DIGITAL ASSIGNMENT-I

APPLIED LINEAR ALGEBRA (MAT3004)

Vivek Shaji Daniel
19BME0303

1. Let $V = \mathbb{R}^2$, $B = \{(1,2), (3,4)\}$, $C = \{(7,3), (4,2)\}$ and $V = (2,5)$ any vector in V

(a) Determine $[V]_B$ and $[V]_C$

(b) Find $P_{C \leftarrow B}$ and $P_{B \leftarrow C}$

(a) To find $[V]_B$, we know $V = (2,5)$

$\therefore a(1,2) + b(3,4) = (2,5)$

$$a + 3b = 2$$
$$2a + 4b = 5$$

$$2a + 6b - (2a + 4b) = 4 - 5 = -1$$

$$2b = -1 \Rightarrow b = \frac{-1}{2} = -0.5$$

$$\therefore a + \left(\frac{-1}{2}\right)3 = 2 \Rightarrow a = 2 + \frac{3}{2} = \frac{7}{2} = 3.5$$

$$[V]_B = \begin{bmatrix} 3.5 \\ -0.5 \end{bmatrix}$$

Similarly to find $[V]_C$

$$c(7,3) + d(4,2) = (2,5)$$

$$7c + 4d = 2$$
$$3c + 2d = 5$$

$$7c + 4d - (6c + 4d) = 2 - 10 = -8$$

$$c = -8$$

$$3(-8) + 2d = 5 \Rightarrow 2d = 29 \Rightarrow 29/2 = 14.5$$

$$[V]_C = \begin{bmatrix} -8 \\ 14.5 \end{bmatrix}$$

b) Now, to find $P_{C\leftarrow B}$, we know that

$$P_{C\leftarrow B} = \left[[v_1]_C, [v_2]_C, \ldots\ldots, [v_n]_C \right]$$

$$(1,2) = a(7,3) + b(4,2) = [w_1]_C$$

$$(3,4) = c(7,3) + d(4,2) = [w_2]_C$$

$$\therefore \quad 7a + 4b = 1$$

$$3a + 2b = 2$$

$$a = -3 \Rightarrow b = 11/2 = 5.5 \Rightarrow [w_1]_C = \begin{bmatrix} -3 \\ 5.5 \end{bmatrix}$$

and,

$$7c + 4d = 3$$

$$3c + 2d = 4$$

$$c = -5 \Rightarrow d = 9.5$$

$$[w_2]_C = \begin{bmatrix} -5 \\ 9.5 \end{bmatrix}$$

$$P_{C\leftarrow B} = \begin{bmatrix} -3 & -5 \\ 5.5 & 9.5 \end{bmatrix}$$

Similarly for $P_{B \leftarrow C}$

$$(7,3) = a(1,2) + b(3,4)$$
$$(4,2) = c(1,2) + d(3,4)$$

$$a + 3b = 7$$
$$2a + 4b = 3$$

$$2b = 11 \Rightarrow b = 5.5$$
$$a = -9.5 \Rightarrow [W_1]_B = \begin{bmatrix} -9.5 \\ 5.5 \end{bmatrix}$$

and

$$c + 3d = 4$$
$$2c + 4d = 2$$

$$2d = 6 \Rightarrow d = 3 \Rightarrow c = -5$$

$$[W_2]_B = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$P_{B \leftarrow C} = \begin{bmatrix} -9.5 & -5 \\ 5.5 & 3 \end{bmatrix}$$

2. Check if the following set is linearly dependent or independent.

$$B = \{(-1,3,1), (2,-4,-3), (-3,8,2)\}$$

∴ For a set ~~of~~ to be linearly dependent, there exists a finite number of distinct vectors $a_1, a_2, a_3 \ldots, a_n$ and scalars $a_1, a_2, \ldots, a_n$ not all zero, such $a_1 u_1 + a_2 u_2 + \ldots + a_n u_n = 0$

∴ Here,

$$a(-1,3,1) + b(2,-4,-3) + c(-3,8,2) = 0$$

$$-a + 2b - 3c = 0 \quad \text{— (1)}$$

$$3a - 4b + 8c = 0 \quad \text{— (2)}$$

$$a + (-3b) + 2c = 0 \quad \text{— (3)}$$

From (1) and (2)

$$(3a - 4b + 8c) + (-3a + 6b - 9c) = 0$$

$$\therefore 2b - c = 0 \Rightarrow 2b = c \Rightarrow b = c/2$$

$$\therefore -a + 2(c/2) - 3c = 0$$

$$\therefore -a - 2c = 0 \Rightarrow 2c = -a \Rightarrow a = -2c$$

$$-(-2c) + 2(c/2) - 3c = 0$$

$$\therefore 2c + c - 3c = 0$$

$$3c - 3c = 0$$

$-2c - 3(c/2) + 2c = 0$

$\therefore -3c/2 = 0 \Rightarrow c = 0 \Rightarrow a = 0 \Rightarrow b = 0$

$\therefore$ there is non-zero scalar

$\therefore$ B is linearly independent.