

**Homework 2**  
**"Applied Methods of Linear Algebra"**  
**variant 50**

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## Task 1

### Description

Find the best approximation matrix  $A_1$  of rank 2 of the matrix  $A$  in the norm  $\|\cdot\|_2$  and find  $\|A - A_1\|_2$ , where

$$A = \begin{pmatrix} -38 & -102 & -48 & -73 \\ -22 & 120 & 12 & 58 \\ 32 & -42 & 12 & -62 \end{pmatrix}$$

### Solution

As far as the norm  $\|\cdot\|_2$  is unitary invariant, according to Eckart-Young-Mirski theorem:

$$A = U \cdot \Sigma \cdot V^* \Rightarrow A_1 = U \cdot \Sigma_{(r)} \cdot V^* \text{ - best low rank approximation}$$

Therefore:

$$\begin{aligned} U &= \begin{pmatrix} 2/3 & -2/3 & 1/3 \\ -2/3 & -1/3 & 2/3 \\ 1/3 & 2/3 & 2/3 \end{pmatrix}, \Sigma = \begin{pmatrix} 198 & 0 & 0 & 0 \\ 0 & 66 & 0 & 0 \\ 0 & 0 & 33 & 0 \end{pmatrix}, V = \begin{pmatrix} 0 & 9/11 & -2/11 & -6/11 \\ -9/11 & 0 & 6/11 & -2/11 \\ -2/11 & 6/11 & 0 & 9/11 \\ -6/11 & -2/11 & -9/11 & 0 \end{pmatrix} \Rightarrow \\ \Rightarrow \Sigma_{(r=2)} &= \begin{pmatrix} 198 & 0 & 0 & 0 \\ 0 & 66 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow A_1 = \begin{pmatrix} -36 & -108 & -48 & -64 \\ -18 & 108 & 12 & 76 \\ 36 & -54 & 12 & -44 \end{pmatrix} \Rightarrow \\ \Rightarrow \|A - A_1\|_2 &= \left\| U \cdot (\Sigma - \Sigma_{(r=2)}) \cdot V^* \right\|_2 \stackrel{\text{unitary invariant}}{=} \|\Sigma - \Sigma_{(r=2)}\|_2 = \\ &= \left\| \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 33 & 0 \end{pmatrix} \right\|_2 = 33 \end{aligned}$$

### Answer

The best approximation matrix is:

$$A_1 = \begin{pmatrix} -36 & -108 & -48 & -64 \\ -18 & 108 & 12 & 76 \\ 36 & -54 & 12 & -44 \end{pmatrix}$$

$$\|A - A_1\|_2 = 33$$

## Task 2

### Description

Estimate the relative error of the approximate solution  $(1; 1)$  of the system  $AX = b$  in the norms  $|\cdot|_1$  and  $|\cdot|_2$  using the condition number of the matrix  $A$ , where

$$A = \begin{pmatrix} 3.9 & 0.03 \\ -4.11 & -7.91 \end{pmatrix}, \quad b = \begin{pmatrix} 4.19 \\ -12.1 \end{pmatrix}$$

### Solution

According to the lecture:

$$\begin{cases} \delta x \leq \frac{\text{cond}(A)}{1 - \text{cond}(A) \cdot \delta A} \cdot (\delta A + \delta b) \\ \text{cond}(A) = \|A\| \cdot \|A^{-1}\| \\ \delta A = \frac{\|\hat{A} - A\|}{\|A\|} \\ \delta b = \frac{\|\hat{b} - b\|}{\|b\|} \end{cases}$$

To start with:

$$\hat{A} = \begin{pmatrix} 4 & 0 \\ -4 & -8 \end{pmatrix}, \quad \hat{b} = \begin{pmatrix} 4 \\ -12 \end{pmatrix}$$

where  $(1; 1)$  is an exact solution. Then:

$$\begin{cases} \delta b_1 = \frac{|\hat{b} - b|_1}{|b|_1} = \frac{\sum_i |\hat{b}_i - b_i|}{\sum_i |b_i|} = \frac{0.29}{16.29} \approx 0.0178 \\ \delta b_2 = \frac{|\hat{b} - b|_2}{|b|_2} = \frac{\sqrt{\sum_i (\hat{b}_i - b_i)^2}}{\sqrt{\sum_i b_i^2}} \approx \frac{0.2147}{12.8049} \approx 0.0168 \end{cases}$$

and:

$$\begin{cases} \delta A_1 = \frac{|\hat{A} - A|_1}{|A|_1} = \frac{\max_j \left( \sum_i |\hat{A}_{ij} - A_{ij}| \right)}{\max_j \left( \sum_i |A_{ij}| \right)} = \frac{0.21}{8.01} \approx 0.0262 \\ \delta A_2 = \frac{|\hat{A} - A|_2}{|A|_2} = \frac{\max_i \left( \sigma_i(\hat{A} - A) \right)}{\max_i \left( \sigma_i(A) \right)} \approx \frac{0.1733}{9.1293} \approx 0.0190 \end{cases}$$

Furthermore:

$$A^{-1} \approx \begin{pmatrix} 0.2574 & 0.0010 \\ -0.1338 & -0.1269 \end{pmatrix} \Rightarrow \begin{cases} \text{cond}_1(A) = |A|_1 \cdot |A^{-1}|_1 \approx 8.01 \cdot 0.3912 \approx 3.1335 \\ \text{cond}_2(A) = |A|_2 \cdot |A^{-1}|_2 \approx 9.1293 \cdot 0.2971 \approx 2.7123 \end{cases}$$

