biomechanics
and Motor Control of Human Movement

Fourth Edition

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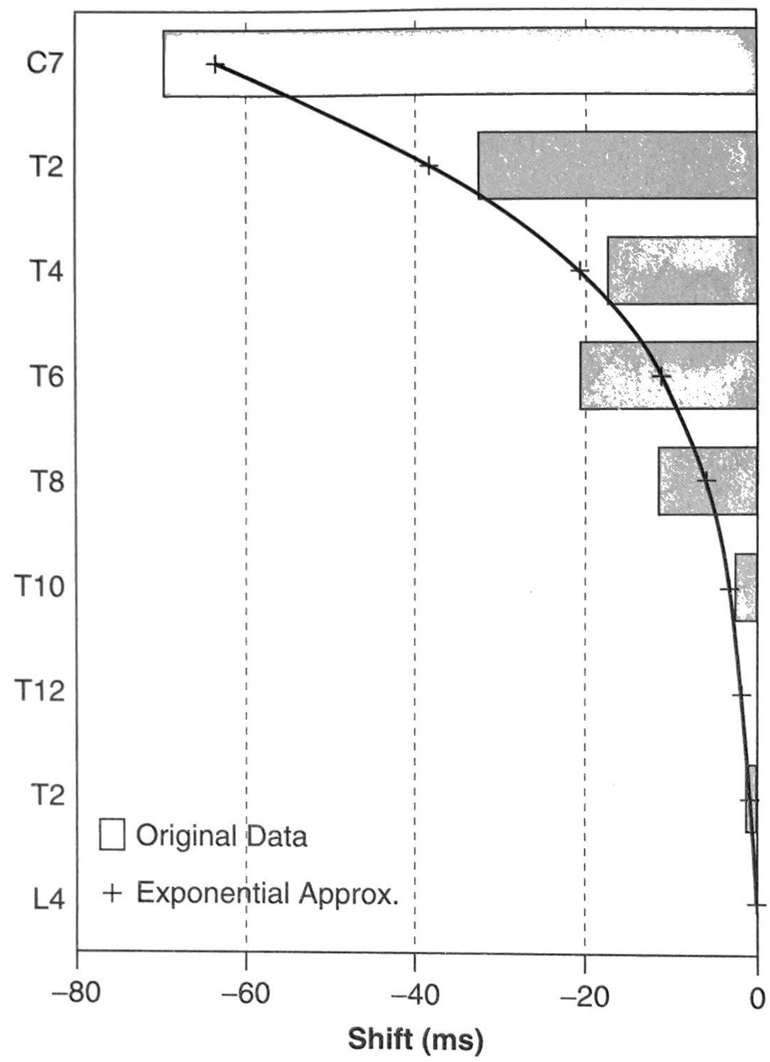

Figure 2.8 Phase shift (ms) of the activation profiles of the paraspinal muscles relative to the profile at the L_4 level. The negative shift indicates the activation was in advance of L_4. The curve fit was exponential. (Reproduced by permission from *Gait and Posture*)

et al. (2008) reported a study of left and right gluteus medius patterns during a long duration standing manual task. Because these patterns are an excellent example of motor synergies and are also related to another medial/lateral postural strategy their details are presented in Chapter 11.

2.2 FREQUENCY ANALYSIS

2.2.1 Introduction—Time Domain vs. Frequency Domain

All the signals that we measure and analyze have a characteristic frequency content, which we refer to as the signal spectrum; this is a plot of all the harmonics in the signal from the lowest to the highest. The purpose of this section is to provide a conceptual background with sufficient mathematical derivations to help the student collect and process data and be an intelligent collector and consumer of commercial software. Frequency domain analysis uses a powerful transform called the Fourier transform, named after Baron

Jean-Baptiste-Joseph Fourier, a French mathematician who developed the technique in 1807.

The knowledge of the frequency spectrum of any given signal is mandatory in making decisions about collection and processing of any given signal. The spectrum decides the sampling rate you must chose before an analog-to-digital conversion is done, and it also decides the length of record that must be converted. Also, the spectrum influences the frequency of filtering of the data to remove undesirable noise and movement artifacts. All these factors will be discussed in the sections to come.

2.2.2 Discrete Fourier (Harmonic) Analysis

1. *Alternating Signals*. An alternating signal (often called ac, for alternating current) is one that continuously changes over time. It may be periodic or completely random, or a combination of both. Also, any signal may have a dc (direct current) component, which may be defined as the bias value about which the ac component fluctuates. Figure 2.9 shows example signals.
2. *Frequency Content*. Any of these signals can also be discussed in terms of their frequency content. A sine (or cosine) waveform is a single frequency; any other waveform can be the sum of a number of sine and cosine waves.

Note that the Fourier transformation (see Figure 2.10) of periodic signals has discrete frequencies, while nonperiodic signals have a continuous spectrum defined by its lowest frequency, f_1, and its highest frequency, f_2. To analyze a periodic signal, we must express the frequency content in multiples

