CSE4019 - Image Processing

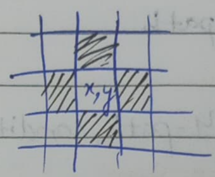
Before what's written here is in Blue-Orange Youva notebook.

* Relationship b/w pixels

· Neighborhood.

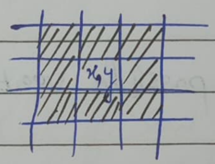
→ 4 neighbors of p :

$N_4(p) = \left\{ \begin{matrix} (x-1, y) \\ (x+1, y) \\ (x, y-1) \\ (x, y+1) \end{matrix} \right\}$



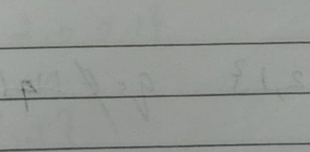
→ 8 neighbors of p :

$N_8(p) =$



→ $N_D(P)$:
↳ Diagonal

$N_D(P) =$



· Connectivity

→ 4-conn.
$q \in N_4(p)$

→ 8-conn.
$q \in N_8(p)$

→ m-conn. (OR mixed conn.)
$q \in N_4(p)$ OR
$q \in N_D(p)$ and $N_4(p) \cap N_4(q) = \phi$

↓ Intersection

NOTE
Here null means that the answer should not be in the given vertex set

Q. Find the shortest 4-, 8- and m-path b/w p and q for $V = \{0, 1\}$ and $V = \{1, 2\}$

$$\begin{array}{cccc} 3 & 1 & 2 & 1(q) \\ 2 & 2 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ (p)\,1 & 0 & 1 & 2 \end{array}$$

Ans. A) For $V = \{0, 1\}$:-

- 4-path

For 4-path, condition is :-

$q \in N_4(p)$

$$\begin{array}{ccccc} & 3 & 1 & 2 & 1\,q \\ & 2 & 2 & 0 & 2 \\ & 1 & 2 & 1 & 1 \\ p & 1_{p_1} & 0_{q_1} & 1_{p_2} & 2 \end{array}$$

Checking for all possible vertices :-

$N_4(p_1) = \{1, \underset{q_1}{0}\}$ $\quad \therefore q_1 \in N_4(p_1)$

$N_4(p_2) = \{1, 2, \underset{q_2}{1}\}$

$N_4(p_3) = \{0, \underset{q_3}{1}, 2\}$

$N_4(p_4) = \{2, \underset{q_4}{0}, 1, 1\}$

$N_4(p_5) = \{2, 2, 2, 1\}$ $\quad q_5 \notin N_4(p_5)$

$\therefore$ 4-path does not exist for $V = \{0, 1\}$ from p to q.

Eye
Spectrum
Image format] Not discussed in class but check these out

- 8-path

$$\begin{array}{ccccc} & 3 & 1 & 2 & 1\ q \\ & 2 & 2 & 0 & 2 \\ & 1 & 2 & 1 & 1 \\ p & 1 & 0 & 1 & 2 \end{array}$$

$q \in N8(p)$

$N8(p_1) = \{1, 2, \underset{q_2}{0}\}$

$N8(p_2) = \{1, 1, 2, 1, 1\}$

$N8(p_3) = \{2, 2, 2, 0, 2, 1, 2, 1\}$

$N8(p_4) = \{2, 2, 1, 2, \underset{q}{1}, 2, 1, 1\}$

$\therefore$ 8-path is $1-0-1-0-1$ from p to q through $V = \{0, 1\}$
~~All other~~ two 8-paths should also be solved and presented.

- m-path

we know $q \in N4(p)$ failed ~~for $V = \{0, 1\}$~~ till 0 vertex.
After that we check for 'or' condition:-
$\therefore$ ii) $q \in ND(p)$ and $N4(p) \cap N4(q) = \phi$

checking $q \in ND(p)$:-

$ND(p)) = \{1, 2, 1, \underset{2}{1}\}$ $q \in ND(p)$

and $N4(p) = \{2, 2, 2, 1\}$

$N4(q) = \{2, 2\}$

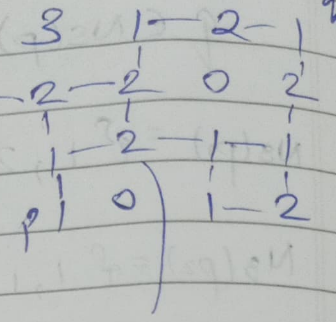
$N4(p) \cap N4(q) = \{2, 2\} = \phi$ since it doesn't have $\{0, 1\}$.

