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# 1 Common

## 1 Setup

1. Terminal: font Monospace 12
2. Gedit: Oblivion, font Monospace 12, auto indent, display line numbers, tab 4, highlight matching brackets, highlight current line, F9 (side panel)
3. `./bashrc: export CXXFLAGS='-Wall -Wshadow -Wextra -Wconversion -Wno-unused-result -Wno-deprecated-declarations -O2 -std=gnu++11 -g -DLOCAL'`
4. `for i in {A..K}; do mkdir $i; cp main.cpp $i/$i.cpp; done`

## 2 Template

```
#include <bits/stdc++.h>

using namespace std;

#define pb push_back
#define mp make_pair
#define fst first
#define snd second
#define sz(x) ((x).size())
#define forn(i, n) for (int i = 0; i < (n); ++i)
#define fornR(i, n) for (int i = (n) - 1; i >= 0; --i)
#define forab(i, a, b) for (int i = (a); i < (b); ++i)
#define all(c) (c).begin(), (c).end()

using ll = long long;
using vi = vector<int>;
using pii = pair<int, int>;

#define FNAME ""

int main() {
#ifdef LOCAL
    freopen(FNAME".in", "r", stdin);
    freopen(FNAME".out", "w", stdout);
#endif
    cin.tie(0);
    ios_base::sync_with_stdio(0);

    return 0;
}
```

## 3 Stress

```
#!/bin/bash

for ((i = 0;; i++)); do
    ./gen $i >in || exit
    ./main <in >out1 || exit
    ./stupid <in >out2 || exit
    diff out1 out2 || exit
    echo $i OK
done
```

## 4 Java

```
import java.io.BufferedReader;
import java.io.FileNotFoundException;
import java.io.FileReader;
import java.io.IOException;
import java.io.InputStreamReader;
import java.io.PrintWriter;
import java.util.*;

public class Main {
    FastScanner in;
    PrintWriter out;

    void solve() {
        int a = in.nextInt();
        int b = in.nextInt();
        out.print(a + b);
    }

    void run() {
```

```
try {
    in = new FastScanner("input.txt");
    out = new PrintWriter("output.txt");
    solve();
    out.flush();
    out.close();
} catch (FileNotFoundException e) {
    e.printStackTrace();
    System.exit(1);
}
}

class FastScanner {
    BufferedReader br;
    StringTokenizer st;

    public FastScanner() {
        br = new BufferedReader(new InputStreamReader(System.in));
    }

    public FastScanner(String s) {
        try {
            br = new BufferedReader(new FileReader(s));
        } catch (FileNotFoundException e) {
            e.printStackTrace();
        }
    }

    String nextToken() {
        while (st == null || !st.hasMoreElements()) {
            try {
                st = new StringTokenizer(br.readLine());
            } catch (IOException e) {
                e.printStackTrace();
            }
        }
        return st.nextToken();
    }

    int nextInt() {
        return Integer.parseInt(nextToken());
    }

    long nextLong() {
        return Long.parseLong(nextToken());
    }

    double nextDouble() {
        return Double.parseDouble(nextToken());
    }

    char nextChar() {
        try {
            return (char) (br.read());
        } catch (IOException e) {
            e.printStackTrace();
        }
        return 0;
    }

    String nextLine() {
        try {
            return br.readLine();
        } catch (IOException e) {
            e.printStackTrace();
        }
        return "";
    }
}

public static void main(String[] args) {
    new Main().run();
}
```

