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1 Common

1 Setup

- 1. Terminal: font Monospace 12
- 2. Gedit: Oblivion, font Monospace 12, auto indent, display line numbers, tab 4, highlight matching brackets, highlight current line, F9 (side panel)
- /.bashrc: export CXXFLAGS='-Wall -Wshadow -Wextra -Wconversion -Wno-unused-result -Wno-deprecated-declarations -O2 -std=gnu++11 -g -DLOCAL'
- 4. for i in {A..K}; do mkdir \$i; cp main.cpp \$i/\$i.cpp; done

Template

```
main.cpp:
#define FNAME ""
#undef __STRICT_ANSI__
\#ifdef\ LOCAL
  #define _GLIBCXX_DEBUG
#endif
#include <bits/stdc++.h>
using namespace std;
#define pb push_back
#define mp make_pair
#define fs first
#define sc second
#define fst first
#define snd second
#define sz(x) (int)((x).size())
#define form(i,n) for (int i = 0; (i) < (n); ++i)
#define form f(i,n) for f(i,n) for f(i,n) = f(i,n) = 0; --i)
#define forab(i,a,b) for (int i = (a); (i) < (b); ++i)
#define forba(i,a,b) for (int i = (int)(b) - 1; (i) >= (a);
#define forit(it,c) for(__typeof((c).begin()) it =
\leftrightarrow (c). begin(); it != (c).end(); ++it)
#define all(c) (c).begin(),(c).end()
  #define eprintf(...) fprintf(stderr, __VA_ARGS__),
  \hookrightarrow fflush(stderr)
#else
  #define eprintf(...) (void) 0;
#endif
typedef long long LL;
typedef unsigned long long ULL;
typedef double dbl;
typedef long double LD;
typedef unsigned int uint;
typedef vector <int> vi;
typedef pair <int, int> pii;
int main() {
  return 0:
```

Stress

stress.sh:

```
!/bin/bash
for ((i = 0;; i++)); do
 ./gen $i >in || exit
  ./main <in >out1 || exit
  ./stupid <in >out2 || exit
 diff out2 out2 || exit
 echo $i OK
```

Java

```
Java template:
import java.io.BufferedReader;
import java.io.FileNotFoundException;
import java.io.FileReader;
import java.io.IOException;
import java.io.InputStreamReader;
import java.io.PrintWriter;
import java.util.*;
public class Main {
  FastScanner in;
  PrintWriter out;
  void solve() {
   int a = in.nextInt();
   int b = in.nextInt();
   out.print(a + b);
  void run() {
   try {
     in = new FastScanner("input.txt");
     out = new PrintWriter("output.txt");
     solve():
     out.flush();
     out.close():
    } catch (FileNotFoundException e) {
     e.printStackTrace();
     System.exit(1);
   }
  class FastScanner {
    BufferedReader br;
    StringTokenizer st;
    public FastScanner() {
     br = new BufferedReader(new
      public FastScanner(String s) {
       br = new BufferedReader(new FileReader(s));
     } catch (FileNotFoundException e) {
        e.printStackTrace();
   }
    String nextToken() {
     while (st == null || !st.hasMoreElements()) {
         st = new StringTokenizer(br.readLine());
        } catch (IOException e) {
         e.printStackTrace();
```

memcpy(a, x.a, sizeof(a[0]) * len);

len = x.len;

```
return st.nextToken();
  int nextInt() {
   return Integer.parseInt(nextToken());
  long nextLong() {
    return Long.parseLong(nextToken());
  double nextDouble() {
   return Double.parseDouble(nextToken());
  char nextChar() {
   trv {
     return (char) (br.read());
    } catch (IOException e) {
     e.printStackTrace();
    return 0;
  String nextLine() {
   trv {
     return br.readLine();
    } catch (IOException e) {
     e.printStackTrace();
    return "";
 }-
}
public static void main(String[] args) {
 new Main().run();
```

2 Big numbers

5 Big uint

}

```
#include <cstdio>
#include <cassert>
#include <cstring>
#include <cmath>
#include <algorithm>
#define form(i, n) for (int i = 0; i < (int)(n); i++)
typedef long long 11;
const int BASE_LEN = 9;
const int NUM_LEN = 50000 / BASE_LEN + 2; // LEN <= NUM_LEN *</pre>
\hookrightarrow \quad \textit{BASE\_LEN}
const int BASE = pow(10, BASE_LEN);
const ll INF = 8e18, ADD = INF / BASE;
struct num {
 11 a[NUM_LEN];
 int len; // always > 0
 inline const ll& operator [] ( int i ) const { return a[i];
 inline ll& operator [] ( int i ) { return a[i]; }
 num& operator = ( const num &x ) { // copy
```

```
return *this;
num( const num &x ) { *this = x; } // copy
num() \{ len = 1, a[0] = 0; \} // 0
num( 11 x ) { // x
 len = 0:
  while (!len || x) {
   assert(len < NUM_LEN); // to catch overflow
    a[len++] = x \% BASE, x /= BASE;
num& cor() {
  while (a[len - 1] >= BASE) {
    assert(len < NUM_LEN); // to catch overflow
    if (a[len - 1] >= 2 * BASE)
     a[len] = a[len - 1] / BASE, a[len - 1] %= BASE;
     a[len] = 1, a[len - 1] -= BASE;
   len++;
  }
 while (len > 1 && !a[len - 1])
   len--;
 return *this;
int length() {
 if (!len)
   return 0;
  int x = a[len - 1], res = 0;
  assert(x);
 while (x \mid \mid !res)
   x /= 10, res++;
  return res + (len - 1) * BASE_LEN;
void out() const {
 int i = len - 1;
  printf("%d", (int)a[i--]);
 while (i >= 0)
   printf("%0*d", BASE_LEN, (int)a[i--]);
 puts("");
void init( const char *s ) {
 int sn = strlen(s);
  while (sn && s[sn - 1] <= 32)
   sn--:
  len = (sn + BASE_LEN - 1) / BASE_LEN;
  memset(a, 0, sizeof(a[0]) * len);
  forn(i, sn) {
   11 &r = a[(sn - i - 1) / BASE_LEN];
    r = r * 10 + (s[i] - '0');
  }
bool read() {
 static const int L = NUM_LEN * BASE_LEN + 1;
  static char s[L];
 if (!fgets(s, L, stdin))
   return 0;
  assert(!s[L - 2]);
 init(s):
 return 1;
void mul2() {
```

```
forn(i, len)
                                                                  }
    a[i] <<= 1;
                                                                };
  forn(i, len - 1)
    if (a[i] >= BASE)
                                                                num& operator += ( num &a, const num &b ) {
                                                                  while (a.len < b.len)
      a[i + 1]++, a[i] -= BASE;
                                                                    a[a.len++] = 0;
  cor();
}
                                                                   forn(i, b.len)
                                                                    a[i] += b[i];
void div2() {
                                                                   forn(i, a.len - 1)
                                                                    if (a[i] >= BASE)
  for (int i = len - 1; i >= 0; i--) {
    if (i && (a[i] & 1))
                                                                      a[i] = BASE, a[i + 1]++;
     a[i - 1] += BASE;
                                                                  return a.cor();
   a[i] >>= 1;
 }
                                                                num& operator -= ( num &a, const num &b ) {
  cor();
                                                                   while (a.len < b.len)
                                                                    a[a.len++] = 0;
static ll cmp( ll *a, ll *b, int n ) {
                                                                   forn(i, b.len)
                                                                    a[i] -= b[i];
  while (n--)
    if (a[n] != b[n])
                                                                   forn(i, a.len - 1)
                                                                    if (a[i] < 0)
     return a[n] - b[n];
                                                                      a[i] += BASE, a[i + 1]--;
                                                                   assert(a[a.len - 1] >= 0); // a >= b
                                                                   return a.cor();
int cmp( const num &b ) const {
 if (len != b.len)
   return len - b.len;
                                                                num& operator *= ( num &a, int k ) {
  for (int i = len - 1; i >= 0; i--)
                                                                  if (k == 1)
   if (a[i] != b[i])
                                                                    return a;
     return a[i] - b[i];
                                                                  if (k == 0) {
                                                                    a.len = 0;
  return 0;
}
                                                                    return a;
bool zero() {
                                                                   forn(i, a.len)
 return len == 1 && !a[0];
                                                                    a[i] *= k;
                                                                   forn(i, a.len - 1)
                                                                    if (a[i] >= BASE)
/** c = this/b, this %= b */
                                                                      a[i + 1] += a[i] / BASE, a[i] %= BASE;
num &div( num b, num &c ) {
                                                                   return a.cor();
  c.len = len - b.len;
  for (int i = c.len; i >= 0; i--) {
    int k = (1.0L * a[len - 1] * BASE + (len >= 2 ? a[len -
                                                                num& operator /= ( num &a, int k ) {
    \rightarrow 2] : 0)) / (1.0L * b[b.len - 1] * BASE + (b.len >=
                                                                  if (k == 1)
     \hookrightarrow 2 ? b[b.len - 2] + 1 : 0));
                                                                   return a:
                                                                   assert(k != 0);
    c[i] = 0;
                                                                   for (int i = a.len - 1; i > 0; i--)
    if (k > 0) {
      c[i] += k;
                                                                   a[i - 1] += (11)(a[i] \% k) * BASE, a[i] /= k;
      forn(j, b.len)
                                                                   a[0] /= k;
        a[i + j] = (11)b[j] * k;
                                                                   return a.cor();
      forn(j, b.len)
        if (a[i + j] < 0) {
         11 k = (-a[i + j] + BASE - 1) / BASE;
                                                                num\& mul(const num \&a, const num \&b, num \&x) {
          a[i + j] += k * BASE, a[i + j + 1] -= k;
                                                                  assert(a.len + b.len - 1 <= NUM_LEN);</pre>
                                                                   memset(x.a, 0, sizeof(x[0]) * (a.len + b.len - 1));
                                                                  forn(i, a.len)
                                                                    forn(j, b.len)
                                                                      if ((x[i + j] += a[i] * b[j]) >= INF)
      len--, a[len - 1] += a[len] * BASE, a[len] = 0;
    else if (cmp(a, b.a, b.len) >= 0) {
                                                                        x[i + j + 1] += ADD, x[i + j] -= INF;
      c[0]++;
                                                                   x.len = a.len + b.len - 1;
      forn(j, b.len)
                                                                   forn(i, x.len - 1)
       if ((a[j] -= b[j]) < 0)
                                                                    if (x[i] >= BASE)
          a[j] += BASE, a[j + 1]--;
                                                                      x[i + 1] += x[i] / BASE, x[i] %= BASE;
    }
                                                                  return x.cor();
  if (c.len < 0)
    c[c.len = 0] = 0;
                                                                bool operator == ( const num &a, const num &b ) { return
  forn(i, c.len)
                                                                 \hookrightarrow a.cmp(b) == 0; }
   if (c[i] >= BASE)
                                                                bool operator != ( const num &a, const num &b ) { return
      c[i + 1] += c[i] / BASE, c[i] %= BASE;
                                                                 \hookrightarrow a.cmp(b) != 0; }
  c.len += (!c.len || c[c.len]);
                                                                bool operator < ( const num &a, const num &b ) { return
  return cor();
                                                                 \hookrightarrow a.cmp(b) < 0; }
```

```
bool operator > ( const num &a, const num &b ) { return
\rightarrow a.cmp(b) > 0; }
                                                                   inline Num operator *(dbl k) const{
bool operator <= ( const num &a, const num &b ) { return</pre>
                                                                    return Num(x*k, y*k);
\hookrightarrow a.cmp(b) <= 0; }
                                                                   inline Num operator *(const Num &B) const{
bool operator >= ( const num &a, const num &b ) { return
\hookrightarrow a.cmp(b) >= 0; }
                                                                    return Num(x*B.x - y*B.y, x*B.y + y*B.x);
num& add( const num &a, const num &b, num &c ) { c = a; c +=
inline void operator +=(const Num &B){
num\& sub( const num \&a, const num \&b, num \&c ) { c = a; c -=
                                                                    x+=B.x, y+=B.y;
num& mul( const num &a, int k, num &c )
                                              \{c = a: c *=
                                                                   inline void operator /=(dbl k){
x/=k, y/=k;
                                              { c = a; c /=
                                                                   }
num& div( const num &a, int k, num &c )
\hookrightarrow k; return c; }
                                                                  inline void operator *=(const Num &B){
                                                                    *this = *this * B;
num& operator *= ( num &a, const num &b ) {
 static num tmp;
                                                                };
 mul(a, b, tmp);
                                                                 inline Num sqr(const Num &x){ return x * x; }
 return a = tmp;
                                                                 inline Num conj(const Num &x){ return Num(x.real(),
                                                                 → -x.imag()); }
num operator \hat{\ } ( const num &a, int k ) {
 num res(1);
                                                                inline int getN(int n){
                                                                  int k = 1;
 forn(i, k)
   res *= a;
                                                                  while(k < n)
 return res;
                                                                    k <<= 1;
                                                                  return k;
num& gcd_binary( num &a, num &b ) {
 int cnt = 0;
                                                                 const int LOG = 18;
                                                                 const int MAX_N = 1 << LOG;</pre>
 while (!a.zero() && !b.zero()) {
   while (!(b[0] & 1) && !(a[0] & 1))
     cnt++, a.div2(), b.div2();
                                                                 Num rt[MAX_N];
   while (!(b[0] & 1))
                                                                 int rev[MAX_N];
     b.div2();
   while (!(a[0] & 1))
                                                                 void fft(Num *a, int n){
                                                                   assert(rev[1]); // don't forget to init
     a.div2();
   if (a.cmp(b) < 0)
                                                                   int q = MAX_N / n;
     b -= a;
                                                                  forn(i, n)
   else
                                                                    if(i < rev[i] / q)
     a -= b;
                                                                       swap(a[i], a[rev[i] / q]);
                                                                   for(int k = 1; k < n; k <<= 1)
 if (a.zero())
                                                                    for(int i = 0; i < n; i += 2 * k)
    std::swap(a, b);
                                                                      forn(j, k){
 while (cnt)
                                                                         const Num z = a[i + j + k] * rt[j + k];
                                                                        a[i + j + k] = a[i + j] - z;
   a.mul2(), cnt--;
                                                                        a[i + j] += z;
 return a;
                                                                 }
num& gcd( num &a, num &b ) {
                                                                 void fft_inv(Num *a, int n){
 static num tmp;
 return b.zero() ? a : gcd(b, a.div(b, tmp));
                                                                  fft(a, n);
                                                                   reverse(a + 1, a + n);
                                                                   forn(i, n)
                                                                     a[i] /= n;
6 FFT
//typedef complex <dbl > Num;
                                                                 void double_fft(Num *a, Num *fa, Num *fb, int n){ // only if
struct Num{

→ you need it

 dbl x, y;
                                                                  fft(a, n):
 Num(){}
                                                                   const int n1 = n - 1;
 Num(dbl _x, dbl _y):x(_x),y(_y){}
                                                                  forn(i, n){
                                                                    const Num &z0 = a[i], &z1 = a[(n - i) & n1];
 inline dbl real() const{ return x; }
                                                                    fa[i] = Num(z0.real() + z1.real(), z0.imag() - z1.imag())
 inline dbl imag() const{ return y; }
                                                                    fb[i] = Num(z0.imag() + z1.imag(), z1.real() - z0.real())
 inline Num operator +(const Num &B) const{

→ * 0.5;

   return Num(x+B.x, y+B.y);
                                                                  }
                                                                }
 inline Num operator -(const Num &B) const{
   return Num(x-B.x, y-B.y);
```

```
Num tmp[MAX_N];
template < class T>
void mult(T *a, T *b, T *r, int n){ // n = 2^k
   tmp[i] = Num((dbl)a[i], (dbl)b[i]);
 fft(tmp, n);
 const int n1 = n - 1;
 const Num c = Num(0, -0.25 / n);
 fornr(i, n / 2 + 1){
   const int j = (n - i) & n1;
   const Num z0 = sqr(tmp[i]), z1 = sqr(tmp[j]);
   tmp[i] = (z1 - conj(z0)) * c;
   tmp[j] = (z0 - conj(z1)) * c;
 }
 fft(tmp, n);
 forn(i, n)
   r[i] = (T)round(tmp[i].real());
void init(){ // don't forget to init
 forn(i, MAX N)
   rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (LOG - 1));
 rt[1] = Num(1, 0);
 for(int k = 1, p = 2; k < LOG; ++k, p *= 2){
   const Num x(cos(M_PI / p), sin(M_PI / p));
   for(int i = p / 2; i < p; ++i)
     rt[2 * i] = rt[i], rt[2 * i + 1] = rt[i] * x;
```

