Homework 2
"Applied Methods of Linear Algebra"
variant 50

Tatarinov Nikita BSE196

by 13.12.2021

Task 1

Description

Find the best approximation matrix A_1 of rank 2 of the matrix A in the norm $||\cdot||_2$ and find $||A-A_1||_2$, where

$$A = \begin{pmatrix} -38 & -102 & -48 & -73 \\ -22 & 120 & 12 & 58 \\ 32 & -42 & 12 & -62 \end{pmatrix}$$

Solution

As far as the norm $||\cdot||_2$ is unitary invariant, according to Eckart-Young-Mirski theorem:

$$A = U\cdot\Sigma\cdot V^* \quad \Rightarrow \quad A_1 = U\cdot\Sigma_{(r)}\cdot V^* \text{ - best low rank approximation}$$

Therefore:

$$U = \begin{pmatrix} {}^2/_3 & -{}^2/_3 & {}^1/_3 \\ -{}^2/_3 & -{}^1/_3 & {}^2/_3 \\ {}^1/_3 & {}^2/_3 & {}^2/_3 \end{pmatrix}, \Sigma = \begin{pmatrix} 198 & 0 & 0 & 0 \\ 0 & 66 & 0 & 0 \\ 0 & 0 & 33 & 0 \end{pmatrix}, V = \begin{pmatrix} 0 & {}^9/_{11} & -{}^2/_{11} & -{}^6/_{11} \\ -{}^9/_{11} & 0 & {}^6/_{11} & -{}^2/_{11} \\ -{}^2/_{11} & {}^6/_{11} & 0 & {}^9/_{11} \\ -{}^6/_{11} & -{}^2/_{11} & -{}^9/_{11} & 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \quad \Sigma_{(r=2)} = \begin{pmatrix} 198 & 0 & 0 & 0 \\ 0 & 66 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad A_1 = \begin{pmatrix} -36 & -108 & -48 & -64 \\ -18 & 108 & 12 & 76 \\ 36 & -54 & 12 & -44 \end{pmatrix} \quad \Rightarrow$$

$$\Rightarrow \quad ||A-A_1||_2 = \left|\left|U\cdot\left(\Sigma-\Sigma_{(r=2)}\right)\cdot V^*\right|\right|_2 \overset{unitary\,invariant}{=} ||\Sigma-\Sigma_{(r=2)}||_2 =$$

$$= \left\|\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 33 & 0 \end{matrix}\right\|_2 = 33$$

Answer

The best approximation matrix is:

$$A_1 = \begin{pmatrix} -36 & -108 & -48 & -64 \\ -18 & 108 & 12 & 76 \\ 36 & -54 & 12 & -44 \end{pmatrix}$$

$$||A-A_1||_2 = 33$$

Task 2

Description

Estimate the relative error of the approximate solution $(1;1)$ of the system $AX = b$ in the norms $\left|\cdot\right|_1$ and $\left|\cdot\right|_2$ using the condition number of the matrix A, where

$$A = \begin{pmatrix} 3.9 & 0.03 \\ -4.11 & -7.91 \end{pmatrix}, \quad b = \begin{pmatrix} 4.19 \\ -12.1 \end{pmatrix}$$

Solution

According to the lecture:

$$\begin{cases} \delta x \leqslant \dfrac{cond(A)}{1-cond(A)\cdot\delta A}\cdot\left(\delta A+\delta b\right) \\ cond(A) = ||A||\cdot||A^{-1}|| \\ \delta A = \dfrac{||\widehat{A}-A||}{||A||} \\ \delta b = \dfrac{||\widehat{b}-b||}{||b||} \end{cases}$$

To start with:

$$\widehat{A} = \begin{pmatrix} 4 & 0 \\ -4 & -8 \end{pmatrix}, \quad \widehat{b} = \begin{pmatrix} 4 \\ -12 \end{pmatrix}$$

where $(1;1)$ is an exact solution. Then:

$$\begin{cases} \delta b_1 = \dfrac{\left|\widehat{b}-b\right|_1}{\left|b\right|_1} = \dfrac{\sum\limits_i \left|\widehat{b}_i-b_i\right|}{\sum\limits_i \left|b_i\right|} = \dfrac{0.29}{16.29} \approx 0.0178 \\ \delta b_2 = \dfrac{\left|\widehat{b}-b\right|_2}{\left|b\right|_2} = \dfrac{\sqrt{\sum\limits_i \left(\widehat{b}_i-b_i\right)^2}}{\sqrt{\sum\limits_i {b_i}^2}} \approx \dfrac{0.2147}{12.8049} \approx 0.0168 \end{cases}$$

and:

$$\begin{cases} \delta A_1 = \dfrac{\left|\widehat{A}-A\right|_1}{\left|A\right|_1} = \dfrac{\max\limits_j\left(\sum\limits_i \left|\widehat{A}_{ij}-A_{ij}\right|\right)}{\max\limits_j\left(\sum\limits_i \left|A_{ij}\right|\right)} = \dfrac{0.21}{8.01} \approx 0.0262 \\ \delta A_2 = \dfrac{\left|\widehat{A}-A\right|_2}{\left|A\right|_2} = \dfrac{\max\limits_i\left(\sigma_i(\widehat{A}-A)\right)}{\max\limits_i\left(\sigma_i(A)\right)} \approx \dfrac{0.1733}{9.1293} \approx 0.0190 \end{cases}$$

Furthermore:

$$A^{-1} \approx \begin{pmatrix} 0.2574 & 0.0010 \\ -0.1338 & -0.1269 \end{pmatrix} \Rightarrow \begin{cases} cond_1(A) = \left|A\right|_1\cdot\left|A^{-1}\right|_1 \approx 8.01\cdot0.3912 \approx 3.1335 \\ cond_2(A) = \left|A\right|_2\cdot\left|A^{-1}\right|_2 \approx 9.1293\cdot0.2971 \approx 2.7123 \end{cases}$$

All in all:

$$\begin{cases} \delta x_1 \leqslant \dfrac{cond_1(A)}{1-cond_1(A)\cdot\delta A_1}\cdot\left(\delta A_1+\delta b_1\right) \approx 0.1502 \\ \delta x_2 \leqslant \dfrac{cond_2(A)}{1-cond_2(A)\cdot\delta A_2}\cdot\left(\delta A_2+\delta b_2\right) \approx 0.1024 \end{cases}$$

Answer

The relative error is:

$$\delta x \lesssim \begin{bmatrix} 0.15 & \text{in the norm } \left| \cdot \right|_1 \\ 0.10 & \text{in the norm } \left| \cdot \right|_2 \end{bmatrix}$$

Task 3

Description

Solve the system approximately and estimate the relative error of the solution in the norms $\left|\cdot\right|_1$, $\left|\cdot\right|_2$, $\left|\cdot\right|_\infty$:

$$\begin{cases} 4\cdot(-8+\varepsilon_1)\cdot x+4\cdot(-4+\varepsilon_2)\cdot y = -5+\varepsilon_3 \\ -5\cdot x+(4+\varepsilon_1)\cdot y = -1+\varepsilon_4 \end{cases}$$

where the unknown numbers ε_j satisfy the conditions $\left|\varepsilon_j\right| < 0.05$ for all $j=\overline{1,4}$.