All in all:

$$\begin{cases} \delta x_1 \leq \frac{\text{cond}_1(A)}{1 - \text{cond}_1(A) \cdot \delta A_1} \cdot (\delta A_1 + \delta b_1) \approx 0.1502 \\ \delta x_2 \leq \frac{\text{cond}_2(A)}{1 - \text{cond}_2(A) \cdot \delta A_2} \cdot (\delta A_2 + \delta b_2) \approx 0.1024 \end{cases}$$

## Answer

The relative error is:

$$\delta x \lesssim \begin{cases} 0.15 & \text{in the norm } \|\cdot\|_1 \\ 0.10 & \text{in the norm } \|\cdot\|_2 \end{cases}$$

## Task 3

### Description

Solve the system approximately and estimate the relative error of the solution in the norms  $|\cdot|_1, |\cdot|_2, |\cdot|_\infty$ :

$$\begin{cases} 4 \cdot (-8 + \varepsilon_1) \cdot x + 4 \cdot (-4 + \varepsilon_2) \cdot y = -5 + \varepsilon_3 \\ -5 \cdot x + (4 + \varepsilon_1) \cdot y = -1 + \varepsilon_4 \end{cases}$$

where the unknown numbers  $\varepsilon_j$  satisfy the conditions  $|\varepsilon_j| < 0.05$  for all  $j = \overline{1, 4}$ .

### Solution

The approximate system is:

$$\begin{cases} 4 \cdot (-8) \cdot x + 4 \cdot (-4) \cdot y = -5 \\ -5 \cdot x + 4 \cdot y = -1 \end{cases}$$

therefore:

$$\hat{A} = \begin{pmatrix} -32 & -16 \\ -5 & 4 \end{pmatrix}, \quad \hat{b} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}, \quad \hat{x} = \begin{pmatrix} \frac{9}{52} \\ -\frac{7}{208} \end{pmatrix}$$

Next several steps are the same as in the previous task:

$$\begin{cases} \delta b_1 = \frac{|\hat{b} - b|_1}{|b|_1} = \frac{|\varepsilon_3| + |\varepsilon_4|}{6 - \varepsilon_3 - \varepsilon_4} \leq \frac{0.05 + 0.05}{6 - 0.05 - 0.05} \approx 0.0169 \\ \delta b_2 = \frac{|\hat{b} - b|_2}{|b|_2} = \frac{\sqrt{\varepsilon_3^2 + \varepsilon_4^2}}{\sqrt{(5 - \varepsilon_3)^2 + (1 - \varepsilon_4)^2}} \leq \frac{\sqrt{0.05^2 + 0.05^2}}{\sqrt{(5 - 0.05)^2 + (1 - 0.05)^2}} \approx 0.0140 \\ \delta b_\infty = \frac{|\hat{b} - b|_\infty}{|b|_\infty} = \frac{\max_i (|\hat{b}_i - b_i|)}{\max_i (|b_i|)} = \frac{\max(|\varepsilon_3|, |\varepsilon_4|)}{5 - \varepsilon_3} \leq \frac{0.05}{5 - 0.05} \approx 0.0101 \end{cases}$$

and:

$$\begin{cases} \delta A_1 = \frac{|\hat{A} - A|_1}{|A|_1} = \frac{\max(4 \cdot |\varepsilon_1|, |\varepsilon_1| + 4 \cdot |\varepsilon_2|)}{37 - 4 \cdot \varepsilon_1} \leq \frac{0.05 + 4 \cdot 0.05}{37 - 4 \cdot 0.05} \approx 0.0068 \\ \delta A_2 = \frac{|\hat{A} - A|_2}{|A|_2} \approx \frac{\sqrt{18 \cdot \varepsilon_1^2 + 4 \cdot \varepsilon_2^2}}{\sqrt{1321/2}} \leq \sqrt{\frac{18 \cdot 0.05^2 + 4 \cdot 0.05^2}{1321/2}} \approx 0.0091 \\ \delta A_\infty = \frac{|\hat{A} - A|_\infty}{|A|_\infty} = \frac{\max_i \left( \sum_j |\hat{A}_{ij} - A_{ij}| \right)}{\max_i \left( \sum_j |A_{ij}| \right)} = \frac{4 \cdot |\varepsilon_1| + 4 \cdot |\varepsilon_2|}{48 - 4 \cdot \varepsilon_1 - 4 \cdot \varepsilon_2} \leq \frac{4 \cdot 0.05 + 4 \cdot 0.05}{48 - 4 \cdot 0.05 - 4 \cdot 0.05} \approx 0.0084 \end{cases}$$

and:

$$A^{-1} = \frac{1}{4 \cdot (\varepsilon_1^2 - 4 \cdot \varepsilon_1 + 5 \cdot \varepsilon_2 - 52)} \cdot \begin{pmatrix} \varepsilon_1 + 4 & 4 \cdot (4 - \varepsilon_2) \\ 5 & -4 \cdot (8 - \varepsilon_1) \end{pmatrix}$$

In case we use this inverse matrix, calculation of  $\text{cond}(A)$  and, eventually,  $\delta x$ , is going to be too complicated (both calculating the norm and, moreover, maximization). Instead, it is better to use the following approximation:

$$\text{cond}(A) \approx \text{cond}(\hat{A}) \Rightarrow \begin{cases} \text{cond}_1(A) = |\hat{A}|_1 \cdot |\hat{A}^{-1}|_1 = 37 \cdot \frac{3}{13} \approx 8.5385 \\ \text{cond}_2(A) = |\hat{A}|_2 \cdot |\hat{A}^{-1}|_2 \approx 33.8803 \cdot 0.1725 \approx 6.1832 \\ \text{cond}_\infty(A) = |\hat{A}|_\infty \cdot |\hat{A}^{-1}|_\infty = 48 \cdot \frac{37}{208} \approx 8.5385 \end{cases}$$