FFT by mod and FFT with digits up to 10^{6}

```
Num ta[MAX_N], tb[MAX_N], tf[MAX_N], tg[MAX_N];
const int HALF = 15;
void mult(int *a, int *b, int *r, int N, int mod){
 int tw = (1 << HALF) - 1;</pre>
  forn(i, N){
   int x = int(a[i] % mod);
    ta[i] = Num(x & tw, x >> HALF);
  forn(i, N){
   int x = int(b[i] % mod);
    tb[i] = Num(x & tw, x >> HALF);
  fft(ta, N);
  fft(tb, N);
  forn(i, N){
    int j = (N - i) & (N - 1);
    Num a1 = (ta[i] + conj(ta[j])) * Num(0.5, 0);
    Num a2 = (ta[i] - conj(ta[j])) * Num(0, -0.5);
    Num b1 = (tb[i] + conj(tb[j])) * Num(0.5 / N, 0);
    Num b2 = (tb[i] - conj(tb[j])) * Num(0, -0.5 / N);
    tf[j] = a1 * b1 + a2 * b2 * Num(0, 1);
    tg[j] = a1 * b2 + a2 * b1;
  fft(tf, N);
  fft(tg, N);
  forn(i, N){
   LL aa = LL(tf[i].x + 0.5);
    LL bb = LL(tg[i].x + 0.5);
   LL cc = LL(tf[i].y + 0.5);
   r[i] = int((aa + ((bb \% mod) << 15) + ((cc \% mod) << 30))
     \rightarrow % mod);
```

```
int tc[MAX_N], td[MAX_N];
const int MOD1 = 1.5e9, MOD2 = MOD1 + 1;
void multLL(int *a, int *b, LL *r, int N){
 mult(a, b, tc, N, MOD1);
 mult(a, b, td, N, MOD2);
 forn(i, N)
   r[i] = tc[i] + (td[i] - tc[i] + (LL)MOD2) * MOD1 % MOD2 *

→ MOD1:
```

Data Structures 3

Centroid Decomposition

```
vector <int> g[MAX_N];
int d[MAX_N], par[MAX_N], centroid;
//d u par - в дереве центроидов
int find(int v, int p, int total) {
 int size = 1, ok = 1;
  for (int to : g[v])
   if (d[to] == -1 && to != p) {
     int s = find(to, v, total);
      if (s > total / 2) ok = 0;
      size += s;
  if (ok && size > total / 2)
   centroid = v:
  return size;
void calc_in_component(int v, int p, int level) {
 // do something
  for (int to : g[v])
   if (d[to] == -1 && to != p)
      calc_in_component(to, v, level);
//decompose(0, -1, 0)
void decompose(int root, int parent, int level) {
 find(root, -1, find(root, -1, INF));
 int c = centroid;
  par[c] = parent;
  d[c] = level;
  calc_in_component(centroid, -1, level);
  for(int to : g[c])
   if(d[to] == -1)
      decompose(to, c, level + 1);
}
    Convex Hull Trick
```

```
struct Line {
  int k, b;
  Line() {}
  Line(int kk, int bb): k(kk), b(bb) {}
  LL get(int x) {
   return b + k * 111 * x;
 bool operator <(const Line &1) const {</pre>
    return k < 1.k; //знак на > если в другую сторону
  }
//проверяет, что пересечение (a,b) левее (a,c)
inline bool check(Line a, Line b, Line c) {
  return (a.b - b.b) * 111 * (c.k - a.k) < (a.b - c.b) * 111 *
  \hookrightarrow (b.k - a.k);
```

++k:

return k;

void insert(const H &hash, int x){

}

```
int k = get(hash);
                                                                       if(!ht[k])
vector <Line> st;
                                                                         ht[k] = hash, data[k] = x;
inline void add(Line 1) {
   while(sz(st) >= 2 \&\& ! check(st[sz(st) - 2], st[sz(st) - 1], 
                                                                    bool count(const H &hash, int x){
  → 1))
                                                                       int k = get(hash);
                                                                       return ht[k] != 0;
   st.pop_back();
  st.pb(1);
                                                                     }
                                                                   }
int get(int x) {
  int 1 = 0, r = sz(st);
  while (r - 1 > 1) {
    int m = (1 + r) / 2; //знак на > если в другую сторону
    if ((st[m - 1].b - st[m].b) < x * 111 * (st[m].k - st[m - 1])
                                                                     vi g[MAX_N];
    \hookrightarrow 1].k))
     1 = m;
    else
      r = m;
 }
  return 1:
                                                                         if (vl != vr - 1)
void buildConvexHull(vector <Line> lines) {
                                                                           forn (j, 2)
 sort(all(lines));
  for(Line 1 : lines)
    add(1);
                                                                     }
     Fenwick Tree
namespace FenwickTree {
  int t[MAX_N];
                                                                       if (vl == vr - 1)
  int n:
  int get(int ind) {
                                                                       if (ind >= vm)
    int res = 0;
    for (; ind >= 0; ind &= (ind + 1), ind--)
      res += t[ind];
     return res;
                                                                      \hookrightarrow int val) {
  void add(int ind, int x) {
     for (; ind < n; ind |= (ind + 1))
       t[ind] += x;
                                                                       }
  int sum(int 1, int r) { //[l, r)
    return get(r - 1) - get(1 - 1);
                                                                        return;
}
     Hash Table
namespace HashTable {
  typedef long long H;
  const int HT_SIZE = 1<<20, HT_AND = HT_SIZE - 1, HT_SIZE_ADD</pre>
  \hookrightarrow = HT_SIZE / 100;
  H ht[HT_SIZE + HT_SIZE_ADD];
  int data[HT_SIZE + HT_SIZE_ADD];
                                                                     int getHLD(int v) {
  int get(const H &hash){
   11 k = ((11) hash) & HT_AND;
    while(ht[k] && ht[k] != hash)
```

```
Heavy Light Decomposition
```

```
namespace HeavyLightDecomposition {
  int size[MAX_N], comp[MAX_N], num[MAX_N], top[MAX_N],
  \ \hookrightarrow \ \ pr[\texttt{MAX}_N] \,, \ tin[\texttt{MAX}_N] \,, \ tout[\texttt{MAX}_N] \,;
  vi t[MAX_N], toPush[MAX_N], lst[MAX_N];
  int curPath = 0, curTime = 0;
  void pushST(int path, int v, int vl, int vr) {
    if (toPush[path][v] != -1) {
          toPush[path][2 * v + j] = toPush[path][v];
        t[path][v] = toPush[path][v];
      toPush[path][v] = -1;
  int getST(int path, int v, int vl, int vr, int ind) {
    pushST(path, v, vl, vr);
     return t[path][v];
    int vm = (v1 + vr) / 2;
      return getST(path, 2 * v + 1, vm, vr, ind);
    return getST(path, 2 * v, v1, vm, ind);
  void setST(int path, int v, int vl, int vr, int l, int r,
    if (v1 >= 1 && vr <= r) {
      toPush[path][v] = val;
      pushST(path, v, vl, vr);
    pushST(path, v, vl, vr);
   if (vl >= r || l >= vr)
    int vm = (v1 + vr) / 2;
    setST(path, 2 * v, v1, vm, 1, r, val);
    setST(path, 2 * v + 1, vm, vr, 1, r, val);
    t[path][v] = min(t[path][2 * v], t[path][2 * v + 1]);
  bool isUpper(int v, int u) {
    return tin[v] <= tin[u] && tout[v] >= tout[u];
    return getST(comp[v], 1, 0, sz(t[comp[v]]) / 2, num[v]);
  int setHLD(int v, int u, int val) {
    int ans = 0, w = 0;
    forn (i, 2) {
       while (!isUpper(w = top[comp[v]], u))
         setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, 0, num[v] +
          \hookrightarrow 1, val), v = pr[w];
```

```
swap(v, u);
                                                                 typedef Node *pNode;
  setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, min(num[v],

    num[u]), max(num[v], num[u]) + 1, val);

                                                                 struct Node{
                                                                     static pNode null;
  return ans;
                                                                     pNode 1, r;
                                                                     int y, val, size, m;
void dfs(int v, int p) {
  tin[v] = curTime++;
  size[v] = 1;

→ m(INF){}

  pr[v] = p;
  for (int u : g[v])
                                                                     \rightarrow m(v){}
    if (u != p) {
      dfs(u, v);
                                                                     void calc(){
      size[v] += size[u];
  tout[v] = curTime++;
                                                                };
void build(int v) {
  if (v == 0 \mid \mid size[v] * 2 < size[pr[v]]) {
    top[curPath] = v;
    comp[v] = curPath;
                                                                     if(1 == Node::null)
                                                                        t = r;
    num[v] = 0;
    curPath++;
                                                                        t = 1;
  else {
                                                                     else if(1->y < r->y)
    comp[v] = comp[pr[v]];
    num[v] = num[pr[v]] + 1;
  lst[comp[v]].pb(v);
                                                                 }
  for (int u : g[v])
       if (u != pr[v])
                                                                     if(t == Node::null)
      build(u);
                                                                        1 = r = t;
void initHLD() {
  dfs(0, 0);
  build(0);
                                                                          \hookrightarrow t)->calc();
  forn (i, curPath) {
                                                                }
    int curSize = 1;
    while (curSize < sz(lst[i]))</pre>
      curSize *= 2;
    t[i].resize(curSize * 2);
                                                                   split(root, root, r, k);
    toPush[i] = vi(curSize * 2, -1);
                                                                   merge(root, root, n);
    //initialize t[i]
                                                                   merge(root, root, r);
  }
}
                                                                   pNode 1, n;
                                                                   split(root, 1, root, k);
    Next Greater in Segment Tree
                                                                   split(root, n, root, 1);
```

```
//на позиции строго большей чем роз число строго большее w
int nextGreaterX(int v, int l, int r, int pos, int x) {
    if (r <= pos + 1 || tree[v] <= x)
        return INF;
    if (v >= tSize)
        return v - tSize;
    int ans = nextGreaterX(2 * v, l, (l + r) / 2, pos, x);
    if (ans == INF)
        ans = nextGreaterX(2 * v + 1, (l + r) / 2, r, pos, x);
    return ans;
}
```

