1. Normalization

The first step is to normalize both 3d and 2d points. For this we subtract the mean x and y (and z) from the respective coordinates. Then we multiply all points by α where:

$$\alpha_{2d} = \frac{\sqrt{2}}{\frac{1}{N}\sum x_i^2 + y_i^2} \; ; \; \alpha_{3d} = \frac{\sqrt{3}}{\frac{1}{N}\sum x_i^2 + y_i^2 + z_i^2}$$

This provides us with points centered around the origin and with the desired average distance. These two transformations are implemented in a matrix form using homogeneous coordinates.

2. DLT

The DLT algorithm finds P in $x = PX$. It is implemented as described in the course by building the given 2Nx12 matrix and solving it using SVD. The solution is directly given as the last column of the V matrix. Then we need to factorize this result into $P = K[R|t]$. For this we do a RQ decomposition of the top left 3x3 part of P and then $t = K^{-1}P(:,4)$. Finally, the error is computed as :

$$error = \frac{1}{N}\sum norm(xy_i - xy_{i_{projected}})$$

3. Gold Standard Algorithm

For the gold standard algorithm, we start by repeating the DLT and then we use that result as the starting point of a non-linear cost optimization where the variables are the elements of P and the cost is the error described above.

4. Results

The algorithms described above were run with 6 points and then with 19 points to improve the results. The following figures present the results.

