Eternity: Numbers

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SOEN 6481
Software Requirements Specifications

Under the guidance of
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Abstract

The document tries to put some light on the understanding of an irrational number, The Champernowne Constant. A brief description of the constant and some of its applications are included in the report. A research and interview on the constant was conducted with a resource who is familiar with Champernowne constant. My interviewee tried to answer most common questions about the constant in a questionnaire that also describes some applications of Champernowne constant. Based on the understanding of Champernowne Constant, a calculator application is to be built, that uses this constant to perform certain operations. This document also gives the basic design details on how the product would look like,what operations is it capable of, its algorithm and use cases.

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Chapter 1

Introduction

1.1 Champernowne's Constant Definition

Champernowne Constant is a real number whose decimal digits are obtained by concatenating the decimal expansions of the successive positive integers:

$$C_{10} = 0.12345678910111213141516\ldots$$

It is named after economist and mathematician David G. Champernowne, who published it as an undergraduate in 1933.

1.2 Champernowne Constant as an Infinite series

The Champernowne constants can be expressed exactly as infinite series:

$$C_m = \sum_{n=1}^{\infty} \frac{n}{10_b^{\left(\sum_{k=1}^{n} \lceil log_{10_b}(k+1) \rceil\right)}}$$

where $\lceil x \rceil$ = ceiling(x), ${10_b}^x = b^x$ in base 10, $log_{10_b}(x) = log_{b_{10}}(x)$ and b is the base of the constant.

1.3 Continued Fraction Expansion

The Continued Fraction Expansion (CFE) of the Champernowne constant turns out to be a set large numbers with various spikes. Kurt Mahler showed that the constant is transcendental; therefore, its continued fraction does not terminate and is aperiodic (because it is not an irreducible quadratic). The terms in the continued fraction expansion exhibit very erratic behaviour, with extremely large terms appearing between many small ones. The CFE begins with 0; 8, 9, 1, 149083, 1, 1, 1, 4, 1, 1, 1, 3, 4, 1, 1, 1, 15, and the coefficient in position 18 has 166 digits. The large number at position 18 has 166 digits,

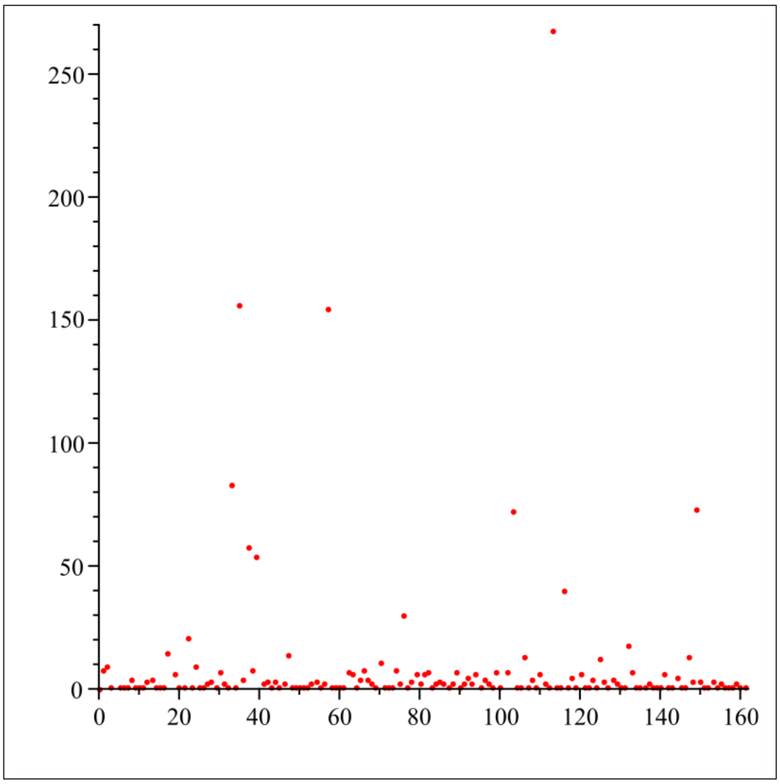

Figure 1.1: The first 161 quotients of the Champernowne constant.

and the next very large number at position 40 of the continued fraction has 2504 digits. The fact that there are such large numbers as terms of the continued fraction expansion is equivalent to saying that the convergents obtained by stopping before these large numbers provide an exceptionally good approximation of the Champernowne constant.

