Assignment 2 : group12

Nikita Pai and Abhilash Hemaraj

QUESTION 1

```r
# Problem 1: Perform principal component analysis on NHL.xlsx, which contains statistics of 30
# teams in the National Hockey League. The description of the variables is provided in the
# 'Description' sheet of the file. Focus only on the variables 12 through 25, and create a new data
# frame.
library(readxl)
nhl <- read_excel("C:/Users/abhil/Downloads/NHL.xlsx")
```

```
## New names:
## * `` -> ...1
```

```r
View(nhl)
colnames(nhl)
```

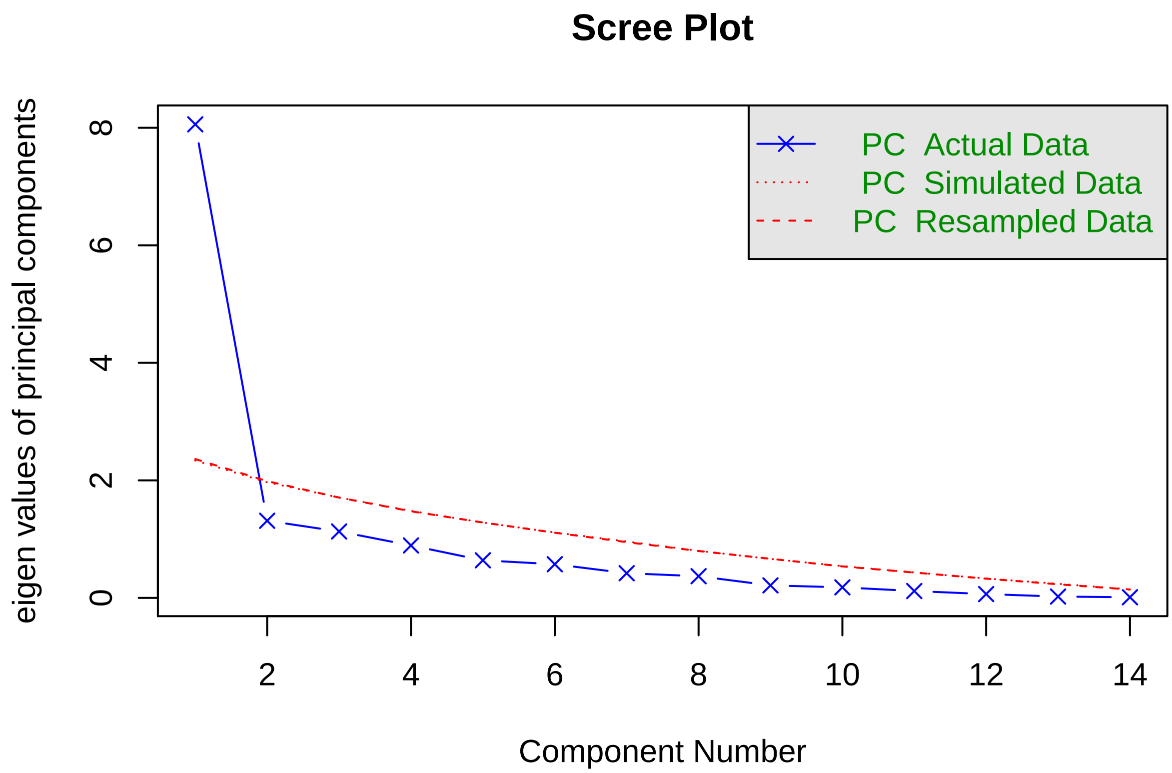
```
##  [1] "...1"   "rank"   "team"   "played" "wins"   "losses" "OTL"    "pts"
##  [9] "ROW"    "HROW"   "RROW"   "p\rpc"  "gg"     "gag"    "five"   "PPP"
## [17] "PKP"    "shots"  "sag"    "sc1"    "tr1"    "lead1"  "lead2"  "wop"
## [25] "wosp"   "face"
```

```r
nhl_df <- nhl[,12:25]
```

```r
# Input the new data frame to fa.parallel() function to determine the number of components to extract
# Input the new data frame to principal() function to extract the components. If raw data is input, the

library(psych)
plot.new()
fa.parallel(nhl_df[,], fa = "pc", n.iter = 100, main = "Scree Plot", show.legend = TRUE)
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```




```
## Parallel analysis suggests that the number of factors =  NA  and the number of components =  1
```

```
principal(nhl_df)
```

```
## Principal Components Analysis
## Call: principal(r = nhl_df)
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PC1   h2    u2 com
## p\rpc  0.97 0.94 0.057   1
## gg     0.83 0.69 0.308   1
## gag   -0.82 0.67 0.327   1
## five   0.92 0.84 0.162   1
## PPP    0.14 0.02 0.980   1
## PKP    0.69 0.48 0.519   1
## shots  0.59 0.34 0.656   1
## sag   -0.62 0.39 0.612   1
## sc1    0.81 0.66 0.338   1
## tr1    0.76 0.58 0.422   1
## lead1  0.81 0.65 0.351   1
## lead2  0.74 0.55 0.452   1
## wop    0.71 0.51 0.491   1
## wosp   0.86 0.73 0.267   1
##
##                 PC1
## SS loadings    8.06
## Proportion Var 0.58
##
```

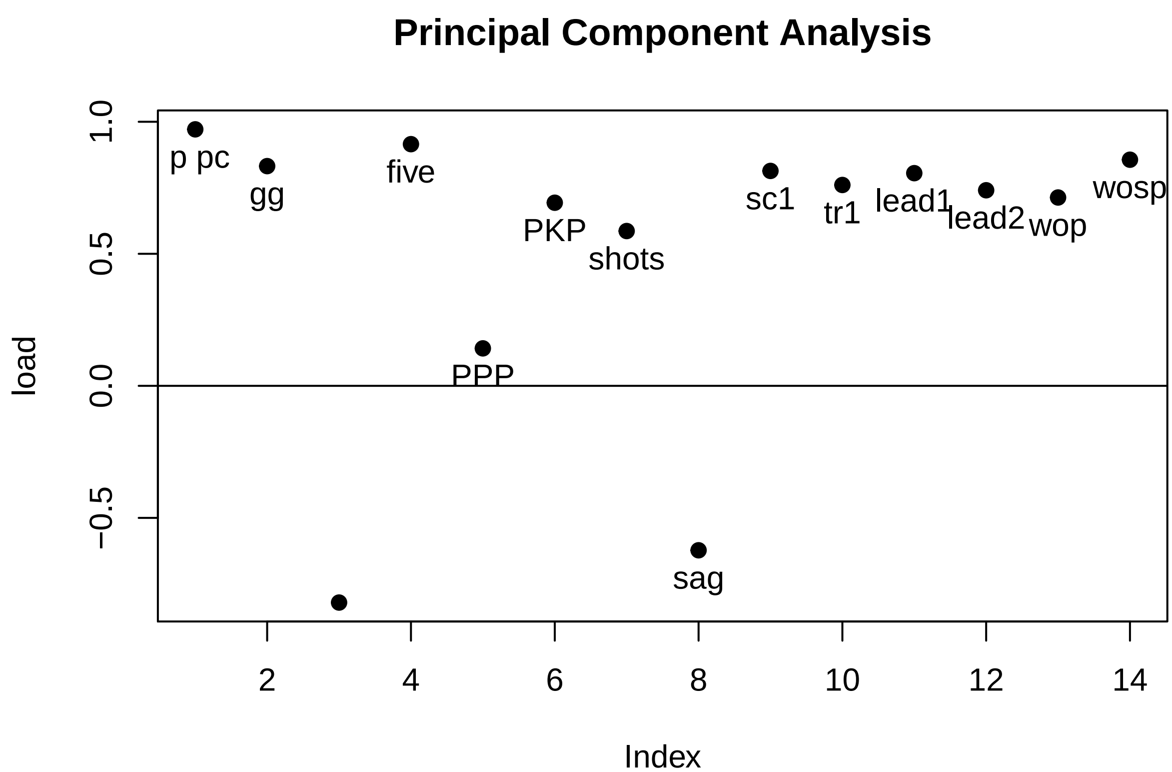
```
## Mean item complexity =  1
## Test of the hypothesis that 1 component is sufficient.
##
## The root mean square of the residuals (RMSR) is  0.1
##  with the empirical chi square  52.29  with prob <  0.99
##
## Fit based upon off diagonal values = 0.97
```

```r
# Rotate the components
rotate_components <- principal(r = nhl_df, nfactors = 1, residuals = TRUE, rotate = "varimax", covar = F
```

```r
# Compute component scores
component_scores <- (principal(r = nhl_df, nfactors = 1, residuals = TRUE, rotate = "varimax", covar = F
```

```r
# Graph an orthogonal solution using factor.plot()
factor.plot(rotate_components, labels = rownames(rotate_components$loadings))
```

```
## Warning in text.default(load, labels, pos = pos, ...): font width unknown for
## character 0xd
```




```r
# Interpret the results

# According to the principal component Analysis, the number of principal components we need to
#extract is only one. Due to the same, we are not able to rotate the components.
#It is also to be noted that the proportionality variance is 0.58
```

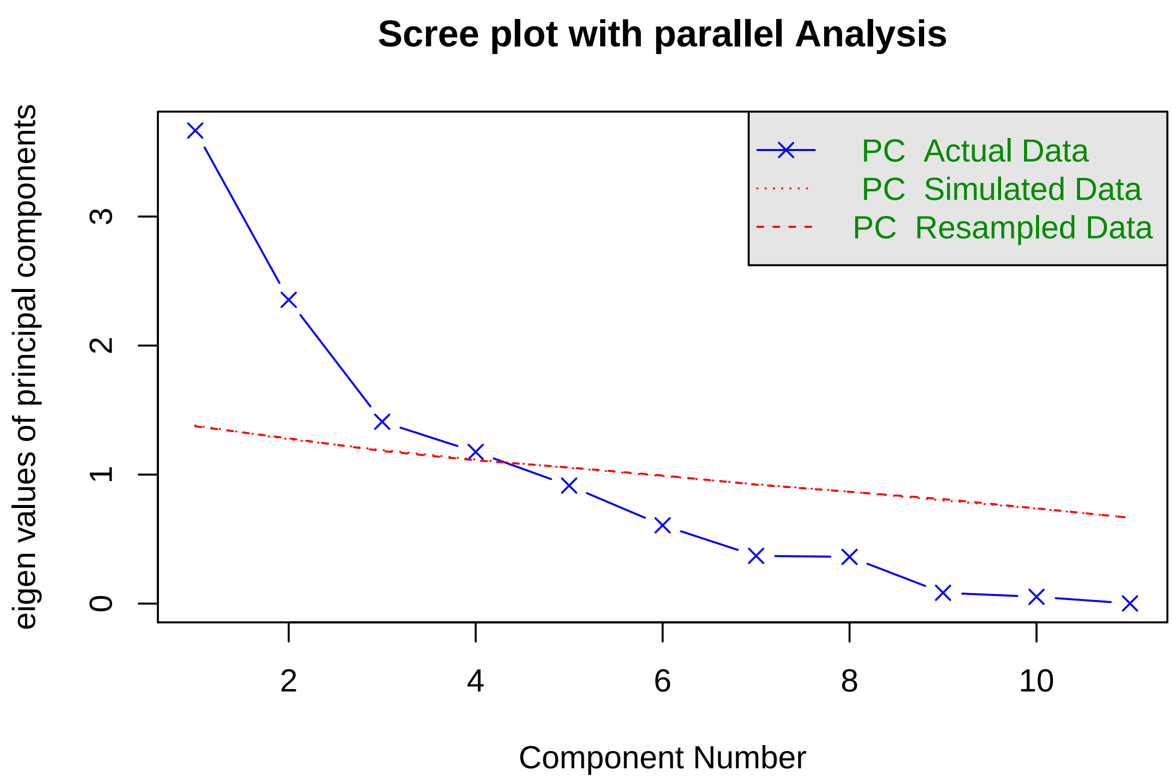
Question 2

```r
#Problem 2: Perform principal component analysis on Glass Identification Data.xlsx
# Input the raw data matrix to fa.parallel() function to determine the number of components to extract
# Input the raw data matrix to principal() function to extract the components. If raw data
#is input, the correlation matrix is automatically calculated by principal() function.
# Rotate the components
# Compute component scores
# Graph an orthogonal solution using factor.plot()
# Interpret the results

library(readxl)
GID <- read_excel("C:/Users/abhil/Downloads/Glass Identification Data(1).xlsx")
View(GID)
library(psych)
#Input the raw data matrix to fa.parallel() function to determine the number of components to extract
fa.parallel(GID, fa = 'pc', n.iter=50, show.legend = TRUE, main = "Scree plot with parallel Analysis")
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
```




```
## Parallel analysis suggests that the number of factors =  NA  and the number of components =  4
```

```r
# Input the raw data matrix to principal() function to extract the components. If raw data is input, th
pc <- principal(GID, nfactors = 4, covar = TRUE, rotate = "none")
pc
```

```
## Principal Components Analysis
## Call: principal(r = GID, nfactors = 4, rotate = "none", covar = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PC1   PC2   PC3   PC4   h2    u2 com
## ID     0.84  0.20  0.02  0.10 0.76 0.244 1.1
## RI    -0.28  0.91  0.11 -0.16 0.95 0.051 1.3
## Na     0.55 -0.06 -0.42 -0.58 0.81 0.185 2.9
## Mg    -0.77 -0.43 -0.02 -0.31 0.87 0.126 1.9
## Al     0.73 -0.25  0.42  0.08 0.77 0.226 1.9
## Si     0.15 -0.39 -0.56  0.69 0.96 0.040 2.7
## K      0.05 -0.41  0.78  0.07 0.78 0.218 1.5
## CA    -0.11  0.92  0.00  0.28 0.94 0.060 1.2
## Ba     0.69  0.13  0.13 -0.25 0.57 0.429 1.4
## Fe    -0.22  0.18  0.32  0.29 0.27 0.731 3.4
## Class  0.95  0.11 -0.06  0.05 0.92 0.083 1.0
##
##                        PC1  PC2  PC3  PC4
## SS loadings           3.67 2.35 1.41 1.18
## Proportion Var        0.33 0.21 0.13 0.11
## Cumulative Var        0.33 0.55 0.68 0.78
## Proportion Explained  0.43 0.27 0.16 0.14
## Cumulative Proportion 0.43 0.70 0.86 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.07
##  with the empirical chi square  116.03  with prob <  8.8e-17
##
## Fit based upon off diagonal values = 0.96
```