Solution

The approximate system is:

$$\begin{cases} 4\cdot(-8)\cdot x+4\cdot(-4)\cdot y = -5 \\ -5\cdot x+4\cdot y = -1 \end{cases}$$

therefore:

$$\widehat{A} = \begin{pmatrix} -32 & -16 \\ -5 & 4 \end{pmatrix}, \quad \widehat{b} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}, \quad \widehat{x} = \begin{pmatrix} {}^{9}/_{52} \\ -{}^{7}/_{208} \end{pmatrix}$$

Next several steps are the same as in the previous task:

$$\begin{cases} \delta b_1 = \dfrac{\left|\widehat{b}-b\right|_1}{\left|b\right|_1} = \dfrac{\left|\varepsilon_3\right|+\left|\varepsilon_4\right|}{6-\varepsilon_3-\varepsilon_4} \leqslant \dfrac{0.05+0.05}{6-0.05-0.05} \approx 0.0169 \\ \delta b_2 = \dfrac{\left|\widehat{b}-b\right|_2}{\left|b\right|_2} = \dfrac{\sqrt{{\varepsilon_3}^2+{\varepsilon_4}^2}}{\sqrt{\left(5-\varepsilon_3\right)^2+\left(1-\varepsilon_4\right)^2}} \leqslant \dfrac{\sqrt{0.05^2+0.05^2}}{\sqrt{\left(5-0.05\right)^2+\left(1-0.05\right)^2}} \approx 0.0140 \\ \delta b_\infty = \dfrac{\left|\widehat{b}-b\right|_\infty}{\left|b\right|_\infty} = \dfrac{\max\limits_i\left(\left|\widehat{b}_i-b_i\right|\right)}{\max\limits_i\left(\left|b_i\right|\right)} = \dfrac{max\left(\left|\varepsilon_3\right|,\left|\varepsilon_4\right|\right)}{5-\varepsilon_3} \leqslant \dfrac{0.05}{5-0.05} \approx 0.0101 \end{cases}$$

and:

$$\begin{cases} \delta A_1 = \dfrac{\left|\widehat{A}-A\right|_1}{\left|A\right|_1} = \dfrac{max\left(4\cdot\left|\varepsilon_1\right|,\left|\varepsilon_1\right|+4\cdot\left|\varepsilon_2\right|\right)}{37-4\cdot\varepsilon_1} \leqslant \dfrac{0.05+4\cdot 0.05}{37-4\cdot 0.05} \approx 0.0068 \\ \delta A_2 = \dfrac{\left|\widehat{A}-A\right|_2}{\left|A\right|_2} \approx \dfrac{\sqrt{18\cdot{\varepsilon_1}^2+4\cdot{\varepsilon_2}^2}}{\sqrt{{}^{1321}/_{2}}} \leqslant \sqrt{\dfrac{18\cdot 0.05^2+4\cdot 0.05^2}{{}^{1321}/_{2}}} \approx 0.0091 \\ \delta A_\infty = \dfrac{\left|\widehat{A}-A\right|_\infty}{\left|A\right|_\infty} = \dfrac{\max\limits_i\left(\sum\limits_j\left|\widehat{A}_{ij}-A_{ij}\right|\right)}{\max\limits_i\left(\sum\limits_j\left|A_{ij}\right|\right)} = \dfrac{4\cdot\left|\varepsilon_1\right|+4\cdot\left|\varepsilon_2\right|}{48-4\cdot\varepsilon_1-4\cdot\varepsilon_2} \leqslant \dfrac{4\cdot 0.05+4\cdot 0.05}{48-4\cdot 0.05-4\cdot 0.05} \approx 0.0084 \end{cases}$$

and:

$$A^{-1} = \frac{1}{4\cdot\left({\varepsilon_1}^2-4\cdot\varepsilon_1+5\cdot\varepsilon_2-52\right)}\cdot\begin{pmatrix} \varepsilon_1+4 & 4\cdot(4-\varepsilon_2) \\ 5 & -4\cdot(8-\varepsilon_1) \end{pmatrix}$$

In case we use this inverse matrix, calculation of $cond(A)$ and, eventually, δx, is going to be too complicated (both calculating the norm and, moreover, maximization). Instead, it is better to use the following approximation:

$$cond(A) \approx cond(\widehat{A}) \quad \Rightarrow \quad \begin{cases} cond_1(A) = \left|\widehat{A}\right|_1\cdot\left|\widehat{A}^{-1}\right|_1 = 37\dfrac{3}{13} \approx= 8.5385 \\ cond_2(A) = \left|\widehat{A}\right|_2\cdot\left|\widehat{A}^{-1}\right|_2 \approx 33.88030.1725 \approx= 6.1832 \\ cond_\infty(A) = \left|\widehat{A}\right|_\infty\cdot\left|\widehat{A}^{-1}\right|_\infty = 48\cdot\dfrac{37}{208} \approx= 8.5385 \end{cases}$$

One more simplification in comparison with the previous task is the formula for δx. Let us suppose that the denominator $1 - cond(A) \cdot \delta A$ is equal to 1 due to a very low value of δA. Consequently:

$$\begin{cases} \delta x_1 \leqslant cond_1(A) \cdot (\delta A_1 + \delta b_1) \leqslant 8.5385 \cdot (0.0068 + 0.0169) \approx 0.2024 \\ \delta x_2 \leqslant cond_2(A) \cdot (\delta A_2 + \delta b_2) \leqslant 6.1832 \cdot (0.0091 + 0.0140) \approx 0.1428 \\ \delta x_\infty \leqslant cond_\infty(A) \cdot (\delta A_\infty + \delta b_\infty) \leqslant 8.5385 \cdot (0.0084 + 0.0101) \approx 0.1580 \end{cases}$$

Answer

The approximate solution is:

$$\widehat{x} = \begin{pmatrix} {}^{9}\!/_{52} \\ -{}^{7}\!/_{208} \end{pmatrix}$$

The relative error is:

$$\delta x \lesssim \begin{bmatrix} 0.20 & \text{in the norm } \left|\cdot\right|_1 \\ 0.14 & \text{in the norm } \left|\cdot\right|_2 \\ 0.15 & \text{in the norm } \left|\cdot\right|_\infty \end{bmatrix}$$

Task 4

Description

Find the approximate inverse matrix to the matrix A and evaluate the approximation error with respect to the uniform norm $\|\cdot\|_1$ if the elements of the matrix A are known with an absolute error of 0.01:

$$A \approx \begin{pmatrix} 2 & 8 \\ -9 & -6 \end{pmatrix}$$

Solution

For matrix

$$A = \begin{pmatrix} 2+\varepsilon_1 & 8+\varepsilon_2 \\ -9+\varepsilon_3 & -6+\varepsilon_4 \end{pmatrix}, \quad |\varepsilon_i| < 0.01 \ \forall i \in \overline{1,4}$$

the approximate matrix is:

$$\widehat{A} = \begin{pmatrix} 2 & 8 \\ -9 & -6 \end{pmatrix}$$

therefore, the approximate inverse matrix of A is:

$$\widehat{A}^{-1} = \frac{1}{60}\begin{pmatrix} -6 & -8 \\ 9 & 2 \end{pmatrix}$$

According to the lectures:

$$\delta A^{-1} \leqslant \frac{cond(A)\cdot\delta A}{1-cond(A)\cdot\delta A}$$

Having looked at the steps from the previous task:

$$\begin{cases} cond(A) \approx cond(\widehat{A}) = \|\widehat{A}\|_1 \cdot \|\widehat{A}^{-1}\|_1 = 14\cdot\dfrac{15}{60} = 3.5 \\ \delta A = \dfrac{\|\widehat{A}-A\|_1}{\|A\|} = \dfrac{max\Big(|\varepsilon_1|+|\varepsilon_3|, |\varepsilon_2|+|\varepsilon_4|\Big)}{14+\varepsilon_2-\varepsilon_4} \end{cases}$$

therefore:

$$\delta A^{-1} \leqslant \frac{3.5\cdot max\Big(|\varepsilon_1|+|\varepsilon_3|, |\varepsilon_2|+|\varepsilon_4|\Big)}{14+\varepsilon_2-\varepsilon_4-3.5\cdot max\Big(|\varepsilon_1|+|\varepsilon_3|, |\varepsilon_2|+|\varepsilon_4|\Big)} \leqslant \frac{3.5\cdot(0.01+0.01)}{14-0.01-0.01-3.5\cdot(0.01+0.01)} \approx 0.0050$$

Answer

The approximate inverse matrix:

$$\widehat{A}^{-1} = \frac{1}{60}\cdot\begin{pmatrix} -6 & -8 \\ 9 & 2 \end{pmatrix}$$

The approximation error:

$$\delta A^{-1} \leqslant 0.0050$$

Task 5

Description

Use simple iteration method for finding the solution of the given linear system

$$\begin{cases} 26x+9y+8z = 9 \\ 6x+26y+3z = 1 \\ x+y+21z = 7 \end{cases}$$

Determine the iteration number after which the approximation error for each coordinate does not exceed 0.01 and find the corresponding approximate solution. Start with $x_0 = [0;0;0]^T$

