

SPACETRACK REPORT NO. 3

Models for Propagation of NORAD Element Sets

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DECEMBER 1980

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31 December 1988

General perturbations element sets generated by NORAD can be used to predict position and velocity of Earth-orbiting objects. To do this one must be careful to use a prediction method which is compatible with the way in which the elements were generated. Equations for five compatible models are given here along with corresponding FORTRAN IV computer code. With this information a user will be able to make satellite predictions which are completely compatible with NORAD predictions.

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1 INTRODUCTION

NORAD maintains general perturbation element sets on all resident space objects. These element sets are periodically refined so as to maintain a reasonable prediction capability on all space objects. In turn, these element sets are provided to users. The purpose of this report is to provide the user with a means of propagating these element sets in time to obtain a position and velocity of the space object.

The most important point to be noted is that not just any prediction model will suffice. The NORAD element sets are “mean” values obtained by removing periodic variations in a particular way. In order to obtain good predictions, these periodic variations must be reconstructed (by the prediction model) in exactly the same way they were removed by NORAD. Hence, inputting NORAD element sets into a different model (even though the model may be more accurate or even a numerical integrator) will result in degraded predictions. The NORAD element sets must be used with one of the models described in this report in order to retain maximum prediction accuracy.

All space objects are classified by NORAD as near-Earth (period less than 225 minutes) or deep-space (period greater than or equal 225 minutes). Depending on the period, the NORAD element sets are automatically generated with the near-Earth or deep-space model. The user can then calculate the satellite period and know which prediction model to use.

2 THE PROPAGATION MODELS

Five mathematical models for prediction of satellite position and velocity are available. The first of these, SGP, was developed by Hilton & Kuhlman (1966) and is used for near-Earth satellites. This model uses a simplification of the work of Kozai (1959) for its gravitational model and it takes the drag effect on mean motion as linear in time. This assumption dictates a quadratic variation of mean anomaly with time. The drag effect on eccentricity is modeled in such a way that perigee height remains constant.

The second model, SGP4, was developed by Ken Cranford in 1970 (see Lane and Hoots 1979) and is used for near-Earth satellites. This model was obtained by simplification of the more extensive analytical theory of Lane and Cranford (1969) which uses the solution of Brouwer (1959) for its gravitational model and a power density function for its atmospheric model (see Lane, et al. 1962).

The next model, SDP4, is an extension of SGP4 to be used for deep-space satellites. The deep-space equations were developed by Hujsak (1979) and model the gravitational effects of the moon and sun as well as certain sectoral and tesseral Earth harmonics which are of particular importance for half-day and one-day period orbits.

The SGP8 model (see Hoots 1980) is used for near-Earth satellites and is obtained by simplification of an extensive analytical theory of Hoots (to appear) which uses the same gravitational and atmospheric models as Lane and Cranford did but integrates the differential equations in a much different manner.

Finally, the SDP8 model is an extension of SGP8 to be used for deep-space satellites. The deep-space effects are modeled in SDP8 with the same equations used in SDP4.

3 COMPATIBILITY WITH NORAD ELEMENT SETS

The NORAD element sets are currently generated with either SGP4 or SDP4 depending on whether the satellite is near-Earth or deep-space. For element sets sent to external users, the value of mean motion is altered slightly and a pseudo-drag term ($\dot{n}/2$) is generated. These changes allow an SGP user to make compatible predictions in the following manner. If the satellite is near-Earth, then the pseudo-drag term used in SGP simulates the drag effect of the SGP4 model. If the satellite is deep-space, then the pseudo-drag term used in SGP simulates the deep-space secular effects of SDP4.

For SGP4 and SDP4 users, the mean motion is first recovered from its altered form and the drag effect is obtained from the SGP4 drag term (B^*) with the pseudo-drag term being ignored. The value of the mean motion can be used to determine whether the satellite is near-Earth or deep-space (and hence whether SGP4 or SDP4 was used to generate the element set). From this information the user can decide whether to use SGP4 or SDP4 for propagation and hence be assured of agreement with NORAD predictions.

The SGP8 and SDP8 models have the same gravitational and atmospheric models as SGP4 and SDP4, although the form of the solution equations is quite different. Additionally, SGP8 and SDP8 use a ballistic coefficient (B term) in the drag equations rather than the B^* drag term. However, compatible predictions can be made with NORAD element sets by first calculating a B term from the SGP4 B^* drag term.

At the present time consideration is being given to replacing SGP4 and SDP4 by SGP8 and SDP8 as the NORAD satellite models. In such a case the new NORAD element sets would still give compatible predictions for SGP, SGP4, and SDP4 users and, for SGP8 and SDP8 users, would give agreement with NORAD predictions.

4 GENERAL PROGRAM DESCRIPTION

The five ephemeris packages cited in Section Two have each been programmed in FORTRAN IV as stand-alone subroutines. They each access the two function subroutines ACTAN and FMOD2P and the deep-space equations access the function subroutine THETAG. The function subroutine ACTAN is a two argument (quadrant preserving) arctangent subroutine which has been specifically designed to return the angle within the range of 0 to 2π . The function subroutine FMOD2P takes an angle and returns the modulo by 2π of that angle. The function subroutine THETAG calculates the epoch time in days since 1950 Jan 0.0 UTC, stores this in COMMON, and returns the right ascension of Greenwich at epoch.

One additional subroutine DEEP is accessed by SDP4 and SDP8 to obtain the deep-space perturbations to be added to the main equations of motion.

The main program DRIVER reads the input NORAD 2-line element set in either G-card internal format or T-card transmission format and calls the appropriate ephemeris package as specified by the user. The DRIVER converts the elements to the units of radians and minutes before calling the appropriate subroutine. The ephemeris package returns position and velocity in units of Earth radii and minutes. These are converted by the DRIVER to kilometers and seconds for printout.

All physical constants are contained in the constants COMMON C1 and can be changed through the data statements in the DRIVER. The one exception is the physical constants used only in DEEP which are set in the data statements in DEEP.

In the following sections the equations and program listing are given for each ephemeris model. Every effort has been made to maintain a strict parallel structure between the equations and the computer code.

5 THE SGP MODEL

The NORAD mean element sets can be used for prediction with SGP. All symbols not defined below are defined in the list of symbols in Section Twelve. Predictions are made by first calculating the constants

$$a_1 = \left(\frac{k_e}{n_o} \right)^{\frac{2}{3}}$$

$$\delta_1 = \frac{3}{4} J_2 \frac{a_E^2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$a_o = a_1 \left[1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3 \right]$$

$$p_o = a_o (1 - e_o^2)$$

$$q_o = a_o (1 - e_o)$$

$$L_o = M_o + \omega_o + \Omega_o$$

$$\frac{d\Omega}{dt} = -\frac{3}{2} J_2 \frac{a_E^2}{p_o^2} n_o \cos i_o$$

$$\frac{d\omega}{dt} = \frac{3}{4} J_2 \frac{a_E^2}{p_o^2} n_o (5 \cos^2 i_o - 1).$$

The secular effects of atmospheric drag and gravitation are included through the equations

$$a = a_o \left\{ \frac{n_o}{n_o + 2 \left(\frac{\dot{n}_o}{2} \right) (t - t_o) + 3 \left(\frac{\ddot{n}_o}{6} \right) (t - t_o)^2} \right\}^{\frac{2}{3}}$$

$$e = \begin{cases} 1 - \frac{q_o}{a}, & \text{for } a > q_o \\ 10^{-6}, & \text{for } a \leq q_o \end{cases}$$

$$p = a(1 - e^2)$$

$$\Omega_{s_o} = \Omega_o + \frac{d\Omega}{dt}(t - t_o)$$

$$\omega_{s_o} = \omega_o + \frac{d\omega}{dt}(t - t_o)$$

$$L_s = L_o + \left(n_o + \frac{d\omega}{dt} + \frac{d\Omega}{dt} \right) (t - t_o) + \frac{\dot{n}_o}{2} (t - t_o)^2 + \frac{\ddot{n}_o}{6} (t - t_o)^3$$

where $(t - t_o)$ is time since epoch.

Long-period periodics are included through the equations

$$a_{yNSL} = e \sin \omega_{s_o} - \frac{1}{2} \frac{J_3}{J_2} \frac{a_E}{p} \sin i_o$$

$$L = L_s - \frac{1}{4} \frac{J_3}{J_2} \frac{a_E}{p} a_{xNSL} \sin i_o \left[\frac{3 + 5 \cos i_o}{1 + \cos i_o} \right]$$

where

$$a_{xNSL} = e \cos \omega_{s_o}.$$

Solve Kepler's equation for $E + \omega$ (by iteration to the desired accuracy), where

$$(E + \omega)_{i+1} = (E + \omega)_i + \Delta(E + \omega)_i$$

with

$$\Delta(E + \omega)_i = \frac{U - a_{yNSL} \cos(E + \omega)_i + a_{xNSL} \sin(E + \omega)_i - (E + \omega)_i}{-a_{yNSL} \sin(E + \omega)_i - a_{xNSL} \cos(E + \omega)_i + 1}$$

$$U = L - \Omega_{s_o}$$

and

$$(E + \omega)_1 = U.$$

Then calculate the intermediate (partially osculating) quantities

$$e \cos E = a_{xNSL} \cos(E + \omega) + a_{yNSL} \sin(E + \omega)$$

$$e \sin E = a_{xNSL} \sin(E + \omega) - a_{yNSL} \cos(E + \omega)$$

$$e_L^2 = (a_{xNSL})^2 + (a_{yNSL})^2$$

$$p_L = a(1 - e_L^2)$$

$$r = a(1 - e \cos E)$$

$$\dot{r} = k_e \frac{\sqrt{a}}{r} e \sin E$$

$$r \dot{v} = k_e \frac{\sqrt{p_L}}{r}$$

$$\sin u = \frac{a}{r} \left[\sin(E + \omega) - a_{yNSL} - a_{xNSL} \frac{e \sin E}{1 + \sqrt{1 - e_L^2}} \right]$$

$$\cos u = \frac{a}{r} \left[\cos(E + \omega) - a_{xNSL} + a_{yNSL} \frac{e \sin E}{1 + \sqrt{1 - e_L^2}} \right]$$

$$u = \tan^{-1} \left(\frac{\sin u}{\cos u} \right).$$

Short-period perturbations are now included by

$$r_k = r + \frac{1}{4} J_2 \frac{a_E^2}{p_L} \sin^2 i_o \cos 2u$$

$$u_k = u - \frac{1}{8} J_2 \frac{a_E^2}{p_L^2} (7 \cos^2 i_o - 1) \sin 2u$$

$$\Omega_k = \Omega_{s_o} + \frac{3}{4} J_2 \frac{a_E^2}{p_L^2} \cos i_o \sin 2u$$

$$i_k = i_o + \frac{3}{4} J_2 \frac{a_E^2}{p_L^2} \sin i_o \cos i_o \cos 2u.$$

Then unit orientation vectors are calculated by

$$\mathbf{U} = \mathbf{M} \sin u_k + \mathbf{N} \cos u_k$$

$$\mathbf{V} = \mathbf{M} \cos u_k - \mathbf{N} \sin u_k$$

where

$$\mathbf{M} = \left\{ \begin{array}{l} M_x = -\sin \Omega_k \cos i_k \\ M_y = \cos \Omega_k \cos i_k \\ M_z = \sin i_k \end{array} \right\}$$

$$\mathbf{N} = \left\{ \begin{array}{l} N_x = \cos \Omega_k \\ N_y = \sin \Omega_k \\ N_z = 0 \end{array} \right\}.$$

Then position and velocity are given by

$$\mathbf{r} = r_k \mathbf{U}$$

and

$$\dot{\mathbf{r}} = \dot{r} \mathbf{U} + (r\dot{v}) \mathbf{V}.$$

A FORTRAN IV computer code listing of the subroutine SGP is given below.

```

*          SGP                                     31 OCT 80
SUBROUTINE SGP(IFLAG,TSINCE)
COMMON/E1/XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT20,XNDD60,BSTAR,
1      X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
1      XJ3,XKE,XKMPER,XMNPDA,AE
DOUBLE PRECISION EPOCH, DS50

IF(IFLAG.EQ.0) GO TO 19

*          INITIALIZATION

C1= CK2*1.5
C2= CK2/4.0
C3= CK2/2.0
C4= XJ3*AE**3/(4.0*CK2)
COSIO=COS(XINCL)
SINIO=SIN(XINCL)
A1=(XKE/XNO)**TOTHRD
D1=      C1/A1/A1*(3.*COSIO*COSIO-1.)/(1.-EO*EO)**1.5
AO=A1*(1.-1./3.*D1-D1*D1-134./81.*D1*D1*D1)
PO=AO*(1.-EO*EO)
QO=AO*(1.-EO)
XLO=XMO+OMEGAO+XNODEO
D10= C3 *SINIO*SINIO
D20= C2 *(7.*COSIO*COSIO-1.)
D30=C1*COSIO
D40=D30*SINIO
PO2NO=XNO/(PO*PO)
OMGDT=C1*PO2NO*(5.*COSIO*COSIO-1.)
XNODOT=-2.*D30*PO2NO
C5=.5*C4*SINIO*(3.+5.*COSIO)/(1.+COSIO)
C6=C4*SINIO
IFLAG=0

*          UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG

19 A=XNO+(2.*XNDT20+3.*XNDD60*TSINCE)*TSINCE
A=AO*(XNO/A)**TOTHRD
E=E6A
IF(A.GT.QO) E=1.-QO/A
P=A*(1.-E*E)
XNODES= XNODEO+XNODOT*TSINCE
OMGAS= OMEGAO+OMGDT*TSINCE
XLS=FMOD2P(XLO+(XNO+OMGDT+XNODOT+(XNDT20+XNDD60*TSINCE)*
1 TSINCE)*TSINCE)

*          LONG PERIOD PERIODICS

```

```

    AXNSL=E*COS(OMGAS)
    AYNSL=E*SIN(OMGAS)-C6/P
    XL=FMOD2P(XLS-C5/P*AXNSL)

*      SOLVE KEPLERS EQUATION

    U=FMOD2P(XL-XNODES)
    ITEM3=0
    E01=U
    TEM5=1.
20  SINE01=SIN(E01)
    COSE01=COS(E01)
    IF(ABS(TEM5).LT.E6A) GO TO 30
    IF(ITEM3.GE.10) GO TO 30
    ITEM3=ITEM3+1
    TEM5=1.-COSE01*AXNSL-SINE01*AYNSL
    TEM5=(U-AYNSL*COSE01+AXNSL*SINE01-E01)/TEM5
    TEM2=ABS(TEM5)
    IF(TEM2.GT.1.) TEM5=TEM2/TEM5
    E01=E01+TEM5
    GO TO 20

*      SHORT PERIOD PRELIMINARY QUANTITIES

30  ECOSE=AXNSL*COSE01+AYNSL*SINE01
    ESINE=AXNSL*SINE01-AYNSL*COSE01
    EL2=AXNSL*AXNSL+AYNSL*AYNSL
    PL=A*(1.-EL2)
    PL2=PL*PL
    R=A*(1.-ECOSE)
    RDOT=XKE*SQRT(A)/R*ESINE
    RVDOT=XKE*SQRT(PL)/R
    TEMP=ESINE/(1.+SQRT(1.-EL2))
    SINU=A/R*(SINE01-AYNSL-AXNSL*TEMP)
    COSU=A/R*(COSE01-AXNSL+AYNSL*TEMP)
    SU=ACTAN(SINU,COSU)

*      UPDATE FOR SHORT PERIODICS

    SIN2U=(COSU+COSU)*SINU
    COS2U=1.-2.*SINU*SINU
    RK=R+D10/PL*COS2U
    UK=SU-D20/PL2*SIN2U
    XNODEK=XNODES+D30*SIN2U/PL2
    XINCK =XINCL+D40/PL2*COS2U

*      ORIENTATION VECTORS

```

```

SINUK=SIN(UK)
COSUK=COS(UK)
SINNOK=SIN(XNODEK)
COSNOK=COS(XNODEK)
SINIK=SIN(XINCK)
COSIK=COS(XINCK)
XMX=-SINNOK*COSIK
XMY=COSNOK*COSIK
UX=XMX*SINUK+COSNOK*COSUK
UY=XMY*SINUK+SINNOK*COSUK
UZ=SINIK*SINUK
VX=XMX*COSUK-COSNOK*SINUK
VY=XMY*COSUK-SINNOK*SINUK
VZ=SINIK*COSUK

```

* POSITION AND VELOCITY

```

X=RK*UX
Y=RK*UY
Z=RK*UZ
XDOT=RDOT*UX
YDOT=RDOT*UY
ZDOT=RDOT*UZ
XDOT=RVDOT*VX+XDOT
YDOT=RVDOT*VY+YDOT
ZDOT=RVDOT*VZ+ZDOT

