Volatility Smile, Term Structure and Surface

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ABSTRACT

Options are an important vehicle in the derivatives market used for the purpose of speculation and risk management. Because of their pragmatic side, options take great importance in the market for investors and also traders. This study aims to price premiums and implied volatilities, graph volatility smiles, term structure of volatility and volatility surfaces. The objective of the tool is to analyze market sentiment through price discrepancy and volatility prediction.

Keywords

Options, Call, Put, In the Money (ITM), Out of the Money (OTM), At the Money (ATM), Black-Scholes Model, Premium, Basis Point.

1. INTRODUCTION

An option on a security is an agreement to buy or sell the asset at a later time at a price known as the strike price. Analyzing the pricing dynamics of the option market will yield additional useful information on the underlying asset of the option.

In 1973, Fischer Black and Myron Scholes published their renowned paper, "The pricing of Options and Corporate Liabilities", in which they outlined a valuation formula for options using a partial differential equation now colloquially known as the Black Scholes model. Much of the work done by Black, Scholes, and Merton is still extensively used in the financial industry today. Our study and work thus far is an extension of the closed form of the partial differential equation of Black-Scholes.

One of the most important variables in the Black-Scholes formula is volatility, a statistical measure of the dispersion of returns for a given security or market index. Simply put, volatility indicates how likely an asset will fluctuate within a statistical bound.

Volatility alongside underlying asset price, strike price, time to maturity, and the risk free interest rate are the inputs used to yield the theoretical price of an option. This theoretical price can often be approximately 5% off the market price of the option's premiums.

The difference between the market price and the theoretical price of an option is an indication of the market sentiment or bias on the performance or return of the underlying asset in conjunction with the time the option expires. This discrepancy shows the implied volatility of the market.

Implied Volatility can be found through an iterative process of increasing or decreasing the input variable of volatility by one basis point (0.01%) until the model yields the current market price of the option.

Since options can be found at different strike prices or expiry dates we can display a skew or relationship in implied volatility using time and the strike price as parameters. This has led us to adopt the vertical skew iVol as a function of strike, the horizontal skew iVol as a function of maturity, and the iVol surface as a function of both.

Our work so far graphically displays these relationships using python libraries to aid us in drawing useful information on the underlying asset.

The pulldata.py file graphically displays the volatility smile, surface and structure. Additionally, our program provides analysis on the greeks of the option.

2. Black-Scholes Pricing Method

We have implemented the Black-Scholes model extension on options paying continuous yield dividends. This model extension takes a simplifying assumption that dividends are paid continuously and that the dividend amount is always proportional to the level of the index of the option. The smile, structure and surface were mapped using the model's calculated premium and implied volatility for the option.

This study has used the standard extension of the closed partial differential Black-Scholes model for theoretically pricing American options. American options can be exercised at any point before the option expiry date, while European options can only be exercised on the date. We have noticed that our skews were slightly misguided due to the dividend factor. It is likely that longer maturities will experience more dividends, which should lower the asset price at the ex-dividend pay date.

2.1 Price Data Collection

The option price data was collected by scraping the Google Finance website for bids, asks and premiums. Historical volatility was calculated by using maturity dates, open, high, low, close, volume, and adjusted close data acquired by scraping Yahoo.

2.2 Volatility Smile

Given one particular expiry date, options with strikes substantially different from the underlying asset price command higher prices (and also implied volatilities) than what is suggest by standard option pricing methods. Implied volatility is a function of the premium that investors or traders are prepared to pay for an option with a given strike. This measure gives an indication of the underlying supply and demand levels for the option at that strike.

A volatility smile describes the relationship where the implied volatility increases relative to the "moneyness" of the option. Being a function of supply and demand of the option, the smile's shape is affected by volatility relative to strike, which is affected by liquidity. Fig. 1 displays the volatility smile using the extension of the model.

