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EVOLUTION OF CLOSE NEUTRON STAR BINARIES

JOHN PAUL ADRIAN CLARK* AND DOUGLAS M. EARDLEY†
Observatory and Department of Physics, Yale University
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ABSTRACT

In binary systems consisting of two neutron stars, the orbit decays by gravitational radiation. A crude model shows that the less massive star may suffer either immediate tidal disruption or slow mass stripping when it reaches its Roche radius, depending on the initial masses and on the details of mass exchange or mass loss. Typical energy releases are 4×10^{52} ergs in gravitational waves before the onset of stripping, 2×10^{52} ergs in gravitational waves after the onset of stripping, 2×10^{53} ergs in neutrinos after the onset of stripping. The stripping process always ends in tidal disruption of the less massive star after a few seconds or a few hundred revolutions.

As the endpoint of binary stellar evolution, such events are estimated to occur only every ~ 100 yr out to a radius of 15 Mpc, and are thus less important than supernovae as sources of gravitational waves; the observed wave amplitude would be $h \sim 10^{-21}$. Such events may occur in Type II supernovae, if the collapsing stellar core rotates rapidly enough to fission into two neutron stars.

Subject headings: gravitation — stars: binaries — stars: evolution — stars: neutron

I. INTRODUCTION

According to Einstein's theory, a binary star system radiates gravitational waves, which carry away both energy and angular momentum. If the orbital period exceeds about half a day, the time scale for decay of the orbit is greater than the age of the universe. For a shorter period, gravitational radiation will eventually drive the members toward their tidal interaction radii. In this paper we will concern ourselves with the final stages of this evolutionary process when both members are neutron stars; the period will be a few milliseconds! Dyson (1963) estimated that the final coalescence of the two neutron stars would result in a burst of gravitational waves with energy 3×10^{52} ergs and duration 2 s, at a frequency of ~ 200 Hz. We shall find in addition a burst of neutrinos with a greater energy, typically $\sim 2 \times 10^{53}$ ergs. Also, as much as a few tenths of a solar mass of neutronized matter may be ejected to infinity. Our work is in many respects parallel to that of Lattimer and Schramm (1974, 1976), who studied binaries consisting of a neutron star and a black hole.

Although we shall refer to the system as evolving from an ordinary massive binary system, the main application of our results may well be to single Type II supernovae, in which a rapidly rotating core might fission or fragment to form an extremely close neutron star binary as it collapses. For this application, our paper is one among many recent works which attempt to estimate gravitational and neutrino radiation in stellar collapse; e.g., Rees, Ruffini, and Wheeler (1974), Thuan and Ostriker (1974), Wiita and Press (1976), Kazanas and Schramm (1976), Shapiro (1976), Ostriker, Smarr, and Thuan (1976), Novikov (1975). Also, such binaries may form in dense star clusters (Zel'dovich and Podurets 1965). Since close binaries tend to absorb a major fraction of the binding energy of the cluster (Aarseth 1974), decay of the neutron star binary may control cluster evolution.

Once the orbital radius decreases to the point where the lighter member is at its Roche radius (or, more properly, its tidal radius), one of two things can happen. The system may remain stable by means of mass transfer to the heavier member or to infinity; we will call this process "mass stripping." On the other hand, the lighter star may be inevitably pulled within its Roche radius despite mass transfer; then it will suffer "immediate tidal disruption."

This paper treats evolution through gravitational radiation and mass transfer using simple and sometimes radical approximations, with the neglect of all but dominant effects. Section II is concerned with outlining details of our model, and in justifying many of the simplifying assumptions. Section III considers the behavior of the system, as a function of various physical parameters. Section IV details the response of gravitational-wave and neutrino detectors to signals from these phenomena, and the likelihood of their occurrence. Section V treats the general question of stability of two-body circular orbits, justifying our assumption of stability.

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II. INTERACTION MODEL

We summarize the approximations, physical assumptions, and formulae that go into our numerical model.

a) *Binaries as Gravitational Radiation Sources*

For general discussion see Misner, Thorne, and Wheeler (1973), Press and Thorne (1972), Zel'dovich and Novikov (1971, § 1.13). Shortly after the system has formed, following the second supernova, the orbit may well be highly eccentric. However, by the time the orbit approaches the tidal interaction separation, reaction forces, which are greatest at periastron, will have circularized the orbit. Therefore we assume a circular orbit.

The total power radiated as gravitational waves (point mass approximation; weak field limit) is

$$L = 32G^4 m_1^2 m_2^2 (m_1 + m_2)/(5c^5 a^5) = (1.63 \times 10^{51}\ \mathrm{ergs\ s^{-1}}) m_1^2 m_2^2 (m_1 + m_2)(a/100\ \mathrm{km})^{-5}; \quad (1)$$

here and throughout, m_1 and m_2 are the masses of the members ($m_1 < m_2$) in units of 1 $M_\odot$, and a is the separation between centers.

Clark (1977) has shown that the correction to equation (1), in allowing for the finite size of the more extended component, is at most 3%. This correction is not incorporated because we are neglecting other larger effects.

The polarization of the waves is a function of the orbital inclination of the binary. The wave frequency in the dominant quadrupole mode is twice the orbital frequency.

Because of the energy and angular momentum carried away by the waves, the stars spiral inward. The time until collision, i.e., $a = 0$ (denoted by τ throughout this paper), as the components spiral inward from the separation a, is

$$\tau = 5c^5 a^4/[256G^3 m_1 m_2 (m_1 + m_2)] = (2.0\ \mathrm{s}) m_1^{-1} m_2^{-1} (m_1 + m_2)^{-1} (a/100\ \mathrm{km})^4 . \quad (2)$$

These formulae, and all of our results, will depend crucially on the correctness of Einstein's theory of general relativity, due to the dominant effects of dipole gravitational radiation in alternative theories of gravitation (Eardley 1975*a*; Will 1977; Will and Eardley 1977).

b) *Onset of Mass Transfer*

Although the tidal interaction processes occur in a region where the weak field approximation is dubious, equation (1) will still be used. Lattimer and Schramm (1976) show this to be allowable for $a \geqslant 6\ Gm_2/c^2$, a condition always fulfilled in our models.

Orbital mechanics is performed using Newtonian theory, with qualitative corrections invoked when predictions differ too strongly from those in Schwarzschild geometry. We argue in § V that the orbit is stable as long as, again, $a \geqslant 6Gm_2/c^2$. The main effect of the Schwarzschild corrections would therefore be to cause the less massive star to spiral inward slightly faster, thus making immediate tidal disruption slightly more likely than is found in § III.

