GATE 2022 IN 60

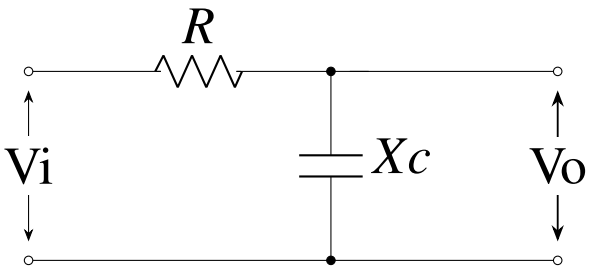
EE23BTECH11213 - MUTHYALA NIKHITHA SRI

Question: A 1kHz sine wave generator having an internal resistance of 50Ω generates an open-circuit voltage of $10V_p$. When a capacitor is connected across the output terminals, the voltage drops to $8V_p$. The capacitance of the capacitor (in microfarads) is (GATE IN 2022)

Solution:

Parameter	Description	Value
V_i	Input voltage	$10V_p$
V_o	Output voltage	$8V_p$
R	Internal resistance	50Ω
f	Frequency of sine wave	1kHz
ω	Angular frequency	$2\pi f$
C	Capicatance of capacitor	?
X_c	Reactance of capicator	$\frac{1}{j\omega C}$

TABLE I
INPUT PARAMETERS



$$V_o = \frac{X_c}{\sqrt{R^2 + X_c^2}} \cdot V_i \tag{1}$$

$$8V_p = \frac{\frac{1}{j\omega C}}{\sqrt{R^2 + \left(\frac{1}{j\omega C}\right)^2}} \cdot 10V_p \tag{2}$$

$$\frac{64}{100} = \frac{1}{1 - \omega^2 R^2 C^2} \tag{3}$$

$$\omega^2 R^2 C^2 = \frac{-9}{16} \tag{4}$$

$$\omega RC = \frac{3j}{4} \tag{5}$$

$$C = \frac{3j}{4 \cdot 50 \cdot 2\pi \cdot 10^3} \tag{6}$$

$$\implies C = 2.387 j\mu F \tag{7}$$