APPLICATION OF SEMI-LOCAL LCS TO STRING APPROXIMATE MATCHING*

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Abstract. We present an application of semi-local lcs to approximate string matching by developing a new algorithm and improving the existing one. Our result is based on the utilization of the underlying algebraic structure of semi-local lcs with the usage of the novel data structure for submatrix maximum queries in Monge matrices. This gives two algorithms with the following running time and space complexity. TODO. The improvement of the existing algorithm not only preserves all properties but also outperforms in practice.

In addition, we show that the algorithm for semi-local lcs based on sticky braid multiplication is not perform well with the current complex recursive structure.

Key words. semi-local lcs, monge matrix, range queries, approximate matching, near-duplicate detection

AMS subject classifications. 68Q25, 68R10, 68U05

1. Introduction. Approximate string matching is an important task in many fields such as computational biology, signal processing, text retrieval and etc. It also refers to a duplicate detection subtask.

In general form it formulates as follows: Given some pattern p and text t need to find all occurrences of pattern p in text t with some degree of similarity.

There are many algorithms that solve the above problem. Nonetheless, the number of algorithms sharply decreases when the algorithm needs to meet some specific requirements imposed by running time, space complexity or specific criterion for the algorithm itself. For example, recently there was developed an approach for interactive duplicate detection for software documentation [2]. The core of this approach is an algorithm that detects approximate clones of a given user pattern with a specified degree of similarity. The main advantage of the algorithm is that it meets a specific requirement of completeness. Nonetheless, it has an unpleasant time complexity.

The algorithm for approximate detection utilizes mainly algorithm for solving the longest commons subsequence (LCS) problem. The longest common subsequence is a well-known fundamental problem in computer science that also has many applications of its own. The major drawback of it that it shows only the global similarity for given input strings. For many tasks, it's simply not enough. The approximate matching is an example of it.

There exist generalazation for LCS called *semi-local LCS* [] which overcome this constraint. The effective theoretical solutions for this generalized problem found applications to various algorithmic problems such as bla bla add cited. For example, there has been developed algorithm for approximate matching in the grammar-compresed strings[].

Although the algorithms for *semi-local LCS* have good theoretical properties, there is unclear how they would behave in practice for a specific task and domain.

To show the applicability of semi-local lcs on practice we developed several algorithms based mainly on it and the underlying algebraic structure. As well as devel-

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oping new algorithms we improve and significantly outperform the existing one for interactive duplicate detection for software documentation []. It should be noted that improvement preserves all properties of this algorithm. Do we need to state that ant algo is slow for current strucute of algorithm

The paper is organized as follows. Blablabla **??**, our new algorithm is in **??**, experimental results are in **??**, and the conclusions follow in **??**.

2. Preliminaries.

2.1. Approximate matching. Describe approximate matching formally

2.2. Semi-local lcs. Describe semi-local lcs (definition), algorithms that solves (steady and and braid reducing)

2.3. Monge matrix. Describe monge property
Say about range queries (about soda12, soda14 and new result that we will be used)

2.4. Near-duplicate detection algorithm. Describe luciv algo

3. Related work. ?????
could mention about approximation. Need discuss

4. Algorithm for near duplicate detection. TODO: third phase also may be imporved/ removed. By filtering in place. Also prserves property.
We now describe an improved version of Luciv et.al. algorithm [2] by utilizing a *semi-local sa* solution. Then we present proof that improved version preserves completneseness property. It is achieved by imitating all phases of the algorithm.

4.1. Algorithm description. NEWALGO.NEED DISCUSS Algorithm comprises of two phases.
At phase one (Line 1) *semi-local sa* problem is solved for the pattern p against whole text t. After that phase we have access to string-substring matrix $H_{p,t}^{str-sub}$ that implicitly stores scores for any substring of text t against pattern p.

At phase two text is processed via sliding window of size $L_w = \frac{|p|}{k}$. If alignment score between substring that corresponds to window position and pattern p is $\geq k_{di}$(ref to previous section described luciv algo) then this we proceed as follows.

We search of largest text fragment within this window

At phase two the largest text fragment that closest

At phase two the search of largest text fragment with highest score within every sliding window of size $L_w = \frac{|p|}{k}$ of text t is performed. More preciesely, we looking for the text fragment that have maximal score with pattern p across all substrings with sizes $|p| * k...\frac{|p|}{k}$ within each sliding window. It corresponds to looking at the square submatrices within $H_{p,t}^{str-sub}$ with $\frac{|p|}{k} - |p| * k$ sizes (see picture). Let's look closely to concrete submatrix (denote it by M). The intresting cells lies in upper-right part Let's denote it by m In other words, we need to find at the submatrix

Also the filtering of duplicate and intersected elements is perfomed in-place during this phase (this corresponds to third phase of [2]).