One more simplification in comparison with the previous task is the formula for  $\delta x$ . Let us suppose that the denominator  $1 - \text{cond}(A) \cdot \delta A$  is equal to 1 due to a very low value of  $\delta A$ . Consequently:

$$\begin{cases} \delta x_1 \leq \text{cond}_1(A) \cdot (\delta A_1 + \delta b_1) \leq 8.5385 \cdot (0.0068 + 0.0169) \approx 0.2024 \\ \delta x_2 \leq \text{cond}_2(A) \cdot (\delta A_2 + \delta b_2) \leq 6.1832 \cdot (0.0091 + 0.0140) \approx 0.1428 \\ \delta x_\infty \leq \text{cond}_\infty(A) \cdot (\delta A_\infty + \delta b_\infty) \leq 8.5385 \cdot (0.0084 + 0.0101) \approx 0.1580 \end{cases}$$

## Answer

The approximate solution is:

$$\hat{x} = \begin{pmatrix} 9/52 \\ -7/208 \end{pmatrix}$$

The relative error is:

$$\delta x \lesssim \begin{cases} 0.20 & \text{in the norm } |\cdot|_1 \\ 0.14 & \text{in the norm } |\cdot|_2 \\ 0.15 & \text{in the norm } |\cdot|_\infty \end{cases}$$

## Task 4

### Description

Find the approximate inverse matrix to the matrix  $A$  and evaluate the approximation error with respect to the uniform norm  $\|\cdot\|_1$  if the elements of the matrix  $A$  are known with an absolute error of 0.01:

$$A \approx \begin{pmatrix} 2 & 8 \\ -9 & -6 \end{pmatrix}$$

### Solution

For matrix

$$A = \begin{pmatrix} 2+\varepsilon_1 & 8+\varepsilon_2 \\ -9+\varepsilon_3 & -6+\varepsilon_4 \end{pmatrix}, \quad |\varepsilon_i| < 0.01 \quad \forall i \in \overline{1,4}$$

the approximate matrix is:

$$\hat{A} = \begin{pmatrix} 2 & 8 \\ -9 & -6 \end{pmatrix}$$

therefore, the approximate inverse matrix of  $A$  is:

$$\hat{A}^{-1} = \frac{1}{60} \begin{pmatrix} -6 & -8 \\ 9 & 2 \end{pmatrix}$$

According to the lectures:

$$\delta A^{-1} \leq \frac{\text{cond}(A) \cdot \delta A}{1 - \text{cond}(A) \cdot \delta A}$$

Having looked at the steps from the previous task:

$$\begin{cases} \text{cond}(A) \approx \text{cond}(\hat{A}) = \|\hat{A}\|_1 \cdot \|\hat{A}^{-1}\|_1 = 14 \cdot \frac{15}{60} = 3.5 \\ \delta A = \frac{\|\hat{A} - A\|_1}{\|\hat{A}\|_1} = \frac{\max(|\varepsilon_1| + |\varepsilon_3|, |\varepsilon_2| + |\varepsilon_4|)}{14 + \varepsilon_2 - \varepsilon_4} \end{cases}$$

therefore:

$$\delta A^{-1} \leq \frac{3.5 \cdot \max(|\varepsilon_1| + |\varepsilon_3|, |\varepsilon_2| + |\varepsilon_4|)}{14 + \varepsilon_2 - \varepsilon_4 - 3.5 \cdot \max(|\varepsilon_1| + |\varepsilon_3|, |\varepsilon_2| + |\varepsilon_4|)} \leq \frac{3.5 \cdot (0.01 + 0.01)}{14 - 0.01 - 0.01 - 3.5 \cdot (0.01 + 0.01)} \approx 0.0050$$

### Answer

The approximate inverse matrix:

$$\hat{A}^{-1} = \frac{1}{60} \cdot \begin{pmatrix} -6 & -8 \\ 9 & 2 \end{pmatrix}$$

The approximation error:

$$\delta A^{-1} \leq 0.0050$$

## Task 5

### Description

Use simple iteration method for finding the solution of the given linear system

$$\begin{cases} 26x+9y+8z = 9 \\ 6x+26y+3z = 1 \\ x+y+21z = 7 \end{cases}$$

Determine the iteration number after which the approximation error for each coordinate does not exceed 0.01 and find the corresponding approximate solution. Start with  $x_0 = [0; 0; 0]^T$

### Solution

To use this method, additional matrix  $P$  and vector  $c$  are required. Let us use one of the methods from lectures:

$$A = \begin{pmatrix} 26 & 9 & 8 \\ 6 & 26 & 3 \\ 1 & 1 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 6 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 26 & 0 & 0 \\ 0 & 26 & 0 \\ 0 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 0 & 9 & 8 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix} = L+D+R \quad \rightarrow$$
$$\rightarrow \begin{cases} P = -D^{-1} \cdot (L+R) = \begin{pmatrix} 0 & -9/26 & -8/26 \\ -3/13 & 0 & -3/26 \\ -1/21 & -1/21 & 0 \end{pmatrix} \\ c = D^{-1} \cdot b = \begin{pmatrix} 9/26 \\ 1/26 \\ 1/3 \end{pmatrix} \end{cases}$$

So, according to the chosen method:

$$\begin{cases} x_0, P, c \text{ are set} \\ x_i = P \cdot x_{i-1} + c \quad i > 0 \end{cases}$$

