№ 1.

$$E_{x,y} E_{X^\ell}(y - a_{X^\ell}(x))^2 = E_{x,y} E_{X^\ell}(y - E_{X^\ell} a_{X^\ell}(x) + E_{X^\ell} a_{X^\ell}(x) - a_{X^\ell}(x))^2 =$$

$$= E_{x,y} E_{X^\ell}(y - E_{X^\ell} a_{X^\ell}(x))^2 + \underbrace{2 E_{x,y} E_{X^\ell}(y - E_{X^\ell} a_{X^\ell}(x))(E_{X^\ell} a_{X^\ell}(x) - a_{X^\ell}(x))}_{0} +$$

$$+ \underbrace{E_{x,y} E_{X^\ell}(a_{X^\ell}(x) - E_{X^\ell} a_{X^\ell}(x))^2}_{variance} = E_{x,y} E_{X^\ell}(y - E_{X^\ell} a_{X^\ell}(x))^2 + variance =$$

$$= E_{x,y} E_{X^\ell}(y - E(y|x) + E(y|x) - E_{X^\ell} a_{X^\ell}(x))^2 + Var =$$

$$= \underbrace{E_{x,y}(y - E(y|x))^2}_{noise} + \underbrace{2 E_{x,y} E_{X^\ell}(y - E(y|x))(E(y|x) - E_{X^\ell} a_{X^\ell}(x))}_{0} +$$

$$+ E_{x,y}(E(y|x) - E_{X^\ell} a_{X^\ell}(x))^2 + variance =$$

$$= noise + bias + variance$$

№2.

$$a(x) = \frac{1}{M}\sum_{m=1}^{M} a_m(x)$$

$$E_{x,y,x,y}(a(x)) = \frac{1}{M} E_{x,y,x,y}(a_m(x)) = E_{x,y,x,y}(a_1(x))$$

$\Rightarrow$ Смещение не изменится.

$$Var_{x,y,x,y}(a(x)^2) = \frac{1}{M^2} Var_{x,y,x,y}\left(\sum_{i,j} a_i(x)\, a_j(x)\right) = \frac{1}{M} Var_{x,y,x}\, a_1(x) +$$

$$+ \frac{1}{M^2}\sum_{i \neq j} cov(a_i(x), a_j(x))$$

Пусть коэфф. корреляции одинаковый у всех пар и равен $\rho \Rightarrow$

$$\Rightarrow \frac{1}{M} Var_{x,y,x}\, a_1(x) + \frac{1}{M^2}\sum_{i\neq j} cov(a_i(x), a_j(x)) = \frac{1}{M} Var_{x,y,x}\, a_1(x) +$$

$$+ \frac{1}{M^2}\sum_{i\neq j} \rho\, Var_{x,y,x}\, a_1(x) = \left(\frac{1}{M} + \frac{\rho(M-1)}{M}\right) Var_{x,y,x}\, a_1(x)$$

№ 3.

$$D\left(\sum_{i=1}^{M} \xi_i / M\right) = E\left(\sum_{i=1}^{M} \xi_i / M - E\left(\sum_{i=1}^{M} \xi_i / M\right)\right)^2 =$$

$$= \frac{1}{M^2} E\left(\sum_{i=1}^{M} (\xi_i - E\xi_i)\right)^2 = \frac{1}{M^2}\left(\sum_{i=1}^{M} E(\xi_i - E\xi_i)\right)^2 \cancel{=}$$

$$= \frac{1}{M^2}\left(\sum_{i=1}^{M} E(\xi_i - E\xi_i)^2 + \sum_{i \neq j}^{M} E\left((\xi_i - E\xi_i)(\xi_j - E\xi_j)\right)\right) =$$

$$= \frac{1}{M^2}\left(M\sigma^2 + M(M-1)\rho\sigma^2\right) = \rho\sigma^2 + \frac{1}{M}(1-\rho)\sigma^2$$