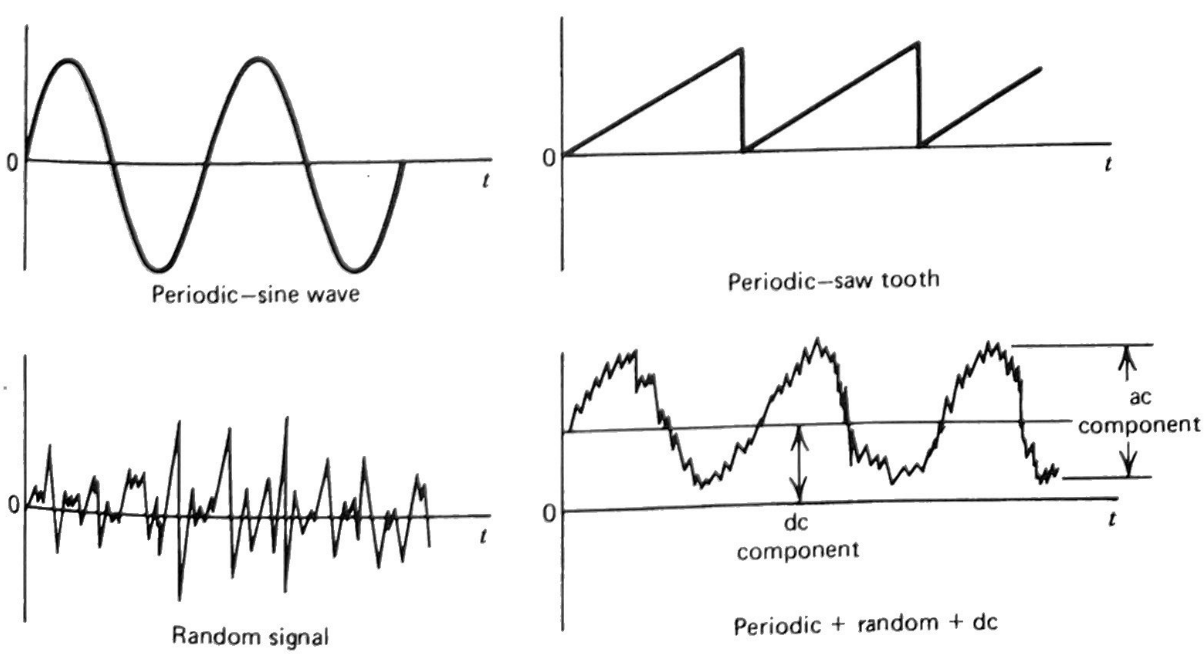

Figure 2.9 Time-related waveforms demonstrate the different types of signals that may be processed.

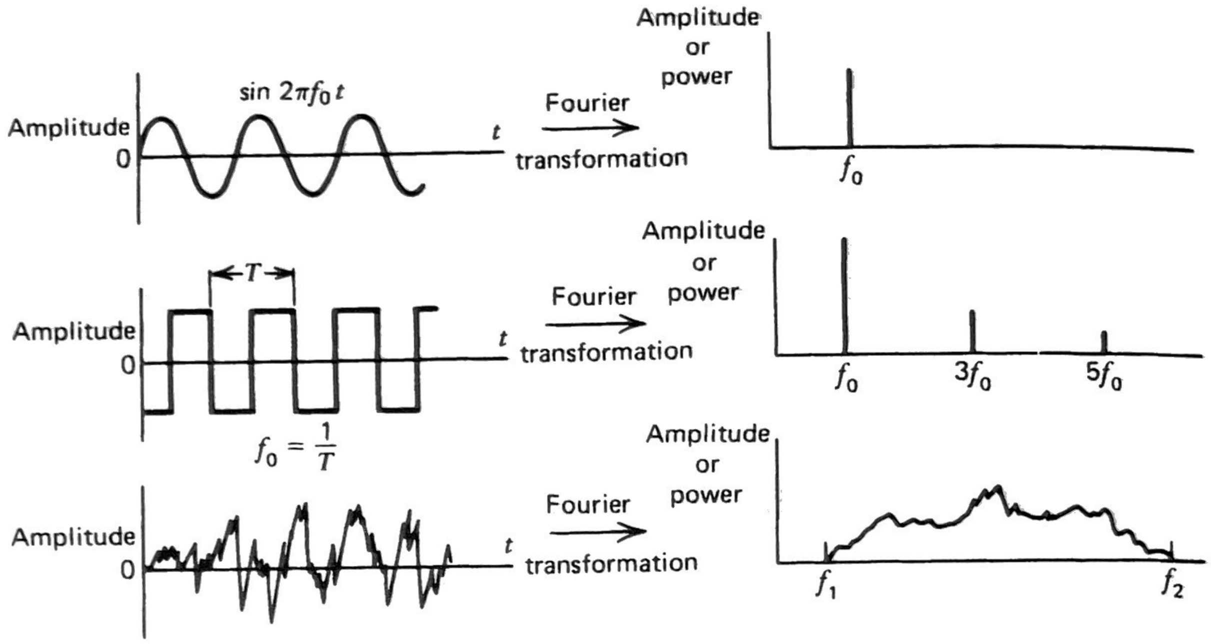

Figure 2.10 Relationship between a signal as seen in the time domain and its equivalent in the frequency domain.

of the fundamental frequency f_0. These higher frequencies are called *harmonics*. The third harmonic is $3f_0$, and the tenth harmonic is $10f_0$. Any perfectly periodic signal can be broken down into its harmonic components. The sum of the proper amplitudes of these harmonics is called a *Fourier series*.

Thus, a given signal $V(t)$ can be expressed as:

$$V(t) = V_{dc} + V_1 \sin(\omega_0 t + \theta_1) + V_2 \sin(2\omega_0 t + \theta_2) + \cdots + V_n \sin(n\omega_0 t + \theta_n) \tag{2.10}$$

where $\omega_0 = 2\pi f_0$, and θ_n is the phase angle of the nth harmonic.

For example, a square wave of amplitude V can be described by the Fourier series of odd harmonics:

$$V(t) = \frac{4V}{\pi}\left(\sin\omega_0 t + \frac{1}{3}\sin 3\,\omega_0 t + \frac{1}{5}\sin 5\,\omega_0 t + \cdots\right) \tag{2.11}$$

A triangular wave of duration $2t$ and repeating itself every T seconds is:

$$V(t) = \frac{2Vt}{T}\left[\frac{1}{2} + \left(\frac{2}{\pi}\right)^2 \cos\omega_0 t + \left(\frac{2}{3\pi}\right)^2 \cos 3\,\omega_0 t + \cdots\right] \tag{2.12}$$

Several names are given to the graph showing these frequency components: *spectral plots, harmonic plots*, and *spectral density functions*. Each shows the amplitude or power of each frequency component plotted against frequency; the mathematical process to accomplish this is called a *Fourier transformation* or *harmonic analysis*. Figure 2.10 shows plots of time-domain signals and their equivalents in the frequency domain.

Care must be used when analyzing or interpreting the results of any harmonic analysis. Such analyses assume that each harmonic component is present with a constant amplitude and phase over the total analysis period. Such consistency is evident in Equation (2.10), where amplitude V_n and phase θ_n are assumed constant. However, in real life each harmonic is not constant in either amplitude or phase. A look at the calculation of the Fourier coefficients is needed for any signal $x(t)$. Over the period of time T, using the *discrete Fourier transform*, we calculate n harmonic coefficients.