The Roche separation was calculated following Chandrasekhar (1969), assuming m_2 to be a point mass and m_1 a homogeneous Roche ellipsoid. The evolution can be followed analytically until the separation decreases to the Roche separation for the lighter star, at which point mass transfer begins and we pick up the system in the numerical model.

c) *Neutron Star Models*

The equation of state was the "best" composite recommended for cold catalyzed matter in the recent review by Canuto (1975). The resultant neutron stars have a maximum mass of 1.75 $M_\odot$, above which collapse sets in; the minimum mass is 0.093 $M_\odot$ below which the star is unstable to free expansion to infinity. The mass-radius relation is shown in Figure 1. The general features of tidal interaction given in § III are not expected to change

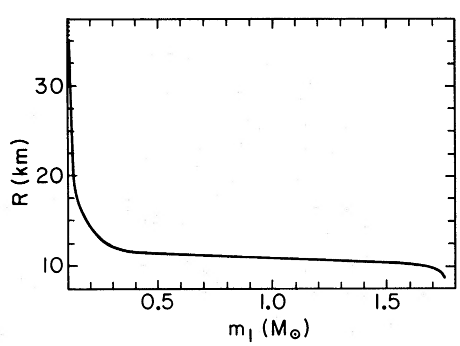

FIG. 1.—Mass-radius relationship for neutron stars from Canuto's (1975) recommended equation of state

much for any of the other hard equations of state. We have assumed zero temperature because energy generation within the stars will be small; this assumption is dubious for stellar collapse.

We will assume that the stellar radius corresponding to a given mass is the equilibrium value, which we justify as follows. The orbital angular velocity for a star of mean density ρ at its Roche radius is (Chandrasekhar 1969)

$$\Omega_{\mathrm{orb}} \approx (0.28 + 0.16\, m_1/m_2)^{1/2}(G\rho)^{1/2} \approx (0.5 \text{ to } 0.7)(G\rho)^{1/2}$$

for $m_1 < m_2$. The angular frequency of radial oscillations in the fundamental mode for a neutron star is (Cameron 1970)

$$\Omega_{\mathrm{osc}} \approx (0.1 \text{ to } 0.7)(\mathrm{G}\rho)^{1/2} .$$

Therefore

$$\Omega_{\mathrm{osc}}/\Omega_{\mathrm{orb}} \approx (0.2 \text{ to } 1.3)$$

which shows that the dynamical time scale $\sim 1/\Omega_{\mathrm{osc}}$ for the star to re-equilibrate its radius during stripping is no greater than the orbital period $2\pi/\Omega_{\mathrm{orb}}$. We shall find that mass stripping occurs on a time scale of $\sim 10^2$ orbital periods; therefore, the assumption of radial equilibrium is reasonably well justified.

d) *Evolution of the System*

i) *Mass Transfer*

Once the system reaches its Roche separation, the stability to immediate tidal disruption of the lighter star is tested, by carrying out a small transfer of mass from the lighter to the heavier neutron star. The system will be stable provided the separation between the stars increases more rapidly than the Roche separation. On the other hand, should the Roche radius increase more rapidly than the separation, immediate tidal disruption occurs.

Assuming that orbital angular momentum and total system mass are conserved during mass transfer (but see below for alternative assumptions that were also tested), a mass transfer dm causes a change of radius da,

$$da/dm = 2a(m_2 - m_1)/(m_1 m_2) . \tag{3}$$

Under the same assumptions, for the Roche limit radius R,

$$dR/dm = -\tfrac{1}{3}R(\rho^{-1}d\rho/dm + \lambda^{-1}\, d\lambda/dm) \tag{4}$$

where ρ is the mean density of the lighter neutron star, and

$$\lambda = 0.09 + 0.05\, m_1/m_2 ; \qquad d\lambda/dm = -(1 + m_1/m_2)/(20 m_2) . \tag{5}$$

The numerical model follows the evolution of the system only as long as mass stripping is stable; the stability criterion is

$$da/dm > dR/dm . \tag{6}$$

When this criterion fails, immediate tidal disruption ensues.

We have made the approximation here that the lighter star is homogeneous with density ρ. Lattimer and Schramm (1976) note that, because the outer regions of the star have lower densities, stripping will occur at somewhat larger separations than our crude model indicates, and will proceed slightly faster. We have not tried to incorporate this refinement in our model.

During mass transfer the trajectory of a test particle is followed from the Lagrange point L_1 to the more massive neutron star, to determine whether an accretion disk forms around the neutron star. Model calculations indicate that formation of an accretion disk is unlikely to occur before the lighter star is tidally disrupted. In the rare cases where one is formed, the total material diverted from the more massive neutron star is less than 0.1 $M_{\odot}$.

The flight time of matter transferred between the two stars is generally of the order of 10^{-2} seconds, and so only a small fraction $\sim 10^{-5}$ of the neutrons have time to decay. Thus, the Eddington limit no longer represents an upper limit to the accretion rate, as it does for normal matter. In any case the dominant cooling mechanism for the accreted matter will be neutrino-pair emission because virial temperatures are high, $10^1 \leqslant kT \leqslant 10^2$ MeV. We do not calculate neutrino cooling, but assume the accretion energy to be emitted efficiently in neutrinos and antineutrinos of energy $E \approx 10^1$ MeV. Neutrino radiation pressure may be important, but we do not expect it to limit the accretion rate.

We have also considered that some matter may be ejected to infinity through hydrodynamic effects or neutrino radiation pressure. The ratio of matter thrown to infinity, to the maximum amount which energetically could reach infinity is denoted by η_1. Fortunately, although it is not possible to calculate the loss of material to infinity, this parameter has only a small influence on the behavior of the system. Should neutronized matter be ejected to infinity, we have an interesting site for nucleosynthesis.

A reasonable upper limit on mass ejection to infinity is dictated by the necessity of the escaping material to have sufficient energy to climb out of the potential well of the more massive star. If the mass loss is denoted by dm_L, and the radii of the stars by r_1, and r_2, respectively, then for a mass transfer dm,

$$dm_L \approx \eta_1 dm(1 - m_1 r_2/m_2 r_1)\,, \tag{7}$$

where $0 \leq \eta_1 \leq 1$.

Denoting the angular momentum lost to the system by dJ, and the orbital angular frequency by Ω,

$$dJ \approx -\Omega m_1^2 a^2 (m_1 + m_2)^{-2} dm_L\,. \tag{8}$$

ii) *Angular Momentum*

By the Thorne-Zel'dovich law (see, for example, Zel'dovich and Novikov 1971, § 10.7) the angular momentum carried away by gravitational waves is

$$dJ = dE/\Omega\,, \tag{9}$$

where dE represents the change of the total energy of the system, as a result of the gravitational waves.

It has been assumed that prior to the onset of mass transfer, the two stars were corotating. This is to be expected since tidal coupling with the orbital angular momentum over many orbital periods will spin up the stars.