In terms of string-subsrtng matrix sliding window refers to square submatrix of size L_w. Moreoveor, sliding window with step 1 refers to sliding square windows that goes diagonally with one step (see picture). Thus, it means that diagonal in string-substring matrix of width $|p| * k..\frac{|p|}{k}$ and length $|t|$ corresponds to diagonal in string-substring matrix.

This means that it is sufficiently to visit only $O(\frac{|p|}{k} - |p| * k) * O(|t|) = O(|p||t|)$ cells to gain requiered text fragments. Now we describe how to visit this cells.

END NEW ALGO

The algorithm comprises three phases as in [2]. At phase one (Line 1) semi-local sa problem is solved for the pattern p against whole text t. This solution provides access to the string-substring matrix which allows performing fast queries of sa score for pattern p against every substring of text t.

At the second phase text t is scanning with a sliding window of length L_w with step 1. First, it checks that given substring w that of a maximum possible size of L_w have score that is higher or equal to a given threshold (Line 4). If no, then this interval will not further be proceeded (Line 5) else this interval will be processed as follows. First, for each prefix of text t it finds suffix that has the highest alignment score with the maximal length among all suffixes with that score. It corresponds to the searching row position for each column in string-substring matrix with associated alignment score. Second, among these suffixes, one is selected with the highest score. If several suffixes have the same score the one with maximal length is selected (Line 8). Then if selected suffix has score higher than the threshold, then it is added to set W_2.

The third phase is the same as in [2]. More precisely, on the third phrase, set W_2 is filtered out in a such way that only non-intersected intervals are left. It is simply the sorting of set W_3 by starts of intervals with following one way passage with filtering.

Algorithm 4.1 PATTERN BASED NEAR DUPLICATE SEARCH ALGORITHM VIA SEMI-LOCAL SA

Input: pattern p, text t, similiarity measure $k \in [\frac{1}{\sqrt{3}}, 1]$

Output: Set of non-intersected clones of pattern p in text t

$$(4.1) \qquad\qquad k_{di} = |p| * (\frac{1}{k} + 1)(1 - k^2)$$

$$(4.2) \qquad\qquad L_w = \frac{|p|}{k}$$

Comment: w_i, w_j — start and end positions of w in text t

Pseudocode:

1: $W = semilocalsa(p, t)$
2: $W_2 = \emptyset$
3: **for** $w \in t, |w| = L_w$ **do**
4:    **if** $W.stringSubstring(w_i, w_j) < -k_{di}$ **then**
5:      $continue$
6:    **end if**
7:    $maximums = FindMaxForColumnsBySmawk(w)$
8:    $max = FindMaxWithLenghtConstraint(maximums)$
9:    **if** $max \geq -k_{di}$ **then**
10:      add substring associated with max to W_2
11:    **end if**
12: **end for**
13: $W_3 = UNIQUE(W_2)$ {3rd phase unchanged}
14: **for** $w \in W_3$ **do**
15:    **if** $\exists w^{'} \in W_3 : w \subset w^{'}$ **then**
16:      $remove\ w\ from\ W_3$
17:    **end if**
18: **end for**
19: **return** W_3

THEOREM 4.1. *Algorithm 4.1 could be solved in* $O(|p| * |t|)$ *where p is pattern, t is text.*

Proof it

$$D = \mathrm{diag}(d_1, \ldots, d_n)$$

THEOREM 4.2. *Algorithm 4.1 preserves completnesses property of algorithm [2].*

Proof it

$$D = \mathrm{diag}(d_1, \ldots, d_n)$$

5. CutMax a new approximate mathing algorithm. We now describe several algorithms that heavily based on semi-local lcs and it's underlying algebraic structure.