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\; dt \tag{2.13}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\; dt \tag{2.14}$$

$$c_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) \tag{2.15}$$

It should be noted that a_n and b_n are calculated *average* values over the period of time T. Thus, the amplitude c_n and the phase θ_n of the nth harmonic are average values as well. A certain harmonic may be present only for part of the time T, but the computer analysis will return an average value, assuming that it is present over the entire time. The fact that a_n and b_n are average values is important when we attempt to reconstitute the original signal as is demonstrated in Section 2.2.4.5.

The digital equivalent of the Fourier transform is important to review because it gives us some insight into the number of calculations that are necessary. In digital form, Equations (2.13) and (2.14) for N samples during the period T:

$$a_n = \frac{2}{N}\sum_{i=0}^{N} x_i \cos(n\omega_0 i/N) \tag{2.16}$$

$$b_n = \frac{2}{N}\sum_{i=0}^{N} x_i \sin(n\omega_0 i/N) \tag{2.17}$$

For each of the n harmonics, N calculations are necessary. The number of harmonics that can be analyzed is from the fundamental ($n = 1$) up to the Nyquist frequency, which is when there are two samples per sine or cosine wave or when $n = N/2$. Therefore, for $N/2$ harmonics, there are $N^2/2$ calculations necessary for each of the sine or cosine coefficients. The total number of calculations is N^2. It should be noted that the major expense in computer time is looking up the sine and cosine values for each of the N angles.

2.2.3 Fast Fourier Transform (FFT)

The Fast Fourier Transform (FFT) became necessary because of the extremely large number of calculations necessary in the Discrete Fourier Transform. As early as 1942, Danielson and Lanczos introduced the *Danielson-Lanczos Lemma*, which showed that the Discrete Fourier Transform of length N can be broken into two separate odd and even numbered components of length $N/2$ each. In a similar manner, each $N/2$ component can be broken into two more odd and even numbered components of length $N/4$ each, and each of these can be broken into two more odd and even components of length $N/8$ each. Thus, the basis of the FFT is a data record that must be binary in length. Therefore, if you collect data files that are not binary in length, the FFT can only accept the largest binary length file within your data file. For example, if you collected 1000 data points, the largest binary file length would be 512 points; thus, 488 points would be wasted. Therefore, it is advisable to prearrange data collection files to be binary in length; in the case of the previous example, a data file of 1024 points would be appropriate. With the advent of computers, many FFT algorithms appeared (Bringham, 1974), and in the mid-1960s J. W. Cooley and J. W. Tukey at IBM developed what is probably the best-known FFT algorithm.

One of the major savings in the FFT is to avoid repetitive and time-consuming calculations especially sines and cosines. If we look at Equations (2.16) and (2.17), we see that for the fundamental frequency ($n = 1$), we must calculate N sine and N cosine values. For the second harmonic, we recalculate every second sine and cosine value, and for the third harmonic we recalculate every third sine and cosine value, and so on up to the highest harmonic. What the FFT does is calculate all sine and cosine values for the fundamental and this forms a "look-up" table for the fundamental plus all higher harmonics. Further savings are achieved by clustering all the products of x_i and the same sine value, then summing all the x_i values, and then carrying out one product with the sine value. The number of calculations for the FFT $= N \log_2 N$, which is considerably less than N^2 for the Discrete Fourier Transform. For example, for $N = 4096$ and a CPU cycle time of $0.1\ \mu s$ the DFT would take $4096^2 10^{-7} = 1.67\,\text{s}$, while the FFT would take $4096 \log_2 4096 \cdot 10^{-7} = 49\,\text{ms}$.

2.2.4 Applications of Spectrum Analyses

2.2.4.1 Analog-to-Digital Converters. To students not familiar with electronics, the process that takes place during conversion of a physiological signal into a digital computer can be somewhat mystifying. A short schematic description of that process is now given. An electrical signal representing a

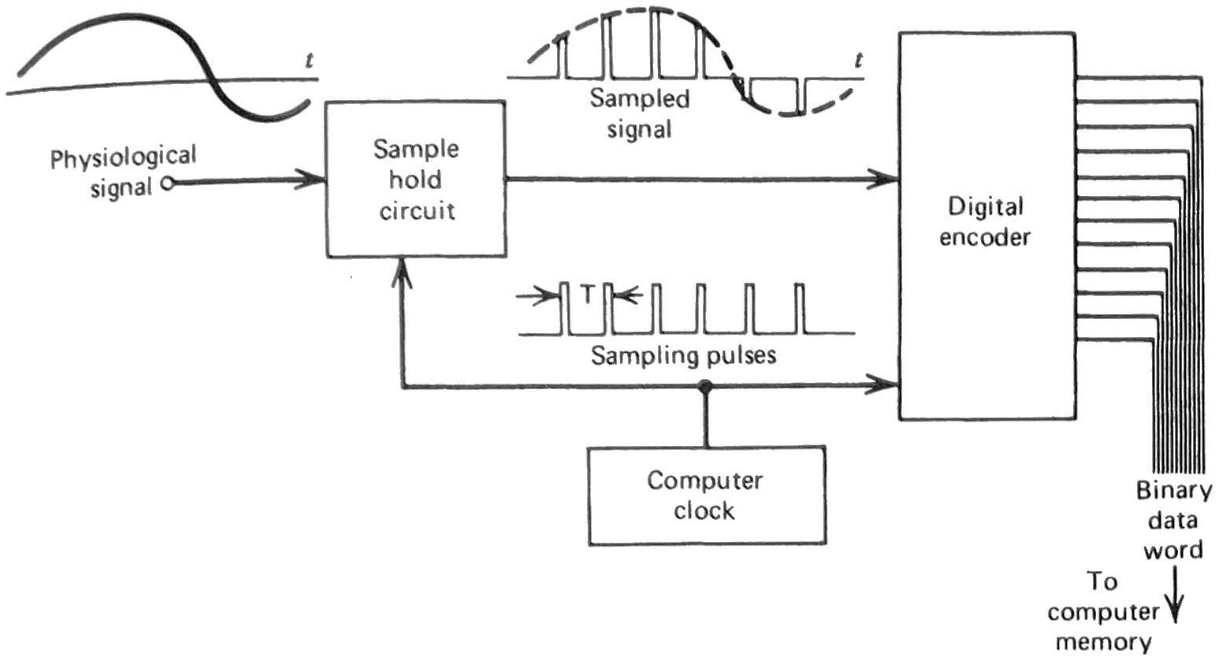

Figure 2.11 Schematic diagram showing the steps involved in an analog-to-digital conversion of a physiological signal.

force, an acceleration, an electromyographic (EMG) potential, or the like is fed into the input terminals of the analog-to-digital converter. The computer controls the rate at which the signal is sampled; the optimal rate is governed by the sampling theorem (see Section 2.2.4.2).