Therefore, the required approximate solution is reached at the  $i=5$  iteration:

$$x^* = \begin{pmatrix} 0.2687302 \\ -0.05955676 \\ 0.32379103 \end{pmatrix}$$

### Answer

The required iteration number:

$$i = 5$$

The required approximate solution:

$$x^* = \begin{pmatrix} 0.2687302 \\ -0.05955676 \\ 0.32379103 \end{pmatrix}$$



## Task 6

### Description

Find the most influential vertex in the graph using the PageRank algorithm with damping factor  $= 1-\beta = 0.85$ , where the graph adjacency matrix is defined as follows

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

### Solution

To use this method, matrix  $Q$  and initial vector  $x_0$  are required. To build  $Q$ , probability matrix  $P$  should be calculated first:

$$P = \begin{pmatrix} 0 & 1/2 & 1/4 & 1/3 & 1/5 \\ 1 & 1/2 & 1/4 & 1/3 & 1/5 \\ 0 & 0 & 1/4 & 0 & 1/5 \\ 0 & 0 & 0 & 1/3 & 1/5 \\ 0 & 0 & 1/4 & 0 & 1/5 \end{pmatrix}$$

Therefore:

$$Q = (1-\beta) \cdot P + \frac{\beta}{n} \cdot \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{pmatrix} = \frac{1}{1200} \cdot \begin{pmatrix} 36 & 546 & 291 & 376 & 240 \\ 1056 & 546 & 291 & 376 & 240 \\ 36 & 36 & 291 & 36 & 240 \\ 36 & 36 & 36 & 376 & 240 \\ 36 & 36 & 291 & 36 & 240 \end{pmatrix}$$
$$x_0 = \frac{1}{n} \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{pmatrix}$$

So, according to PageRank algorithm:

$$\begin{cases} x_i = Q \cdot x_{i-1} & i > 0 \\ x^* = \lim_{n \rightarrow \infty} x_n \end{cases}$$

Therefore, the distribution of vertex influence is approximately equal to (with approximation error 0.001):

$$x^* \approx \begin{pmatrix} 0.298 \\ 0.551 \\ 0.049 \\ 0.053 \\ 0.049 \end{pmatrix}$$

### Answer

The most influential vertex is:

vertex 2

## Task 7

### Description

Find the value  $f(A)$  of the function  $f(l) = \ln(l+6)$ , where

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 4 & -8 & 12 \\ 4 & -14 & 18 \end{pmatrix}$$

### Solution

For the sake of finding  $f(A)$  let us use Lagrange-Silvester polynomial  $r(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2$ , which requires calculating eigenvalues:

$$\det(A - \lambda \cdot I) = -\lambda^3 + 10 \cdot \lambda^2 - 28 \cdot \lambda + 24 = -(x-2)^2 \cdot (x-6) \Rightarrow \begin{cases} \lambda_1 = 6 & k_1 = 1 \\ \lambda_2 = 2 & k_2 = 2 \end{cases}$$

Therefore:

$$\begin{aligned} \begin{cases} r(\lambda_1) = f(\lambda_1) \\ r(\lambda_2) = f(\lambda_2) \\ r'(\lambda_2) = f'(\lambda_2) \end{cases} &\rightarrow \begin{cases} a_0 + 6a_1 + 36a_2 = \ln(12) \\ a_0 + 2a_1 + 4a_2 = \ln(8) \\ a_1 + 4a_2 = \frac{1}{8} \end{cases} \rightarrow \begin{cases} a_0 + 6a_1 + 36a_2 = \ln(3) + 2 \cdot \ln(2) \\ a_0 + 2a_1 + 4a_2 = 3 \cdot \ln(2) \\ a_1 = \frac{1}{8} - 4a_2 \end{cases} \rightarrow \\ &\rightarrow \begin{cases} a_0 = 3 \cdot \ln(2) - 2a_1 - 4a_2 \\ 4a_1 + 32a_2 = \ln(3) - \ln(2) \\ a_1 = \frac{1}{8} - 4a_2 \end{cases} \rightarrow \begin{cases} a_0 = 3 \cdot \ln(2) - 2a_1 - 4a_2 \\ a_1 = \frac{1}{8} - 4a_2 \\ \frac{1}{2} - 16a_2 + 32a_2 = \ln(3) - \ln(2) \end{cases} \rightarrow \\ &\rightarrow \begin{cases} a_0 = 3 \cdot \ln(2) - 2a_1 - 4a_2 \\ a_1 = \frac{1}{8} - 4a_2 \\ a_2 = \frac{2 \cdot \ln(3) - 2 \cdot \ln(2) - 1}{32} \end{cases} \rightarrow \begin{cases} a_0 = 3 \cdot \ln(2) - 2a_1 - 4a_2 \\ a_1 = \frac{1 + \ln(2) - \ln(3)}{4} \\ a_2 = \frac{2 \cdot \ln(3) - 2 \cdot \ln(2) - 1}{32} \end{cases} \rightarrow \begin{cases} a_0 = \frac{22 \cdot \ln(2) + 2 \cdot \ln(3) - 3}{8} \\ a_1 = \frac{1 + \ln(2) - \ln(3)}{4} \\ a_2 = \frac{2 \cdot \ln(3) - 2 \cdot \ln(2) - 1}{32} \end{cases} \end{aligned}$$

