(№1) (с. 405 №5(б))

Построить график $y = x + \frac{7}{x} - \frac{3}{x^2} = f(x)$

$D(f) = \mathbb{R} \setminus \{0\}$

$\begin{cases} f(-x) \neq -f(x) \\ f(x) \neq f(-x) \end{cases}$ $\Big|$ — $f(x)$ — функция общего вида

$f(x)$ не периодична

$$\lim_{x \to 0} \left(x + \frac{7}{x} - \frac{3}{x^2}\right) = \lim_{x \to (-0)} \left(\frac{x^3 + 7x - 3}{x^2}\right) = \frac{-3}{(+0)} = -\infty =$$

$= \lim_{x \to (+0)}$, т.е. $x = 0$ — вертикальная асимптота

и точка разрыва второго рода

$$k_+ = \lim_{x \to +\infty} \left(\frac{f(x)}{x}\right) = \lim_{x \to +\infty} \left(1 + \frac{7}{x^2} - \frac{3}{x^3}\right) = 1$$

$$b_+ = \lim_{x \to +\infty} \left(f(x) - k_+ x\right) = \lim_{x \to +\infty} \left(\frac{7}{x} - \frac{3}{x^2}\right) = 0$$

$y = x$ — наклонная асимптота на $+\infty$

$$k_- = \lim_{x \to -\infty} \left(\frac{f(x)}{x}\right) = \lim_{x \to -\infty} \left(1 + \frac{7}{x^2} - \frac{3}{x^3}\right) = 1$$

$$b_- = \lim_{x \to -\infty} \left(f(x) - k_- x\right) = \lim_{x \to -\infty} \left(\frac{7}{x} - \frac{3}{x^2}\right) = 0$$

$y = x$ — наклонная асимптота на $-\infty$

$$f'(x) = 1 - \frac{7}{x^2} + \frac{6}{x^3} = \frac{x^3 - 7x + 6}{x^3} = \frac{(x-1)(x^2 + x - 6)}{x^3} =$$

