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GATE 2022 IN 60

EE23BTECH11213 - MUTHYALA NIKHITHA SRI

Question: A 1kHz sine wave generator having an internal resistance of 50Ω generates an opencircuit voltage of $10V_p$. When a capacitor is connected across the output terminals, the voltage drops to $8V_p$. The capacitance of the capacitor (in microfarads) is (GATE IN 2022)

Solution:

Parameter	Description	Value
V_i	Input voltage	$10V_p$
V_o	Output voltage	$8V_p$
R	Internal resistance	50Ω
f	Frequency of sine wave	1kHz
ω	Angular frequency	$2\pi f$
С	Capicatance of capacitor	?
X_c	Reactance of capicator	$\frac{1}{i\omega C}$

TABLE I INPUT PARAMETERS

$$V_o = \left| \frac{X_c}{R + X_c} \right| \cdot V_i \tag{1}$$

$$8V_p = \left| \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \right| \cdot 10V_p \tag{2}$$

$$\frac{4}{5} = \left| \frac{1}{1 + j\omega RC} \right| \tag{3}$$

$$\frac{16}{25} = \frac{1}{1 + \omega^2 R^2 C^2} \tag{4}$$

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$$\omega^2 R^2 C^2 = \frac{9}{16}$$
(4)

$$\omega RC = \frac{3}{4} \tag{6}$$

$$C = \frac{3}{4 \cdot 50 \cdot 2\pi \cdot 10^3}$$

$$\implies C = 2.387 \mu F$$
(8)

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