Thus:

$$f(A) = r(A) = a_0 I + a_1 A + a_2 A^2 = \frac{1}{8} \begin{pmatrix} -2 + 24 \cdot \ln(2) & -6 + 14 \cdot \ln(2) + 10 \cdot \ln(3) & -1 + 30 \cdot \ln(2) - 6 \cdot \ln(3) \\ 1 + 22 \cdot \ln(2) + 2 \cdot \ln(3) & 6 + 56 \cdot \ln(2) - 32 \cdot \ln(3) & -7 - 10 \cdot \ln(2) + 34 \cdot \ln(3) \\ 1 + 22 \cdot \ln(2) + 2 \cdot \ln(3) & 3 + 62 \cdot \ln(2) - 38 \cdot \ln(3) & -4 - 16 \cdot \ln(2) + 40 \cdot \ln(3) \end{pmatrix}$$

### Answer

$$f(A) = \frac{1}{8} \begin{pmatrix} -2 + 24 \cdot \ln(2) & -6 + 14 \cdot \ln(2) + 10 \cdot \ln(3) & -1 + 30 \cdot \ln(2) - 6 \cdot \ln(3) \\ 1 + 22 \cdot \ln(2) + 2 \cdot \ln(3) & 6 + 56 \cdot \ln(2) - 32 \cdot \ln(3) & -7 - 10 \cdot \ln(2) + 34 \cdot \ln(3) \\ 1 + 22 \cdot \ln(2) + 2 \cdot \ln(3) & 3 + 62 \cdot \ln(2) - 38 \cdot \ln(3) & -4 - 16 \cdot \ln(2) + 40 \cdot \ln(3) \end{pmatrix}$$

# Calculations

## Task 1

SVD  $\{ \{-38, -102, -48, -73\}, \{-22, 120, 12, 58\}, \{32, -42, 12, -62\} \}$ 
=

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**Input**

singular value decomposition

$$\begin{pmatrix} -38 & -102 & -48 & -73 \\ -22 & 120 & 12 & 58 \\ 32 & -42 & 12 & -62 \end{pmatrix}$$

**Result** Approximate forms

$M = U \cdot \Sigma \cdot V^{\dagger}$

where

$$M = \begin{pmatrix} -38 & -102 & -48 & -73 \\ -22 & 120 & 12 & 58 \\ 32 & -42 & 12 & -62 \end{pmatrix}$$

$$U = \begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 198 & 0 & 0 & 0 \\ 0 & 66 & 0 & 0 \\ 0 & 0 & 33 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} 0 & \frac{9}{11} & -\frac{2}{11} & -\frac{6}{11} \\ -\frac{9}{11} & 0 & \frac{6}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{6}{11} & 0 & \frac{9}{11} \\ -\frac{6}{11} & -\frac{2}{11} & -\frac{9}{11} & 0 \end{pmatrix}$$

$m^{\dagger}$  gives the conjugate transpose of  $m$

Picture calculation.1.1: SVD of  $A$

$\{ \{2/3, -2/3, 1/3\}, \{-2/3, -1/3, 2/3\}, \{1/3, 2/3, 2/3\} \} * \{ \{198, 0, 0, 0\}, \{0, 66, 0, 0\}, \{0, 0, 0, 0\} \}$ 
=

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**Input**

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

$$\begin{pmatrix} 198 & 0 & 0 & 0 \\ 0 & 66 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

**Result** Step-by-step solution

$$\begin{pmatrix} 132 & -44 & 0 & 0 \\ -132 & -22 & 0 & 0 \\ 66 & 44 & 0 & 0 \end{pmatrix}$$

Picture calculation.1.2:  $A_1$  (part 1)

\* (transpose {{0, 9/11, -2/11, -6/11}, {-9/11, 0, 6/11, -2/11}, {-2/11, 6/11, 0, 9/11}, {-6/11, -2/11, -9/11, 0}}) =

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Input

$$\begin{pmatrix} 132 & -44 & 0 & 0 \\ -132 & -22 & 0 & 0 \\ 66 & 44 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & \frac{9}{11} & -\frac{2}{11} & -\frac{6}{11} \\ -\frac{9}{11} & 0 & \frac{6}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{6}{11} & 0 & \frac{9}{11} \\ -\frac{6}{11} & -\frac{2}{11} & -\frac{9}{11} & 0 \end{pmatrix}^T$$

$m^T$  gives the transpose of  $m$

Result

$$2 \begin{pmatrix} -18 & -54 & -24 & -32 \\ -9 & 54 & 6 & 38 \\ 18 & -27 & 6 & -22 \end{pmatrix}$$

Picture calculation.1.3:  $A_1$  (part 2)

## Task 2

SVD  $\{\{3.9, 0.03\}, \{-4.11, -7.91\}\}$ 
=

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**Input**

singular value decomposition  $\begin{pmatrix} 3.9 & 0.03 \\ -4.11 & -7.91 \end{pmatrix}$

**Result**

$M = U \cdot \Sigma \cdot V^\dagger$

where

$M = \begin{pmatrix} 3.9 & 0.03 \\ -4.11 & -7.91 \end{pmatrix}$

$U = \begin{pmatrix} 0.232218 & 0.972664 \\ -0.972664 & 0.232218 \end{pmatrix}$

$\Sigma = \begin{pmatrix} 9.12928 & 0 \\ 0 & 3.36562 \end{pmatrix}$

$V = \begin{pmatrix} 0.537096 & 0.843521 \\ 0.843521 & -0.537096 \end{pmatrix}$

$m^\dagger$  gives the conjugate transpose of  $m$

Picture calculation.2.1: SVD of  $A$

SVD  $\{\{0.1, -0.03\}, \{0.11, -0.09\}\}$ 
=

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**Input**

singular value decomposition  $\begin{pmatrix} 0.1 & -0.03 \\ 0.11 & -0.09 \end{pmatrix}$