The first algorithm 5.2 refers to following constraint. There should be found all non-intersected clones τ_i of pattern p from text t that has the highest similarity score on the uncovered part of the text t i.e algorithm should perform greedy choice at each step. This is a more intuitive approach i.e like looking for the most similar one every

123   time. Formally:

124   (5.1)
$$\tau_i = \underset{l,r \in (t \cap (\cup_{j=1}^{i-1} \tau_j)), l < r, t_{l,r} \cap (\cup_{j=1}^{i-1} \tau_j) = \emptyset}{\arg\max} sa(t_{l,r}, p)$$

125   The algorithm proceeds as follows. First, upon string-substring Monge matrix
126   M of semi-local solution is built data structure for performing range queries on it
127   denoted by $rmq2D$ (Lines 1).
128   Second, algorthm make recursive call to subroutine *greedy*. The *greedy* routine
129   perfoms greedy choice of τ_i with maximal alignment within the current uncovered
130   part of the text $t_{i,j}$. More precisely, it refers to searching maximum value with
131   corresponding position (row and column) in matrix M within $t_{i,j}$ (starting at ith
132   position and ending at jth position of text t. It is solved via range queries. When
133   detected *interval* has alignment score less then threshold it means that no clones
134   of pattern p are presented in this part of text $t_{i,j}$, and further processing should be
135   skipped. Otherwise, the founded clone is added to final result and the current part
136   of the text splits on two smaller parts and processed in the same way. Finally, the
137   algorithm outputs a set of the non-intersected intervals of clones of pattern p in text
138   t.

Algorithm 5.1 Greedy subroutine

Input: $rmq2D$— range maximum query data structure for perfoming range queries
on monge matrix M, h — theshold value, i, j — start and end positions of current
text $t_{i,j}$
Output: Set of non-intersected intervals from $t_{i,j}$
Pseudocode:
$greedy(rmq2D, h, i, j, t_{i,j}) :$

1: $interval = rmq2D.query(i, j, i, j)$
2: $result = \emptyset$
3: **if** $interval.score < h$ **then**
4:     **return** $result$
5: **end if**
6: **if** $interval.i - i \geq 1$ **then**
7:     $cl = greedy(rmq2D, h, t_{i,interval.i})$
8:     $result.add(cl)$
9: **end if**
10: **if** $j - interval.j \geq 1$ **then**
11:    $cl = greedy(rmq2D, h, t_{j,interval.j})$
12:    $result.add(cl)$
13: **end if**
14: **return** $result$

Algorithm 5.2 GREEDY-PATTERN BASED NEAR DUPLICATE SEARCH AL-
GORITHM

Input: monge matrix M that correspond to string-substring matrix for pattern p and
text t, theshold value h

Output: Set of non-intersected clones of pattern p in text t

Pseudocode:

$GreedyMathing(M, h, t)$

  1: $rmq2D = buildRMQStructure(M)$
  2: $result = greedy(rmq2D, 0, |t|, t)$
  3: **return** $result$

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139   The second algorithm 5.3 uses a less sophisticated approach and a more light-
140 weight one but found fewer duplicates of pattern p(see example **??**). The algorithm
141 also follows a greedy approach but instead of looking at the uncovered part of text t
142 at each step it looks at the text t and chooses the first available substring with the
143 highest score that doesn't intersect with already taken substrings. More formally, it
144 approximates algorithm 5.2.

145   *Algorithm description.* First, the *semi-local sa* problem is solved (Line 1). Then
146 we solve *complete approximate matching problem* (Line 3) i.e for each prefix of text t
147 we find the shortest suffix that has the highest similarity score with pattern p (Line
148 3):

149 (5.2)
$$a[j] = \max_{i \in 0..j} sa(p, t[i, j]), j \in 0..|t|$$

150   Further, we remove suffixes whose similarity is below the given threshold h (Line
151 4). Then remaining suffixes are sorted in descending order (Line 5) and the interval
152 tree is built upon them (Lines 7-11). The building process comprises from checking
153 that current substring *candidate* not intersected with already added substrings to
154 *tree* and adding it to *tree*. Finally, algorithm output set of non-intersected substrings
155 (clones) of pattern p in text t.

Algorithm 5.3 Greedy approximate

Input: pattern p, text t, theshold value h

Output: Set of non-intersected clones of pattern p in text t

Pseudocode:

  1: $sa = semilocalsa(p, t)$
  2: $matrix = sa.getStringSubstringMatrix()$
  3: $colmax = smawk(matrix)$
  4: $colmax = colmax.filter(it.score >= h)$
  5: $colmax = colmax.sortedByDescending(it.score)$
  6: $tree = buildIntervalTree()$
  7: **for** $candidate \in colmax$ **do**
  8:    **if** $candidate \cap tree = \emptyset$ **then**
  9:      $tree.add(candidate)$
10:    **end if**
11: **end for**
12: $result = tree.toList()$
13: **return** $result$