Chapter 2

Interview

2.1 Brief introduction about the Interviewee

Name : Megha Kamath
Qualification : Masters in Mathematics
Reason behind opting the interviewee : My Interviewee is pursuing Masters in Mathematics and it was obvious for me to work with her to better understand about the Champernowne constant and its applications.

2.2 Interview Questions on Champernowne Constant

1. Could you mention some areas where Champernowne constant can be applied?
The Champernowne constant has seemingly random numbers which are clearly well determined. This property could be useful in functional programming contexts where one would need explicit randomness which is entirely deterministic. For example, the C2 (Base 2) value of the Champernowne constant can be used as a binary random number to trigger true or false conditions randomly.

2. Could you mention the properties of the Champernowne constant?

   (a) *The constant given by 0.123456789101112 . . . is normal in base ten.*

   (b) *The constant is transcendental.*

   (c) *The constant also has a peculiar continued fraction expansion. It namely contains exceptionally large terms throughout the expansion.*

3. Is Champernowne constant a Liouville number?
No. Champernowne constant and Liouville numbers are both transcendental but Champernowne constant is irrational and Liouville numbers are almost rational and can be approximated quite closely by sequences of rational numbers than any algebraic irrational numbers.

4. Is Champernowne constant as useful as π constant?
The Champernowne constant is quite useful. But Pi is a constant which is there almost everywhere in mathematics.

5. Does it appear in any sort of geometric solutions like π does?
   The Champernowne constant cannot be represented as a finite number due to which there have been no reported utilization in geometry.

6. How often do you use the Champernowne constant?
   As a student of pure mathematics I do not use the Champernowne constant quite often.

7. Has the Champernowne constant ever been used in any major proofs?
   No.

8. Can it be expressed in terms of e, π or both?
   No.

9. What works of Alan Turing and David Champernowne made use of the Champernowne constant?
   Notably, There were two major works of Alan Turing and David Champernowne, the TuroChamp and Round the house Chess. But the Champernowne constant is not mentioned to have been in any of these machines.

10. Of all the base versions of Champernowne constants which particular Champernowne sequence has been vastly used?
    Base 2 (C_2) and Base 10 (C_{10}).

11. Who and How was Champernowne constant proved Transcendental?
    The Champernowne constant was shown to be transcendental by Kurt Mahler in 1937.

12. What are the alternatives available for Champernowne constant?
    None.

13. Is Champernowne constant used in Turochamp or Round the house Chess? if Yes, how does it fit into the algorithms of these games?
    No.

14. Do you think of any problem where Champernowne Constant would be a perfect fit to be used?
    I can actually think of 2, One being a Random Binary digit generator where Champernowne Constant of Base 2 would be a perfect fit and Two, Given a sequence of numbers (to base 10), How to calculate the position of any random number. I think Champernowne constant fits well into these scenarios.

2.3 Analysis of the Interview

The Interviewee had moderate knowledge about the Champernowne constant. She was able to explain the basic characteristics of the constant with logical examples (mentioned in the response to the interview questions). The practical applications of the constant have been limited and the interviewee was able to explain one particular area where the constant (its base 2 value) is widely used to generate the randomized binary numbers.

Chapter 3

User model: Persona Template

Private Information

- Name: Megha Kamath
- Qualification: Currently pursuing Masters in Mathematics
- University: Mangalore University, Karnataka, India
- Megha lives in Karnataka, India with her parents.
- She loves to read fictional novels and also cooking is her favourite pass-time.

Use of Number, relation to Number

Based on the conversation with interviewee -

- The Champernowne constant has seemingly random numbers which are clearly well determined. This property could be useful in functional programming contexts where one would need explicit randomness which is entirely deterministic.
- Base 2 of the Champernowne constant is in functional programming as a Random Binary Digit Generator.

Description of work or daily life

- Megha is in her First Year of Masters in Mathematics. Her specialization is Pure Mathematics.
- She has moderate knowledge about the Champernowne Constant.
- She does not use the Champernowne constant quite often as her specialization is Pure Mathematics.