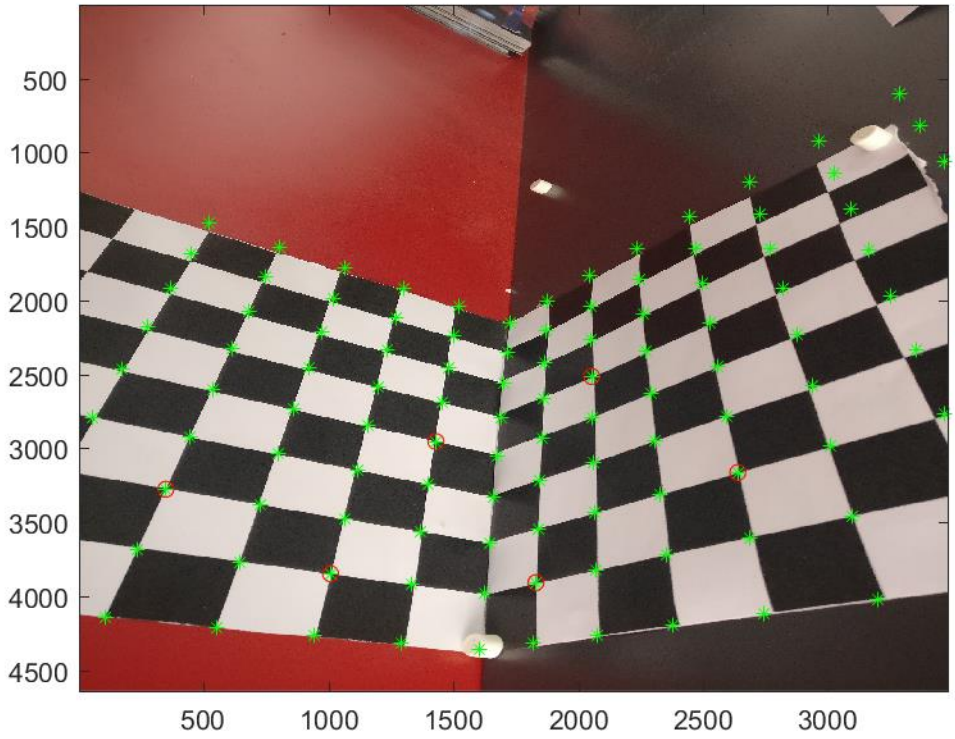


Figure 1: DLT 6 points

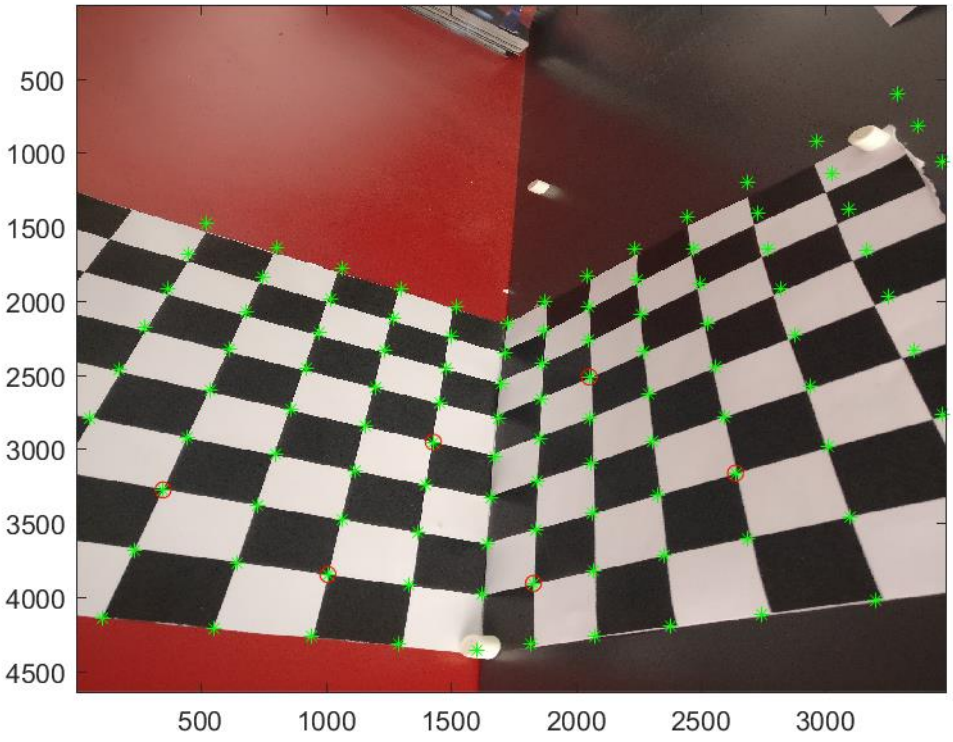


Figure 2: Gold standard 6 points

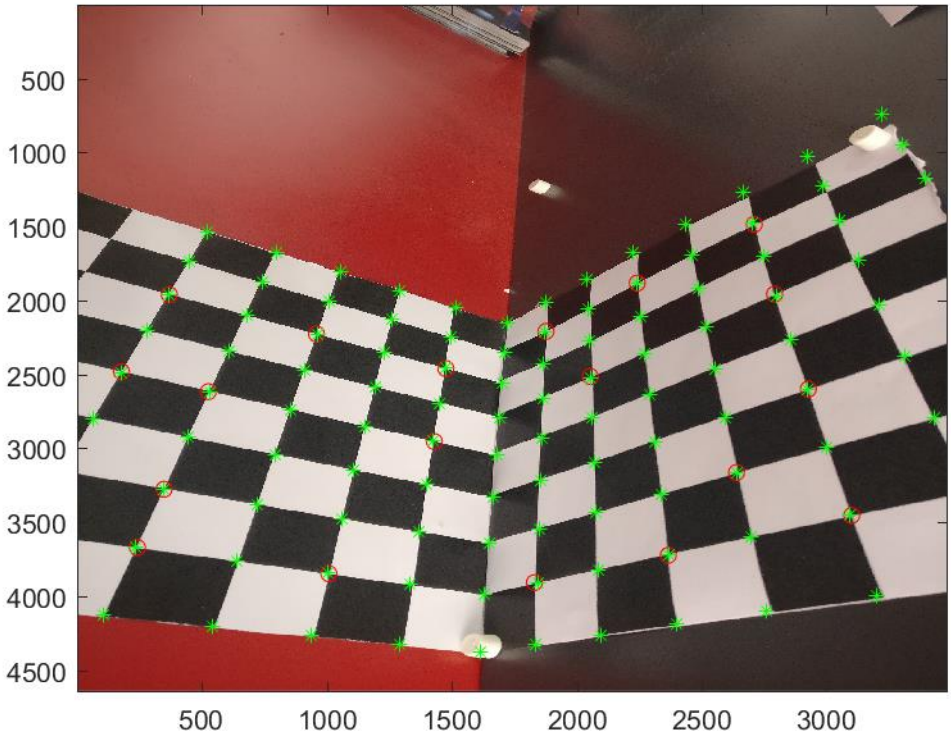


Figure 3: DLT 19 points

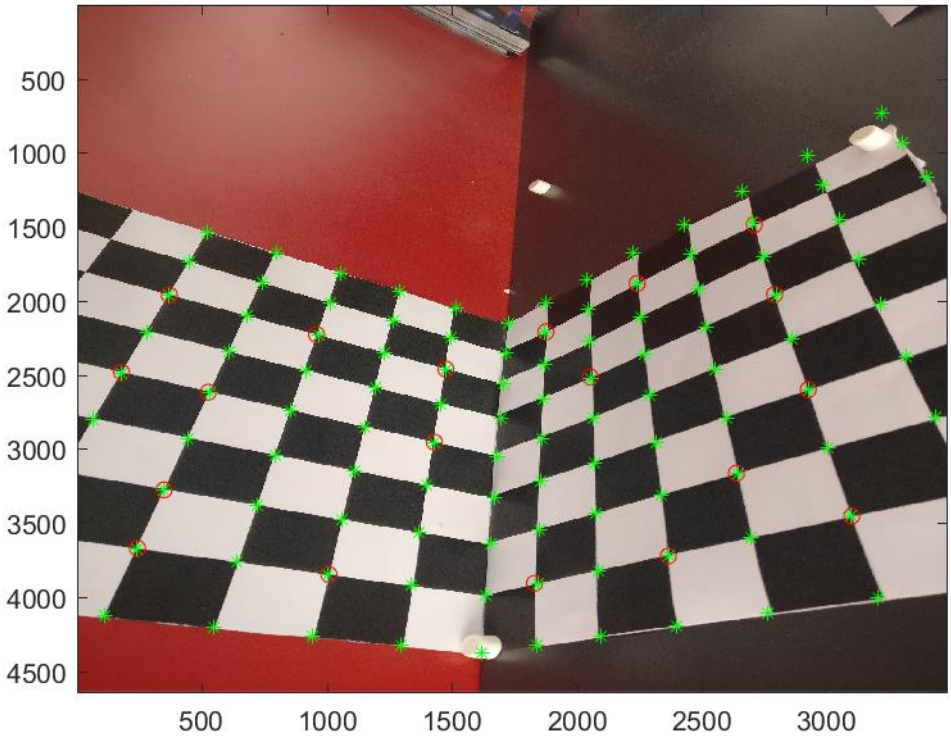


Figure 2: Gold standard 19 points

	ERROR
DLT 6 POINTS	0.1076
GOLDEN STANDARD 6 POINTS	0.1026
DLT 19 POINTS	6.1196
GOLDEN STANDARD 19 POINTS	5.6280

We can see that there is a big difference in the error between 6 and 19 points. Actually, after looking at other projected corners we see that the 19 points work better even though the calculated error on the known points is worse. This can be explained by the fact that the 6 points version overfits the errors in the given points and as such predicts worse other corners.

There is not a big difference between DLT and Gold standard. Nevertheless, Gold standard performs slightly better in both cases in terms of error and on the final image.