```

```

RETURN
END

```

6 THE SGP4 MODEL

The NORAD mean element sets can be used for prediction with SGP4. All symbols not defined below are defined in the list of symbols in Section Twelve. The original mean motion (n_o'') and semimajor axis (a_o'') are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o} \right)^{\frac{2}{3}}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3 \right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$n_o'' = \frac{n_o}{1 + \delta_o}$$

$$a_o'' = \frac{a_o}{1 - \delta_o}.$$

For perigee between 98 kilometers and 156 kilometers, the value of the constant s used in SGP4 is changed to

$$s^* = a_o''(1 - e_o) - s + a_E$$

For perigee below 98 kilometers, the value of s is changed to

$$s^* = 20/\text{XKMPER} + a_E.$$

If the value of s is changed, then the value of $(q_o - s)^4$ must be replaced by

$$(q_o - s^*)^4 = \left[[(q_o - s)^4]^{\frac{1}{4}} + s - s^* \right]^4.$$

Then calculate the constants (using the appropriate values of s and $(q_o - s)^4$)

$$\theta = \cos i_o$$

$$\xi = \frac{1}{a_o'' - s}$$

$$\beta_o = (1 - e_o^2)^{\frac{1}{2}}$$

$$\eta = a_o'' e_o \xi$$

$$C_2 = (q_o - s)^4 \xi^4 n_o'' (1 - \eta^2)^{-\frac{7}{2}} \left[a_o'' \left(1 + \frac{3}{2} \eta^2 + 4e_o \eta + e_o \eta^3 \right) + \frac{3}{2} \frac{k_2 \xi}{(1 - \eta^2)} \left(-\frac{1}{2} + \frac{3}{2} \theta^2 \right) (8 + 24\eta^2 + 3\eta^4) \right]$$

$$C_1 = B^* C_2$$

$$C_3 = \frac{(q_o - s)^4 \xi^5 A_{3,0} n_o'' a_E \sin i_o}{k_2 e_o}$$

$$C_4 = 2n_o'' (q_o - s)^4 \xi^4 a_o'' \beta_o^2 (1 - \eta^2)^{-\frac{7}{2}} \left(\left[2\eta(1 + e_o \eta) + \frac{1}{2} e_o + \frac{1}{2} \eta^3 \right] - \frac{2k_2 \xi}{a_o'' (1 - \eta^2)} \times \left[3(1 - 3\theta^2) \left(1 + \frac{3}{2} \eta^2 - 2e_o \eta - \frac{1}{2} e_o \eta^3 \right) + \frac{3}{4} (1 - \theta^2) (2\eta^2 - e_o \eta - e_o \eta^3) \cos 2\omega_o \right] \right)$$

$$C_5 = 2(q_o - s)^4 \xi^4 a_o'' \beta_o^2 (1 - \eta^2)^{-\frac{7}{2}} \left[1 + \frac{11}{4} \eta(\eta + e_o) + e_o \eta^3 \right]$$

$$D_2 = 4a_o'' \xi C_1^2$$

$$D_3 = \frac{4}{3} a_o'' \xi^2 (17a_o'' + s) C_1^3$$

$$D_4 = \frac{2}{3} a_o'' \xi^3 (221a_o'' + 31s) C_1^4.$$

The secular effects of atmospheric drag and gravitation are included through the equations

$$M_{DF} = M_o + \left[1 + \frac{3k_2(-1 + 3\theta^2)}{2a_o''^2 \beta_o^3} + \frac{3k_2^2(13 - 78\theta^2 + 137\theta^4)}{16a_o''^4 \beta_o^7} \right] n_o''(t - t_o)$$

$$\omega_{DF} = \omega_o + \left[-\frac{3k_2(1 - 5\theta^2)}{2a_o''^2 \beta_o^4} + \frac{3k_2^2(7 - 114\theta^2 + 395\theta^4)}{16a_o''^4 \beta_o^8} + \frac{5k_4(3 - 36\theta^2 + 49\theta^4)}{4a_o''^4 \beta_o^8} \right] n_o''(t - t_o)$$

$$\Omega_{DF} = \Omega_o + \left[-\frac{3k_2\theta}{a_o''^2\beta_o^4} + \frac{3k_2^2(4\theta - 19\theta^3)}{2a_o''^4\beta_o^8} + \frac{5k_4\theta(3 - 7\theta^2)}{2a_o''^4\beta_o^8} \right] n_o''(t - t_o)$$

$$\delta\omega = B^*C_3(\cos \omega_o)(t - t_o)$$

$$\delta M = -\frac{2}{3}(q_o - s)^4 B^* \xi^4 \frac{a_E}{e_o \eta} [(1 + \eta \cos M_{DF})^3 - (1 + \eta \cos M_o)^3]$$

$$M_p = M_{DF} + \delta\omega + \delta M$$

$$\omega = \omega_{DF} - \delta\omega - \delta M$$

$$\Omega = \Omega_{DF} - \frac{21}{2} \frac{n_o'' k_2 \theta}{a_o''^2 \beta_o^2} C_1 (t - t_o)^2$$

$$e = e_o - B^*C_4(t - t_o) - B^*C_5(\sin M_p - \sin M_o)$$

$$a = a_o''[1 - C_1(t - t_o) - D_2(t - t_o)^2 - D_3(t - t_o)^3 - D_4(t - t_o)^4]^2$$

$$\begin{aligned} \mathcal{L} = & M_p + \omega + \Omega + n_o'' \left[\frac{3}{2} C_1 (t - t_o)^2 + (D_2 + 2C_1^2)(t - t_o)^3 \right. \\ & + \frac{1}{4} (3D_3 + 12C_1 D_2 + 10C_1^3)(t - t_o)^4 \\ & \left. + \frac{1}{5} (3D_4 + 12C_1 D_3 + 6D_2^2 + 30C_1^2 D_2 + 15C_1^4)(t - t_o)^5 \right] \end{aligned}$$

$$\beta = \sqrt{(1 - e^2)}$$

$$n = k_e / a^{\frac{3}{2}}$$

where $(t - t_o)$ is time since epoch. It should be noted that when epoch perigee height is less than 220 kilometers, the equations for a and \mathcal{L} are truncated after the C_1 term, and the terms involving C_5 , $\delta\omega$, and δM are dropped.

Add the long-period periodic terms

$$a_{xN} = e \cos \omega$$

$$\mathcal{I}_L = \frac{A_{3,0} \sin i_o}{8k_2 a \beta^2} (e \cos \omega) \left(\frac{3 + 5\theta}{1 + \theta} \right)$$

$$a_{yNL} = \frac{A_{3,0} \sin i_o}{4k_2 a \beta^2}$$

$$\mathcal{I}_T = \mathcal{I} + \mathcal{I}_L$$

$$a_{yN} = e \sin \omega + a_{yNL}.$$

Solve Kepler's equation for $(E + \omega)$ by defining

$$U = \mathcal{I}_T - \Omega$$

and using the iteration equation

$$(E + \omega)_{i+1} = (E + \omega)_i + \Delta(E + \omega)_i$$

with

$$\Delta(E + \omega)_i = \frac{U - a_{yN} \cos(E + \omega)_i + a_{xN} \sin(E + \omega)_i - (E + \omega)_i}{-a_{yN} \sin(E + \omega)_i - a_{xN} \cos(E + \omega)_i + 1}$$

and

$$(E + \omega)_1 = U.$$

The following equations are used to calculate preliminary quantities needed for short-period periodics.

$$e \cos E = a_{xN} \cos(E + \omega) + a_{yN} \sin(E + \omega)$$

$$e \sin E = a_{xN} \sin(E + \omega) - a_{yN} \cos(E + \omega)$$

$$e_L = (a_{xN}^2 + a_{yN}^2)^{\frac{1}{2}}$$

$$p_L = a(1 - e_L^2)$$

$$r = a(1 - e \cos E)$$

$$\dot{r} = k_e \frac{\sqrt{a}}{r} e \sin E$$

$$r \dot{f} = k_e \frac{\sqrt{p_L}}{r}$$

$$\cos u = \frac{a}{r} \left[\cos(E + \omega) - a_{xN} + \frac{a_{yN}(e \sin E)}{1 + \sqrt{1 - e_L^2}} \right]$$

$$\sin u = \frac{a}{r} \left[\sin(E + \omega) - a_{yN} - \frac{a_{xN}(e \sin E)}{1 + \sqrt{1 - e_L^2}} \right]$$

$$u = \tan^{-1} \left(\frac{\sin u}{\cos u} \right)$$

$$\Delta r = \frac{k_2}{2p_L} (1 - \theta^2) \cos 2u$$

$$\Delta u = -\frac{k_2}{4p_L^2} (7\theta^2 - 1) \sin 2u$$

$$\Delta \Omega = \frac{3k_2\theta}{2p_L^2} \sin 2u$$

$$\Delta i = \frac{3k_2\theta}{2p_L^2} \sin i_o \cos 2u$$

$$\Delta \dot{r} = -\frac{k_2 n}{p_L} (1 - \theta^2) \sin 2u$$

$$\Delta r \dot{f} = \frac{k_2 n}{p_L} \left[(1 - \theta^2) \cos 2u - \frac{3}{2} (1 - 3\theta^2) \right]$$

The short-period periodics are added to give the osculating quantities

$$r_k = r \left[1 - \frac{3}{2} k_2 \frac{\sqrt{1 - e_L^2}}{p_L^2} (3\theta^2 - 1) \right] + \Delta r$$

$$u_k = u + \Delta u$$

$$\Omega_k = \Omega + \Delta\Omega$$

$$i_k = i_o + \Delta i$$

$$\dot{r}_k = \dot{r} + \Delta\dot{r}$$

$$r\dot{f}_k = r\dot{f} + \Delta r\dot{f}.$$

Then unit orientation vectors are calculated by

$$\mathbf{U} = \mathbf{M} \sin u_k + \mathbf{N} \cos u_k$$

$$\mathbf{V} = \mathbf{M} \cos u_k - \mathbf{N} \sin u_k$$

where

$$\mathbf{M} = \left\{ \begin{array}{l} M_x = -\sin \Omega_k \cos i_k \\ M_y = \cos \Omega_k \cos i_k \\ M_z = \sin i_k \end{array} \right\}$$

$$\mathbf{N} = \left\{ \begin{array}{l} N_x = \cos \Omega_k \\ N_y = \sin \Omega_k \\ N_z = 0 \end{array} \right\}.$$

Then position and velocity are given by

$$\mathbf{r} = r_k \mathbf{U}$$

and

$$\dot{\mathbf{r}} = \dot{r}_k \mathbf{U} + (r\dot{f})_k \mathbf{V}.$$

A FORTRAN IV computer code listing of the subroutine SGP4 is given below. These equations contain all currently anticipated changes to the SCC operational program. These changes are scheduled for implementation in March, 1981.

```

*          SGP4                                     3 NOV 80
SUBROUTINE SGP4(IFLAG,TSINCE)
COMMON/E1/XMO,XNODEO,OMEGAO,E0,XINCL,XNO,XNDT20,
1          XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
1          XJ3,XKE,XKMPER,XMNPDA,AE
DOUBLE PRECISION EPOCH, DS50

IF (IFLAG .EQ. 0) GO TO 100

*          RECOVER ORIGINAL MEAN MOTION (XNDP) AND SEMIMAJOR AXIS (AODP)
*          FROM INPUT ELEMENTS

A1=(XKE/XNO)**TOTHRD
COSIO=COS(XINCL)
THETA2=COSIO*COSIO
X3THM1=3.*THETA2-1.
EOSQ=E0*E0
BETA02=1.-EOSQ
BETA0=SQRT(BETA02)
DEL1=1.5*CK2*X3THM1/(A1*A1*BETA0*BETA02)
AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
DELO=1.5*CK2*X3THM1/(AO*AO*BETA0*BETA02)
XNDP=XNO/(1.+DELO)
AODP=AO/(1.-DELO)

*          INITIALIZATION

*          FOR PERIGEE LESS THAN 220 KILOMETERS, THE ISIMP FLAG IS SET AND
*          THE EQUATIONS ARE TRUNCATED TO LINEAR VARIATION IN SQRT A AND
*          QUADRATIC VARIATION IN MEAN ANOMALY.  ALSO, THE C3 TERM, THE
*          DELTA OMEGA TERM, AND THE DELTA M TERM ARE DROPPED.

ISIMP=0
IF((AODP*(1.-E0)/AE) .LT. (220./XKMPER+AE)) ISIMP=1

*          FOR PERIGEE BELOW 156 KM, THE VALUES OF
*          S AND QOMS2T ARE ALTERED

S4=S
QOMS24=QOMS2T
PERIGE=(AODP*(1.-E0)-AE)*XKMPER
IF(PERIGE .GE. 156.) GO TO 10
S4=PERIGE-78.
IF(PERIGE .GT. 98.) GO TO 9
S4=20.
9 QOMS24=((120.-S4)*AE/XKMPER)**4
S4=S4/XKMPER+AE

```

```

10 PINVSQ=1./(AODP*AODP*BETA02*BETA02)
   TSI=1./(AODP-S4)
   ETA=AODP*EO*TSI
   ETASQ=ETA*ETA
   EETA=EO*ETA
   PSISQ=ABS(1.-ETASQ)
   COEF=QOMS24*TSI**4
   COEF1=COEF/PSISQ**3.5
   C2=COEF1*XNODP*(AODP*(1.+1.5*ETASQ+EETA*(4.+ETASQ))+.75*
1       CK2*TSI/PSISQ*X3THM1*(8.+3.*ETASQ*(8.+ETASQ)))
   C1=BSTAR*C2
   SINIO=SIN(XINCL)
   A3OVK2=-XJ3/CK2*AE**3
   C3=COEF*TSI*A3OVK2*XNODP*AE*SINIO/EO
   X1MTH2=1.-THETA2
   C4=2.*XNODP*COEF1*AODP*BETA02*(ETA*
1       (2.+5*ETASQ)+EO*(.5+2.*ETASQ)-2.*CK2*TSI/
2       (AODP*PSISQ)*(-3.*X3THM1*(1.-2.*EETA+ETASQ*
3       (1.5-.5*EETA))+.75*X1MTH2*(2.*ETASQ-EETA*
4       (1.+ETASQ))*COS(2.*OMEGA0)))
   C5=2.*COEF1*AODP*BETA02*(1.+2.75*(ETASQ+EETA)+EETA*ETASQ)
   THETA4=THETA2*THETA2
   TEMP1=3.*CK2*PINVSQ*XNODP
   TEMP2=TEMP1*CK2*PINVSQ
   TEMP3=1.25*CK4*PINVSQ*PINVSQ*XNODP
   XMDOT=XNODP+.5*TEMP1*BETA0*X3THM1+.0625*TEMP2*BETA0*
1       (13.-78.*THETA2+137.*THETA4)
   X1M5TH=1.-5.*THETA2
   OMGDOT=-.5*TEMP1*X1M5TH+.0625*TEMP2*(7.-114.*THETA2+
1       395.*THETA4)+TEMP3*(3.-36.*THETA2+49.*THETA4)
   XHDOT1=-TEMP1*COSIO
   XNODOT=XHDOT1+(.5*TEMP2*(4.-19.*THETA2)+2.*TEMP3*(3.-
1       7.*THETA2))*COSIO
   OMGCOF=BSTAR*C3*COS(OMEGA0)
   XMCOF=-TOTHDRD*COEF*BSTAR*AE/EETA
   XNODCF=3.5*BETA02*XHDOT1*C1
   T2COF=1.5*C1
   XLCOF=.125*A3OVK2*SINIO*(3.+5.*COSIO)/(1.+COSIO)
   AYCOF=.25*A3OVK2*SINIO
   DELMO=(1.+ETA*COS(XMO))**3
   SINMO=SIN(XMO)
   X7THM1=7.*THETA2-1.
   IF(ISIMP.EQ. 1) GO TO 90
   C1SQ=C1*C1
   D2=4.*AODP*TSI*C1SQ
   TEMP=D2*TSI*C1/3.
   D3=(17.*AODP+S4)*TEMP
   D4=.5*TEMP*AODP*TSI*(221.*AODP+31.*S4)*C1

```

```

      T3COF=D2+2.*C1SQ
      T4COF=.25*(3.*D3+C1*(12.*D2+10.*C1SQ))
      T5COF=.2*(3.*D4+12.*C1*D3+6.*D2*D2+15.*C1SQ*(
1          2.*D2+C1SQ))
90 IFLAG=0

*      UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG

100 XMDF=XMO+XMDOT*TSINCE
      OMGADF=OMGAO+OMGDOT*TSINCE
      XNODDF=XNODEO+XNODOT*TSINCE
      OMEGA=OMGADF
      XMP=XMDF
      TSQ=TSINCE*TSINCE
      XNODE=XNODDF+XNODCF*TSQ
      TEMPA=1.-C1*TSINCE
      TEMPE=BSTAR*C4*TSINCE
      TEMPL=T2COF*TSQ
      IF(ISIMP .EQ. 1) GO TO 110
      DELOMG=OMGCOF*TSINCE
      DELM=XMCOF*((1.+ETA*COS(XMDF))**3-DELMO)
      TEMP=DELOMG+DELM
      XMP=XMDF+TEMP
      OMEGA=OMGADF-TEMP
      TCUBE=TSQ*TSINCE
      TFOUR=TSINCE*TCUBE
      TEMPA=TEMPA-D2*TSQ-D3*TCUBE-D4*TFOUR
      TEMPE=TEMPE+BSTAR*C5*(SIN(XMP)-SINMO)
      TEMPL=TEMPL+T3COF*TCUBE+
1          TFOUR*(T4COF+TSINCE*T5COF)
110 A=AODP*TEMPA**2
      E=EO-TEMPE
      XL=XMP+OMEGA+XNODE+XNODP*TEMPL
      BETA=SQRT(1.-E*E)
      XN=XKE/A**1.5

*      LONG PERIOD PERIODICS

      AXN=E*COS(OMEGA)
      TEMP=1./(A*BETA*BETA)
      XLL=TEMP*XLCOF*AXN
      AYNL=TEMP*AYCOF
      XLT=XL+XLL
      AYN=E*SIN(OMEGA)+AYNL

*      SOLVE KEPLERS EQUATION

      CAPU=FMOD2P(XLT-XNODE)

```

```

TEMP2=CAPU
DO 130 I=1,10
SINEPW=SIN(TEMP2)
COSEPW=COS(TEMP2)
TEMP3=AXN*SINEPW
TEMP4=AYN*COSEPW
TEMP5=AXN*COSEPW
TEMP6=AYN*SINEPW
EPW=(CAPU-TEMP4+TEMP3-TEMP2)/(1.-TEMP5-TEMP6)+TEMP2
IF(ABS(EPW-TEMP2) .LE. E6A) GO TO 140
130 TEMP2=EPW

```

* SHORT PERIOD PRELIMINARY QUANTITIES

```

140 ECOSE=TEMP5+TEMP6
ESINE=TEMP3-TEMP4
ELSQ=AXN*AXN+AYN*AYN
TEMP=1.-ELSQ
PL=A*TEMP
R=A*(1.-ECOSE)
TEMP1=1./R
RDOT=XKE*SQRT(A)*ESINE*TEMP1
RFDOT=XKE*SQRT(PL)*TEMP1
TEMP2=A*TEMP1
BETAL=SQRT(TEMP)
TEMP3=1./(1.+BETAL)
COSU=TEMP2*(COSEPW-AXN+AYN*ESINE*TEMP3)
SINU=TEMP2*(SINEPW-AYN-AXN*ESINE*TEMP3)
U=ACTAN(SINU,COSU)
SIN2U=2.*SINU*COSU
COS2U=2.*COSU*COSU-1.
TEMP=1./PL
TEMP1=CK2*TEMP
TEMP2=TEMP1*TEMP