14 Treap (Rope)

```
#include<bits/stdc++.h>
const int INF=1e9;
using namespace std;
```

```
struct Node:
    Node(): l(this), r(this), y(-1), val(INF), size(0),
    Node(int v): l(null), r(null), y(rand()), val(v), size(1),
       size = 1 + 1->size + r->size;
        m = min(val, min(1->m, r->m));
pNode Node::null = new Node();
void merge(pNode &t, pNode 1, pNode r){
    else if(r == Node::null)
       merge(l->r, l->r, r), (t = 1)->calc();
       merge(r - > 1, 1, r - > 1), (t = r) -> calc();
void split(pNode t, pNode &1, pNode &r, int k){
    else if(t->1->size >= k)
       split(t->1, 1, t->1, k), (r = t)->calc();
       split(t->r, t->r, r, k - t->1->size - 1), (1 =
void insert(pNode &root, int k, int x){
 pNode r, n = new Node(x);
void erase(pNode &root, int k){
 merge(root, 1, root);
pNode build(int k){
   if(k == 1)
       return new Node(0);
    pNode root;
   merge(root, build(k / 2), build((k + 1) / 2));
    return root;
void print(pNode t, bool root = 1){
    if(t != Node::null){
       print(t->1, 0);
        printf("%d ", t->val);
        print(t->r ,0);
    if(root)
```

```
puts("");
                                                                        if(L0<=L && R<=R0){
                                                                          need->Add(L1,R1,x);
                                                                          val->Add(L1,R1,x*LL(R-L));
int main(){
                                                                          return;
 pNode r=Node::null; // r is an empty tree
                                                                        }
                                                                        int a=max(L0,L), b=min(R0,R), M=(L+R)>>1;
  // work with r
                                                                        1 -> Add(L0,R0,L1,R1,x,L,M), r -> Add(L0,R0,L1,R1,x,M,R);
                                                                        val->Add(L1,R1,x*LL(b-a));
      Segtree 2D
15
                                                                      }
                                                                    };
const int ST_SZ=1<<10, ST_SZ2=2*ST_SZ;</pre>
struct Node_1d{
                                                                    int main(){
  Node_1d *1,*r;
                                                                      Node_2d *Z=new Node_2d();
  LL val.need:
  Node_1d():1(NULL),r(NULL),val(0),need(0){}
                                                                     int x,y,Q;
  inline void norm(){
                                                                      scanf("%d%d%d",&x,&y,&Q);
   if(l==NULL)
                                                                      forn(i,Q){
      l=new Node_1d();
                                                                        int type,a,b,c,d;
   if(r==NULL)
                                                                        scanf("%d%d%d%d%d", &type, &a, &b, &c, &d);
      r=new Node_1d();
                                                                        --a, --b;
                                                                       if(type==1){
  LL Get(int L0,int R0,int L=0,int R=ST_SZ){
                                                                          int w;
    if(L0>=R \mid | L>=R0)
                                                                          scanf("%d",&w);
     return 0;
                                                                          Z->Add(a,c,b,d,w);
    if(L0<=L && R<=R0)
                                                                        }else
     return val;
                                                                          printf(11d"\n",Z->Get(a,c,b,d));
    int a=max(L0,L), b=min(R0,R), M=(L+R)>>1;
                                                                      }
   norm():
                                                                    }
    return 1->Get(L0,R0,L,M)+r->Get(L0,R0,M,R)+need*LL(b-a);
 }
  void Add(int L0,int R0,int x,int L=0,int R=ST_SZ){
                                                                          Segtree 2D — Fenwick
                                                                    16
    if(L0>=R || L>=R0)
                                                                    const int MAX_N=1002;
      return;
    if(L0 \le L \&\& R \le R0){
                                                                    LL F[4][MAX_N][MAX_N];
     need+=x;
                                                                    int N.M:
      val+=x*LL(R-L);
                                                                    inline int Z(int a){
     return:
                                                                      return a\&^{(a-1)};
   int M=(L+R)>>1;
    norm();
                                                                    inline void add(int k,int x,int y,LL a){
    1 \rightarrow Add(L0,R0,x,L,M), r \rightarrow Add(L0,R0,x,M,R);
                                                                      for(;x<=N;x+=Z(x))
    val=l->val+r->val+need*(R-L);
 }
                                                                        for(int j=y;j<=M;j+=Z(j))</pre>
};
                                                                          F[k][x][j]+=a;
                                                                    }
struct Node_2d{
                                                                    inline LL get(int k,int x,int y){
  Node_2d *1,*r;
                                                                      LL s=0;
  Node_1d *val,*need;
                                                                      for(;x>0;x-=Z(x))
  Node_2d():1(NULL),r(NULL),val(new Node_1d()),need(new
                                                                        for(int j=y;j>0;j-=Z(j))
                                                                          s+=F[k][x][j];
  \rightarrow Node_1d()){}
  inline void norm(){
                                                                      return s:
   if(l==NULL)
                                                                    }
      1=new Node_2d();
   if(r==NULL)
                                                                    inline LL Get(int a,int b){
                                                                      return LL(a+1)*(b+1)*get(0,a,b)-(b+1)*get(1,a,b)
      r=new Node_2d();
                                                                          -(a+1)*get(2,a,b)+get(3,a,b);
  LL Get(int L0,int R0,int L1,int R1,int L=0,int R=ST_SZ){
    if(L0>=R | L>=R0)
                                                                    inline void Add(int a, int b, LL w){
     return 0:
    if(L0 \le L \&\& R \le R0)
                                                                      add(0,a,b,w);
     return val->Get(L1.R1):
                                                                      add(1,a,b,w*a);
    int a=max(L0,L), b=min(R0,R), M=(L+R)>>1;
                                                                      add(2,a,b,w*b);
    norm():
                                                                      add(3,a,b,w*a*b);
     \rightarrow 1-> Get(L0,R0,L1,R1,L,M)+r-> Get(L0,R0,L1,R1,M,R)+need-> Get(L1,R1)*LL(b-a);
                                                                   inline LL Get(int a,int b,int c,int d){
  void Add(int L0,int R0,int L1,int R1,int x,int L=0,int
                                                                      return Get(c,d)-Get(a-1,d)-Get(c,b-1)+Get(a-1,b-1);
  \hookrightarrow R=ST SZ){
    if(L0>=R | L>=R0)
                                                                    inline void Add(int a,int b,int c,int d,LL w){
      return;
```

```
Add(a,b,w);
  if(d<M)
    Add(a,d+1,-w);
  if(c<N)
   Add(c+1,b,-w);
  if(c<N && d<M)
    Add(c+1,d+1,w);
int main(){
  int Q;
  scanf("%d%d%d",&N,&M,&Q);
  forn(i,Q){
   int type,a,b,c,d;
    scanf("%d%d%d%d", &type, &a, &b, &c, &d);
   if(type==1){
     int w;
      scanf("%d",&w);
      Add(a,b,c,d,w);
      printf(lld"\n",Get(a,b,c,d));
}
```

4 Flows

17 Utilities

```
//for directed unweighted graph
struct Edge {
 int v, u, c, f;
 Edge() {}
 Edge(int v, int u, int c): v(v), u(u), c(c), f(0) {}
vector <Edge> edges;
inline void addFlow(int e, int flow) {
   edges[e].f += flow;
   edges[e ^ 1].f -= flow;
inline void addEdge(int v, int u, int c) {
 g[v].pb(sz(edges));
 edges.pb(Edge(v, u, c));
 g[u].pb(sz(edges));
 edges.pb(Edge(u, v, 0)); //for undirected O should be c
void read(int m) {
   forn (i, m) \{
     int v, u, c;
     scanf("%d%d%d", &v, &u, &c);
     addEdge(v - 1, u - 1, c);
  }
}
```

18 Ford-Fulkerson

```
namespace FordFulkerson {
  int used[MAX_N], pr[MAX_N];
  vi g[MAX_N];
  int curTime = 1;

#include "Utilities.cpp"

int dfs(int v, int can, int toPush, int t) {
  if (v == t)
    return can;
  used[v] = curTime;
  for (int edge : g[v]) {
```

```
auto &e = edges[edge];
       if (used[e.u] != curTime && e.c - e.f >= toPush) {
          int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
          if (flow > 0) {
           addFlow(edge, flow);
           pr[e.u] = edge;
           return flow;
        }
      }
     }
     return 0;
  int fordFulkerson(int n, int m, int s, int t) {
   read(m);
   int ansFlow = 0, flow = 0;
   //Without scaling
   while ((flow = dfs(s, INF, 1, t)) > 0) {
     ansFlow += flow;
      curTime++;
   //With scaling
   fornr (i, INF_LOG)
     for (curTime++; (flow = dfs(s, INF, (1 << i), t)) > 0;
   curTime++)
       ansFlow += flow;
    return ansFlow;
 }
}
```

19 Dinic

```
namespace Dinic {
 int pr[MAX_N], d[MAX_N], q[MAX_N], first[MAX_N];
  vector <int> g[MAX_N];
  #include "Utilities.cpp"
  int dfs(int v, int can, int toPush, int t) {
   if (v == t)
     return can;
   int sum = 0;
   for (; first[v] < (int) g[v].size(); first[v]++) {</pre>
     auto &e = edges[g[v][first[v]]];
     if (d[e.u] != d[v] + 1 || e.c - e.f < toPush)
        continue;
      int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
      addFlow(g[v][first[v]], flow);
      can -= flow, sum += flow;
      if (!can)
       return sum;
   }
    return sum;
  bool bfs(int n, int s, int t, int curPush) {
   for (int i = 0; i < n; i++)
     d[i] = INF, first[i] = 0;
   int head = 0, tail = 0;
   q[tail++] = s;
   d[s] = 0;
   while (tail - head > 0) {
     int v = q[head++];
     for (int edge : g[v]) {
       auto &e = edges[edge];
       if (d[e.u] > d[v] + 1 && e.c - e.f >= curPush) {
          d[e.u] = d[v] + 1;
          q[tail++] = e.u;
        }
```

