$$\frac{\int D ||x||}{|E||x||} \rightarrow 1$$

 $X = (X_1, --, X_n) \qquad X_i \sim U[0, 1]$

USTT
$$\int_{\mathbb{N}} \left(\frac{\| \times_{1} \|^{2}}{n} - \mathbb{E} \times_{1}^{2} \right) \xrightarrow{n \to \infty} N(o, D \times_{1}^{2})$$

$$\int_{\mathbb{N}} \left(\frac{\| \times_{1} \|^{2}}{n} - \int_{\mathbb{E}} \times_{1}^{2} \right) \xrightarrow{n \to \infty} N(o, C)$$

$$\| \times_{1} \| - \int_{\mathbb{N}} \mathbb{E} \times_{1}^{2} \xrightarrow{n \to \infty} N(o, C)$$

II xII - N (In Ex,2, C)

 $\frac{\int D \parallel \times \parallel}{E \parallel \times \parallel} = \int \frac{E \parallel \times \parallel^2 - (E \parallel \times \parallel)^2}{(E \parallel \times \parallel)^2} = \int \frac{E \parallel \times \parallel^2}{(E \parallel \times \parallel)^2} - 1$

 $|| \times ||^2 = \chi_1^2 + \dots + \chi_n^2$

.

.

3 agara
$$X = X_1 - X_n$$
 $X_i \in \mathbb{R}^p$ bundopka

i) Procepours $h: X \to Y = (Y_1 - Y_n)$ $Y_i \in \mathbb{R}^d$ (Embedding)

3) Proction to the
$$\mathbb{R}^D \to \mathbb{R}^d$$

$$g: \mathbb{R}^d \to \mathbb{R}^D \quad \times \approx g(h(x))$$

PCA:
$$X = (X_1, ..., X_n)$$
 $X_i \in \mathbb{R}^D$

Haûtu $\mathcal{L} = \{x \in \mathbb{R}^D \mid x = x_0 + \sum_{i=1}^d y_i e_i, y_0 \in \mathbb{R}^2 \}$

kotopoe xopomo npudnimaet namy budopny

 $x_0, e_i \in \mathbb{R}^D$ — napametpu nognp-ba

проекции не направленные о

 $\sum = \frac{1}{h} \sum_{i=1}^{h} (x_i - \overline{x})^i (x_i - \overline{x})^{\top}$

$$e_{i}$$



$$D(\alpha^{T}X) = \alpha^{T}DX; \quad \alpha = \alpha^{T}\sum \alpha \Rightarrow \hat{G}_{\alpha} = \alpha^{T}\hat{\sum} \alpha$$

$$\sum_{i=1}^{n} (\alpha^{T}X_{i} - \alpha^{T}\overline{X})(\alpha^{T}X_{i} - \alpha^{T}\overline{X})^{T} =$$

= $a^{\tau} \stackrel{\kappa}{\Sigma} a$

$$\sum_{i=1}^{\infty} \sum_{i=1}^{\infty} (a^i X_i - a^i X_i)(a^i X_i - a^i X_i)$$

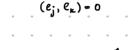
$$= \underbrace{\bot}_{\Sigma} \overset{\mathbf{n}}{\Sigma} a^{\mathsf{T}} (x_{i} - \overline{x}_{i}) (a^{\mathsf{T}} (x_{i} - \overline{x}_{i}))^{\mathsf{T}} =$$

$$= \frac{1}{h} \sum_{i=1}^{h} \alpha^{T} (x_{i} - \overline{x}) \left(\alpha^{T} (x_{i} - \overline{x}) \right)^{T} =$$

•
$$e_k$$
 rake \hat{u} 270 \hat{G}_{e_k} \rightarrow max

 $\|e_k\| = 1$
 $(e_j, e_k) = 0$

$$\begin{cases}
e_i^{\mathsf{T}} \hat{\Sigma} e_i & \rightarrow \text{ max} \\
e_i & \rightarrow
\end{cases}
\begin{cases}
e_i^{\mathsf{T}} \hat{\Sigma} e_i & \rightarrow \text{ max} \\
e_i^{\mathsf{T}} e_i & = 1
\end{cases}$$



 $L = e_i^T \hat{\Sigma} e_i = \lambda e_i^T e_i$

 $\frac{\partial L}{\partial L} = 2 \hat{\Sigma} e_1 - 2 \lambda e_1 = 0$

$$\hat{\Sigma}e_i = \lambda e_i$$

$$= \hat{\Sigma}e_i - coderb. ben \hat{\Sigma}c - coderb zuar \lambda$$

$$\widehat{\sigma}_{e_1} = e_1^{\mathsf{T}} \widehat{\Sigma} e_1 = e_1^{\mathsf{T}} \lambda e_1 = \lambda e_1^{\mathsf{T}} e_1 - \lambda \rightarrow \max$$