```

* UPDATE FOR SHORT PERIODICS

```

RK=R*(1.-1.5*TEMP2*BETAL*X3THM1)+.5*TEMP1*X1MTH2*COS2U
UK=U-.25*TEMP2*X7THM1*SIN2U
XNODEK=XNODE+1.5*TEMP2*COSIO*SIN2U
XINCK=XINCL+1.5*TEMP2*COSIO*SINIO*COS2U
RDOTK=RDOT-XN*TEMP1*X1MTH2*SIN2U
RFDOTK=RFDOT+XN*TEMP1*(X1MTH2*COS2U+1.5*X3THM1)

```

* ORIENTATION VECTORS

```

SINUK=SIN(UK)
COSUK=COS(UK)

```

```

SINIK=SIN(XINCK)
COSIK=COS(XINCK)
SINNOK=SIN(XNODEK)
COSNOK=COS(XNODEK)
XMX=-SINNOK*COSIK
XMY=COSNOK*COSIK
UX=XMX*SINUK+COSNOK*COSUK
UY=XMY*SINUK+SINNOK*COSUK
UZ=SINIK*SINUK
VX=XMX*COSUK-COSNOK*SINUK
VY=XMY*COSUK-SINNOK*SINUK
VZ=SINIK*COSUK

```

```

*      POSITION AND VELOCITY

```

```

X=RK*UX
Y=RK*UY
Z=RK*UZ
XDOT=RDOTK*UX+RFDOTK*VX
YDOT=RDOTK*UY+RFDOTK*VY
ZDOT=RDOTK*UZ+RFDOTK*VZ

```

```

RETURN
END

```

7 THE SDP4 MODEL

The NORAD mean element sets can be used for prediction with SDP4. All symbols not defined below are defined in the list of symbols in Section Twelve. The original mean motion (n_o'') and semimajor axis (a_o'') are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o} \right)^{\frac{2}{3}}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3 \right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$n_o'' = \frac{n_o}{1 + \delta_o}$$

$$a_o'' = \frac{a_o}{1 - \delta_o}.$$

For perigee between 98 kilometers and 156 kilometers, the value of the constant s used in SDP4 is changed to

$$s^* = a_o''(1 - e_o) - s + a_E.$$

For perigee below 98 kilometers, the value of s is changed to

$$s^* = 20/\text{XKMPER} + a_E.$$

If the value of s is changed, then the value of $(q_o - s)^4$ must be replaced by

$$(q_o - s^*)^4 = \left[[(q_o - s)^4]^{\frac{1}{4}} + s - s^* \right]^4.$$

Then calculate the constants (using the appropriate values of s and $(q_o - s)^4$)

$$\theta = \cos i_o$$

$$\xi = \frac{1}{a_o'' - s}$$

$$\beta_o = (1 - e_o^2)^{\frac{1}{2}}$$

$$\eta = a_o'' e_o \xi$$

$$C_2 = (q_o - s)^4 \xi^4 n_o'' (1 - \eta^2)^{-\frac{7}{2}} \left[a_o'' (1 + \frac{3}{2} \eta^2 + 4e_o \eta + e_o \eta^3) + \frac{3}{2} \frac{k_2 \xi}{(1 - \eta^2)} \left(-\frac{1}{2} + \frac{3}{2} \theta^2 \right) (8 + 24\eta^2 + 3\eta^4) \right]$$

$$C_1 = B^* C_2$$

$$C_4 = 2n_o'' (q_o - s)^4 \xi^4 a_o'' \beta_o^2 (1 - \eta^2)^{-\frac{7}{2}} \left(\left[2\eta(1 + e_o \eta) + \frac{1}{2} e_o + \frac{1}{2} \eta^3 \right] - \frac{2k_2 \xi}{a_o'' (1 - \eta^2)} \times \left[3(1 - 3\theta^2) \left(1 + \frac{3}{2} \eta^2 - 2e_o \eta - \frac{1}{2} e_o \eta^3 \right) + \frac{3}{4} (1 - \theta^2) (2\eta^2 - e_o \eta - e_o \eta^3) \cos 2\omega_o \right] \right)$$

$$\dot{M} = \left[1 + \frac{3k_2(-1 + 3\theta^2)}{2a_o''^2 \beta_o^3} + \frac{3k_2^2(13 - 78\theta^2 + 137\theta^4)}{16a_o''^4 \beta_o^7} \right] n_o''$$

$$\dot{\omega} = \left[-\frac{3k_2(1 - 5\theta^2)}{2a_o''^2 \beta_o^4} + \frac{3k_2^2(7 - 114\theta^2 + 395\theta^4)}{16a_o''^4 \beta_o^8} + \frac{5k_4(3 - 36\theta^2 + 49\theta^4)}{4a_o''^4 \beta_o^8} \right] n_o''$$

$$\dot{\Omega}_1 = -\frac{3k_2 \theta}{a_o''^2 \beta_o^4} n_o''$$

$$\dot{\Omega} = \dot{\Omega}_1 + \left[\frac{3k_2^2(4\theta - 19\theta^3)}{2a_o''^4 \beta_o^8} + \frac{5k_4 \theta(3 - 7\theta^2)}{2a_o''^4 \beta_o^8} \right] n_o''.$$

At this point SDP4 calls the initialization section of DEEP which calculates all initialized quantities needed for the deep-space perturbations (see Section Ten).

The secular effects of gravity are included by

$$M_{DF} = M_o + \dot{M}(t - t_o)$$

$$\omega_{DF} = \omega_o + \dot{\omega}(t - t_o)$$

$$\Omega_{DF} = \Omega_o + \dot{\Omega}(t - t_o)$$

where $(t - t_o)$ is time since epoch. The secular effect of drag on longitude of ascending node is included by

$$\Omega = \Omega_{DF} - \frac{21}{2} \frac{n_o'' k_2 \theta}{a_o''^2 \beta_o^2} C_1 (t - t_o)^2.$$

Next, SDP4 calls the secular section of DEEP which adds the deep-space secular effects and long-period resonance effects to the six classical orbital elements (see Section Ten).

The secular effects of drag are included in the remaining elements by

$$a = a_{DS} [1 - C_1 (t - t_o)]^2$$

$$e = e_{DS} - B^* C_4 (t - t_o)$$

$$\mathcal{I}L = M_{DS} + \omega_{DS} + \Omega_{DS} + n_o'' \left[\frac{3}{2} C_1 (t - t_o)^2 \right]$$

where a_{DS} , e_{DS} , M_{DS} , ω_{DS} , and Ω_{DS} , are the values of n_o , e_o , M_{DF} , ω_{DF} , and Ω after deep-space secular and resonance perturbations have been applied.

Here SDP4 calls the periodics section of DEEP which adds the deep-space lunar and solar periodics to the orbital elements (see Section Ten). From this point on, it will be assumed that n , e , I , ω , Ω , and M are the mean motion, eccentricity, inclination, argument of perigee, longitude of ascending node, and mean anomaly after lunar-solar periodics have been added.

Add the long-period periodic terms

$$a_{xN} = e \cos \omega$$

$$\beta = \sqrt{(1 - e^2)}$$

$$\mathcal{I}L_L = \frac{A_{3,0} \sin i_o}{8k_2 a \beta^2} (e \cos \omega) \left(\frac{3 + 5\theta}{1 + \theta} \right)$$

$$a_{yNL} = \frac{A_{3,0} \sin i_o}{4k_2 a \beta^2}$$

$$\mathcal{I}L_T = \mathcal{I}L + \mathcal{I}L_L$$

$$a_{yN} = e \sin \omega + a_{yNL}.$$

Solve Kepler's equation for $(E + \omega)$ by defining

$$U = \mathbb{L}_T - \Omega$$

and using the iteration equation

$$(E + \omega)_{i+1} = (E + \omega)_i + \Delta(E + \omega)_i$$

with

$$\Delta(E + \omega)_i = \frac{U - a_{yN} \cos(E + \omega)_i + a_{xN} \sin(E + \omega)_i - (E + \omega)_i}{-a_{yN} \sin(E + \omega)_i - a_{xN} \cos(E + \omega)_i + 1}$$

and

$$(E + \omega)_1 = U.$$

The following equations are used to calculate preliminary quantities needed for short-period periodics.

$$e \cos E = a_{xN} \cos(E + \omega) + a_{yN} \sin(E + \omega)$$

$$e \sin E = a_{xN} \sin(E + \omega) - a_{yN} \cos(E + \omega)$$

$$e_L = (a_{xN}^2 + a_{yN}^2)^{\frac{1}{2}}$$

$$p_L = a(1 - e_L^2)$$

$$r = a(1 - e \cos E)$$

$$\dot{r} = k_e \frac{\sqrt{a}}{r} e \sin E$$

$$r \dot{f} = k_e \frac{\sqrt{p_L}}{r}$$

$$\cos u = \frac{a}{r} \left[\cos(E + \omega) - a_{xN} + \frac{a_{yN}(e \sin E)}{1 + \sqrt{1 - e_L^2}} \right]$$

$$\sin u = \frac{a}{r} \left[\sin(E + \omega) - a_{yN} - \frac{a_{xN}(e \sin E)}{1 + \sqrt{1 - e_L^2}} \right]$$

$$u = \tan^{-1} \left(\frac{\sin u}{\cos u} \right)$$

$$\Delta r = \frac{k_2}{2p_L} (1 - \theta^2) \cos 2u$$

$$\Delta u = -\frac{k_2}{4p_L^2} (7\theta^2 - 1) \sin 2u$$

$$\Delta \Omega = \frac{3k_2\theta}{2p_L^2} \sin 2u$$

$$\Delta i = \frac{3k_2\theta}{2p_L^2} \sin i_o \cos 2u$$

$$\Delta \dot{r} = -\frac{k_2 n}{p_L} (1 - \theta^2) \sin 2u$$

$$\Delta r \dot{f} = \frac{k_2 n}{p_L} \left[(1 - \theta^2) \cos 2u - \frac{3}{2} (1 - 3\theta^2) \right]$$

The short-period periodics are added to give the osculating quantities

$$r_k = r \left[1 - \frac{3}{2} k_2 \frac{\sqrt{1 - e_L^2}}{p_L^2} (3\theta^2 - 1) \right] + \Delta r$$

$$u_k = u + \Delta u$$

$$\Omega_k = \Omega + \Delta \Omega$$

$$i_k = I + \Delta i$$

$$\dot{r}_k = \dot{r} + \Delta \dot{r}$$

$$r \dot{f}_k = r \dot{f} + \Delta r \dot{f}.$$

Then unit orientation vectors are calculated by

$$\mathbf{U} = \mathbf{M} \sin u_k + \mathbf{N} \cos u_k$$

$$\mathbf{V} = \mathbf{M} \cos u_k - \mathbf{N} \sin u_k$$

where

$$\mathbf{M} = \left\{ \begin{array}{l} M_x = -\sin \Omega_k \cos i_k \\ M_y = \cos \Omega_k \cos i_k \\ M_z = \sin i_k \end{array} \right\}$$

$$\mathbf{N} = \left\{ \begin{array}{l} N_x = \cos \Omega_k \\ N_y = \sin \Omega_k \\ N_z = 0 \end{array} \right\}.$$

Then position and velocity are given by

$$\mathbf{r} = r_k \mathbf{U}$$

and

$$\dot{\mathbf{r}} = \dot{r}_k \mathbf{U} + (r \dot{f})_k \mathbf{V}.$$

A FORTRAN IV computer code listing of the subroutine SDP4 is given below. These equations contain all currently anticipated changes to the SCC operational program. These changes are scheduled for implementation in March, 1981.

```

*      SDP4
*      SUBROUTINE SDP4(IFLAG,TSINCE)
*      COMMON/E1/XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT20,
1      XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
*      COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
1      XJ3,XKE,XKMPER,XMNPDA,AE
*      DOUBLE PRECISION EPOCH, DS50

*      IF (IFLAG .EQ. 0) GO TO 100

*      RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
*      FROM INPUT ELEMENTS

A1=(XKE/XNO)**TOTHRD
COSIO=COS(XINCL)
THETA2=COSIO*COSIO
X3THM1=3.*THETA2-1.
EOSQ=EO*EO
BETA02=1.-EOSQ
BETA0=SQRT(BETA02)
DEL1=1.5*CK2*X3THM1/(A1*A1*BETA0*BETA02)
AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
DELO=1.5*CK2*X3THM1/(AO*AO*BETA0*BETA02)
XNODP=XNO/(1.+DELO)
AODP=AO/(1.-DELO)

*      INITIALIZATION

*      FOR PERIGEE BELOW 156 KM, THE VALUES OF
*      S AND QOMS2T ARE ALTERED

S4=S
QOMS24=QOMS2T
PERIGE=(AODP*(1.-EO)-AE)*XKMPER
IF(PERIGE .GE. 156.) GO TO 10
S4=PERIGE-78.
IF(PERIGE .GT. 98.) GO TO 9
S4=20.
9 QOMS24=((120.-S4)*AE/XKMPER)**4
S4=S4/XKMPER+AE
10 PINVSQ=1./(AODP*AODP*BETA02*BETA02)
SING=SIN(OMEGAO)
COSG=COS(OMEGAO)
TSI=1./(AODP-S4)
ETA=AODP*EO*TSI
ETASQ=ETA*ETA
EETA=EO*ETA
PSISQ=ABS(1.-ETASQ)

```

```

COEF=QOMS24*TSI**4
COEF1=COEF/PSISQ**3.5
C2=COEF1*XNODP*(AODP*(1.+1.5*ETASQ+EETA*(4.+ETASQ))+.75*
1      CK2*TSI/PSISQ*X3THM1*(8.+3.*ETASQ*(8.+ETASQ)))
C1=BSTAR*C2
SINIO=SIN(XINCL)
A3OVK2=-XJ3/CK2*AE**3
X1MTH2=1.-THETA2
C4=2.*XNODP*COEF1*AODP*BETA02*(ETA*
1      (2.+5*ETASQ)+EO*(.5+2.*ETASQ)-2.*CK2*TSI/
2      (AODP*PSISQ)*(-3.*X3THM1*(1.-2.*EETA+ETASQ*
3      (1.5-.5*EETA))+.75*X1MTH2*(2.*ETASQ-EETA*
4      (1.+ETASQ))*COS(2.*OMEGAO)))
THETA4=THETA2*THETA2
TEMP1=3.*CK2*PINVSQ*XNODP
TEMP2=TEMP1*CK2*PINVSQ
TEMP3=1.25*CK4*PINVSQ*PINVSQ*XNODP
XMDOT=XNODP+.5*TEMP1*BETA0*X3THM1+.0625*TEMP2*BETA0*
1      (13.-78.*THETA2+137.*THETA4)
X1M5TH=1.-5.*THETA2
OMGDOT=-.5*TEMP1*X1M5TH+.0625*TEMP2*(7.-114.*THETA2+
1      395.*THETA4)+TEMP3*(3.-36.*THETA2+49.*THETA4)
XHDOT1=-TEMP1*COSIO
XNODOT=XHDOT1+(.5*TEMP2*(4.-19.*THETA2)+2.*TEMP3*(3.-
1      7.*THETA2))*COSIO
XNODCF=3.5*BETA02*XHDOT1*C1
T2COF=1.5*C1
XLCOF=.125*A3OVK2*SINIO*(3.+5.*COSIO)/(1.+COSIO)
AYCOF=.25*A3OVK2*SINIO
X7THM1=7.*THETA2-1.
90 IFLAG=0
CALL DPINIT(EOSQ,SINIO,COSIO,BETA0,AODP,THETA2,
1      SING,COSG,BETA02,XMDOT,OMGDOT,XNODOT,XNODP)

*      UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG

100 XMDF=XMO+XMDOT*TSINCE
OMGADF=OMEGAO+OMGDOT*TSINCE
XNODDF=XNODEO+XNODOT*TSINCE
TSQ=TSINCE*TSINCE
XNODE=XNODDF+XNODCF*TSQ
TEMPA=1.-C1*TSINCE
TEMPE=BSTAR*C4*TSINCE
TEMPL=T2COF*TSQ
XN=XNODP
CALL DPSEC(XMDF,OMGADF,XNODE,EM,XINC,XN,TSINCE)
A=(XKE/XN)**TOTHDR*TEMPA**2
E=EM-TEMPE

```

```

XMAM=XMDF+XNODP*TEMPL
CALL DPPER(E,XINC,OMGADF,XNODE,XMAM)
XL=XMAM+OMGADF*XNODE
BETA=SQRT(1.-E*E)
XN=XKE/A**1.5

*      LONG PERIOD PERIODICS

AXN=E*COS(OMGADF)
TEMP=1./(A*BETA*BETA)
XLL=TEMP*XLCOF*AXN
AYNL=TEMP*AYCOF
XLT=XL+XLL
AYN=E*SIN(OMGADF)+AYNL

*      SOLVE KEPLERS EQUATION

CAPU=FMOD2P(XLT-XNODE)
TEMP2=CAPU
DO 130 I=1,10
SINEPW=SIN(TEMP2)
COSEPW=COS(TEMP2)
TEMP3=AXN*SINEPW
TEMP4=AYN*COSEPW
TEMP5=AXN*COSEPW
TEMP6=AYN*SINEPW
EPW=(CAPU-TEMP4+TEMP3-TEMP2)/(1.-TEMP5-TEMP6)+TEMP2
IF(ABS(EPW-TEMP2) .LE. E6A) GO TO 140
130 TEMP2=EPW

*      SHORT PERIOD PRELIMINARY QUANTITIES

140 ECOSE=TEMP5+TEMP6
ESINE=TEMP3-TEMP4
ELSQ=AXN*AXN+AYN*AYN
TEMP=1.-ELSQ
PL=A*TEMP
R=A*(1.-ECOSE)
TEMP1=1./R
RDOT=XKE*SQRT(A)*ESINE*TEMP1
RFDOT=XKE*SQRT(PL)*TEMP1
TEMP2=A*TEMP1
BETAL=SQRT(TEMP)
TEMP3=1./(1.+BETAL)
COSU=TEMP2*(COSEPW-AXN+AYN*ESINE*TEMP3)
SINU=TEMP2*(SINEPW-AYN-AXN*ESINE*TEMP3)
U=ACTAN(SINU,COSU)
SIN2U=2.*SINU*COSU