};

```
}
                                                                  vector <Edge> edges;
   return d[t] != INF;
                                                                  inline void addFlow(int e, int flow) {
                                                                      edges[e].f += flow;
                                                                      edges[e ^ 1].f -= flow;
 int dinic(int n, int m, int s, int t) {
   read(m);
   int ansFlow = 0;
   //Without scaling
                                                                  inline void addEdge(int v, int u, int c, int w) {
   while(bfs(n, s, t, 1))
                                                                    g[v].pb(sz(edges));
     ansFlow += dfs(s, INF, 1, t);
                                                                    edges.pb(Edge(v, u, c, w));
    //With scaling
                                                                    g[u].pb(sz(edges));
                                                                    edges.pb(Edge(u, v, 0, -w));
   fornr (j, INF_LOG)
     while (bfs(n, s, t, 1 \ll j))
       ansFlow += dfs(s, INF, 1 << j, t);
                                                                  void read(int m) {
                                                                     forn (i, m) {
   return ansFlow;
                                                                        int v, u, c, w;
                                                                        scanf("%d%d%d", &v, &u, &c, &w);
                                                                        addEdge(v - 1, u - 1, c, w);
                                                                  }
      Hungarian
                                                                  int dijkstra(int n, int s, int t) {
const int INF = 1e9;
                                                                    forn (i, n)
int a[MAX_N][MAX_N];
                                                                      used[i] = 0, d[i] = INF;
                                                                    d[s] = 0;
// min = sum of a[pa[i], i]
                                                                    while (1) {
// you may optimize speed by about 15%, just change all
                                                                       int v = -1;

    vectors to static arrays

                                                                       forn (i, n)
vi Hungrian(int n) {
                                                                         if (!used[i] && (v == -1 \mid \mid d[v] > d[i]))
 vi pa(n + 1, -1), row(n + 1, 0), col(n + 1, 0), la(n + 1);
                                                                           v = i;
  forn(k, n) {
                                                                        if (v == -1 || d[v] == INF)
   vi u(n + 1, 0), d(n + 1, INF);
                                                                         break:
   pa[n] = k;
                                                                         used[v] = 1;
    int 1 = n, x;
                                                                       for (int edge : g[v]) {
    while ((x = pa[1]) != -1) {
                                                                          auto &e = edges[edge];
     u[1] = 1;
                                                                          11 w = e.w + pot[v] - pot[e.u];
     int minn = INF, tmp, 10 = 1;
                                                                          if (e.c > e.f \&\& d[e.u] > d[v] + w)
     forn(j, n)
                                                                            d[e.u] = d[v] + w, pr[e.u] = edge;
       if (!u[j]) {
         if ((tmp = a[x][j] + row[x] + col[j]) < d[j])
                                                                    }
          d[j] = tmp, la[j] = 10;
                                                                    if (d[t] == INF)
         if (d[j] < minn)</pre>
                                                                      return d[t];
           minn = d[j], 1 = j;
                                                                    forn (i, n)
       }
                                                                      pot[i] += d[i];
     forn(j, n + 1)
                                                                    return pot[t];
       if (u[j])
         col[j] += minn, row[pa[j]] -= minn;
                                                                  int fordBellman(int n, int s, int t) {
         d[j] -= minn;
                                                                    forn (i, n)
                                                                     d[i] = INF;
   while (1 != n)
                                                                    int head = 0, tail = 0;
     pa[1] = pa[la[1]], 1 = la[1];
                                                                    d[s] = 0;
                                                                    q[tail++] = s;
 return pa;
                                                                    in[s] = 1;
                                                                    while (tail - head > 0) {
                                                                      int v = q[head++];
     Min Cost Max Flow
                                                                      in[v] = 0;
                                                                      for (int edge : g[v]) {
namespace MinCostMaxFlow {
                                                                        auto &e = edges[edge];
  const int MAX_M = 1e4;
                                                                        if (e.c > e.f \&\& d[e.u] > d[v] + e.w) {
 d[e.u] = d[v] + e.w;
  \hookrightarrow d[MAX_N], pot[MAX_N];
                                                                          pr[e.u] = edge;
 vi g[MAX_N];
                                                                          if (!in[e.u])
                                                                            in[e.u] = 1, q[tail++] = e.u;
 struct Edge {
                                                                        }
   int v, u, c, f, w;
                                                                      }
   Edge() {}
                                                                    }
   Edge(int v, int u, int c, int w): v(v), u(u), c(c), f(0),
                                                                    return d[t];
    \hookrightarrow w(w) \{\}
```

5 Geometry

22 ClosestPoints (SweepLine)

```
#include "header.h"
const int N = 2e5;
struct Pnt {
  int x, y, i;
  bool operator <(const Pnt &p) const{</pre>
   return mp(y, i) < mp(p.y, p.i);
};
LL d2 = 8e18, d = (LL)sqrt(d2) + 1;
Pnt p[N];
inline LL sqr(int x){
  return (LL)x * x;
inline void relax(const Pnt &a, const Pnt &b){
  LL tmp = sqr(a.x - b.x) + sqr(a.y - b.y);
  if (tmp < d2)
    d2 = tmp, d = (LL)(sqrt(d2) + 1 - 1e-9); // round up
inline bool xless(const Pnt &a, const Pnt &b){
 return a.x < b.x;
int main() {
 int n:
  scanf("%d", &n);
  forn(i, n)
   scanf("%d%d", &p[i].x, &p[i].y), p[i].i = i;
  sort(p, p + n, xless);
  set <Pnt> s;
  int 1 = 0:
  forn(r, n){
    set<Pnt>::iterator it_r = s.lower_bound(p[r]), it_1 =
    \hookrightarrow it_r;
    for (; it_r != s.end() && it_r->y - p[r].y < d; ++it_r)
     relax(*it_r, p[r]);
    while (it_1 != s.begin() && p[r].y - (--it_1)->y < d)
     relax(*it_l, p[r]);
    s.insert(p[r]);
    while (1 <= r \&\& p[r].x - p[1].x >= d)
      s.erase(p[1++]);
  }
```

```
printf("%.9f\n", sqrt(d2));
return 0;
}
```

23 ConvexHull

```
typedef vector<Pnt> vpnt;
inline bool by Angle (const Pnt &a, const Pnt &b) {
  dbl x = a \% b;
 return eq(x, 0) ? a.len2() < b.len2() : x < 0;
vpnt convexHull(vpnt p){
 int n = sz(p);
  assert(n > 0);
  swap(p[0], *min_element(all(p)));
  forab(i, 1, n)
  p[i] = p[i] - p[0];
  sort(p.begin() + 1, p.end(), byAngle);
/* Если надо оставить развёрнутые углы, нужно (1) и (2)
  int k = p.size() - 1;
  while(k > 0 \& eq((p[k-1]-p.back()) \% p.back(), 0))
    --k:
  reverse(pi.begin() + k, pi.end()); */
 int rn = 0;
  vpnt r(n);
  r[rn++] = p[0];
  forab(i, 1, n){
   Pnt q = p[i] + p[0];
    while(rn >= 2 && geq((r[rn - 1] - r[rn - 2]) % (q - r[rn -
    \leftrightarrow 2]), 0)) // (2) ge
      --rn;
    r[rn++] = q;
 }
  r.resize(rn);
  return r;
```

24 GeometryBase

```
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
typedef long double LD;
typedef double dbl;
const dbl EPS = 1e-9;
const int PREC = 20:
inline bool eq(dbl a, dbl b){ return abs(a-b)<=EPS; }</pre>
inline bool gr(dbl a, dbl b){ return a>b+EPS; }
inline bool geq(dbl a, dbl b){ return a>=b-EPS; }
inline bool ls(dbl a, dbl b){ return a < b - EPS; }</pre>
inline bool leq(dbl a, dbl b){ return a<=b+EPS; }</pre>
struct Pnt{
    dbl x,y;
    Pnt():x(0),y(0){}
    Pnt(dbl _x,dbl _y):x(_x),y(_y){}
    inline Pnt operator +(const Pnt &B) const{ return
      \rightarrow Pnt(x+B.x, y+B.y); }
    inline Pnt operator -(const Pnt &B) const{ return
     \hookrightarrow Pnt(x-B.x, y-B.y); }
    inline dbl operator *(const Pnt &B) const{ return x*B.x +
     \hookrightarrow y*B.y; } // LL
```

```
inline dbl operator %(const Pnt &B) const{ return x*B.y -
    \hookrightarrow y*B.x; } // LL
                                                                   };
    inline Pnt operator *(dbl k) const{ return Pnt(x*k, y*k);
    inline Pnt operator /(dbl k) const{ return Pnt(x/k, y/k);
    → }
    inline Pnt operator -() const{ return Pnt(-x, -y); }
                                                                       1.c = r1;
    inline void operator +=(const Pnt &B) { x+=B.x, y+=B.y; }
    inline void operator -=(const Pnt &B){ x-=B.x, y-=B.y; }
    inline void operator *=(dbl k){ x*=k, y*=k; }
    inline bool operator ==(const Pnt &B) { return
    \rightarrow abs(x-B.x)<=EPS && abs(y-B.y)<=EPS; }
    inline bool operator !=(const Pnt &B){ return
    \rightarrow abs(x-B.x)>EPS || abs(y-B.y)>EPS; }
    inline bool operator <(const Pnt &B){ return
    \rightarrow abs(x-B.x)<=EPS ? y<B.y-EPS : x<B.x; }
                                                                   struct Circle {
                                                                      Pnt p;
    inline dbl angle() const{ return atan2(y, x); } // LD
                                                                       dbl r;
    inline dbl len2() const{ return x*x+y*y; } // LL
                                                                   };
    inline dbl len() const{ return sqrt(x*x+y*y); } // LL, LD
    inline Pnt getNorm() const{
       auto 1 = len();
        return Pnt(x/1, y/1);
    }
    inline void normalize(){
        auto 1 = len();
        x/=1, y/=1;
    inline Pnt getRot90() const{ //counter-clockwise
       return Pnt(-y, x);
    inline Pnt getRot(dbl a) const{ // LD
        dbl si = sin(a), co = cos(a);
        return Pnt(x*co - y*si, x*si + y*co);
                                                                   }
    inline void read(){
        int _x,_y;
        scanf("%d%d",\&_x,\&_y);
        x=_x, y=_y;
                                                                           return:
    7
    inline void write() const{
        printf("%.*f %.*f ", PREC, (double)x, PREC,
         }
}:
                                                                       }
struct Line{
                                                                   }
    dbl a, b, c;
    Line():a(0),b(0),c(0){}
    Line(dbl _a, dbl _b, dbl _c):a(_a),b(_b),c(_c){}
    Line(const Pnt &A, const Pnt &B){ // it normalizes (a,b),
    \hookrightarrow important in d(), normalToP()
        Pnt n = (B-A).getRot90().getNorm();
        a = n.x, b = n.y, c = -(a*A.x + b*A.y);
    inline dbl d(const Pnt &p) const{ return a*p.x + b*p.y +
    inline Pnt no() const {return Pnt(a, b);}
                                                                   }
    inline Pnt normalToP(const Pnt &p) const{ return Pnt(a,b)
    \rightarrow * (a*p.x + b*p.y + c); }
    inline void write() const{
       printf("%.*f %.*f %.*f ", PREC, (double)a, PREC,
         \hookrightarrow (double)b, PREC, (double)c);
```