$\therefore$ m-path is :- $1-0-1-1-0-1$.

B) $V = \{1, 2\}$

- 4-path:

1,3,2



1,2,1,0

$q \in N_4(p)$

$N_4(p_1) = \{1, 0\}$

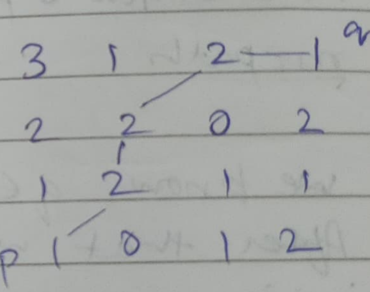
$N_4(p_2) = \{\underset{q}{2}, \underset{q}{2}, 1\}$

~~N4(p) N4~~ 4-path = 1-1-2-2-1-2-1

1-1-2-2-1-2-1

1-1-2-1-1-2-1

- 8-path:



- m-path:

Exists ✓

~~We don't~~

Theory something from ppt

we don't need grey intensity to find the distance b/w two pixels, just coordinates are needed.

* D_4 dist. (city-block)

$$D_4(p,q) = |x-s| + |y-t|$$

* D_8 dist. (chessboard)

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

Ques.

16	12	14	12
8	9^p	6	2
25	14	10	7
19	13	6_q	4

$p = (1,1)$
$q = (3,2)$

find $D_e(p,q)$, $D_4(p,q)$, $D_8(p,q)$

Ans.

$$D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$$
$$= [(3-1)^2 + (2-1)^2]^{1/2}$$
$$= [2^2 + 1^2]^{1/2}$$
$$= \sqrt{5}.$$

$D_4(p,q) = |x-s| + |y-t| = |3-1| + |2-1| = 2+1 = 3$
$D_8(p,q) = \max(2,1) = 2.$

Date 14/07/22
Page

Digital IP

Not so imp.

* Color models

$(2^8)^3 \rightarrow 16$ million.

Motive ✓

Cover human eye portion from textbook.
& spectrum

Rod, cones
Blind spot

* Color fundamentals

Basic quan. to describe the quality of light source:-
- Radiance
- luminance
- Brightness

This part more imp

R	G	B
65%	33%	~~2%~~

gens. color percieved

amount of white

Brightness, Hue and Saturation are responsible for distinguishing one color from another.

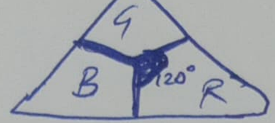
Definitions (2-5m)

Hue + Saturation → chromaticity.

Human eye.
HSI color mode
Hue Sat. Int./Brightness

Sec. colors → CMYK (Cyan, Magenta, Yellow, Black)
color printing.

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



prism.

The HSI Color models

- Converting colors from RGB to HSI

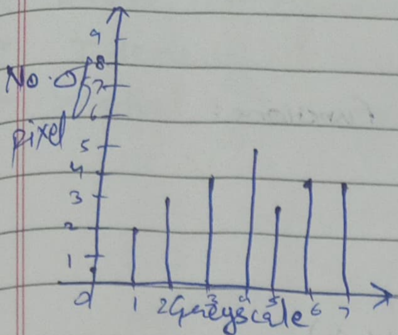
$$H = \begin{cases} \theta & \text{if } B \le G \\ 360 - \theta & \text{if } B > G \end{cases}$$

$$\theta = \cos^{-1}\left[\frac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2+(R-B)(G-B)]^{1/2}} \right]$$

$$S = 1 - \frac{3}{(R+G+B)} [\min$$

18/07/22.

* Histogram

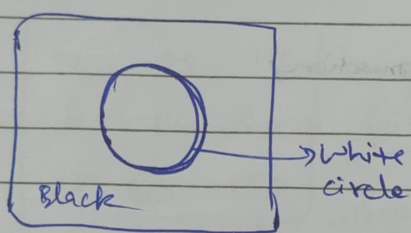


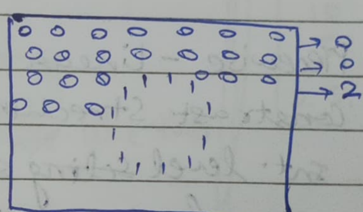
$$\begin{matrix} 5 & 7 & 6 & 4 & 3 \\ 2 & 3 & 4 & 6 & 7 \\ 1 & 7 & 7 & 6 & 4 \\ 3 & 6 & 5 & 4 & 2 \\ 1 & 2 & 3 & 4 & 5 \end{matrix}$$

$2^8 = 256$ $\quad$ $2^3 = 8$

36p

* Entropy
  ↳ No. of transitions in each row & column.