```



```

COS2U=2.*COSU*COSU-1.
TEMP=1./PL
TEMP1=CK2*TEMP
TEMP2=TEMP1*TEMP

*      UPDATE FOR SHORT PERIODICS

RK=R*(1.-1.5*TEMP2*BETAL*X3THM1)+.5*TEMP1*X1MTH2*COS2U
UK=U-.25*TEMP2*X7THM1*SIN2U
XNODEK=XNODE+1.5*TEMP2*COSIO*SIN2U
XINCK=XINC+1.5*TEMP2*COSIO*SINIO*COS2U
RDOTK=RDOT-XN*TEMP1*X1MTH2*SIN2U
RFDOTK=RFDOT+XN*TEMP1*(X1MTH2*COS2U+1.5*X3THM1)

*      ORIENTATION VECTORS

SINUK=SIN(UK)
COSUK=COS(UK)
SINIK=SIN(XINCK)
COSIK=COS(XINCK)
SINNOK=SIN(XNODEK)
COSNOK=COS(XNODEK)
XMX=-SINNOK*COSIK
XMY=COSNOK*COSIK
UX=XMX*SINUK+COSNOK*COSUK
UY=XMY*SINUK+SINNOK*COSUK
UZ=SINIK*SINUK
VX=XMX*COSUK-COSNOK*SINUK
VY=XMY*COSUK-SINNOK*SINUK
VZ=SINIK*COSUK

*      POSITION AND VELOCITY

X=RK*UX
Y=RK*UY
Z=RK*UZ
XDOT=RDOTK*UX+RFDOTK*VX
YDOT=RDOTK*UY+RFDOTK*VY
ZDOT=RDOTK*UZ+RFDOTK*VZ

RETURN
END

```

8 THE SGP8 MODEL

The NORAD mean element sets can be used for prediction with SGP8. All symbols not defined below are defined in the list of symbols in Section Twelve. The original mean motion (n_o'') and semimajor axis (a_o'') are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o} \right)^{\frac{2}{3}}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1^2 - \frac{134}{81} \delta_1^3 \right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$n_o'' = \frac{n_o}{1 + \delta_o}$$

$$a_o'' = \frac{a_o}{1 - \delta_o}.$$

The ballistic coefficient (B term) is then calculated from the B^* drag term by

$$B = 2B^*/\rho_o$$

where

$$\rho_o = (2.461 \times 10^{-5}) \text{ XKMPER kg/m}^2/\text{Earth radii}$$

is a reference value of atmospheric density.

Then calculate the constants

$$\beta^2 = 1 - e^2$$

$$\theta = \cos i$$

$$\dot{M}_1 = -\frac{3}{2} \frac{n'' k_2}{a''^2 \beta^3} (1 - 3\theta^2)$$

$$\dot{\omega}_1 = -\frac{3}{2} \frac{n'' k_2}{a''^2 \beta^4} (1 - 5\theta^2)$$

$$\dot{\Omega}_1 = -3 \frac{n'' k_2}{a''^2 \beta^4} \theta$$

$$\dot{M}_2 = \frac{3}{16} \frac{n'' k_2^2}{a''^4 \beta^7} (13 - 78\theta^2 + 137\theta^4)$$

$$\dot{\omega}_2 = \frac{3}{16} \frac{n'' k_2^2}{a''^4 \beta^8} (7 - 114\theta^2 + 395\theta^4) + \frac{5}{4} \frac{n'' k_4}{a''^4 \beta^8} (3 - 36\theta^2 + 49\theta^4)$$

$$\dot{\Omega}_2 = \frac{3}{2} \frac{n'' k_2^2}{a''^4 \beta^8} \theta (4 - 19\theta^2) + \frac{5}{2} \frac{n'' k_4}{a''^4 \beta^8} \theta (3 - 7\theta^2)$$

$$\dot{\ell} = n'' + \dot{M}_1 + \dot{M}_2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$\dot{\Omega} = \dot{\Omega}_1 + \dot{\Omega}_2$$

$$\xi = \frac{1}{a'' \beta^2 - s}$$

$$\eta = es\xi$$

$$\psi = \sqrt{1 - \eta^2}$$

$$\alpha^2 = 1 + e^2$$

$$C_o = \frac{1}{2} B \rho_o (q_o - s)^4 n'' a'' \xi^4 \alpha^{-1} \psi^{-7}$$

$$C_1 = \frac{3}{2} n'' \alpha^4 C_o$$

$$D_1 = \xi\psi^{-2}/a''\beta^2$$

$$D_2 = 12 + 36\eta^2 + \frac{9}{2}\eta^4$$

$$D_3 = 15\eta^2 + \frac{5}{2}\eta^4$$

$$D_4 = 5\eta + \frac{15}{4}\eta^3$$

$$D_5 = \xi\psi^{-2}$$

$$B_1 = -k_2(1 - 3\theta^2)$$

$$B_2 = -k_2(1 - \theta^2)$$

$$B_3 = \frac{A_{3,0}}{k_2} \sin i$$

$$C_2 = D_1 D_3 B_2$$

$$C_3 = D_4 D_5 B_3$$

$$\dot{n}_o = C_1 \left(2 + 3\eta^2 + 20e\eta + 5e\eta^3 + \frac{17}{2}e^2 + 34e^2\eta^2 + D_1 D_2 B_1 + C_2 \cos 2\omega + C_3 \sin \omega \right)$$

$$C_4 = D_1 D_7 B_2$$

$$C_5 = D_5 D_8 B_3$$

$$D_6 = 30\eta + \frac{45}{2}\eta^3$$

$$D_7 = 5\eta + \frac{25}{2}\eta^3$$

$$D_8 = 1 + \frac{27}{4}\eta^2 + \eta^4$$

$$\dot{e}_o = -C_o \left(4\eta + \eta^3 + 5e + 15e\eta^2 + \frac{31}{2}e^2\eta + 7e^2\eta^3 + D_1D_6B_1 + C_4 \cos 2\omega + C_5 \sin \omega \right)$$

$$\dot{\alpha}/\alpha = e\dot{e}\alpha^{-2}$$

$$C_6 = \frac{1}{3} \frac{\dot{n}}{n''}$$

$$\dot{\xi}/\xi = 2a''\xi(C_6\beta^2 + e\dot{e})$$

$$\dot{\eta} = (\dot{e} + e\dot{\xi}/\xi)s\xi$$

$$\dot{\psi}/\psi = -\eta\dot{\eta}\psi^{-2}$$

$$\dot{C}_o/C_o = C_6 + 4\dot{\xi}/\xi - \dot{\alpha}/\alpha - 7\dot{\psi}/\psi$$

$$\dot{C}_1/C_1 = \dot{n}/n'' + 4\dot{\alpha}/\alpha + \dot{C}_o/C_o$$

$$D_9 = 6\eta + 20e + 15e\eta^2 + 68e^2\eta$$

$$D_{10} = 20\eta + 5\eta^3 + 17e + 68e\eta^2$$

$$D_{11} = 72\eta + 18\eta^3$$

$$D_{12} = 30\eta + 10\eta^3$$

$$D_{13} = 5 + \frac{45}{4}\eta^2$$

$$D_{14} = \dot{\xi}/\xi - 2\dot{\psi}/\psi$$

$$D_{15} = 2(C_6 + e\dot{e}\beta^{-2})$$

$$\dot{D}_1 = D_1(D_{14} + D_{15})$$

$$\dot{D}_2 = \dot{\eta}D_{11}$$

$$\dot{D}_3 = \dot{\eta} D_{12}$$

$$\dot{D}_4 = \dot{\eta} D_{13}$$

$$\dot{D}_5 = D_5 D_{14}$$

$$\dot{C}_2 = B_2(\dot{D}_1 D_3 + D_1 \dot{D}_3)$$

$$\dot{C}_3 = B_3(\dot{D}_5 D_4 + D_5 \dot{D}_4)$$

$$\dot{\omega} = -\frac{3}{2} \frac{n'' k_2}{a''^2 \beta^4} (1 - 5\theta^2)$$

$$D_{16} = D_9 \dot{\eta} + D_{10} \dot{e} + B_1(\dot{D}_1 D_2 + D_1 \dot{D}_2) + \dot{C}_2 \cos 2\omega + \dot{C}_3 \sin \omega + \dot{\omega}(C_3 \cos \omega - 2C_2 \sin 2\omega)$$

$$\ddot{n}_o = \dot{n} \dot{C}_1 / C_1 + C_1 D_{16}$$

$$\begin{aligned} \ddot{e}_o = & \dot{e} \dot{C}_o / C_o - C_o \left\{ \left(4 + 3\eta^2 + 30e\eta + \frac{31}{2}e^2 + 21e^2\eta^2 \right) \dot{\eta} + (5 + 15\eta^2 + 31e\eta + 14e\eta^3) \dot{e} \right. \\ & + B_1 \left[\dot{D}_1 D_6 + D_1 \dot{\eta} \left(30 + \frac{135}{2}\eta^2 \right) \right] + B_2 \left[\dot{D}_1 D_7 + D_1 \dot{\eta} \left(5 + \frac{75}{2}\eta^2 \right) \right] \cos \omega \\ & \left. + B_3 \left[\dot{D}_5 D_8 + D_5 \eta \dot{\eta} \left(\frac{27}{2} + 4\eta^2 \right) \right] \sin \omega + \dot{\omega}(C_5 \cos \omega - 2C_4 \sin 2\omega) \right\} \end{aligned}$$

$$D_{17} = \ddot{n}/n'' - (\dot{n}/n'')^2$$

$$\ddot{\xi}/\xi = 2(\dot{\xi}/\xi - C_6)\dot{\xi}/\xi + 2a''\xi \left(\frac{1}{3}D_{17}\beta^2 - 2C_6e\dot{e} + \dot{e}^2 + e\ddot{e} \right)$$

$$\ddot{\eta} = (\ddot{e} + 2\dot{e}\dot{\xi}/\xi)s\xi + \eta\ddot{\xi}/\xi$$

$$D_{18} = \ddot{\xi}/\xi - (\dot{\xi}/\xi)^2$$

$$D_{19} = -(\dot{\psi}/\psi)^2(1 + \eta^{-2}) - \eta\ddot{\eta}\psi^{-2}$$

$$\ddot{D}_1 = \dot{D}_1(D_{14} + D_{15}) + D_1 \left(D_{18} - 2D_{19} + \frac{2}{3}D_{17} + 2\alpha^2\dot{e}^2\beta^{-4} + 2e\ddot{e}\beta^{-2} \right)$$

$$\begin{aligned}
\ddot{n}_o = & \dot{n} \left[\frac{4}{3} D_{17} + 3\dot{e}^2 \alpha^{-2} + 3e\ddot{e} \alpha^{-2} - 6(\dot{\alpha}/\alpha)^2 + 4D_{18} - 7D_{19} \right] \\
& + \ddot{n} \dot{C}_1 / C_1 + C_1 \left\{ D_{16} \dot{C}_1 / C_1 + D_9 \ddot{\eta} + D_{10} \ddot{e} + \dot{\eta}^2 (6 + 30e\eta + 68e^2) \right. \\
& + \dot{\eta} \dot{e} (40 + 30\eta^2 + 272e\eta) + \dot{e}^2 (17 + 68\eta^2) \\
& + B_1 [\ddot{D}_1 D_2 + 2\dot{D}_1 \dot{D}_2 + D_1 (\ddot{\eta} D_{11} + \dot{\eta}^2 (72 + 54\eta^2))] \\
& + B_2 [\ddot{D}_1 D_3 + 2\dot{D}_1 \dot{D}_3 + D_1 (\ddot{\eta} D_{12} + \dot{\eta}^2 (30 + 30\eta^2))] \cos 2\omega \\
& + B_3 \left[(\dot{D}_5 D_{14} + D_5 (D_{18} - 2D_{19})) D_4 + 2\dot{D}_4 \dot{D}_5 + D_5 \left(\ddot{\eta} D_{13} + \frac{45}{2} \eta \dot{\eta}^2 \right) \right] \sin \omega \\
& + \dot{\omega} [(7C_6 + 4e\dot{e}\beta^{-2})(C_3 \cos \omega - 2C_2 \sin 2\omega) + 2C_3 \cos \omega \\
& \left. - 4C_2 \sin 2\omega - \dot{\omega} (C_3 \sin \omega + 4C_2 \cos 2\omega)] \right\}
\end{aligned}$$

$$p = \frac{2\ddot{n}_o^2 - \dot{n}_o \ddot{\ddot{n}}_o}{\ddot{n}_o^2 - \dot{n}_o \ddot{\ddot{n}}_o}$$

$$\gamma = -\frac{\ddot{\ddot{n}}_o}{\ddot{n}_o} \frac{1}{(p-2)}$$

$$n_D = \frac{\dot{n}_o}{p\gamma}$$

$$q = 1 - \frac{\ddot{e}_o}{\dot{e}_o \gamma}$$

$$e_D = \frac{\dot{e}_o}{q\gamma}$$

where all quantities are epoch values.

The secular effects of atmospheric drag and gravitation are included by

$$n = n_o'' + n_D [1 - (1 - \gamma(t - t_o))^p]$$

$$e = e_o + e_D [1 - (1 - \gamma(t - t_o))^q]$$

$$\omega = \omega_o + \dot{\omega}_1 \left[(t - t_o) + \frac{7}{3} \frac{1}{n_o''} Z_1 \right] + \dot{\omega}_2 (t - t_o)$$

$$\Omega = \Omega_o'' + \dot{\Omega}_1 \left[(t - t_o) + \frac{7}{3} \frac{1}{n_o''} Z_1 \right] + \dot{\Omega}_2 (t - t_o)$$

$$M = M_o + n_o''(t - t_o) + Z_1 + \dot{M}_1 \left[(t - t_o) + \frac{7}{3} \frac{1}{n_o''} Z_1 \right] + \dot{M}_2 (t - t_o)$$

where

$$Z_1 = \frac{\dot{n}_o}{p\gamma} \left\{ (t - t_o) + \frac{1}{\gamma(p+1)} [(1 - \gamma(t - t_o))^{p+1} - 1] \right\}.$$

If drag is very small ($\frac{\dot{n}}{n_o''}$ less than $1.5 \times 10^{-6}/\text{min}$) then the secular equations for n , e , and Z_1 should be replaced by

$$n = n_o'' + \dot{n}(t - t_o)$$

$$e = e_o'' + \dot{e}(t - t_o)$$

$$Z_1 = \frac{1}{2} \dot{n}_o (t - t_o)^2$$

where $(t - t_o)$ is time since epoch and where

$$\dot{e} = -\frac{2}{3} \frac{\dot{n}_o}{n_o''} (1 - e_o).$$

Solve Kepler's equation for E by using the iteration equation

$$E_{i+1} = E_i + \Delta E_i$$

with

$$\Delta E_i = \frac{M + e \sin E_i - E_i}{1 - e \cos E_i}$$

and

$$E_1 = M + e \sin M + \frac{1}{2} e^2 \sin 2M.$$

The following equations are used to calculate preliminary quantities needed for the short-period periodics.

$$a = \left(\frac{k_e}{n} \right)^{\frac{2}{3}}$$

$$\beta = (1 - e^2)^{\frac{1}{2}}$$

$$\sin f = \frac{\beta \sin E}{1 - e \cos E}$$

$$\cos f = \frac{\cos E - e}{1 - e \cos E}$$

$$u = f + \omega$$

$$r'' = \frac{a\beta^2}{1 + e \cos f}$$

$$\dot{r}'' = \frac{nae}{\beta} \sin f$$

$$(r\dot{f})'' = \frac{na^2\beta}{r}$$

$$\delta r = \frac{1}{2} \frac{k_2}{a\beta^2} [(1 - \theta^2) \cos 2u + 3(1 - 3\theta^2)] - \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \sin u$$

$$\delta \dot{r} = -n \left(\frac{a}{r} \right)^2 \left[\frac{k_2}{a\beta^2} (1 - \theta^2) \sin 2u + \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \cos u \right]$$

$$\delta I = \theta \left[\frac{3}{2} \frac{k_2}{a^2\beta^4} \sin i_o \cos 2u - \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} e \sin \omega \right]$$

$$\delta(r\dot{f}) = -n \left(\frac{a}{r} \right)^2 \delta r + na \left(\frac{a}{r} \right) \frac{\sin i_o}{\theta} \delta I$$

$$\begin{aligned} \delta u = & \frac{1}{2} \frac{k_2}{a^2\beta^4} \left[\frac{1}{2} (1 - 7\theta^2) \sin 2u - 3(1 - 5\theta^2)(f - M + e \sin f) \right] \\ & - \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} \left[\sin i_o \cos u (2 + e \cos f) + \frac{1}{2} \frac{\theta^2}{\sin i_o/2 \cos i_o/2} e \cos \omega \right] \end{aligned}$$

$$\begin{aligned}\delta\lambda = & \frac{1}{2} \frac{k_2}{a^2 \beta^4} \left[\frac{1}{2} (1 + 6\theta - 7\theta^2) \sin 2u - 3(1 + 2\theta - 5\theta^2)(f - M + e \sin f) \right] \\ & + \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} \sin i_o \left[\frac{e\theta}{1 + \theta} \cos \omega - (2 + e \cos f) \cos u \right]\end{aligned}$$

