Operations Research Project

1 Mathematical Modelization

Some of the notations are taken from the subject.

1.1 Variables

We define the following variables :

x_s	1 if the production site s is build, else 0
a_s	1 if the production site s is automatized , else 0
y_s	1 if the distribution site s is build, else 0
$ma_{s\,i}$	1 if client i ∈ I is delivered by the automatized production site s, else 0
$mna_{s\,i}$	1 if client i ∈ I is delivered by the non automatized production site s, else 0
$B_{s\,s'}$	1 if distribution site $s^{'}$ is linked to production site s, else 0
$ba_{s\,s'\,i}$	1 if client i is delivered by s' ∈ D linked to s ∈ S automatized, else 0
$bna_{s\,s'\,i}$	1 if client i is delivered by s' ∈ D linked to s ∈ S non automatized, else 0

1.2 Objective Function

The objective is to minimize the sum of the following costs :

Building costs

$$BC = \sum_{s\,\in S} c_P^b\,x_s + c_A^b\,a_s + c_D^b\,y_s$$

Production costs

$$PC = \sum_{i\,\in i} d_i \left[\sum_{s\,\in\,S} (ma_{s\,i}\,(c_P^p - c_A^p) + c_P^p\,mna_{s\,i}) + \sum_{s,\,s'\,\in\,S} ba_{s\,s'\,i}\,(c_D^p - c_A^p + c_P^p) + bna_{s\,s'\,i}\,(c_D^p + c_P^p) \right]$$

Distribution costs

$$DC = \sum_{s\,\in\,S,\,i\,\in\,I} c_2^r\,\Delta(s,i)\,(ma_{s\,i} + mna_{s\,i}) \quad + \sum_{s\,s'\in\,S,\,i\,\in\,I} (c_2^r\,\Delta(s',i) + c_1^r\,\Delta(s,s'))\,(ba_{s\,s'\,i} + bna_{s\,s'\,i})$$

Capacity costs

We denote C_s the exceeding production of site s. This quantity is negative if the site s does not exceed its capacity.

$$C_s = \sum_{i\,\in\,I} d_i\,(ma_{s\,i} + mna_{s\,i} + \sum_{s'\,\in\,S} ba_{s\,s'\,i} + bna_{s\,s'\,i}) - \left(u_P\; + a_s\,u_A\right)$$

The capacity cost is thus

$$CC = \sum_{s \in S} c^u \; [C_s]^+$$

To ensure linearity of the model, we introduce a dummy variable T :

$$\begin{aligned} T_s &\geqslant 0 \qquad && \forall s \in S \\ T_s &\geqslant C_s && \forall s \in S \end{aligned}$$

Therefore, in the model, the capacity cost that will be minmized is :

$$CC = \sum_{s \in S} c^u \; T_s$$

1.3 Contraints

We cannot build different types of site at the same place :

$$x_s + y_s \leqslant 1 \qquad \forall \, s \in S$$

Each client must be delivered by one and only one site :

$$\sum_{s \in S} (ma_{s\,i} + mna_{s\,i}) + \sum_{s\,s' \in S} ba_{s\,s'\,i} + bna_{s\,s'\,i} = 1 \qquad \forall \, i \in I$$

Each distribution site must be linked to one and only one production site.

$$\sum_{s \in S} B_{s\,s'} = y_{s'} \qquad \forall \, s' \in S$$

We must now add constraints that link variables.

For instance, if a distribution site s' is linked to a production site s , the two site must be build :

$$0.5 \; (x_s + y_{\,s'}) \geqslant B_{s\,s'} \qquad \forall \, s \, , s' \in \; S$$

Another example is that if a client i is linked to an automatized production site s . The latter must be automatized :

$$a_s \geqslant ma_{s\,i} \qquad \forall \, s \in S, \, i \in I$$

After considering these constraints, we obtain the following model :

1.4 MILP Description

$$
\begin{array}{lll}
\min & BC + PC + RC + DC & \\
\text{s.c} & x_s + y_s \leqslant 1 & \forall\, s \in S \\
& \displaystyle\sum_{s \in S} (ma_{s\,i} + mna_{s\,i}) + \sum_{s,\, s' \in S} (ba_{s\,s'\,i} + bna_{s\,s'\,i}) = 1 & \forall\, i \in I \\
& \displaystyle\sum_{s \in S} B_{s\,s'} = y_{s'} & \forall\, s' \in S \\
& T_s \geqslant 0 & \forall\, s \in S \\
& T_s \geqslant C_s & \forall\, s \in S \\
& x_s \geqslant a_s & \forall\, s \in S \\
& 0.5\,(x_s + y_{s'}) \geqslant B_{s\,s'} & \forall\, s\,, s' \in S \\
& a_s \geqslant ma_{s\,i} & \forall\, s \in S,\ i \in I \\
& 1 - a_s \geqslant mna_{s\,i} & \forall\, s \in S,\ i \in I \\
& a_s \geqslant ba_{s\,s'\,i} & \forall\, s, s' \in S,\ i \in I \\
& 1 - a_s \geqslant bna_{s\,s'\,i} & \forall\, s, s' \in S,\ i \in I \\
& B_{s\,s'} \geqslant ba_{s\,s'\,i} + bna_{s\,s'\,i} & \forall\, s\,, s' \in S,\ i \in I
\end{array}
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