* Mathematical opers on pixel

(Done for enhancement purpose)

$$\begin{bmatrix} 0 & 100 & 5 \\ 3 & 18 & 150 \\ 25 & 220 & 10 \end{bmatrix} \begin{bmatrix} 10 & 110 & 15 \\ 25 & 200 & 16 \\ 0 & 17 & 20 \end{bmatrix} \quad \cancel{255}$$

4.563

4 $\quad$ 5

Round off to any value

$$S(x,y) = f(x,y) \pm g(x,y)$$

$*$

$\%$

if $x > 255$
then, $x = 255$

if $x < 0$
then $x = 0$

* Point operations
  All formula (whole topic numerical).

small letters → spatial domain

$g(x,y) = T[f(x,y)]$

capital letters → freq. domain

* Image Enhancement in the spatial domain.
  subjected to human only

* Basic Intensity Transformation Functions:

- Image -ves $s = (L-1) - r$
- Log transformations $s = c \log(1+r)$
- Power-law (Gamma) transformations $S = c r^{\wedge}\gamma$

$$S = \left\{255 * \left(\frac{r}{255}\right)^{\wedge}(1/\gamma)\right\}$$

- Piecewise - Linear transformations

Constrast Streching

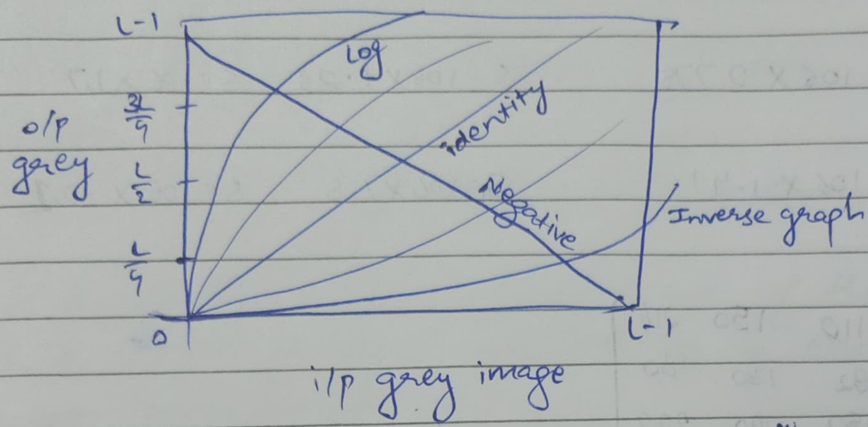
Int. level ~~scling~~ slicing

Bit plane "

Eve
18/07/22

Enhancement

* Direct transformation of pixels
* Intensity transformation
* Spatial filtering → Neighbourhood



Q.
$$\begin{bmatrix} 10 & 25 & 110 \\ 5 & 16 & 50 \\ 25 & 65 & 150 \end{bmatrix}$$

if given : 3 bit ~~then use~~ like this

otherwise ~~2n~~

→ Highest 2^8 → 256

$$\begin{bmatrix} 10 & 25 & 10 \\ 5 & 16 & 50 \\ 25 & 63 & 50 \end{bmatrix}$$

↓ max

$2^6 = 64$

∴ 6 bits

$S = (L-1) - r$

$S = (256-1) - r$

$= 255 - r$

$$= \begin{bmatrix} 245 & 230 & 145 \\ 250 & 239 & 205 \\ 230 & 190 & 105 \end{bmatrix}$$

log transformation for that same Q.