Other uses or relations to the Number

- As per Megha's research, the Champernowne constant was never used in major proofs.
- Due to the constant's inefficient nature, the constant is mostly avoided in mathematical calculations.

Influencers that surround the persona and that may influence choices

- Professors
- Friends
- Classmates

Chapter 4

Problem Domain Model

4.1 UML Class Diagram

Figure 4.1 shows the Class diagram of the Eternity: Numbers system which shows the different concepts, properties of each concepts and the relationship between the concepts

- **Interface: Calculator.java:** Interface class that defines the calculate method signature for which is implemented by the classes that perform operations on the Eternity: Numbers.
- **Class: EventProcessor.java:** Is the main class of the calculator. It is the starting point of the product, Eternity: Numbers and it does the following operations:
  - Allows user to pick a constant to perform calculations,
  - Invokes the relevant calculator class to process user requests and
  - Display the processed/calculated result to user.
- **Class: EulersCalculator.java:** This class performs key operations on Euler's constant. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.
- **Class: GaussianCalculator.java:** This class performs key operations on Gaussian Integral constant. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.
- **Class: GelfondCalculator.java:** This class performs key operations on Gelfond's constant. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.
- **Class: GoldenRatioCalculator.java:** This class performs key operations on Golden Ratio. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.
- **Class: LiouvilleCalculator.java:** This class performs key operations on Liouville constant. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.

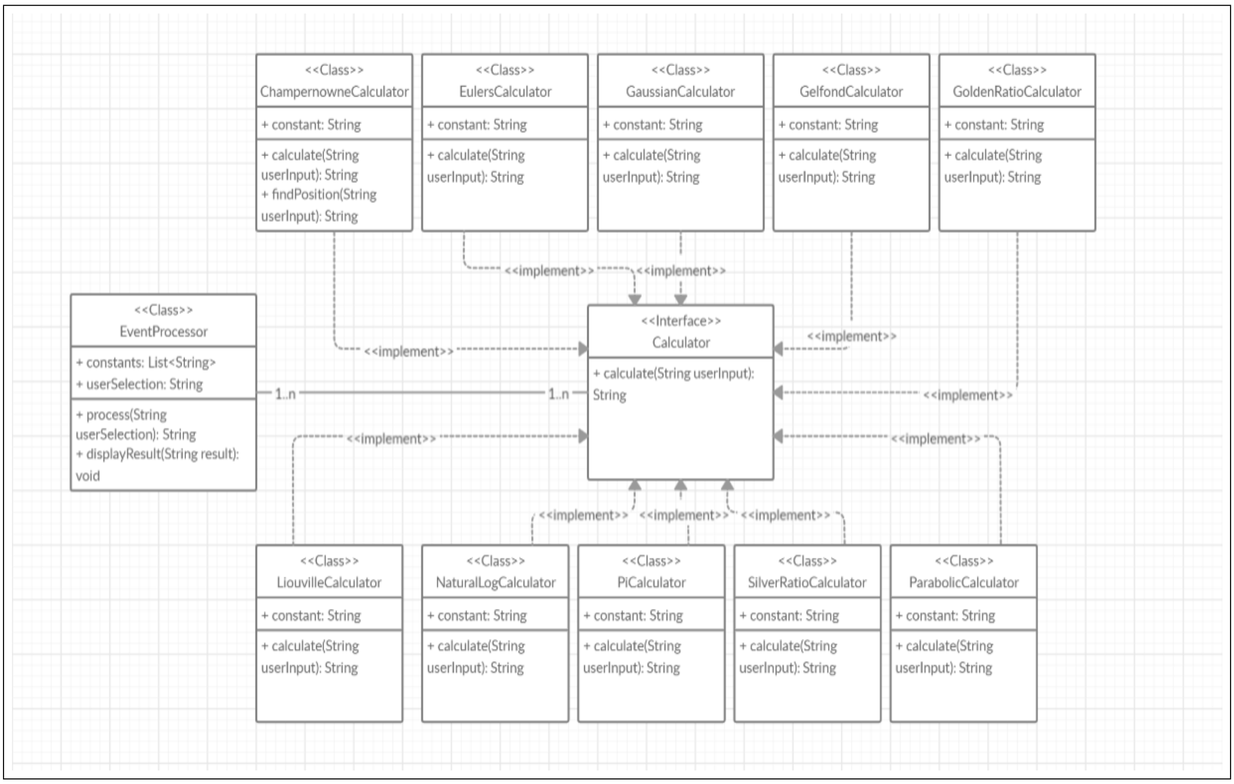

Figure 4.1: UML Class Diagram of Eternity: Numbers

- **Class: NaturalLogCalculator.java:** This class performs key operations on Natural Logarithm. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.