25 GeometryInterTangent

```
void buildTangent(Pnt p1, dbl r1, Pnt p2, dbl r2, Line &1) {
\hookrightarrow // r1, r2 = radius with sign
    Pnt p = p2 - p1;
    dbl c2 = p.len2(), c1 = sqrt(c2 - sqr(r2));
    1.a = (-p.x * (r1 - r2) + p.y * c1) / c2;
    1.b = (-p.y * (r1 - r2) - p.x * c1) / c2;
    1.c -= 1.no() * p1;
    assert(eq(1.d(p1), r1));
    assert(eq(1.d(p2), r2));
vector<Pnt> v; // to store intersection
// Intersection of two lines
int line_line(const Line &1, const Line &m){
    dbl z = m.a * 1.b - 1.a * m.b,
            x = m.c * 1.b - 1.c * m.b,
            y = m.c * 1.a - 1.c * m.a;
    if(fabs(z) > EPS){
        v.pb(Pnt(-x/z, y/z));
        return 1;
    }else if(fabs(x) > EPS || fabs(y) > EPS)
       return 0; // parallel lines
        return 2; // same lines
// Intersection of Circle and line
void circle_line(const Circle &c, const Line &l){
    db1 d = 1.d(c.p);
    if(fabs(d) > c.r + EPS)
    if(fabs(fabs(d) / c.r - 1) < EPS)
       v.pb(c.p - 1.no() * d);
        dbl s = sqrt(fabs(sqr(c.r) - sqr(d)));
        v.pb(c.p - 1.no() * d + 1.no().getRot90() * s);
v.pb(c.p - 1.no() * d - 1.no().getRot90() * s);
// Intersection of two circles
void circle_circle(const Circle &a, const Circle &b){
    circle_line(a, Line((b.p - a.p) * 2, a.p.len2() -
    \rightarrow b.p.len2() + sqr(b.r) - sqr(a.r)));
// Squared distance between point p and segment [a..b]
dbl dist2(Pnt p, Pnt a, Pnt b){
    if ((p - a) * (b - a) < 0) return (p - a).len2();
if ((p - b) * (a - b) < 0) return (p - b).len2();
    return fabs((p - a) % (b - a)) / (b - a).len2();
      GeometrySimple
```

```
int sign(dbl a){ return (a > EPS) - (a < -EPS); }
// Checks, if point is inside the segment</pre>
```

```
inline bool inSeg(const Pnt &p, const Pnt &a, const Pnt &b) {
                                                                     return
    return eq((p - a) \% (p - b), 0) && leq((p - a) * (p - b),
                                                                             gr((p[b] - p[a]) % (p[i] - p[a]), 0) &&
                                                                             gr((p[c] - p[b]) % (p[i] - p[b]), 0) &&
    \rightarrow 0);
                                                                             gr((p[a] - p[c]) % (p[i] - p[c]), 0);
                                                                 }
// Checks, if two intervals (segments without ends) intersect
\hookrightarrow AND do not lie on the same line
                                                                        Halfplanes Intersection
                                                                 27
inline bool subIntr(const Pnt &a, const Pnt &b, const Pnt &c,
\hookrightarrow const Pnt &d){
                                                                 const int maxn = (int)4e5 + 9;
   return
                                                                 const dbl eps = 1e-12;
            sign((b - a) \% (c - a)) * sign((b - a) \% (d - a))
            dbl sqr( dbl x ) { return x * x; }
            sign((d - c) \% (a - c)) * sign((d - c) \% (b - c))
            struct pnt{
}
                                                                   LL operator * ( pnt p ) { return (LL)x * p.y - (LL)y * p.x;
                                                                    → }
// Checks, if two segments (ends are included) has an
                                                                   LL operator ^ ( pnt p ) { return (LL)x * p.x + (LL)y * p.y;
inline bool checkSegInter(const Pnt &a, const Pnt &b, const
                                                                   pnt ort() { return pnt(-y, x); }
→ Pnt &c, const Pnt &d) {
                                                                   dbl ang() { return atan2(y, x); }
    return inSeg(c, a, b) || inSeg(d, a, b) || inSeg(a, c, d)
                                                                   LL d2() { return x * x + y * y; }
    \rightarrow || inSeg(b, c, d) || subIntr(a, b, c, d);
                                                                 }:
                                                                 pnt st, v, p[maxn];
inline dbl area(vector<Pnt> p){
                                                                 int n, sp, ss[maxn], ind[maxn], no[maxn], cnt[maxn], k = 0,
   dbl s = 0;
                                                                  \rightarrow a[maxn], b[maxn];
   int n = p.size();
                                                                 dbl ang[maxn];
   p.pb(p[0]);
                                                                 pnt Norm( int k ){    return (p[a[k]] - p[b[k]]).ort();}
      s += p[i + 1] % p[i];
   p.pop_back();
                                                                 void AddPlane( int i, int j ){
    return abs(s) / 2;
                                                                   a[k] = i, b[k] = j, ind[k] = k;
                                                                   ang[k] = Norm(k).ang();
                                                                   k++;
// Check if point p is inside polygon <n, q[]>
                                                                 }
int contains_slow(Pnt p, Pnt *z, int n){
    int cnt = 0;
                                                                 bool angLess( int i, int j ){ return ang[i] < ang[j];}</pre>
    forn(j, n){
       Pnt a = z[j], b = z[(j + 1) % n];
                                                                 void Unique()
        if (inSeg(p, a, b))
           return -1; // border
                                                                   int i = 0, k2 = 0;
        if (min(a.y, b.y) - EPS \le p.y \&\& p.y \le max(a.y, b.y)
                                                                   while (i < k)
        cnt += (p.x < a.x + (p.y - a.y) * (b.x - a.x) /
                                                                     int ma = ind[i], st = i;
            \hookrightarrow (b.y - a.y));
                                                                     pnt no = Norm(ma);
    return cnt & 1; // O = outside, 1 = inside
                                                                     for (i++; i < k \&\& fabs(ang[ind[st]] - ang[ind[i]]) < eps;
}
                                                                       if ((no ^ p[a[ma]]) < (no ^ p[a[ind[i]]]))
                                                                         ma = ind[i];
//for convex polygon
                                                                     ind[k2++] = ma;
//assume polygon is counterclockwise-ordered
                                                                   }
bool contains_fast(Pnt p, Pnt *z, int n) {
                                                                   k = k2:
   Pnt o = z[0];
    if(gr((p - o) \% (z[1] - o), 0) || ls((p - o) \% (z[n - 1] -
    \rightarrow o), 0))
                                                                 dbl xx, yy, tmp;
       return 0;
   int 1 = 0, r = n - 1;
                                                                 #define BUILD(a1, b1, c1, i) \
    while(r - l > 1){
                                                                   dbl \ a1 = Norm(i).x;
        int m = (1 + r) / 2;
                                                                   if(gr((p - o) % (z[m] - o), 0))
                                                                   tmp = sqrt(a1 * a1 + b1 * b1); \
          r = m;
                                                                   a1 /= tmp, b1 /= tmp; \
        else
                                                                   dbl c1 = -(a1 * p[a[i]].x + b1 * p[a[i]].y);
           1 = m:
                                                                 void FindPoint( int i, int j, dbl step = 0.0 ){
    return leq((p - z[1]) % (z[r] - z[1]), 0);
                                                                   BUILD(a1, b1, c1, i);
                                                                   BUILD(a2, b2, c2, j);
// Checks, if point "i" is in the triangle "abc" IFF triangle
                                                                   xx = -(c1 * b2 - c2 * b1) / (a1 * b2 - a2 * b1);
\hookrightarrow in CCW order
                                                                   yy = (c1 * a2 - c2 * a1) / (a1 * b2 - a2 * b1);
inline int isInTr(int i, int a, int b, int c){
```

```
dbl no = sqrt(sqr(a1 + a2) + sqr(b1 + b2));
 xx += (a1 + a2) * step / no;
 yy += (b1 + b2) * step / no;
void TryShiftPoint( int i, int j, dbl step )
 FindPoint(i, j, step);
 forn(i, k){
   BUILD(a1, b1, c1, ind[i]);
   if (a1 * xx + b1 * yy + c1 < eps)
     return;
 puts("Possible");
 printf("%.201f %.201f\n", (double)xx, (double)yy);
 exit(0);
void PushPlaneIntoStack( int i )
 while (sp \ge 2 \&\& ang[i] - ang[ss[sp - 2]] + eps < M_PI){
   FindPoint(i, ss[sp - 2]);
   BUILD(a1, b1, c1, ss[sp - 1]);
   if ((a1 * xx + b1 * yy + c1) < -eps)
     break;
   sp--;
 ss[sp++] = i;
int main()
 scanf("%d", &n);
 forn(i, n)
   scanf("%d%d", &p[i].x, &p[i].y);
 p[n] = p[0];
  // Find set of planes
 forn(i, sp)
   AddPlane(max(ss[i], ss[i + 1]), min(ss[i], ss[i + 1]));
 forn(i, n - 1)
   AddPlane(i + 1, i);
 sort(ind, ind + k, angLess);
 int oldK = k;
 Unique();
 forn(i, oldK)
   no[i] = i;
 forn(i, k){
   int j = oldK + i, x = ind[i];
   ang[j] = ang[x] + 2 * M_PI;
   a[j] = a[x];
   b[j] = b[x];
   ind[i + k] = j, no[j] = x;
 sp = 0;
 forn(i, 2 * k)
   PushPlaneIntoStack(ind[i]);
 forn(t, sp)
   if (++cnt[no[ss[t]]] > 1){
     TryShiftPoint(ss[t], ss[t - 1], 1e-5);
     break;
 return 0;
```

6 Graphs

28 2-SAT

```
//MAX_N - 2 * vars
vector <int> g[MAX_N], rg[MAX_N], tsort;
vector <bool> values;
int used[MAX_N], comp[MAX_N];
void dfs(int v) {
 used[v] = 1;
 for(int to : g[v])
   if (!used[to])
     dfs(to):
  tsort.pb(v);
void rdfs(int v, int num) {
 used[v] = 1;
  comp[v] = num;
  for(int to : rg[v])
   if (!used[to])
     rdfs(to, num);
void addEdge(int a, int b) {
  g[a ^ 1].pb(b);
  g[b ^ 1] pb(a);
  rg[b].pb(a ^ 1);
 rg[a].pb(b ^ 1);
//n - удвоенное
bool sat2(const vector <pii> &v, int n) {
  forn(i, sz(v)) {
    addEdge(v[i].fst, v[i].snd);
  memset(used, 0, sizeof(used));
  forn(i, n)
   if (!used[i])
     dfs(i);
  memset(used, 0, sizeof(used));
  int num = 0;
  fornr(i, n) {
    int u = tsort[i];
    if (!used[u])
      rdfs(u, num), num++;
  values.resize(n);
  for(int i = 0; i < n; i += 2)
   if (comp[i] == comp[i ^ 1])
     return 0;
    else if (comp[i] > comp[i ^ 1])
      values[i] = 1, values[i ^ 1] = 0;
      values[i] = 0, values[i ^ 1] = 1;
  return 1;
29
     Bridges
struct Edge {
  int to, id;
  Edge(int aa, int bb) : to(aa), id(bb) {}
int up[MAX_N], tin[MAX_N], timer;
vector <Edge> g[MAX_N];
vector <vector <int>> comp;
vector <int> st;
void newComp(int size = 0) {
  comp.emplace_back(); // новая пустая
```

