Никита Осипов, ПМ - 1801, 1.1.9. а) Методы ортогонализации. Ортогонализация столбцов матрицы

In[]:=
```
A = {{1, 0.17, -0.25, 0.54}, {0.47, 1, 0.67, -0.32},
    {-0.11, 0.35, 1, -0.74}, {0.55, 0.43, 0.36, 1}};
F = {{0.3, 0, 0, 0}, {0, 0.5, 0, 0}, {0, 0, 0.7, 0}, {0, 0, 0, 0.9}};
```

In[]:=
```
(*Грамм - Шмидт*)
gramSchmidt[a_] := Module[{Q = {}, k = 2, A = Transpose@a, b, β = {}},
  AppendTo[Q, A[[1]] / (N@Sqrt[A[[1]].A[[1]]])]; AppendTo[β, N@Sqrt[A[[1]].A[[1]]]];
  While[k ≤ Length@A,
   For[i = 1, i < k, i++, b = A[[k]] - Sum[(A[[k]].Q[[i]] / Q[[i]].Q[[i]]) * Q[[i]], {i, 1, k - 1}]];
   AppendTo[Q, b / (N@Sqrt[b.b])]; AppendTo[β, N@Sqrt[b.b]];
   k++];
  {Transpose@Q, β}]

(*Решение системы*)
```

In[]:=
```
mainFun[A_, F_] := Module[{Q, G, β, GH = gramSchmidt[A], n = Length@A, X, Qt, At},
  X = ConstantArray[0, n];
  Q = GH[[1]];
  β = GH[[2]];
  Qt = Transpose@Q;
  G = Qt.F;
  At = Transpose@A;
  X[[n]] = G[[n]] / β[[n]];
  For[i = n - 1, i ≥ 1, i--,
   X[[i]] = (G[[i]] - Sum[(At[[k]].Qt[[i]]) * X[[k]], {k, i, n}]) / β[[i]]];
  {A.X, X}]
```

In[]:=
```
Grid[{{"A", , "X"},
  {MatrixForm@A, Style["*", 28], mainFun[A, F][[2]] // MatrixForm, Style["=", 28]},
  {"", "", "F", ""},
  {Style["=", 28], SpanFromLeft, mainFun[A, F][[1]] // MatrixForm, SpanFromLeft}}]
```

Out[]=

$$\underset{A}{\begin{pmatrix} 1 & 0.17 & -0.25 & 0.54 \\ 0.47 & 1 & 0.67 & -0.32 \\ -0.11 & 0.35 & 1 & -0.74 \\ 0.55 & 0.43 & 0.36 & 1 \end{pmatrix}} * \underset{X}{\begin{pmatrix} 0.430284 & -0.170815 & 0.500592 & -0.319172 \\ -0.274752 & 0.800349 & -0.919699 & 0.0310711 \\ 0.0440578 & -0.382234 & 0.920615 & 0.584359 \\ -0.134374 & -0.112597 & -0.211276 & 0.851815 \end{pmatrix}} =$$

$$= \underset{F}{\begin{pmatrix} 0.3 & -2.77556\times10^{-17} & 1.38778\times10^{-17} & 1.11022\times10^{-16} \\ -1.38778\times10^{-17} & 0.5 & 8.32667\times10^{-17} & -5.55112\times10^{-17} \\ -6.93889\times10^{-17} & -8.32667\times10^{-17} & 0.7 & -1.11022\times10^{-16} \\ -2.77556\times10^{-17} & 0. & -8.32667\times10^{-17} & 0.9 \end{pmatrix}}$$

In[•]:=
```
Row[{MatrixForm[A.mainFun[A, F][[2]]], Style["==", 28], MatrixForm@F}]
```

Out[•]=

$$\begin{pmatrix} 0.3 & -2.77556\times10^{-17} & 1.38778\times10^{-17} & 1.11022\times10^{-16} \\ -1.38778\times10^{-17} & 0.5 & 8.32667\times10^{-17} & -5.55112\times10^{-17} \\ -6.93889\times10^{-17} & -8.32667\times10^{-17} & 0.7 & -1.11022\times10^{-16} \\ -2.77556\times10^{-17} & 0. & -8.32667\times10^{-17} & 0.9 \end{pmatrix} == \begin{pmatrix} 0.3 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.9 \end{pmatrix}$$