**Result**

$M = U \cdot \Sigma \cdot V^\dagger$

where

$M = \begin{pmatrix} 0.1 & -0.03 \\ 0.11 & -0.09 \end{pmatrix}$

$U = \begin{pmatrix} -0.582492 & 0.812837 \\ -0.812837 & -0.582492 \end{pmatrix}$

$\Sigma = \begin{pmatrix} 0.173256 & 0 \\ 0 & 0.0328993 \end{pmatrix}$

$V = \begin{pmatrix} -0.852272 & 0.523099 \\ 0.523099 & 0.852272 \end{pmatrix}$

$m^\dagger$  gives the conjugate transpose of  $m$

Picture calculation.2.2: SVD of  $(\hat{A} - A)$

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Input

$$\begin{pmatrix} 3.9 & 0.03 \\ -4.11 & -7.91 \end{pmatrix}^{-1} \text{ (matrix inverse)}$$

Result

$$\begin{pmatrix} 0.257439 & 0.000976381 \\ -0.133764 & -0.12693 \end{pmatrix}$$

Step-by-step solution

Picture calculation.2.3: Inverse of  $A$

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Input

singular value decomposition

$$\begin{pmatrix} 0.2574 & -0.1338 \\ 0.001 & -0.1269 \end{pmatrix}$$

Result

$M = U \cdot \Sigma \cdot V^T$

where

$$M = \begin{pmatrix} 0.2574 & -0.1338 \\ 0.001 & -0.1269 \end{pmatrix}$$

$$U = \begin{pmatrix} -0.972644 & 0.232302 \\ -0.232302 & -0.972644 \end{pmatrix}$$







$$\Sigma = \begin{pmatrix} 0.297109 & 0 \\ 0 & 0.109489 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.84343 & 0.53724 \\ 0.53724 & 0.84343 \end{pmatrix}$$

Picture calculation.2.4: SVD of  $A^{-1}$

### Task 3

eigenvalues  $(\{\{4a, 4b\}, \{0, a\}\} * \{\{4a, 0\}, \{b, a\}\})$

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Input

eigenvalues  $\begin{pmatrix} 4a & 4b \\ 0 & a \end{pmatrix} \cdot \begin{pmatrix} 4a & 0 \\ b & a \end{pmatrix}$

Results Step-by-step solution

$\lambda_1 = \frac{1}{2} \left( 17a^2 - \sqrt{225a^4 + 136a^2b^2 + 16b^4} + 4b^2 \right)$

$\lambda_2 = \frac{1}{2} \left( 17a^2 + \sqrt{225a^4 + 136a^2b^2 + 16b^4} + 4b^2 \right)$

Picture calculation.3.1: Eigenvalues of  $(\hat{A} - A)$

eigenvalues  $(\{\{-32 + 4a, -16 + 4b\}, \{-5, 4 + a\}\} * \{\{-32 + 4a, -5\}, \{-16 + 4b, 4 + a\}\})$

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Input

eigenvalues  $\begin{pmatrix} -32 + 4a & -16 + 4b \\ -5 & 4 + a \end{pmatrix} \cdot \begin{pmatrix} -32 + 4a & -5 \\ -16 + 4b & 4 + a \end{pmatrix}$

Results Step-by-step solution

$\lambda_1 =$

$\frac{1}{2} \left( 17a^2 - \sqrt{(225a^4 - 7920a^3 + 544a^2b^2 - 4992a^2b + 112050a^2 - 7936ab^2 + 66048ab - 681840a + 256b^4 - 4096b^3 + 57056b^2 - 304896b + 1571985) - 248a + 16b^2 - 128b + 1321} \right)$

$\lambda_2 =$

$\frac{1}{2} \left( 17a^2 + \sqrt{(225a^4 - 7920a^3 + 544a^2b^2 - 4992a^2b + 112050a^2 - 7936ab^2 + 66048ab - 681840a + 256b^4 - 4096b^3 + 57056b^2 - 304896b + 1571985) - 248a + 16b^2 - 128b + 1321} \right)$

Picture calculation.3.2: Eigenvalues of  $A$

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Input

$$\begin{pmatrix} -32 + 4a & -16 + 4b \\ -5 & 4 + a \end{pmatrix}^{-1} \text{ (matrix inverse)}$$

Result

☒ Step-by-step solution

$$\frac{1}{4(a^2 - 4a + 5b - 52)} \begin{pmatrix} a + 4 & 4(4 - b) \\ 5 & -4(8 - a) \end{pmatrix}$$

Picture calculation.3.3: Inverse of  $A$

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Input

singular value decomposition

$$\begin{pmatrix} -32 & -16 \\ -5 & 4 \end{pmatrix}$$

Result

$$M = U \cdot \Sigma \cdot V^\dagger$$

where

$$M = \begin{pmatrix} -32 & -16 \\ -5 & 4 \end{pmatrix}$$

$$U = \begin{pmatrix} -0.997047 & -0.0767947 \\ -0.0767947 & 0.997047 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 35.8803 & 0 \\ 0 & 5.79706 \end{pmatrix}$$

$$V = \begin{pmatrix} 0.899923 & -0.436049 \\ 0.436049 & 0.899923 \end{pmatrix}$$

$m^\dagger$  gives the conjugate transpose of  $m$

Picture calculation.3.4: SVD of  $\hat{A}$



SVD  $\{(-4/208, -16/208), (-5/208, 32/208)\}$



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Input

singular value decomposition

$$\begin{pmatrix} -\frac{4}{208} & -\frac{16}{208} \\ -\frac{5}{208} & \frac{32}{208} \end{pmatrix}$$