The short-period periodics are added to give the osculating quantities

$$r = r'' + \delta r$$

$$\dot{r} = \dot{r}'' + \delta \dot{r}$$

$$r\dot{f} = (r\dot{f})'' + \delta(r\dot{f})$$

$$y_4 = \sin \frac{i_o}{2} \sin u + \cos u \sin \frac{i_o}{2} \delta u + \frac{1}{2} \sin u \cos \frac{i_o}{2} \delta I$$

$$y_5 = \sin \frac{i_o}{2} \cos u - \sin u \sin \frac{i_o}{2} \delta u + \frac{1}{2} \cos u \cos \frac{i_o}{2} \delta I$$

$$\lambda = u + \Omega + \delta\lambda.$$

Unit orientation vectors are calculated by

$$U_x = 2y_4(y_5 \sin \lambda - y_4 \cos \lambda) + \cos \lambda$$

$$U_y = -2y_4(y_5 \cos \lambda + y_4 \sin \lambda) + \sin \lambda$$

$$U_z = 2y_4 \cos \frac{I}{2}$$

$$V_x = 2y_5(y_5 \sin \lambda - y_4 \cos \lambda) - \sin \lambda$$

$$V_y = -2y_5(y_5 \cos \lambda + y_4 \sin \lambda) + \cos \lambda$$

$$V_z = 2y_5 \cos \frac{I}{2}$$

where

$$\cos \frac{I}{2} = \sqrt{1 - y_4^2 - y_5^2}.$$

Position and velocity are given by

$$\mathbf{r} = r\mathbf{U}$$

$$\dot{\mathbf{r}} = \dot{r}\mathbf{U} + r\dot{f}\mathbf{V}.$$

A FORTRAN IV computer code listing of the subroutine SGP8 is given below.

```

*          SGP8                                     14 NOV 80
SUBROUTINE SGP8(IFLAG,TSINCE)
COMMON/E1/XMO,XNODEO,OMEGAO,E0,XINCL,XNO,XNDT20,
1          XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
1          XJ3,XKE,XKMPER,XMNPDA,AE
DOUBLE PRECISION EPOCH, DS50
DATA RHO/.15696615/

IF (IFLAG .EQ. 0) GO TO 100

*          RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
*          FROM INPUT ELEMENTS ----- CALCULATE BALLISTIC COEFFICIENT
*          (B TERM) FROM INPUT B* DRAG TERM

A1=(XKE/XNO)**TOTHRD
COSI=COS(XINCL)
THETA2=COSI*COSI
TTHMUN=3.*THETA2-1.
EOSQ=E0*E0
BETA02=1.-EOSQ
BETA0=SQRT(BETA02)
DEL1=1.5*CK2*TTHMUN/(A1*A1*BETA0*BETA02)
AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
DELO=1.5*CK2*TTHMUN/(AO*AO*BETA0*BETA02)
AODP=AO/(1.-DELO)
XNODP=XNO/(1.+DELO)
B=2.*BSTAR/RHO

*          INITIALIZATION

ISIMP=0
PO=AODP*BETA02
POM2=1./(PO*PO)
SINI=SIN(XINCL)
SING=SIN(OMEGAO)
COSG=COS(OMEGAO)
TEMP=.5*XINCL
SINIO2=SIN(TEMP)
COSIO2=COS(TEMP)
THETA4=THETA2**2
UNM5TH=1.-5.*THETA2
UNMTH2=1.-THETA2
A3COF=-XJ3/CK2*AE**3
PARDT1=3.*CK2*POM2*XNODP
PARDT2=PARDT1*CK2*POM2
PARDT4=1.25*CK4*POM2*POM2*XNODP
XMDT1=.5*PARDT1*BETA0*TTHMUN

```

```

XGDT1=-.5*PARDT1*UNM5TH
XHDT1=-PARDT1*COSI
XLLDOT=XNODP+XMDT1+
2      .0625*PARDT2*BETA0*(13.-78.*THETA2+137.*THETA4)
OMGDT=XGDT1+
1      .0625*PARDT2*(7.-114.*THETA2+395.*THETA4)+PARDT4*(3.-36.*
2      THETA2+49.*THETA4)
XNODOT=XHDT1+
1      (.5*PARDT2*(4.-19.*THETA2)+2.*PARDT4*(3.-7.*THETA2))*COSI
TSI=1./(P0-S)
ETA=E0*S*TSI
ETA2=ETA**2
PSIM2=ABS(1./(1.-ETA2))
ALPHA2=1.+EOSQ
EETA=E0*ETA
COS2G=2.*COSG**2-1.
D5=TSI*PSIM2
D1=D5/P0
D2=12.+ETA2*(36.+4.5*ETA2)
D3=ETA2*(15.+2.5*ETA2)
D4=ETA*(5.+3.75*ETA2)
B1=CK2*TTHMUN
B2=-CK2*UNMTH2
B3=A3COF*SINI
C0=.5*B*RHO*QOMS2T*XNODP*AODP*TSI**4*PSIM2**3.5/SQRT(ALPHA2)
C1=1.5*XNODP*ALPHA2**2*C0
C4=D1*D3*B2
C5=D5*D4*B3
XNDT=C1*(
1  (2.+ETA2*(3.+34.*EOSQ)+5.*EETA*(4.+ETA2)+8.5*EOSQ)+
1  D1*D2*B1+ C4*COS2G+C5*SING)
XNDTN=XNDT/XNODP

*      IF DRAG IS VERY SMALL, THE ISIMP FLAG IS SET AND THE
*      EQUATIONS ARE TRUNCATED TO LINEAR VARIATION IN MEAN
*      MOTION AND QUADRATIC VARIATION IN MEAN ANOMALY

IF(ABS(XNDTN*XMNPDA) .LT. 2.16E-3) GO TO 50
D6=ETA*(30.+22.5*ETA2)
D7=ETA*(5.+12.5*ETA2)
D8=1.+ETA2*(6.75+ETA2)
C8=D1*D7*B2
C9=D5*D8*B3
EDOT=-C0*(
1  ETA*(4.+ETA2+EOSQ*(15.5+7.*ETA2))+E0*(5.+15.*ETA2)+
1  D1*D6*B1 +
1  C8*COS2G+C9*SING)
D20=.5*TOTHRD*XNDTN

```

```

ALDTAL=E0*EDOT/ALPHA2
TSDTTS=2.*AODP*TSI*(D20*BETA02+E0*EDOT)
ETDT=(EDOT+E0*TSDTTS)*TSI*S
PSDTPS=-ETA*ETDT*PSIM2
SIN2G=2.*SING*COSG
CODTC0=D20+4.*TSDTTS-ALDTAL-7.*PSDTPS
C1DTC1=XNDTN+4.*ALDTAL+CODTC0
D9=ETA*(6.+68.*EOSQ)+E0*(20.+15.*ETA2)
D10=5.*ETA*(4.+ETA2)+E0*(17.+68.*ETA2)
D11=ETA*(72.+18.*ETA2)
D12=ETA*(30.+10.*ETA2)
D13=5.+11.25*ETA2
D14=TSDTTS-2.*PSDTPS
D15=2.*(D20+E0*EDOT/BETA02)
D1DT=D1*(D14+D15)
D2DT=ETDT*D11
D3DT=ETDT*D12
D4DT=ETDT*D13
D5DT=D5*D14
C4DT=B2*(D1DT*D3+D1*D3DT)
C5DT=B3*(D5DT*D4+D5*D4DT)
D16=
1      D9*ETDT+D10*EDOT +
1      B1*(D1DT*D2+D1*D2DT) +
1      C4DT*COS2G+C5DT*SING+XGDT1*(C5*COSG-2.*C4*SIN2G)
XNDDT=C1DTC1*XNDT+C1*D16
EDDOT=CODTC0*EDOT-C0*(
1      (4.+3.*ETA2+30.*EETA+EOSQ*(15.5+21.*ETA2))*ETDT+(5.+15.*ETA2
',
      +EETA*(31.+14.*ETA2))*EDOT +
1      B1*(D1DT*D6+D1*ETDT*(30.+67.5*ETA2)) +
1      B2*(D1DT*D7+D1*ETDT*(5.+37.5*ETA2))*COS2G+
1      B3*(D5DT*D8+D5*ETDT*ETA*(13.5+4.*ETA2))*SING+XGDT1*(C9*
',
      COSG-2.*C8*SIN2G))
D25=EDOT**2
D17=XNDDT/XNODP-XNDTN**2
TSDDTS=2.*TSDTTS*(TSDTTS-D20)+AODP*TSI*(TOTHRD*BETA02*D17-4.*D20*
',
      E0*EDOT+2.*(D25+E0*EDDOT))
ETDDT =(EDDOT+2.*EDOT*TSDTTS)*TSI*S+TSDDTS*ETA
D18=TSDDTS-TSDTTS**2
D19=-PSDTPS**2/ETA2-ETA*ETDDT*PSIM2-PSDTPS**2
D23=ETDT*ETDT
D1DDT=D1DT*(D14+D15)+D1*(D18-2.*D19+TOTHRD*D17+2.*(ALPHA2*D25
',
      /BETA02+E0*EDDOT)/BETA02)
XNTRDT=XNDT*(2.*TOTHRD*D17+3.*
1      (D25+E0*EDDOT)/ALPHA2-6.*ALDTAL**2 +
1      4.*D18-7.*D19 ) +
1      C1DTC1*XNDDT+C1*(C1DTC1*D16+
1      D9*ETDDT+D10*EDDOT+D23*(6.+30.*EETA+68.*EOSQ)+

```

```

1  ETD*EDOT*(40.+30.*
'   ETA2+272.*ETA)+D25*(17.+68.*ETA2) +
1    B1*(D1DDT*D2+2.*D1DT*D2DT+D1*(ETDDT*D11+D23*(72.+54.*ETA2))) +
1    B2*(D1DDT*D3+2.*D1DT*D3DT+D1*(ETDDT*D12+D23*(30.+30.*ETA2))) *
1    COS2G+
1    B3*((D5DT*D14+D5*(D18-2.*D19)) *
1 D4+2.*D4DT*D5DT+D5*(ETDDT*D13+22.5*ETA*D23)) *SING+XGDT1*
1    ((7.*D20+4.*EO*EDOT/BETA02)*
'    (C5*COSG-2.*C4*SIN2G)
'    +((2.*C5DT*COSG-4.*C4DT*SIN2G)-XGDT1*(C5*SING+4.*
'    C4*COS2G)))
TMNDDT=XNDDT*1.E9
TEMP=TMNDDT**2-XNDT*1.E18*XNTRDT
PP=(TEMP+TMNDDT**2)/TEMP
GAMMA=-XNTRDT/(XNDDT*(PP-2.))
XND=XNDT/(PP*GAMMA)
QQ=1.-EDDOT/(EDOT*GAMMA)
ED=EDOT/(QQ*GAMMA)
OVGPP=1./(GAMMA*(PP+1.))
GO TO 70
50 ISIMP=1
   EDOT=-TOTHDR*XNDTN*(1.-EO)
70 IFLAG=0

*      UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG

100 XMAM=FMOD2P(XM0+XLLDOT*TSINCE)
   OMGASM=OMGAO+OMGDT*TSINCE
   XNODES=XNODEO+XNODOT*TSINCE
   IF(ISIMP.EQ. 1) GO TO 105
   TEMP=1.-GAMMA*TSINCE
   TEMP1=TEMP**PP
   XN=XNODP+XND*(1.-TEMP1)
   EM=EO+ED*(1.-TEMP**QQ)
   Z1=XND*(TSINCE+OVGPP*(TEMP*TEMP1-1.))
   GO TO 108
105 XN=XNODP+XNDT*TSINCE
   EM=EO+EDOT*TSINCE
   Z1=.5*XNDT*TSINCE*TSINCE
108 Z7=3.5*TOTHDR*Z1/XNODP
   XMAM=FMOD2P(XMAM+Z1+Z7*XMDT1)
   OMGASM=OMGASM+Z7*XGDT1
   XNODES=XNODES+Z7*XHDT1

*      SOLVE KEPLERS EQUATION

ZC2=XMAM+EM*SIN(XMAM)*(1.+EM*COS(XMAM))
DO 130 I=1,10

```

```

SINE=SIN(ZC2)
COSE=COS(ZC2)
ZC5=1./(1.-EM*COSE)
CAPE=(XMAM+EM*SINE-ZC2)*
1   ZC5+ZC2
   IF(ABS(CAPE-ZC2) .LE. E6A) GO TO 140
130 ZC2=CAPE

```

* SHORT PERIOD PRELIMINARY QUANTITIES

```

140 AM=(XKE/XN)**TOTHRD
   BETA2M=1.-EM*EM
   SINOS=SIN(OMGASM)
   COSOS=COS(OMGASM)
   AXNM=EM*COSOS
   AYNM=EM*SINOS
   PM=AM*BETA2M
   G1=1./PM
   G2=.5*CK2*G1
   G3=G2*G1
   BETA=SQRT(BETA2M)
   G4=.25*A3COF*SINI
   G5=.25*A3COF*G1
   SNF=BETA*SINE*ZC5
   CSF=(COSE-EM)*ZC5
   FM=ACTAN(SNF,CSF)
   SNFG=SNF*COSOS+CSF*SINOS
   CSFG=CSF*COSOS-SNF*SINOS
   SN2F2G=2.*SNFG*CSFG
   CS2F2G=2.*CSFG**2-1.
   ECOSF=EM*CSF
   G10=FM-XMAM+EM*SNF
   RM=PM/(1.+ECOSF)
   AOVR=AM/RM
   G13=XN*AOVR
   G14=-G13*AOVR
   DR=G2*(UNMTH2*CS2F2G-3.*TTHMUN)-G4*SNFG
   DIWC=3.*G3*SINI*CS2F2G-G5*AYNM
   DI=DIWC*COSI

```

* UPDATE FOR SHORT PERIOD PERIODICS

```

SNI2DU=SINI02*(
1   G3*(.5*(1.-7.*THETA2)*SN2F2G-3.*UNM5TH*G10)-G5*SINI*CSFG*(2.+
2   ECOSF))- .5*G5*THETA2*AXNM/COSI02
   XLAMB=FM+OMGASM+XNODES+G3*(.5*(1.+6.*COSI-7.*THETA2)*SN2F2G-3.*
1   (UNM5TH+2.*COSI)*G10)+G5*SINI*(COSI*AXNM/(1.+COSI)-(2.
2   +ECOSF)*CSFG)

```



```

Y4=SINI02*SNFG+CSFG*SNI2DU+.5*SNFG*COSI02*DI
Y5=SINI02*CSFG-SNFG*SNI2DU+.5*CSFG*COSI02*DI
R=RM+DR
RDOT=XN*AM*EM*SNF/BETA+G14*(2.*G2*UNMTH2*SN2F2G+G4*CSFG)
RVDOT=XN*AM**2*BETA/RM+
1      G14*DR+AM*G13*SINI*DIWC

```

* ORIENTATION VECTORS

```

SNLAMB=SIN(XLAMB)
CSLAMB=COS(XLAMB)
TEMP=2.*(Y5*SNLAMB-Y4*CSLAMB)
UX=Y4*TEMP+CSLAMB
VX=Y5*TEMP-SNLAMB
TEMP=2.*(Y5*CSLAMB+Y4*SNLAMB)
UY=-Y4*TEMP+SNLAMB
VY=-Y5*TEMP+CSLAMB
TEMP=2.*SQRT(1.-Y4*Y4-Y5*Y5)
UZ=Y4*TEMP
VZ=Y5*TEMP

```

* POSITION AND VELOCITY

```

X=R*UX
Y=R*UY
Z=R*UZ
XDOT=RDOT*UX+RVDOT*VX
YDOT=RDOT*UY+RVDOT*VY
ZDOT=RDOT*UZ+RVDOT*VZ

```

```

RETURN
END

```

9 THE SDP8 MODEL

The NORAD mean element sets can be used for prediction with SDP8. All symbols not defined below are defined in the list of symbols in Section Twelve. The original mean motion (n_o'') and semimajor axis (a_o'') are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o} \right)^{\frac{2}{3}}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1^2 - \frac{134}{81} \delta_1^3 \right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{\frac{3}{2}}}$$

$$n_o'' = \frac{n_o}{1 + \delta_o}$$

$$a_o'' = \frac{a_o}{1 - \delta_o}.$$

The ballistic coefficient (B term) is then calculated from the B^* drag term by

$$B = 2B^*/\rho_o$$

where

$$\rho_o = (2.461 \times 10^{-5}) \text{ XKMPER kg/m}^2/\text{Earth radii}$$

is a reference value of atmospheric density.

Then calculate the constants

$$\beta^2 = 1 - e^2$$

$$\theta = \cos i$$

$$\dot{M}_1 = -\frac{3}{2} \frac{n'' k_2}{a''^2 \beta^3} (1 - 3\theta^2)$$

$$\dot{\omega}_1 = -\frac{3}{2} \frac{n'' k_2}{a''^2 \beta^4} (1 - 5\theta^2)$$

$$\dot{\Omega}_1 = -3 \frac{n'' k_2}{a''^2 \beta^4} \theta$$

$$\dot{M}_2 = \frac{3}{16} \frac{n'' k_2^2}{a''^4 \beta^7} (13 - 78\theta^2 + 137\theta^4)$$

$$\dot{\omega}_2 = \frac{3}{16} \frac{n'' k_2^2}{a''^4 \beta^8} (7 - 114\theta^2 + 395\theta^4) + \frac{5}{4} \frac{n'' k_4}{a''^4 \beta^8} (3 - 36\theta^2 + 49\theta^4)$$

$$\dot{\Omega}_2 = \frac{3}{2} \frac{n'' k_2^2}{a''^4 \beta^8} \theta (4 - 19\theta^2) + \frac{5}{2} \frac{n'' k_4}{a''^4 \beta^8} \theta (3 - 7\theta^2)$$

$$\dot{\ell} = n''_o + \dot{M}_1 + \dot{M}_2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$\dot{\Omega} = \dot{\Omega}_1 + \dot{\Omega}_2$$

$$\xi = \frac{1}{a'' \beta^2 - s}$$

$$\eta = es\xi$$

$$\psi = \sqrt{1 - \eta^2}$$

$$\alpha^2 = 1 + e^2$$

$$C_o = \frac{1}{2} B \rho_o (q_o - s)^4 n'' a'' \xi^4 \alpha^{-1} \psi^{-7}$$

$$C_1 = \frac{3}{2} n'' \alpha^4 C_o$$

$$D_1 = \xi\psi^{-2}/a''\beta^2$$

$$D_2 = 12 + 36\eta^2 + \frac{9}{2}\eta^4$$

$$D_3 = 15\eta^2 + \frac{5}{2}\eta^4$$

$$D_4 = 5\eta + \frac{15}{4}\eta^3$$

$$D_5 = \xi\psi^{-2}$$

$$B_1 = -k_2(1 - 3\theta^2)$$

$$B_2 = -k_2(1 - \theta^2)$$

$$B_3 = \frac{A_{3,0}}{k_2} \sin i$$

$$C_2 = D_1 D_3 B_2$$

$$C_3 = D_4 D_5 B_3$$

$$\dot{n}_o = C_1 \left(2 + 3\eta^2 + 20e\eta + 5e\eta^3 + \frac{17}{2}e^2 + 34e^2\eta^2 + D_1 D_2 B_1 + C_2 \cos 2\omega + C_3 \sin \omega \right)$$

$$\dot{e}_o = -\frac{2}{3} \frac{\dot{n}}{n''} (1 - e)$$

where all quantities are epoch values.