```r
# Rotate the components
rotation <- principal(GID, nfactors = 4, covar = FALSE, scores = TRUE, rotate = "varimax")
rotation
```

```
## Principal Components Analysis
## Call: principal(r = GID, nfactors = 4, rotate = "varimax", covar = FALSE,
##     scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##         RC1   RC2   RC3   RC4   h2    u2 com
## ID     0.86  0.03 -0.14 -0.06 0.76 0.244 1.1
## RI    -0.05  0.82  0.19  0.48 0.95 0.051 1.7
## Na     0.32 -0.04 -0.84  0.10 0.81 0.185 1.3
## Mg    -0.88 -0.27 -0.06  0.13 0.87 0.126 1.2
## Al     0.71 -0.50  0.08  0.07 0.77 0.226 1.8
## Si     0.06 -0.12 -0.02 -0.97 0.96 0.040 1.0
## K      0.11 -0.66  0.49  0.30 0.78 0.218 2.3
## CA     0.17  0.88  0.36  0.09 0.94 0.060 1.4
## Ba     0.66 -0.08 -0.26  0.25 0.57 0.429 1.7
## Fe    -0.06  0.10  0.50  0.06 0.27 0.731 1.1
## Class  0.91 -0.04 -0.27 -0.11 0.92 0.083 1.2
```

```
##
##                         RC1  RC2  RC3  RC4
## SS loadings           3.43 2.26 1.53 1.39
## Proportion Var        0.31 0.21 0.14 0.13
## Cumulative Var        0.31 0.52 0.66 0.78
## Proportion Explained  0.40 0.26 0.18 0.16
## Cumulative Proportion 0.40 0.66 0.84 1.00
##
## Mean item complexity =  1.5
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.07
##  with the empirical chi square  116.03  with prob <  8.8e-17
##
## Fit based upon off diagonal values = 0.96
```

```r
# Compute component scores
score.GID <- principal(GID, nfactors=4, scores = TRUE)
score.GID$scores
```

```
##                RC1          RC2           RC3           RC4
##    [1,] -1.34333631  0.338743060 -0.9397920286  1.1055228407
##    [2,] -1.04642244 -0.483267798 -0.7722930735  0.1090181212
##    [3,] -0.96124029 -0.675177807 -0.5016910636 -0.3323209634
##    [4,] -1.04370052 -0.420994384 -0.2248294574  0.1150912794
##    [5,] -1.05040872 -0.419812824 -0.2617201074 -0.3510410163
##    [6,] -0.74010608 -0.782278634  1.1493481640 -0.3898310189
##    [7,] -1.07242412 -0.357000030 -0.2797084041 -0.3671424336
##    [8,] -1.10111315 -0.273100865 -0.1813700455 -0.5442442072
##    [9,] -0.97648940 -0.260500721 -0.8036292448  0.8901444887
##   [10,] -0.89063623 -0.374999875  0.4176556986 -0.3021950475
##   [11,] -0.71495942 -0.767197928  1.1674595279 -0.6555586745
##   [12,] -0.96919147 -0.322828935  0.1780237176 -0.3474019871
##   [13,] -0.77271105 -0.682145446  1.0207621567 -0.7081535468
##   [14,] -0.87689916 -0.306074166  0.7069520354 -0.5765288901
##   [15,] -0.92183329 -0.328626709  0.3438898213 -0.6499817950
##   [16,] -0.95609193 -0.314390323  0.1579545075 -0.5843982248
##   [17,] -0.98420904 -0.212532099  0.2671660081 -0.4666118311
##   [18,] -1.21461933  0.668391778 -1.3674749328  1.6224042259
##   [19,] -1.07199367  0.245715723 -0.9769196409  0.5903701595
##   [20,] -0.71578694 -0.530622523  0.3379854597 -0.0385754037
##   [21,] -0.73867808 -0.385570017  0.8529291590 -0.1277625167
##   [22,] -1.47507961  0.786368089 -1.8968206394  0.7210322151
##   [23,] -0.90549857 -0.318720111  0.1927077338 -0.1870903430
##   [24,] -0.86121977 -0.359190942  0.2253211206 -0.3622465301
##   [25,] -0.97434637 -0.261206408 -0.3548560075 -0.2086392273
##   [26,] -0.91325872 -0.304498858  0.0689891693 -0.3165861523
##   [27,] -0.83257595 -0.366682520 -0.1067045047  0.0963256314
##   [28,] -0.85741080 -0.377505840  0.1244964913 -0.4284734764
##   [29,] -0.79765497 -0.356025232  0.4145419879 -0.5250208029
##   [30,] -0.86870040 -0.296040281 -0.0205774463 -0.1625711399
##   [31,] -0.77653717 -0.260118818  0.8389544657 -0.4651728162
##   [32,] -0.91475365 -0.216944758  0.1283640837 -0.6629798794
##   [33,] -0.72997546 -0.240622836  0.9119274445 -0.3047643965
##   [34,] -0.75062780 -0.333356295  0.6537431517 -0.7709551902
```

```
## [35,] -0.80894781 -0.246345144  0.3038862347 -0.3268552283
## [36,] -0.88307104 -0.444888993 -0.2395229350 -0.2442041748
## [37,] -0.81489952 -0.093497829 -0.7178511680  1.0848118045
## [38,] -0.77573346 -0.283915575  0.3113624089 -0.2923664904
## [39,] -1.27456592  1.051550537 -1.3054103831  1.1082687033
## [40,] -1.27049776  1.052171182 -1.3052584994  1.1075693617
## [41,] -0.86451194 -0.151216093  0.2183637602 -0.3955112571
## [42,] -0.82026588 -0.221625965  0.2750421995 -0.6107888175
## [43,] -0.77109059 -0.276291537 -0.0846643955 -0.0671714220
## [44,] -1.11899751  0.931546468 -0.8236667007  1.0854596143
## [45,] -0.66464507 -0.134924718  1.3333113511 -0.3666366558
## [46,] -0.76285743 -0.064635158 -0.3185299143  0.8273809157
## [47,] -0.72443818 -0.008340599  0.4826113720 -0.0125470273
## [48,] -1.03223960  1.558362042 -0.6766118902  1.6343212357
## [49,] -1.02443650  1.018146050 -0.3764879067  0.7653187836
## [50,] -0.77405851 -0.032276504 -0.3847557145  0.7025382569
## [51,] -1.06769412  1.317406226 -0.2888877825  1.0846723583
## [52,] -0.65481293  0.054077083  0.3702292502  0.3899084518
## [53,] -0.66068756  0.028321504 -0.1711959181 -0.1868313781
## [54,] -0.60200204  0.024613895  0.0781620231 -0.2126910823
## [55,] -0.55310933 -0.009422235  0.3306778009 -0.3955709950
## [56,] -0.41993533  0.013565805  1.5248629043 -1.2182743709
## [57,] -0.71003761 -0.810214482  1.0234678304 -0.8610067248
## [58,] -0.74623248 -0.256340843  0.1419782817 -0.2659311018
## [59,] -0.88863138 -0.384592218 -0.4295801341 -0.2402784832
## [60,] -0.79642415 -0.305978974  0.0431722885 -0.1000204817
## [61,] -0.89254320  0.248610932 -0.6871688897  0.0277902358
## [62,] -0.58275931  0.126003556 -1.0551620489  1.3097956907
## [63,] -0.92605254  0.750842782 -0.2219634648  1.0200602242
## [64,] -1.04581652  1.008228934 -1.2581959761  1.4954258576
## [65,] -0.89840193  0.805327786 -0.3159758294  0.7827776100
## [66,] -0.83165063  0.625207319 -0.7226352318  0.8055078049
## [67,] -0.80946992  0.881483447  0.4064869156  0.4627123363
## [68,] -0.80249026  0.887052584  0.4179803116  0.3690182240
## [69,] -0.77660002  0.844719376  0.3557541934  0.5078763838
## [70,] -0.86031002  1.171124642 -0.2791084624  0.8135909574
## [71,] -0.54912717 -0.768440121 -1.3125235141  0.8040417990
## [72,] -0.55497557 -0.222801665  0.4956182317  0.7689809121
## [73,] -0.52610844 -0.792435839 -0.0438173588 -0.5141996728
## [74,] -0.50593453 -0.728601819 -0.2472703986 -0.2383599210
## [75,] -0.48982979 -0.804579821  0.0574297004 -0.5183610572
## [76,] -0.50290305 -0.765109698  0.0377584279 -0.5501716834
## [77,] -0.51260798 -0.713120255 -0.3329932779  0.2492134794
## [78,] -0.49211381 -0.687553848  0.0004349709 -0.2794686208
## [79,] -0.58915157 -0.407270329 -0.3942838949 -0.3232341082
## [80,] -0.30121205 -0.955477405  0.2896573783 -0.2854924773
## [81,] -0.20270593 -1.065256762  0.3136298816 -0.0523210825
## [82,] -0.50772736 -0.681753414 -0.1930194317 -0.6087593082
## [83,] -0.58823640 -0.537999309 -0.3322233619 -0.2020259187
## [84,] -0.39944110 -0.736419082  0.3094106037 -0.3194918178
## [85,] -0.16454760 -1.646095315 -0.6031549748  0.5232058368
## [86,] -0.49750013 -0.523673736 -0.3476653761 -0.1919605880
## [87,] -0.49332792 -0.550732333 -0.2766489408 -0.7897608230
## [88,] -0.39221347 -0.650350365  0.1007270374 -0.0287438921
```

