GATE 2023 BM 33

EE23BTECH11213 - MUTHYALA NIKHITHA SRI

Question: A continuous time, band-limited signal $x(t)$ has its Fourier transform described by:

$$X(f) = \begin{cases} 1 - \frac{|f|}{200} & \text{if } |f| \le 200 \\ 0 & \text{if } |f| > 200 \end{cases} \tag{1}$$

The signal is uniformly sampled at a sampling rate of 600 Hz. The Fourier transform of the signal is $X_s(f)$. What is the value of $\frac{X_s(600)}{X_s(500)}$?

Solution:

Parameter	Description	Value
$X(f)$	Fourier transform of $x(t)$	$\begin{cases} 1 - \frac{\lvert f\rvert}{200} & \text{if } \lvert f\rvert \le 200 \\ 0 & \text{if } \lvert f\rvert > 200 \end{cases}$
$X_s(f)$	Fourier transform of sampled signal	?

TABLE I
INPUT PARAMETERS

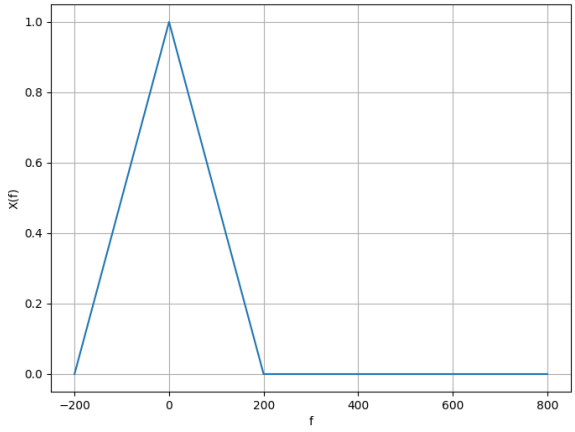

Fig. 1. Plot of $X(f)$

$$X_s(f) = \frac{1}{600} \sum_{k=-\infty}^{\infty} X(f - 600k) \tag{2}$$

$$\implies X_s(f + 600) = \frac{X(f)}{600} \tag{3}$$

(1)

$$X_s(600) = \frac{X(0)}{600} \tag{4}$$

$$\implies X_s(600) = \frac{1}{600} \tag{5}$$

$$X_s(500) = \frac{X(-100)}{600} \tag{6}$$

$$\implies X_s(500) = \frac{1}{2 \cdot 600} \tag{7}$$

$$\frac{X_s(600)}{X_s(500)} = 2 \tag{8}$$

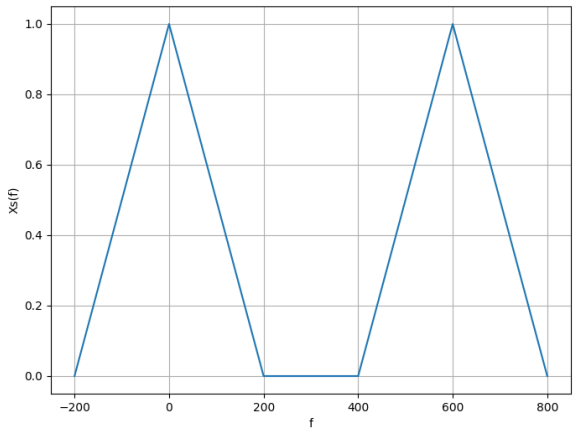

Fig. 2. Plot of $X_s(f)$