++x, ++compCnt;

```
while (sz(st) > size) {
                                                                                                                                                       void dfs(int v, int p);
         comp.back().pb(st.back());
                                                                                                                                                       void calcTree(int v, int p) {
         st.pop_back();
                                                                                                                                                           used[v] = 1;
}
                                                                                                                                                           for (int u : g[v])
                                                                                                                                                                if (u != p \&\& !isForbidden(v, u)) {
void find_bridges(int v, int parentEdge = -1) {
                                                                                                                                                                    dfs(u, v);
    if (up[v]) // уже были
                                                                                                                                                                     //calc dp
        return;
                                                                                                                                                                1
    up[v] = tin[v] = ++timer;
     st.pb(v); // st - stack
    for (Edge e : g[v]) {
                                                                                                                                                       void calcCycle(int v, int p) {
        if (e.id == parentEdge)
                                                                                                                                                           int c = inCycle[v];
                                                                                                                                                           for (int u : cycles[c])
             continue;
         int u = e.to;
                                                                                                                                                               inProcess[u] = 1;
        if (!tin[u]) {
                                                                                                                                                           for (int u : cycles[c])
            int size = sz(st);
                                                                                                                                                               for (int w : g[u])
             find_bridges(u, e.id) ;
                                                                                                                                                                     if (w != p && inCycle[w] != c)
             if (up[u] > tin[v])
                                                                                                                                                                         dfs(w, u), sons[u].pb(w);
                  newComp(size);
                                                                                                                                                            //calc dp on cactus
                                                                                                                                                            for (int u : cycles[c])
         up[v] = min(up[v], up[u]);
                                                                                                                                                                 inProcess[u] = 0, used[u] = 1;
    }
}
                                                                                                                                                       void dfs(int v, int p) {
 //после вызова find_bridges newComp() для корня
void run(int n) {
                                                                                                                                                           if (used[v])
    forn(i, n) {
        if (!up[i]) {
                                                                                                                                                            if (!inProcess[v] && inCycle[v] != -1)
            find_bridges(i);
                                                                                                                                                                calcCycle(v, p);
            newComp();
                                                                                                                                                           else
        }
                                                                                                                                                                 calcTree(v, p);
    }
                                                                                                                                                       int init(int n) {
                                                                                                                                                           forn (i, n)
              Cactuses
 30
                                                                                                                                                               inCycle[i] = -1;
                                                                                                                                                            getCycles(0, -1);
namespace Cactus {
                                                                                                                                                           forn (i, n)
    int used[MAX_N], inCycle[MAX_N], dp[MAX_N],
                                                                                                                                                              used[i] = 0;
      \hookrightarrow inProcess[MAX_N];
                                                                                                                                                           dfs(0, -1);
    vi g[MAX_N], sons[MAX_N], st, cycle;
                                                                                                                                                           return dp[0];
    set<pii> forbidden;
    vector<vi> cycles;
                                                                                                                                                  }
    int curCycle = 0;
    void getCycles(int v, int p) {
                                                                                                                                                   31
                                                                                                                                                                 Cut Points
        used[v] = 1;
         st.pb(v);
                                                                                                                                                   struct Edge {
         for (int u : g[v])
                                                                                                                                                       int to, id;
             if (u != p && used[u] == 1) {
                                                                                                                                                       Edge(int aa, int bb) : to(aa), id(bb) {}
                  cycle.clear();
                  formr (i, sz(st)) {
                      cycle.pb(st[i]);
                                                                                                                                                   vector<Edge> g[MAX_N]; // (to, id)
                                                                                                                                                   vector<int> st; // stack
                      inCycle[st[i]] = curCycle;
                                                                                                                                                   bool used[MAX_M];
                      if (st[i] == u)
                                                                                                                                                   int tin[MAX_N], timer, is_cut[MAX_N], color[MAX_M], compCnt;
                          break;
                  curCycle++;
                                                                                                                                                   int dfs(int v, int parent = -1) {
                                                                                                                                                      tin[v] = ++timer;
                 reverse(all(cycle));
                  cycles.pb(cycle);
                                                                                                                                                       int up = tin[v], x = 0, y = (parent != -1);
                                                                                                                                                       for (Edge p : g[v]){
                                                                                                                                                           int u = p.to, id = p.id;
             else if (u != p && !used[u])
                                                                                                                                                           if (id != parent) {
                  getCycles(u, v);
                                                                                                                                                                int t, size = sz(st);
         st.pop_back();
         used[v] = 2;
                                                                                                                                                                if (!used[id]){
                                                                                                                                                                     st.push_back(id);
                                                                                                                                                                     used[id] = 1;
    bool isForbidden(int v, int u) {
           \texttt{return forbidden.count}(\texttt{mp}(\texttt{v, u})) \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ | \ | \ \texttt{forbidden.count}(\texttt{mp}(\texttt{u, u})) \ | \ \texttt{forbidden.count}
                                                                                                                                                                if (!tin[u]) { // not visited yet
             \rightarrow v)):
                                                                                                                                                                     t = dfs(u, id);
                                                                                                                                                                     if (t >= tin[v]){
```

used[v] = 1;

for(int to : g[v])

```
while(sz(st) != size){
                                                                    if (paired[to] == -1 || dfs(paired[to])) {
            color[st.back()] = compCnt;
                                                                      paired[to] = v;
                                                                      paired[v] = to;
            st.pop_back();
         }
                                                                      return true;
       }
                                                                    }
     } else
                                                                  return false;
       t = tin[u];
     up = min(up, t);
   }
                                                                int kuhn() {
 }
                                                                  int ans = 0;
 if(x + y >= 2)
                                                                  forn(i, n + m)
                                                                   paired[i] = -1;
   is_cut[v] = 1; // v is cut vertex
                                                                  for (int run = 1; run;) {
 return up;
                                                                    run = 0;
                                                                    memset(used, 0, sizeof(used));
                                                                    forn(i, n)
32
    DP tree
                                                                      if (!used[i] && paired[i] == -1 && dfs(i)) {
int dfs(int v) {
                                                                        run = 1;
 forn(i, n + 1)
   dp[v][i] = -INF;
                                                                  }
 dp[v][1] = num[v];
                                                                  return ans;
 int mxsz = 1;
                                                                }
 for (int to : g[v]) {
                                                                //Стартуем из вершин без пары из первой доли, ходим из первой
   int size = dfs(to);
                                                                 → доли, из второй - только по парсочу.
   forba(i, 1, mxsz + 1)
                                                                //Мах независимое - А+, В-
     fornr(j, size + 1)
       dp[v][i + j] = max(dp[v][i + j], dp[v][i] +
                                                                //Min покрытие - A-, B+
        \hookrightarrow dp[to][j]);
                                                                vector <int> minCover, maxIndependent;
   mxsz += size;
                                                                void dfsCoverIndependent(int v) {
 return mxsz;
                                                                  if (used[v])
                                                                   return;
                                                                  used[v] = 1;
33 Eulerian Cycle
                                                                  for(int to : g[v])
                                                                    if (!used[to])
struct Edge {
                                                                      used[to] = 1, dfsCoverIndependent(paired[to]);
 int to, used;
                                                                }
 Edge(): to(-1), used(0) {}
 Edge(int v): to(v), used(0) {}
                                                                //Сперва Куна
                                                                void findCoverIndependent() {
                                                                  memset(used, 0, sizeof(used));
vector <Edge> edges[MAX_M];
                                                                  forn(i, n)
vector <int> g[MAX_N], res, ptr;
                                                                    if (paired[i] == -1)
//не забывать чистить ptr
                                                                      dfsCoverIndependent(i);
                                                                  forn(i, n)
void dfs(int v) {
                                                                    if(used[i])
 for(; ptr[v] < sz(g[v]);) {
                                                                      maxIndependent.pb(i);
   int id = g[v][ptr[v]++];
                                                                    else
   if (!edges[id].used) {
                                                                      minCover.pb(i);
     edges[id].used = edges[id ^ 1].used = 1;
                                                                  forab(i, n, n + m)
     dfs(edges[id].to);
                                                                    if (used[i])
       res.pb(id); // для ребер
                                                                      minCover.pb(i);
   }
                                                                    else
 }
                                                                      maxIndependent.pb(i);
 res.pb(v); // в res вершины
                                                                }
                                                                      Math
                                                                7
34 Kuhn's algorithm
//первая доля – п вершин вторая доля – т вершин
                                                                35 CRT (KTO)
//нумерация сквозная
const int MAX_N = 1e5 + 100;
                                                                namespace Math {
                                                                  vi crt(vi a, vi mod) {
int n, m, paired[2 * MAX_N], used[2 * MAX_N];
                                                                    int n = sz(a);
vector <int> g[MAX_N];
                                                                    vi x(n);
                                                                    forn (i, n) {
bool dfs(int v) {
                                                                      x[i] = a[i];
 if (used[v])
                                                                      forn (j, i) {
   return false;
                                                                         x[i] = inverse(mod[j], mod[i]) * (x[i] - x[j]) %
```

mod[i];
if (x[i] < 0)</pre>

```
x[i] += mod[i];
}
return x;
}
```

36 Discrete Logariphm

```
namespace Math {
  int discreteLogariphm(int a, int b, int mod) { //returns x:
  \rightarrow a^x = b (mod mod) or -1, if no such x exists
    int sq = sqrt(mod);
    int sq2 = mod / sq + (mod % sq ? 1 : 0);
    vector<pii> powers(sq2);
    forn (i, sq2)
     powers[i] = mp(power(a, (i + 1) * sq, mod), i + 1);
    sort(all(powers));
    forn (i, sq + 1) {
       int cur = power(a, i, mod);
       cur = (cur * 111 * b) % mod;
       auto it = lower_bound(all(powers), mp(cur, 0));
       if (it != powers.end() && it->fs == cur)
        return it->sc * sq - i;
   }
    return -1;
 }
}
```

37 Discrete Root

```
namespace Math {
   //returns x: x^k = a mod mod, mod is prime
   int discreteRoot(int a, int k, int mod) {
     if (a == 0)
        return 0;
     int g = primitiveRoot(mod);
     int y = discreteLogariphm(power(g, k, mod), a, mod);
     return power(g, y, mod);
   }
}
```

38 Eratosthenes

```
namespace Math {
  vi eratosthenes(int n) {
    vi minDiv(n + 1, 0);
    minDiv[1] = 1;
    for (int i = 2; i <= n; i++)
      if (minDiv[i] == 0)
        for (int j = i; j \le n; j += i)
          if (minDiv[j] == 0)
           minDiv[j] = i;
    return minDiv;
  vi eratosthenesFast(int n) {
    vi minDiv(n + 1, 0);
    vi primes;
    minDiv[1] = 1;
    for (int i = 2; i <= n; i++) {
      if (minDiv[i] == 0) {
       minDiv[i] = i;
       primes.pb(i);
      for (int j = 0; j < sz(primes) && primes[j] <= minDiv[i]
       \hookrightarrow && i * primes[j] <= n; j++)
        minDiv[i * primes[j]] = primes[j];
    return minDiv;
```

```
}
```

39 Factorial

```
namespace Math {
  //returns pair (rem, deg), where rem = n! % mod,
  //deg = k: mod^k | n!, mod is prime, O(mod log mod)
  pii fact(int n, int mod) {
   int rem = 1, deg = 0;
    int n2 = n;
    while (n2)
     n2 /= mod, deg += n2;
    while (n > 1) {
     rem = (rem * ((n / mod) % 2 ? -1 : 1) + mod) % mod;
      for (int i = 2; i <= n % mod; i++)
       rem = (rem * 111 * i) % mod;
     n /= mod;
   }
    return mp(rem % p, deg);
}
```

40 Gauss

```
namespace Math {
  const double EPS = 1e-9;
  int gauss(double **a, int n, int m) { //n is number of
  int row = 0, col = 0;
   vi par(m, -1);
   vector<double> ans(m, 0);
   for (col = 0; col < m && row < n; col++) {
     int best = row;
     for (int i = row; i < n; i++)
       if (abs(a[i][col]) > abs(a[best][col]))
         best = i;
     if (abs(a[best][col]) < EPS)</pre>
       continue:
     par[col] = row;
     for (int i = 0; i \le m; i++)
       swap(a[row][i], a[best][i]);
     for (int i = 0; i < n; i++)
       if (i != row) {
         long double k = a[i][col] / a[row][col];
         for (int j = col; j \le m; j++)
           a[i][j] -= k * a[row][j];
       }
     row++;
   int single = 1;
   for (int i = 0; i < m; i++)
     if (par[i] != -1)
       ans[i] = a[par[i]][m] / a[par[i]][i];
     else
       single = 0;
   for (int i = 0; i < n; i++) {
     long double cur = 0;
     for (int j = 0; j < m; j++)
       cur += ans[j] * a[i][j];
      if (abs(cur - a[i][m]) > EPS)
       return 0;
   if (!single)
     return 2;
   return 1;
 }
}
```

int g = gcd(a, mod, x, y);