```
##  [89,] -0.45279864 -0.623924588 -0.0042775799 -0.3738765492
##  [90,] -0.20722800 -0.790046265  0.8258089561 -0.6837702097
##  [91,] -0.49507027 -0.030764388  0.7986707147  0.3038008625
##  [92,] -0.44665256 -0.481476166  0.0182733585 -0.6412253994
##  [93,] -0.32342637 -0.277013079  0.4090328004 -0.9385852260
##  [94,] -0.42252022 -0.470987426 -0.2380670212 -0.6536381085
##  [95,] -0.35666823 -0.590858482  0.3347290449 -0.7790300167
##  [96,] -0.41936860 -0.174158983 -0.2754154559  0.4228610552
##  [97,] -0.48913080  0.048885163  0.5173909042  0.2397850139
##  [98,] -0.30357437 -0.063050571  1.5638153980 -1.1498160898
##  [99,] -0.11793885 -0.540041270  0.5678068343 -0.6632979621
## [100,] -0.21379868 -0.163813532  0.1721520545 -0.2699206332
## [101,] -0.08744290 -0.260440562  1.1251974585 -0.8245904000
## [102,] -0.06703170 -0.264116137  0.8423830451 -0.4353579061
## [103,] -0.34423450  0.532994279  1.0319681386 -1.4687081124
## [104,] -0.52358161  2.126337675 -0.6616953901  2.4192821546
## [105,] -0.29834142  1.376538808 -0.5938663456  1.6972279857
## [106,]  1.11134205  1.697236364  3.7750693123  0.3534052383
## [107,]  2.20956342  2.051319814  3.0377176073  4.3196576689
## [108,]  0.77610095  4.172876799  2.4804558626  2.7157378652
## [109,]  0.34731859  1.834746282 -0.2081884634 -0.1116019824
## [110,]  0.09901864  1.582260438 -0.1333697715 -2.3497006742
## [111,]  0.50454761  3.253890155  2.2472975060 -0.9863312595
## [112,]  0.52231226  3.407507250  2.4280995584 -0.8506738492
## [113,]  0.41065496  3.291196954  1.0904955807  0.4934360975
## [114,] -0.44985638 -0.188779225  0.0637823496  0.2206304649
## [115,] -0.55383771 -0.210205561 -0.1834959276  0.2318410830
## [116,] -0.49772663 -0.254187998 -0.4289945149  0.3317604477
## [117,] -0.40447343 -0.316488079  0.0917951171  0.3519372523
## [118,] -0.25980626 -0.792871061 -0.5047421905  0.6872582262
## [119,] -0.20866372 -0.603541476  0.8079063270  0.0632675245
## [120,] -0.36866432 -0.643664285 -0.4454830443  0.1704451636
## [121,] -0.43405154 -0.239741626 -0.2402568125  0.2852203891
## [122,] -0.15907820 -0.638931659  0.8679395317 -0.4069315488
## [123,] -0.32969099 -0.513183433 -0.1814476341 -0.2592099453
## [124,] -0.21966507 -0.665745195 -0.2895701536  0.1621145811
## [125,] -0.45111011  0.272391096 -0.1777835882  0.1896047908
## [126,] -0.20934085 -0.259672420  0.5401483724  0.1641827442
## [127,] -0.39103047 -0.313386775  0.0081896752 -0.2983188818
## [128,]  0.11236998  0.565297991  0.3428544028  0.7518854196
## [129,]  0.34815597  0.344275313  0.5366790912  0.6841525159
## [130,]  0.43207320  0.740599811  0.4240733680  0.8299780474
## [131,]  0.35293011  1.247974683  0.0187055256  0.4195579886
## [132,]  0.75521820  2.477101734  0.7482064464  1.4487246135
## [133,] -0.51426316 -0.284914160 -0.4725227860  0.2043288811
## [134,] -0.28797878 -0.429151575 -0.1030371663  0.9056811040
## [135,] -0.44891912 -0.268701734 -0.3713496031 -0.1084098863
## [136,] -0.28320043 -0.235584967  0.7466230573  0.2556117952
## [137,] -0.41494669 -0.116405239  0.3316480451 -0.4829969685
## [138,] -0.22831801 -0.556079282  0.1297726921 -0.3806618180
## [139,] -0.21624445 -0.637450421  0.2370782885 -0.7942657213
## [140,] -0.17620326 -0.667241916  0.1939858587 -0.5608884741
## [141,] -0.19002630 -0.636139140 -0.1675042901  0.0951818168
## [142,] -0.31365380 -0.047191837  0.3498629365 -0.1672853879
```

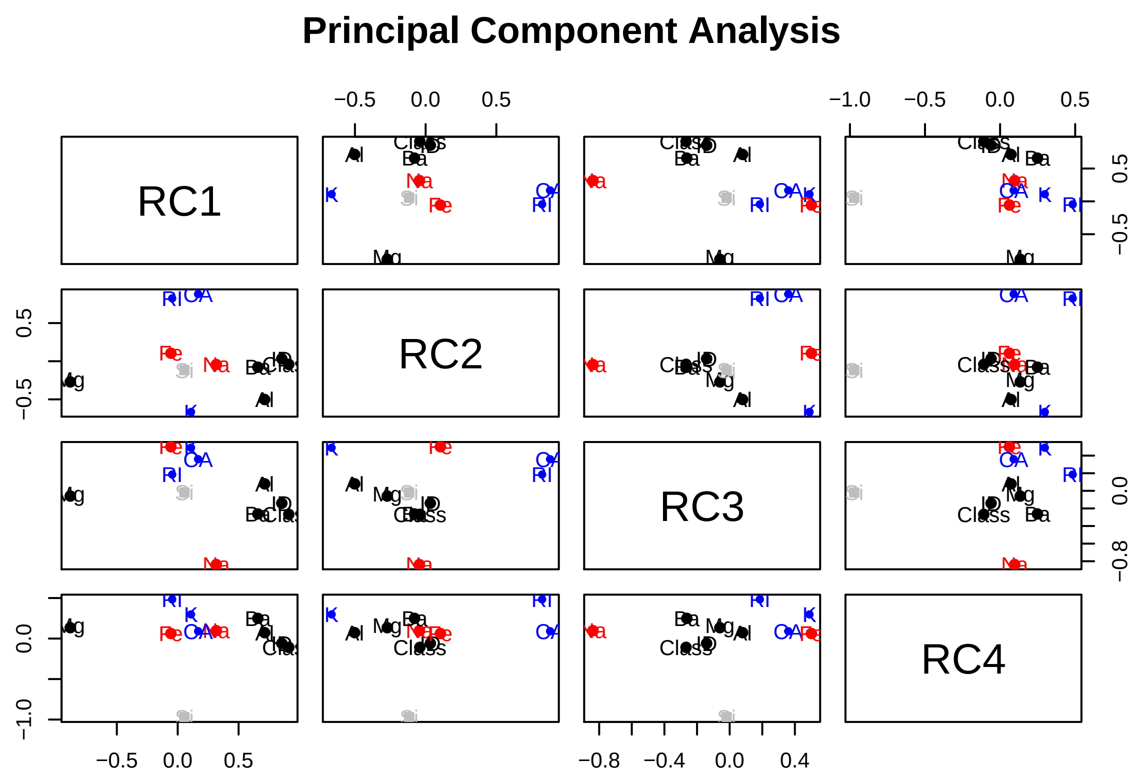
```
## [143,] -0.08771107 -0.505930256  1.0569973944 -0.4877711036
## [144,] -0.06954650 -0.661184273  0.1561268894 -0.1029087956
## [145,] -0.10679848 -0.159769698  0.9312782652 -0.5783451283
## [146,] -0.15051508 -0.085255835  1.3524930662 -0.0130993349
## [147,] -0.32872631  0.149215984 -0.8226735295 -0.3264127913
## [148,] -0.16161226 -0.449872748 -0.3188974066 -0.2318650905
## [149,] -0.08679051 -0.364407554  0.1373770517 -0.2241276356
## [150,] -0.11120305 -0.387150887  0.5770139243 -0.6259311356
## [151,]  0.14755703 -0.576706875  0.5858000443  0.0226686750
## [152,] -0.45881994  0.896222063 -1.4341742762  1.1979040319
## [153,] -0.48701977  0.508427756 -0.9185916472 -0.6489295829
## [154,] -0.16307293 -0.392639453 -0.3901532977 -0.2604107010
## [155,] -0.10888485 -0.265697337  0.1040746258 -0.1897865505
## [156,] -0.11160784 -0.256985152 -0.0670171908 -0.6247246805
## [157,] -0.10506378 -0.245557471 -0.3494281081 -0.2050005856
## [158,] -0.48304597  1.014418756 -1.1773048838  0.8429667681
## [159,]  0.01419201 -0.252281387 -0.3497988948  0.5434955903
## [160,]  0.11996558 -0.258562541  0.0218656386  0.6511252323
## [161,]  0.06539384 -0.128915280 -0.1556726247  0.4449662549
## [162,] -0.20963104  0.584678781 -0.0261171551 -0.1008268708
## [163,] -0.17203022  0.790933891  0.0845225021  1.4413292746
## [164,]  1.92691819 -3.024288251 -0.6622928996  4.0589749626
## [165,]  0.88161123  0.260995080  0.7409213059 -0.2749423155
## [166,]  0.85308764  1.078523410  1.6473029226 -0.5625522347
## [167,]  0.94270413  1.085642735  2.1992584359 -1.2009466330
## [168,]  1.28260952  1.162325815  1.1569629160 -1.5773107730
## [169,]  1.33186088  0.075259913  1.1653631168 -1.6337844807
## [170,]  1.31113303  1.012522781  0.6911683381 -0.7111476351
## [171,]  1.27462047  1.792741368  0.5264294260  0.2898827348
## [172,]  2.22958344 -4.790504181  3.5670878810  3.5004057124
## [173,]  2.22907811 -4.772742540  3.5954085912  3.2933946339
## [174,]  1.21186973  1.594331688  0.5162580722 -0.0793404089
## [175,]  1.41984996  0.140440048  2.5283910807  0.5079391735
## [176,]  1.25913732  1.461289614  1.6288727617 -1.1518817000
## [177,]  0.66503199  0.561184980 -0.8110405114  0.0048611385
## [178,]  0.52592816  0.835958761 -0.7100755960 -0.4384835182
## [179,]  0.69960271  0.396792679 -1.1566643799  0.0173136433
## [180,]  0.74960512  0.429442569 -0.8315945994 -0.3004356016
## [181,]  0.68475353 -0.422513497 -1.1762517320 -2.4933908815
## [182,]  1.11253225  0.755381008 -1.2041132870 -0.0898365309
## [183,]  1.51956529  0.939172001 -0.2329156293 -0.4886412756
## [184,]  0.86995987  1.855024413 -0.8944762554 -1.3354465815
## [185,]  0.41814219 -0.012306007 -3.5731433524 -3.2604629726
## [186,]  1.08208898 -2.610564964 -0.6099356652  0.2327732082
## [187,]  1.38113299 -1.884510324 -1.2346789456  2.5741939394
## [188,]  0.53696298  0.531531795 -0.3725823949  0.5026303076
## [189,]  1.16159102  0.273207403 -0.9805535593  2.7373518200
## [190,]  1.32846745  0.636641430 -2.6847219852  3.2788699321
## [191,]  1.27161307 -0.047253132 -0.9041395676 -0.6594675706
## [192,]  1.90614100 -0.078454902 -0.8170127506 -0.7892917294
## [193,]  2.05422233 -0.184809913 -0.0686248783 -1.0834423641
## [194,]  2.04308105  0.077019250 -1.2013836157 -0.1143539115
## [195,]  2.03366977  0.054520957 -1.0801921383 -0.4442446193
## [196,]  2.07157645 -0.255474347 -0.2482289305 -1.0137148649
```

```
## [197,]  2.09320310 -0.154990174 -0.2255963522 -0.8305712447
## [198,]  1.89778158  0.110267740 -1.0018302851 -0.7070434696
## [199,]  2.04514632 -0.252421979 -0.6649702024 -0.7162879110
## [200,]  1.92142337 -0.148584046 -1.1609874150 -0.5462020768
## [201,]  1.80446899 -0.218855848 -1.4416607779 -1.0348715544
## [202,]  1.59535877 -0.848630795  2.4860999263 -2.5353440448
## [203,]  1.87966951 -0.269162837 -1.1210897831 -1.2924326894
## [204,]  2.06266640 -0.059442248 -1.5900274212 -0.1859807061
## [205,]  1.87266287 -0.027971722 -1.2532678983 -0.7828379018
## [206,]  1.95244413  0.208679032 -1.6708188930 -0.0950675805
## [207,]  1.93430237  0.117737965 -1.5851317397 -0.3494566402
## [208,]  2.45668197 -1.166979123 -1.1276638799  1.1759640629
## [209,]  2.10858341 -0.051138853 -0.5827339750 -0.4511440519
## [210,]  2.33962551 -0.308002117 -0.5305148358 -0.0002604651
## [211,]  2.05766686  0.015473860 -1.6262671757 -0.1535647189
## [212,]  2.12909297  0.419725988 -1.1373606956 -0.2270472498
## [213,]  2.06181060  0.040602873 -1.1866961587 -0.8162752377
## [214,]  2.17167380  0.053335226 -1.0571193724 -0.4957023730
```

```r
head(score.GID$scores)
```

```
##             RC1        RC2        RC3        RC4
## [1,] -1.3433363  0.3387431 -0.9397920  1.1055228
## [2,] -1.0464224 -0.4832678 -0.7722931  0.1090181
## [3,] -0.9612403 -0.6751778 -0.5016911 -0.3323210
## [4,] -1.0437005 -0.4209944 -0.2248295  0.1150913
## [5,] -1.0504087 -0.4198128 -0.2617201 -0.3510410
## [6,] -0.7401061 -0.7822786  1.1493482 -0.3898310
```

```r
# Graph an orthogonal solution using factor.plot()
factor.plot(rotation, labels = rownames(rotation$loadings))
```

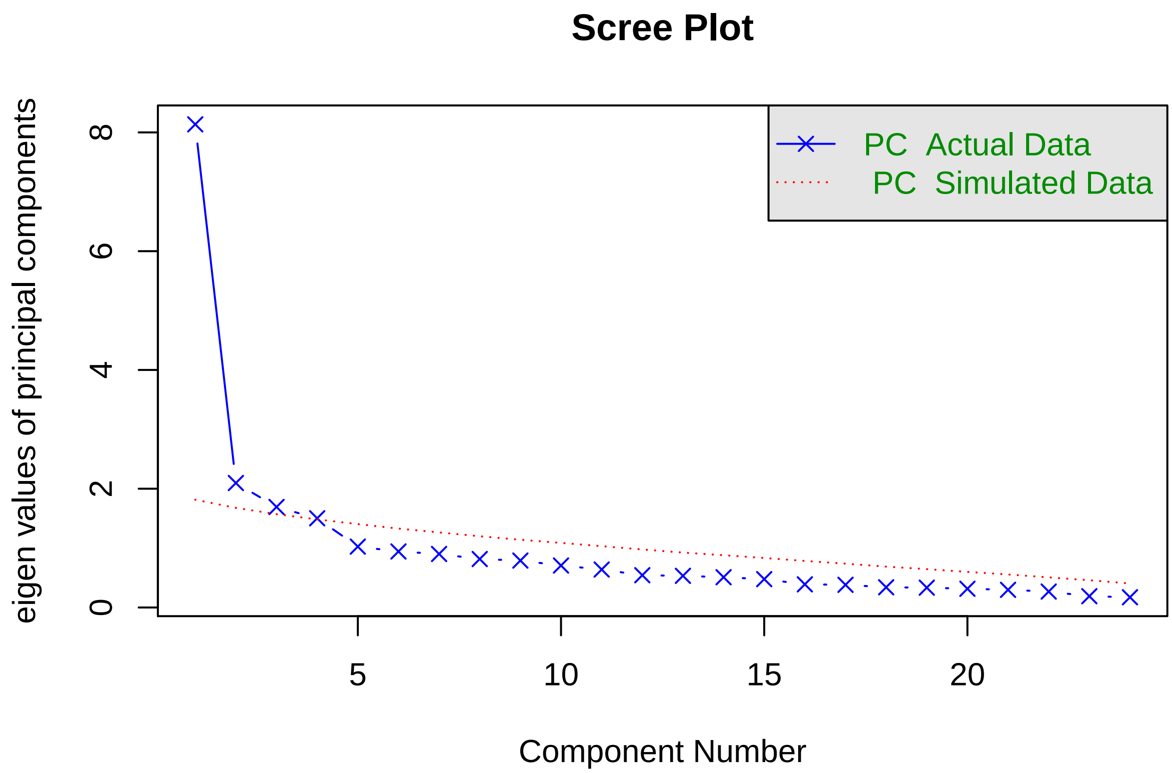



