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Вариант 3
(номер 21)

№1

$r = a(1-\sin\varphi)$, $\varphi \in [-\frac{\pi}{2}; -\frac{\pi}{6}]$

$r' = a' - (a\sin\varphi)' = -a\cos\varphi$

$$L = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \sqrt{r^2 + r'^2}\, d\varphi = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \sqrt{a^2 - 2a^2\sin\varphi + a^2\sin^2\varphi + a^2\cos^2\varphi}\, d\varphi =$$

$$= \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \sqrt{2a^2 - 2a^2\sin\varphi}\, d\varphi = \left\langle \begin{array}{l} \alpha = \frac{\pi}{4} - \frac{\varphi}{2} \\ d\alpha = -\frac{1}{2}d\varphi \end{array} \right\rangle =$$

$$= -2\int_{0}^{\frac{\pi}{6}} \sqrt{2a^2\left(1-\sin\left(\frac{\pi}{2}-2\alpha\right)\right)}\, d\alpha = -\frac{2}{1}\int_{0}^{\frac{\pi}{6}} \sqrt{2a^2(1-\cos 2\alpha)}\, d\alpha =$$

$$= -\frac{2}{1}\int_{0}^{\frac{\pi}{6}} \sqrt{4a^2 \cdot \sin^2\alpha}\, d\alpha = -4|a|\int_{0}^{\frac{\pi}{6}} \sqrt{\sin^2\alpha}\, d\alpha$$

При $\alpha \in [0; \frac{\pi}{6}]$ $\sin\alpha > 0$

$$L = -4|a|\int_{0}^{\frac{\pi}{6}} \sin\alpha\, d\alpha = -4|a|\left(-\cos\alpha\Big|_0^{\frac{\pi}{6}}\right) =$$

$$= -4|a|\left(-\frac{\sqrt{3}}{2} + 1\right) = 2|a|(\sqrt{3} - 2) -$$

— отрицательная, т.е. нужно взять модуль

$$\boxed{\text{Ответ: } 2|a|(2-\sqrt{3})}$$

№2

$$\int_0^{\pi/2} \frac{dx}{1+\sin x+\cos x} = \left\langle \begin{array}{ll} t = tg\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ dx = \frac{2dt}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \\ x\in[0;\frac{\pi}{2}] \to t\in[0;1] & \end{array} \right\rangle =$$

$$= \int_0^1 \frac{\left(\frac{2dt}{1+t^2}\right)}{\left(1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}\right)} = \int_0^1 \frac{2dt}{1+\cancel{t^2}+2t+1-\cancel{t^2}} = \int_0^1 \frac{2dt}{2+2t} =$$

$$= \int_0^1 \frac{dt}{1+t} = \int_0^1 \frac{d(t+1)}{t+1} = \ln|t+1|\Big|_0^1 = \ln 2 - \ln 1 = \ln 2$$

$$\boxed{\text{Ответ: } \ln 2}$$

№4

№3

$$\int_0^1 x\sqrt{1+x}\,dx = \left\langle \begin{matrix} t = 1+x \\ x = t-1 \\ dt = dx \end{matrix} \right\rangle = \int_1^2 (t-1)\sqrt{t}\,dt =$$

$$= \int_1^2 t^{\frac{3}{2}}\,dt - \int_1^2 t^{\frac{1}{2}}\,dt = \frac{2}{5} t^{\frac{5}{2}}\Big|_1^2 -$$

$$- \frac{2}{3} t^{\frac{3}{2}}\Big|_1^2 = \left(\frac{2}{5}\cdot\sqrt{2^5} - \frac{2}{5}\sqrt{1^5}\right) -$$

$$- \left(\frac{2}{3}\sqrt{2^3} - \frac{2}{3}\sqrt{1^3}\right) = \frac{8}{5}\sqrt{2} - \frac{2}{5} - \frac{4}{3}\sqrt{2} +$$

$$+ \frac{2}{3} = \frac{24}{15}\sqrt{2} - \frac{20}{15}\sqrt{2} + \frac{10}{15} - \frac{6}{15} =$$

$$= \frac{4\sqrt{2}+4}{15}$$

$$\boxed{\text{Ответ: } \frac{4}{15}\left(1+\sqrt{2}\right)}$$

№4

$$\int_1^2 \frac{\operatorname{arctg}(x-1)}{x-\sqrt{x}}\,dx = \left\langle \begin{array}{l} \cancel{\lim_{x\to 1} \operatorname{arctg}(x-1) = x} \\ \operatorname{arctg}(x-1) \sim (x-1), \text{ т.к. } (x-1) \to 0 \end{array} \right\rangle =$$

$$= \int_1^2 \frac{x-1}{x-\sqrt{x}}\,dx = \left\langle \begin{array}{l} t^2 = x \\ dx = 2t\,dt \end{array} \right\rangle = \int_1^{\sqrt{2}} \frac{t^2-1}{t^2-t}\cdot 2t\,dt =$$

$$= \int_1^{\sqrt{2}} \frac{t^2-1}{(t-1)}\,dt = \int_1^{\sqrt{2}} (t+1)\,dt = \int_1^{\sqrt{2}} (t+1)^1\,d(t+1) =$$

$$= \frac{(t+1)^2}{2}\Big|_1^{\sqrt{2}} = \frac{(1+\sqrt{2})^2}{2} - \frac{2^2}{2} =$$

$$= \frac{(\sqrt{2}-1)(\sqrt{2}+3)}{2} = \frac{2+3\sqrt{2}-\sqrt{2}-3}{2} = 2\sqrt{2} - \frac{1}{2}$$

Ответ: сходится

№5

$$\lim_{x\to+\infty}\frac{\left(\frac{x\cos 7x}{x^2+2x+2}\right)}{\left(\frac{\cos 7x}{x}\right)} = \lim_{x\to+\infty}\frac{x^2}{x^2+2x+2} = 1,$$ т. е.

сходимости одинаковы

Воспользуемся пр. сходимости

Дирихле:

$\int_0^{+\infty}\frac{\cos 7x}{x}\,dx$, $f(x)=\cos 7x$ – первообразная ограничена и непрерывна

$g(x)=\frac{1}{x}\to 0$ при $x\to\infty$, непрерывна дифференцируема и монотонна =>

$\int_0^{+\infty} f(x)\cdot g(x)\,dx = \int_0^{+\infty}\frac{\cos 7x}{x}\,dx$ сходится,

т. е. искомый тоже

$\boxed{\text{Ответ: сходится}}$