LPF

$$h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H(e^{j\omega})\,e^{j\omega n}d\omega=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{jn}e^{j\omega n}d\omega$$
$$=\frac{1}{2\pi jn}\int_{-\omega_c}^{\omega_c}e^{j\omega n}d\omega=\frac{1}{2\pi jn}\left(e^{j\omega_c n}-e^{-j\omega_c n}\right)=\frac{\sin(\omega_c n)}{\pi n}$$

$$h(0)=\lim_{n\to 0}\frac{\sin(\omega_c n)}{\pi n}=\frac{\omega_c}{\pi}$$

HPF

$$h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H\left(e^{j\omega}\right)e^{j\omega n}d\omega=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{jn}e^{j\omega n}d\omega$$
$$=\frac{1}{2\pi jn}\int_{-\pi}^{-\omega_c}e^{j\omega n}d\omega+\int_{\omega_c}^{\pi}e^{j\omega n}d\omega=\frac{1}{2\pi jn}\left(e^{-j\omega_c n}-e^{-j\pi n}+e^{j\pi n}-e^{j\omega_c n}\right)$$
$$=\frac{\sin(\pi n)-\sin(\omega_c n)}{\pi n}$$

$$h(0)=\lim_{n\to 0}\frac{\sin(\pi n)-\sin(\omega_c n)}{\pi n}=1-\frac{\omega_c}{\pi}$$

BPF

$$h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H\left(e^{j\omega}\right)e^{j\omega n}d\omega=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{jn}e^{j\omega n}d\omega$$
$$=\frac{1}{2\pi jn}\int_{-\omega_2}^{-\omega_1}e^{j\omega n}d\omega+\int_{\omega_1}^{\omega_2}e^{j\omega n}d\omega=\frac{1}{2\pi jn}\left(e^{-j\omega_1 n}-e^{-j\omega_2 n}+e^{j\omega_2 n}-e^{j\omega_1 n}\right)$$
$$=\frac{\sin(\omega_2 n)-\sin(\omega_1 n)}{\pi n}$$

$$h(0)=\lim_{n\to 0}\frac{\sin(\omega_2 n)-\sin(\omega_1 n)}{\pi n}=\frac{\omega_2-\omega_1}{\pi}$$

BSF

$$
\begin{aligned}
h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H\left(e^{j\omega}\right) e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{jn} e^{j\omega n} d\omega \\
= \frac{1}{2\pi jn}\int_{-\pi}^{-\omega_2} e^{j\omega n} d\omega + \int_{-\omega_1}^{\omega_1} e^{j\omega n} d\omega + \int_{\omega_2}^{\pi} e^{j\omega n} d\omega \\
= \frac{1}{2\pi jn}\left(e^{-j\omega_2 n} - e^{-j\pi n} + e^{j\omega_1 n} - e^{-j\omega_1 n} + e^{j\pi n} - e^{j\omega_2 n}\right) \\
= \frac{\sin(\omega_1 n) - \sin(\omega_2 n) + \sin(\pi n)}{\pi n}
\end{aligned}
$$

$$
h(0) = \lim_{n\to 0} \frac{\sin(\omega_1 n) - \sin(\omega_2 n) + \sin(\pi n)}{\pi n} = 1 + \frac{\omega_1 - \omega_2}{\pi}
$$