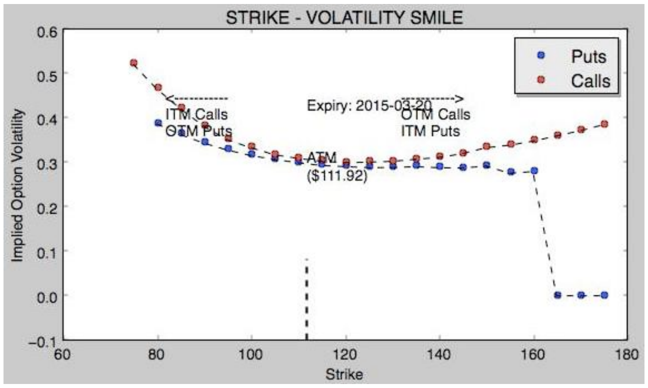

Figure 1. AAPL 4-month Maturity Volatility Smile

2.3 Term Structure of Volatility

The term structure describes the relationship between volatility and time to expiry. The volatility of contracts closer to expiration will be higher than those further out. As the maturity date gets closer, the complex supply and demand fundamentals became clearer and so shocks in the fundamentals have a greater impact on the prices closer to maturity. The term structure is a tool for investors and traders to gauge between cheap and expensive options.

Options that mature earlier exhibit a larger swing in implied volatility than options with longer maturities. Usually, implied volatility will rise during the period prior to earnings announcement and then fall again after the underlying stock price reflects the new information. Fig. 2 displays the term structure of volatility calculated by our program.

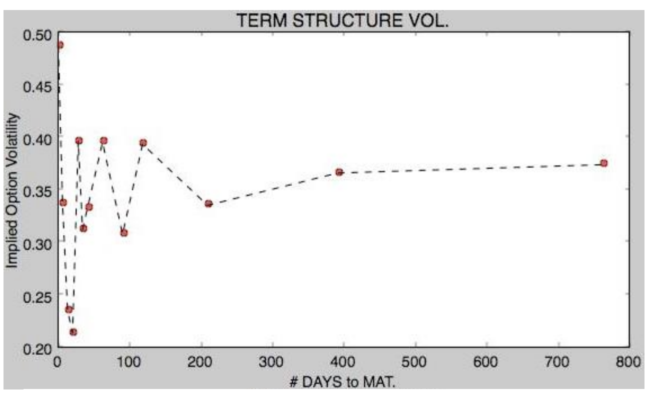

Figure 2. AAPL Term Structure of Volatility

2.4 Volatility Surface

The volatility surface is a combination of the volatility smile and the term structure of volatility for a given underlying. It is a three-dimensional curved surface where the current market implied volatility (Z-axis) for call and put options on the underlying is plotted against the price (Y-axis) and time to maturity (X-axis).

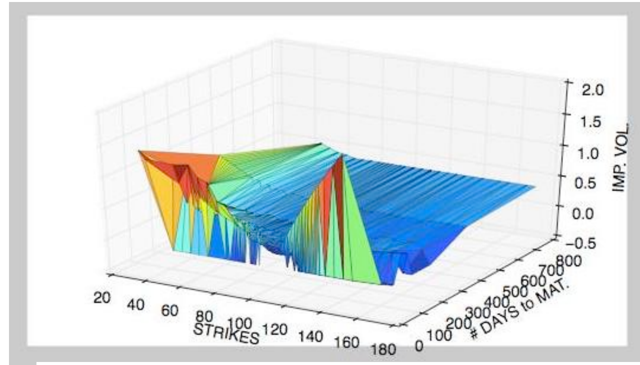

Figure 3. AAPL 3D Volatility Surface

In general, the surface is used to see where the implied volatility swings out of line. In essence this could be used in arbitrage models. Fig. 3 shows our program's generated surface using the smile and structure from Fig. 1 and 2.

3. Results and Analysis

The table below shows the problem of the significant discrepancy between the model's calculated premium and the actual market premiums. Call and put premium discrepancy was averaged for each maturity period.

Expiry	ATM	ITM	OTM
1 week	7.60%	5.64%	83.56%
2 weeks	7.66%	6.68%	84.93%
1 month	16.57%	9.63%	95.57%
3 months	28.18%	10.05%	96.46%

Table 1. Table of % Discrepancy to Actual Premium

4. Improvements and Conclusion

As previously mentioned, our skews were slightly misguided due to the dividend factor. Because Black Scholes is implemented as a continuous function, the presence of a dividend is factored during the whole existence of the dividend when in reality it needs to be represented as a single event in time.

A fix to this problem is to implement a discrete binomial distribution of the asset price and account for the dividend by deducting it from the stock price nodes at the point of time the dividend is issued to stockholders.

5. ACKNOWLEDGMENTS

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6. REFERENCES

[1] Hull, John. *Options, Futures, and Other Derivatives*. Boston: Prentice Hall, 2012. Print.