$$= \frac{(x+3)(x-1)(x-2)}{x^3}$$

$\nearrow +$ $\searrow -$ $\nearrow +$ $\searrow -$ $\nearrow +$

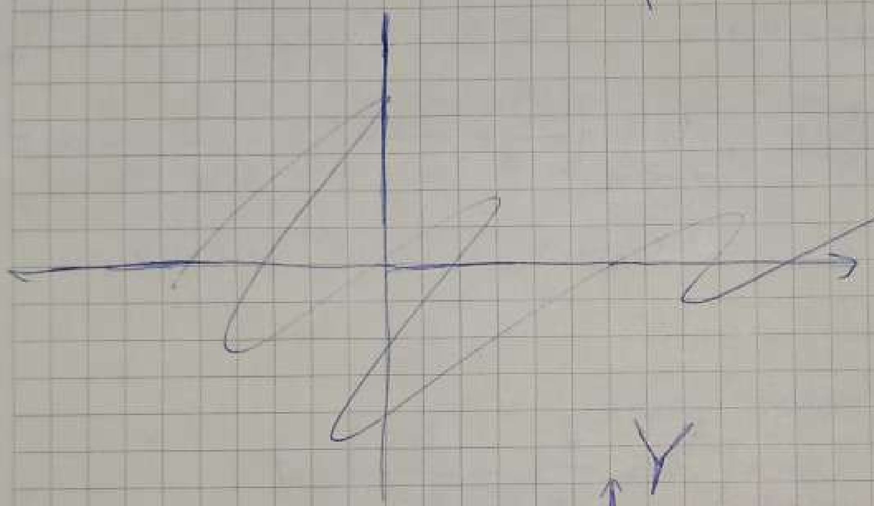
-3 0 1 2 X

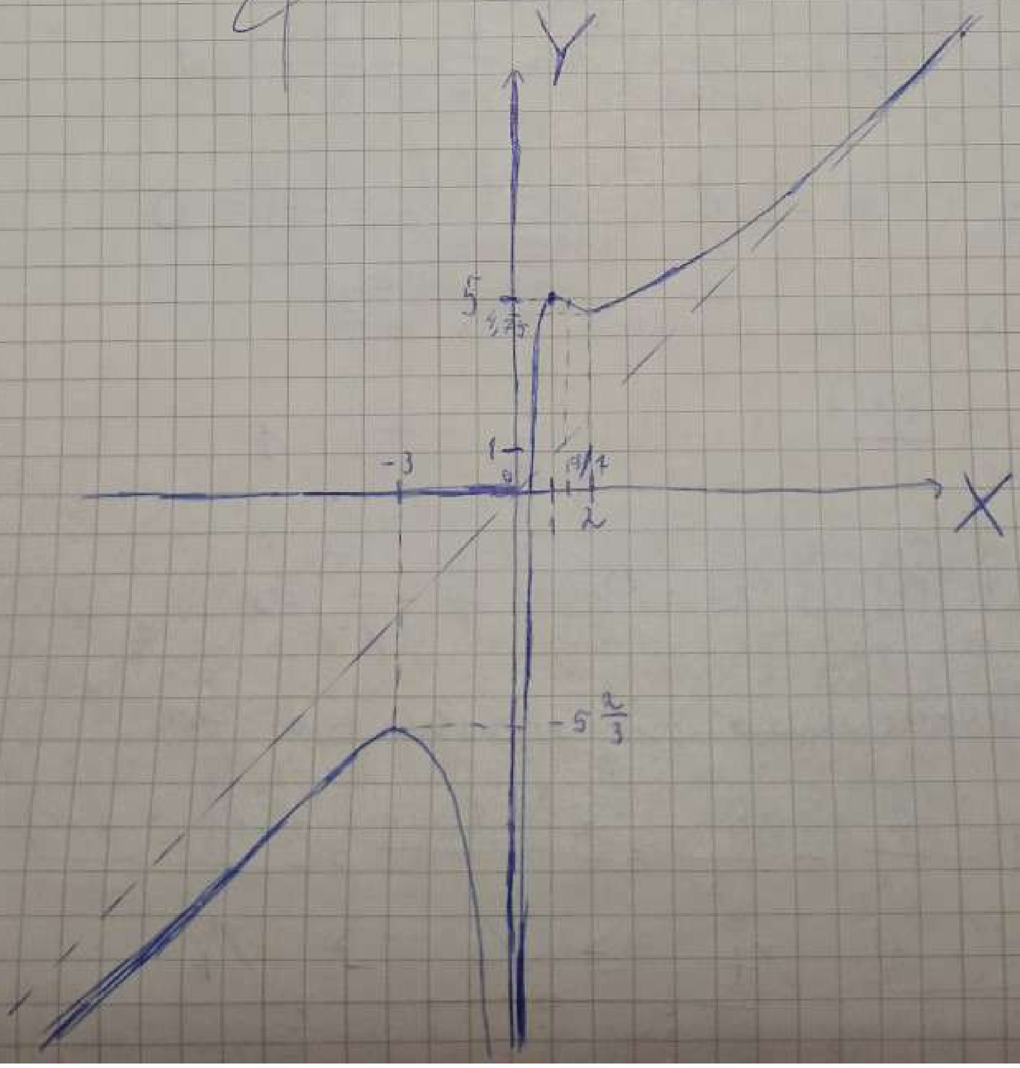
$x=3$ и $x=1$ – точки локального максимума,

$x=2$ – точки локального минимума.

$$f''(x) = \frac{14}{x^3} - \frac{18}{x^4} = \frac{14x-18}{x^4} = 14\,\frac{x-\frac{9}{7}}{x^4}$$