Solution

To use this method, additional matrix P and vector c are required. Let us use one of the methods from lectures:

$$A = \begin{pmatrix} 26 & 9 & 8 \\ 6 & 26 & 3 \\ 1 & 1 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 6 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 26 & 0 & 0 \\ 0 & 26 & 0 \\ 0 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 0 & 9 & 8 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix} = L+D+R \quad \rightarrow$$

$$\rightarrow \quad \begin{cases} P = -D^{-1}\cdot(L+R) = \begin{pmatrix} 0 & -{}^{9}/_{26} & -{}^{4}/_{13} \\ -{}^{3}/_{13} & 0 & -{}^{3}/_{26} \\ -{}^{1}/_{21} & -{}^{1}/_{21} & 0 \end{pmatrix} \\ c = D^{-1}\cdot b = \begin{pmatrix} {}^{9}/_{26} \\ {}^{1}/_{26} \\ {}^{1}/_{3} \end{pmatrix} \end{cases}$$

So, according to the chosen method:

$$\begin{cases} x_0, P, c \text{ are set} \\ x_i = P\cdot x_{i-1}+c \quad i>0 \end{cases}$$

Therefore, the required approximate solution is reached at the $i=5$ iteration:

$$x^* = \begin{pmatrix} 0.2687302 \\ -0.05955676 \\ 0.32379103 \end{pmatrix}$$

Answer

The required iteration number:

$$i = 5$$

The required approximate solution:

$$x^* = \begin{pmatrix} 0.2687302 \\ -0.05955676 \\ 0.32379103 \end{pmatrix}$$

Task 6

Description

Find the most influential vertex in the graph using the PageRank algorithm with damping factor $= 1{-}\beta = 0.85$, where the graph adjacency matrix is defined as follows

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Solution

To use this method, matrix Q and initial vector x_0 are required. To build Q, probability matrix P should be calculated first:

$$P = \begin{pmatrix} 0 & 1/2 & 1/4 & 1/3 & 1/5 \\ 1 & 1/2 & 1/4 & 1/3 & 1/5 \\ 0 & 0 & 1/4 & 0 & 1/5 \\ 0 & 0 & 0 & 1/3 & 1/5 \\ 0 & 0 & 1/4 & 0 & 1/5 \end{pmatrix}$$

Therefore:

$$Q = (1{-}\beta)\cdot P + \frac{\beta}{n}\cdot \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix} = \frac{1}{1200}\cdot \begin{pmatrix} 36 & 546 & 291 & 376 & 240 \\ 1056 & 546 & 291 & 376 & 240 \\ 36 & 36 & 291 & 36 & 240 \\ 36 & 36 & 36 & 376 & 240 \\ 36 & 36 & 291 & 36 & 240 \end{pmatrix}$$

$$x_0 = \frac{1}{n}\cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{pmatrix}$$

So, according to PageRank algorithm:

$$\begin{cases} x_i = Q\cdot x_{i-1} & i > 0 \\ x^* = \lim\limits_{n\to\infty} x_n \end{cases}$$

Therefore, the distribution of vertex influence is approximately equal to (with approximation error 0.001):

$$x* \approx \begin{pmatrix} 0.298 \\ 0.551 \\ 0.049 \\ 0.053 \\ 0.049 \end{pmatrix}$$

Answer

The most influential vertex is:

vertex 2

Task 7

Description

Find the value $f(A)$ of the function $f(l) = ln(l{+}6)$, where

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 4 & -8 & 12 \\ 4 & -14 & 18 \end{pmatrix}$$

Solution

For the sake of finding $f(A)$ let us use Lagrange-Silvester polynomial $r(x) = a_0{+}a_1{\cdot}x{+}a_2{\cdot}x^2$, which requires calculating eigenvalues:

$$det\Big(A-\lambda{\cdot}I\Big) = -\lambda^3+10{\cdot}\lambda^2-28{\cdot}\lambda+24 = -(x-2)^2{\cdot}(x-6) \quad \Rightarrow \begin{cases} \lambda_1 = 6 & k_1 = 1 \\ \lambda_2 = 2 & k_2 = 2 \end{cases}$$

Therefore:

$$\begin{cases} r(\lambda_1) = f(\lambda_1) \\ r(\lambda_2) = f(\lambda_2) \\ r'(\lambda_2) = f'(\lambda_2) \end{cases} \rightarrow \begin{cases} a_0+6a_1+36a_2 = ln(12) \\ a_0+2a_1+4a_2 = ln(8) \\ a_1+4a_2 = \dfrac{1}{8} \end{cases} \rightarrow \begin{cases} a_0+6a_1+36a_2 = ln(3)+2{\cdot}ln(2) \\ a_0+2a_1+4a_2 = 3{\cdot}ln(2) \\ a_1 = \dfrac{1}{8}-4a_2 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} a_0 = 3{\cdot}ln(2)-2a_1-4a_2 \\ 4a_1+32a_2 = ln(3)-ln(2) \\ a_1 = \dfrac{1}{8}-4a_2 \end{cases} \rightarrow \begin{cases} a_0 = 3{\cdot}ln(2)-2a_1-4a_2 \\ a_1 = \dfrac{1}{8}-4a_2 \\ \dfrac{1}{2}-16a_2+32a_2 = ln(3)-ln(2) \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} a_0 = 3{\cdot}ln(2)-2a_1-4a_2 \\ a_1 = \dfrac{1}{8}-4a_2 \\ a_2 = \dfrac{2{\cdot}ln(3)-2{\cdot}ln(2)-1}{32} \end{cases} \rightarrow \begin{cases} a_0 = 3{\cdot}ln(2)-2a_1-4a_2 \\ a_1 = \dfrac{1+ln(2)-ln(3)}{4} \\ a_2 = \dfrac{2{\cdot}ln(3)-2{\cdot}ln(2)-1}{32} \end{cases} \rightarrow \begin{cases} a_0 = \dfrac{22{\cdot}ln(2)+2{\cdot}ln(3)-3}{8} \\ a_1 = \dfrac{1+ln(2)-ln(3)}{4} \\ a_2 = \dfrac{2{\cdot}ln(3)-2{\cdot}ln(2)-1}{32} \end{cases}$$

Thus:

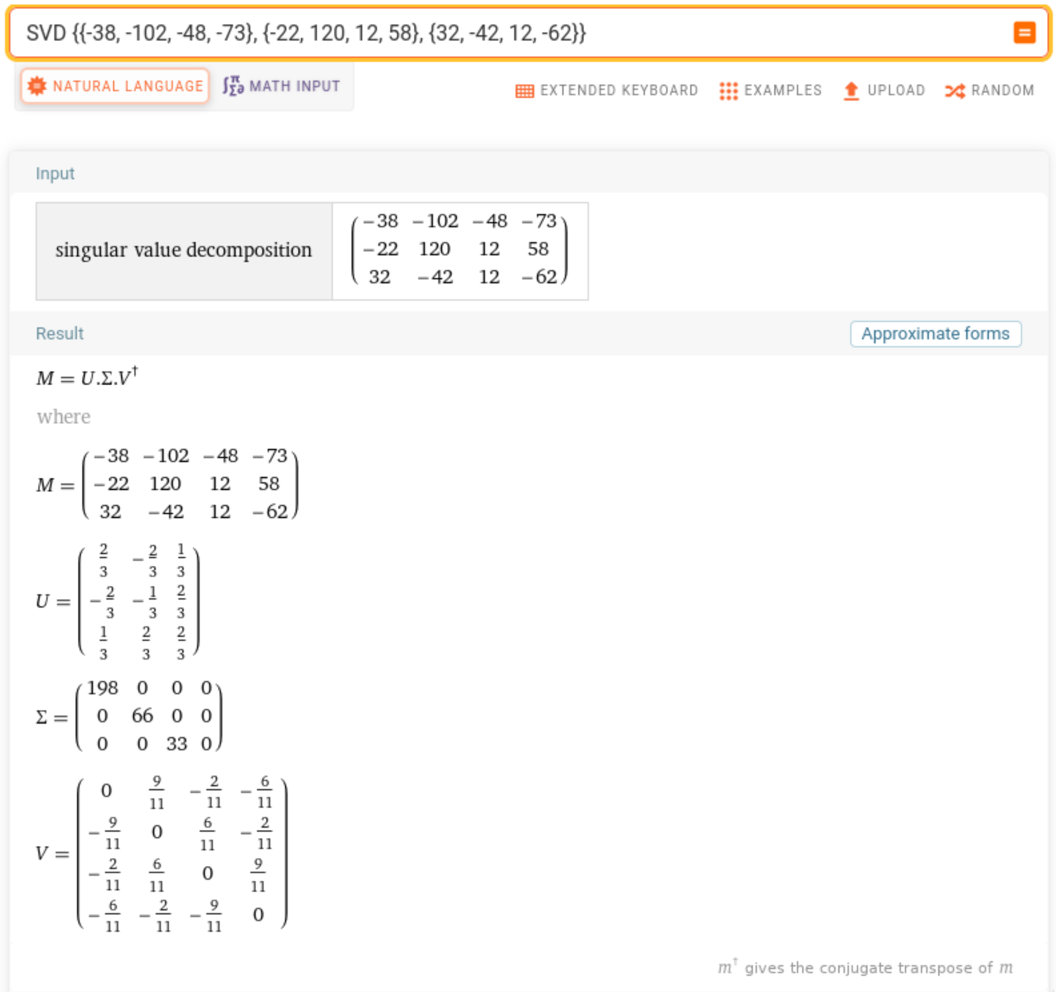
$$f(A) = r(A) = a_0{\cdot}I+a_1{\cdot}A+a_2{\cdot}A^2 = \frac{1}{8}\begin{pmatrix} -2+24{\cdot}ln(2) & -6+14{\cdot}ln(2)+10{\cdot}ln(3) & -1+30{\cdot}ln(2)-6{\cdot}ln(3) \\ 1+22{\cdot}ln(2)+2{\cdot}ln(3) & 6+56{\cdot}ln(2)-32{\cdot}ln(3) & -7-10{\cdot}ln(2)+34{\cdot}ln(3) \\ 1+22{\cdot}ln(2)+2{\cdot}ln(3) & 3+62{\cdot}ln(2)-38{\cdot}ln(3) & -4-16{\cdot}ln(2)+40{\cdot}ln(3) \end{pmatrix}$$

