1. i $A \cup B = \{1, 2, 3, 4, 5\}$

$A \cap B \cap C = \emptyset$

$(A \cap C) \cup B = \{1, 2, 3, 4, 5\}$

ii $U = \{1, 2, 3, 4, 5, 6\}$

$(B \cup C) = \{1, 2, 3, 4, 5\}$

$(B \cup C)^c = \{6\}$

$A^c = \{5, 6\}$, $B^c = \{2, 4, 6\}$, $C^c = \{1, 3, 6\}$

$A^c \cap B^c \cap C^c = \{6\}$

$U^c = \emptyset$

iii $(B \cap C) = \{5\}$

$(B \cap C)^c = \{1, 2, 3, 4, 6\}$, 5 elements

$(B \cup C) = \{1, 2, 3, 4, 5\}$

$(B \cup C)^c = \{6\}$, 1 element

$\{x : x \subseteq B\}$: 8 elements.

$\{x : x \subseteq A$ and x has at most two elements$\}$:

0 element : 1

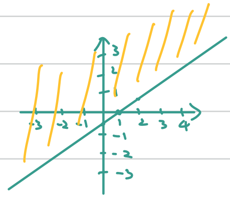
1 element : 4

2 elements : 6.

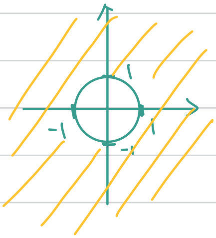
Sum : 11 elements

2. i $x \le 2y+1$

$\frac{x-1}{2} \le y$. $y \ge \frac{x-1}{2}$



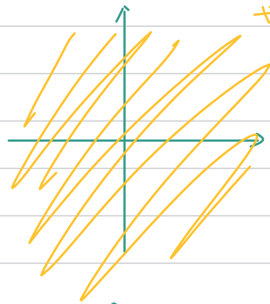
ii



iii

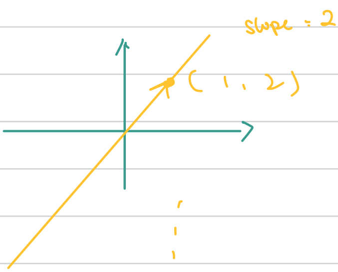
$f(x) = e^x$

vi

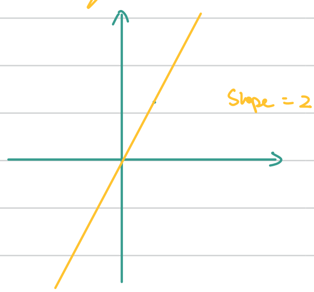


the entire plane

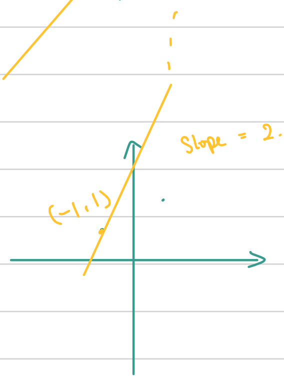
iv



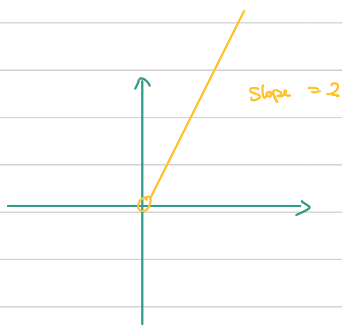
vii



V.



Viii



3. i $\{ (2x)^2 \mid x \in \mathbb{N},\ x > 0 \}$

ii $\{ \frac{2}{3^x} \mid x \in \mathbb{Z} \}$

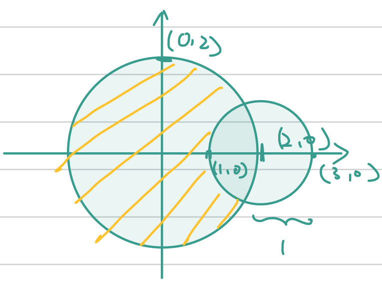
iii $\{ 3(2x+1) \mid x \in \mathbb{Z} \}$

4. i a. $B \setminus C = \{1, 3\}$

$C \setminus B = \{2, 4\}$

$A \setminus (B \cup C) = \emptyset$

b.