Once mass transfer has begun, we expect some fraction of the available angular momentum, denoted by η_2, to spin up the massive star. The resultant loss of angular momentum from the orbit is

$$dJ \approx -\eta_2 dm \Omega m_2^2 a^2 (m_1 + m_2)^{-2}\,. \tag{10}$$

III. RESULTS

a) *Spin-up of Massive Neutron Star*

Calculation of the amount of orbital angular momentum lost to spin-up the more massive neutron star is extremely difficult. In particular, it is a function of the star's initial spin rate, the trajectory followed by matter hitting it, and the rate of tidal coupling of angular momentum back to the orbit. The last quantity is the most crucial, and unfortunately we have no way of calculating it.

Instead we will find it convenient to define upper and lower limits on the rate of spin-up. The upper limit will occur when the tidal coupling back to the orbit is negligible and the star retains all the angular momentum imparted to it by the transferring matter. The lower limit on η_2 is slightly less than zero, because as the stars spiral apart the orbital angular frequency decreases, and the original spin angular momentum of the star can be coupled to the orbit. We will take the lower limit to be zero, a reasonable approximation.

Figure 2 shows the variation of η_2 for two different initial configurations, as they evolve.

b) *Mass Stripping—Immediate Tidal Disruption*

Since immediate tidal disruption implies the end of the evolution of the system, it is desirable to map out the "equipotentials" of quasi-stability as a function of the system's parameters. Figures 3 and 4 do this for all possible values of the masses, and representative values of η_1, and η_2. Mass stripping occurs for systems occupying the region between the arms of the V-shaped contours. Outside this, immediate tidal disruption occurs.

These diagrams make it clear that the stability of the system is far more sensitive to the parameter η_2 than to η_1, this being true because η_2 directly governs a large loss of angular momentum from the orbit. It is interesting to note that although increasing η_1 increases the loss of angular momentum from a system, it moves a system toward greater stability. This occurs because the system's tendency toward instability, due to angular momentum loss, is more than offset by the mass loss.

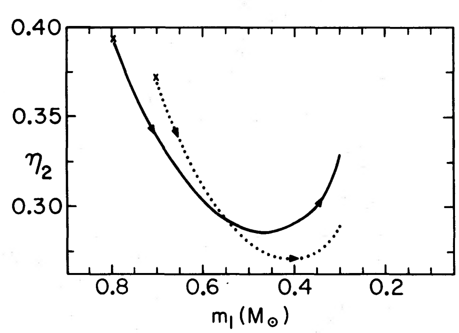

FIG. 2.—Upper limit of η_2. Crosses represent initial masses; arrows indicate direction of evolution of system. *Solid line*, initially 0.8 and 1.2 $M_\odot$. *Dashed line*, initially 0.7 and 0.9 $M_\odot$.

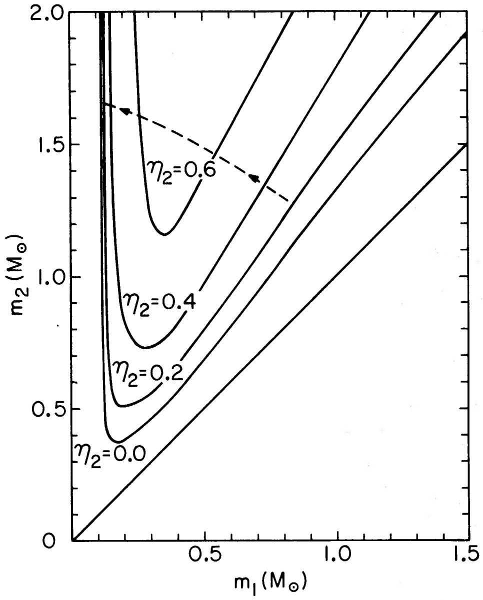

FIG. 3

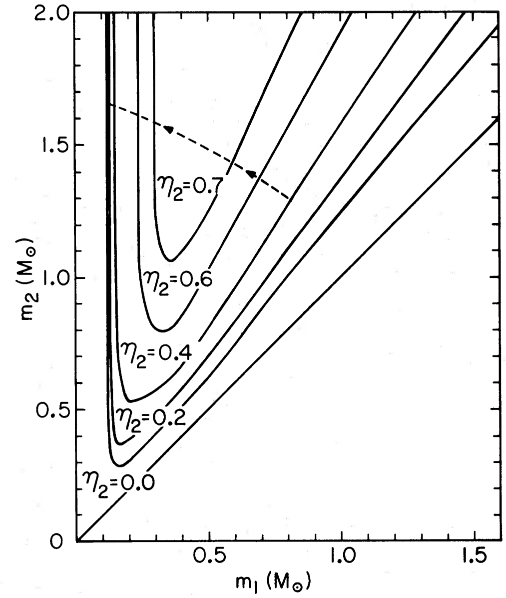

FIG. 4

FIG. 3.—"Equipotentials" of quasistability for $\eta_1 = 0.0$. The dashed line represents the track followed by the system of § III*e*. System is stable as long as it lies above the curve.

FIG. 4.—"Equipotentials" of quasistability for $\eta_1 = 1.0$. Dashed line as in Fig. 3.

The track followed by the model of § III*e* is shown on both diagrams, and is representative of any possible system. A system will stop evolving tidally and suffer immediate tidal disruption when it reaches the point of quasi-stability on the left side of its V contour.

c) *Collapse of m_2 to a Black Hole*

Should the total mass of the system be large enough initially, then at some stage during mass stripping the heavier neutron star will become unstable to collapse. This result is of some significance, since current thinking has it that formation of black holes by accretion is not an important mechanism, because of slow accretion rates imposed by the Eddington limit, $M^{-1}\, dM/dt \lesssim 10^{-8}\ \mathrm{yr}^{-1}$.

The collapse of the more massive star will be accompanied by a burst of gravitational radiation, whose magnitude depends principally on the departure of the star from spherical symmetry. The resulting object will be a Kerr black hole, with $J/M^2 \lesssim 0.4$.

So long as the black holes does not approach $J/M^2 \sim 1$ (possible only for large initial combined masses), η_2 will be approximately equal to the upper limit calculated in § III*a*, because tidal coupling of spin angular momentum back to the orbit is no longer significant (cf. Hartle and Hawking 1972; Press 1972).

Following collapse of the massive star, the neutrino flux may decrease. Similarly, the fashion in which material is thrown to infinity may change from "splattering" off the neutron star surface, to stuff carrying away excess angular momentum.

d) *Gravitational and Neutrino Radiation*

Prior to the system's reaching its Roche separation, the gravitational wave flux is defined by equation (1), and there is no neutrino flux.