Strangely I do not notice any difference when using unnormalized points. I assume in some cases unnormalized points can lead to errors in the SVD decomposition and as such in the final solution.

For the best result (gold standard 19 points) the obtained parameters are :

K=

3554.003	-14.939	1736.278
0.000	3607.961	2579.023
0.000	0.000	1.000

R=

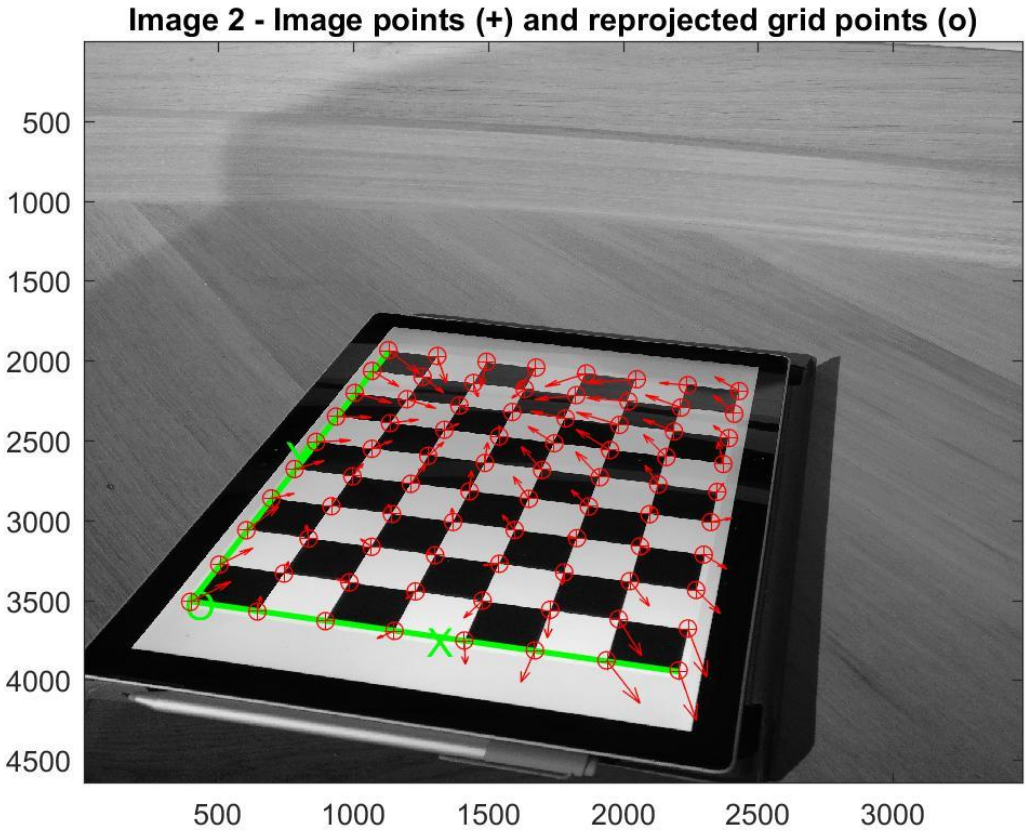
-0.8147	0.5790	0.0309
-0.3607	-0.4644	-0.8088
-0.4540	-0.6701	0.5872

t= [-0.00806;0.12941;0.25956]

5. Bouget's Calibration Toolbox

When using the toolbox with 6 images the following result is obtained:

We we



We see that the reprojection error is much smaller than in our previous implementation.

We also get:

K=

3696.86488	0	1761.97395
0	3687.19125	2420.27978
0	0	1

The calibration result (K) is reasonably close to the one we obtained before.