$s = c \log_{10}(1+r)$

~~where c = 1/log~~

$c = L / \log(1+L)$

Assume, $c = 1$

$S = c \log_{10}(1+r)$

$$\begin{bmatrix} \cancel{1.041} & \cancel{1.41} & \cancel{2.045} \\ 1 & 1 & 2 \\ \cancel{0.778} & \cancel{1.23} & \cancel{1.707} \\ 1 & 1 & 2 \\ \cancel{1.41} & \cancel{1.81} & \cancel{2.178} \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 2 & 3 \end{bmatrix}$$

$$c = \frac{256}{\log_{10}(257)} = \frac{256}{2.409} = 106$$

$s = 106 \times 1.041$ $s = 106 \times 1.41$ $s = 106 \times 2.04$

$s = 106 \times 0.778$ $s = 106 \times 1.23$ $s = 106 \times 1.7$

$s = 106 \times 1.41$ $s = 106 \times 1.8$ $s = 106 \times 2.1$

$$= \begin{bmatrix} 110 & 150 & 216 \\ 82 & 130 & 180 \\ 150 & 190 & 222 \end{bmatrix}$$

* Power-Law (gamma) transformations $s = cr^\wedge \gamma$

$$S = \left\{ 255 * \left(\frac{r}{255}\right)^\wedge \left(\frac{1}{\gamma}\right) \right\}$$

→ Exact value u take

r = [1 10 20 30 40 210 220 230 240 250 255]

s(γ = .4) = [0 0 0 1 2 157 176 197 219 243 255]

s(γ = 2.5) = [28 70 92 108 122 236 200 245 249 253 255]

19/July/2022

* Contrast streching.

Eg. For an image, intensity range [50-150]
What should (r_1, s_1) & (r_2, s_2) be to increase the dynamic range of the image to [0-255]?

$s_1 = 50 - 50 + a = 0$
$s_2 = (150 - 50)\left[\frac{255}{100}\right] = 255$

Ans. $S = (r - c)\,[(b - a)/(d - c)] + a$

where $a = 0$, $b = 255$, $r_1 = 50$ (c), $s_1 = 0$, $r_2 = 150$ (d), $s_2 = 255$

c & d are the lower & upper limit of the itensity.
S & r are the o/p & i/p intensity of each pixel of an image resp.
Here, $c = 50$ & $d = 150$.

Ans. 50 - 150
13 - 255

$$S = (55 - 50)\left[\frac{255 - 0}{150 - 50}\right] + 0$$

$$\begin{bmatrix} 55 & 70 & 80 \\ 100 & 95 & 130 \\ 110 & 60 & 150 \end{bmatrix}$$

$r_1 \to 50$ $r_2 \to 150$

$s_1 \to$ $s_2 \to$

---

* Contrast streching & thresholding. (No need)

* Gray-level slicing / Intensity slicing
Highlighting a specific range of grey levels in an image.

* Bit-plane slicing.

128	64	32	16	8	4	2	1
0	0	1	1	0	1	1	1

MSB ← → LSB

$$\begin{bmatrix} 55 & 70 & 80 \\ 100 & 95 & 130 \\ 110 & 60 & 150 \end{bmatrix}$$

Refer PPT.

understood

* Intensity level slicing
* Histogram equalisation

} Camera notes Yash Rao

Practice

—X—

NOTE Smoothing freq. filter is totally diff.

23/07/22.

* Spatial filtering — Filters — Smoothing : blur, noise reduction & Low pass → Allows only low freq. (high blocked); Sharpening : High pass → " " high freq. (low blocked) — Mask, Template, window

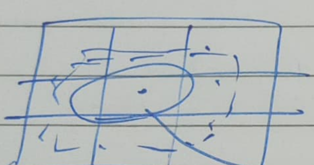
filter out un-necessary freq.

Filters →

Low pass
Allows only low freq.
High freq. blocked
Smoothing filter
Blurs the image
Noise reduction

(Mean filter) Averaging, weighted avg.
Examples
Smoothing filter — Linear smoothing spacial filter; Non linear — max filter, median filter, min filter

High pass
Allows only high freq.
Low freq. blocked
Sharpening filter.
Sharpen the image
highlight edges



$f(x,y)$
Neighbourhood averaging.
→ To make changes to this by spatial filtering, we will need to change something for all its neighbouring pixels.

$g(x,y) = T[f(x,y)]$

Band-pass / Band reject — we won't study.
Types of filters

* $m \times n$ Dim^n of image

$m = 2a + 1$
$n = 2b + 1$

where $a, b > 0$

$\therefore$ min. possible values for a & b is 1.
$\therefore$ $m = 3$
$n = 3$.

Always, m and n will be odd numbers cuz we need a centre pixel.