In the event that the system suffers immediate tidal disruption on reaching its Roche separation, we expect the remains of the lighter star to accrete or disperse in a few orbits. Although details of the evolution of the remains are unclear, it is likely that the gravitational flux would be much smaller than expected in the case of stripping off, while the neutrino flux may remain large.

In the stripping-off case we expect large fluxes of both gravitational waves and neutrinos, the time dependence of which we calculate numerically. Generally, the gravitational flux is a decreasing function of time. The neutrino flux is a decreasing function of time, except just before tidal disruption, when it may increase slightly.

Figure 5 illustrates the minimum separation ($a_{\min}$) reached by a binary as a function of the masses of its two components, while Figure 6 tabulates the peak gravitational wave flux emitted.

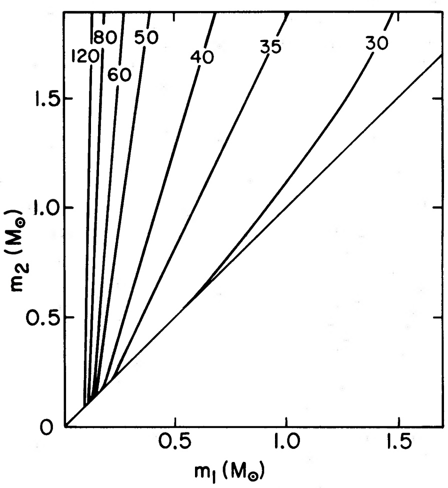

FIG. 5

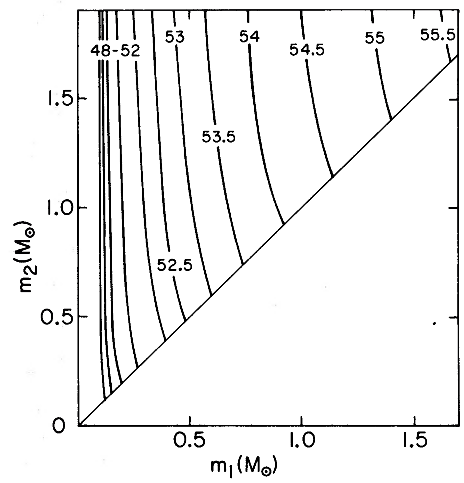

FIG. 6

FIG. 5.—Minimum separation (in km) of components as a function of their masses

FIG. 6.—Log of peak luminosity L_{GW} (in ergs s^{-1}) of gravitational waves, as a function of the components' masses

e) *A Sample Model*

To illustrate the behavior of a system undergoing mass stripping, details of all physical quantities have been followed for a representative example. The results are illustrated in Figures 7 and 8. The time coordinate t has been chosen so that mass stripping begins at $t = 0$.[1] Parameters $\eta_1 = 0.5$, $\eta_2 = 0.25$ were adopted. Initial masses were $m_1 = 0.8\ M_\odot$, $m_2 = 1.3\ M_\odot$. Some calculated numbers are:

Minimum separation between centers:	33.5 km
Peak wave frequency:	865 Hz
Peak gravitational flux:	8.7×10^{53} ergs s^{-1}
Peak neutrino flux:	4.0×10^{55} ergs s^{-1}
Duration of mass stripping:	1.7 s
Mass lost to infinity:	$0.23\ M_\odot$
Gravitational wave energy release:	
Before onset of stripping:	4.1×10^{52} ergs
After onset of stripping:	1.6×10^{52} ergs
Total neutrino energy release:	2.0×10^{53} ergs .

IV. DETECTION OF AN EVENT

a) *Density of Systems*

Lattimer and Schramm (1976) estimate that a lower limit on the birthrate of progenitors of double compact binaries in the galactic plane is approximately 2×10^{-12} pc^{-2} yr^{-1}. Since only a small fraction, denoted by β, may be expected to survive disruption by the second supernova, to become double neutron star binaries; and since the lifetime of the latter will probably be less than the age of the Galaxy, the rate of events in our Galaxy will be

$$\frac{dN}{dt} \sim 6 \times 10^{-6}\left(\frac{\beta}{10^{-2}}\right) \mathrm{yr}^{-1} . \tag{11}$$

Although no binary neutron star systems have yet been positively identified, PSR 1913+16 (Hulse and Taylor 1975) is a strong candidate, with a lifetime[2] of $\sim 10^8$ yr (Smarr and Blandford 1976).

Taking β to be 10^{-2} for illustrative purposes (see Lattimer and Schramm 1976 for difficulty in estimating β), we expect approximately one event every 80 years out to the radius of the Virgo cluster ($\sim$15 Mpc).

b) *Gravitational Radiation*

The fractional motion h of a detector due to a passing gravitational wave is given by

$$F = (c^3/16\pi G)\langle (dh_+/dt)^2 + (dh_\times/dt)^2 \rangle \tag{12}$$

[1] The times t and τ are related by $\tau + t =$ (value of τ in eq. [2] for $a = a_{\min}$).

[2] Assuming the companion to be compact. Taylor *et al.* (1976) show that its mass is at least 0.56 times that of the pulsar.

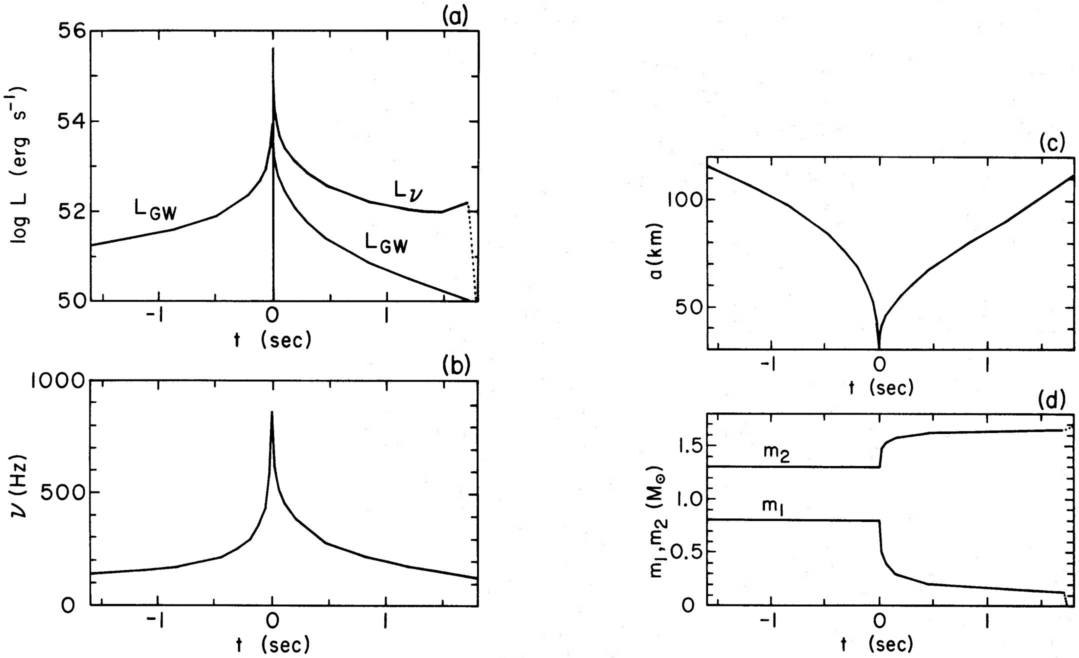

FIG. 7—Time evolution of a system with initial masses 0.8 and 1.3 $M_{\odot}$. (*a*) Neutrino and gravitational wave luminosities. (*b*) Frequency of gravitational wave. (*c*) Separation of components. (*d*) Masses of stars.

