NCERT Mathematics 11.9.3 Q32

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Question: If A.M. and G.M. of roots of a quadratic equation are 8 and 5,respectively,then obtain the quadratic equation.

Solution:

Parameter	Description	Value
x_1, x_2	Roots of a quadratic equation	?
$\frac{x_1+x_2}{2}$	A.M. of roots	8
$\sqrt{x_1 \cdot x_2}$	G.M. of roots	5

TABLE I
INPUT PARAMETERS

$$x_1 \cdot x_2 = 25 \tag{1}$$

$$x_1 + x_2 = 16 \tag{2}$$

$$\implies x^2 - 16x + 25 = 0 \tag{3}$$

$$\implies x_1 = 8 + \sqrt{39} \tag{4}$$

$$\implies x_2 = 8 - \sqrt{39} \tag{5}$$

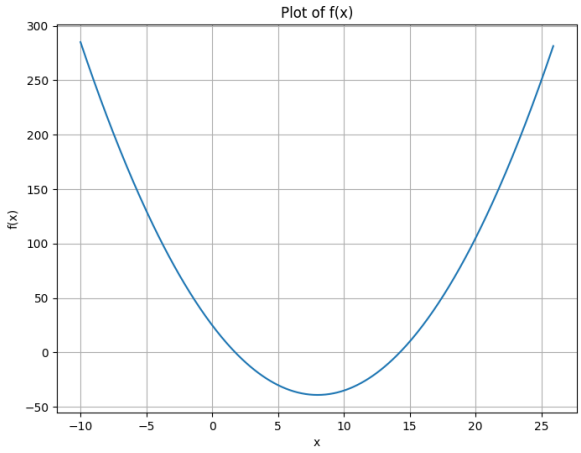

Fig. 1. Plot of $f(x) = x^2 - 16x + 25 = 0$

For AP,

$$x(n) = \left(8 + \sqrt{39} + n\left(-2\sqrt{39}\right)\right) u(n) \tag{6}$$

$$X(z) = \frac{8 + \sqrt{39}}{1 - z^{-1}} + \frac{\left(-2\sqrt{39}\right) \cdot z^{-1}}{\left(1 - z^{-1}\right)^2} \quad |z| > 1 \tag{7}$$

$$\implies X(z) = \frac{8 + \sqrt{39} - \left(8 + 3\sqrt{39}\right) \cdot z^{-1}}{\left(1 - z^{-1}\right)^2} \quad |z| > 1 \tag{8}$$

For GP,

$$x(n) = \left(\left(8 + \sqrt{39}\right) \cdot \left(\frac{8 - \sqrt{39}}{8 + \sqrt{39}}\right)^n\right) u(n) \tag{9}$$

$$X(z) = \frac{8 + \sqrt{39}}{1 - \frac{\left(8 - \sqrt{39}\right) z^{-1}}{8 + \sqrt{39}}} \quad |z| > \frac{103 - 16\sqrt{39}}{25} \tag{10}$$

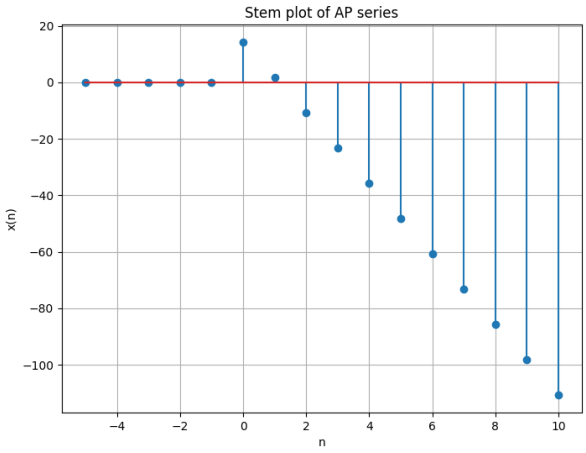

Fig. 2. Plot of $x(n) = \left(8 + \sqrt{39} + n\left(-2\sqrt{39}\right)\right) u(n)$

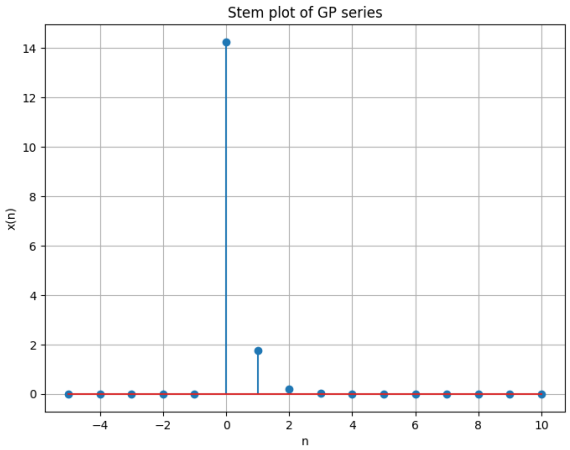

Fig. 3. Plot of $x(n) = \left(\left(8 + \sqrt{39}\right) \cdot \left(\frac{8-\sqrt{39}}{8+\sqrt{39}}\right)^n\right) u(n)$