Figure 2.11 depicts the various stages in the conversion process. The first is a sample/hold circuit in which the analog input signal is changed into a series of short-duration pulses, each one equal in amplitude to the original analog signal at the time of sampling. The final stage of conversion is to translate the amplitude and polarity of the sampled pulse into digital format. This is usually a binary code in which the signal is represented by a number of bits. For example, a 12-bit code represents $2^{12} = 4096$ levels. This means that the original sampled analog signal can be broken into 4096 discrete amplitude levels with a unique code representing each of these levels. Each coded sample (consisting of 0s and 1s) forms a 12-bit "word," which is rapidly stored in computer memory for recall at a later time. If a 5-s signal were converted at a sampling rate of 100 times per second, there would be 500 data words stored in memory to represent the original 5-s signal.

2.2.4.2 *Deciding the Sampling Rate—The Sampling Theorem.* In the processing of any time-varying data, no matter what their source, the sampling theorem must not be violated. Without going into the mathematics of the sampling process, the theorem states that "the process signal must be sampled at a frequency at least twice as high as the highest frequency present in the signal itself." If we sample a signal at too low a frequency, we get aliasing errors. This results in false frequencies, frequencies that were not present in the original signal, being generated in the sample data. Figure 2.12 illustrates

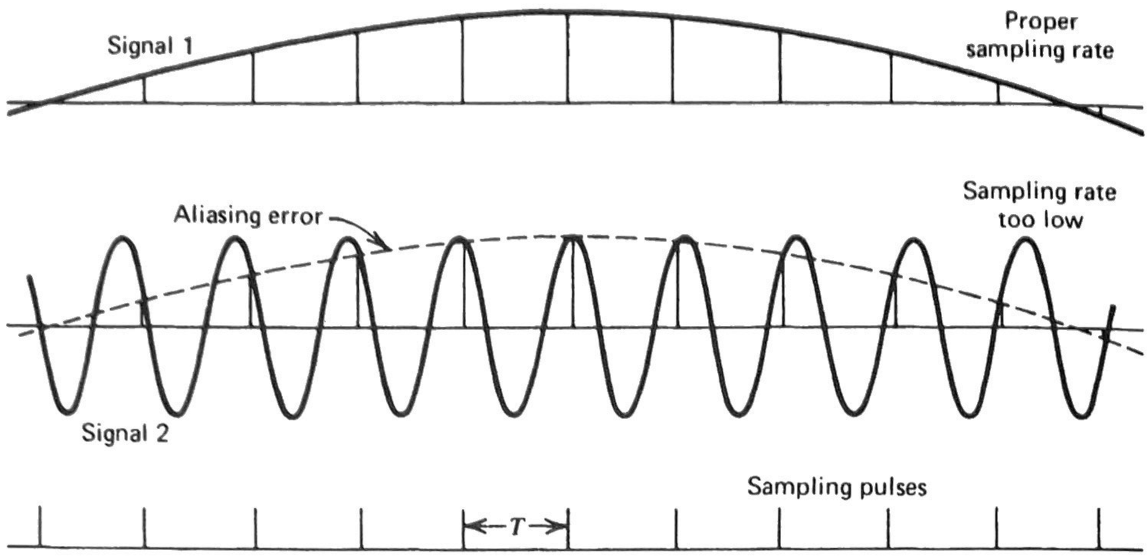

Figure 2.12 Sampling of two signals, one at a proper rate, the other at too low a rate. Signal 2 is sampled at a rate less than twice its frequency, such that its sampled amplitudes are the same as for signal 1. This represents a violation of the sampling theorem and results in an error called *aliasing*.

this effect. Both signals are being sampled at the same interval T. Signal 1 is being sampled about 10 times per cycle, while signal 2 is being sampled less than twice per cycle. Note that the amplitudes of the samples taken from signal 2 are identical to those sampled from signal 1. A false set of sampled data has been generated from signal 2 because the sample rate is too low—the sampling theorem has been violated.

The tendency of those using film is to play it safe and film at too high a rate. Usually, there is a cost associated with such a decision. The initial cost is probably in the equipment required. A high-speed movie camera can cost four or five times as much as a standard model (24 frames per second). Or a special optoelectric system complete with the necessary computer can be a $70,000 decision. In addition to these capital costs, there are the higher operational costs of converting the data and running the necessary kinematic and kinetic computer programs. Except for high-speed running and athletic movements, it is quite adequate to use a standard movie or television camera. For normal and pathological gait studies, it has been shown that kinetic and energy analyses can be done with negligible error using a standard 24–frame per second movie camera (Winter, 1982). Figure 2.13 compares the results of kinematic analysis of the foot during normal walking, where a 50-Hz film rate was compared with 25 Hz. The data were collected at 50 Hz, and the acceleration of the foot was calculated using every frame of data, then reanalyzed again, using every second frame of converted data. It can be seen that the difference between the curves is minimal; only at the peak negative acceleration was there a noticeable difference. The final decision as to whether this error is acceptable should not rest in this curve, but in your final goal. If, for example, the final analysis was a hip and knee torque analysis, the acceleration of the foot segment may not be too important, as is evident from