## 2 Big numbers

### 5 Big Int

```
constexpr int BASE = 1000000000;
constexpr int BASE_DIGITS = 9;

struct BigInt {
    // value == 0 is represented by empty z
    vi z; // digits
    // sign == 1/-1 <==> value >=< 0
    int sign;
    BigInt(): sign(1) {}
    BigInt(ll v) { *this = v; }
    BigInt& operator=(ll v) {
        sign = v < 0 ? -1 : 1; v *= sign;
        z.clear(); for (; v > 0; v = v / BASE) z.pb((int) (v %
        BASE));
        return *this;
    }
    BigInt& operator+=(const BigInt& other) {
        if (sign == other.sign) {
            for (int i = 0, carry = 0; i < sz(other.z) || carry; ++i) {
                if (i == sz(z)) z.pb(0);
                z[i] += carry + (i < sz(other.z) ? other.z[i] : 0);
                carry = z[i] >= BASE;
                if (carry) z[i] -= BASE;
            }
        } else if (other != 0 /* prevent infinite loop */) {
            *this -= -other;
        }
        return *this;
    }
    friend BigInt operator+(BigInt a, const BigInt& b) { return a
    += b; }
    BigInt& operator-=(const BigInt& other) {
        if (sign == other.sign) {
            if ((sign == 1 && *this >= other) || (sign == -1 && *this
            <= other)) {
                for (int i = 0, carry = 0; i < sz(other.z) || carry; ++i)
                {
                    z[i] -= carry + (i < sz(other.z) ? other.z[i] : 0);
                    carry = z[i] < 0;
                    if (carry)
                        z[i] += BASE;
                }
                trim();
            } else {
                *this = other - *this;
                this->sign = -this->sign;
            }
        } else
            *this += -other;
        return *this;
    }
    friend BigInt operator-(BigInt a, const BigInt& b) { return a
    -= b; }
    BigInt& operator*=(int v) {
        if (v < 0) sign = -sign, v = -v;
        for (int i = 0, carry = 0; i < sz(z) || carry; ++i) {
            if (i == sz(z))
                z.pb(0);
            ll cur = (ll) z[i] * v + carry;
            carry = (int) (cur / BASE);
            z[i] = (int) (cur % BASE);
        }
        trim();
        return *this;
    }
    BigInt operator*(int v) const { return BigInt(*this) *= v; }
    friend pair<BigInt, BigInt> divmod(const BigInt& a1, const
    BigInt& b1) {
        int norm = BASE / (b1.z.back() + 1);
        BigInt a = a1.abs() * norm;
        BigInt b = b1.abs() * norm;
        BigInt q, r;
        q.z.resize(sz(a.z));
        fornr (i, sz(a.z)) {
            r *= BASE, r += a.z[i];
            int s1 = sz(b.z) < sz(r.z) ? r.z[sz(b.z)] : 0;
```

```
            int s2 = sz(b.z) - 1 < sz(r.z) ? r.z[sz(b.z) - 1] : 0;
            int d = (int) (((ll) s1 * BASE + s2) / b.z.back());
            r -= b * d;
            while (r < 0) r += b, --d;
            q.z[i] = d;
        }
        q.sign = a1.sign * b1.sign, r.sign = a1.sign;
        q.trim(), r.trim();
        return {q, r / norm};
    }
    BigInt operator/(const BigInt& v) const { return divmod(*this,
    v).fst; }
    BigInt operator%(const BigInt& v) const { return divmod(*this,
    v).snd; }
    BigInt& operator/=(int v) {
        if (v < 0) sign = -sign, v = -v;
        int rem = 0;
        fornr (i, sz(z)) {
            ll cur = z[i] + rem * (ll) BASE;
            z[i] = (int) (cur / v);
            rem = (int) (cur % v);
        }
        trim();
        return *this;
    }
    BigInt operator/(int v) const { return BigInt(*this) /= v; }
    int operator%(int v) const {
        if (v < 0) v = -v;
        int m = 0;
        fornr (i, sz(z))
            m = (int) ((z[i] + m * (ll) BASE) % v);
        return m * sign;
    }
    BigInt& operator*=(const BigInt& v) { return *this = *this *
    v; }