```
# Interpret the results
#With the help of the parallel analysis, we are able to affirm that the number of
#components we need to extract.The cumulative variance captured by all the four components
#is 0.78.
```

Question 3

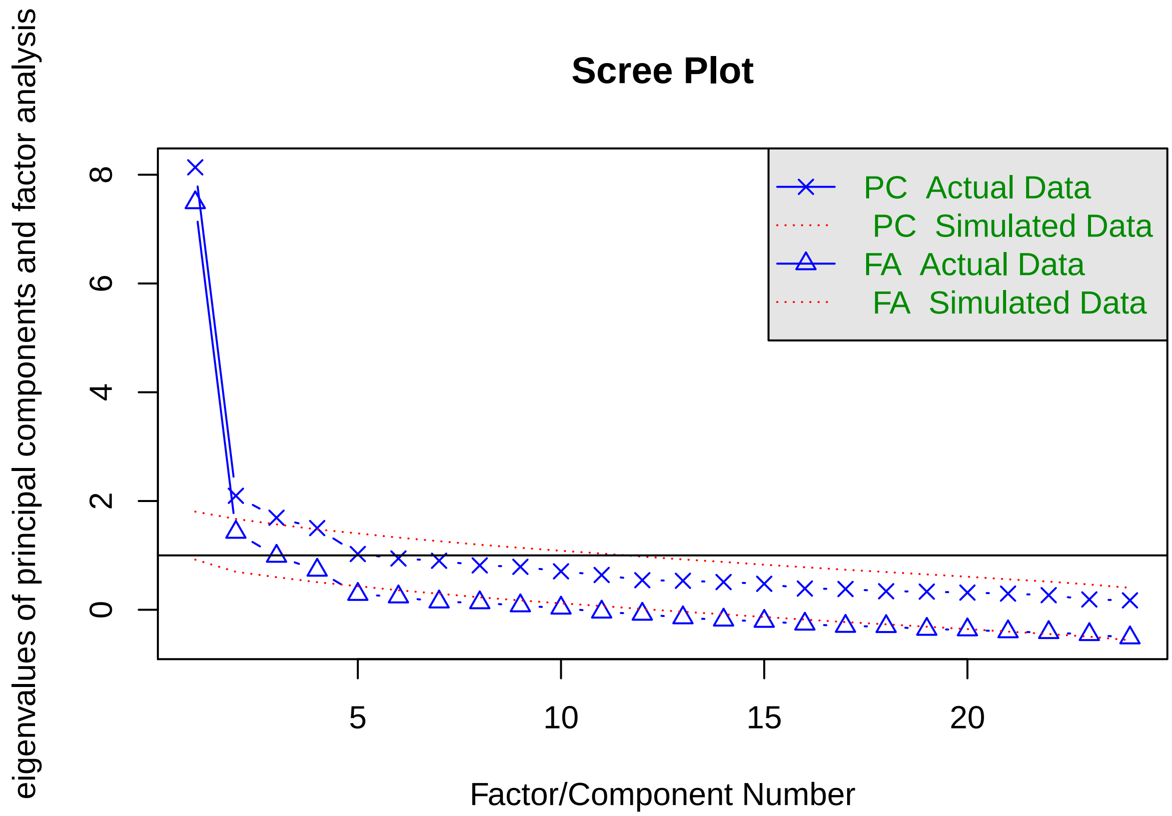
```
# Problem 3: Perform factor analysis on Herman74.cor, which is a data structure available in the
# base installation (A correlation matrix of 24 psychological tests given to 145 seventh and eightgrade
# Input the correlation matrix to fa.parallel() function to determine the number of components to extra
# Compute factor scores
# Graph an orthogonal solution using factor.plot()
# Graph an oblique solutions using fa.diagram()
# Interpret the results


library(datasets)
data("Harman74.cor")
View(Harman74.cor)
H_corr <- cov2cor(Harman74.cor$cov)
fa.parallel(Harman74.cor$cov, fa = "pc", n.iter = 100, n.obs = 150, main = "Scree Plot", show.legend = '
```




```
## Parallel analysis suggests that the number of factors =  NA  and the number of components =  3
```

```r
fa.parallel(H_corr, fa = "both", n.iter = 100, n.obs = 150, main = "Scree Plot", show.legend = TRUE)
```




```
## Parallel analysis suggests that the number of factors =  4  and the number of components =  3
```

```r
# Input the correlation matrix to fa() function to extract the components. If raw data is
#input, the correlation matrix is automatically calculated by fa() function.

fa(Harman74.cor[["cov"]], 4, rotate = "none", fm = "pa")
```

```
## Factor Analysis using method =  pa
## Call: fa(r = Harman74.cor[["cov"]], nfactors = 4, rotate = "none",
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                         PA1   PA2   PA3   PA4   h2   u2 com
## VisualPerception       0.60  0.03  0.38 -0.22 0.55 0.45 2.0
## Cubes                  0.37 -0.03  0.26 -0.15 0.23 0.77 2.2
## PaperFormBoard         0.42 -0.12  0.36 -0.13 0.34 0.66 2.3
## Flags                  0.48 -0.11  0.26 -0.19 0.35 0.65 2.0
## GeneralInformation     0.69 -0.30 -0.27 -0.04 0.64 0.36 1.7
## PargraphComprehension  0.69 -0.40 -0.20  0.08 0.68 0.32 1.8
## SentenceCompletion     0.68 -0.41 -0.30 -0.08 0.73 0.27 2.1
## WordClassification     0.67 -0.20 -0.09 -0.11 0.51 0.49 1.3
## WordMeaning            0.70 -0.45 -0.22  0.08 0.74 0.26 2.0
## Addition               0.47  0.53 -0.48 -0.10 0.74 0.26 3.1
## Code                   0.56  0.36 -0.16  0.09 0.47 0.53 2.0
## CountingDots           0.47  0.50 -0.14 -0.24 0.55 0.45 2.6
## StraightCurvedCapitals 0.60  0.26  0.01 -0.29 0.51 0.49 1.9
## WordRecognition        0.43  0.06  0.01  0.42 0.36 0.64 2.0
## NumberRecognition      0.39  0.10  0.09  0.37 0.31 0.69 2.2
```

```
## FigureRecognition      0.51  0.09  0.35  0.25 0.45 0.55 2.4
## ObjectNumber           0.47  0.21 -0.01  0.39 0.41 0.59 2.4
## NumberFigure           0.52  0.32  0.16  0.14 0.41 0.59 2.1
## FigureWord             0.44  0.10  0.10  0.13 0.23 0.77 1.4
## Deduction              0.62 -0.13  0.14  0.04 0.42 0.58 1.2
## NumericalPuzzles       0.59  0.21  0.07 -0.14 0.42 0.58 1.4
## ProblemReasoning       0.61 -0.10  0.12  0.03 0.40 0.60 1.1
## SeriesCompletion       0.69 -0.06  0.15 -0.10 0.51 0.49 1.2
## ArithmeticProblems     0.65  0.17 -0.19  0.00 0.49 0.51 1.3
##
##                        PA1  PA2  PA3  PA4
## SS loadings           7.65 1.69 1.22 0.92
## Proportion Var        0.32 0.07 0.05 0.04
## Cumulative Var        0.32 0.39 0.44 0.48
## Proportion Explained  0.67 0.15 0.11 0.08
## Cumulative Proportion 0.67 0.81 0.92 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective function was  11.44
## The degrees of freedom for the model are 186  and the objective function was  1.72
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                    PA1  PA2  PA3  PA4
## Correlation of (regression) scores with factors   0.97 0.91 0.86 0.79
## Multiple R square of scores with factors          0.94 0.82 0.75 0.62
## Minimum correlation of possible factor scores     0.89 0.65 0.50 0.24
```

```r
# Rotating the factors
rotate <- fa(Harman74.cor$cov, 4, rotate = "varimax", fm = "pa")
rotate
```

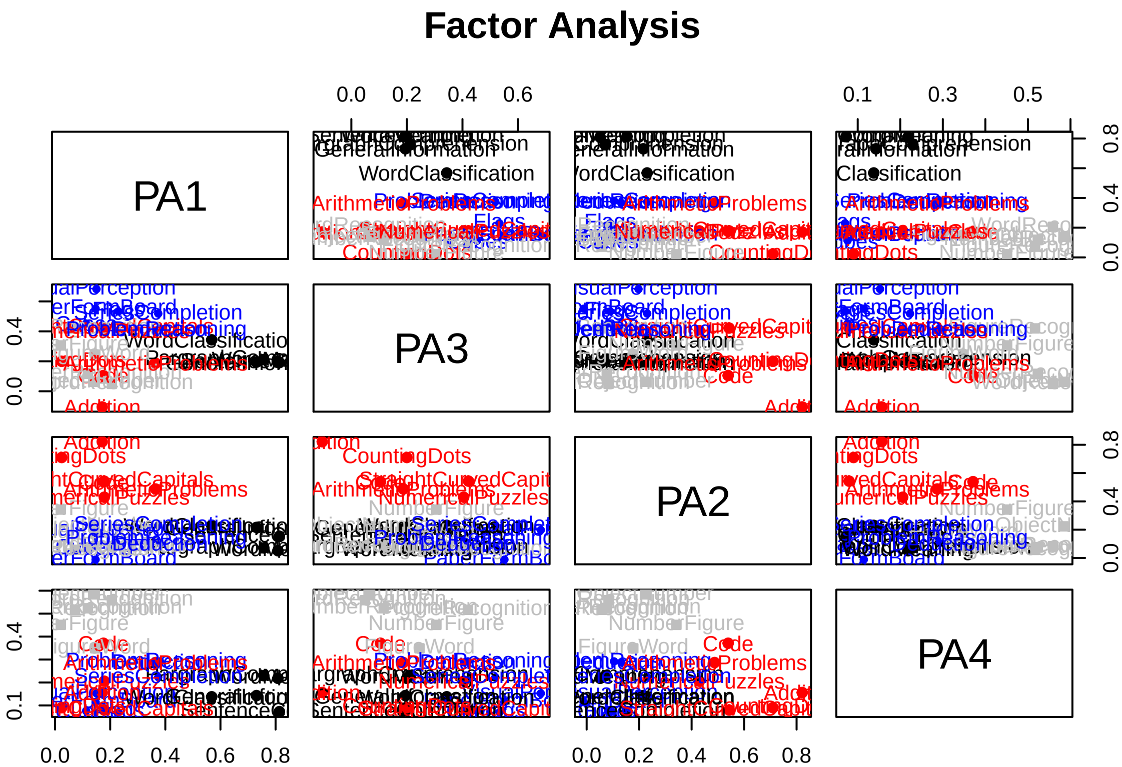
```
## Factor Analysis using method =  pa
## Call: fa(r = Harman74.cor$cov, nfactors = 4, rotate = "varimax", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                          PA1   PA3   PA2  PA4   h2   u2 com
## VisualPerception        0.15  0.68  0.20 0.15 0.55 0.45 1.4
## Cubes                   0.11  0.45  0.08 0.08 0.23 0.77 1.3
## PaperFormBoard          0.15  0.55 -0.01 0.11 0.34 0.66 1.2
## Flags                   0.23  0.53  0.09 0.07 0.35 0.65 1.5
## GeneralInformation      0.73  0.19  0.22 0.14 0.64 0.36 1.4
## PargraphComprehension   0.76  0.21  0.07 0.23 0.68 0.32 1.4
## SentenceCompletion      0.81  0.19  0.15 0.07 0.73 0.27 1.2
## WordClassification      0.57  0.34  0.23 0.14 0.51 0.49 2.2
## WordMeaning             0.81  0.20  0.05 0.22 0.74 0.26 1.3
## Addition                0.17 -0.10  0.82 0.16 0.74 0.26 1.2
## Code                    0.18  0.10  0.54 0.37 0.47 0.53 2.1
## CountingDots            0.02  0.20  0.71 0.09 0.55 0.45 1.2
## StraightCurvedCapitals 0.18  0.42  0.54 0.08 0.51 0.49 2.2
## WordRecognition         0.21  0.05  0.08 0.56 0.36 0.64 1.3
```

```
## NumberRecognition      0.12  0.12  0.08 0.52 0.31 0.69 1.3
## FigureRecognition      0.07  0.42  0.06 0.52 0.45 0.55 2.0
## ObjectNumber           0.14  0.06  0.22 0.58 0.41 0.59 1.4
## NumberFigure           0.02  0.31  0.34 0.45 0.41 0.59 2.7
## FigureWord             0.15  0.25  0.18 0.35 0.23 0.77 2.8
## Deduction              0.38  0.42  0.10 0.29 0.42 0.58 2.9
## NumericalPuzzles       0.18  0.40  0.43 0.21 0.42 0.58 2.8
## ProblemReasoning       0.37  0.41  0.13 0.29 0.40 0.60 3.0
## SeriesCompletion       0.37  0.52  0.23 0.22 0.51 0.49 2.7
## ArithmeticProblems     0.36  0.19  0.49 0.29 0.49 0.51 2.9
##
##                        PA1  PA3  PA2  PA4
## SS loadings           3.64 2.93 2.67 2.23
## Proportion Var        0.15 0.12 0.11 0.09
## Cumulative Var        0.15 0.27 0.38 0.48
## Proportion Explained  0.32 0.26 0.23 0.19
## Cumulative Proportion 0.32 0.57 0.81 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective function was  11.44
## The degrees of freedom for the model are 186  and the objective function was  1.72
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                    PA1  PA3  PA2  PA4
## Correlation of (regression) scores with factors   0.93 0.87 0.91 0.82
## Multiple R square of scores with factors          0.87 0.76 0.82 0.68
## Minimum correlation of possible factor scores     0.74 0.52 0.65 0.36
```

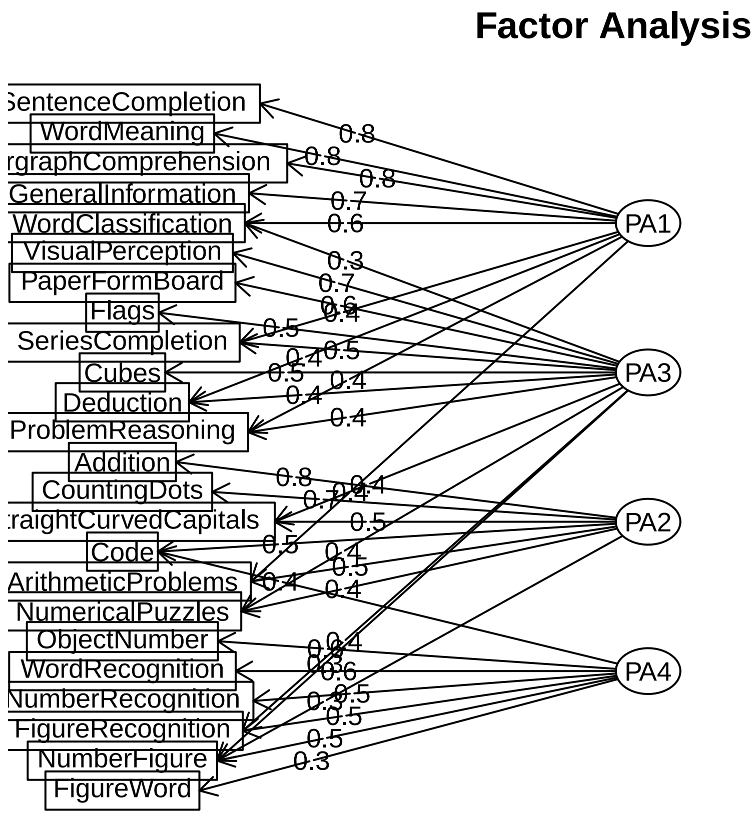
```r
# Compute factor scores
(fa(Harman74.cor[["cov"]], 4, rotate = "varimax", fm = "pa", score = TRUE))$score
```