Result

Exact forms

$$M = U \cdot \Sigma \cdot V^\dagger$$

where

$$M = \begin{pmatrix} -0.0192308 & -0.0769231 \\ -0.0240385 & 0.153846 \end{pmatrix}$$

$$U = \begin{pmatrix} -0.436049 & -0.899923 \\ 0.899923 & -0.436049 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 0.172501 & 0 \\ 0 & 0.0278705 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.0767947 & 0.997047 \\ 0.997047 & 0.0767947 \end{pmatrix}$$

$m^\dagger$  gives the conjugate transpose of  $m$

Picture calculation.3.5: SVD of  $\widehat{A}^{-1}$

## Task 4

inverse  $\{\{2, 8\}, \{-9, -6\}\}$

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Input

$\begin{pmatrix} 2 & 8 \\ -9 & -6 \end{pmatrix}^{-1}$  (matrix inverse)

Result

$\frac{1}{60} \begin{pmatrix} -6 & -8 \\ 9 & 2 \end{pmatrix}$

Expanded form

$\begin{pmatrix} -\frac{1}{10} & -\frac{2}{15} \\ \frac{3}{20} & \frac{1}{30} \end{pmatrix}$

Picture calculation.4.1:  $\hat{A}$

## Task 5

$\{ \{1/26, 0, 0\}, \{0, 1/26, 0\}, \{0, 0, 1/21\} \} * \{ \{0, 9, 8\}, \{6, 0, 3\}, \{1, 1, 0\} \}$

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Input

$$\begin{pmatrix} \frac{1}{26} & 0 & 0 \\ 0 & \frac{1}{26} & 0 \\ 0 & 0 & \frac{1}{21} \end{pmatrix} \cdot \begin{pmatrix} 0 & 9 & 8 \\ 6 & 0 & 3 \\ 1 & 1 & 0 \end{pmatrix}$$

Result

[Approximate forms](#)
[Step-by-step solution](#)

$$\begin{pmatrix} 0 & \frac{9}{26} & \frac{4}{13} \\ \frac{3}{13} & 0 & \frac{3}{26} \\ \frac{1}{21} & \frac{1}{21} & 0 \end{pmatrix}$$

Picture calculation.5.1: Matrix  $P$

## Task 5

```
In [2]: P = np.array([[0, -9/26, -4/13], [-3/13, 0, -3/26], [-1/21, -1/21, 0]])
c = np.array([[9/26], [1/26], [1/3]])
x0 = np.array([[0], [0], [0]])
```

```
In [3]: xi = P.dot(x0) + c
i = 1
print('x_0: ', x0)
print('x_1', xi)
while np.any(np.abs(xi - x0) > 0.01):
    x0 = xi
    xi = P.dot(x0) + c
    i += 1
    print('x_', i, ': ', xi, sep='')
print('delta: ', xi - x0)
```

```
x_0: [[0]
[0]
[0]]
x_1 [[0.34615385]
[0.03846154]
[0.33333333]]
x_2: [[ 0.23027613]
[-0.07988166]
[ 0.31501832]]
x_3: [[ 0.27687648]
[-0.05102737]
[ 0.32617169]]
x_4: [[ 0.26345665]
[-0.06306823]
[ 0.32257861]]
x_5: [[ 0.2687302 ]
[-0.05955676]
[ 0.32379103]]
delta: [[0.00527355]
[0.00351147]
[0.00121241]]
```

Picture calculation.5.2: Approximate solution and iteration number

Task 6

85/100\*{{0,1/2,1/4,1/3,1/5},{1,1/2,1/4,1/3,1/5},{0,0,1/4,0,1/5},{0,0,0,1/3,1/5},{0,0,1/4,0,1/5}}+3/100\*{{1,1,1,1,1},{1,1,1,1,1},{1,1,1,1,1},{1,1,1,1,1},{1,1,1,1,1}}

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Input

$$\frac{85}{100} \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{3} & \frac{1}{5} \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{3} & \frac{1}{5} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{5} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{5} \end{pmatrix} + \frac{3}{100} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Result

Step-by-step solution

$$\begin{pmatrix} \frac{3}{100} & \frac{91}{200} & \frac{97}{400} & \frac{47}{150} & \frac{1}{5} \\ \frac{22}{25} & \frac{91}{200} & \frac{97}{400} & \frac{47}{150} & \frac{1}{5} \\ \frac{3}{100} & \frac{3}{100} & \frac{97}{400} & \frac{3}{100} & \frac{1}{5} \\ \frac{3}{100} & \frac{3}{100} & \frac{3}{100} & \frac{47}{150} & \frac{1}{5} \\ \frac{3}{100} & \frac{3}{100} & \frac{97}{400} & \frac{3}{100} & \frac{1}{5} \end{pmatrix}$$

Picture calculation.6.1: Matrix  $Q$

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## Task 6

```
In [4]: Q = 1 / 1200 * np.array([[36, 546, 291, 376, 240], [1056, 546, 291, 376, 240],
                                [36, 36, 291, 36, 240], [36, 36, 36, 376, 240],
                                [36, 36, 291, 36, 240]])
x0 = np.array([[1/5], [1/5], [1/5], [1/5], [1/5]])
```

```
In [5]: print('x_0: ', x0)
xi = Q.dot(x0)
i = 1
print('x_1', xi)
while np.any(np.abs(xi - x0) > 0.001):
    x0 = xi
    xi = Q.dot(x0)
    i += 1
    print('x_', i, ': ', xi, sep='')
print('delta: ', xi - x0)
```

```
x_0: [[0.2]
      [0.2]
      [0.2]
      [0.2]
      [0.2]]
x_1 [[0.24816667]
     [0.41816667]
     [0.1065    ]
     [0.12066667]
     [0.1065    ]]
x_2: [[0.28264597]
     [0.49358764]
     [0.07073625]
     [0.08229389]
     [0.07073625]]
x_3: [[0.29014796]
     [0.53039704]
     [0.05705662]
     [0.06534176]
     [0.05705662]]
x_4: [[0.2957564 ]
     [0.54238217]
     [0.05182416]
     [0.05821312]
     [0.05182416]]
```