    BigInt& operator/=(const BigInt& v) { return *this = *this / v;
    }
    bool operator<(const BigInt& v) const {
        if (sign != v.sign) return sign < v.sign;
        if (sz(z) != sz(v.z)) return sz(z) * sign < sz(v.z) * v.sign;
        fornr (i, sz(z))
            if (z[i] != v.z[i])
                return z[i] * sign < v.z[i] * sign;
        return false;
    }
    bool operator>(const BigInt& v) const { return v < *this; }
    bool operator<=(const BigInt& v) const { return !(v < *this); }
    bool operator>=(const BigInt& v) const { return !(*this < v); }
    bool operator==(const BigInt& v) const { return !(*this < v)
    && !(v < *this); }
    bool operator!=(const BigInt& v) const { return *this < v || v
    < *this; }
    void trim() {
        while (!z.empty() && z.back() == 0) z.pop_back();
        if (z.empty()) sign = 1;
    }
    bool isZero() const { return z.empty(); }
    friend BigInt operator-(BigInt v) {
        if (!v.z.empty()) v.sign = -v.sign;
        return v;
    }
    BigInt abs() const {
        return sign == 1 ? *this : -*this;
    }
    void read(const string& s) {
        sign = 1, z.clear();
        int pos = 0;
        while (pos < sz(s) && (s[pos] == '-' || s[pos] == '+')) {
            if (s[pos] == '-') sign = -sign;
            ++pos;
        }
        for (int i = sz(s) - 1; i >= pos; i -= BASE_DIGITS) {
            int x = 0;
            forab (j, max(pos, i - BASE_DIGITS + 1), i)
                x = x * 10 + s[j] - '0';
            z.pb(x);
        }
        trim();
    }
    friend ostream &operator<<(ostream& stream, const BigInt& v) {
```

```

    if (v.sign == -1)
        stream << '-';
    stream << (v.z.empty() ? 0 : v.z.back());
    fornr (i, sz(v.z) - 1)
        stream << setw(BASE_DIGITS) << setfill('0') << v.z[i];
    return stream;
}

static vi convertBase(const vi& a, int oldDigits, int
→ newDigits) {
    vector<ll> p(max(oldDigits, newDigits) + 1);
    p[0] = 1;
    for (int i = 1; i < sz(p); i++)
        p[i] = p[i - 1] * 10;
    vi res;
    ll cur = 0;
    int curDigits = 0;
    for (int v : a) {
        cur += v * p[curDigits];
        curDigits += oldDigits;
        while (curDigits >= newDigits) {
            res.pb(int(cur % p[newDigits]));
            cur /= p[newDigits];
            curDigits -= newDigits;
        }
    }
    res.pb((int) cur);
    while (!res.empty() && res.back() == 0) res.pop_back();
    return res;
}

static vll karatsubaMultiply(const vll& a, const vll& b) {
    int n = sz(a);
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }

    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k), a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k), b2(b.begin() + k, b.end());
    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);
    forn (i, k) a2[i] += a1[i];
    forn (i, k) b2[i] += b1[i];

    vll r = karatsubaMultiply(a2, b2);
    forn (i, sz(a1b1)) r[i] -= a1b1[i];
    forn (i, sz(a2b2)) r[i] -= a2b2[i];
    forn (i, sz(r)) res[i + k] += r[i];
    forn (i, sz(a1b1)) res[i] += a1b1[i];
    forn (i, sz(a2b2)) res[i + n] += a2b2[i];
    return res;
}

BigInt operator*(const BigInt& v) const {
    vi a6 = convertBase(this->z, BASE_DIGITS, 6);
    vi b6 = convertBase(v.z, BASE_DIGITS, 6);
    vll a(all(a6)), b(all(b6));
    while (sz(a) < sz(b)) a.pb(0);
    while (sz(b) < sz(a)) b.pb(0);
    while (sz(a) & (sz(a) - 1)) a.pb(0), b.pb(0);
    vll c = karatsubaMultiply(a, b);
    BigInt res;
    res.sign = sign * v.sign;
    int carry = 0;
    forn (i, sz(c)) {
        ll cur = c[i] + carry;
        res.z.push_back((int) (cur % 1000000));
        carry = (int) (cur / 1000000);
    }
    res.z = convertBase(res.z, 6, BASE_DIGITS);
    res.trim();
    return res;
}
};