$x = \frac{9}{7}$ – точка перегиба





№2 (с. 18 N 28(4))

$$\int \frac{\operatorname{arctg}\sqrt{x}}{\sqrt{x+1}}\,dx = \left\langle \begin{matrix} u = \operatorname{arctg}\sqrt{x} & du = \frac{1}{x+1}\cdot\frac{1}{2\sqrt{x}}\,dx \\ dv = \frac{dx}{\sqrt{x+1}} & v = 2\sqrt{x+1} \end{matrix} \right\rangle =$$

$$= 2\sqrt{x+1}\,\operatorname{arctg}\sqrt{x} - \int 2\sqrt{x+1}\cdot\frac{1}{x+1}\cdot\frac{1}{2\sqrt{x}}\,dx =$$

$$= 2\sqrt{x+1}\,\operatorname{arctg}\sqrt{x} - \cancel{\int\frac{dx}{\sqrt{x+1}}}\ \int\frac{dx}{\sqrt{x}}\cdot\frac{1}{\sqrt{x+1}} = 2\sqrt{x+1}\,\operatorname{arctg}\sqrt{x} -$$

$$- \int\frac{d(2\sqrt{x})}{\sqrt{x+1}} = 2\sqrt{x+1}\,\operatorname{arctg}\sqrt{x} - 2\int\frac{d\sqrt{x}}{\sqrt{x+1}} =$$

$$= 2\sqrt{x+1}\,\operatorname{arctg}\sqrt{x} - 2\ln\left|\sqrt{x}+\sqrt{x+1}\right|$$

№3 (с. 31 №3(3))

$$\int \frac{dx}{x^3-x^2-x+1}$$

$$\begin{array}{r|l} x^3-x^2-x+1 & x-1 \\ -\;x^3-x^2 & \overline{x^2-1} \\ \hline -x+1 & \\ -\;-x+1 & \\ \hline 0 & \end{array}$$

$x^3-x^2-x+1=(x-1)(x^2-1)=$
$=(x-1)^2(x+1)$

$$\frac{1}{x^3-x^2-x+1}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+1)}=\frac{A(x-1)(x+1)}{(x-1)^2(x+1)}+$$

$$+\frac{B(x+1)+C(x-1)^2}{(x-1)^2(x+1)}=\frac{(A+C)x^2+(B-2C)x+(-A+B+C)}{(x-1)^2(x+1)}$$

$$\begin{cases} A+C=0 \\ B-2C=0 \\ -A+B+C=1 \end{cases} \Leftrightarrow \begin{cases} A=-C \\ B=2C \\ C+2C+C=1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} A=-\frac{1}{4} \\ B=\frac{1}{2} \\ C=\frac{1}{4} \end{cases}$$

$$\int \frac{dx}{x^3-x^2-x+1}=-\frac{1}{4}\int\frac{dx}{(x-1)}+\frac{1}{2}\int\frac{dx}{(x-1)^2}+\frac{1}{4}\int\frac{dx}{(x+1)}=$$

$$=-\frac{1}{4}\cdot \ln|x-1|+\frac{1}{2}(x-1)^{-1}\cdot(-1)+\frac{1}{4}\ln|x+1|+C=$$

$$=\boxed{-\frac{1}{2}(x-1)^{-1}+\frac{1}{4}\ln\left|\frac{x+1}{x-1}\right|+C}$$

№4 (с. 45 №8(2))

$$\int \frac{x^4\,dx}{\sqrt{x^2+4x+5}}$$

$$\int \frac{x^4\,dx}{\sqrt{x^2+4x+5}} = (Ax^3+Bx^2+Cx+D)\sqrt{x^2+4x+5} + \lambda \int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$\frac{x^4}{\sqrt{x^2+4x+5}} = (3Ax^2+2Bx+C)\sqrt{x^2+4x+5} + \frac{(Ax^3+Bx^2+Cx+D)(2x+4)+\lambda}{\sqrt{x^2+4x+5}}$$