Date
Page

Size of the matrix & mask filter (w) ~~should~~ must be of same size.

averaging ~~filter~~ filtering ques.

$$f = \frac{1}{9}\begin{bmatrix} 15\times1 & 16\times1 & 25\times1 \\ 5\times1 & 7\times1 & 2\times1 \\ 65\times1 & 9\times1 & 15\times1 \end{bmatrix} \qquad w = \frac{1}{9}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Mean filter

overlapping

$\frac{1}{9} \rightarrow$ sum of pixels

Box filter

$\rightarrow \dfrac{\text{sum of all multiplications}}{9}$ ~~→ no. of pixels~~

$$\frac{159}{9} = 17.66 \approx 18.$$

$$\begin{bmatrix} 15 & 16 & 25 \\ 5 & 18 & 2 \\ 65 & 9 & 15 \end{bmatrix}$$

You need to mask center pixel of mask filter (w) with every pixel in f matrix.

For that expand your matrix. You have 3 processes for the same →

1) Zero padding ✓ most easy; use this if nothings mentioned in the ques.
2) Pixel replication
3) Pixel ~~packing~~ wrapping

- Zero padding:

0	0	0	0	0
0	15	26	35	0
0	122	53	69	0
0	75	86	110	0
0	0	0	0	0

- Pixel replication

15	15	26	35	35
15	15	26	35	35
122	122	53	69	69
75	75	86	110	110
75	75	86	110	110

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 15 & 26 & 35 \\ 1 & 122 & 53 & 69 \\ 2 & 75 & 86 & 110 \end{array}$$

Q.

3 X 3 filter $\longrightarrow$ $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

mean/box/average filter

Ans. Zero padding ✓ Pixel repl. ✓ Pixel wrapping :

This is wrongly solved

110	75	86	110	75
35	15^{66}	26^{71}	35^{71}	15
69	122^{76}	53^{75}	69^{77}	122
110	75	86	110	75
35	15	26	35	15

Don't use changed/ updated values of center pixel, use prev value only

3X3 image to 5X5 image

Now, we need to smooth 9 pixels (original).

1st pixel $\rightarrow$ $\frac{591}{9} = 65.66 \approx 66$

2nd $\rightarrow$ $\frac{642}{9} = 71.33 \approx 71$

3rd $\rightarrow$ $\frac{636}{9} = 70.66 \approx 71$

4th $\rightarrow$ $\frac{687}{9} = 76.33 \approx 76$

5th $\rightarrow$ $\frac{677}{9} = 75.22 \approx 75$

6th $\rightarrow$ $\frac{694}{9} = 77.11 = 77$

less than 0 & greater than 255 ~~>255~~ Round it to 255 or 0 accordingly

Final answer $\rightarrow$ 3 X 3 matrix with updated values.

7th → $\frac{567}{9} = 63$

8th → $\frac{563}{9} = 62.55 \approx 63$

9th → $\frac{598}{9} = 66.44 \approx 66$

$$\text{matrix} = \begin{bmatrix} 66 & 71 & 71 \\ 76 & 75 & 77 \\ 63 & 63 & 66 \end{bmatrix}$$

→ smoothen image
(not much variation in pixels)

* weighted avg. filter

mean filter → $\frac{1}{10}\begin{bmatrix} \cancel{1} & \cancel{1} & \cancel{1} \\ \cancel{1} & \cancel{2} & \cancel{1} \\ 1 & 1 & 1 \end{bmatrix}$ This is also correct BUT use this only

3X3

(*) $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

centre pixel is square of its neighbours
(neighbours are square root)

Rest procedure is same.

* 5 X 5 weighted avg. filter

$$\frac{1}{60}\begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 1 & 2 & 4 & 2 & 1 \\ 2 & 4 & 16 & 4 & 2 \\ 1 & 2 & 4 & 2 & 1 \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix}$$

1
2
4
16

25 July 2022.

* Non-linear filters

Min, max & median filter

$$\min \begin{cases} z_i \\ i = 1 \text{ to } 9 \end{cases}$$

Matrix →

$$\begin{matrix} 110 & 75 & 86 & 110 & 75 \\ 35 & 15 & 26 & 35 & 15 \\ 69 & 122 & 53 & 69 & 122 \\ 110 & 75 & 86 & 110 & 75 \\ 35 & 15 & 26 & 35 & 15 \end{matrix}$$

min filter →

$$\begin{vmatrix} 15 & 15 & 15 \\ 15 & 15 & 15 \\ 15 & 15 & 15 \end{vmatrix}$$

max filter → similarly max.