Answer

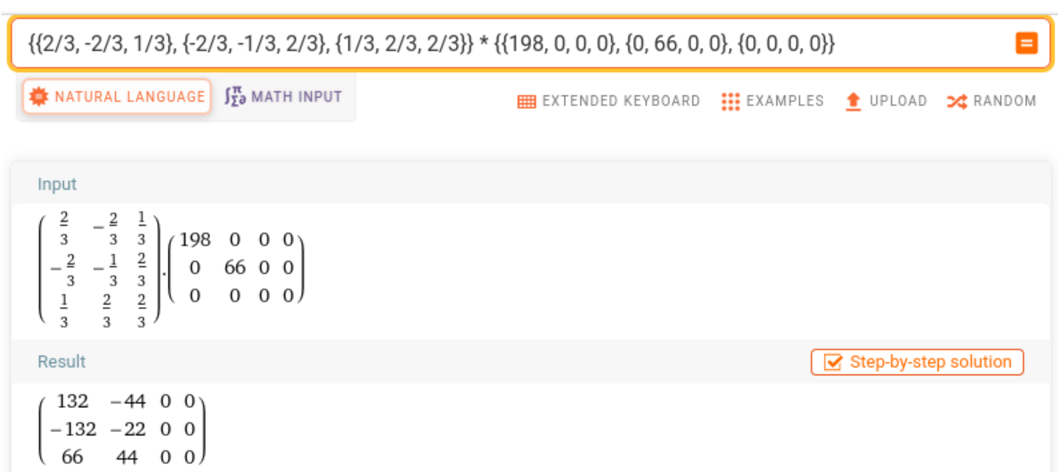
$$f(A) = \frac{1}{8}\begin{pmatrix} -2+24{\cdot}ln(2) & -6+14{\cdot}ln(2)+10{\cdot}ln(3) & -1+30{\cdot}ln(2)-6{\cdot}ln(3) \\ 1+22{\cdot}ln(2)+2{\cdot}ln(3) & 6+56{\cdot}ln(2)-32{\cdot}ln(3) & -7-10{\cdot}ln(2)+34{\cdot}ln(3) \\ 1+22{\cdot}ln(2)+2{\cdot}ln(3) & 3+62{\cdot}ln(2)-38{\cdot}ln(3) & -4-16{\cdot}ln(2)+40{\cdot}ln(3) \end{pmatrix}$$

Calculations

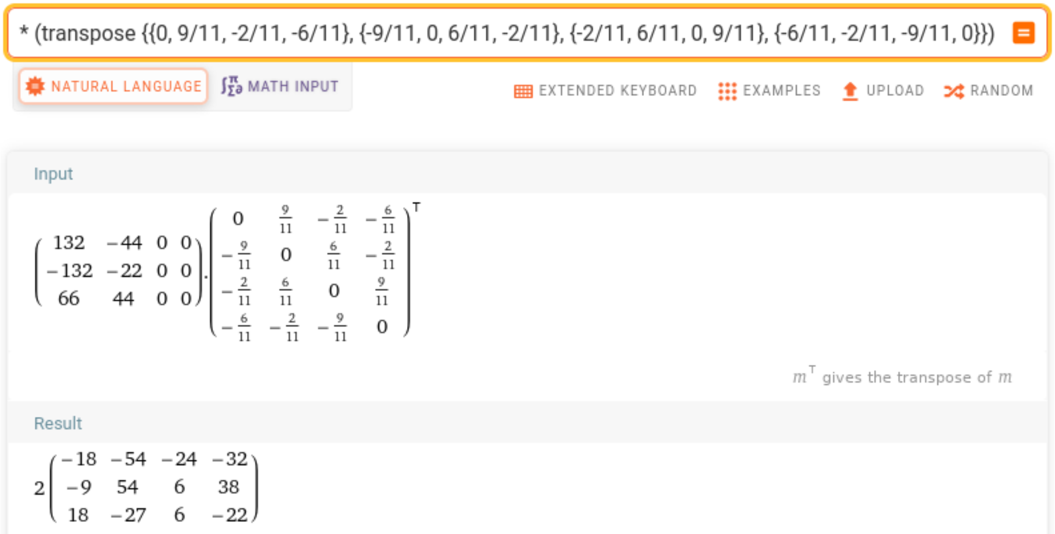
Task 1



Picture calculation.1.1: SVD of A

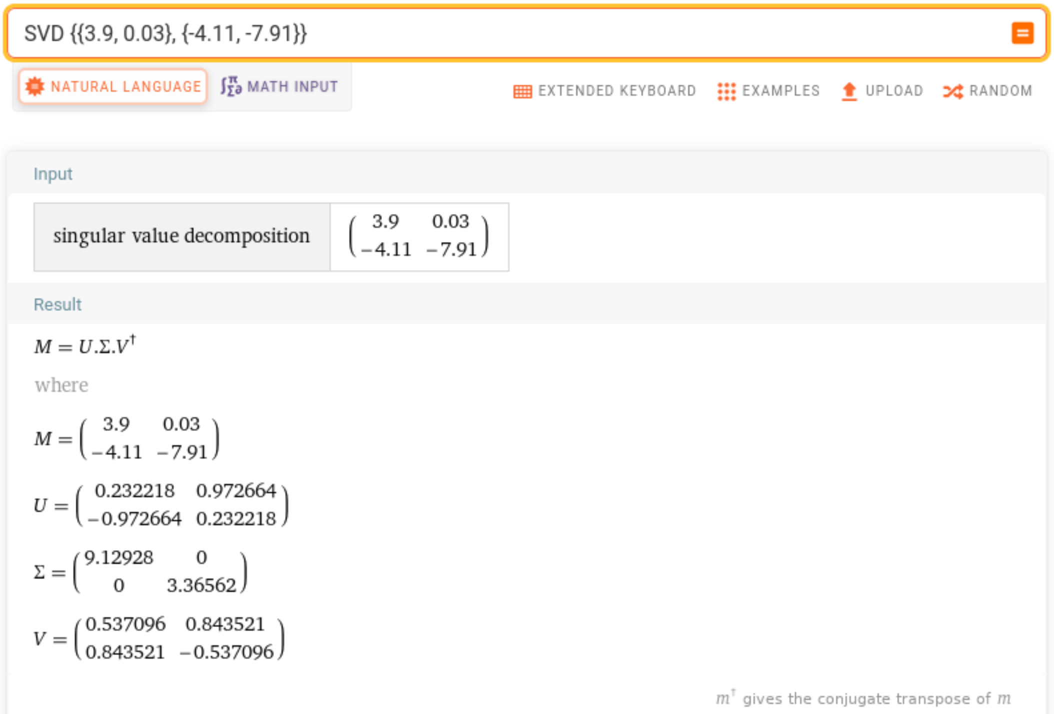


Picture calculation.1.2: A_1 (part 1)