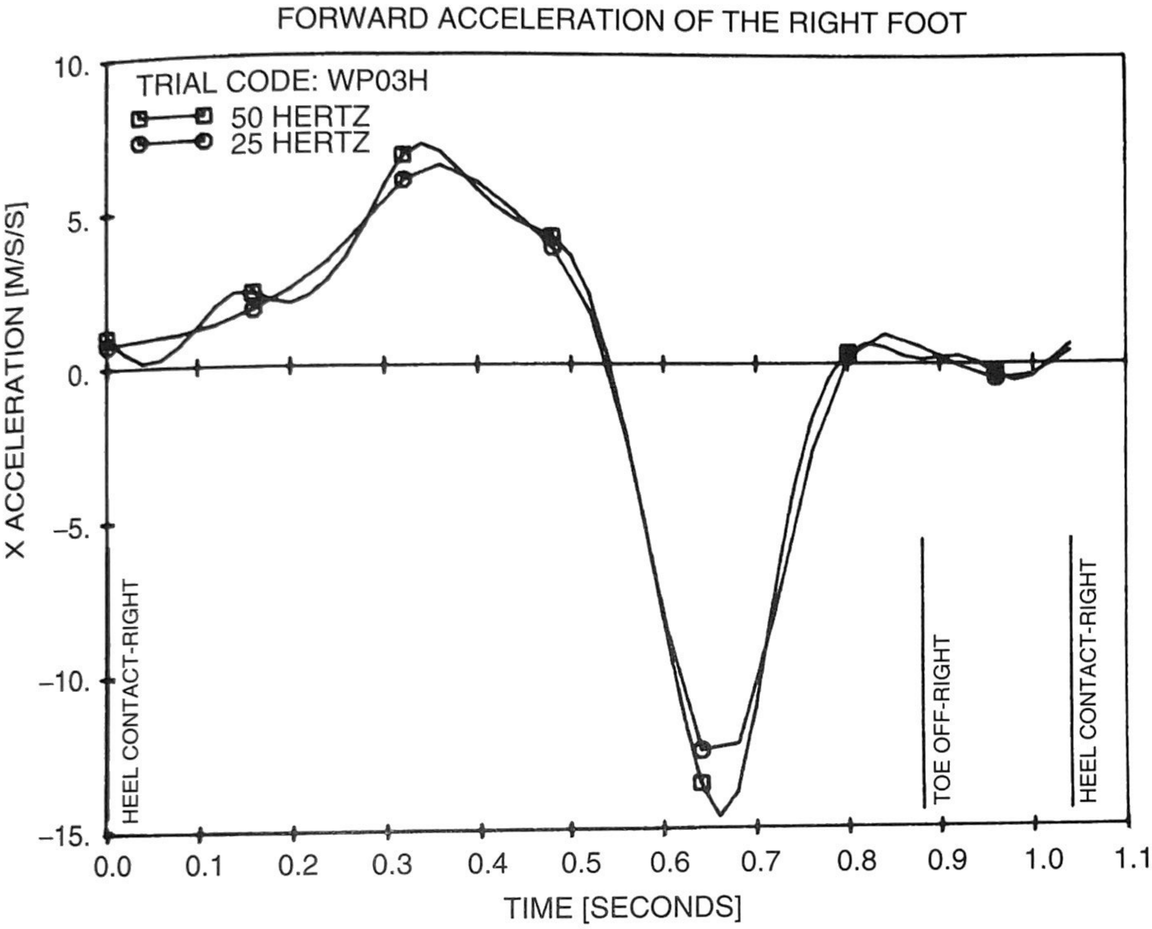

Figure 2.13 Comparison of the forward acceleration of the right foot during walking using the same data sampled at 50 Hz and at 25 Hz (using data from every second frame). The major pattern is maintained with minor errors at the peaks.

another walking trial, shown in Figure 2.14. The minor differences in no way interfere with the general pattern of joint torques over the stride period, and the assessment of the motor patterns would be identical. Thus, for movements such as walking or for slow movements, an inexpensive camera at 24 frames per second appears to be quite adequate.

2.2.4.3 ***Deciding the Record Length.*** The duration of record length is decided by the lowest frequency present in the signal. In cyclical events such as walking, cycling, or swimming the lowest frequency is easy to determine; it is the stride frequency or how often each segment of the body repeats itself. For example, if a patient is walking at 105 steps/min the step frequency is $105/60 = 1.75$ steps/s $= 0.875$ strides/s. Thus, the fundamental frequency is 0.875 Hz. However, there are a number of noncyclical movements, which do not have a defined lowest frequency. One such "movement" is standing either quietly or in a work-related task. In quiet standing, we model the total body as an inverted pendulum (Gage et al., 2004), which simplifies the total body into a single weighted-average center of mass (COM) and which can be compared with the center of pressure (COP) measured from the force plate. Figure 2.15 presents a typical FFT of the COP and COM in the anterior/posterior direction