if (n % i == 0) {

41 Gauss mod 2

int x, y;

```
return g == 1 ? (x % mod + mod) % mod : -1;
namespace Math {
 const int MAX = 1024;
                                                                  vi inverseForAll(int mod) {
 int gaussMod2(vector<bitset<MAX>> a, int n, int m) {
                                                                    vi r(mod, 0);
   int row = 0, col = 0;
                                                                     r[1] = 1;
    vi par(m, -1);
                                                                     for (int i = 2; i < mod; i++)
   for (col = 0; col < m && row < n; col++) {
                                                                       r[i] = (mod - r[mod % i]) * (mod / i) % mod;
     int best = row;
                                                                     return r;
     for (int i = row; i < n; i++)
                                                                  }
       if (a[i][col] > a[best][col])
         best = i;
      if (a[best][col] == 0)
                                                                43
                                                                       Gray
       continue;
     par[col] = row;
                                                                namespace Math {
     swap(a[row], a[best]);
                                                                  int gray(int n) {
     for (int i = 0; i < n; i++)
                                                                   return n ^ (n >> 1);
       if (i != row) {
         if (a[i][col])
           a[i] ^= a[row];
                                                                  int revGray(int n) {
       }
                                                                     int k = 0;
     row++;
                                                                     for (; n; n >>= 1)
   }
                                                                      k = n;
    vi ans(m, 0);
                                                                     return k;
   for (int i = 0; i < m; i++)
                                                                  }
     if (par[i] != -1)
                                                                }
       ans[i] = a[par[i]][n] / a[par[i]][i];
    bool ok = 1;
   for (int i = 0; i < n; i++) {
                                                                       Miller-Rabin Test
                                                                44
     int cur = 0;
                                                                namespace Math {
     for (int j = 0; j < m; j++)
                                                                  vector <int> primes = {2,3,5,7,11,13,17,19,23};
       cur ^= (ans[j] & a[i][j]);
      if (cur != a[i][n])
       ok = 0:
                                                                  bool isPrimeMillerRabin(ll n) {
                                                                    int k = 0;
                                                                    11 t = n - 1;
   return ok;
                                                                    while (t \% 2 == 0)
                                                                     k++, t /= 2;
}
                                                                    for (auto p : primes) {
                                                                      ll g = __gcd(n, (ll) p);
42
     \operatorname{Gcd}
                                                                      if (g > 1 && g < n)
                                                                       return 0;
namespace Math {
                                                                      if (g == n)
 int gcd(int a, int b) {
                                                                       return 1;
   return b ? gcd(b, a % b) : a;
                                                                      11 b = power(p, t, n);
                                                                      ll last = n - 1;
                                                                      bool was = 0;
 int gcd(int a, int b, int &x, int &y) {
                                                                      forn (i, k + 1) {
    if (b == 0) {
                                                                       if (b == 1 && last != n - 1)
     x = 1, y = 0;
                                                                          return 0:
      return a;
                                                                        if (b == 1) {
                                                                         was = 1;
    else {
     int g = gcd(b, a % b, x, y);
                                                                         break;
     int newX = y;
                                                                        last = b;
     y = x - a / b * y;
                                                                        b = mul(b, b, n);
     x = newX;
                                                                      }
     return g;
                                                                      if (!was)
                                                                        return 0:
                                                                    return 1;
  void diophant(int a, int b, int c, int &x, int &y) {
                                                                  }
   int g = gcd(a, b, x, y);
                                                                }
   if (c % g != 0)
    return;
   x *= c / g, y *= c / g;
                                                                45 Phi
   //next solutions: x += b / g, y -= a / g
                                                                namespace Math {
                                                                  int phi(int n) {
                                                                    int result = n;
 int inverse(int a, int mod) { //returns -1, if a and mod are
                                                                    for (int i = 2; i * i <= n; i++)

→ not coprime
```

46 Pollard

```
namespace Math {
  inline void pollardFoo(11 &x, 11 mod) {
    x = (mul(x, x, mod) + 1) \% mod;
  vector <pair <11, int> > factorize(11 n) {
    if (n == 1)
     return {};
    if (isPrimeMillerRabin(n))
     return {mp(n, 1)};
    if (n \le 100) {
      vector <pair <11, int> > ans;
      for (int i = 2; i * i <= n; i++)
        if (n % i == 0) {
          int cnt = 0;
          while (n \% i == 0)
           n /= i, cnt++;
          ans.pb(mp(i, cnt));
        }
      if (n != 1)
        ans.pb(mp(n, 1));
      sort(all(ans)):
      return ans;
    while (1) {
      ll a = rand() % n, b = a;
      while (1) {
        pollardFoo(a, n), pollardFoo(b, n), pollardFoo(b, n);
        11 g = \__gcd(abs(a-b), n);
        if (g != 1) {
          if (g == n)
            break;
          else {
            auto ans1 = factorize(g);
            auto ans2 = factorize(n / g);
            vector <pair <11, int> > ans;
            ans1.insert(ans1.end(), all(ans2));
            sort(all(ans1));
            for (auto np : ans1)
              if (sz(ans) == 0 \mid \mid np.fs \mid = ans.back().fs)
                ans.pb(np);
              else
                ans.back().sc += np.sc;
            return ans;
          }
        }
     }
    }
   assert(0);
 }
}
```

47 Primitive Root

```
namespace Math {
  int primitiveRoot(int mod) { //returns -1 if no primitive
    vi fact;
     int ph = phi(mod);
     int n = mod;
     for (int i = 2; i * i <= n; i++) {
       if (n % i == 0) {
         fact.pb(i);
           while (n % i == 0)
             n /= i;
        }
     }
     if (n > 1)
       fact.pb(n);
     forab (i, 2, mod + 1) {
       bool ok = 1;
        for (int j = 0; j < sz(fact) && ok; <math>j++)
          ok &= power(i, ph / fact[j], mod) != 1;
        if (ok)
          return i;
     }
     return -1;
  }
}
```

48 Simpson

8 Mix

49 Fast allocation (operator new)

```
#include <cassert>
/** Begin fast allocation */
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX_MEM];
inline void * operator new ( size_t n ) {
 char *res = mem + mpos;
 mpos += n;
  assert(mpos <= MAX_MEM);</pre>
 return (void *)res:
inline void operator delete ( void * ) { }
/** End fast allocation */
inline void * operator new [] ( size_t ) { assert(0); }
inline void operator delete [] ( void * ) { assert(0); }
    Fast I/O (short)
50
inline int readChar();
inline int readInt();
```

template <class T> inline void writeInt(T x);

```
inline int readChar() {
 int c = getchar();
 while (c <= 32)
   c = getchar();
 return c;
inline int readInt() {
 int s = 0, c = getchar(), x = 0;
 if (c == '-')
  s = 1, c = getchar();
 while ('0' <= c && c <= '9')
   x = x * 10 + c - '0', c = getchar();
 return s ? -x : x;
template <class T> inline void writeInt( T x ) {
 if(x < 0)
   putchar('-'), x = -x;
 char s[24];
 int n = 0;
 while (x \mid | \mid n)
   s[n++] = 0 + x \% 10, x /= 10;
 while (n--)
   putchar(s[n]);
      Masks tricks
forn(mask, 1 \ll d) {
 dp[mask][d] = 1;
```

```
fornr(i, d) {
    dp[mask][i] = dp[mask][i + 1];
    if ((1 << i) & mask)
      dp[mask][i] += dp[mask ^ (1 << i)][i + 1];</pre>
  cout << mask << " -> " << dp[mask][0] << '\n';
int num[64];
for (ULL i = 0; i < 64; ++i) {
```

Hash of pair

num[(1ull << i) % 67] = i;

```
struct MyHash {
 size_t operator()(const pair<int,int> &t) const {
   return t.first * 239017 + t.second;
};
```

9 **Strings**

Aho-Corasick

```
const int ALPHA = 26;
const int MAX_N = 1e5;
struct Node {
 int next[ALPHA], term; //Bop
 int go[ALPHA], suf, p, pch; //Asmomam
 Node() {
   memset(next, -1, sizeof(next));
   term = 0;
   memset(go, -1, sizeof(go));
   suf = p = -1;
 }
};
```

```
Node g[MAX_N];
int last;
void add(const string &s) {
 int now = 0;
  for(char x : s) { // x = a
   if (g[now].next[x] == -1) {
     g[now].next[x] = ++last;
      g[last].p = now;
     g[last].pch = x;
   now = g[now].next[x];
  g[now].term = 1;
int go(int v, int c);
int get_link(int v) {
 if (g[v].suf == -1) {
   if (!v || !g[v].p)
     g[v].suf = 0;
     g[v].suf = go(get_link(g[v].p), g[v].pch);
 }
  return g[v].suf;
int go(int v, int c) {
 if (g[v].go[c] == -1) {
   if (g[v].next[c] != -1)
     g[v].go[c] = g[v].next[c];
    else
     g[v].go[c] = !v ? 0 : go(get_link(v), c);
 }
 return g[v].go[c];
}
```

54 Prefix-function

```
// pr[len] - для префикса длины len
int k = 0;
pr[0] = pr[1] = 0;
for (int i = 2; i <= n; i++) {
 k = pr[i - 1];
  while (k && s[k] != s[i - 1])
   k = pr[k];
  if (s[k] == s[i - 1])
   k++;
 pr[i] = k;
```

Z-function

```
//z[i] - с позиции i сколько матчится
int 1 = -1, r = -1;
z[0] = 0;
for (int i = 1; i < n; i++) {
 int k = 0;
 if (r >= i)
   k = min(z[i - 1], r - i);
 while (i + k < n \&\& s[i + k] == s[k])
   k++;
  z[i] = k;
 if (i + z[i] > r)
    1 = i, r = i + z[i];
```

if (r > i + t) {

56 Hash

forn(t, 2) {

forn(i, n - t) {

int k = 0;

int *z = t ? z1 : z0, 1 = -1, r = -1; // [l...r]

```
int j = 1 + (r - i - t);
#include < bits/stdc++.h>
                                                                            k = min(z[j], j - 1);
typedef long long LL;
                                                                          while (i - k >= 0 \&\& i + k + t < n \&\& s[i - k] == s[i +
                                                                           \hookrightarrow k + t])
inline int byMod(int a, int m){
                                                                           k++;
    return a \ge m ? a - m : a;
                                                                          z[i] = k;
                                                                          if (k && i + k + t > r)
                                                                            1 = i - k + 1, r = i + k + t - 1;
const int MX = 1e9 + 9, MY = 1e9 + 7;
                                                                      }
//typedef unsigned long long H;
                                                                    }
struct H{
    int x, y;
                                                                    const int N = 1e5;
    H(): x(0), y(0){}
   H(int _x): x(_x), y(_x){}
                                                                    int n, r0[N], r1[N];
    H(int _x, int _y): x(_x), y(_y){}
                                                                    char s[N + 1];
    inline H operator +(const H &B) const{return H(byMod(x +
    \rightarrow B.x, MX), byMod(y + B.y, MY));}
                                                                    int main() {
    inline H operator -(const H &B) const{return H(byMod(x +
                                                                     assert(freopen("palindrome.in", "rt", stdin));
    \rightarrow MX - B.x, MX), byMod(y + MY - B.y, MY));}
                                                                      assert(freopen("palindrome.out", "wt", stdout));
    inline H operator *(LL k) const{return H(int((x * k) % k))
    \hookrightarrow MX), int((y * k) % MY));}
                                                                      gets(s);
    inline H operator *(const H &B) const{return H(int((LL(x)
                                                                      n = strlen(s);

    * B.x) % MX), int((LL(y) * B.y) % MY));}

                                                                      manaker(n, s, r0, r1);
    inline bool operator ==(const H &B) const{return x == B.x
                                                                      cout << accumulate(r0, r0 + n, OLL) + accumulate(r1, r1 + n,</pre>
    \hookrightarrow && y == B.y;}
                                                                      \hookrightarrow 0LL) - n << endl;
    inline bool operator !=(const H &B) const{return x != B.x
                                                                      return 0;
     \hookrightarrow || y != B.y;}
                                                                    }
    inline bool operator <(const H &B) const{return x < B.x | |</pre>
    \hookrightarrow (x == B.x && y < B.y);}
    explicit inline operator LL() const{return (LL)x * MY + y}
                                                                          Palindromic Tree
                                                                    58
     → + 1;} // > 0
                                                                    \#define\ fill(a,\ x)\ memset(a,\ x,\ sizeof(a))
const int P = 239017, MAX_N = 1e6 + 10;
                                                                    template < const int N>
H deg[MAX_N], h[MAX_N];
                                                                    struct PalindromeTree {
char s[MAX_N];
                                                                      struct Vertex {
                                                                        int suf, len, next[26];
inline H Get(int a, int 1){
    return h[a + 1] - h[a] * deg[1];
                                                                      int vn, v;
                                                                      Vertex t[N + 2];
int main(){
                                                                      int n, s[N];
#ifdef LOCAL
    assert(freopen("test.in", "r", stdin));
                                                                      int get( int i ) { return i < 0 ? -1 : s[i]; }</pre>
    assert(freopen("test.out", "w", stdout));
#endif
                                                                      void init() {
                                                                       fill(t, 0);
    gets(s);
                                                                        t[0].len = -1, vn = 2, v = 0, n = 0;
    int L = strlen(s);
    deg[0] = 1;
    for(int i = 0; i < L; ++i)
                                                                      void add( int ch ) {
       h[i + 1] = h[i] * P + s[i], deg[i + 1] = deg[i] * P;
                                                                        s[n++] = ch;
                                                                        while (v != 0 && ch != get(n - t[v].len - 2))
    return 0;
                                                                          v = t[v].suf;
                                                                        int &r = t[v].next[ch];
                                                                        if (!r) {
                                                                          t[vn].len = t[v].len + 2;
     Manaker
                                                                          if (!v)
#include <bits/stdc++.h>
                                                                            t[vn].suf = 1;
                                                                          else {
                                                                            v = t[v].suf;
using namespace std;
                                                                            while (v != 0 \&\& ch != get(n - t[v].len - 2))
#define form(i, n) for (int i = 0; i < (int)(n); i++)
                                                                              v = t[v].suf;
                                                                            t[vn].suf = t[v].next[ch];
void manaker( int n, char *s, int *z0, int *z1 ) {
                                                                          }
```

r = vn++;