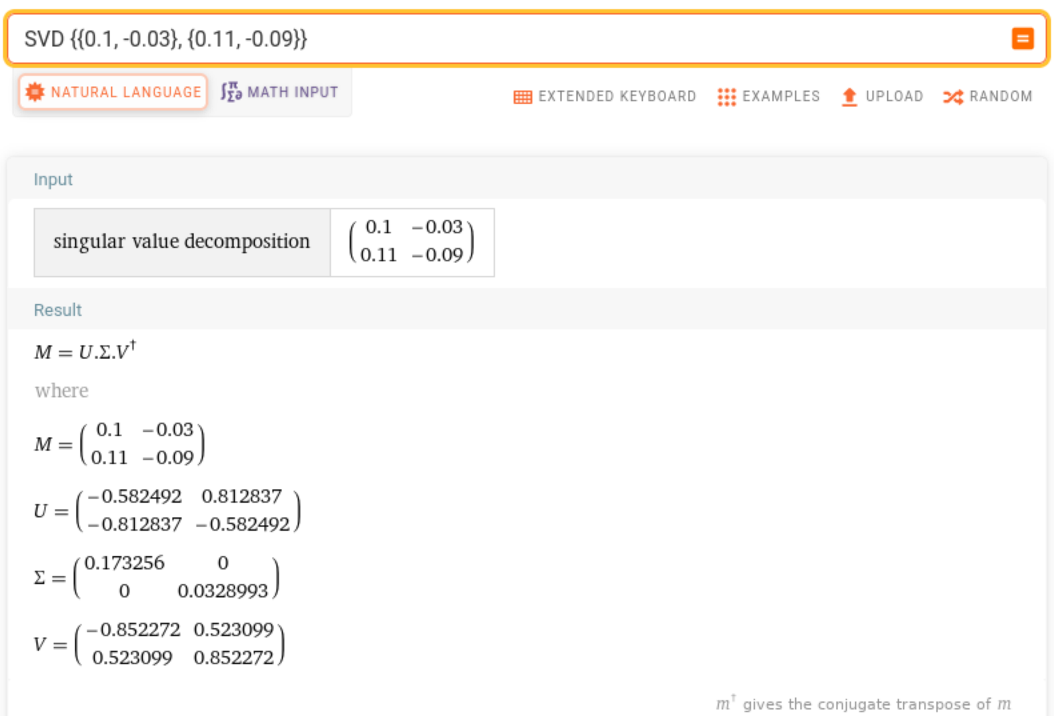


Picture calculation.1.3: A_1 (part 2)