where F is the energy flux at the Earth. Then, a binary neutron star system, with inclination θ, will produce a peak amplitude

$$h(t) \approx 6.6 \times 10^{-24}(1 + 6\cos^2\theta + \cos^4\theta)^{1/2} m_1^{3/4} m_2^{3/4} (m_1 + m_2)^{-1/4} (-\tau/1^{\mathrm{d}})^{-1/4} (d/15\ \mathrm{Mpc})^{-1} . \quad (13)$$

(Note that Press and Thorne 1972 give a value for h too large by a factor of 2.) The observed frequency ν is

$$\nu(t) \approx (2.1\ \mathrm{Hz}) m_1^{-3/8} m_2^{-3/8} (m_1 + m_2)^{1/8} (-\tau/1^{\mathrm{d}})^{-3/8} , \quad (14)$$

$$d\nu/dt \approx (9.2 \times 10^{-6}\ \mathrm{Hz\ s^{-1}}) m_1^{-3/8} m_2^{-3/8} (m_1 + m_2)^{1/8} (-\tau/1^{\mathrm{d}})^{-11/8} . \quad (15)$$

Representative values of h and ν at peak radiation are $\sim 10^{-21}$ and $\sim 10^3$, thus giving a flux comparable to the strongest known normal close binaries (cf. Braginsky 1965), for a distance of 15 Mpc.

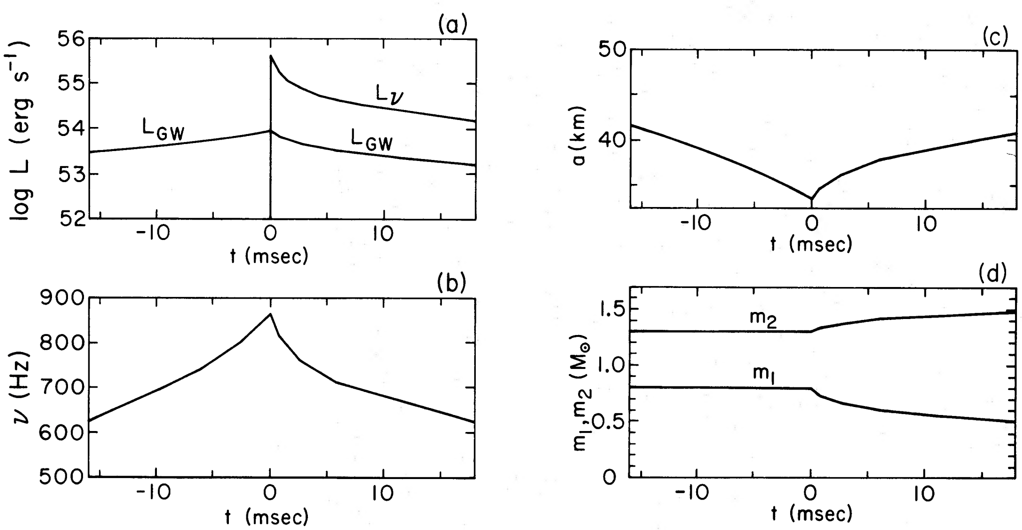

FIG. 8.—Same as Fig. 5, displayed on an expanded time axis

c) Response of a Resonant Gravitational-Wave Detector

Several types of rapidly rotating astrophysical systems, of which our binary is one, will emit a waveform which is a sine wave of slowly changing frequency and amplitude,

$$h(t) = H(t) \sin [2\pi\nu(t)t], \quad H' \ll \nu H, \nu' \ll \nu^2 ; \tag{16}$$

here a prime denotes d/dt. Consider a resonant detector tuned to a frequency $\omega/2\pi$, with quality factor $Q \gg 1$. The equation of motion for the amplitude $B(t)$ of detector motion is (see Press and Thorne 1972; Misner, Thorne, and Wheeler 1973)

$$B'' + B'/\tau + \omega^2 B = Lh'' , \tag{17}$$

where $\tau = Q/\omega$ is damping time and L (dimensions of length) is a detector-dependent parameter: roughly, $L \sim$ (size of antenna). We neglect thermal noise. It has become commonplace to employ, instead of $B(t)$, the complex amplitude $z(t)$ in the phase plane of the detector mode,

$$z = (B - iB'/\omega)e^{-i\omega t} .$$

In terms of z the Green's-function solution to equation (17) takes the simple form

$$z(t) = -iL\omega^{-1} \int_{-\infty}^{t} dt' \exp[-(t - t')/2\tau]h''(t') \exp(-i\omega t') . \tag{18}$$

With the neglect of damping ($\tau \to \infty$), free ringing of the detector corresponds to constant z.

We approximate the waveform, equation (16), by a truncated Taylor expansion about $t = 0$, taken as the instant when $2\pi\nu(t) = \omega$:

$$h(t) \approx A \sin [\omega t \pm \tfrac{1}{2}\pi(\omega t/q)^2] \tag{19}$$

where $A \equiv H(0)$, upper sign means $\nu'(0) > 0$ and lower sign means $\nu'(0) < 0$, and

$$q \equiv \pi\nu(0)/(|\nu'(0)|)^{1/2} .$$

The dimensionless parameter q may be thought of as "the Q of the signal"; at $\omega t = 0$ the waveform is in phase with a pure sine wave of frequency ω, while by the time that $\omega t = q$ the waveform and that sine wave have fallen out of phase by $\pi/2$ radians. For our binary, equations (15) and (16) yield

$$q(t) = 2200(-\tau/1^{\mathrm{d}})^{5/16} m_1^{-3/16} m_2^{-3/16} (m_1 + m_2)^{1/16} = 62(-\tau/1^{\mathrm{s}})^{5/16} m_1^{-3/16} m_2^{-3/16} (m_1 + m_2)^{1/16} . \tag{20}$$

Since $Q \sim 10^5$ for presently operating detectors, the approximation $Q \gg q \gg 1$ will be made.