At this point SDP8 calls the initialization section of DEEP which calculates all initialized quantities needed for the deep-space perturbations (see Section Ten).

The secular effect of gravity is included in mean anomaly by

$$M_{DF} = M_o + \dot{\ell}(t - t_o)$$

and the secular effects of gravity and atmospheric drag are included in argument of perigee and longitude of ascending node by

$$\omega = \omega_o + \dot{\omega}(t - t_o) + \dot{\omega}_1 Z_7$$

$$\Omega = \Omega_o + \dot{\Omega}(t - t_o) + \dot{\Omega}_1 Z_7$$

where

$$Z_7 = \frac{7}{3} Z_1 / n_o''$$

with

$$Z_1 = \frac{1}{2} \dot{n}_o (t - t_o)^2.$$

Next, SDP8 calls the secular section of DEEP which adds the deep-space secular effects and long-period resonance effects to the six classical orbital elements (see Section Ten).

The secular effects of drag are included in the remaining elements by

$$n = n_{DS} + \dot{n}_o (t - t_o)$$

$$e = e_{DS} + \dot{e}_o (t - t_o)$$

$$M = M_{DS} + Z_1 + \dot{M}_1 Z_7$$

where n_{DS} , e_{DS} , M_{DS} are the values of n_o , e_o , M_{DF} after deep-space secular and resonance perturbations have been applied.

Here, SDP8 calls the periodics section of DEEP which adds the deep-space lunar and solar periodics to the orbital elements (see Section Ten). From this point on, it will be assumed that n , e , I , ω , Ω , and M are the mean motion, eccentricity, inclination, argument of perigee, longitude of ascending node, and mean anomaly after lunar-solar periodics have been added.

Solve Kepler's equation for E by using the iteration equation

$$E_{i+1} = E_i + \Delta E_i$$

with

$$\Delta E_i = \frac{M + e \sin E_i - E_i}{1 - e \cos E_i}$$

and

$$E_1 = M + e \sin M + \frac{1}{2} e^2 \sin 2M.$$

The following equations are used to calculate preliminary quantities needed for the short-period periodics.

$$a = \left(\frac{k_e}{n} \right)^{\frac{2}{3}}$$

$$\beta = (1 - e^2)^{\frac{1}{2}}$$

$$\sin f = \frac{\beta \sin E}{1 - e \cos E}$$

$$\cos f = \frac{\cos E - e}{1 - e \cos E}$$

$$u = f + \omega$$

$$r'' = \frac{a\beta^2}{1 + e \cos f}$$

$$\dot{r}'' = \frac{nae}{\beta} \sin f$$

$$(r\dot{f})'' = \frac{na^2\beta}{r}$$

$$\delta r = \frac{1}{2} \frac{k_2}{a\beta^2} [(1 - \theta^2) \cos 2u + 3(1 - 3\theta^2)] - \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \sin u$$

$$\delta \dot{r} = -n \left(\frac{a}{r} \right)^2 \left[\frac{k_2}{a\beta^2} (1 - \theta^2) \sin 2u + \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \cos u \right]$$

$$\delta I = \theta \left[\frac{3}{2} \frac{k_2}{a^2\beta^4} \sin i_o \cos 2u - \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} e \sin \omega \right]$$

$$\delta(r\dot{f}) = -n \left(\frac{a}{r} \right)^2 \delta r + na \left(\frac{a}{r} \right) \frac{\sin i_o}{\theta} \delta I$$

$$\begin{aligned} \delta u = & \frac{1}{2} \frac{k_2}{a^2\beta^4} \left[\frac{1}{2} (1 - 7\theta^2) \sin 2u - 3(1 - 5\theta^2)(f - M + e \sin f) \right] \\ & - \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} \left[\sin i_o \cos u (2 + e \cos f) + \frac{1}{2} \frac{\theta^2}{\sin i_o / 2 \cos i_o / 2} e \cos \omega \right] \end{aligned}$$

$$\begin{aligned}\delta\lambda = & \frac{1}{2} \frac{k_2}{a^2 \beta^4} \left[\frac{1}{2} (1 + 6\theta - 7\theta^2) \sin 2u - 3(1 + 2\theta - 5\theta^2)(f - M + e \sin f) \right] \\ & + \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} \sin i_o \left[\frac{e\theta}{1 + \theta} \cos \omega - (2 + e \cos f) \cos u \right]\end{aligned}$$

The short-period periodics are added to give the osculating quantities

$$r = r'' + \delta r$$

$$\dot{r} = \dot{r}'' + \delta \dot{r}$$

$$r \dot{f} = (r \dot{f})'' + \delta(r \dot{f})$$

$$y_4 = \sin \frac{I}{2} \sin u + \cos u \sin \frac{i_o}{2} \delta u + \frac{1}{2} \sin u \cos \frac{i_o}{2} \delta I$$

$$y_5 = \sin \frac{I}{2} \cos u - \sin u \sin \frac{i_o}{2} \delta u + \frac{1}{2} \cos u \cos \frac{i_o}{2} \delta I$$

$$\lambda = u + \Omega + \delta \lambda.$$

Unit orientation vectors are calculated by

$$U_x = 2y_4(y_5 \sin \lambda - y_4 \cos \lambda) + \cos \lambda$$

$$U_y = -2y_4(y_5 \cos \lambda + y_4 \sin \lambda) + \sin \lambda$$

$$U_z = 2y_4 \cos \frac{I}{2}$$

$$V_x = 2y_5(y_5 \sin \lambda - y_4 \cos \lambda) - \sin \lambda$$

$$V_y = -2y_5(y_5 \cos \lambda + y_4 \sin \lambda) + \cos \lambda$$

$$V_z = 2y_5 \cos \frac{I}{2}$$

where

$$\cos \frac{I}{2} = \sqrt{1 - y_4^2 - y_5^2}.$$

Position and velocity are given by

$$\mathbf{r} = r\mathbf{U}$$

$$\dot{\mathbf{r}} = \dot{r}\mathbf{U} + r\dot{f}\mathbf{V}.$$

A FORTRAN IV computer code listing of the subroutine SDP8 is given below.


```

*          SDP8                                     14 NOV 80
SUBROUTINE SDP8(IFLAG,TSINCE)
COMMON/E1/XMO,XNODEO,OMEGAO,E0,XINCL,XNO,XNDT20,
1          XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
1          XJ3,XKE,XKMPER,XMNPDA,AE
DOUBLE PRECISION EPOCH, DS50
DATA RHO/.15696615/

IF (IFLAG .EQ. 0) GO TO 100

*          RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
*          FROM INPUT ELEMENTS ----- CALCULATE BALLISTIC COEFFICIENT
*          (B TERM) FROM INPUT B* DRAG TERM

A1=(XKE/XNO)**TOTHRD
COSI=COS(XINCL)
THETA2=COSI*COSI
TTHMUN=3.*THETA2-1.
EOSQ=E0*E0
BETA02=1.-EOSQ
BETA0=SQRT(BETA02)
DEL1=1.5*CK2*TTHMUN/(A1*A1*BETA0*BETA02)
AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
DELO=1.5*CK2*TTHMUN/(AO*AO*BETA0*BETA02)
AODP=AO/(1.-DELO)
XNODP=XNO/(1.+DELO)
B=2.*BSTAR/RHO

*          INITIALIZATION

PO=AODP*BETA02
POM2=1./(PO*PO)
SINI=SIN(XINCL)
SING=SIN(OMEGAO)
COSG=COS(OMEGAO)
TEMP=.5*XINCL
SINIO2=SIN(TEMP)
COSIO2=COS(TEMP)
THETA4=THETA2**2
UNM5TH=1.-5.*THETA2
UNMTH2=1.-THETA2
A3COF=-XJ3/CK2*AE**3
PARDT1=3.*CK2*POM2*XNODP
PARDT2=PARDT1*CK2*POM2
PARDT4=1.25*CK4*POM2*POM2*XNODP
XMDT1=.5*PARDT1*BETA0*TTHMUN
XGDT1=-.5*PARDT1*UNM5TH

```

```

XHDT1=-PARDT1*COSI
XLLDOT=XNODP+XMDT1+
2      .0625*PARDT2*BETA0*(13.-78.*THETA2+137.*THETA4)
OMGDT=XGDT1+
1      .0625*PARDT2*(7.-114.*THETA2+395.*THETA4)+PARDT4*(3.-36.*
2      THETA2+49.*THETA4)
XNODOT=XHDT1+
1      (.5*PARDT2*(4.-19.*THETA2)+2.*PARDT4*(3.-7.*THETA2))*COSI
TSI=1./(P0-S)
ETA=E0*S*TSI
ETA2=ETA**2
PSIM2=ABS(1./(1.-ETA2))
ALPHA2=1.+EOSQ
EETA=E0*ETA
COS2G=2.*COSG**2-1.
D5=TSI*PSIM2
D1=D5/P0
D2=12.+ETA2*(36.+4.5*ETA2)
D3=ETA2*(15.+2.5*ETA2)
D4=ETA*(5.+3.75*ETA2)
B1=CK2*TTHMUN
B2=-CK2*UNMTH2
B3=A3COF*SINI
C0=.5*B*RHO*QOMS2T*XNODP*AODP*TSI**4*PSIM2**3.5/SQRT(ALPHA2)
C1=1.5*XNODP*ALPHA2**2*C0
C4=D1*D3*B2
C5=D5*D4*B3
XNDT=C1*(
1  (2.+ETA2*(3.+34.*EOSQ)+5.*EETA*(4.+ETA2)+8.5*EOSQ)+
1  D1*D2*B1+ C4*COS2G+C5*SING)
XNDTN=XNDT/XNODP
EDOT=-TOTHDRD*XNDTN*(1.-E0)
IFLAG=0
CALL DPINIT(EOSQ,SINI,COSI,BETA0,AODP,THETA2,SING,COSG,
1      BETA02,XLLDOT,OMGDT,XNODOT,XNODP)

*      UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG

100 Z1=.5*XNDT*TSINCE*TSINCE
Z7=3.5*TOTHDRD*Z1/XNODP
XMAMDF=XM0+XLLDOT*TSINCE
OMGASM=OMEGA0+OMGDT*TSINCE+Z7*XGDT1
XNODES=XNODE0+XNODOT*TSINCE+Z7*XHDT1
XN=XNODP
CALL DPSEC(XMAMDF,OMGASM,XNODES,EM,XINC,XN,TSINCE)
XN=XN+XNDT*TSINCE
EM=EM+EDOT*TSINCE
XMAM=XMAMDF+Z1+Z7*XMDT1

```

```

CALL DPPER(EM,XINC,OMGASM,XNODES,XMAM)
XMAM=FMOD2P(XMAM)

*      SOLVE KEPLERS EQUATION

ZC2=XMAM+EM*SIN(XMAM)*(1.+EM*COS(XMAM))
DO 130 I=1,10
SINE=SIN(ZC2)
COSE=COS(ZC2)
ZC5=1./(1.-EM*COSE)
CAPE=(XMAM+EM*SINE-ZC2)*
1  ZC5+ZC2
IF(ABS(CAPE-ZC2) .LE. E6A) GO TO 140
130 ZC2=CAPE

*      SHORT PERIOD PRELIMINARY QUANTITIES

140 AM=(XKE/XN)**TOTHRD
BETA2M=1.-EM*EM
SINOS=SIN(OMGASM)
COSOS=COS(OMGASM)
AXNM=EM*COSOS
AYNM=EM*SINOS
PM=AM*BETA2M
G1=1./PM
G2=.5*CK2*G1
G3=G2*G1
BETA=SQRT(BETA2M)
G4=.25*A3COF*SINI
G5=.25*A3COF*G1
SNF=BETA*SINE*ZC5
CSF=(COSE-EM)*ZC5
FM=ACTAN(SNF,CSF)
SNFG=SNF*COSOS+CSF*SINOS
CSFG=CSF*COSOS-SNF*SINOS
SN2F2G=2.*SNFG*CSFG
CS2F2G=2.*CSFG**2-1.
ECOSF=EM*CSF
G10=FM-XMAM+EM*SNF
RM=PM/(1.+ECOSF)
AOVR=AM/RM
G13=XN*AOVR
G14=-G13*AOVR
DR=G2*(UNMTH2*CS2F2G-3.*TTHMUN)-G4*SNFG
DIWC=3.*G3*SINI*CS2F2G-G5*AYNM
DI=DIWC*COSI
SINI2=SIN(.5*XINC)

```

```

*      UPDATE FOR SHORT PERIOD PERIODICS

      SNI2DU=SINIO2*(
1      G3*(.5*(1.-7.*THETA2)*SN2F2G-3.*UNM5TH*G10)-G5*SINI*CSFG*(2.+
2      ECOSF))- .5*G5*THETA2*AXNM/COSIO2
      XLAMB=FM+OMGASM+XNODES+G3*(.5*(1.+6.*COSI-7.*THETA2)*SN2F2G-3.*
1      (UNM5TH+2.*COSI)*G10)+G5*SINI*(COSI*AXNM/(1.+COSI)-(2.*
2      +ECOSF)*CSFG)
      Y4=SINI2*SNFG+CSFG*SNI2DU+.5*SNFG*COSIO2*DI
      Y5=SINI2*CSFG-SNFG*SNI2DU+.5*CSFG*COSIO2*DI
      R=RM+DR
      RDOT=XN*AM*EM*SNF/BETA+G14*(2.*G2*UNMTH2*SN2F2G+G4*CSFG)
      RVDOT=XN*AM**2*BETA/RM+
1      G14*DR+AM*G13*SINI*DIWC

*      ORIENTATION VECTORS

      SNLAMB=SIN(XLAMB)
      CSLAMB=COS(XLAMB)
      TEMP=2.*(Y5*SNLAMB-Y4*CSLAMB)
      UX=Y4*TEMP+CSLAMB
      VX=Y5*TEMP-SNLAMB
      TEMP=2.*(Y5*CSLAMB+Y4*SNLAMB)
      UY=-Y4*TEMP+SNLAMB
      VY=-Y5*TEMP+CSLAMB
      TEMP=2.*SQRT(1.-Y4*Y4-Y5*Y5)
      UZ=Y4*TEMP
      VZ=Y5*TEMP

*      POSITION AND VELOCITY

      X=R*UX
      Y=R*UY
      Z=R*UZ
      XDOT=RDOT*UX+RVDOT*VX
      YDOT=RDOT*UY+RVDOT*VY
      ZDOT=RDOT*UZ+RVDOT*VZ

      RETURN
      END

```

10 THE DEEP-SPACE SUBROUTINE

The two deep-space models, SDP4 and SDP8, both access the subroutine DEEP to obtain the deep-space perturbations of the six classical orbital elements. The perturbation equations are quite extensive and will not be repeated here. Rather, this section will concentrate on a general description of the flow between the main program and the deep-space subroutines. A specific listing of the equations is available in Hujsak (1979) or Hujsak and Hoots (1977).

The first time the deep-space subroutine is accessed is during the initialization portion of SDP4/SDP8 and is via the entry DPINIT. Through this entry, certain constants already calculated in SDP4/SDP8 are passed to the deep-space subroutine which in turn calculates all initialized (time independent) quantities needed for prediction in deep space. Additionally, a determination is made and flags are set concerning whether the orbit is synchronous and whether the orbit experiences resonance effects.

The next access to the deep-space subroutine occurs during the secular update portion of SDP4/SDP8 and is via the entry DPSEC. Through this entry, the current secular values of the "mean" orbital elements are passed to the deep-space subroutine which in turn adds the appropriate deep-space secular and long-period resonance effects to these mean elements.

The last access to the deep-space subroutine occurs at the beginning of the osculation portion (periodics application) of SDP4/SDP8 and is via the entry DPPER. Through this entry, the current values of the orbital elements are passed to the deep-space subroutine which in turn adds the appropriate deep-space lunar and solar periodics to the orbital elements.

During initialization the deep-space subroutine calls the function subroutine THETAG to obtain the location of Greenwich at epoch and to convert epoch to minutes since 1950. All physical constants which are unique to the deep-space subroutine are set via data statements in DEEP rather than being passed through a COMMON.