```

## 6 FFT

```

int rev[N];

//using Num = complex<dbl>;
struct Num {
    dbl x, y;
    Num() {}
    Num(dbl _x, dbl _y): x(_x), y(_y) {}
    inline dbl real() const { return x; }
    inline dbl imag() const { return y; }
    inline Num operator+(const Num &B) const { return Num(x + B.x, y
→ + B.y); }
    inline Num operator-(const Num &B) const { return Num(x - B.x, y
→ - B.y); }
    inline Num operator*(dbl k) const { return Num(x * k, y * k); }
    inline Num operator*(const Num &B) const { return Num(x * B.x -
→ y * B.y, x * B.y + y * B.x); }
    inline void operator+=(const Num &B) { x += B.x, y += B.y; }
    inline void operator/=(dbl k) { x /= k, y /= k; }
    inline void operator*=(const Num &B) { *this = *this * B; }
};

Num rt[N];

inline Num sqr(const Num &x) { return x * x; }
inline Num conj(const Num &x) { return Num(x.real(), -x.imag());
→ }

inline int getN(int n) {
    int k = 1;
    while(k < n)
        k <<= 1;
    return k;
}

void fft(Num *a, int n) {
    assert(rev[1]); // don't forget to init
    int q = N / n;
    forn (i, n)
        if(i < rev[i] / q)
            swap(a[i], a[rev[i] / q]);
    for (int k = 1; k < n; k <<= 1)
        for (int i = 0; i < n; i += 2 * k)
            forn (j, k) {
                const Num z = a[i + j + k] * rt[j + k];
                a[i + j + k] = a[i + j] - z;
                a[i + j] += z;
            }
}

void fftInv(Num *a, int n) {
    fft(a, n);
    reverse(a + 1, a + n);
    forn (i, n)
        a[i] /= n;
}

void doubleFft(Num *a, Num *fa, Num *fb, int n) { // only if you
→ need it
    fft(a, n);
    const int n1 = n - 1;
    forn (i, n) {
        const Num &z0 = a[i], &z1 = a[(n - i) & n1];
        fa[i] = Num(z0.real() + z1.real(), z0.imag() - z1.imag()) *
→ 0.5;
        fb[i] = Num(z0.imag() + z1.imag(), z1.real() - z0.real()) *
→ 0.5;
    }
}

Num tmp[N];
template<class T>
void mult(T *a, T *b, T *r, int n) { // n = 2^k
    forn (i, n)
        tmp[i] = Num((dbl) a[i], (dbl) b[i]);
    fft(tmp, n);
    const int n1 = n - 1;
    const Num c = Num(0, -0.25 / n);
    fornr (i, n / 2 + 1) {

```

```

    const int j = (n - i) & n1;
    const Num z0 = sqr(tmp[i]), z1 = sqr(tmp[j]);
    tmp[i] = (z1 - conj(z0)) * c;
    tmp[j] = (z0 - conj(z1)) * c;
}
fft(tmp, n);
forn (i, n)
    r[i] = (T) round(tmp[i].real());
}

void init() { // don't forget to init
    forn(i, N)
        rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (LOG - 1));

    rt[1] = Num(1, 0);
    for (int k = 1, p = 2; k < LOG; k++, p *= 2) {
        const Num x(cos(PI / p), sin(PI / p));
        forab (i, p / 2, p)
            rt[2 * i] = rt[i], rt[2 * i + 1] = rt[i] * x;
    }
}

```

## 7 FFT by mod and FFT with digits up to $10^6$

Num ta[N], tb[N], tf[N], tg[N];

```

const int HALF = 15;

void mult(int *a, int *b, int *r, int n, int mod) {
    int tw = (1 << HALF) - 1;
    forn (i, n) {
        int x = int(a[i] % mod);
        ta[i] = Num(x & tw, x >> HALF);
    }
    forn (i, n) {
        int x = int(b[i] % mod);
        tb[i] = Num(x & tw, x >> HALF);
    }

    fft(ta, n), fft(tb, n);
    forn (i, n) {
        int j = (n - i) & (n - 1);
        Num a1 = (ta[i] + conj(ta[j])) * Num(0.5, 0);
        Num a2 = (ta[i] - conj(ta[j])) * Num(0, -0.5);
        Num b1 = (tb[i] + conj(tb[j])) * Num(0.5 / n, 0);
        Num b2 = (tb[i] - conj(tb[j])) * Num(0, -0.5 / n);
        tf[j] = a1 * b1 + a2 * b2 * Num(0, 1);
        tg[j] = a1 * b2 + a2 * b1;
    }

    fft(tf, n), fft(tg, n);
    forn (i, n) {
        ll aa = ll(tf[i].x + 0.5);
        ll bb = ll(tg[i].x + 0.5);
        ll cc = ll(tf[i].y + 0.5);
        r[i] = int((aa + ((bb % mod) << HALF) + ((cc % mod) << (2 *
↪ HALF))) % mod);
    }
}

int tc[N], td[N];

const int MOD1 = 1.5e9, MOD2 = MOD1 + 1;
void multLL(int *a, int *b, ll *r, int n){
    mult(a, b, tc, n, MOD1), mult(a, b, td, n, MOD2);
    forn(i, n)
        r[i] = tc[i] + (td[i] - tc[i] + (ll)MOD2) * MOD1 % MOD2 *
↪ MOD1;
}