```
##           [,1]      [,2]      [,3]      [,4]
## [1,] 1.0000000 0.6063244 0.4793939 0.4265543
## [2,] 0.6063244 1.0000000 0.5092860 0.5309466
## [3,] 0.4793939 0.5092860 1.0000000 0.5275798
## [4,] 0.4265543 0.5309466 0.5275798 1.0000000
```

```r
# Graph an orthogonal solution using factor.plot()
factor.plot(rotate, labels = rownames(rotate$loadings))
```




```r
# Graph an oblique solutions using fa.diagram()
fa.diagram(rotate, simple = FALSE)
```




```
#Interpret
#According to the parallel analysis th enumber of factors turn out to be 4.
#According to the standardized loadings based on the correlation matrix, the cumulative variance captur
#Which means all the 4 components are able to capture 48% variance in the data.
#The test of hypothesis also affirms that 4 components are enough to capture the essence of the data.
```

Question 4

```
#Problem 4: Perform factor analysis on breast-cancer-wisconsin.xlsx, is a multivariate
#dataset that is used to predict whether a cancer is malignant or benign from biopsy details of
#699 patients with 11 attributes. Create a new data frame by removing the variable "BN".
#• Calculate the correlation matrix from the new data frame. Visualize the correlation
#matrix using pairs.panels function of the "psych" package. How would you interpret
#the result in terms of correlation among the variables?
#• Input the correlation matrix to fa.parallel() function to determine the number of
#components to extract
#• Input the correlation matrix to fa() function to extract the components. If raw data is
#input, the correlation matrix is automatically calculated by fa() function.
#• Rotate the factors
#• Compute factor scores
#• Graph an orthogonal solution using factor.plot()
#• Graph an oblique solutions using fa.diagram()
#• Interpret the results

library(readxl)
```

```r
ac <- read_excel("C:/Users/abhil/Downloads/breast-cancer-wisconsin.xlsx")
View(ac)
library(psych)
library(corrplot)
```

```
## corrplot 0.84 loaded
```

```r
#creating new dataframe by removing BC
cc <- ac[,-7]
cc
```

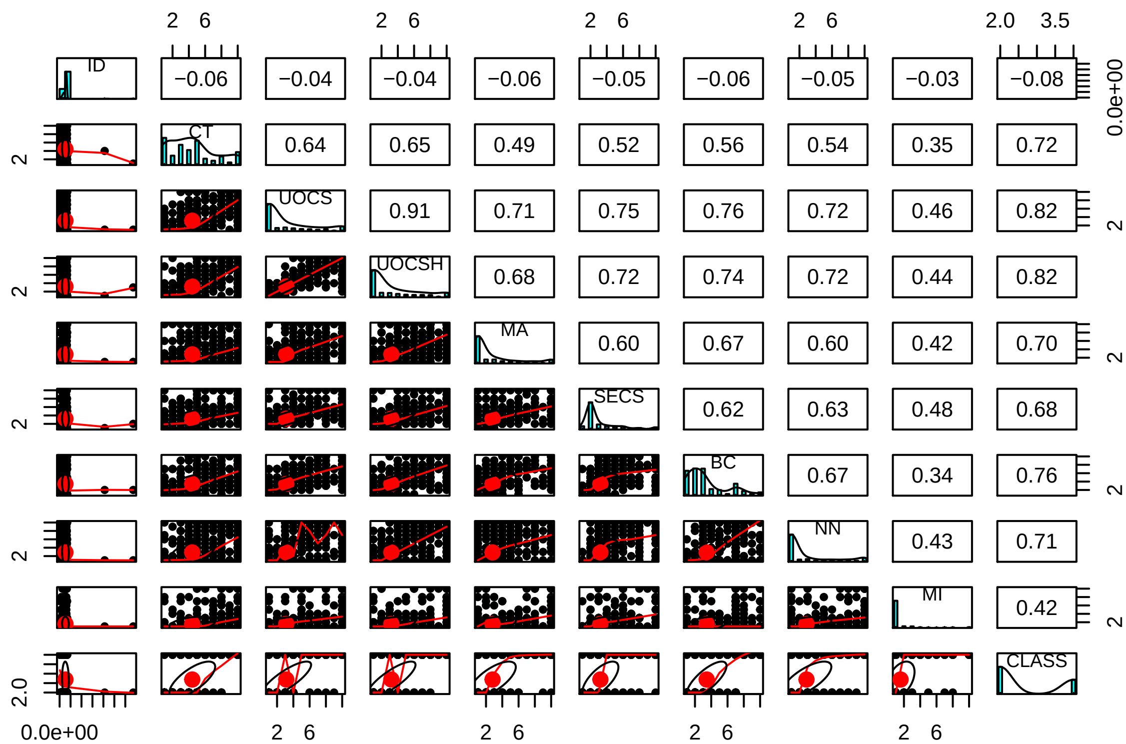
```
## # A tibble: 699 x 10
##         ID    CT  UOCS UOCSH    MA  SECS    BC    NN    MI CLASS
##      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
##  1 1000025     5     1     1     1     2     3     1     1     2
##  2 1002945     5     4     4     5     7     3     2     1     2
##  3 1015425     3     1     1     1     2     3     1     1     2
##  4 1016277     6     8     8     1     3     3     7     1     2
##  5 1017023     4     1     1     3     2     3     1     1     2
##  6 1017122     8    10    10     8     7     9     7     1     4
##  7 1018099     1     1     1     1     2     3     1     1     2
##  8 1018561     2     1     2     1     2     3     1     1     2
##  9 1033078     2     1     1     1     2     1     1     5     2
## 10 1033078     4     2     1     1     2     2     1     1     2
## # ... with 689 more rows
```

```r
View(cc)
#Calculate the correlation matrix from the new data frame.
correlation_cc <- cor(cc, method = "pearson", use = "complete.obs")
correlation_cc
```

```
##                ID          CT        UOCS       UOCSH          MA        SECS
## ID     1.00000000 -0.05530844 -0.04160334 -0.04157607 -0.06487808 -0.04552828
## CT    -0.05530844  1.00000000  0.64491250  0.65458908  0.48635624  0.52181622
## UOCS  -0.04160334  0.64491250  1.00000000  0.90688191  0.70558181  0.75179913
## UOCSH -0.04157607  0.65458908  0.90688191  1.00000000  0.68307920  0.71966844
## MA    -0.06487808  0.48635624  0.70558181  0.68307920  1.00000000  0.59959907
## SECS  -0.04552828  0.52181622  0.75179913  0.71966844  0.59959907  1.00000000
## BC    -0.06005053  0.55842816  0.75572098  0.73594845  0.66671533  0.61610184
## NN    -0.05207195  0.53583455  0.72286482  0.71944632  0.60335241  0.62888069
## MI    -0.03490066  0.35003386  0.45869315  0.43891093  0.41763278  0.47910148
## CLASS -0.08022565  0.71600136  0.81790374  0.81893374  0.69680021  0.68278453
##                BC          NN          MI       CLASS
## ID    -0.06005053 -0.05207195 -0.03490066 -0.08022565
## CT     0.55842816  0.53583455  0.35003386  0.71600136
## UOCS   0.75572098  0.72286482  0.45869315  0.81790374
## UOCSH  0.73594845  0.71944632  0.43891093  0.81893374
## MA     0.66671533  0.60335241  0.41763278  0.69680021
## SECS   0.61610184  0.62888069  0.47910148  0.68278453
## BC     1.00000000  0.66587781  0.34416950  0.75661615
## NN     0.66587781  1.00000000  0.42833575  0.71224362
## MI     0.34416950  0.42833575  1.00000000  0.42317026
## CLASS  0.75661615  0.71224362  0.42317026  1.00000000
```

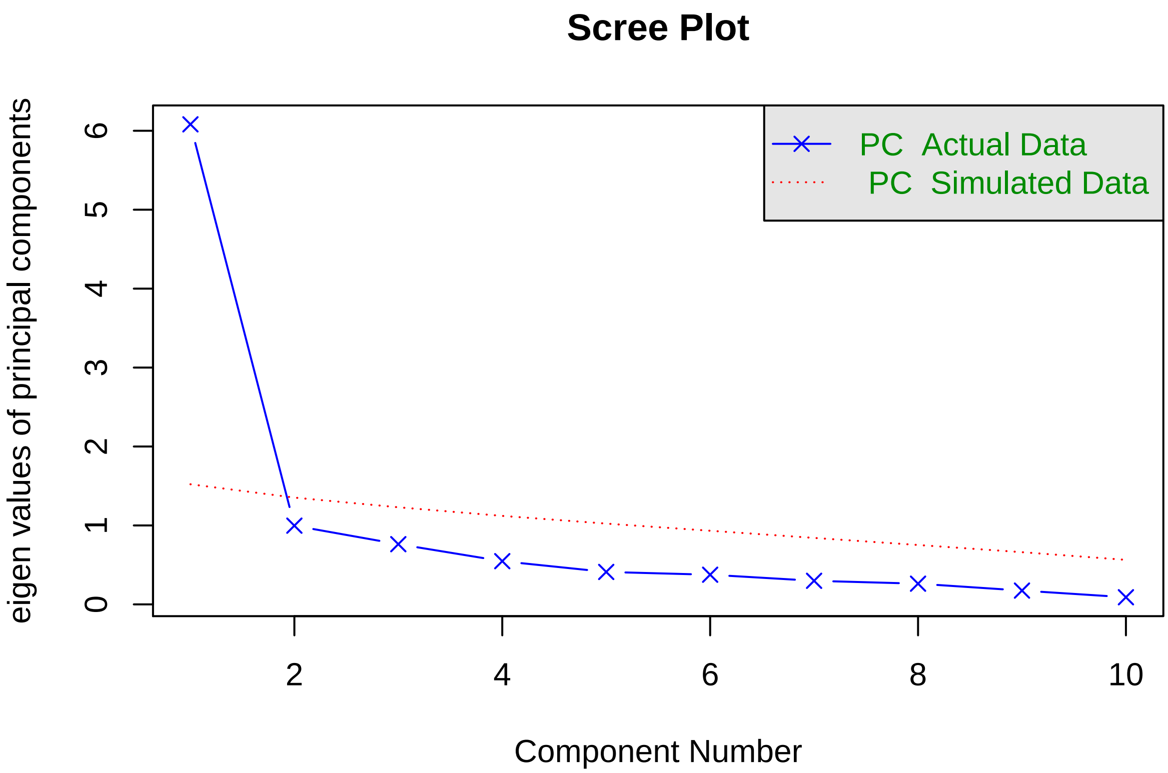
```r
#Visualize the correlation matrix using pairs.panels function of the "psych" package.
#How would you interpret the result in terms of correlation among the variables?
```

```r
# The most striking observation we can make from the correlation matrix is that all the variables are p
pairs.panels(cc)
```




```r
# Input the correlation matrix to fa.parallel() function to determine the number of
#components to extract
fa.parallel(correlation_cc, fa = "pc", n.iter = 100, main = "Scree Plot", show.legend = TRUE)
```

```
## Warning in fa.parallel(correlation_cc, fa = "pc", n.iter = 100, main = "Scree
## Plot", : It seems as if you are using a correlation matrix, but have not
## specified the number of cases. The number of subjects is arbitrarily set to be
## 100
```




```
## Parallel analysis suggests that the number of factors =  NA  and the number of components =  1
```

```r
# Input the correlation matrix to fa() function to extract the components. If raw data is
#input, the correlation matrix is automatically calculated by fa() function.

fa(correlation_cc, 1, rotate = "none", fm = "pa")
```

```
## Factor Analysis using method =  pa
## Call: fa(r = correlation_cc, nfactors = 1, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PA1     h2   u2 com
## ID    -0.07 0.0044 1.00   1
## CT     0.70 0.4887 0.51   1
## UOCS   0.93 0.8721 0.13   1
## UOCSH  0.92 0.8438 0.16   1
## MA     0.76 0.5837 0.42   1
## SECS   0.79 0.6168 0.38   1
## BC     0.81 0.6614 0.34   1
## NN     0.79 0.6261 0.37   1
## MI     0.50 0.2532 0.75   1
## CLASS  0.91 0.8195 0.18   1
##
##                 PA1
## SS loadings    5.77
## Proportion Var 0.58
##
## Mean item complexity =  1
```