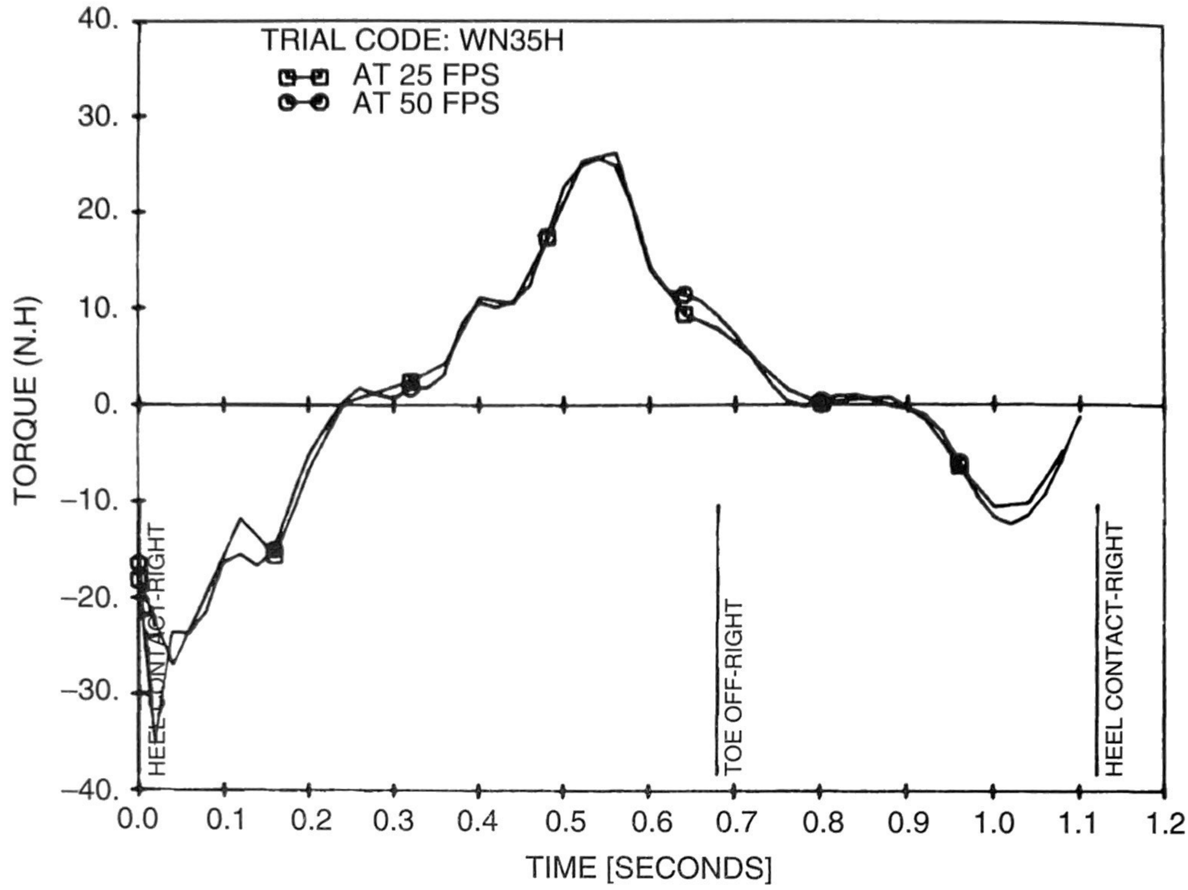

Figure 2.14 Comparison of the hip moment of force during level walking using the same data sampled at 50 Hz and at 25 Hz. The residual error is quite small because the joint reaction forces dominate the inertial contributions to the net moment of force.

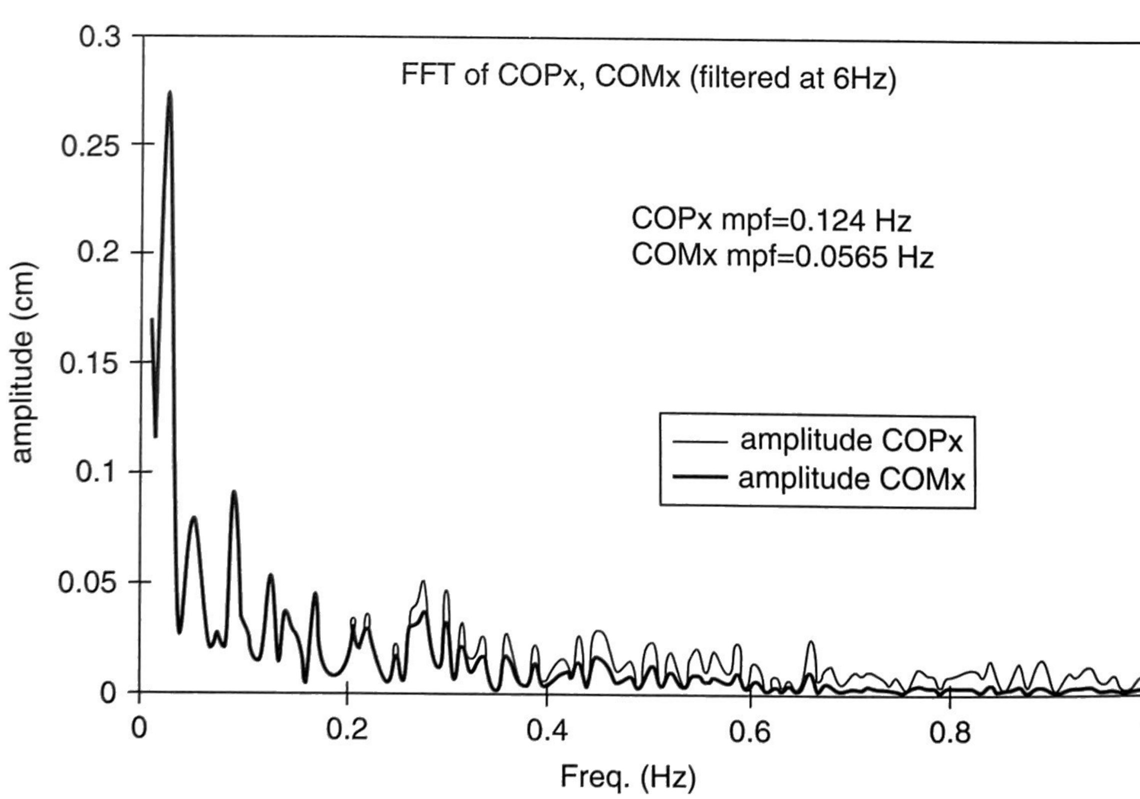

Figure 2.15 FFT of the COM and COP in the anterior/posterior direction of a subject standing quietly for 137 seconds. (Winter, D. A., A.B.C. *(Anatomy, Biomechanics, and Control) of Balance During Standing and Walking*. Waterloo Biomechanics. 1995)

for a subject standing quietly for 137 seconds (8192 samples @ 60 Hz). Note that the FFT plots the amplitude of each harmonic from 0.0073 Hz to 1 Hz. Also note the dominant low frequency components of both COM and COP below 0.2 Hz. The length of record must be at least a minute or longer. This long record may be compromised when studying patients with balance disorders because they may not be able to stand quietly for that length of time. However, for studies on normal subjects Carpenter, et al. (2001) found that records of at least one minute were required for acceptable reliability.