A FORTRAN IV computer code listing of the subroutine DEEP is given below. These equations contain all currently anticipated changes to the SCC operational program. These changes are scheduled for implementation in March, 1981.

```

*      DEEP SPACE                                     31 OCT 80
SUBROUTINE DEEP
COMMON/E1/XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT20,
1      XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
1      XJ3,XKE,XKMPER,XMNPDA,AE
COMMON/C2/DE2RA,PI,PIO2,TWOPI,X3PIO2
DOUBLE PRECISION EPOCH, DS50
DOUBLE PRECISION
*      DAY,PREEP,XNODCE,ATIME,DELT,SAVTSN,STEP2,STEPP,STEPP
DATA      ZNS,      C1SS,      ZES/
A      1.19459E-5,      2.9864797E-6, .01675/
DATA      ZNL,      C1L,      ZEL/
A      1.5835218E-4,      4.7968065E-7, .05490/
DATA      ZCOSIS,      ZSINIS,      ZSINGS/
A      .91744867,      .39785416,      -.98088458/
DATA      ZCOSGS,      ZCOSHS,      ZSINHS/
A      .1945905,      1.0,      0.0/
DATA Q22,Q31,Q33/1.7891679E-6,2.1460748E-6,2.2123015E-7/
DATA G22,G32/5.7686396,0.95240898/
DATA G44,G52/1.8014998,1.0508330/
DATA G54/4.4108898/
DATA ROOT22,ROOT32/1.7891679E-6,3.7393792E-7/
DATA ROOT44,ROOT52/7.3636953E-9,1.1428639E-7/
DATA ROOT54/2.1765803E-9/
DATA THDT/4.3752691E-3/

*      ENTRANCE FOR DEEP SPACE INITIALIZATION

ENTRY DPINIT(EQSQ,SINI,QCOSIQ,RTEQSQ,AO,COSQ2,SINOMO,COSOMO,
1      BSQ,XLLDOT,OMGDT,XNODOT,XNODP)
THGR=THETAG(EPOCH)
EQ = EO
XNQ = XNODP
AQNV = 1./AO
XQNCL = XINCL
XMAO=XMO
XPIDOT=OMGDT+XNODOT
SINQ = SIN(XNODEO)
COSQ = COS(XNODEO)
OMEGAQ = OMEGAO

*      INITIALIZE LUNAR SOLAR TERMS

5 DAY=DS50+18261.5D0
IF (DAY.EQ.PREEP) GO TO 10
PREEP = DAY
XNODCE=4.5236020-9.2422029E-4*DAY

```

```

STEM=DSIN (XNODCE)
CTEM=DCOS (XNODCE)
ZCOSIL=.91375164-.03568096*CTEM
ZSINIL=SQRT (1.-ZCOSIL*ZCOSIL)
ZSINHL= .089683511*STEM/ZSINIL
ZCOSHL=SQRT (1.-ZSINHL*ZSINHL)
C=4.7199672+.22997150*DAY
GAM=5.8351514+.0019443680*DAY
ZMOL = FMOD2P(C-GAM)
ZX= .39785416*STEM/ZSINIL
ZY= ZCOSHL*CTEM+0.91744867*ZSINHL*STEM
ZX=ACTAN(ZX,ZY)
ZX=GAM+ZX-XNODCE
ZCOSGL=COS (ZX)
ZSINGL=SIN (ZX)
ZMOS=6.2565837D0+.017201977D0*DAY
ZMOS=FMOD2P(ZMOS)

```

* DO SOLAR TERMS

```

10 LS = 0
SAVTSN=1.D20
ZCOSG=ZCOSGS
ZSING=ZSINGS
ZCOSI=ZCOSIS
ZSINI=ZSINIS
ZCOSH=COSQ
ZSINH=SINQ
CC=C1SS
ZN=ZNS
ZE=ZES
ZMO=ZMOS
XNOI=1./XNQ
ASSIGN 30 TO LS
20 A1=ZCOSG*ZCOSH+ZSING*ZCOSI*ZSINH
A3=-ZSING*ZCOSH+ZCOSG*ZCOSI*ZSINH
A7=-ZCOSG*ZSINH+ZSING*ZCOSI*ZCOSH
A8=ZSING*ZSINI
A9=ZSING*ZSINH+ZCOSG*ZCOSI*ZCOSH
A10=ZCOSG*ZSINI
A2= COSIQ*A7+ SINIQ*A8
A4= COSIQ*A9+ SINIQ*A10
A5=- SINIQ*A7+ COSIQ*A8
A6=- SINIQ*A9+ COSIQ*A10

```

C

```

X1=A1*COSOMO+A2*SINOMO
X2=A3*COSOMO+A4*SINOMO
X3=-A1*SINOMO+A2*COSOMO

```

```

X4=-A3*SINOMO+A4*COSOMO
X5=A5*SINOMO
X6=A6*SINOMO
X7=A5*COSOMO
X8=A6*COSOMO

```

C

```

Z31=12.*X1*X1-3.*X3*X3
Z32=24.*X1*X2-6.*X3*X4
Z33=12.*X2*X2-3.*X4*X4
Z1=3.*(A1*A1+A2*A2)+Z31*EQSQ
Z2=6.*(A1*A3+A2*A4)+Z32*EQSQ
Z3=3.*(A3*A3+A4*A4)+Z33*EQSQ
Z11=-6.*A1*A5+EQSQ*(-24.*X1*X7-6.*X3*X5)
Z12=-6.*(A1*A6+A3*A5)+EQSQ*(-24.*(X2*X7+X1*X8)-6.*(X3*X6+X4*X5))
Z13=-6.*A3*A6+EQSQ*(-24.*X2*X8-6.*X4*X6)
Z21=6.*A2*A5+EQSQ*(24.*X1*X5-6.*X3*X7)
Z22=6.*(A4*A5+A2*A6)+EQSQ*(24.*(X2*X5+X1*X6)-6.*(X4*X7+X3*X8))
Z23=6.*A4*A6+EQSQ*(24.*X2*X6-6.*X4*X8)
Z1=Z1+Z1+BSQ*Z31
Z2=Z2+Z2+BSQ*Z32
Z3=Z3+Z3+BSQ*Z33
S3=CC*XNOI
S2=-.5*S3/RTEQSQ
S4=S3*RTEQSQ
S1=-15.*EQ*S4
S5=X1*X3+X2*X4
S6=X2*X3+X1*X4
S7=X2*X4-X1*X3
SE=S1*ZN*S5
SI=S2*ZN*(Z11+Z13)
SL=-ZN*S3*(Z1+Z3-14.-6.*EQSQ)
SGH=S4*ZN*(Z31+Z33-6.)
SH=-ZN*S2*(Z21+Z23)
IF(XQNCL.LT.5.2359877E-2) SH=0.0
EE2=2.*S1*S6
E3=2.*S1*S7
XI2=2.*S2*Z12
XI3=2.*S2*(Z13-Z11)
XL2=-2.*S3*Z2
XL3=-2.*S3*(Z3-Z1)
XL4=-2.*S3*(-21.-9.*EQSQ)*ZE
XGH2=2.*S4*Z32
XGH3=2.*S4*(Z33-Z31)
XGH4=-18.*S4*ZE
XH2=-2.*S2*Z22
XH3=-2.*S2*(Z23-Z21)
GO TO LS

```


* DO LUNAR TERMS

```
30 SSE = SE
   SSI=SI
   SSL=SL
   SSH=SH/SINI
   SSG=SGH-COSIQ*SSH
   SE2=EE2
   SI2=XI2
   SL2=XL2
   SGH2=XGH2
   SH2=XH2
   SE3=E3
   SI3=XI3
   SL3=XL3
   SGH3=XGH3
   SH3=XH3
   SL4=XL4
   SGH4=XGH4
   LS=1
   ZCOSG=ZCOSGL
   ZSING=ZSINGL
   ZCOSI=ZCOSIL
   ZSINI=ZSINIL
   ZCOSH=ZCOSHL*COSQ+ZSINHL*SINQ
   ZSINH=SINQ*ZCOSHL-COSQ*ZSINHL
   ZN=ZNL
   CC=C1L
   ZE=ZEL
   ZMO=ZMOL
   ASSIGN 40 TO LS
   GO TO 20
40 SSE = SSE+SE
   SSI=SSI+SI
   SSL=SSL+SL
   SSG=SSG+SGH-COSIQ/SINI*SH
   SSH=SSH+SH/SINI
```

* GEOPOTENTIAL RESONANCE INITIALIZATION FOR 12 HOUR ORBITS

```
IRESFL=0
ISYNFL=0
IF(XNQ.LT.(.0052359877).AND.XNQ.GT.(.0034906585)) GO TO 70
IF (XNQ.LT.(8.26E-3) .OR. XNQ.GT.(9.24E-3)) RETURN
IF (EQ.LT.0.5) RETURN
IRESFL =1
EOC=EQ*EQSQ
G201=-.306-(EQ-.64)*.440
```



```

D3222 = TEMP*F322*G322
TEMP1 = TEMP1*AQNV
TEMP = 2.*TEMP1*ROOT44
D4410 = TEMP*F441*G410
D4422 = TEMP*F442*G422
TEMP1 = TEMP1*AQNV
TEMP = TEMP1*ROOT52
D5220 = TEMP*F522*G520
D5232 = TEMP*F523*G532
TEMP = 2.*TEMP1*ROOT54
D5421 = TEMP*F542*G521
D5433 = TEMP*F543*G533
XLAMO = XMAO+XNODEO+XNODEO-THGR-THGR
BFACT = XLLDOT+XNODOT+XNODOT-THDT-THDT
BFACT=BFACT+SSL+SSH+SSH
GO TO 80

```

* SYNCHRONOUS RESONANCE TERMS INITIALIZATION

```

70 IRESFL=1
   ISYNFL=1
   G200=1.0+EQSQ*(-2.5+.8125*EQSQ)
   G310=1.0+2.0*EQSQ
   G300=1.0+EQSQ*(-6.0+6.60937*EQSQ)
   F220=.75*(1.+COSIQ)*(1.+COSIQ)
   F311=.9375*SINIQ*SINIQ*(1.+3.*COSIQ)-.75*(1.+COSIQ)
   F330=1.+COSIQ
   F330=1.875*F330*F330*F330
   DEL1=3.*XNQ*XNQ*AQNV*AQNV
   DEL2=2.*DEL1*F220*G200*Q22
   DEL3=3.*DEL1*F330*G300*Q33*AQNV
   DEL1=DEL1*F311*G310*Q31*AQNV
   FASX2=.13130908
   FASX4=2.8843198
   FASX6=.37448087
   XLAMO=XMAO+XNODEO+OMEGAO-THGR
   BFACT = XLLDOT+XPIDOT-THDT
   BFACT=BFACT+SSL+SSG+SSH
80 XFACT=BFACT-XNQ

```

C

C

INITIALIZE INTEGRATOR

C

```

XLI=XLAMO
XNI=XNQ
ATIME=0.D0
STEPP=720.D0
STEPN=-720.D0
STEP2 = 259200.D0

```

```

        RETURN

*      ENTRANCE FOR DEEP SPACE SECULAR EFFECTS

      ENTRY DPSEC(XLL,OMGASM,XNODES,EM,XINC,XN,T)
      XLL=XLL+SSL*T
      OMGASM=OMGASM+SSG*T
      XNODES=XNODES+SSH*T
      EM=EO+SSE*T
      XINC=XINCL+SSI*T
      IF(XINC .GE. 0.) GO TO 90
      XINC = -XINC
      XNODES = XNODES + PI
      OMGASM = OMGASM - PI
90    IF(IRESFL .EQ. 0) RETURN
100   IF (ATIME.EQ.0.DO)      GO TO 170
      IF(T.GE.(0.DO).AND.ATIME.LT.(0.DO)) GO TO 170
      IF(T.LT.(0.DO).AND.ATIME.GE.(0.DO)) GO TO 170
105   IF(DABS(T).GE.DABS(ATIME)) GO TO 120
      DELT=STEPP
      IF (T.GE.0.DO)      DELT = STEPN
110   ASSIGN 100 TO IRET
      GO TO 160
120   DELT=STEPN
      IF (T.GT.0.DO)      DELT = STEPP
125   IF (DABS(T-ATIME).LT.STEPP)      GO TO 130
      ASSIGN 125 TO IRET
      GO TO 160
130   FT = T-ATIME
      ASSIGN 140 TO IRET
      GO TO 150
140   XN = XNI+XNDOT*FT+XNDDT*FT*FT*0.5
      XL = XLI+XLDOT*FT+XNDOT*FT*FT*0.5
      TEMP = -XNODES+THGR+T*THDT
      XLL = XL-OMGASM+TEMP
      IF (ISYNFL.EQ.0)      XLL = XL+TEMP+TEMP
      RETURN

C
C      DOT TERMS CALCULATED
C
150   IF (ISYNFL.EQ.0)      GO TO 152
      XNDOT=DEL1*SIN (XLI-FASX2)+DEL2*SIN (2.*(XLI-FASX4))
1    +DEL3*SIN (3.*(XLI-FASX6))
      XNDDT = DEL1*COS(XLI-FASX2)
      *      +2.*DEL2*COS(2.*(XLI-FASX4))
      *      +3.*DEL3*COS(3.*(XLI-FASX6))
      GO TO 154
152   XOMI = OMEGAQ+OMGDT*ATIME

```

```

X2OMI = XOMI+XOMI
X2LI = XLI+XLI
XNDOT = D2201*SIN(X2OMI+XLI-G22)
*      +D2211*SIN(XLI-G22)
*      +D3210*SIN(XOMI+XLI-G32)
*      +D3222*SIN(-XOMI+XLI-G32)
*      +D4410*SIN(X2OMI+X2LI-G44)
*      +D4422*SIN(X2LI-G44)
*      +D5220*SIN(XOMI+XLI-G52)
*      +D5232*SIN(-XOMI+XLI-G52)
*      +D5421*SIN(XOMI+X2LI-G54)
*      +D5433*SIN(-XOMI+X2LI-G54)
XNDDT = D2201*COS(X2OMI+XLI-G22)
*      +D2211*COS(XLI-G22)
*      +D3210*COS(XOMI+XLI-G32)
*      +D3222*COS(-XOMI+XLI-G32)
*      +D5220*COS(XOMI+XLI-G52)
*      +D5232*COS(-XOMI+XLI-G52)
*      +2.*(D4410*COS(X2OMI+X2LI-G44)
*      +D4422*COS(X2LI-G44)
*      +D5421*COS(XOMI+X2LI-G54)
*      +D5433*COS(-XOMI+X2LI-G54))
154 XLDOT=XNI*XFACT
    XNDDT = XNDDT*XLDOT
    GO TO IRETN
C
C    INTEGRATOR
C
160 ASSIGN 165 TO IRETN
    GO TO 150
165 XLI = XLI+XLDOT*DELT+XNDOT*STEP2
    XNI = XNI+XNDOT*DELT+XNDDT*STEP2
    ATIME=ATIME+DELT
    GO TO IRET
C
C    EPOCH RESTART
C
170 IF (T.GE.0.D0)      GO TO 175
    DELT=STEPPN
    GO TO 180
175 DELT = STEPP
180 ATIME = 0.D0
    XNI=XNQ
    XLI=XLAMO
    GO TO 125
C
C    ENTRANCES FOR LUNAR-SOLAR PERIODICS
C

```

```

C
ENTRY DPPER(EM,XINC,OMGASM,XNODES,XLL)
SINIS = SIN(XINC)
COSIS = COS(XINC)
IF (DABS(SAVTSN-T).LT.(30.DO)) GO TO 210
SAVTSN=T
ZM=ZMOS+ZNS*T
205 ZF=ZM+2.*ZES*SIN (ZM)
SINZF=SIN (ZF)
F2=.5*SINZF*SINZF-.25
F3=-.5*SINZF*COS (ZF)
SES=SE2*F2+SE3*F3
SIS=SI2*F2+SI3*F3
SLS=SL2*F2+SL3*F3+SL4*SINZF
SGHS=SGH2*F2+SGH3*F3+SGH4*SINZF
SHS=SH2*F2+SH3*F3
ZM=ZMOL+ZNL*T
ZF=ZM+2.*ZEL*SIN (ZM)
SINZF=SIN (ZF)
F2=.5*SINZF*SINZF-.25
F3=-.5*SINZF*COS (ZF)
SEL=EE2*F2+E3*F3
SIL=XI2*F2+XI3*F3
SLL=XL2*F2+XL3*F3+XL4*SINZF
SGHL=XGH2*F2+XGH3*F3+XGH4*SINZF
SHL=XH2*F2+XH3*F3
PE=SES+SEL
PINC=SIS+SIL
PL=SLS+SLL
210 PGH=SGHS+SGHL
PH=SHS+SHL
XINC = XINC+PINC
EM = EM+PE
IF(XQNCL.LT.(.2)) GO TO 220
GO TO 218

C
C APPLY PERIODICS DIRECTLY
C
218 PH=PH/SINIQ
PGH=PGH-COSIQ*PH
OMGASM=OMGASM+PGH
XNODES=XNODES+PH
XLL = XLL+PL
GO TO 230

C
C APPLY PERIODICS WITH LYDDANE MODIFICATION
C
220 SINOK=SIN(XNODES)

```

```

COSOK=COS(XNODES)
ALFDP=SINIS*SINOK
BETDP=SINIS*COSOK
DALF=PH*COSOK+PINC*COSIS*SINOK
DBET=-PH*SINOK+PINC*COSIS*COSOK
ALFDP=ALFDP+DALF
BETDP=BETDP+DBET
XLS = XLL+OMGASM+COSIS*XNODES
DLS=PL+PGH-PINC*XNODES*SINIS
XLS=XLS+DLS
XNODES=ACTAN(ALFDP,BETDP)
XLL = XLL+PL
OMGASM = XLS-XLL-COS(XINC)*XNODES
230 CONTINUE
RETURN
END

```

11 DRIVER AND FUNCTION SUBROUTINES

The DRIVER controls the input and output function and the selection of the model. The input consists of a program card which specifies the model to be used and the output times and either a G-card or T-card element set.

The DRIVER reads and converts the input elements to units of radians and minutes. These are communicated to the prediction model through the COMMON E1. Values of the physical and mathematical constants are set and communicated through the COMMONs C1 and C2, respectively.

The program card indicates the mathematical model to be used and the start and stop time of prediction as well as the increment of time for output. These times are in minutes since epoch.

In the interest of efficiency the DRIVER sets a flag (IFLAG) the first time the model is called. This flag tells the model to calculate all initialized (time independent) quantities. After initialization, the model subroutine turns off the flag so that all subsequent calls only access the time dependent part of the model. This mode continues until another input case is encountered.

The DRIVER takes the output from the mathematical model (communicated through the COMMON E1) and converts it to units of kilometers and seconds for printout.