```
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  45  and the objective function was  7.64
## The degrees of freedom for the model are 35  and the objective function was  0.36
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.03
##
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    PA1
## Correlation of (regression) scores with factors   0.98
## Multiple R square of scores with factors          0.96
## Minimum correlation of possible factor scores     0.92
```

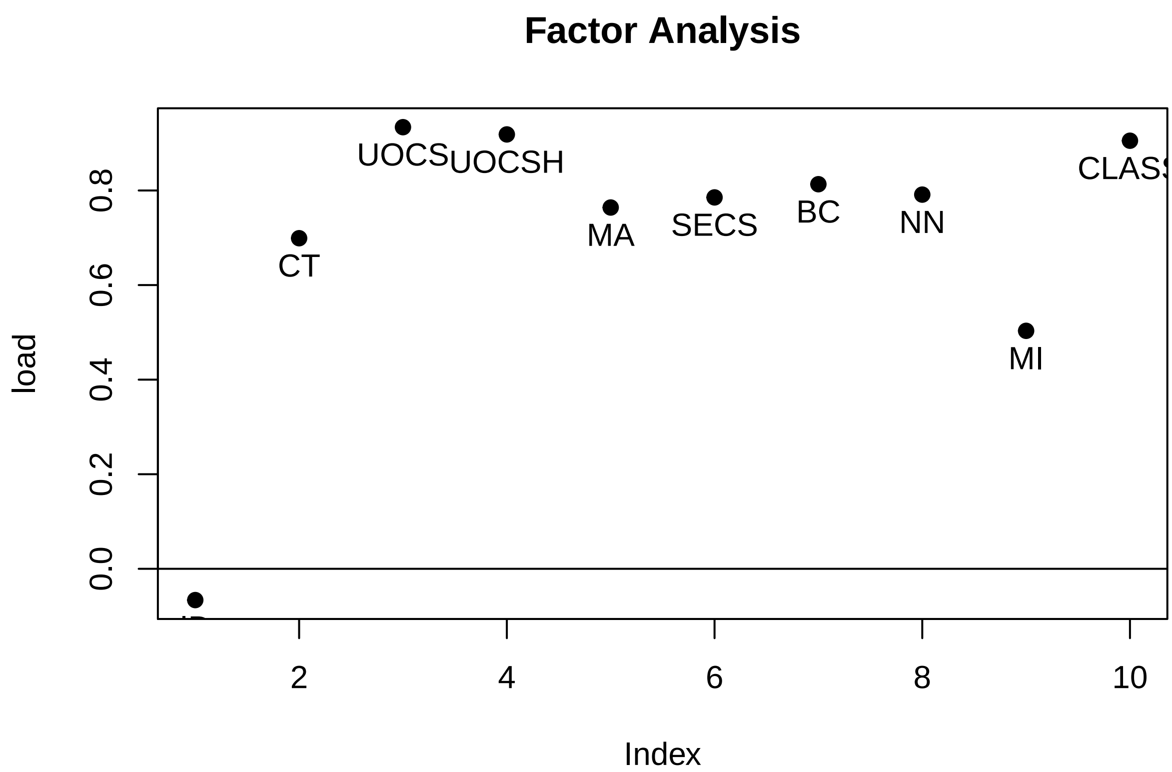
```r
# Rotate the factors
rotate_cc <- fa(correlation_cc, 1, rotate = "varimax", fm = "pa", scores = TRUE)
rotate_cc
```

```
## Factor Analysis using method =  pa
## Call: fa(r = correlation_cc, nfactors = 1, rotate = "varimax", scores = TRUE,
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PA1     h2   u2 com
## ID    -0.07 0.0044 1.00   1
## CT     0.70 0.4887 0.51   1
## UOCS   0.93 0.8721 0.13   1
## UOCSH  0.92 0.8438 0.16   1
## MA     0.76 0.5837 0.42   1
## SECS   0.79 0.6168 0.38   1
## BC     0.81 0.6614 0.34   1
## NN     0.79 0.6261 0.37   1
## MI     0.50 0.2532 0.75   1
## CLASS  0.91 0.8195 0.18   1
##
##                 PA1
## SS loadings    5.77
## Proportion Var 0.58
##
## Mean item complexity =  1
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  45  and the objective function was  7.64
## The degrees of freedom for the model are 35  and the objective function was  0.36
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.03
##
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    PA1
## Correlation of (regression) scores with factors   0.98
## Multiple R square of scores with factors          0.96
## Minimum correlation of possible factor scores     0.92
```

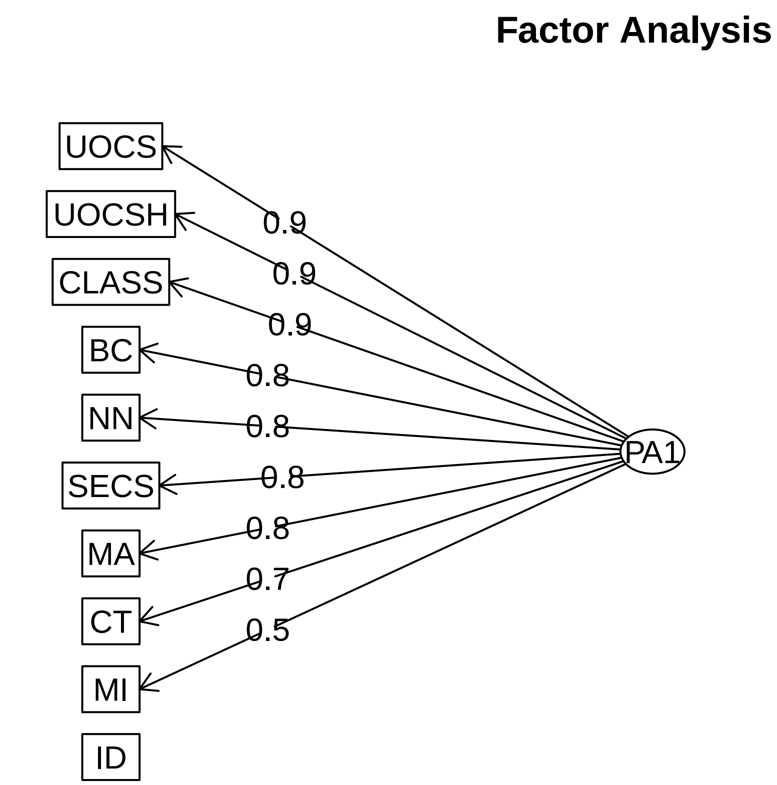
```r
# Compute factor scores
rotate_cc$score
```

```
## NULL
```

```r
# Graph an orthogonal solution using factor.plot()
factor.plot(rotate_cc, labels = rownames(rotate_cc$loadings))
```




```r
# Graph an oblique solutions using fa.diagram()
fa.diagram(rotate_cc, simple = FALSE)
```




```r
# Interpret the results

# It is evident from the scree plot that we need only 1 principal component extracted from the data.
#and the number of factors as 1.
#Varimax roattion was applied to the factors.
#The RMSR came to be 0.03.
#The test of hypothesis also states that the 1 factor is sufficient.
#The proportion variance captured is 0.58
```

Question 5

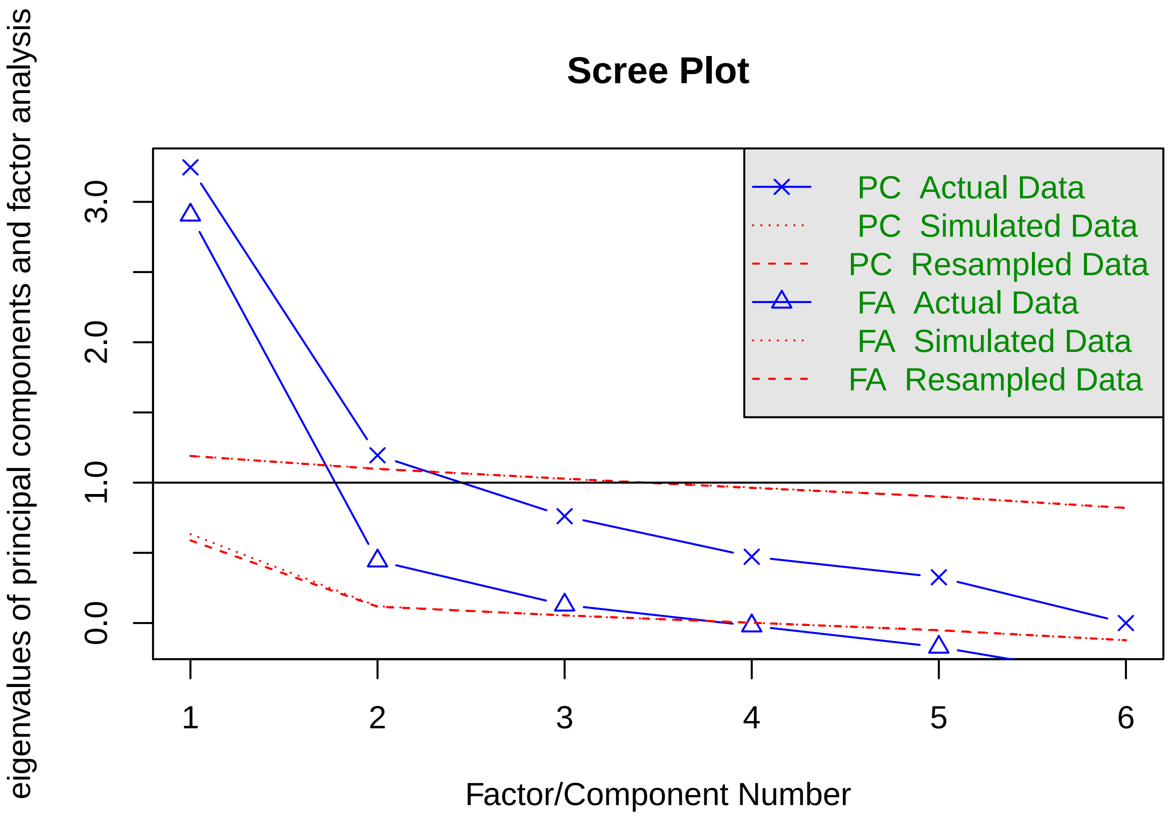
```r
# Problem 5. Perform multidimensional scaling on Vertebral Column Data.xlsx
vcd <- read_excel("C:/Users/abhil/Downloads/Vertebral Column Data(1).xlsx")
```

```r
# Input the raw data matrix to fa.parallel() function to determine the number of components to extract
fa.parallel(vcd[,1:6], fa = "both", n.iter = 200, main = "Scree Plot", show.legend = TRUE)
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.

## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully

## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```




```
## Parallel analysis suggests that the number of factors =  3  and the number of components =  2
```

```r
# Input the raw data matrix to cmdscale() function to perform multidimensional scaling.
# cmdscale() function which is available in the base installation performs a classical multidimensional
vcd_df <- dist(vcd[,1:6])
cmdscale(vcd_df, eig = TRUE, k = 3)
```

```
## $points
##              [,1]        [,2]         [,3]
##   [1,] -25.2126401  13.2042059 -15.89167139
##   [2,] -37.5502849 -18.9516208 -11.83917146
##   [3,] -21.9508658  23.0636140  -6.31851555
##   [4,] -10.8470948  13.9179838 -12.97106847
##   [5,] -27.7330523  -7.5890054 -18.43533164
##   [6,] -39.7480044 -22.9598414   2.54552855
##   [7,] -25.7002456  -4.4898798  -1.90156226
##   [8,] -46.2677698  -5.1806989  -7.09556361
##   [9,] -22.7776433 -14.0844558   4.69378434
##  [10,] -35.6460154   1.0457484 -28.38458751
##  [11,] -40.7578858   0.5109254 -12.90636830
##  [12,] -49.0398317 -32.7127927  -5.61904171
##  [13,] -25.7732384  -8.4297528   1.76764397
##  [14,] -26.1148599  -5.2410716  -9.09261463
##  [15,] -21.5356988   3.0431869   3.47719166
##  [16,] -31.1661702 -12.7858878   1.01500971
##  [17,] -20.7636038  18.2482578   0.77137950
##  [18,] -39.9936916 -23.8992110   1.71101012
```

```
## [19,] -39.2213539 -18.3279008   0.16094198
## [20,] -39.8602114 -12.4680456  -6.47068928
## [21,] -30.4347547 -16.4892176  10.73118508
## [22,] -27.0946622  -1.7657634   6.96841210
## [23,]  -7.3981484  10.1576593  -3.67883216
## [24,] -41.1289322  -7.9832200  -5.42210769
## [25,] -45.9095465 -19.4984976  -0.56007522
## [26,] -30.7815216  -6.0470012   3.90632894
## [27,] -60.8285159 -32.4577810  -3.46474272
## [28,] -25.1110755  -5.6634192  -3.21177430
## [29,] -36.7837502 -14.6374156  -8.49149760
## [30,] -22.4104478  17.0752596   0.81943947
## [31,] -20.9584740  -3.3935078  -5.87215225
## [32,] -29.5242601  -5.0275397 -22.01508895
## [33,] -33.5268294 -16.1581822  -5.46364606
## [34,] -35.4179274 -14.6096502   5.60198289
## [35,] -24.9876616   3.8551534   9.24258183
## [36,] -41.7337403 -24.9287273 -11.83297701
## [37,] -36.5779951 -24.8383541   7.42215610
## [38,] -46.4225583 -31.8656190  10.33512315
## [39,] -25.3068178   0.2003045  12.04544787
## [40,] -30.5322648  -3.5610683  -3.21522685
## [41,] -49.1788167 -21.2120863 -19.55481403
## [42,] -34.4229812  -9.1679827 -11.52432666
## [43,] -27.7394786  -7.4473101  -1.48204624
## [44,] -22.5936627  12.1453447   6.70148995
## [45,] -23.7144593  17.1093482   6.03465089
## [46,] -32.0151217   1.7202873   5.98791735
## [47,] -28.8257840 -10.3662693  -0.22693424
## [48,] -43.0174770 -11.0436320  -5.97974688
## [49,] -40.2567820 -14.2930081   2.27638154
## [50,] -40.6570535 -21.5503749  -7.53080813
## [51,] -28.0719018  -3.1873437  -4.88875450
## [52,] -20.3341711   5.7735744  -8.19252761
## [53,] -29.5440643 -14.0609578  11.75485751
## [54,] -33.7553252 -25.0743360  -6.22946979
## [55,] -46.8189739  -8.4926524  -1.15692171
## [56,] -27.7692867  -1.9128714 -19.32038794
## [57,] -33.0049258 -12.9856215 -14.21768217
## [58,] -32.5176922  -6.0449544  -2.86916124
## [59,] -40.0115525 -12.5306511   6.07687813
## [60,] -27.3528326 -11.7035005   0.66969369
## [61,]  39.1346938  -0.5371560  33.66711795
## [62,]  79.9640882   1.7427290  17.36623267
## [63,]  -5.7922219 -18.1030288  12.59895137
## [64,]  13.2447555  20.9744228   1.25818674
## [65,]   3.4796941  34.4020905  20.17671248
## [66,]  13.6059042  19.5551626   9.11249402
## [67,]  12.5262038  17.6632230   2.09184554
## [68,]  49.6234120   5.1802161 -19.14945186
## [69,] -15.9482416  22.4043028  -2.61500969
## [70,]   2.5857163   7.3856268 -21.62461545
## [71,]  11.4606737   7.1808756  -2.91168880
## [72,]  75.9358241 -25.8486219   3.48046613
```

```
##  [73,]  55.5055809   -4.3144469   6.48353176
##  [74,]   1.6639371  -14.7019793   1.39489622
##  [75,]  43.3222553    8.1933017  30.97998409
##  [76,] 109.8132112  -56.5906570  26.79848620
##  [77,]  89.8204072  -21.2315089  18.06867249
##  [78,]  22.8789772   -3.9000478 -22.11541841
##  [79,]  12.3231638    9.1890590  -3.87018212
##  [80,] -14.1800237  -13.9050513  -5.73387822
##  [81,]  51.4966621    0.3433561   1.43287685
##  [82,]  50.1124013  -10.5707464  -2.90503855
##  [83,]  34.4769479    4.9370016  -3.49123809
##  [84,]  50.7541803   -0.2749626  32.97244478
##  [85,]  28.0204460   -4.1484594  13.67790154
##  [86,] -15.9134568  -26.0721896  32.14968981
##  [87,]  -1.8467964    4.7446701   5.55802490
##  [88,]  -0.6422532  -24.4117027   4.68658678
##  [89,]  -7.2651397  -10.3222967   3.22196902
##  [90,]   3.3840566    1.8791228  -2.14279401
##  [91,]  15.9433983   17.2452772   0.55460752
##  [92,]  14.0712118   11.5838336   1.98586585
##  [93,]  62.1581539    6.1550276   1.52323529
##  [94,]  28.1970639    3.3182919   6.76131451
##  [95,]  46.3510715   25.7593787  -7.66711429
##  [96,] 101.5414366  -74.0820150  12.06610707
##  [97,]  63.2828865   31.6546987   8.19729414
##  [98,]  23.0468629   27.3233794   5.28313231
##  [99,]  51.4128422   17.1576805  18.75598408
## [100,]  -0.2745349   -7.4489721  -8.94334587
## [101,]  15.0150022   27.9826416  -0.34278794
## [102,]  24.3802880   17.4351349  -2.66539899
## [103,]   8.3139429   19.9207973 -12.11571984
## [104,]  -4.9340230   -2.7572380  -7.86844239
## [105,]  69.3542843  -15.4580100  -0.14559432
## [106,] -15.4642150   10.2552950   4.89509820
## [107,]  25.8213587    5.2795442 -23.10355275
## [108,]  58.7118606   -7.3171332  25.71495263
## [109,]  20.1790025    5.7895561  -8.70061303
## [110,]  19.1687198    5.3828234   4.77696466
## [111,]  18.4520910    3.3328754  -2.58862447
## [112,]  60.0472714    9.6962581  17.16941967
## [113,]  -1.5829978    0.9878689  -4.98463501
## [114,]  25.8484652    0.5019259 -22.76828665
## [115,]  70.4305569   -5.4152697  33.08225923
## [116,] 375.7163521 -126.0132204 -79.30169542
## [117,]  35.5360741    1.9702958  -6.59912232
## [118,]  41.0306181   -4.8200620   1.48024790
## [119,]  -8.9324554   -1.9921745  14.64447144
## [120,]   2.4596558   -4.9681621  -9.76189737
## [121,]   6.3021554  -15.9226389  -8.68975088
## [122,]  69.2784722    8.8530401  13.04816364
## [123,]  43.6088534   -9.2165806   0.70890609
## [124,]  28.9410454   -1.7072457 -12.85527934
## [125,]  11.5872215    0.9887161 -24.66379502
## [126,]   2.3897671   -0.5052937 -18.64848851
```

```
## [127,]   8.9867591  15.4692131  -8.59686982
## [128,]  42.3548649  10.2603673  -3.13697828
## [129,]  48.9989237  21.9710807 -10.37047444
## [130,]  20.2309453   9.2066274 -13.07606254
## [131,]  -9.7960339 -11.4528691 -25.69301541
## [132,]  -0.1021163  15.9407953   0.37962458
## [133,]  29.4365794  -6.1498334  -4.34203564
## [134,]  41.9281785  12.7911261   2.74664360
## [135,]  27.6979657 -13.0101162  26.33192406
## [136,]  63.7687861  -0.3225496   2.79859306
## [137,]  47.9353443  18.9140542  14.73853536
## [138,]  39.0436942  20.1242626   7.46092565
## [139,]  40.2906890  11.6827902   2.71688681
## [140,]  35.2763854  38.4381584   6.93007603
## [141,]  15.2071247  22.8319287  -2.68347257
## [142,]  97.4908031 -23.0690787  20.92327448
## [143,]  56.3342456  33.6071351   8.64099801
## [144,]  71.4362967 -31.7010865  -2.04050639
## [145,]   5.8594681   5.9004683  -8.95034137
## [146,]  33.8326585  24.8588060   8.13940082
## [147,]  56.5426590  12.5199650   5.94738380
## [148,]   5.3788762   3.4268567 -14.65417090
## [149,]  14.8598972   4.1355216 -18.02475778
## [150,]  15.5782187   9.4946996 -23.24135838
## [151,]  26.3477010  22.2408192 -19.58478187
## [152,]   3.8979182 -10.0367812   3.99909107
## [153,]   5.6473178   0.7083169  -9.75091100
## [154,]   5.7105853   2.7877824   4.17312363
## [155,] -10.3085291 -14.0157654 -17.09896712
## [156,]  24.1258440  20.9458858 -22.52247908
## [157,]  45.4662193   6.3318923   5.48909524
## [158,]   5.6307133 -17.5079709 -16.25611351
## [159,]  21.7752785  -7.2999175 -24.12229032
## [160,]  19.5974671  14.5401379 -14.43983550
## [161,]  50.3266490  20.9245963   8.34895107
## [162,]  18.8273592   7.3837411 -24.81735235
## [163,]  70.7397762  31.6906087 -38.84732601
## [164,]  82.9985062  32.8085616  -8.37624925
## [165,]  -6.6249382  -9.5023762  -1.85390764
## [166,]  56.6485261   9.3153841   8.61667296
## [167,]  10.0202743   0.1757717  -9.59509199
## [168,]  -2.0008395  36.9269166 -35.11348553
## [169,]  58.9792860  44.8902511  -9.70971784
## [170,] -12.7794368  -9.2364023 -26.28413154
## [171,]   0.6016506  12.3969395  -3.20232170
## [172,]   6.7239151  42.0248137  -1.31047789
## [173,]  -7.8757023   3.8234420 -23.35861946
## [174,]  -4.6578338  10.5306006 -29.53806777
## [175,]  -7.5825366  -1.4221743 -13.19136839
## [176,]  20.5669617  -6.2322753 -20.90722857
## [177,]  15.3779579  16.1757500 -11.13687822
## [178,]  29.9178438  22.6030846 -23.76236228
## [179,]  35.1581408   6.2168481   2.40104088
## [180,]  30.4836462  -9.2726905  24.80904854
```

```
## [181,] -13.9481009  -46.5442480  15.55928358
## [182,]  12.3689006   19.0001537 -16.50039276
## [183,]  43.1406358   18.6662453  10.18184296
## [184,]  49.1523468   -1.5665153  24.66003819
## [185,]  32.7990628   29.7462842  20.53156394
## [186,]  48.5311225   29.4392149   8.20452337
## [187,]  39.3534003   28.7906434 -14.39004958
## [188,]   7.2415050    8.5071433  -9.90413089
## [189,]  51.4304129   13.8630674  17.84130616
## [190,]  49.6986922   11.5196989   7.91592707
## [191,]   5.2843675   -9.6845458 -24.23808419
## [192,]  40.4929391   13.1410182  -9.60823612
## [193,]  94.8141502  -41.2545829  22.48049107
## [194,]  18.7928824   19.8707307   6.04700115
## [195,]  10.4679943   19.3039963   4.90047413
## [196,]  27.8200840   27.3163626   0.61382405
## [197,]  32.7722697    6.0038510 -22.01250595
## [198,]  97.6029829  -17.3072185  50.55159216
## [199,]  17.3026742   16.5757335  -3.24825213
## [200,]  15.3679561   15.0304960   0.43511634
## [201,]  15.3463754    0.4067369  17.08108374
## [202,]  68.3740704  -13.5923056  42.55547866
## [203,]  82.8710298  -12.0333799  31.46985777
## [204,]  12.3057384   25.1632638 -17.01864856
## [205,]  -3.8824344   -0.7273452 -16.56743060
## [206,]  82.5911273   -8.3120127  17.13383853
## [207,]  62.6527069    7.7999744 -12.84722730
## [208,]  54.7887973   -1.9941387  11.60899308
## [209,]  70.4053899   19.4589915  11.52280730
## [210,]  -8.7096201  -12.0266120 -23.22961293
## [211,] -32.8887382  -21.8877223   6.79133596
## [212,] -24.4272059    3.8279552  11.07973090
## [213,] -28.0378499   -7.4245248   8.47154675
## [214,] -31.5031440   -2.9097309  12.98665757
## [215,] -36.0645040   -5.3287970   8.62163308
## [216,] -45.5199458  -27.0277239  19.94603428
## [217,] -34.0679120   -3.2348169 -10.23928699
## [218,] -28.9781601  -11.7751128  12.68278639
## [219,] -31.8973313   -7.6561907   4.97421738
## [220,] -18.7661999   15.2459644  -3.17926615
## [221,] -37.9912617  -11.6148589  -3.77816640
## [222,] -25.0134162    9.1728240   6.03116008
## [223,]   5.5942610    1.3837759   2.23598313
## [224,] -10.7251322   37.5519938   2.39620976
## [225,]   7.3512665   61.0575752   3.46948942
## [226,] -18.5396713   12.6926901   5.07001506
## [227,] -12.6008456   11.1171563  25.17748286
## [228,] -17.9202055   10.9188613   4.48208334
## [229,] -38.4098379  -21.3756850  -4.54213537
## [230,] -39.1819781  -13.0652983  11.25521332
## [231,] -20.9855687   21.1358158  15.32781177
## [232,] -25.3269374   -1.0787149  -4.39364377
## [233,] -34.6138831  -11.1742801   8.05227845
## [234,] -41.6174946  -18.6581278  10.21794800
```

```
## [235,] -25.8041538  -22.1154565  12.02136599
## [236,] -34.0701700   15.5785882  -4.76525902
## [237,] -16.9364105   19.4802576   9.52886577
## [238,] -15.8079739   10.9895304  15.12872302
## [239,]  -7.5502651   15.0602046   5.90965787
## [240,] -29.8248372   -1.7036964  17.74065295
## [241,] -41.9274640   -6.1045248  -4.76599372
## [242,] -20.4771710  -12.4257590   0.10230083
## [243,] -41.6211114  -19.6047301   0.64891166
## [244,] -41.9423479  -11.3438841   5.34457242
## [245,] -15.8714439    9.4509133   3.94995787
## [246,] -28.7481963    2.6497716   9.69003422
## [247,] -25.7828918   -9.8845012  -0.01035463
## [248,] -34.0409806  -11.0100254  -3.88970147
## [249,] -32.5070377   -6.4322255  -4.42417691
## [250,] -30.8538792   -6.8760850   2.83501557
## [251,] -41.8124021  -17.5262545   4.92618024
## [252,] -32.3644655   -8.0190488  15.48839637
## [253,] -30.9802045    7.8649076   1.65245891
## [254,]  -6.3979964   15.4397851  11.04890711
## [255,] -12.5831143   29.3690814   0.17309806
## [256,]   5.4321631    8.3793035   4.11179792
## [257,] -19.2440987    2.2355530   1.55178281
## [258,] -36.8108783    3.8259792   1.81169056
## [259,] -36.8900003  -13.3888664   5.34736987
## [260,] -22.0421403   14.5893002   0.10847680
## [261,] -28.2572471    2.6119544   0.70116364
## [262,] -12.4233517   29.9351553 -10.84180921
## [263,] -39.7459267  -19.8755079   1.36888243
## [264,] -45.4881506  -22.8385511  -2.05296184
## [265,] -29.9781657   12.0314633  -7.63545252
## [266,] -28.1231265  -10.4198167   9.15906149
## [267,] -32.8977312   -3.0662742   1.01480757
## [268,] -26.0356198    3.2388839   2.16272632
## [269,] -22.6876038    4.1780103  -4.84756564
## [270,] -37.4020800  -27.6975616  -0.57861527
## [271,] -27.3239769    8.2410059  10.35005088
## [272,] -28.4990935  -12.3661632   3.53291480
## [273,] -35.8536041  -11.2743268  -4.62093969
## [274,] -38.4962792  -10.3837915   3.85186537
## [275,] -36.8498160  -14.8626099  -3.66414218
## [276,] -15.5807180    9.5031875  15.10143867
## [277,] -31.7414769   -9.1788973   4.35087300
## [278,] -23.9267789   11.4342243   4.64700015
## [279,] -43.3209498   -7.8493743 -10.14883402
## [280,] -20.5954205   -9.2763885  21.67588582
## [281,] -30.5307045   -6.2467431  -4.42669145
## [282,] -17.6193887   10.4186146  -6.11752223
## [283,] -26.5967213    2.4620846  -5.82412336
## [284,] -28.2158341    1.0733531  10.27158233
## [285,] -23.4207688    5.3614623  -0.35200784
## [286,] -17.1835228   11.6673733  -2.81980454
## [287,] -20.9478288    8.7250475   2.03499080
## [288,] -37.6052603  -28.5975506 -14.63761620
```

```
## [289,] -12.0828276  32.5180667  -7.94472669
## [290,] -40.4515676 -13.2985326   6.88990161
## [291,] -44.9137127 -26.4652732  -1.94538411
## [292,] -26.2283026  -5.5958640  -6.56796399
## [293,] -42.4803473 -19.8425025  -1.95081023
## [294,] -10.0845732  -7.7314691  14.12537520
## [295,] -38.8268011  -6.1690024   3.95693987
## [296,] -28.5696408  -3.5068858   8.47647267
## [297,] -43.0408542 -14.9887902  11.73145851
## [298,] -33.5630637 -11.2280645  -2.58656261
## [299,] -24.9644921   1.8580085   1.62096602
## [300,]  -4.5475561  31.1425051   0.61483166
## [301,] -32.1459772  -1.0972687  -9.56633477
## [302,]   2.3153768  42.9283926   2.95013231
## [303,] -34.3669919  -4.9077874  -3.95420808
## [304,] -44.2946998 -17.4339491   0.52479898
## [305,] -41.2700012  -8.2767648  25.02631631
## [306,] -37.5343327  -3.2656500  -3.41994687
## [307,] -33.6618308  -4.5600797  -7.45763822
## [308,] -27.0125654   7.8027891   9.06262553
## [309,] -32.9610331  -3.6892415  -2.23394970
## [310,] -40.7187165 -16.6016629   0.61818143
##
## $eig
##   [1] 5.503172e+05 1.067048e+05 5.833205e+04 3.275852e+04 2.738089e+04
##   [6] 2.942013e-03 7.032194e-10 4.898211e-10 4.312057e-10 3.631628e-10
##  [11] 2.398488e-10 2.396298e-10 1.595477e-10 1.469035e-10 4.755321e-11
##  [16] 2.132445e-11 2.091854e-11 1.546792e-11 1.416156e-11 1.347003e-11
##  [21] 1.234092e-11 1.221587e-11 1.081441e-11 8.850349e-12 7.321424e-12
##  [26] 6.891075e-12 6.786715e-12 6.659225e-12 6.348727e-12 5.389669e-12
##  [31] 4.412734e-12 4.010057e-12 3.920417e-12 3.788308e-12 3.322099e-12
##  [36] 3.215708e-12 2.776303e-12 2.715802e-12 2.667428e-12 2.527044e-12
##  [41] 2.135655e-12 2.125102e-12 2.117223e-12 2.053923e-12 1.894841e-12
##  [46] 1.801999e-12 1.777611e-12 1.616113e-12 1.328305e-12 1.247216e-12
##  [51] 1.187367e-12 1.168439e-12 1.144651e-12 1.137671e-12 9.945385e-13
##  [56] 9.708681e-13 9.520230e-13 9.494081e-13 9.200417e-13 8.800521e-13
##  [61] 8.561331e-13 8.431170e-13 8.318222e-13 8.291758e-13 7.563771e-13
##  [66] 7.362068e-13 7.302199e-13 7.298761e-13 7.193850e-13 6.757138e-13
##  [71] 6.622077e-13 6.334616e-13 6.279447e-13 5.868291e-13 5.781714e-13
##  [76] 5.729774e-13 5.694643e-13 5.498693e-13 5.470381e-13 5.368081e-13
##  [81] 5.291224e-13 5.282675e-13 5.180582e-13 5.168896e-13 5.012817e-13
##  [86] 4.900136e-13 4.824346e-13 4.695359e-13 4.551563e-13 4.534240e-13
##  [91] 4.401463e-13 4.393949e-13 4.210132e-13 4.086747e-13 4.024641e-13
##  [96] 3.990193e-13 3.905471e-13 3.662669e-13 3.417964e-13 3.308595e-13
## [101] 3.293568e-13 3.270819e-13 3.257113e-13 3.249792e-13 3.242890e-13
## [106] 3.212508e-13 2.986648e-13 2.964983e-13 2.950959e-13 2.814635e-13
## [111] 2.739412e-13 2.727669e-13 2.683601e-13 2.675831e-13 2.661030e-13
## [116] 2.643020e-13 2.593789e-13 2.569328e-13 2.456413e-13 2.450898e-13
## [121] 2.401156e-13 2.368422e-13 2.314245e-13 2.285802e-13 1.823820e-13
## [126] 1.784003e-13 1.669430e-13 1.588764e-13 1.300457e-13 1.130335e-13
## [131] 1.079903e-13 1.041017e-13 9.840781e-14 9.112945e-14 9.097576e-14
## [136] 8.963278e-14 8.535276e-14 8.115336e-14 7.880255e-14 7.750252e-14
## [141] 7.694293e-14 6.885414e-14 6.668926e-14 6.529406e-14 6.363150e-14
## [146] 6.257301e-14 5.968367e-14 5.953112e-14 5.505936e-14 4.528071e-14
```