median filter.

matrix →

$$\begin{matrix} 110 & 75 & 86 & 110 & 75 \\ 35 & 0 & 226 & 35 & 0 \\ 69 & 122 & 53 & 69 & 122 \\ 110 & 75 & 86 & 110 & 75 \\ 35 & 0 & 226 & 35 & 0 \end{matrix}$$

1st mat → 0. 35 53 69 $\boxed{75}$ 86 110 122 226

2nd → 0 35 53 69 $\boxed{75}$ 86 110 122 226

median filter →

$$\begin{vmatrix} 75 & 75 & 75 \\ 75 & 75 & 75 \\ 75 & 75 & 75 \end{vmatrix}$$

classmate Date Page

* Sharpening Spatial Filters

Reverse of the smoothing process.

→ Removing blurring from images
→ Highlight edges

• First derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

• Second derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Q. 5 5 6 1 0 5

$$\frac{\partial f}{\partial x} = 0 \quad 1 \quad -5 \quad -1 \quad 5$$

$$\frac{\partial^2 f}{\partial x^2} = 6+5-10 \quad 6-10 \quad 6-10 \quad 6-0$$

$$= 1 \quad -4 \quad -4 \quad 6$$

Graph is V. Imp values will be given, draw graph

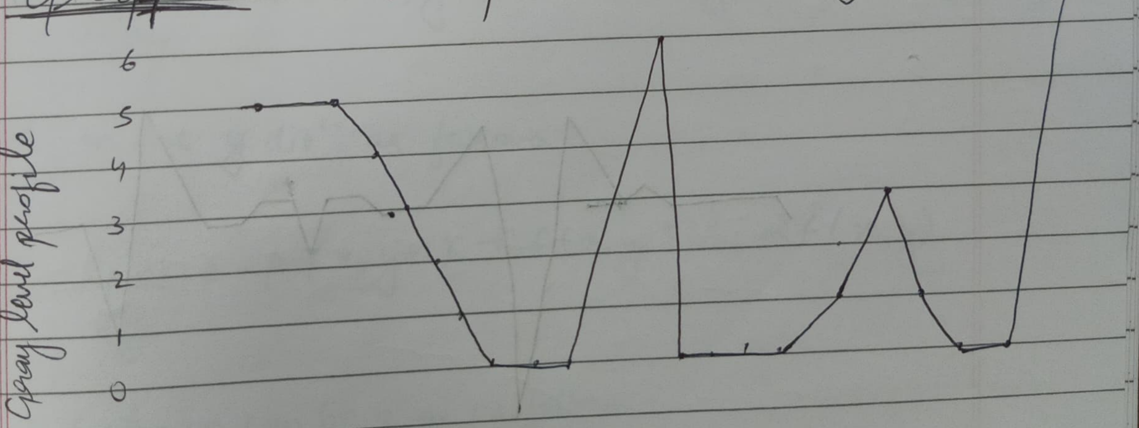


Image strip 5 5

Ramp   Step   Flat segment   Isolated (alone) v. high   Thin line like isolated but not much variation

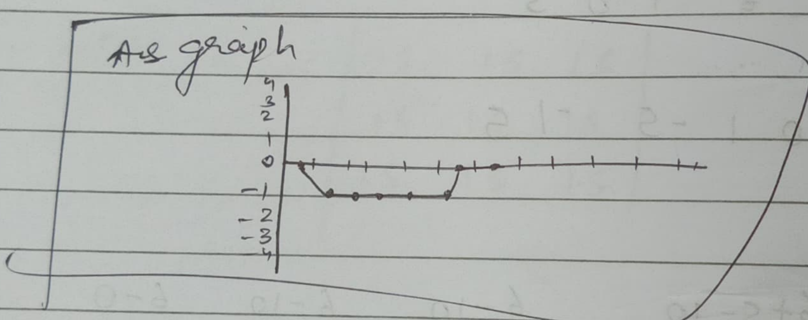
Q. Given a graph & image strip, find sharpened image.