```

## 3 Data Structures

### 8 Centroid Decomposition

```

vi g[N];
int d[N], par[N], centroid;
//d and par - in centroid tree

int find(int v, int p, int total) {

```

```

    int size = 1, ok = 1;
    for (int to : g[v])
        if (d[to] == -1 && to != p) {
            int s = find(to, v, total);
            if (s > total / 2) ok = 0;
            size += s;
        }
    if (ok && size > total / 2) centroid = v;
    return size;
}

void calcInComponent(int v, int p, int level) {
    // do something
    for (int to : g[v])
        if (d[to] == -1 && to != p)
            calcInComponent(to, v, level);
}

//fill(d, d + n, -1)
//decompose(0, -1, 0)
void decompose(int root, int parent, int level) {
    find(root, -1, find(root, -1, INF));
    int c = centroid;
    par[c] = parent, d[c] = level;
    calcInComponent(centroid, -1, level);
    for (int to : g[c])
        if (d[to] == -1)
            decompose(to, c, level + 1);
}

```

## 9 Convex Hull Trick

```

struct Line {
    int k, b;
    Line() {}
    Line(int _k, int _b): k(_k), b(_b) {}
    ll get(int x) { return b + k * 1ll * x; }
    bool operator<(const Line &l) const { return k < l.k; } //
↪ change to > in case of different order
};

// Checks if intersection of (a, b) is on the left from (a, c).
inline bool check(Line a, Line b, Line c) {
    return (a.b - b.b) * 1ll * (c.k - a.k) < (a.b - c.b) * 1ll *
↪ (b.k - a.k);
}

struct Convex {
    vector<Line> st;
    inline void add(Line l) {
        while (sz(st) >= 2 && !check(st[sz(st) - 2], st[sz(st) - 1],
↪ l))
            st.pop_back();
        st.pb(l);
    }
    int get(int x) {
        int l = 0, r = sz(st);
        while (r - l > 1) {
            int m = (l + r) / 2; // change to > in case of different
↪ order
            if (st[m - 1].get(x) < st[m].get(x))
                l = m;
            else
                r = m;
        }
        return l;
    }
    Convex() {}
    Convex(vector<Line> &lines) {
        st.clear();
        for(Line &l : lines)
            add(l);
    }
    Convex(Line line) { st.pb(line); }
    Convex(const Convex &a, const Convex &b) {
        vector<Line> lines;
        lines.resize(sz(a.st) + sz(b.st));
        merge(all(a.st), all(b.st), lines.begin());
        st.clear();
        for(Line &l : lines)

```

```

        add(1);
    }
};

```

## 10 DSU

```

int pr[N];

int get(int v) {
    return v == pr[v] ? v : pr[v] = get(pr[v]);
}

bool unite(int v, int u) {
    v = get(v), u = get(u);
    if (v == u) return 0;
    pr[u] = v;
    return 1;
}

void init(int n) {
    for (i, n) pr[i] = i;
}

```

## 11 Fenwick Tree

```

int t[N];

int get(int ind) {
    int res = 0;
    for (; ind >= 0; ind &= (ind + 1), ind--)
        res += t[ind];
    return res;
}

void add(int ind, int n, int val) {
    for (; ind < n; ind |= (ind + 1))
        t[ind] += val;
}

int sum(int l, int r) { // [l, r)
    return get(r - 1) - get(l - 1);
}

```

## 12 Hash Table

```

using H = ll;
const int HT_SIZE = 1<<20, HT_AND = HT_SIZE - 1, HT_SIZE_ADD =
    HT_SIZE / 100;
H ht[HT_SIZE + HT_SIZE_ADD];
int data[HT_SIZE + HT_SIZE_ADD];

int get(const H &hash){
    int k = ((ll) hash) & HT_AND;
    while (ht[k] && ht[k] != hash) ++k;
    return k;
}

void insert(const H &hash, int x){
    int k = get(hash);
    if (!ht[k]) ht[k] = hash, data[k] = x;
}

bool count(const H &hash){
    int k = get(hash);
    return ht[k] != 0;
}