The function subroutine ACTAN is passed the values of sine and cosine in that order and it returns the angle in radians within the range of 0 to 2π . The function subroutine FMOD2P is passed an angle in radians and returns the angle in radians within the range of 0 to 2π . The function subroutine THETAG is passed the epoch time exactly as it appears on the input element cards.¹ The routine converts this time to days since 1950 Jan 0.0 UTC, stores this in the COMMON E1, and returns the right ascension of Greenwich at epoch (in radians).

FORTTRAN IV computer code listings of the routines DRIVER, ACTAN, FMOD2P, and THETAG are given below.

¹If only one year digit is given (as on standard G-cards) the program assumes the 80 decade. This may be overridden by putting a 2 digit year in columns 30-31 of the first G-card.


```

*          DRIVER                                          3 NOV 80

* WGS-72 PHYSICAL AND GEOPOTENTIAL CONSTANTS
*          CK2= .5*J2*AE**2      CK4=-.375*J4*AE**4

      DOUBLE PRECISION EPOCH,DS50
      COMMON/E1/XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT20,XNDD60,BSTAR,
1      X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
      COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
1      XJ3,XKE,XKMPER,XMNPDA,AE
      COMMON/C2/DE2RA,PI,PIO2,TWOPI,X3PIO2
      DATA IHG/1HG/
      DATA DE2RA,E6A,PI,PIO2,QO,SO,TOTHRD,TWOPI,X3PIO2,XJ2,XJ3,
1      XJ4,XKE,XKMPER,XMNPDA,AE/.174532925E-1,1.E-6,
2      3.14159265,1.57079633,120.0,78.0,.66666667,
4      6.2831853,4.71238898,1.082616E-3,-.253881E-5,
5      -1.65597E-6,.743669161E-1,6378.135,1440.,1./
      DIMENSION ISET(5)
      CHARACTER ABUF*80(2)
      DATA (ISET(I),I=1,5)/3HSGP,4HSGP4,4HSDP4,4HSGP8,4HSDP8/

* SELECT EPHEMERIS TYPE AND OUTPUT TIMES

      CK2=.5*XJ2*AE**2
      CK4=-.375*XJ4*AE**4
      QOMS2T=((QO-SO)*AE/XKMPER)**4
      S=AE*(1.+SO/XKMPER)
2 READ (5,700) IEPT, TS,TF,DELT
      IF(IEPT.LE.0) STOP
      IDEEP=0

* READ IN MEAN ELEMENTS FROM 2 CARD T(TRANS) OR G(INTERN) FORMAT

      READ (5,706) ABUF
      DECODE(ABUF(1),707) ITYPE
      IF(ITYPE.EQ.IHG) GO TO 5
      DECODE (ABUF,702) EPOCH,XNDT20,XNDD60,IEXP,BSTAR,IBEXP,XINCL,
1      XNODEO,EO,OMEGAO,XMO,XNO
      GO TO 7
5 DECODE(ABUF,701) EPOCH,XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT20,
1      XNDD60,IEXP,BSTAR,IBEXP
7 IF(XNO.LE.0.) STOP
      WRITE(6,704) ABUF,ISET(IEPT)
      IF(IEPT.GT.5) GO TO 900
      XNDD60=XNDD60*(10.**IEXP)
      XNODEO=XNODEO*DE2RA
      OMEGAO=OMEGAO*DE2RA
      XMO=XMO*DE2RA

```

```

XINCL=XINCL*DE2RA
TEMP=TWOPI/XMNPDA/XMNPDA
XNO=XNO*TEMP*XMNPDA
XNDT20=XNDT20*TEMP
XNDD60=XNDD60*TEMP/XMNPDA

*   INPUT CHECK FOR PERIOD VS EPHEMERIS SELECTED
*   PERIOD GE 225 MINUTES  IS DEEP SPACE

A1=(XKE/XNO)**TOTHRD
TEMP=1.5*CK2*(3.*COS(XINCL)**2-1.)/(1.-EO*EO)**1.5
DEL1=TEMP/(A1*A1)
AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
DELO=TEMP/(AO*AO)
XNODP=XNO/(1.+DELO)
IF((TWOPI/XNODP/XMNPDA) .GE. .15625) IDEEP=1

BSTAR=BSTAR*(10.**IBEXP)/AE
TSINCE=TS
IFLAG=1
IF(IDEEP .EQ. 1 .AND. (IEPT .EQ. 1 .OR. IEPT .EQ. 2
1      .OR. IEPT .EQ. 4)) GO TO 800
9 IF(IDEEP .EQ. 0 .AND. (IEPT .EQ. 3 .OR. IEPT .EQ. 5))
1      GO TO 850
10 GO TO (21,22,23,24,25), IEPT
21 CALL SGP(IFLAG,TSINCE)
   GO TO 60
22 CALL SGP4(IFLAG,TSINCE)
   GO TO 60
23 CALL SDP4(IFLAG,TSINCE)
   GO TO 60
24 CALL SGP8(IFLAG,TSINCE)
   GO TO 60
25 CALL SDP8(IFLAG,TSINCE)
60 X=X*XKMPER/AE
   Y=Y*XKMPER/AE
   Z=Z*XKMPER/AE
   XDOT=XDOT*XKMPER/AE*XMNPDA/86400.
   YDOT=YDOT*XKMPER/AE*XMNPDA/86400.
   ZDOT=ZDOT*XKMPER/AE*XMNPDA/86400.
   WRITE(6,705) TSINCE,X,Y,Z,XDOT,YDOT,ZDOT
   TSINCE=TSINCE+DELT
   IF(ABS(TSINCE) .GT. ABS(TF)) GO TO 2
   GO TO 10
700 FORMAT(I1,3F10.0)
701 FORMAT(29X,D14.8,1X,3F8.4,/,6X,F8.7,F8.4,1X,2F11.9,1X,F6.5,I2,
1      4X,F8.7,I2)
702 FORMAT(18X,D14.8,1X,F10.8,2(1X,F6.5,I2),/,7X,2(1X,F8.4),1X,

```

```

1          F7.7,2(1X,F8.4),1X,F11.8)
703 FORMAT(79X,A1)
704 FORMAT(1H1,A80,/,1X,A80,/,1X,A4,7H TSINCE,
1 14X,1HX,16X,1HY,16X,1HZ,14X,
1 4HXDOT,13X,4HYDOT,13X,4HZDOT,/)
705 FORMAT(7F17.8)
706 FORMAT(A80)
707 FORMAT(79X,A1)
930 FORMAT("SHOULD USE DEEP SPACE EPHEMERIS")
940 FORMAT("SHOULD USE NEAR EARTH EPHEMERIS")
950 FORMAT("EPHEMERIS NUMBER",I2," NOT LEGAL, WILL SKIP THIS CASE")
800 WRITE(6,930)
      GO TO 9
850 WRITE(6,940)
      GO TO 10
900 WRITE(6,950) IEPT
      GO TO 2
      END

```

```

FUNCTION  ACTAN(SINX,COSX)
COMMON/C2/DE2RA,PI,PIO2,TWOPI,X3PIO2
ACTAN=0.
IF (COSX.EQ.0. ) GO TO 5
IF (COSX.GT.0. ) GO TO 1
ACTAN=PI
GO TO 7
1 IF (SINX.EQ.0. ) GO TO 8
  IF (SINX.GT.0. ) GO TO 7
  ACTAN=TWOPI
  GO TO 7
5 IF (SINX.EQ.0. ) GO TO 8
  IF (SINX.GT.0. ) GO TO 6
  ACTAN=X3PIO2
  GO TO 8
6 ACTAN=PIO2
  GO TO 8
7 TEMP=SINX/COSX
  ACTAN=ACTAN+ATAN(TEMP)
8 RETURN
END

```

```
FUNCTION FMOD2P(X)
COMMON/C2/DE2RA,PI,PIO2,TWOPI,X3PIO2
FMOD2P=X
I=FMOD2P/TWOPI
  FMOD2P=FMOD2P-I*TWOPI
IF (FMOD2P.LT.0) FMOD2P=FMOD2P+TWOPI
RETURN
END
```

```

FUNCTION THETAG(EP)
COMMON /E1/XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT2O,XNDD6O,BSTAR,
1 X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS5O
DOUBLE PRECISION EPOCH,D,THETA,TWOPI,YR,TEMP,EP,DS5O
TWOPI=6.28318530717959D0
YR=(EP+2.D-7)*1.D-3
JY=YR
YR=JY
D=EP-YR*1.D3
IF(JY.LT.10) JY=JY+80
N=(JY-69)/4
IF(JY.LT.70) N=(JY-72)/4
DS5O=7305.D0 + 365.D0*(JY-70) +N + D
THETA=1.72944494D0 + 6.3003880987D0*DS5O
TEMP=THETA/TWOPI
I=TEMP
TEMP=I
THETAG=THETA-TEMP*TWOPI
IF(THETAG.LT.O.D0) THETAG=THETAG+TWOPI
RETURN
END

```

12 USERS GUIDE, CONSTANTS, AND SYMBOLS

The first input card is the program card. The format is as follows:

Column	Format	Description
1	I1	Ephemeris program desired 1 = SGP 2 = SGP4 3 = SDP4 4 = SGP8 5 = SDP8
2-11	F10.0	Prediction start time
12-21	F10.0	Prediction stop time
22-31	F10.0	Time increment

All times are in minutes since epoch and can be positive or negative. The second and third input cards consist of either a 2-card transmission or 2-card G type element set. Either type can be used with the only condition being that the two cards must be in the correct order. For reference a format sheet for the T-card and G-card element sets follows this section.

The values of the physical and mathematical constants used in the program are given below.

<u>Variable name</u>	<u>Definition</u>	<u>Value</u>
CK2	$\frac{1}{2}J_2a_E^2$	5.413080E-4
CK4	$-\frac{3}{8}J_4a_E^4$.62098875E-6
E6A	10^{-6}	1.0 E-6
QOMS2T	$(q_o - s)^4 \text{ (er)}^4$	1.88027916E-9
S	$s \text{ (er)}$	1.01222928
TOTHRD	$2/3$.66666667
XJ3	J_3	-.253881E-5
XKE	$k_e \left(\frac{\text{er}}{\text{min}} \right)^{\frac{3}{2}}$.743669161E-1
XKMPER	kilometers/Earth radii	6378.135
XMNPDA	time units/day	1440.0

AE	distance units/Earth radii	1.0
DE2RA	radians/degree	.174532925E-1
PI	π	3.14159265
PIO2	$\pi/2$	1.57079633
TWOPI	2π	6.2831853
X3PIO2	$3\pi/2$	4.71238898

where er = Earth radii. Except for the deep-space models, all ephemeris models are independent of units. Thus, units input or output as well as physical constants can be changed by making the appropriate changes in only the DRIVER program.

Following is a list of symbols commonly used in this report.

n_o = the SGP type “mean” mean motion at epoch

e_o = the “mean” eccentricity at epoch

i_o = the “mean” inclination at epoch

M_o = the “mean” mean anomaly at epoch

ω_o = the “mean” argument of perigee at epoch

Ω_o = the “mean” longitude of ascending node at epoch

\dot{n}_o = the time rate of change of “mean” mean motion at epoch

\ddot{n}_o = the second time rate of change of “mean” mean motion at epoch

B^* = the SGP4 type drag coefficient

$k_e = \sqrt{GM}$ where G is Newton’s universal gravitational constant and M is the mass of the Earth

a_E = the equatorial radius of the Earth

J_2 = the second gravitational zonal harmonic of the Earth

J_3 = the third gravitational zonal harmonic of the Earth

J_4 = the fourth gravitational zonal harmonic of the Earth

$(t - t_o)$ = time since epoch

$$k_2 = \frac{1}{2}J_2a_E^2$$

$$k_4 = -\frac{3}{8}J_4a_E^4$$

$$A_{3,0} = -J_3a_E^3$$

q_o = parameter for the SGP4/SGP8 density function

s = parameter for the SGP4/SGP8 density function

$B = \frac{1}{2}C_D\frac{A}{m}$, the ballistic coefficient for SGP8 where C_D is a dimensionless drag coefficient and A is the average cross-sectional area of the satellite of mass m

13 SAMPLE TEST CASES

For reference a sample test case is given for each of the five models.¹ The input used was standard T-cards and the output is given at 360 minute intervals in units of kilometers and seconds.

When implemented on a given computer, the accuracies with which the test cases are duplicated will be dominated by the accuracy of the epoch mean motion. If, after reading and converting, the epoch mean motion has an error $\Delta n = j \times 10^{-k}$ radians/time, then the predicted positions at time t may differ from the test cases by numbers on the order of

$$\Delta r = \Delta n(t - t_o)(6,378.135) \text{ kilometers}$$

¹The test cases were generated on a machine with 8 digits of accuracy. After a one day prediction, the test cases have only 5 to 6 digits of accuracy.

```

1 88888U      80275.98708465 .00073094 13844-3 66816-4 0    8
2 88888 72.8435 115.9689 0086731 52.6988 110.5714 16.05824518 105

```

SGP	TSINCE	X	Y	Z
	0.	2328.96594238	-5995.21600342	1719.97894287
	360.00000000	2456.00610352	-6071.94232177	1222.95977784
	720.00000000	2567.39477539	-6112.49725342	713.97710419
	1080.00000000	2663.03179932	-6115.37414551	195.73919105
	1440.00000000	2742.85470581	-6079.13580322	-328.86091614

XDOT	YDOT	ZDOT
2.91110113	-0.98164053	-7.09049922
2.67852119	-0.44705850	-7.22800565
2.43952477	0.09884824	-7.31889641
2.19531813	0.65333930	-7.36169147
1.94707947	1.21346101	-7.35499924

1	88888U		80275.98708465	.00073094	13844-3	66816-4	0	8
2	88888	72.8435	115.9689	0086731	52.6988	110.5714	16.05824518	105

SGP4 TSINCE	X	Y	Z
0.	2328.97048951	-5995.22076416	1719.97067261
360.00000000	2456.10705566	-6071.93853760	1222.89727783
720.00000000	2567.56195068	-6112.50384522	713.96397400
1080.00000000	2663.09078980	-6115.48229980	196.39640427
1440.00000000	2742.55133057	-6079.67144775	-326.38095856

XDOT	YDOT	ZDOT
2.91207230	-0.98341546	-7.09081703
2.67938992	-0.44829041	-7.22879231
2.44024599	0.09810869	-7.31995916
2.19611958	0.65241995	-7.36282432
1.94850229	1.21106251	-7.35619372

```

1 11801U      80230.29629788 .01431103 00000-0 14311-1
2 11801 46.7916 230.4354 7318036 47.4722 10.4117 2.28537848

```

```

SDP4 TSINCE      X      Y      Z

      0.      7473.37066650      428.95261765      5828.74786377
    360.00000000    -3305.22537232    32410.86328125    -24697.17675781
    720.00000000    14271.28759766    24110.46411133    -4725.76837158
   1080.00000000   -9990.05883789    22717.35522461    -23616.89062501
   1440.00000000    9787.86975097    33753.34667969   -15030.81176758

```

```

      XDOT      YDOT      ZDOT

      5.10715413      6.44468284     -0.18613096
     -1.30113538     -1.15131518     -0.28333528
     -0.32050445      2.67984074     -2.08405289
     -1.01667246     -2.29026759      0.72892364
     -1.09425066      0.92358845     -1.52230928

```

1	88888U		80275.98708465	.00073094	13844-3	66816-4	0	8
2	88888	72.8435	115.9689	0086731	52.6988	110.5714	16.05824518	105

SGP8	TSINCE	X	Y	Z
	0.	2328.87265015	-5995.21289063	1720.04884338
	360.00000000	2456.04577637	-6071.90490722	1222.84086609
	720.00000000	2567.68383789	-6112.40881348	713.29282379
	1080.00000000	2663.49508667	-6115.18182373	194.62816810
	1440.00000000	2743.29238892	-6078.90783691	-329.73434067

XDOT	YDOT	ZDOT
2.91210661	-0.98353850	-7.09081554
2.67936245	-0.44820847	-7.22888553
2.43992555	0.09893919	-7.32018769
2.19525236	0.65453661	-7.36308974
1.94680957	1.21500109	-7.35625595

1	11801U		80230.29629788	.01431103	00000-0	14311-1
2	11801	46.7916	230.4354	7318036	47.4722	10.4117 2.28537848

SDP8 TSINCE	X	Y	Z
0.	7469.47631836	415.99390792	5829.64318848
360.00000000	-3337.38992310	32351.39086914	-24658.63037109
720.00000000	14226.54333496	24236.08740234	-4856.19744873
1080.00000000	-10151.59838867	22223.69848633	-23392.39770508
1440.00000000	9420.08203125	33847.21875000	-15391.06469727

XDOT	YDOT	ZDOT
5.11402285	6.44403201	-0.18296110
-1.30200730	-1.15603013	-0.28164955
-0.33951668	2.65315416	-2.08114153
-1.00112480	-2.33532837	0.76987664
-1.11986055	0.85410149	-1.49506933

14 SAMPLE IMPLEMENTATION

These FORTRAN IV routines have been implemented on a Honeywell-6000 series computer. This machine has a processing speed in the 1MIPS range and a 36 bit floating point word providing 8 significant figures of accuracy in single precision. The information in the following table is provided to allow a comparison of the relative size and speed of the different models¹.

Model	core used (words)	CPU time per call (milliseconds)	
		Initialize	Continue
SGP	541	.8	2.7
SGP4	1,041	1.9	2.5
SDP4	3,095	5.1	3.6
SGP8	1,601	1.8	2.2
SDP8	3,149	5.4	3.2

¹The timing results are for the test cases in Section Thirteen with a one day prediction. Times may vary slightly with orbital characteristics and, for deep-space satellites, with prediction interval.

ACKNOWLEDGEMENTS

The authors would like to express our appreciation to Bob Morris for his helpful comments, to Linda Crawford for supplying one of the function subroutines, and to Don Larson for his help in validating the computer programs. Finally, we wish to acknowledge the skill and patience of Maria Gallegos who accomplished the unenviable task of typing this report.

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