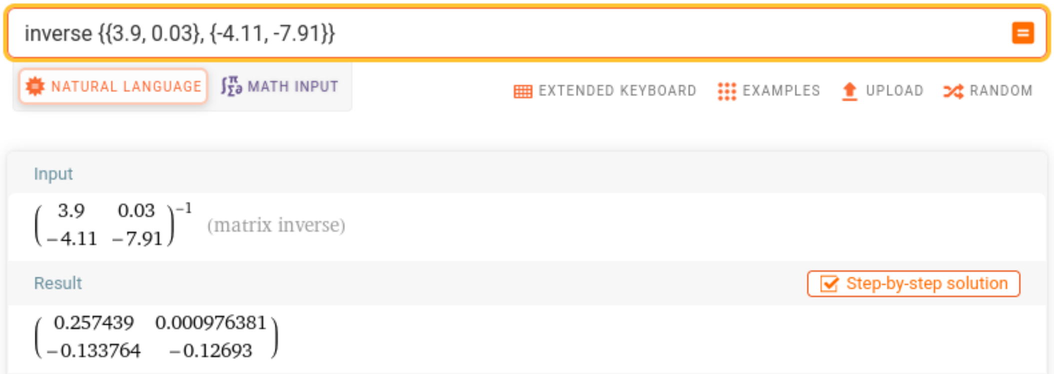
Task 2



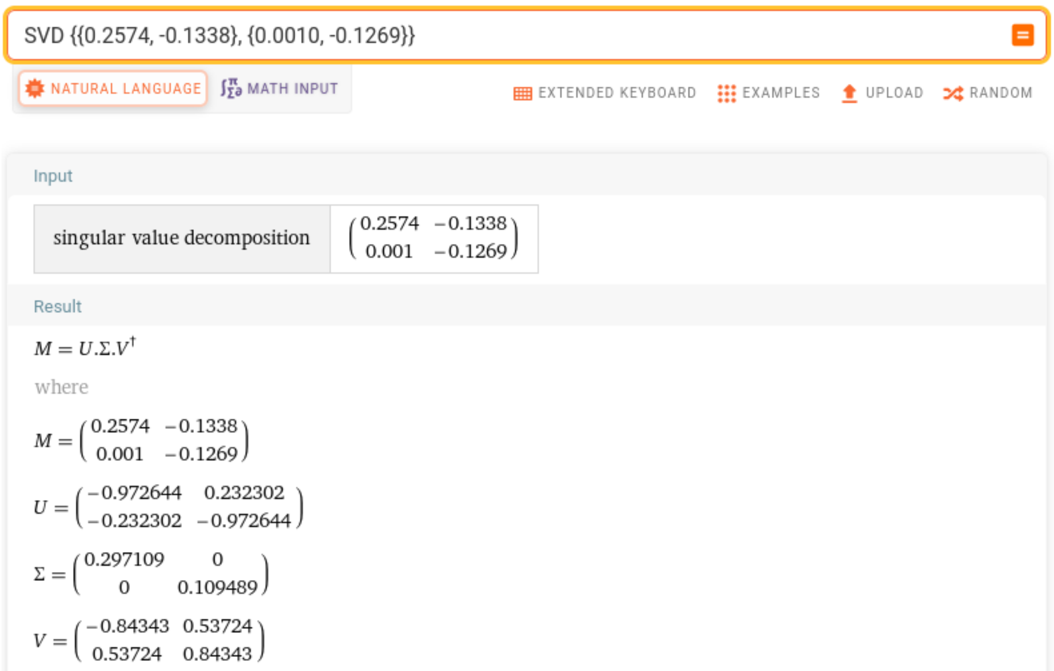
Picture calculation.2.1: SVD of A



Picture calculation.2.2: SVD of $(\widehat{A} - A)$

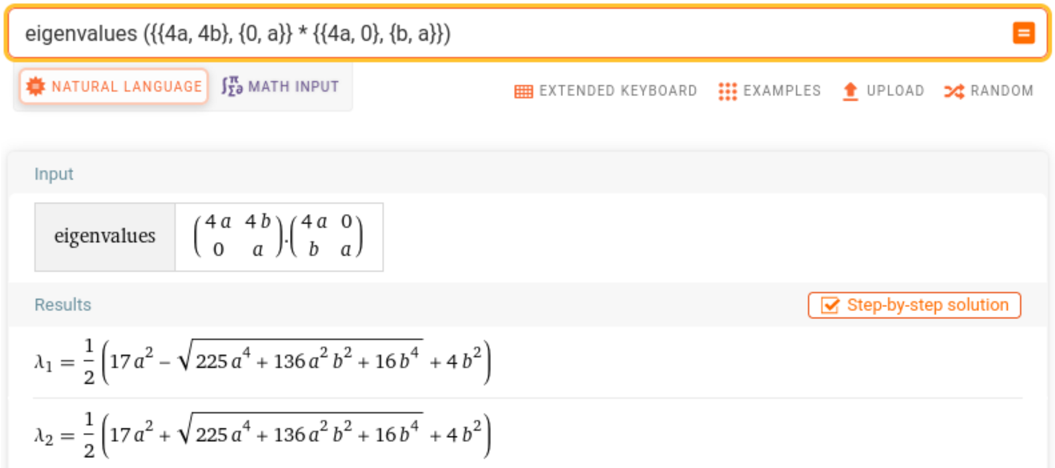


Picture calculation.2.3: Inverse of A

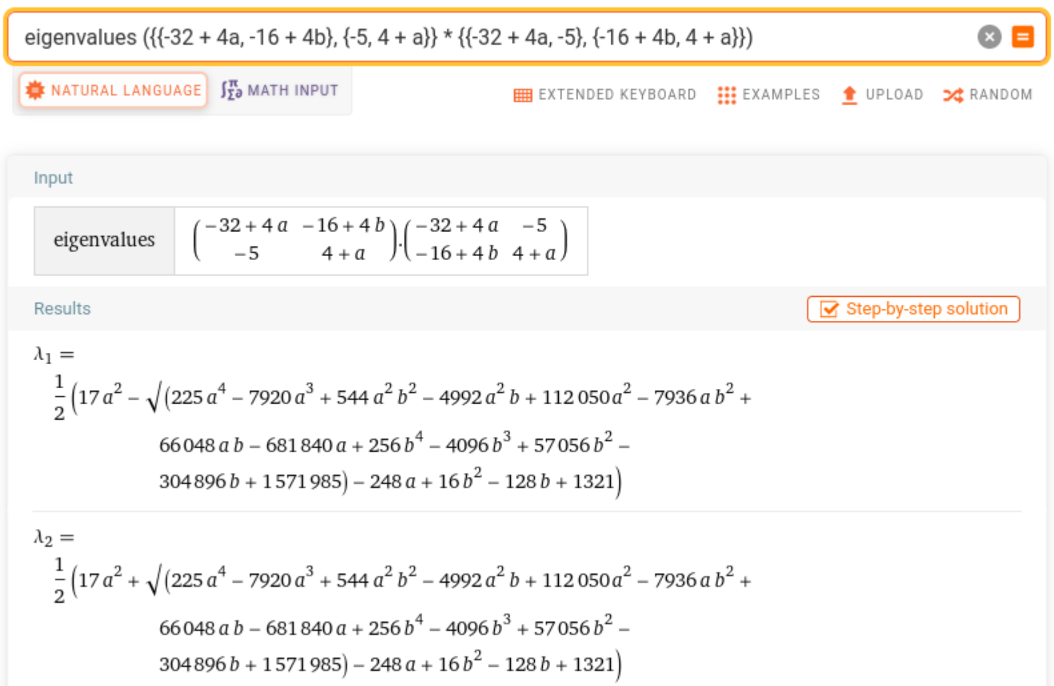


Picture calculation.2.4: SVD of A^{-1}

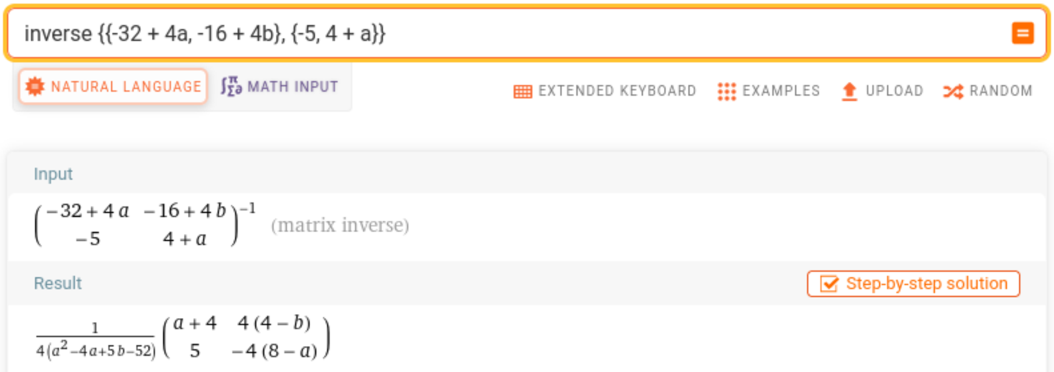
Task 3



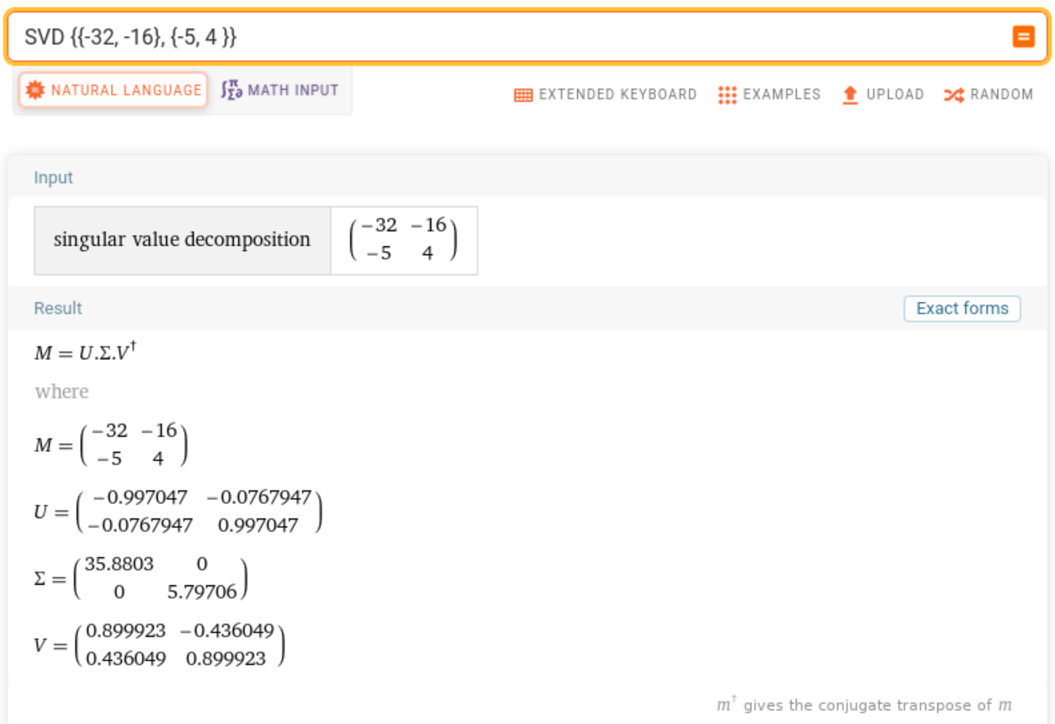
Picture calculation.3.1: Eigenvalues of $(\widehat{A} - A)$



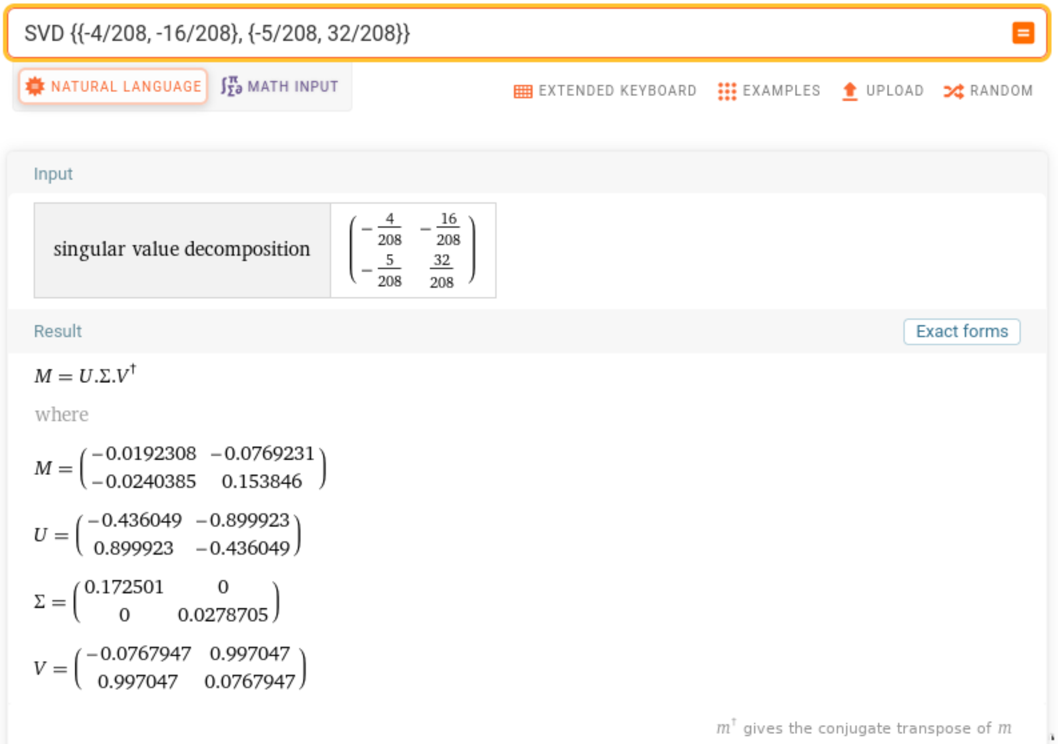
Picture calculation.3.2: Eigenvalues of A