}

v = r;

}:

```
forn(i, n){
                                                                      int j = p2[i];
const int N = 1e5;
                                                                      lcp = max(0, lcp - 1);
                                                                      if (j != n - 1)
                                                                        while (lcp < n \&\& s[(p[j] + lcp) \% n] == s[(p[j + 1] +
PalindromeTree<N> pt:
                                                                         \rightarrow lcp) % n])
char s[N + 1];
                                                                         lcp++;
                                                                      len[j] = lcp;
int main() {
                                                                      if (j != n - 1 && p[j + 1] == n - 1)
                                                                        lcp = 0;
 gets(s);
  int n = strlen(s);
                                                                    }
 pt.init():
 forn(i, n) {
  pt.add(s[i] - 'a');
                                                                  int main()
 printf("%d ", pt.vn - 2);
}
                                                                    scanf("%d%s", &n, s);
 return 0;
}
                                                                    BuildArray();
                                                                    BuildLCP();
      Suffix Array (+stable)
                                                                    // res = sum of all LCP[i,i+1]
                                                                    LL res = 0;
const int MAX_N = 250000;
                                                                    forn(i, n)
                                                                      res += len[i];
int n, num[MAX_N + 1];
                                                                    printf("\%.3f\n", (double)res / (n - 1));
char s[MAX_N + 1];
                                                                    return 0;
int p[MAX_N], col[MAX_N], p2[MAX_N], len[MAX_N];
void BuildArray(){
 int ma = max(n, 256);
                                                                         Suffix Automaton
                                                                  60
  forn(i, n)
                                                                  #include <bits/stdc++.h>
   col[i] = s[i], p[i] = i;
  for (int k2 = 1; k2 / 2 < n; k2 *= 2){
                                                                  struct Vx{
    int k = k2 / 2;
                                                                      static const int AL = 26;
    memset(num, 0, sizeof(num));
                                                                      int len. suf:
   forn(i, n)
                                                                      int next[AL];
     num[col[i] + 1]++;
                                                                      Vx(){}
                                                                      Vx(int 1, int s):len(1), suf(s){}
    forn(i, ma)
     num[i + 1] += num[i];
    forn(i, n)
     p2[num[col[(p[i] - k + n) % n]]++] = (p[i] - k + n) % n;
                                                                  struct SA{
                                                                      static const int MAX_LEN = 1e5 + 100, MAX_V = 2 * MAX_LEN;
    int cc = 0;
                                                                      int last, vcnt;
    forn(i, n){
                                                                      Vx v[MAX_V];
      if (i && (col[p2[i]] != col[p2[i - 1]] ||
      col[(p2[i] + k) \% n] != col[(p2[i - 1] + k) \% n]))
                                                                      SA(){
                                                                           vcnt = 1;
       cc++;
                                                                           last = newV(0, 0); // root = vertex with number 1
      num[p2[i]] = cc;
    forn(i, n)
                                                                      int newV(int len, int suf){
      p[i] = p2[i], col[i] = num[i];
                                                                           v[vcnt] = Vx(len, suf);
                                                                           return vcnt++;
  // make it stable
  memset(num, 0, sizeof(num));
                                                                      int add(char ch){
                                                                           int p = last, c = ch - 'a';
  forn(i, n)
                                                                           last = newV(v[last].len + 1, 0);
   num[col[i] + 1]++;
  forn(i, ma)
                                                                           while(!v[p].next[c])
   num[i + 1] += num[i];
                                                                               v[p].next[c] = last, p = v[p].suf;
  forn(i, n)
                                                                           if(!p)
   p2[num[col[i]]++] = i;
                                                                               v[last].suf = 1;
                                                                           else{
  forn(i, n)
   p[i] = p2[i];
                                                                               int q = v[p].next[c];
                                                                               if (v[q].len == v[p].len + 1)
  // calc inverse permutation
                                                                                  v[last].suf = q;
  forn(i, n)
                                                                               elsef
    p2[p[i]] = i;
                                                                                  int r = newV(v[p].len + 1, v[q].suf);
                                                                                   v[last].suf = v[q].suf = r;
                                                                                   memcpy(v[r].next, v[q].next,
void BuildLCP(){

    sizeof(v[r].next));
                                                                                   \label{eq:while(p && v[p].next[c] == q)} while(p && v[p].next[c] == q)
  int lcp = 0;
```

```
v[p].next[c] = r, p = v[p].suf;
        }
        return last;
   }
};
      Suffix Tree
61
const int MAX_L=1e5+10;
char S[MAX_L];
int L;
struct Node;
struct Pos:
typedef Node *pNode;
typedef map<char,pNode> mapt;
struct Node{
 pNode P,link;
  int L,R;
 mapt next;
  Node():P(NULL),link(this),L(0),R(0){}
  Node(pNode P,int L,int R):P(P),link(NULL),L(L),R(R){}
  inline int elen() const{return R-L;}
  inline pNode add_edge(int L,int R){return next[S[L]]=new
  → Node(this,L,R);}
struct Pos{
 pNode V;
  int up;
  Pos():V(NULL),up(0){}
  Pos(pNode V, int up): V(V), up(up){}
  pNode split_edge() const{
    if(!up)
     return V:
    int L=V->L, M=V->R-up;
    pNode P=V->P, n=new Node(P,L,M);
    P \rightarrow next[S[L]] = n;
   n->next[S[M]]=V;
    V \rightarrow P=n, V \rightarrow L=M;
    return n;
  Pos next_char(char c) const{
    if(up)
      return S[V->R-up]==c ? Pos(V,up-1) : Pos();
    elsef
      mapt::iterator it=V->next.find(c);
      return it==V->next.end() ? Pos()
       \hookrightarrow Pos(it->snd,it->snd->elen()-1);
    }
 }
};
Pos go_down(pNode V,int L,int R){
  if(L==R)
    return Pos(V,0);
  while(1){
    V=V->next[S[L]];
    L+=V->elen():
    if(L>=R)
      return Pos(V,L-R);
}
inline pNode calc_link(pNode &V){
```

if(!V->link)

```
V \rightarrow link = go_down(V \rightarrow P \rightarrow link, V \rightarrow L + !V \rightarrow P \rightarrow P, V \rightarrow R).split_edge();
  return V->link;
Pos add_char(Pos P,int k){
  while(1){
    Pos p=P.next_char(S[k]);
    if(p.V)
       return p;
     pNode n=P.split_edge();
    n->add_edge(k,MAX_L);
    if(!n->P)
       return Pos(n,0);
    P=Pos(calc_link(n),0);
}
pNode Root;
void make_tree(){
  Root=new Node();
  Pos P(Root,0);
  forn(i,L)
    P=add_char(P,i);
```

10 C++ Tricks

62 Tree

11 Notes

63 Tree

Приемы для работы с деревьями:

- 1. Двоичные подъемы
- 2. Поддеревья как отрезки Эйлерова обхода
- 3. Вертикальные пути в Эйлеровом обходе (на ребрах вниз +k, на ребрах вверх -k).
- Храним в вершине значение функции на пути от корня до нее, дальше LCA.
- 5. Спуск с DFS, поддерживаем ДО на пути до текущей вершины.
- 6. Heavy-light decomposition
- 7. Centroid decomposition

64 DP

Табличка с оптимизациями для динамики:

	Name	Original recurrence	Sufficient Condition	From	То
	Convex Hull Optimization 1	$dp[i] = \min_{j < i} dp[j] + b[j] \cdot a[i]$	$b[j] \geqslant b[j+1] a[i] \leqslant a[i+1]$	$O(n^2)$	O(n)
	Convex Hull Optimization 2	$dp[i][j] = \min_{k < j} dp[i-1][k] + b[k] \cdot a[j]$	$b[k] \geqslant b[k+1] a[j] \leqslant a[j+1]$	$O(kn^2)$	O(kn)
	D&C Optimization	$dp[i][j] = \min_{k < j} dp[i-1][k] + c[k][j]$	$p[i,j] \leqslant p[i,j+1]$	$O(kn^2)$	$O(kn\log n)$
	Knuth Optimization	$dp[i][j] = \min_{i < k < j} dp[i][k] + dp[k][j] + c[i][j]$	$p[i,j-1] \leqslant p[i,j] \leqslant p[i+1,j]$	$O(n^3)$	$O(n^2)$

65 Combinatorics

Биномиальные коэффициенты:

Теорема Люка для биномиальных коэффициентов: Хотим посчитать C_n^k , разложим в р-ичной системе счисления, $n=(n_0,n_1,\ldots),k=(k_0,k_1,\ldots).$ $ans=C_{n_0}^{k_0}\cdot C_{n_1}^{k_1}\cdot\ldots$

Способы вычисления C_n^k :

1. $C_n^k = C_{n-1}^k + C_{n-1}^{k-1}$ precalc: $O(n^2)$, query: O(1).

2. $C_n^k = \frac{n!}{k!(n-k)!}$, предподсчитываем факториалы precalc: $O(n\log n)$, query: $O(\log n)$

3. Теорема Люка precalc: $O(p \log p)$, query: O(log p).

4. $C_n^k = C_n^{k-1} \cdot \frac{n-k+1}{k}$

5. $C_n^k = \frac{n!}{k!(n-k)!}$, для каждого факториала считаем степень вхождения и остаток

precalc: $O(p \log p)$, query: O(log p).

$$C_n^{\frac{n}{2}} = \frac{2^n}{\sqrt{\frac{\pi n}{2}}}$$

66 Masks

Считаем динамику по маскам за $O(2^n \cdot n)$ f[mask] = sum по submask g[submask]. dp[mask][i] — значение динамики для маски mask, если младшие i бит в ней зафиксированы (то есть мы не можем удалять оттуда). Ответ в dp[mask][0]. dp[mask][len] = g[mask]. Если i-ый бит 0, то dp[mask][i] = dp[mask][i+1], иначе $dp[mask][i] = dp[mask][i+1] + dp[mask^{(1)}][i+1]$.

Старший бит: предподсчет.

Младший бит: $x \& \sim (-x)$

Чтобы по степени двойки получить логарифм, можно воспользоваться тем, что все степени двойки имеют разный остаток по модулю 67.

67 Flows

Потоки:

Name	Asympthotic
Ford-Fulkerson	$O(f \cdot E)$
Ford-Fulkerson with scaling	$O(\log f \cdot E^2)$
Edmonds-Karp	$O(V \cdot E^2)$
Dinic	$O(V^2 \cdot E)$
Dinic with scaling	$O(V \cdot E \cdot \log C)$
Dinic on bipartite graph	$O(E\sqrt{V})$
Dinic on unit network	$O(E\sqrt{E})$

L-R потоки: Есть граф с недостатками или избытками в каждой вершине. Создаем фиктивные исток и сток (из истока все ребра в избытки, из недостатков все ребра в сток). Теперь пусть у нас есть L-R граф, для каждого ребра e ($v \to u$) известны L_e и R_e . Добавим в v избыток L_e , в u недостаток L_e , а пропускную способность сделаем $R_e - L_e$. Получили решение задачи о LR-циркуляции. Если у нас обычный граф с истоком и стоком, то добавляем бесконечное ребро из стока в сток и ищем циркуляцию. Таким образом нашли удовлетворяющий условиям LR-поток. Если хотим максимальный поток, то на остаточной сети запускаем поиск максимального потока. В новом графе в прямую сторону пропускная способность равна $R_e - f_e$, в обратную $f_e - L_e$.

MinCostCirculation: Пока есть цикл отрицательного веса, запускаем алгоритм Карпа и пускаем максимальный поток по найденному циклу.

68 Grandi

Теорема Шпрага-Гранди: берем тех всех значений функции Гранди по состояниям, в которые можем перейти из данного. Если сумма независимых игр, то значение функции Гранди равно хог значений функций Гранди по всем играм. Бывает полезно вывести первые п значений и поискать закономерность. Часто сводится к хог по чемунибудь.