Then from equations (18) and (19) we obtain an explicit formula for detector response,

$$z(t) = \tfrac{1}{2}ALq[C(\omega t/q) \pm iS(\omega t/q) + (1 \pm i)/2] , \tag{21}$$

valid as long as damping can be neglected, $\omega t \ll Q$; where C and S are the Fresnel integrals familiar from diffraction theory (e.g., Born and Wolf 1975; Abramowitz and Stegun 1964). The amplitude z therefore describes the shape of Cornu's spiral in the complex plane; see Figure 9. The orientation of the spiral depends on the arbitrarily chosen phase of the local oscillator in the detector, and therefore has no fundamental significance. The direction of traversal along the spiral is forwards compared to the conventional one if $\nu'(0) > 0$, backwards if $\nu'(0) < 0$. Most of the energy is deposited into the detector over the time interval $-q \lesssim \omega t \lesssim q$; the total jump over all time (but again neglecting damping) is

$$|\Delta z| = Alq/\sqrt{2} .$$

For example, we refer to the sample model above in § III*e*, and to a resonant detector with frequency $\omega/2\pi =$ 600 Hz. The detector "sees" a Cornu spiral at $\tau = -19$ ms as ν sweeps up, with $q \sim 22$; it "sees" a backwards Cornu spiral at $\tau = +25$ ms as ν sweeps back down after the onset of mass stripping, with $q \sim 32$. The size $|\Delta z|$ of the total jump happens to be the same for the two events; the phase difference between the two jumps is essentially random. The time scales q/ω for the two jumps are 6 ms and 8 ms.

At the present stage in the development of gravitational wave detectors it seems unlikely that a binary could be detected outside our Galaxy ($d \gtrsim 15$ kpc).

d) Detection of Neutrino Radiation

Typically, 2×10^{53} ergs of neutrinos and antineutrinos will be released during mass stripping, and a similar amount will perhaps be released in the case of immediate tidal disruption. If the distance d is 15 Mpc, Earth will

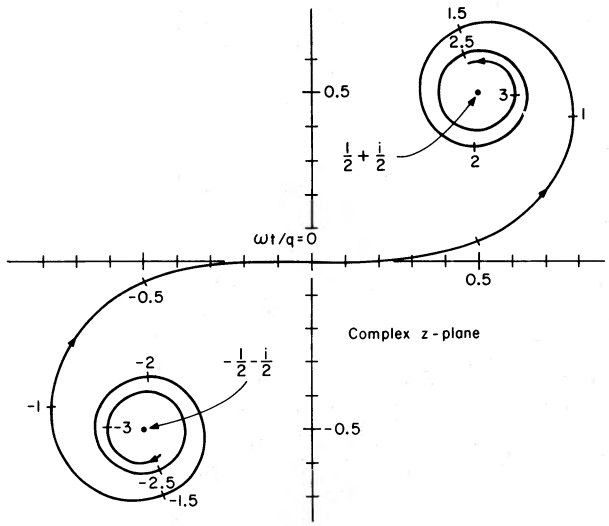

FIG. 9.—Response of a resonant gravitational wave detector, in the phase plane of complex amplitude z. Shape is that of Cornu's spiral.

receive a flux of ~ 4 ergs cm^2. However the cross section for a neutrino to interact with matter is only about

$$\sigma \sim 10^{-42}(E_\nu/10 \text{ MeV})^2 \text{ cm}^2 \text{ nucleon}^{-1} \sim 10^{-18}(E_\nu/10 \text{ MeV})^2 \text{ cm}^2 \text{ g}^{-1} . \quad (22)$$

Therefore, the number of neutrino-induced events in a detector of mass M is

$$N \sim 10^{-6}(E_\nu/10 \text{ MeV})(d/15 \text{ Mpc})^{-2}(M/10^7 \text{ g}) . \quad (23)$$

It seems hopeless to detect the binary with current detectors unless it is in our Galaxy, $d \lesssim 15$ kpc. However, a detector with $M \sim 10^{14}$ g, such as the Deep Underwater Muon and Neutrino Detector proposed by Roberts *et al.* (1976), could detect these events out to $d \sim 10$ Mpc.

We would expect the neutrino flux to be modulated at the first and second harmonics of the orbital frequency, that is, at frequencies $\frac{1}{2}\nu$ and ν, where ν is the frequency of gravitational radiation. It seems unlikely but worth mentioning that there may be a connection with the possible antineutrino event reported by Lande *et al.* (1974). Their detector comprises 1.2×10^7 g of water in Cerenkov counters deep underground. Their event was consistent with $N \geq 24\, \bar{\nu}_e$ interactions in a few milliseconds, with a remarkable time structure: four pulses about 1 ms apart, each only about 1 μs long. The very short duration of the individual pulses tends to be inconsistent with our binary model, but the periodicity is roughly correct.

e) *Comparison of Total Energy Radiated*

There has recently been a good deal of discussion about the relative magnitude of neutrino energy E_ν and gravitational-wave energy E_{GW} radiated in a collapse event (e.g., Kazanas and Schramm 1976; Ostriker, Smarr, and Thuan 1976; J. P. Ostriker, personal communication). We estimate in the present case

$$E_\nu \sim 4E_{\text{GW}} ,$$

as follows.

For neutrinos emitted by accretion of stripped or disrupted mass Δm onto the heavy neutron star,

$$E_\nu \sim (Gm_2/r_2)\Delta mc^2 \sim (0.15\, M_\odot c^2)m_2\Delta m \sim (3 \times 10^{53} \text{ ergs})m_2\Delta m$$

(for our sample model, $m_2 \sim 1.5\, M_\odot$, $\Delta m \sim 0.5\, M_\odot$).

Most of the gravitational waves come *before* the onset of stripping, i.e., before $a = a_{\min}$ (see Fig. 7*a*); the liberated orbital binding energy is

$$E_{\text{GW,before}} \approx \tfrac{1}{2}Gm_1m_2/a_{\min} .$$

From Figure 5 we obtain an empirical formula for $a_{\min}$,

$$a_{\min} \approx (27km)(m_1 - 0.1)^{-1/3}m_2^{1/3} ;$$

therefore,

$$E_{\text{GW,before}} \approx (0.027\, M_\odot c^2)m_1(m_1 - 0.1)^{1/3}m_2^{2/3} \approx (5 \times 10^{52} \text{ ergs})m_1(m_1 - 0.1)^{1/3}m_2^{2/3} .$$

During stripping we typically find

$$E_{\mathrm{GW,during}} \sim 0.4 E_{\mathrm{GW,before}} .$$

We have not tried to calculate E_{GW} *after* disruption, but this contribution seems unlikely to exceed $E_{\mathrm{GW,during}}$. Finally there may be a contribution to E_{GW} upon collapse of m_2 to a black hole. Rotation is probably not a strong influence in this collapse (we find $J/M^2 \sim 0.4$ above), and then there is some reason to believe (Eardley 1975*b* and references cited therein) that

$$E_{\mathrm{GW,collapse}} \lesssim (0.02\, M_\odot c^2) m_2 ,$$

and this is probably a generous upper limit. So, all together, a reasonable guess is $E_{\mathrm{GW}} \sim 1.4\, E_{\mathrm{GW,before}}$, or

$$E_{\mathrm{GW}} \sim (0.04 M_\odot c^2) m_1 (m_1 - 0.1)^{1/3} m_2^{2/3} \sim (7 \times 10^{52}\ \mathrm{ergs}) m_1 (m_1 - 0.1)^{1/3} m_2^{2/3} .$$

Therefore, $E_\nu \sim 4 E_{\mathrm{GW}}$.