Picture calculation.3.3: Inverse of A

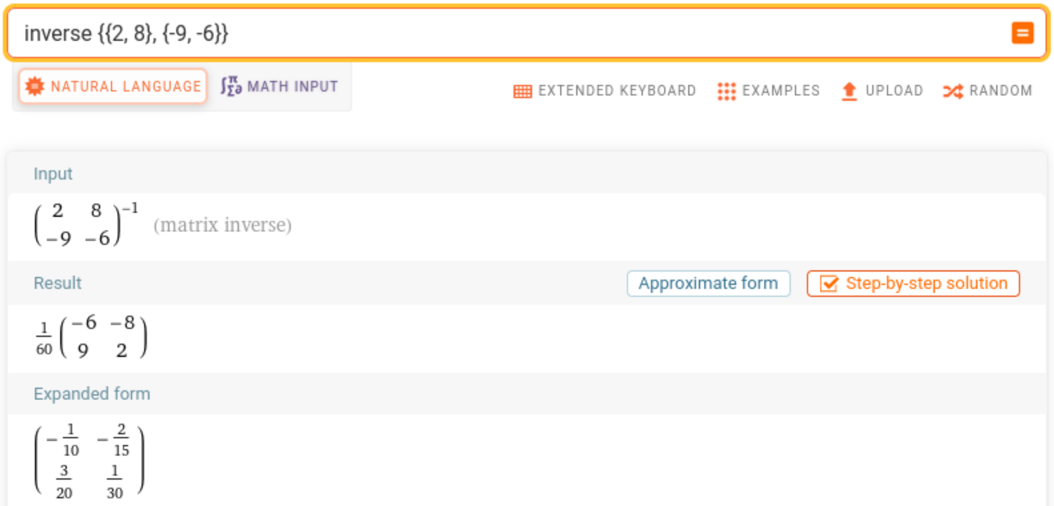


Picture calculation.3.4: SVD of $\widehat{A}$



Picture calculation.3.5: SVD of $\widehat{A^{-1}}$

Task 4



Picture calculation.4.1: $\widehat{A}$

Task 5

{{1 / 26, 0 , 0}, {0, 1 / 26, 0}, {0, 0, 1 / 21}} * {{0, 9, 8}, {6, 0, 3}, {1, 1, 0}}

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD EXAMPLES UPLOAD RANDOM

Input

$$\begin{pmatrix} \frac{1}{26} & 0 & 0 \\ 0 & \frac{1}{26} & 0 \\ 0 & 0 & \frac{1}{21} \end{pmatrix} . \begin{pmatrix} 0 & 9 & 8 \\ 6 & 0 & 3 \\ 1 & 1 & 0 \end{pmatrix}$$

Result

Approximate forms Step-by-step solution

$$\begin{pmatrix} 0 & \frac{9}{26} & \frac{4}{13} \\ \frac{3}{13} & 0 & \frac{3}{26} \\ \frac{1}{21} & \frac{1}{21} & 0 \end{pmatrix}$$

Picture calculation.5.1: Matrix P

Task 5

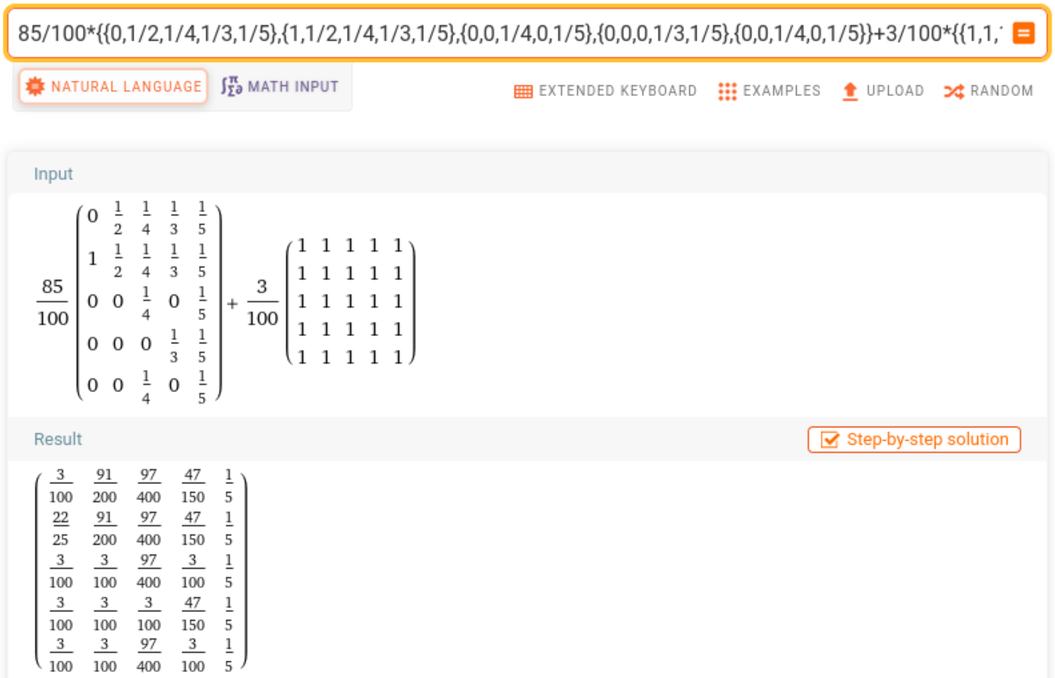
In [2]:
```
P = np.array([[0, -9/26, -4/13], [-3/13, 0, -3/26], [-1/21, -1/21, 0]])
c = np.array([[9/26], [1/26], [1/3]])
x0 = np.array([[0], [0], [0]])
```

In [3]:
```
xi = P.dot(x0) + c
i = 1
print('x_0: ', x0)
print('x_1', xi)
while np.any(np.abs(xi - x0) > 0.01):
    x0 = xi
    xi = P.dot(x0) + c
    i += 1
    print('x_', i, ': ', xi, sep='')
print('delta: ', xi - x0)
```

```
x_0:  [[0]
 [0]
 [0]]
x_1 [[0.34615385]
 [0.03846154]
 [0.33333333]]
x_2: [[ 0.23027613]
 [-0.07988166]
 [ 0.31501832]]
x_3: [[ 0.27687648]
 [-0.05102737]
 [ 0.32617169]]
x_4: [[ 0.26345665]
 [-0.06306823]
 [ 0.32257861]]
x_5: [[ 0.2687302 ]
 [-0.05955676]
 [ 0.32379103]]
delta:  [[0.00527355]
 [0.00351147]
 [0.00121241]]
```

Picture calculation.5.2: Approximate solution and iteration number

Task 6



Picture calculation.6.1: Matrix Q

Task 6

```
In [4]: Q = 1 / 1200 * np.array([[36, 546, 291, 376, 240], [1056, 546, 291, 376, 240],
                                 [36, 36, 291, 36, 240], [36, 36, 36, 376, 240],
                                 [36, 36, 291, 36, 240]])
        x0 = np.array([[1/5], [1/5], [1/5], [1/5], [1/5]])
```

```
In [5]: print('x_0: ', x0)
        xi = Q.dot(x0)
        i = 1
        print('x_1', xi)
        while np.any(np.abs(xi - x0) > 0.001):
            x0 = xi
            xi = Q.dot(x0)
            i += 1
            print('x_', i, ': ', xi, sep='')
        print('delta: ', xi - x0)
```