5 5 4 3 2 1 0 0 0 6 0 0 0 0 1 3 1 0 0 0 0 7 7 7

Ans: 0 -1 -1 -1 -1 0 0 6 -6 0 0 0 1 2 -2 -1 0 0 0 7 0 0

↓ First derivative

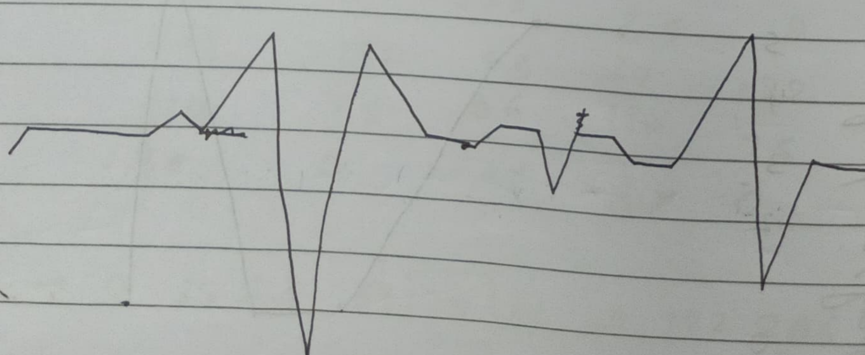
Ques graph



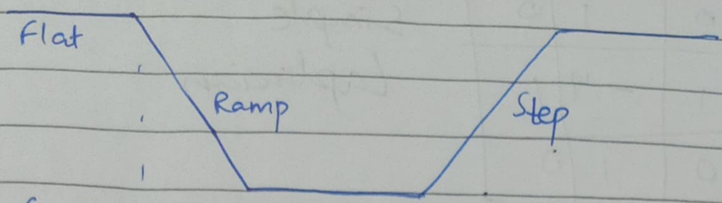
- Second derivative:

Second derivative ↑

-1 0 0 0 0 1 0 6 -12 6 0 0 1 1 -4 1 1 0 0 7 -7 0 0



25/07/22.



6 6 6. 5 4 3 2 1 1 1 1. 6 6 6 6

$\frac{\partial f}{\partial x}$ = 0 0 -1 -1 -1 -1 -1 0 0 0 5 0 0 0

$\frac{\partial^2 f}{\partial x^2}$ = 0 1 -1 0 0 0 0 1 0 0 5 -5 0 0

→ tameez se likhkr observe karna hai ye

★ The Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 2nd order der. in the dirⁿ. x is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

& in the y dirⁿ. as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

So, Laplacian can be given as follows:-

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

We can easily build a filter based on this →

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ Simple Laplacian

for the diagonal :-

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Original image $f(x,y)$ + Laplacian filter = Laplacian filtered image

$$\underset{\substack{\downarrow \\ \text{Expected} \\ \text{Sharpen image}}}{g(x,y)} = f(x,y) - \nabla^2 f \cancel{= 5f(x,y) - f}$$

Q.

5	5	6	7	7
5	5	6	7	7
2	2	4	1	1
3	3	4	6	6
3	3	4	6	6

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g(x,y) = 5f(x,y) - f(x+1,y) - f(x-1,y) - f(x,y+1) - f(x,y+1)$$

Ans Laplacian is :-

$$f(x,y) - \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \text{OR} \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

If diagonal needs to be considered :-

$$\begin{matrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{matrix} \quad \text{OR} \quad \begin{matrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{matrix}$$

- Sharpened filter without diagonal :-

$$\begin{matrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{matrix} \quad \text{OR} \quad \begin{matrix} 0 & 1 & 0 \\ 1 & -5 & 1 \\ 0 & 1 & 0 \end{matrix}$$

- Sharpened filter with diagonal :-

$$\begin{matrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{matrix} \quad \text{OR} \quad \begin{matrix} 1 & 1 & 1 \\ 1 & -9 & 1 \\ 1 & 1 & 1 \end{matrix}$$

26/07/22

Date
Page

★ Un sharp Masking.

1. Blur the image

~~$g_{mask}(x,y) = f(x,y) - f_{lp}(x,y)$~~ ~~High pass filter~~

~~$F_{hb}(x,y) = A$~~ $(\bar{f}(x,y)\ f_{lp}(x,y))$

2. Subtract from the original image.

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

↳ unsharp mask

3. Add the gmask to the original image

$$g(x,y) = f(x,y) + K * g_{mask}$$

↓ sharpened image

$K=1 \rightarrow$ unsharp mask

$K>1 \rightarrow$ High boost filter

$K<1 \rightarrow$ should not be less than 1 (similar to laplacian filter only)

* 1st Derivative filtering.