2.2.4.4 Analog and Digital Filtering of Signals—Noise and Movement Artifacts. The basic approach can be described by analyzing the frequency spectrum of both signal and noise. Figure 2.16*a* shows a schematic plot of a signal and noise spectrum. As can be seen, the signal is assumed to occupy the lower end of the frequency spectrum and overlaps with the noise, which is usually higher frequency. Filtering of any signal is aimed at the selective rejection, or attenuation, of certain frequencies. In the preceding case, the obvious filter is one that passes, unattenuated, the lower-frequency signals, while at the same time attenuating the higher-frequency noise. Such a filter, called a *low-pass filter*, has a frequency response as shown in Figure 2.16*b*. The frequency response of the filter is the ratio of the output $X_o(f)$ of the filter to its input $X_i(f)$ at each frequency present. As can be seen, the response at lower frequencies is 1.0. This means that the input signal passes through the filter unattenuated. However, there is a sharp transition at the cutoff frequency f_c so that the signals above f_c are severely attenuated. The net result of the filtering process can be seen by plotting the spectrum of the output signal $X_o(f)$ as seen in Figure 2.16*c*. Two things should be noted. First, the higher-frequency noise has been severely reduced but not completely rejected. Second, the signal, especially in the region where the signal and noise overlap (usually around f_c) is also slightly attenuated. This results in a slight distortion of the signal. Thus, a compromise has to be made in the selection of the cutoff frequency. If f_c is set too high, less signal distortion occurs, but too much noise is allowed to pass. Conversely, if f_c is too low, the noise is reduced drastically, but at the expense of increased signal distortion. A sharper cutoff filter will improve matters, but at an additional expense. In digital filtering, this means a more complex digital filter and, thus, more computer time.

The first aspect that must be assessed is what the signal spectrum is as opposed to the noise spectrum. This can readily be done, as is seen in the harmonic analysis for 20 subjects presented in Figure 2.17. Here is the harmonic content of the vertical displacement of the toe marker in natural walking (Winter et al., 1974). The highest harmonics were found to be in the toe and heel trajectories, and it was found that 99.7% of the signal power was contained in the lower seven harmonics (below 6 Hz). Above the seventh harmonic, there was still some signal power, but it had the characteristics of "noise." Noise is the term used to describe components of the final signal

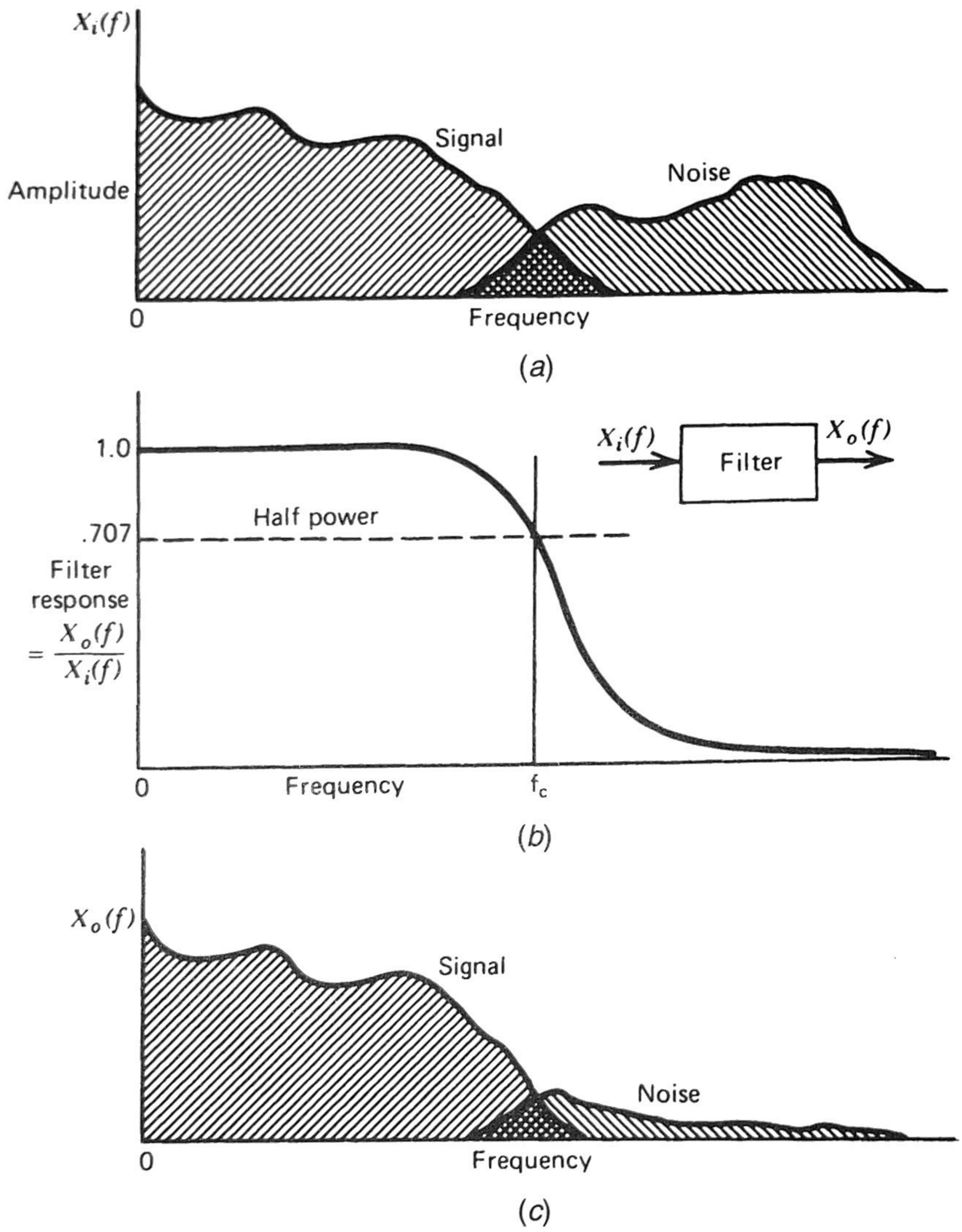

Figure 2.16 (*a*) Hypothetical frequency spectrum of a waveform consisting of a desired signal and unwanted higher-frequency noise. (*b*) Response of low-pass filter $X_o(f)/X_i(f)$, introduced to attenuate the noise. (*c*) Spectrum of the output waveform, obtained by multiplying the amplitude of the input by the filter response at each frequency. Higher-frequency noise is severely attenuated, while the signal is passed with only minor distortion in transition region around f_c.

that are not the result of the process itself (in this case, walking). Noise comes from many sources: electronic noise in optoelectric devices, spatial precision of the TV scan or film digitizing system, and human error in film digitizing. If the total effect of all these errors is random, then the true signal will have an added random component. Usually, the random component is high frequency, as is borne out in Figure 2.17. Here, we see evidence of higher-frequency components extending up to the 20th harmonic, which was the highest frequency analyzed. The presence of the higher-frequency noise is of considerable importance when we consider the problem of trying to