Picture calculation.6.2: Distribution of vertex influence (part 1)

```
x_5: [[0.29682888]
     [0.54822182]
     [0.04982274]
     [0.05530383]
     [0.04982274]]
x_6: [[0.29772089]
     [0.55002543]
     [0.0490572 ]
     [0.05413928]
     [0.0490572 ]]
x_7: [[0.29786465]
     [0.55092741]
     [0.04876438]
     [0.05367919]
     [0.04876438]]
delta: [[ 0.00014376]
        [ 0.00090197]
        [-0.00029282]
        [-0.0004601 ]
        [-0.00029282]]
```

Picture calculation.6.3: Distribution of vertex influence (part 2)

## Task 7

$$\begin{pmatrix} 0 & 1 & -2 \\ 4 & -8 & 12 \\ 4 & -14 & 18 \end{pmatrix}$$

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**Input**

$$\begin{pmatrix} 0 & 1 & -2 \\ 4 & -8 & 12 \\ 4 & -14 & 18 \end{pmatrix}$$

**Dimensions**

3 (rows) × 3 (columns)

**Matrix plot**

**Trace** ☒ Step-by-step solution

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**Determinant** ☒ Step-by-step solution

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**Inverse**  ☒ Step-by-step solution

$$\frac{1}{12} \begin{pmatrix} 12 & 5 & -2 \\ -12 & 4 & -4 \\ -12 & 2 & -2 \end{pmatrix}$$

**Characteristic polynomial** ☒ Step-by-step solution

$-\lambda^3 + 10\lambda^2 - 28\lambda + 24$

[Characteristic polynomial »](#)

Picture calculation.7.1: Eigenvalues of  $A$

$$\{0, 1, -2\}, \{4, -8, 12\}, \{4, -14, 18\}^2$$

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**Input**

$$\begin{pmatrix} 0 & 1 & -2 \\ 4 & -8 & 12 \\ 4 & -14 & 18 \end{pmatrix}^2$$

**Result**

$$4 \begin{pmatrix} -1 & 5 & -6 \\ 4 & -25 & 28 \\ 4 & -34 & 37 \end{pmatrix}$$

Picture calculation.7.2: Eigenvalues of  $A^2$

$$(2 * \ln(3) - 2 * \ln(2) - 1) / 8 * \{-1, 5, -6\}, \{4, -25, 28\}, \{4, -34, 37\}\} + (1 + \ln(2) - \ln(3)) / 4 * \{0, 1, -2\}, \{4, -8, 12\} =$$

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Input

$$\left(\frac{1}{8} (2 \log(3) - 2 \log(2) - 1)\right) \begin{pmatrix} -1 & 5 & -6 \\ 4 & -25 & 28 \\ 4 & -34 & 37 \end{pmatrix} + \left(\frac{1}{4} (1 + \log(2) - \log(3))\right) \begin{pmatrix} 0 & 1 & -2 \\ 4 & -8 & 12 \\ 4 & -14 & 18 \end{pmatrix} + \frac{1}{8} (22 \log(2) + 2 \log(3) - 3)$$

$\log(x)$  is the natural logarithm

Result

$$\left( \begin{array}{ccc} \frac{1}{8} (1 + 2 \log(2) - 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) & \frac{1}{8} (1 + \log(2) - \log(3)) + \frac{3}{8} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) & \frac{1}{8} (-1 - \log(2) + \log(3)) - \frac{3}{8} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) \\ 1 + \log(2) - \log(3) + \frac{1}{2} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) & -2(1 + \log(2) - \log(3)) - \frac{23}{8} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) & 3(1 + \log(2) - \log(3)) + \frac{7}{2} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) \\ 1 + \log(2) - \log(3) + \frac{1}{2} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) & -\frac{7}{2} (1 + \log(2) - \log(3)) - \frac{17}{4} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) & \frac{3}{2} (1 + \log(2) - \log(3)) + \frac{23}{8} (-1 - 2 \log(2) + 2 \log(3)) + \frac{1}{8} (-3 + 22 \log(2) + 2 \log(3)) \end{array} \right)$$

Expanded form

☒ Step-by-step solution

$$\left( \begin{array}{ccc} 3 \log(2) - \frac{1}{4} & -\frac{3}{4} + \frac{7 \log(2)}{4} + \frac{5 \log(3)}{4} & -\frac{1}{8} + \frac{15 \log(2)}{4} - \frac{3 \log(3)}{4} \\ \frac{1}{8} + \frac{11 \log(2)}{4} + \frac{\log(3)}{4} & \frac{3}{4} + 7 \log(2) - 4 \log(3) & -\frac{7}{8} - \frac{5 \log(2)}{4} + \frac{17 \log(3)}{4} \\ \frac{1}{8} + \frac{11 \log(2)}{4} + \frac{\log(3)}{4} & \frac{3}{8} + \frac{31 \log(2)}{4} - \frac{19 \log(3)}{4} & -\frac{1}{2} - 2 \log(2) + 5 \log(3) \end{array} \right)$$

Picture calculation.7.3: Result  $f(A)$