- **Class: PiCalculator.java:** This class performs key operations on Pi constant. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.

- **Class: SilverRatioCalculator.java:** This class performs key operations on Silver Ratio. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.

- **Class: ParabolicCalculator.java:** This class performs key operations on Universal Parabolic constant. Since our focus is more on the Champernowne constant. This class is designed to request user to choose Champernowne constant from the startup menu.

Chapter 5

Use Case Model

5.1 UML Use Case Diagram

Steps in the Use case model of Eternity numbers calculator (Refer to Figure 5.1):

1. User starts the calculator application i.e. Eternity: Numbers constant. The application displays a set of constants(Eternity: Numbers), which a user can pick to perform various operations.

2. User selects the Champernowne Constant.

3. The applications prompts the user to enter any random number between 0 and 180000, for which the user would want to know the position in the Champernowne constant.

4. User enters the number.

5. The system processes user's input and displays the position of the number given by the user.

5.2 UML Activity Diagram

Figure 5.2 shows the Activity diagram representation of the above use case. If the user selects any other constant other than the Champernowne constant, a relevant message is displayed that the feature is not available as this version of the product focuses on Champernowne Constant only.

5.3 Normal Scenario of Use case model

Figure 5.3 shows the normal scenario in the use case model of the Eternity Numbers application

