimate α in Figure 12

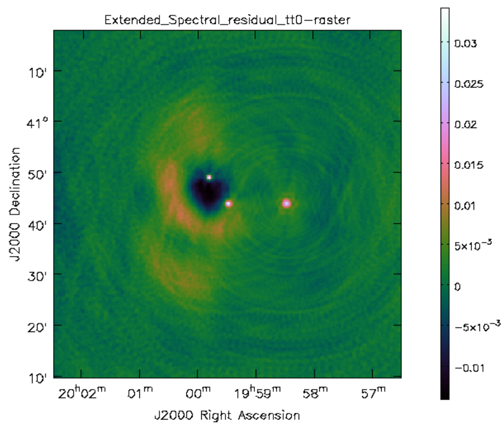

Figure 14. The residual error on Figure 11

4.3. *Using WMCA on Pure Single-Dish Data*

An interesting consequence of the WMCA algorithm's structure is that declaring the interferometer visibility data to be empty and feeding only the single-dish data also gives a good reconstruction of the sky. The images are inherently limited by the resolution of the single-dish telescope, but are extremely responsive to the sources that have large angular scales. Shown below are the output image set of the algorithm being run on pure single-dish data on a flat-spectrum sky.

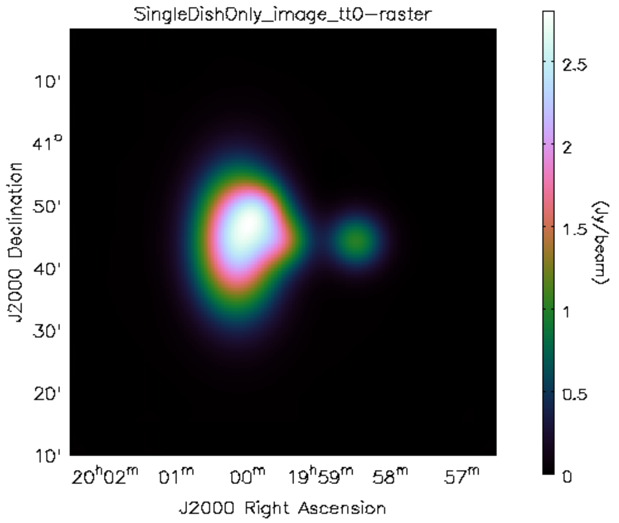

Figure 15. The Reconstructed Sky Image using only single-dish data at the reference frequency.

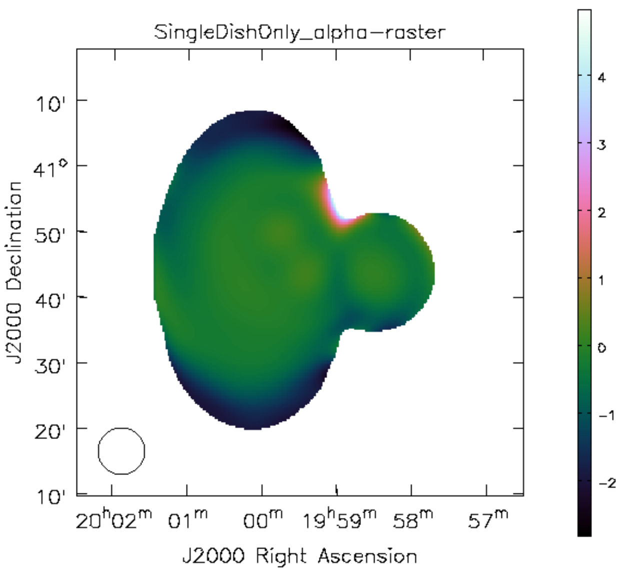

Figure 16. The Spectral map produced by the WMCA algorithm using only single-dish data

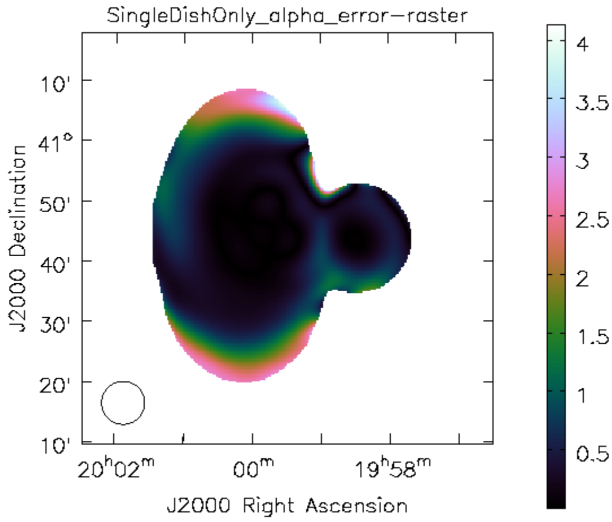

Figure 17. The error on the spectral map produced by the WMCA algorithm using only single-dish data

5. DISCUSSION

5.1. *Summary of Results*

The following goals were accomplished during the development of this algorithm :

1. Solve the short-spacing problem encountered by interferometers.

2. To combine the wideband multi-frequency data from a single-dish radio telescope and interferometer.

3. To evaluate the efficacy of a joint-deconvolution algorithm where the single-dish data was inserted into the interferometer data *before* the creation of an image, as compared to the traditional approach of a *post-facto* combination.

4. To estimate the spectrum of extended sources by creating a wideband image.

5. To prototype and test the working of an algorithm in order to accomplish these tasks.

5.2. *Further Work*

1. Test robustness of this algorithm with respect to weighting scheme interpretation, specifically, the immunity of this algorithm to scaling factors inherent to its operation. One obvious stopping criterion is when the deconvolution PSF begins to get distorted. However, the effect of any other things is still to be evaluated.

2. ??????A-projection???????

3. Primary Beam Correction (?)
4. Using Real data : G55 SNR and CTB80 Wideband Mosaic (?)

6. ACKNOWLEDGEMENTS

This research was conducted under the National Radio Astronomy's Summer Student Research Assistantship programme, under the guidance of Dr. Urvashi Rau Venkata. This project also fulfills NN's compulsory summer training/internship requirements, mandated as part of the B. Tech (Hons.) course in Instrumentation Engineering, under the Dept. of Electrical Engineering, Indian Institute of Technology Kharagpur.
Many thanks to Sanjay Bhatnagar and Kumar Golap for their help in resolving several technical issues. The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.

7. REFERENCES

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