```

## 13 Heavy Light Decomposition

```

vi g[N];
int size[N], comp[N], num[N], top[N], pr[N], tin[N], tout[N];
vi t[N], toPush[N], lst[N];
int curPath = 0, curTime = 0;

void pushST(int path, int v, int vl, int vr) {
    if (toPush[path][v] != -1) {
        if (vl != vr - 1)
            for (j, 2)
                toPush[path][2 * v + j] = toPush[path][v];
    }
}

```

```

        else
            t[path][v] = toPush[path][v];
            toPush[path][v] = -1;
    }
}

int getST(int path, int v, int vl, int vr, int ind) {
    pushST(path, v, vl, vr);
    if (vl == vr - 1)
        return t[path][v];
    int vm = (vl + vr) / 2;
    if (ind >= vm)
        return getST(path, 2 * v + 1, vm, vr, ind);
    return getST(path, 2 * v, vl, vm, ind);
}

void setST(int path, int v, int vl, int vr, int l, int r, int val)
    ↪ {
    if (vl >= l && vr <= r) {
        toPush[path][v] = val;
        pushST(path, v, vl, vr);
        return;
    }
    pushST(path, v, vl, vr);
    if (vl >= r || l >= vr)
        return;
    int vm = (vl + vr) / 2;
    setST(path, 2 * v, vl, vm, l, r, val);
    setST(path, 2 * v + 1, vm, vr, l, r, val);
    t[path][v] = min(t[path][2 * v], t[path][2 * v + 1]);
}

bool isUpper(int v, int u) {
    return tin[v] <= tin[u] && tout[v] >= tout[u];
}

int getHLD(int v) {
    return getST(comp[v], 1, 0, sz(t[comp[v]]) / 2, num[v]);
}

int setHLD(int v, int u, int val) {
    int ans = 0, w = 0;
    for (i, 2) {
        while (!isUpper(w = top[comp[v]], u))
            setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, 0, num[v] + 1,
                ↪ val), v = pr[w];
        swap(v, u);
    }
    setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, min(num[v], num[u]),
        ↪ max(num[v], num[u]) + 1, val);
    return ans;
}

void dfs(int v, int p) {
    tin[v] = curTime++;
    size[v] = 1;
    pr[v] = p;
    for (int u : g[v])
        if (u != p) {
            dfs(u, v);
            size[v] += size[u];
        }
    tout[v] = curTime++;
}

void build(int v) {
    if (v == 0 || size[v] * 2 < size[pr[v]])
        top[curPath] = v, comp[v] = curPath, num[v] = 0, curPath++;
    else
        comp[v] = comp[pr[v]], num[v] = num[pr[v]] + 1;
    lst[comp[v]].pb(v);
    for (int u : g[v])
        if (u != pr[v])
            build(u);
}

void initHLD() {
    dfs(0, 0);
    build(0);
}

```

```

    forn (i, curPath) {
        int curSize = 1;
        while (curSize < sz(lst[i]))
            curSize *= 2;
        t[i].resize(curSize * 2);
        toPush[i] = vi(curSize * 2, -1);
        //initialize t[i]
    }
}

```

## 14 Next Greater in Segment Tree

```

int t[4 * N], tSize = 1;

// Find position > pos with val > x.
int nextGreaterX(int v, int l, int r, int pos, int x) {
    if (r <= pos + 1 || t[v] <= x) return INF;
    if (v >= tSize) return v - tSize;
    int ans = nextGreaterX(2 * v, l, (l + r) / 2, pos, x);
    if (ans == INF)
        ans = nextGreaterX(2 * v + 1, (l + r) / 2, r, pos, x);
    return ans;
}

```

## 15 Sparse Table

```

int st[N][LOG];
int lg[N];

int get(int l, int r) { // [l, r)
    int curLog = lg[r - l];
    return min(st[l][curLog], st[r - (1 << curLog)][curLog]);
}

void initSparseTable(int *a, int n) {
    lg[1] = 0;
    forab (i, 2, n + 1) lg[i] = lg[i / 2] + 1;
    forn (i, n) st[i][0] = a[i];
    forn (j, lg[n])
        forn (i, n - (1 << (j + 1)) + 1)
            st[i][j + 1] = min(st[i][j], st[i + (1 << j)][j]);
}