V. STABILITY OF THE ORBIT

Finally we wish to raise and discuss a point of principle: What is the radius of separation for the innermost stable circular orbit for two bodies in general relativity? If the radius exceeded $6G(m_1 + m_2)/c^2$ by very much, most of our work in this paper would have to be modified. We cannot give a definitive answer, but we shall tentatively suggest that the radius is $\lesssim 6G \max(m_1, m_2)/c^2$, so that all our models above are comfortably stable.

The problem is not even well defined as stated, because gravitational radiative damping will cause the radius to shrink on a time scale somewhat longer than, but comparable with, the period. To make the orbit stationary we must inject energy into the orbital motion, perhaps with laser beams. More elegantly but less realistically, we can place the binary system in a fictitious large cavity which reflects gravitational waves (cf. Thorne 1969) so that ingoing and outgoing waves are equal in strength. Then spacetime inside the cavity will be stationary—will admit a timelike rotating Killing vector—so that the question of stability of the orbit to small perturbations is now a well defined one. This treatment would in principle be a satisfactory one as long as the total energy in the standing waves is sufficiently small. However, the technical means to carry out this analysis are not now available, and we will only give here a crude and tentative estimate based on the post-Newtonian approximation to the motion of two point masses m_1 and m_2.

The gravitational-plus-centrifugal potential energy $E(r, J)$ as a function of separation r (in the Einstein-Infeld-Hoffmann [EIH] gauge) and orbital angular momentum J is given by Landau and Lifshitz (1962) as

$$E(r, J) = J^2/(2\mu r^2) - \mu M/r - J^4(M - 3\mu)/(8M\mu^3 r^4) - J^2(3M + \mu)/(2\mu r^3) + \mu M^2/(2r^2) + \cdots \tag{24}$$

through post-Newtonian order, where $M = m_1 + m_2$, $\mu = m_1 m_2/M$, and we are taking $G = 1 = c$. For given J, the radius r of the circular orbit is determined by the variational formula $\partial E/\partial r = 0$; the innermost stable circular orbit is marked by the additional equation $\partial^2 E/\partial r^2 = 0$. Now when we apply this method to equation (24), we obtain manifestly incorrect results, which means that the marginally stable orbit cannot be unambiguously determined just from post-Newtonian formulas. However, we know some additional global information about the analytic structure of the problem which we now inject: We know the exact form of $E(r, J)$ for the limit $m_1 \to 0$, i.e., for a test particle in the Schwarzschild metric, with J/μ fixed. It is

$$[1 + E(R, J)/\mu]^2 = (1 - 2M/R)[1 + J^2/(\mu^2 R^2)] , \tag{25}$$

where R is the curvature or Schwarzschild radius, which is *not* in EIH gauge. The relation between r and R through first order is

$$R = r + M + \cdots . \tag{26}$$

Our tentative procedure is to substitute R from equation (26) into equation (24) and then define a new quantity $F(R, J)$ through post-Newtonian order as

$$F(R, J) \equiv \{1 + E[r(R), J]/\mu\}^2 = 1 + J^2/(\mu^2 R^2) - 2M/R + 3J^4/(4M\mu^3 R^4) - J^2(2M + \mu)/(\mu^2 R^3) + \cdots . \tag{27}$$

The variational formulae for the innermost stable orbit are then $\partial F/\partial R = 0 = \partial^2 F/\partial R^2$. Now equation (27) reduces to equation (25) as $\mu \to 0$, and the variational formulae give the exact answer in this limit. Therefore we adopt $F(R, J)$ as a more reliable approximation. The resulting expressions for the innermost stable circular orbit for all m_1, m_2 are

$$R = \tfrac{3}{4}[5M - 2\mu + 3M^{1/2}(M - 4\mu)^{1/2}] , \tag{28a}$$

$$E = \mu[F^{1/2} - 1] , \tag{28b}$$

$$F = 1 - (M - 6M^2/R + 24\mu M/R)/27\mu , \tag{28c}$$

$$J^2 = \tfrac{3}{4}\mu M[M^2 + 10\mu M - 2\mu^2 - M^{1/2}(M - 4\mu)^{3/2}] . \tag{28d}$$

These expressions should not be taken too seriously in view of the crudity of our method; the main conclusions from their study can be summarized to $\sim 20\%$ as

$$R \approx 6 \max(m_1, m_2)\,, \tag{29a}$$

$$E \approx -0.05 \min(m_1, m_2)\,, \tag{29b}$$

$$J \approx 3.3\, m_1 m_2\,. \tag{29c}$$

The rule of thumb, therefore, is that the minimum orbital radius is 6 times the gravitational radius of the larger member, *not* of the total system mass. The relative binding energy $|E|/M$ is always small; it is only 2% even for equal masses.

If the bodies are corotating rather than nonrotating, one expects the minimum radius to be smaller. Again, we have some exact information in the limit $\mu \to 0$: A Kerr black hole[3] which corotates synchronously with a test particle in the innermost stable orbit has $a/M \approx 0.359$, and the orbit lies at $R \approx 4.764\, M$. This suggests that equations (29) would become, for the corotating case,

$$R \approx 5 \max(m_1, m_2)\,, \tag{30a}$$

$$E \approx -0.07 \min(m_1, m_2)\,, \tag{30b}$$

$$J \approx 3 m_1 m_2\,. \tag{30c}$$

If after passing through the innermost stable orbit the bodies fall together to form a Kerr black hole, the above results suggest $a/M \leqslant 0.9$ for this hole. Therefore the hole need not radiate very much angular momentum as it forms, because it already obeys the limit $a/M < 1$.

Perhaps binaries made of two *rapidly rotating* ($a \approx M$) black holes will have higher efficiencies for radiation. See Zel'dovich and Novikov (1964) for pioneering work on the questions discussed in this section.