$$x^4 = (3Ax^2+2Bx+C)(x^2+4x+5) + (Ax^3+Bx^2+Cx+D)(2x+4)+\lambda =$$

$$= 3Ax^4+12Ax^3+15Ax^2+2Bx^3+8Bx^2+10Bx+Cx^2+4Cx+5C+$$

$$+2Ax^4+2Bx^3+2Cx^2+2Dx+4Ax^3+4Bx^2+4Cx+4D+\lambda =$$

$$= 5Ax^4+(16A+4B)x^3+(15A+12B+3C)x^2+(10B+8C+$$

$$+2D)x+(5C+4D+\lambda)$$

$$\begin{cases} 5A=1 \\ 16A+4B=0 \\ 15A+12B+3C=0 \\ 10B+8C+2D=0 \\ 5C+4D+\lambda=0 \end{cases} \Leftrightarrow \begin{cases} A=\frac{1}{5} \\ B=-\frac{4}{5} \\ C=\frac{11}{5} \\ D=-\frac{24}{5} \\ \lambda=\frac{41}{5} \end{cases}$$

$$\int \frac{x^4\,dx}{\sqrt{x^2+4x+5}} = \left(\frac{1}{5}x^3-\frac{4}{5}x^2+\frac{11}{5}x-\frac{24}{5}\right)\sqrt{x^2+4x+5}+$$

$$+\frac{41}{5}\int\frac{dx}{\sqrt{(x+2)^2+1}} =$$

$$= \left(\frac{1}{5}x^3-\frac{4}{5}x^2+\frac{11}{5}x-\frac{24}{5}\right)\sqrt{x^2+4x+5}+\frac{41}{5}\ln\left|x+2+\sqrt{x^2+4x+5}\right|+C$$

№5 (с. 57 №9(3))

$$\int \frac{dx}{\sin^2 x \cos^3 x} = \int \frac{\cos x\, dx}{\sin^2 x \cos^4 x} = \int \frac{d(\sin x)}{\sin^2 x \cos^4 x} =$$

$$= \left\langle t = \sin x \right\rangle = \int \frac{dt}{t^2(1-t^2)^2} \cancel{\int \frac{dt}{t^2(1-t)}} = \int \frac{dt}{t^2(1-t)^2(1+t)^2}$$

$$\cancel{\frac{1}{t^2(1-t^2)^2} = \frac{A}{t^2} + \frac{Bx+C}{(1-t^2)}}$$

$$\frac{1}{t^2(1-t)^2(1+t)^2} = \frac{A}{t^2} + \frac{B}{(1-t)} + \frac{C}{(1-t)^2} + \frac{D}{(1+t)} + \frac{E}{(1+t)^2} =$$

$$= \frac{A(1-t)^2(1+t)^2 + Bt^2(1-t)(1+t)^2 + Ct^2(1+t)^2}{t^2(1-t)^2(1+t)^2} +$$

$$+ \frac{Dt^2(1+t)(1-t)^2 + Et^2(1-t)^2}{t^2(1-t)^2(1+t)^2} = \frac{A - 2At^2 + At^4}{t^2(1-t)^2(1+t)^2} +$$

$$+ \frac{Bt^2 + Bt^3 - Bt^4 - Bt^5 + Ct^2 + 2Ct^3 + Ct^4 + Dt^2 + Dt^5}{t^2(1-t)^2(1+t)^2} +$$

$$+ \frac{-Dt^3 - Dt^4 + Et^2 - 2Et^3 + Et^4}{t^2(1-t)^2(1+t)^2} = \frac{t^5(D-B)}{t^2(1-t)^2(1+t)^2} +$$

$$+ \frac{t^4(A - B + C - D + E) + t^3(B + 2C - 2E) + t^2(-2A + B + C + D + E)}{t^2(1-t)^2(1+t)^2} +$$