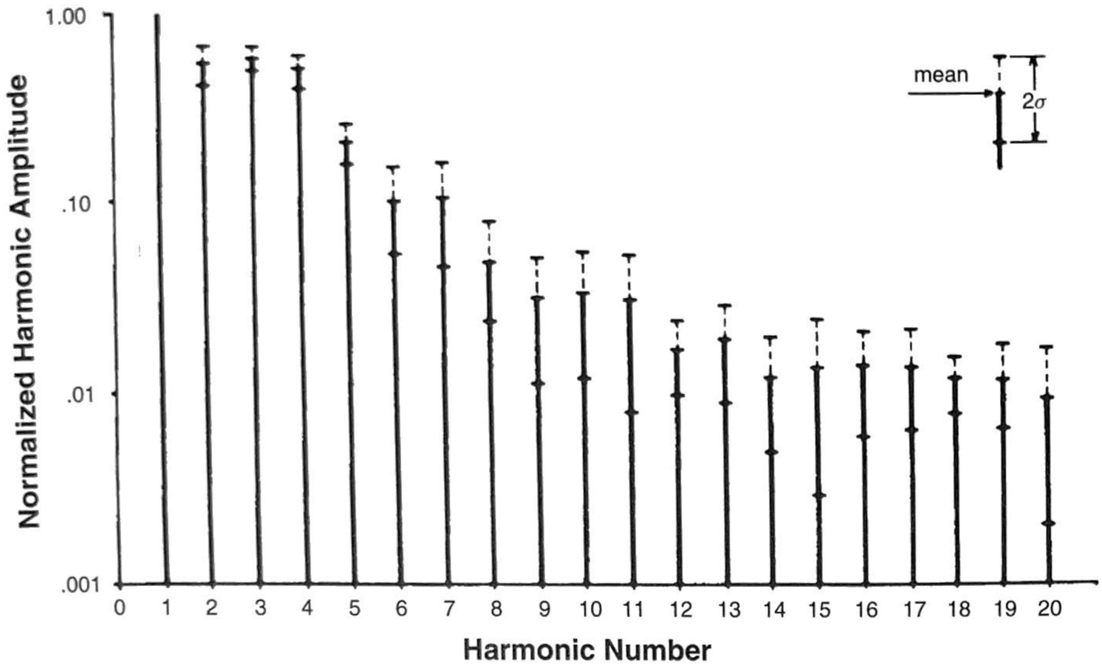

Figure 2.17 Harmonic content of the vertical displacement of a toe marker from 20 subjects during normal walking. Fundamental frequency (harmonic number = 1) is normalized at 1.00. Over 99% of power is contained below the seventh harmonic. (Reproduced by permission from the *Journal of Biomechanics*.)

calculate velocities and accelerations from the displacement data, as will be evident later in Section 3.4.3.

The theory behind digital filtering (Radar and Gold, 1967) will not be covered, but the application of low-pass digital filtering will be described in detail. As a result of the previous discussion for these data on walking, the cutoff frequency of a digital filter should be set at about 6 Hz. The format of a recursive digital filter that processes the raw data in time domain is as follows:

$$\begin{aligned} X^1(nT) &= a_0X(nT) + a_1X(nT - T) + a_2X(nT - 2T) \\ &\quad + b_1X^1(nT - T) + b_2X^1(nT - 2T) \end{aligned} \tag{2.18}$$

where X^1 = filtered output coordinates
X = unfiltered coordinate data
nT = nth sample
$(nT - T)$ = $(n-1)$th sample
$(nT - 2T)$ = $(n-2)$th sample
$a_0, \ldots, b_0, \ldots$ = filter coefficients

These filter coefficients a_0, a_1, a_2, b_1 and b_2 are constants that depend on the type and order of the filter, the sampling frequency, and the cutoff frequency. As can be seen, the filter output $X^1(nT)$ is a weighted version of the immediate and past raw data plus a weighted contribution of past filtered output. The exact equations to calculate the coefficients for a Butterworth or a critically damped filter are as follows:

$$\omega_c = \frac{(\tan(\pi f_c/f_s))}{C} \tag{2.19}$$

where C is the correction factor for number of passes required, to be explained shortly. For a single pass filter $C = 1$.

$K = \sqrt{2}\omega_c$ for a Butterworth filter,
or, 2 ω_c for a critically damped filter

$$K_2 = \omega_c^2, \quad a_0 = \frac{K_2}{(1 + K_1 + K_2)}, \quad a_1 = 2a_0, \quad a_2 = a_0$$

$$K_3 = \frac{2a_0}{K_2}, \quad b_1 = -2a_0 + K_3$$

$$b_2 = 1 - 2a_0 - K_3, \qquad \text{or} \qquad b_2 = 1 - a_0 - a_1 - a_2 - b_1$$

For example, a Butterworth-type low-pass filter of second order is to be designed to cutoff at 6 Hz using film data taken at 60 Hz (60 frames per second). As seen in Equation (2.19) the only thing that is required to determine these coefficients is the ratio of sampling frequency to cutoff frequency. In this case it is 10. The design of such a filter would yield the following coefficients:

$$a_0 = 0.067455, \quad a_1 = 0.13491, \quad a_2 = 0.067455,$$
$$b_1 = 1.14298, \quad b_2 = -0.41280$$

Note that the algebraic sum of all the coefficients equals 1.0000. This gives a response of unity over the passband. Note that the same filter coefficients could be used in many different applications, as long as the ratio f_s/f_c is the same. For example, an EMG signal sampled at 2000 Hz with cutoff desired at 400 Hz would have the same coefficients as one employed for movie film coordinates where the film rate was 30 Hz and cutoff was 6 Hz. The number of passes, C, in Equation (2.17) are important when filtering kinematic data in order to eliminate the phase shift of the filtered data. This aspect of digital filtering of kinematic data will be detailed later in Section 3.4.4.2.

2.2.4.5 Fourier Reconstitution of Original Signal. Figure 2.18 is presented to illustrate a Fourier reconstitution of the vertical trajectory of the heel of an adult walking his or her natural cadence. A total of nine harmonics is represented here because the addition of higher harmonics did not improve the curve of the original data. As can be seen, the harmonic reconstitution is visibly different from the original, sufficiently so as to cause reasonable