```

## 16 Fenwick Tree 2D

```

ll a[4][N][N];
int n, m;

inline int f(int x) { return x & ~(x - 1); }

inline void add(int k, int x, int y, ll val) {
    for (; x <= n; x += f(x))
        for (int j = y; j <= m; j += f(j))
            a[k][x][j] += val;
}

inline ll get(int k, int x, int y) {
    ll s = 0;
    for (; x > 0; x -= f(x))
        for (int j = y; j > 0; j -= f(j))
            s += a[k][x][j];
    return s;
}

inline ll get(int x, int y) {
    return ll(x + 1) * (y + 1) * get(0, x, y) - (y + 1) * get(1, x,
↪ y)
        - (x + 1) * get(2, x, y) + get(3, x, y);
}

inline void add(int x, int y, ll val) {
    add(0, x, y, val);
    add(1, x, y, val * x);
    add(2, x, y, val * y);
    add(3, x, y, val * x * y);
}

inline ll get(int x_1, int y_1, int x_2, int y_2) {
    return get(x_2, y_2) - get(x_1 - 1, y_2) - get(x_2, y_1 - 1) +
↪ get(x_1 - 1, y_1 - 1);
}

```

```

}

// Adds val to corresponding rectangle
inline void add(int x_1, int y_1, int x_2, int y_2, ll val) {
    add(x_1, y_1, val);
    if (y_2 < m) add(x_1, y_2 + 1, -val);
    if (x_2 < n) add(x_2 + 1, y_1, -val);
    if (x_2 < n && y_2 < m) add(x_2 + 1, y_2 + 1, val);
}

```

## 17 Segment Tree 2D

```

int tSize = (1 << 10);

struct Node1D {
    Node1D *l, *r;
    ll val, need;
    Node1D(): l(nullptr), r(nullptr), val(0), need(0) {}
    inline void norm() {
        if(!l) l = new Node1D();
        if(!r) r = new Node1D();
    }
    ll get(int ql, int qr, int vl = 0, int vr = tSize) {
        if(vl >= qr || ql >= vr)
            return 0;
        if(ql <= vl && vr <= qr)
            return val;
        int a = max(vl, ql), b = min(vr, qr), vm = (vl + vr) / 2;
        norm();
        return l->get(ql, qr, vl, vm) + r->get(ql, qr, vm, vr) + need
↪ * ll(b - a);
    }
    void add(int ql, int qr, int x, int vl = 0, int vr = tSize) {
        if (ql >= vr || vl >= qr)
            return;
        if (ql <= vl && vr <= qr){
            need += x;
            val += x * ll(vr - vl);
            return;
        }
        int vm = (vl + vr) / 2;
        norm();
        l->add(ql, qr, x, vl, vm), r->add(ql, qr, x, vm, vr);
        val = l->val + r->val + need * (vr - vl);
    }
};

struct Node2D {
    Node2D *l, *r;
    Node1D *val, *need;
    Node2D(): l(nullptr), r(nullptr), val(new Node1D()), need(new
↪ Node1D()) {}
    inline void norm() {
        if(!l) l = new Node2D();
        if(!r) r = new Node2D();
    }
    ll get(int ql0, int qr0, int ql1, int qr1, int vl = 0, int vr =
↪ tSize) {
        if(vl >= qr0 || ql0 >= vr)
            return 0;
        if(ql0 <= vl && vr <= qr0)
            return val->get(ql1, qr1);
        int a = max(vl, ql0), b = min(vr, qr0), vm = (vl + vr) / 2;
        norm();
        return l->get(ql0, qr0, ql1, qr1, vl, vm) + r->get(ql0, qr0,
↪ ql1, qr1, vm, vr) + need->get(ql1, qr1) * ll(b - a);
    }
    void add(int ql0, int qr0, int ql1, int qr1, int x, int vl = 0,
↪ int vr = tSize) {
        if (ql0 >= vr || vl >= qr0)
            return;
        if (ql0 <= vl && vr <= qr0){
            need->add(ql1, qr1, x);
            val->add(ql1, qr1, x * ll(vr - vl));
            return;
        }
        int a = max(ql0, vl), b = min(qr0, vr), vm = (vl + vr) / 2;
        norm();
        l->add(ql0, qr0, ql1, qr1, x, vl, vm), r->add(ql0, qr0, ql1,
↪ qr1, x, vm, vr);
    }
}