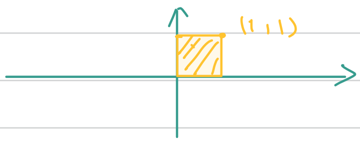


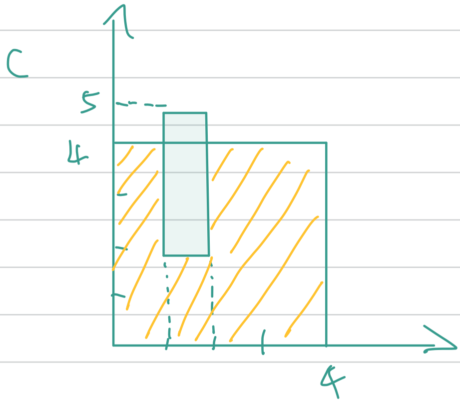
ii a. $C \times B = \{(2,1), (2,3), (2,5),$
$(4,1), (4,3), (4,5),$
$(5,1), (5,3), (5,5)\}$

$B \times C = \{(1,2), (1,4), (1,5),$
$(3,2), (3,4), (3,5),$
$(5,2), (5,4), (5,5)\}$

$A \times \phi = \phi$

b.





5. i false. counterexample:

let $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$C = \{11, 12\}$

$(A \cap B) \cup C = \{3, 11, 12\}$

$A \cap (B \cup C) = \{3\}$

$\therefore$ $(A \cap B) \cup C$ doesn't always equal to $A \cap (B \cup C)$.

ii true. $A \cup (B \cap C)$ means $x : x \in A$ or $x \in B, C$.

$(A \cup B) \cap (A \cup C)$ means $x : x \in A$ or $x \in B$ or $x \in A, B$

and $x \in A$ or $x \in C$ or $x \in A, C$

essentially they both requires x to be in A or B, C,

or A, B, and C.

iii true. Assume $A \not\subseteq B$, $\exists\, x \in A$ such that

$x \notin B$.

Therefore, $B \neq A \cup B$.

$\therefore$ $B = A \cup B$ if and only if $A \subseteq B$

iv. false. $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$.

then $A \setminus B = \{1, 2\}$

$B \setminus A = \{4, 5\}$.

$A \cup B = \{1, 2, 3, 4, 5\}$.

$\therefore (A \setminus B) \cup (B \setminus A) \neq A \cup B$.

v. true. Assume $\exists\, x \in (A \times B)$ but $\notin (A \times C)$, given that $B \subseteq C$. Then this $x \in B$ and $\notin A$. Which means $x \in B$ but $\notin C$, so $B \notin C$.

$\therefore$ if $B \subseteq C$ then $(A \times B) \subseteq (A \times C)$

6. i original: $\forall n \ni \mathbb{Z}\ \exists\, a \in \mathbb{R}\ [a = n]$

negation: $\exists\, n \ni \mathbb{Z}\ \forall\, a \in \mathbb{R}\ [a \neq n]$

ii original: $\forall x > 0,\ \forall y\ [x^2 + y^2 > 0]$

negation: $\exists\, x > 0,\ \exists\, y\ [x^2 + y^2 \leq 0]$

there exist $x > 0$ and a y such that $x^2 + y^2 \leq 0$

7. i There exists some continuous functions that are not differentiable.

ii For any real numbers, there exist a larger real number.

iii For any prime number, there exists a bigger prime number.

8. i ① reflexive :

for $p \in A$, if $p = p$, then by definition it satisfies "$p = q$".

② symmetric :

for all $p, q \in A$,

- if $p = q$, then $q = p$
- if $p \neq q$: as long as the line pq crosses origin, then qp also crosses the origin.

③ transitive :

- if $p = q$, $q = r$, then $p = r$.
- if $p = q$, $q \neq r$ but $q \sim r$, then $p \sim r$.
- if $p \neq q$, $q = r$. and $p \sim q$, then $p \sim r$
- if $p \neq q \neq r$, and $p \sim q$, $q \sim r$, then p, q, r all on the same line that crosses the origin.

$\therefore$ $p \sim r$.

$\therefore$ all three requirements are satisfied.

$\therefore$ $\sim$ defines an equivalence relation on A.

ii For any $(p, q) \in A$, their equivalent class would be the line that passes through p and q.