THEOREM 5.1. *Algorithm 5.3 could be solved in* $\max(O(|p|*|t|*|v|), O(|t|*\log^2|t|v))$ *time with* $O(|t|*v*\log|t|*v)$ *space when* $|p| < |t|$ *where p is pattern, t is text and v is denominator of normalized mismatch score for semi-local sequence alignment* $w_{normalized} = (1, \frac{\mu}{v}, 0)$ *assuming we are storing solution matrix implicitly.*

First phase. *As shown in section 2 the time complexity of solving* $semi-localsa$ *is* $O(|p|*|t|*|v|)$. *The space complexity of storing monge matrix of semi-local solution is* $O(|t|*v*\log|t|*v)$ *at most due to fact that* $v-subbistochasticmatrix$ *has at most v non-zeros in each row and upon these* $v*|t|$ *points we build two dimensional range tree data structure with* $|t|*v*\log|t|*v$ *nodes that have report range sum queries in* $O(\log^2|t|v)$ *time.*

Second phase. $SMAWK$ *algorithm requires* $O(|t|*q)$ *time where q stands for time complexity of random access of monge matrix. Thus, the total time complexity of line 3 is* $O(|t|*\log^2|t|v)$. *Filtering and sorting have at most* $O(|t|)$ *and* $O(|t|*log|t|)$ *time complexity. In Line 6 simple intialization of interval tree is perfomed that requires* $O(1)$.

Third phase *colmax array has as worst case* $O(|t|)$ *elements when filtering does not eliminate any substrings. Thus, adding to interval tree (both operation at most require* $O(\log|t|)$ *time) as well as intersection in (Lines 8-9) will be perfomed at most* $O(|t|)$. *Thus, the total complexity of last phase is* $O(|t|*\log t)$.

As we see, the third phase is dominated by the second phase in terms of running time and second phase is dominated by the space complexity of third phase. Thereby,the total time and space complexity is $\max(O(|p|*|t|*|v|), O(|t|*\log^2|t|v))$ *and* $O(|t|*v*\log|t|*v)$ *respectively.*

COROLLARY 5.2. *Algorithm 5.3 could be solved in* $\max(O(|p|*|t|), O(|t|*\log|t|))$ *when* $v = O(1)$.

When $v = O(1)$ *we will use simple range tree for orthogonal range queries with* $O(log|t|)$ *query time.*

COROLLARY 5.3. *Algorithm 5.3 could be solved in* $O(|p|*|t|)$.

When amount of clones is relatively small and threshold value is set high then after filtering out t intervals (Line 4) sorting is perfomed on s small set of elements. Thus, this part is dominated by calculating semi-local sa solution.

THEOREM 5.4. *Algorithm 5.2 could be solved in* $\max(O(|p|*|t|*v), O(|t|*\log|t|))$ *time with* $O(|t|\log|t|)$ *space when* $|p| < |t|$ *where p is pattern, t is text and v is denominator of normalized mismatch score for semi-local sequence alignment* $w_{normalized} = (1, \frac{\mu}{v}, 0)$.

On the first phase of alg

The first phase of algorithm requires $O(|p|*|t|*v)$ *with* $O(|t|*v)$ *additional space for stroring monge matrix implicitly. We denote this matrix, specifically it's lower-left quadrant that refers to string-substring solution as M with size* $|t| \times |t|$.

Theorema 3.4 First, note that

Building structure for rmq queries for staircase matrix requires Theorem 5.8. Given an n n partial Monge matrix M, a data structure of size O(n) can be constructed in O(n logn) time to answer submatrix maximum queries in O(log logn) time.

Proof it

$$D = \text{diag}(d_1, \ldots, d_n)$$

COROLLARY 5.5. *Algorithm 5.2 could be solved in* $\max(O(|p|*|t|), O(|t|*\log|t|))$ *when* $v = O(1)$.

6. Evaluation.

Semi-local algorithms. Show perfomance between lcs and semi-local lcs??? and poor perfomance of recursive algorithm based on steady ant?

Approximate matching algorithms. Show outperforming for different cases between luciv and our algorithm.

Show quality betwee our new algo and luciv algo (our should be better)

Show that sparse table bad when large?

7. Conclusion. Say may be succesfully be applied on practice (showed by algorithm luciv updated)

Open problem.$->$

Say that need to implement with monge2020 (what we not finished)

Improve algo based on recursive steady ant. Because it's critical for algos based on it.

df[1]

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