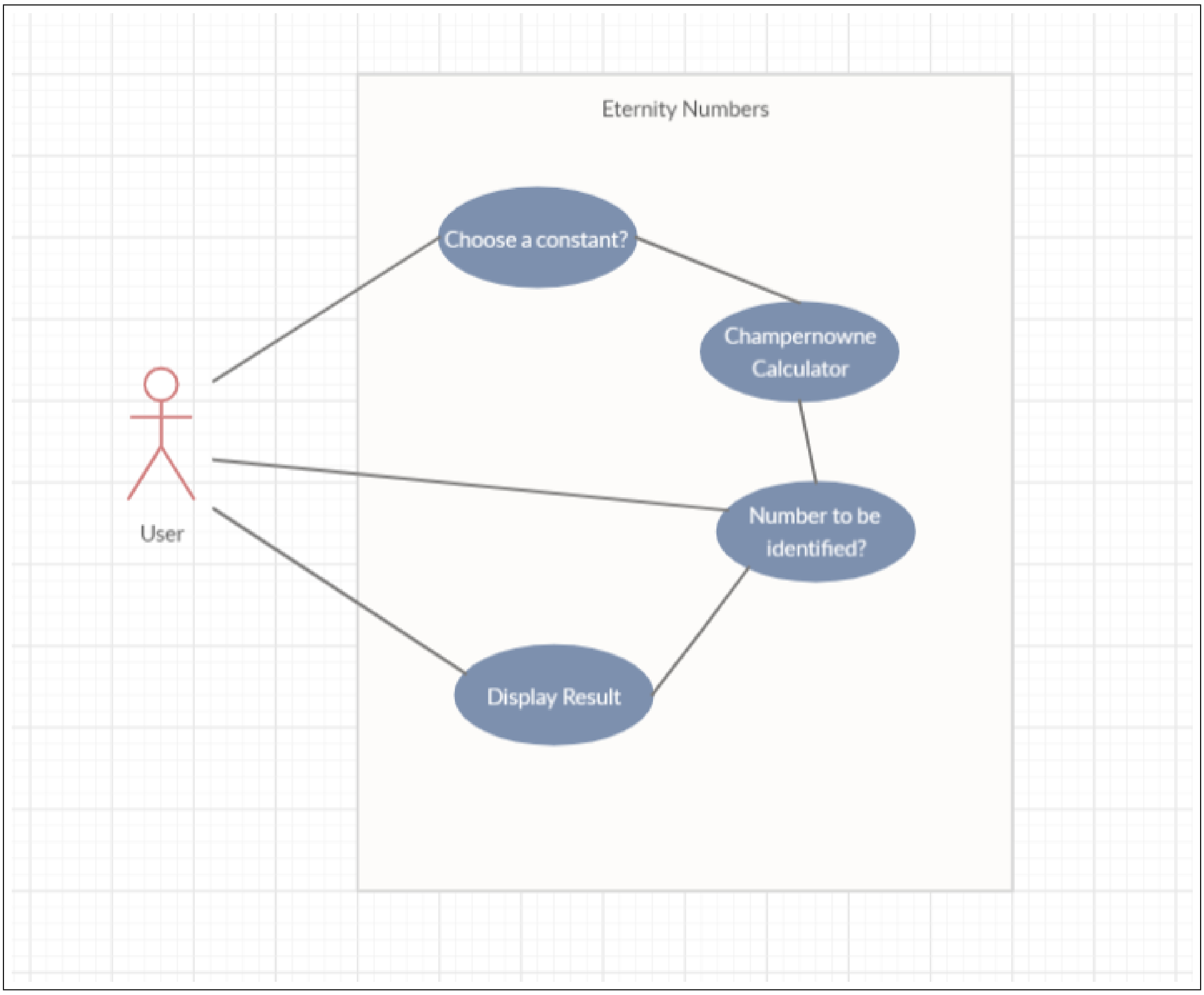

Figure 5.1: UML Use Case diagram of Eternity: Numbers

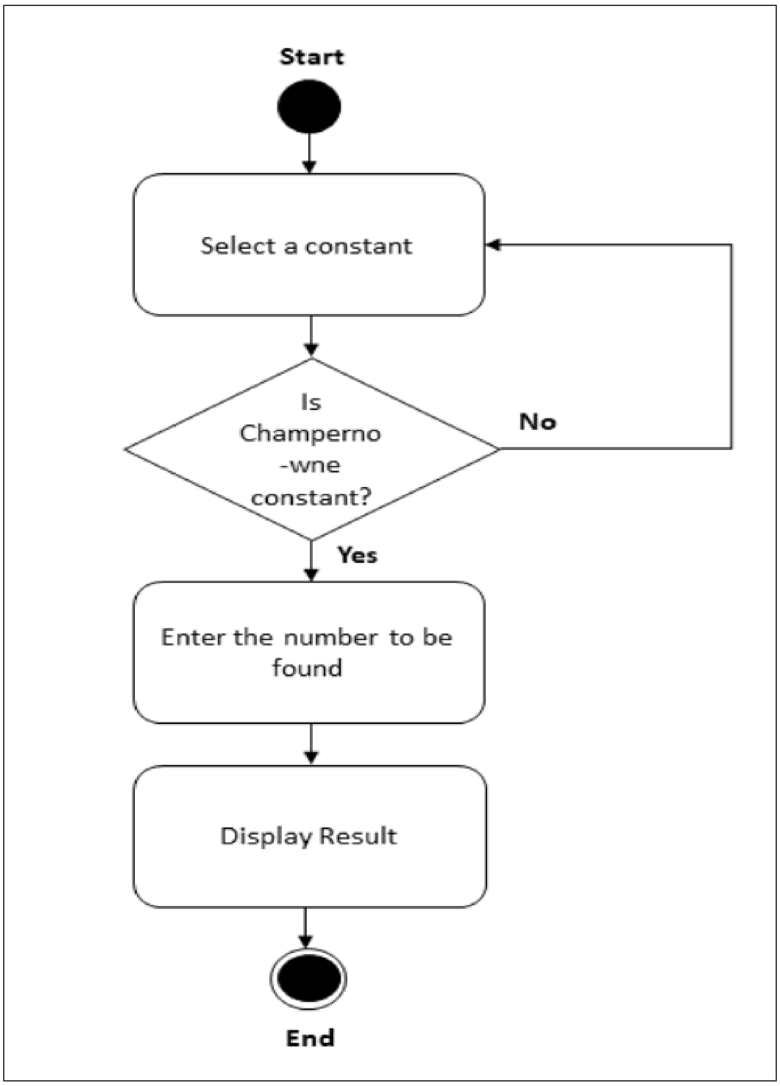

Figure 5.2: UML Activity diagram of Eternity: Numbers