$$+ \frac{t \cdot 0 + A}{t^2(1-t)^2(1+t)^2}$$

$$\begin{cases} D - B = 0 \\ A - B + C - D + E = 0 \\ B + 2C - 2E = 0 \\ -2A + B + C + D + E = 0 \\ A = 1 \end{cases} \iff \begin{cases} A = 1 \\ D = B \\ 1 - 2B + C + E = 0 \\ B + 2C - 2E = 0 \\ -2 + 2B + C + E = 0 \end{cases} \iff$$

$$\iff \begin{cases} A = 1 \\ D = B \\ B = 2E - 2C \\ 1 + 4C - 4E + C + E = 0 \\ -2 + 4E - 4C + C + E = 0 \end{cases} \iff \begin{cases} A = 1 \\ D = B \\ B = 2E - 2C \\ 5C = 3E - 1 \\ 3C = 5E - 2 \end{cases} \iff$$

$$\iff \begin{cases} A = 1 \\ D = B \\ B = 2E - 2C \\ 3C = 5E - 2 \\ 2C = -2E + 1 \end{cases} \iff \begin{cases} A = 1 \\ D = B \\ B = 2E - 2C \\ 2C = -2E + 1 \\ C = 7E - 3 \end{cases} \iff$$

$$\begin{cases} A = 1 \\ D = B \\ B = 2E - 2C \\ C = 7E - 3 \\ 14E - 6 = -2E + 1 \end{cases} \iff \begin{cases} A = 1 \\ D = B \\ B = 2E - 2C \\ C = \frac{1}{16} \\ E = \frac{7}{16} \end{cases} \iff$$

$$\iff \begin{cases} A = 1 \\ B = \frac{3}{4} \\ C = \frac{1}{16} \\ D = \frac{3}{4} \\ E = \frac{7}{16} \end{cases}$$

$$\int \frac{dt}{t^2(1-t)^2(1+t)^2} = \int \frac{dt}{t^2} + \frac{3}{4}\int \frac{dt}{(1-t)} + \frac{1}{16}\int \frac{dt}{(1-t)^2} +$$

$$+ \frac{3}{4}\int \frac{dt}{(1+t)} + \frac{1}{16}\int \frac{dt}{(1+t)^2} = -t^{-1} \cancel{+\frac{3}{4}} - \frac{3}{4}\ln|1-t| +$$

$$+ \frac{1}{16}(1-t)^{-1} + \frac{3}{4}\ln|1+t| + -\frac{1}{16}(1+t)^{-1} + C =$$

$$= \frac{3}{4}\ln\left|\frac{1+t}{1-t}\right| + \frac{t(1+t) - 7t(1-t) - 16(1-t)(1+t)}{t(1-t^2)} + C =$$

$$= \frac{3}{4}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + \frac{t+t^2-7t+7t^2-16+16t^2}{t(1-t^2)} + C =$$

$$= \frac{3}{4}\ln\left|\frac{\sin\frac{\pi}{2}+\sin x}{\sin\frac{\pi}{2}-\sin x}\right| + \frac{24t^2-6t-16}{\sin x\cos^2 x} + C =$$

$$= \frac{3}{4}\ln\left|\frac{\cancel{2}\cdot\sin(x/2+\frac{\pi}{4})\cdot\cos(\frac{x}{2}-\frac{\pi}{4})}{\cancel{2}\cdot\sin(\frac{\pi}{4}-\frac{x}{2})\cdot\cos(\frac{x}{2}+\frac{\pi}{4})}\right| + \frac{24\sin^2 x - 6\sin x - 16}{\sin x\cos^2 x} + C =$$

$$= \frac{3}{4}\ln\left|\operatorname{tg}\left(x+\frac{\pi}{4}\right)\cdot\operatorname{ctg}\left(\frac{x}{2}-\frac{\pi}{4}\right)\right| - \frac{6}{\cos^2 x} - \frac{24}{\sin x} + \frac{8}{\sin x\cos^2 x} + C$$