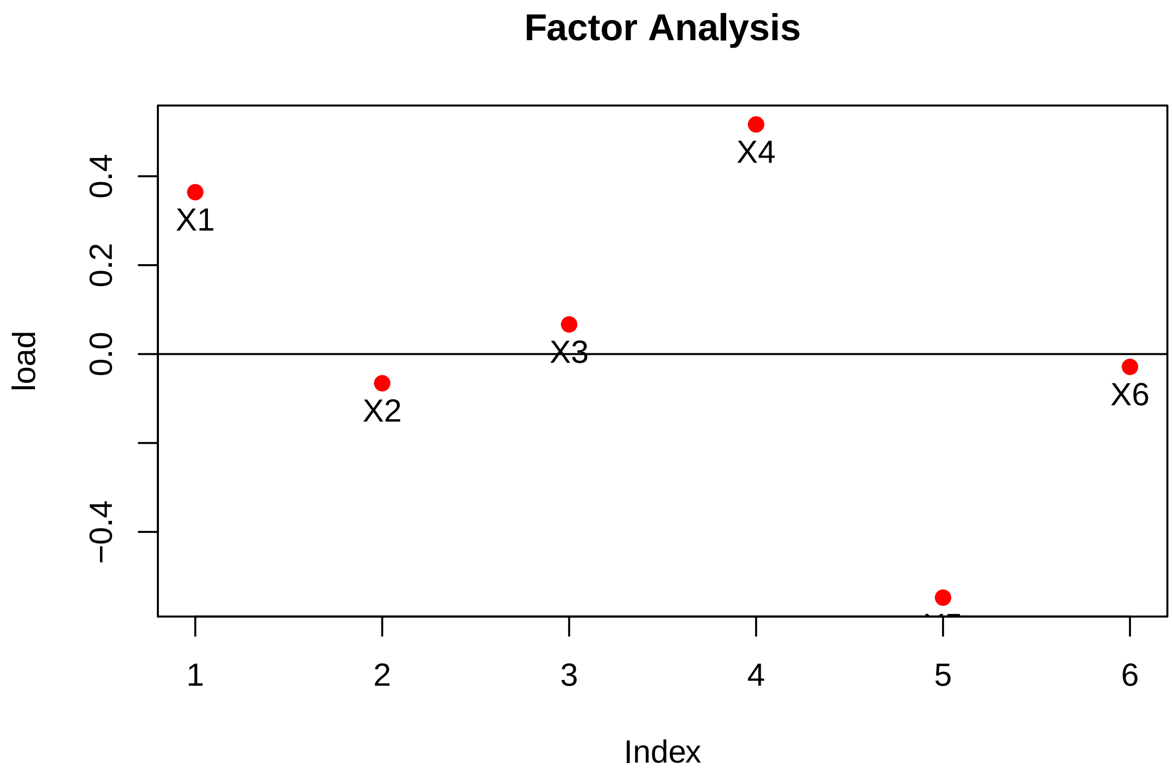
```
## [151]  4.193055e-14  3.847483e-14  2.532178e-14  2.219892e-14  1.634214e-14
## [156]  3.932858e-15 -8.317931e-15 -1.719529e-14 -1.837300e-14 -1.871338e-14
## [161] -1.950851e-14 -2.492496e-14 -2.666354e-14 -3.098185e-14 -3.793382e-14
## [166] -4.175800e-14 -4.628914e-14 -4.829509e-14 -4.926426e-14 -5.095204e-14
## [171] -5.105255e-14 -5.292853e-14 -5.569735e-14 -5.906044e-14 -6.423779e-14
## [176] -7.285366e-14 -7.326098e-14 -7.402400e-14 -7.695724e-14 -7.999553e-14
## [181] -9.464037e-14 -1.073636e-13 -1.107881e-13 -1.122268e-13 -1.155534e-13
## [186] -1.193809e-13 -1.207578e-13 -1.239389e-13 -1.289474e-13 -1.294819e-13
## [191] -1.318266e-13 -1.343856e-13 -1.423935e-13 -1.480119e-13 -1.513403e-13
## [196] -1.587447e-13 -1.821855e-13 -1.826030e-13 -1.925833e-13 -2.074545e-13
## [201] -2.082919e-13 -2.301687e-13 -2.330415e-13 -2.338536e-13 -2.345681e-13
## [206] -2.369920e-13 -2.446556e-13 -2.459546e-13 -2.554023e-13 -2.554464e-13
## [211] -2.603227e-13 -2.637630e-13 -2.713079e-13 -2.790816e-13 -2.834986e-13
## [216] -2.858018e-13 -2.858630e-13 -2.887947e-13 -3.023881e-13 -3.162562e-13
## [221] -3.163760e-13 -3.170161e-13 -3.183964e-13 -3.295577e-13 -3.491188e-13
## [226] -3.546710e-13 -3.601795e-13 -3.607263e-13 -3.638552e-13 -3.827448e-13
## [231] -3.859459e-13 -4.094868e-13 -4.100646e-13 -4.203870e-13 -4.306542e-13
## [236] -4.310942e-13 -4.529272e-13 -4.573478e-13 -4.825641e-13 -4.923205e-13
## [241] -5.005823e-13 -5.161908e-13 -5.293325e-13 -5.350917e-13 -5.505740e-13
## [246] -5.927002e-13 -6.101520e-13 -6.117826e-13 -6.229774e-13 -6.250470e-13
## [251] -6.277077e-13 -6.377376e-13 -6.384238e-13 -6.533066e-13 -7.020028e-13
## [256] -7.087698e-13 -7.497810e-13 -7.636583e-13 -7.922841e-13 -7.975536e-13
## [261] -8.088701e-13 -8.601742e-13 -8.602648e-13 -8.654713e-13 -8.983672e-13
## [266] -9.010069e-13 -1.069837e-12 -1.137094e-12 -1.147437e-12 -1.149312e-12
## [271] -1.192068e-12 -1.204419e-12 -1.255060e-12 -1.540208e-12 -1.668779e-12
## [276] -1.753359e-12 -1.897583e-12 -2.443193e-12 -2.594031e-12 -2.659455e-12
## [281] -3.178396e-12 -3.268728e-12 -3.310298e-12 -3.572711e-12 -4.369728e-12
## [286] -5.290073e-12 -5.559181e-12 -5.623910e-12 -6.846141e-12 -7.844745e-12
## [291] -8.793976e-12 -9.458108e-12 -9.581462e-12 -1.126122e-11 -1.217148e-11
## [296] -1.314970e-11 -1.423785e-11 -1.590360e-11 -1.671355e-11 -2.243079e-11
## [301] -3.585691e-11 -5.415199e-11 -1.070735e-10 -1.505341e-10 -2.222474e-10
## [306] -3.770885e-10 -3.925776e-10 -4.229698e-10 -4.613050e-10 -8.269459e-10
##
## $x
## NULL
##
## $ac
## [1] 0
##
## $GOF
## [1] 0.9224501 0.9224501
```

```r
# Graph an orthogonal solution using factor.plot()
library(GPArotation)
rotate <- fa(cor(vcd[,1:6]), nfactors = 3, rotate = "varimax", fm = "minres")
rotate
```

```
## Factor Analysis using method =  minres
## Call: fa(r = cor(vcd[, 1:6]), nfactors = 3, rotate = "varimax", fm = "minres")
## Standardized loadings (pattern matrix) based upon correlation matrix
##      MR1   MR2   MR3   h2     u2 com
## X1  0.81  0.45  0.36 1.00 0.0024 2.0
## X2  0.27  0.96 -0.07 1.00 0.0041 1.2
## X3  0.71  0.26  0.07 0.57 0.4261 1.3
## X4  0.84 -0.13  0.52 1.00 0.0034 1.7
```

```
## X5 -0.07  0.02 -0.55 0.30 0.6951 1.0
## X6  0.67  0.23 -0.03 0.51 0.4948 1.2
##
##                        MR1  MR2  MR3
## SS loadings           2.40 1.26 0.71
## Proportion Var        0.40 0.21 0.12
## Cumulative Var        0.40 0.61 0.73
## Proportion Explained  0.55 0.29 0.16
## Cumulative Proportion 0.55 0.84 1.00
##
## Mean item complexity =  1.4
## Test of the hypothesis that 3 factors are sufficient.
##
## The degrees of freedom for the null model are  15  and the objective function was  17.61
## The degrees of freedom for the model are 0  and the objective function was  10.01
##
## The root mean square of the residuals (RMSR) is  0
## The df corrected root mean square of the residuals is  NA
##
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                   MR1  MR2  MR3
## Correlation of (regression) scores with factors   0.92 0.99 0.74
## Multiple R square of scores with factors          0.85 0.97 0.55
## Minimum correlation of possible factor scores     0.70 0.94 0.09
```

```r
factor.plot(rotate, choose = 3, cluster = 3, labels = rownames(rotate$loadings))
```




```
# Intrepret the results
# The fa.parallel() function provides us with the number of components and
#number of factors to be extracted as 2 and 3 respectively.
#The 3 components capture 73% of the variance of the data.
```