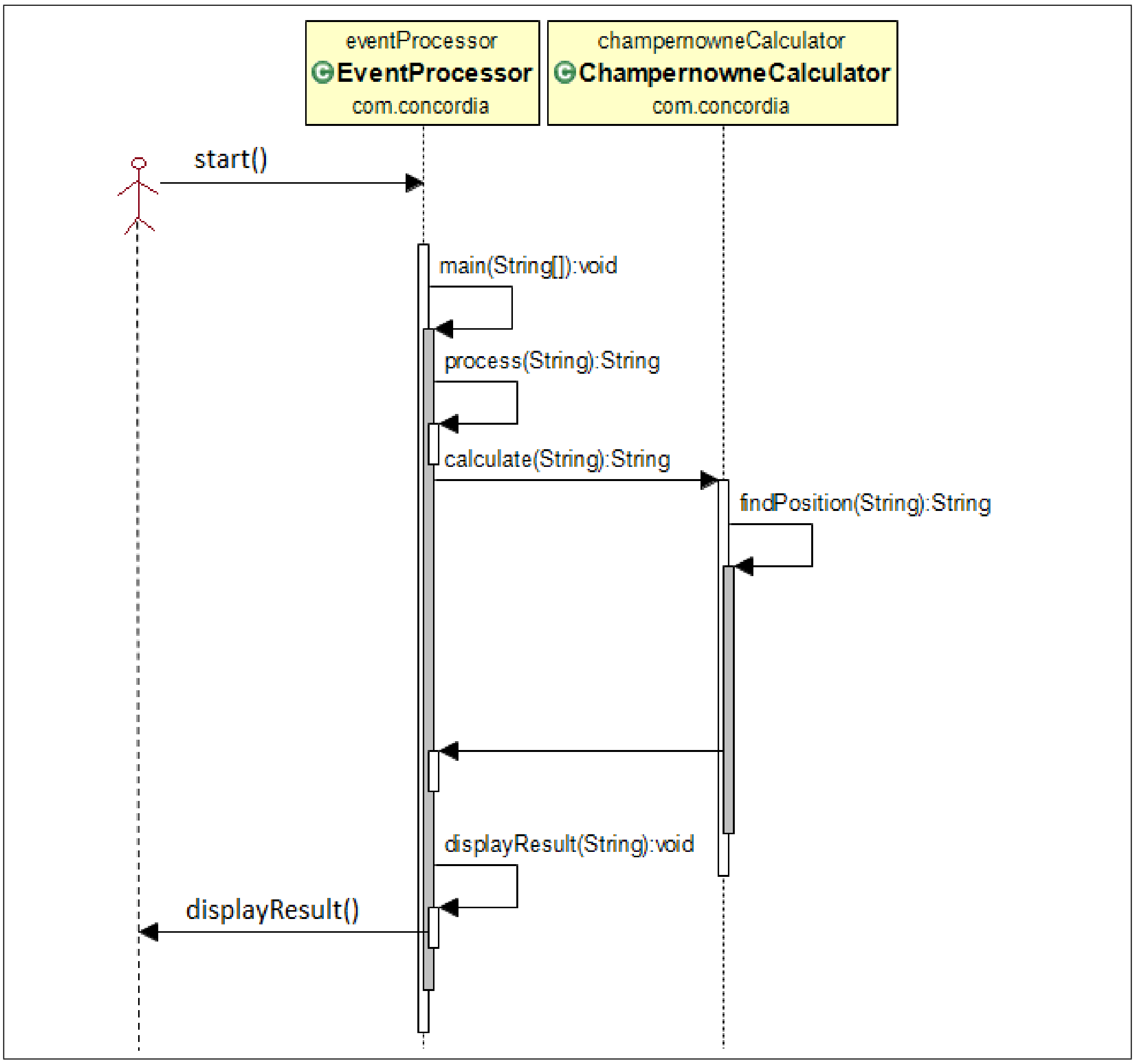

Figure 5.3: UML Sequence diagram of Eternity: Numbers

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