VI. CONCLUSIONS

The fact that stable mass stripping of a light degenerate companion can occur in a close binary is familiar in the case of a degenerate dwarf; cf. Paczyński (1967), Vila (1971), Faulkner, Flannery, and Warner (1972), Pringle and Webbink (1975). In this paper we find a similar result in the case of a degenerate neutron star, as long as the parameter η_2 which measures spin-up of the heavy star is not too large.

In comparing our results with those of Lattimer and Schramm (1974, 1976), the reader may question why we find stable mass stripping while they do not. The answer lies in differing assumptions about the initial masses. For Lattimer and Schramm, the heavy star is a black hole with $m_2 \geqslant 5\, M_\odot$; in this case the Roche radius of the neutron-star companion nearly coincides with the innermost stable circular orbit, so that there is no opportunity for stable mass stripping. In contrast, in our case the mass of the heavy star is much less, e.g., $m_2 \sim 1.3\, M_\odot$, so that the Roche radius is more than twice the radius of the innermost orbit, according to $a_{\rm Roche}/a_{\rm innermost} \propto m_2^{-2/3}$. Hence there is ample room for stripping. We have not calculated how much, if any, of the stripped neutron matter is ejected to infinity; we think it likely that some amount $\geqslant 0.1\, M_\odot$ is ejected, with interesting implications for *r*-process and superheavy nucleosynthesis (cf. Lattimer and Schramm 1974, 1976; Pringle, Dearborn, and Fabian 1976).

We had originally thought that a neutron star binary would be a very efficient source of gravitational radiation. However, in terms of sheer total energy E, we find that neutrinos dominate gravitational waves,

$$E_\nu \approx 4\, E_{\rm GW}\,.$$

But sheer energy itself is only one crude parameter of the event, and we think it essential that observers continue the development of both gravitational-wave and neutrino detectors, since each of the two kinds of radiation contains unique information about the binary evolution or stellar collapse.

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[3] Here a denotes Kerr parameter $a = J_{\rm hole}/M$, not to be confused with our previous usage.

REFERENCES

Aarseth, S. J. 1974, *Astr. Ap.*, **35**, 237.
Abramowitz, M., and Stegun, I. A. 1964, *Handbook of Mathematical Functions* (Washington: Government Printing Office), chap. 7.
Born, M., and Wolf, E. 1975, *Principles of Optics* (5th ed.; Oxford: Pergamon Press), § 8.7.2.
Braginsky, V. B. 1965, *Usp. Fiz. Nauk*, **86**, 433 (English transl. *Soviet Phys.–Uspekhi*, **8**, 513).

Cameron, A. G. W. 1970, *Ann. Rev. Astr. Ap.*, **8**, 179.
Canuto, V. 1975, *Ann. Rev. Astr. Ap.*, **13**, 335.
Chandrasekhar, S. 1969, *Ellipsoidal Figures of Equilibrium* (New Haven: Yale University Press), p. 192.
Clark, J. P. A. 1977, *Ap. Letters*, **18**, 73.
Dyson, F. J. 1963, in *The Search for Extraterrestial Life*, ed. A. G. W. Cameron (New York: Benjamin), p. 119.
Eardley, D. M. 1975*a*, *Ap. J.* (*Letters*), **196**, L59.
———. 1975*b*, *Phys. Rev.* **D12**, 3072.
Faulkner, J., Flannery, B. P., and Warner, B. 1972, *Ap. J.* (*Letters*), **175**, L79.
Hartle, J. B., and Hawking, S. W. 1972, *Commun. Math. Phys.*, **27**, 283.
Hulse, R. A. and Taylor, J. H. 1975, *Ap. J.* (*Letters*), **195**, L51.
Kazanas, D., and Schramm, D. N. 1976, *Nature*, **262**, 671.
Landau, L. D., and Lifshitz, E. M. 1962, *The Classical Theory of Fields* (2d ed.; Reading, Mass.: Addison-Wesley), § 105.
Lande, K., Bozoki, G., Frati, W., Lee, C. K., Fenyves, E., and Saavedra, O. 1974, *Nature*, **251**, 485.
Lattimer, J. M., and Schramm, D. N. 1974, *Ap. J.* (*Letters*), **192**, L145.
———. 1976, *Ap. J.*, **210**, 549.
Misner, C. W., Thorne, K. S., and Wheeler, J. A. 1973, *Gravitation* (San Francisco: Freeman).
Novikov, I. D. 1975, *Astr. Zh.* **55**, 657 (English transl. *Soviet Astr.—AJ*, **19**, 398).
Ostriker, J. P., Smarr, L. L., and Thuan, T. X. 1976, in preparation.
Paczyński, B. 1967, *Acta Astr.*, **17**, 287.
Press, W. H. 1972, *Ap. J.*, **175**, 243.
Press, W. H., and Thorne, K. S. 1972, *Ann. Rev. Astr. Ap.*, **10**, 335.
Pringle, J. E., Dearborn, D. S. P., and Fabian, A. C. 1976, *Nature*, **263**, 114.
Pringle, J. E., and Webbink, R. F. 1975, *M.N.R.A.S.*, **172**, 493.
Rees, M., Ruffini, R., and Wheeler, J. A. 1974, *Black Holes, Gravitational Waves and Cosmology* (New York: Gordon & Breach), § 7.12.
Roberts, A., Blood, M., Learned, J., and Reines, F. 1976, preprint.
Shapiro, S. L. 1976, preprint.
Smarr, L. L., and Blandford, R. 1976, *Ap. J.* **207**, 574.
Taylor, J. H., Hulse, R. A., Fowler, L. A., Gullahorn, G. E., and Rankin, J. M. 1976, *Ap. J.* (*Letters*), **206**, L53.
Thorne, K. S. 1969, *Ap. J.*, **158**, 1.
Thuan, T. X., and Ostriker, J. P. 1974, *Ap. J.* (*Letters*), **191**, L105.
Vila, S. C. 1971, *Ap. J.*, **168**, 217.
Wiita, P. J., and Press, W. H. 1976, *Ap. J.*, **208**, 525.
Will, C. M. 1977, *Ap. J.*, in press.
Will, C. M., and Eardley, D. M. 1977, *Ap. J.* (*Letters*), **212**, L94.
Zel'dovich, Ya. B., and Novikov, I. D. 1964, *Doklady Akad. Nauk SSSR*, **155**, 1033 (English transl. *Soviet Phys.—Doklady*, **9**, 246).
———. 1971, *Relativistic Astrophysics* Vol. **1** (Chicago: University of Chicago Press).
Zel'dovich, Ya. B., and Podurets, M. A. 1965, *Astr. Zh.*, **42**, 963 (English transl. *Soviet Astr.—AJ*, **9**, 742).

Paul Clark: Yale University Observatory, Box 2023 Yale Station, New Haven, CT 06520

Douglas Eardley: Department of Physics, Yale University, 217 Prospect Street, New Haven, CT 06520