Gradient $\rightarrow$ $\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$

Magnitude $\nabla f = mag(\nabla f)$

$$= [G_x^2 + G_y^2]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

For simplification $\rightarrow$ $\nabla f = |G_x| + |G_y|$

- $mag(x,y) = |G_x| + |G_y|$

$$\frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

$$= z_6 - z_5$$

$$\frac{\partial f}{\partial y} =$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

1) Roberts   2) Prewitt   3) Sobel

- Roberts operator

$G_x = (z_6 - z_5)$ and $G_y = (z_8 - z_5)$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- Prewitt operator

$$G_x = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7) \text{ and}$$

$$G_y = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)$$

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

- Sobel operator

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Also, to highlight corners:

$$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

- 1st Der. Filtering.

$$\nabla f \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

Q.

$$\begin{matrix} 50 & 52 & 53 \\ 75 & 60 & 63 \\ 70 & 62 & 50 \end{matrix}$$

Solve using laplacian to get a sharpened image. Diagonal needs to be considered.

Ans.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ filter.

→ By DEFAULT take this filter only

Applying filter over ~~matrix~~ image :-

$$\begin{bmatrix} \cancel{50}\ 70 & 62 & 50\ 70 \\ 53\ 50 & 52 & 53\ 50 \\ 63\ 75 & 60 & 63\ 75 \\ 50\ 70 & 62 & 50\ 70 \\ 53\ 50 & 52 & 53\ 50 \end{bmatrix}$$

1st → 85

2nd → ~~58~~ 67

3 → 58

4 → −140 ≈ 0.

5 → −5 ≈ 0

$6 \rightarrow -32 \approx 0$

$7 \rightarrow -95 \approx 0$

$8 \rightarrow -23 \approx 0$

$9 \rightarrow 85$

Laplacian filtered image $= \begin{bmatrix} 85 & 67 & 58 \\ 0 & 0 & 0 \\ 0 & 0 & 85 \end{bmatrix}$

$\therefore$ Now, to get sharpened image :

$$\begin{bmatrix} 50 & 52 & 53 \\ 75 & 60 & 63 \\ 70 & 62 & 50 \end{bmatrix} - \begin{bmatrix} 85 & 67 & 58 \\ 0 & 0 & 0 \\ 0 & 0 & 85 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 75 & 60 & 63 \\ 70 & 62 & 0 \end{bmatrix}$$

[Convert -ves to zero]

Afternoon
26/07/22

Date
Page

* Freq. Domain Filters.

$f(x,y) \rightarrow F(u,v) \rightarrow H(u,v) \longrightarrow G(u,v)$

preprocessing
multiply by
$(-1)^{x+y}$

transform
DFT
DCT
DWT
FFT

filters
low
high

Inverse
transform
$\downarrow$
$g(x,y)$

V.V.I
* Basic filtering in the freq. Domain

Steps :-

1. Multiply the input image by $(-1)^{x+y}$ to center the transform.
2. Compute $F(u,v)$ the DFT of the image from (1)
3. Multiply $F(u,v)$ by a filter function $H(u,v)$
4.
5.
6.

* Low pass filters
Ideal LPF
Butterworth LPF
Gaussian LPF

* High pass filters

* Ideal LPF

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

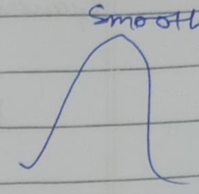
$$D(u,v) = \left[(u-M/2)^2 + (v-N/2)^2\right]^{1/2}$$

Diagram is also imp.

Diagram + Formula

- Butterworth LPF

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

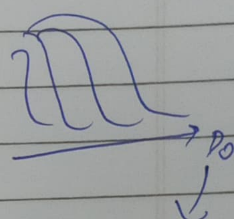


Completely depends on order 'n'

n is lesser → Gaussian
n is larger → Ideal

GRAPH IS MANDATORY.

- Gaussian LPF



$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Depends on the value of D_0.

filtering is even more smoothing.

✱ High Pass Filters.

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

- HPLP filters.
Ideal, Butterworth, Gaussian, Unsharp Masking & High Boosting

- Ideal

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

BW

$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

Gaussian

$$H(u,v) \quad 1 - e^{-D^2(u,v)/2D_0^2}$$