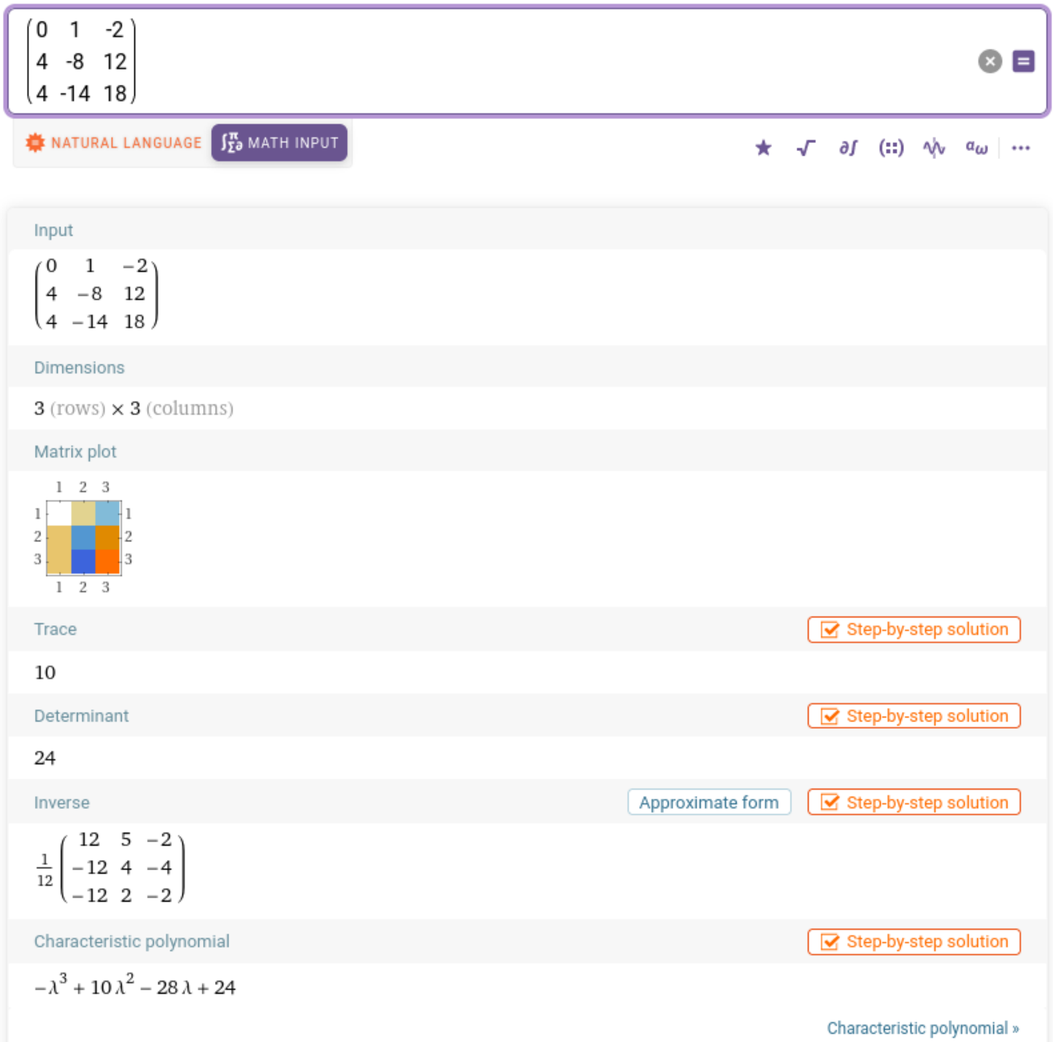
```
x_0:  [[0.2]
 [0.2]
 [0.2]
 [0.2]
 [0.2]]
x_1 [[0.24816667]
 [0.41816667]
 [0.1065    ]
 [0.12066667]
 [0.1065    ]]
x_2: [[0.28264597]
 [0.49358764]
 [0.07073625]
 [0.08229389]
 [0.07073625]]
x_3: [[0.29014796]
 [0.53039704]
 [0.05705662]
 [0.06534176]
 [0.05705662]]
x_4: [[0.2957564 ]
 [0.54238217]
 [0.05182416]
 [0.05821312]
 [0.05182416]]
```

Picture calculation.6.2: Distribution of vertex influence (part 1)

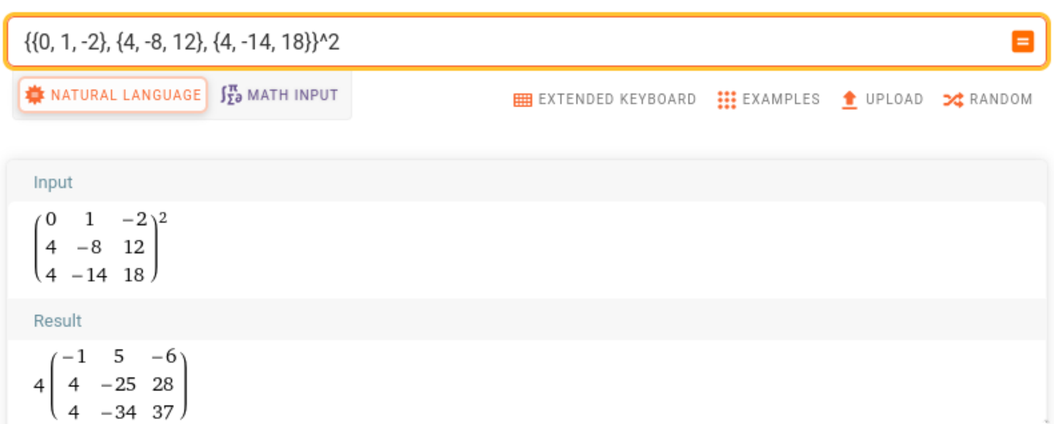
```
x_5: [[0.29682888]
 [0.54822182]
 [0.04982274]
 [0.05530383]
 [0.04982274]]
x_6: [[0.29772089]
 [0.55002543]
 [0.0490572 ]
 [0.05413928]
 [0.0490572 ]]
x_7: [[0.29786465]
 [0.55092741]
 [0.04876438]
 [0.05367919]
 [0.04876438]]
delta:  [[ 0.00014376]
 [ 0.00090197]
 [-0.00029282]
 [-0.0004601 ]
 [-0.00029282]]
```

Picture calculation.6.3: Distribution of vertex influence (part 2)

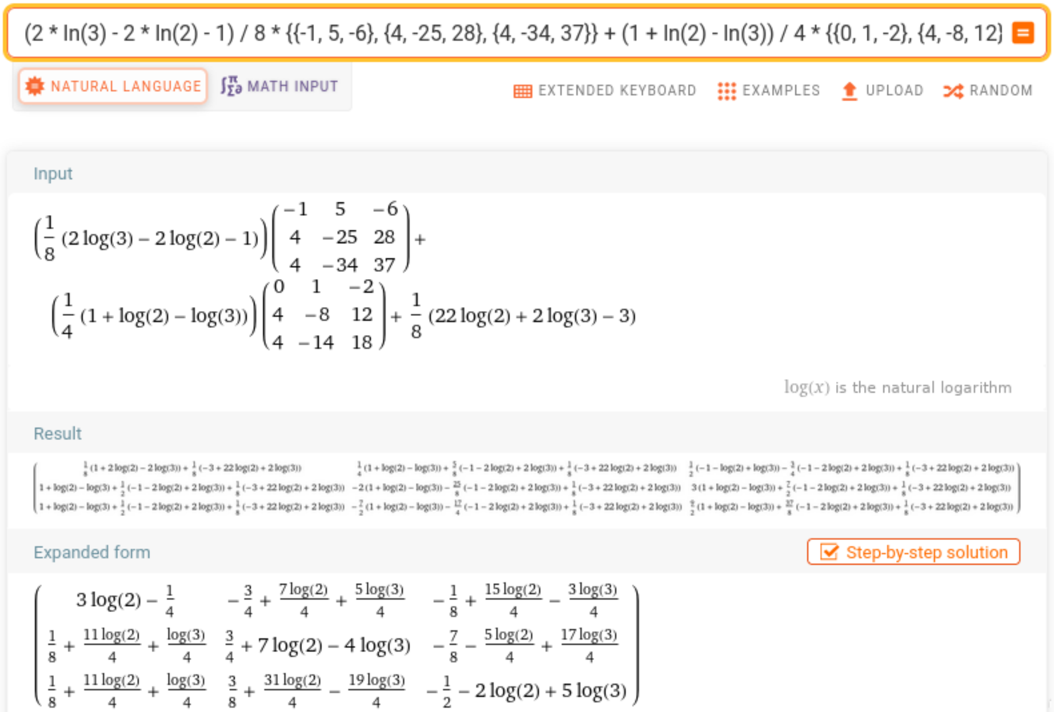
Task 7



Picture calculation.7.1: Eigenvalues of A



Picture calculation.7.2: Eigenvalues of A^2



Picture calculation.7.3: Result $f(A)$