```



```

    val->add(ql1, qr1, x * ll(b - a));
}
};

```

## 4 Dynamic Programming

### 18 LIS

```

int longestIncreasingSubsequence(vi a) {
    int n = sz(a);
    vi d(n + 1, INF);
    d[0] = -INF;
    forn (i, n)
        *upper_bound(all(d), a[i]) = a[i];
    fornr (i, n + 1) if (d[i] != INF) return i;
    return 0;
}

```

### 19 DP tree

```

int dp[N][N], a[N];
vi g[N];

int dfs(int v, int n) {
    forn (i, n + 1)
        dp[v][i] = -INF;
    dp[v][1] = a[v];
    int curSz = 1;
    for (int to : g[v]) {
        int toSz = dfs(to, n);
        for (int i = curSz; i >= 1; i--)
            fornr (j, toSz + 1)
                dp[v][i + j] = max(dp[v][i + j], dp[v][i] + dp[to][j]);
        curSz += toSz;
    }
    return curSz;
}

```

### 20 Masks tricks

```

int dp[(1 << MASK)][MASK];

void calcDP(int n) {
    forn(mask, 1 << n) {
        dp[mask][n] = 1;
        fornr(i, n) {
            dp[mask][i] = dp[mask][i + 1];
            if ((1 << i) & mask)
                dp[mask][i] += dp[mask ^ (1 << i)][i + 1];
        }
    }
}

```

## 5 Flows

### 21 Utilities

```

vi g[N];

// for directed unweighted graph
struct Edge {
    int v, u, c, f;
    Edge() {}
    Edge(int _v, int _u, int _c): v(_v), u(_u), c(_c), f(0) {}
};

vector<Edge> edges;

inline void addFlow(int e, int flow) {
    edges[e].f += flow, edges[e ^ 1].f -= flow;
}

inline void addEdge(int v, int u, int c) {
    g[v].pb(sz(edges)), edges.pb(Edge(v, u, c));
    g[u].pb(sz(edges)), edges.pb(Edge(u, v, 0)); // for undirected 0
    ↪ should be c
}

```

### 22 Ford-Fulkerson

```

int used[N], pr[N];
int curTime = 1;

int dfs(int v, int can, int toPush, int t) {
    if (v == t) return can;
    used[v] = curTime;
    for (int edge : g[v]) {
        auto &e = edges[edge];
        if (used[e.u] != curTime && e.c - e.f >= toPush) {
            int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
            if (flow > 0) {
                addFlow(edge, flow), pr[e.u] = edge;
                return flow;
            }
        }
    }
    return 0;
}

```

```

int fordFulkerson(int s, int t) {
    int ansFlow = 0, flow = 0;
    // Without scaling
    while ((flow = dfs(s, INF, 1, t)) > 0)
        ansFlow += flow, curTime++;
    // With scaling
    fornr (i, INF_LOG)
        for (curTime++; (flow = dfs(s, INF, (1 << i), t)) > 0;
        ↪ curTime++)
            ansFlow += flow;
    return ansFlow;
}

```

### 23 Dinic

```

int pr[N], d[N], q[N], first[N];

int dfs(int v, int can, int toPush, int t) {
    if (v == t) return can;
    int sum = 0;
    for (; first[v] < (int) g[v].size(); first[v]++) {
        auto &e = edges[g[v][first[v]]];
        if (d[e.u] != d[v] + 1 || e.c - e.f < toPush) continue;
        int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
        addFlow(g[v][first[v]], flow);
        can -= flow, sum += flow;
        if (!can) return sum;
    }
    return sum;
}

bool bfs(int n, int s, int t, int curPush) {
    forn (i, n) d[i] = INF, first[i] = 0;
    int head = 0, tail = 0;
    q[tail++] = s;
    d[s] = 0;
    while (tail - head > 0) {
        int v = q[head++];
        for (int edge : g[v]) {
            auto &e = edges[edge];
            if (d[e.u] > d[v] + 1 && e.c - e.f >= curPush)
                d[e.u] = d[v] + 1, q[tail++] = e.u;
        }
    }
    return d[t] != INF;
}

```

```

int dinic(int n, int s, int t) {
    int ansFlow = 0;
    // Without scaling
    while (bfs(n, s, t, 1))
        ansFlow += dfs(s, INF, 1, t);
    // With scaling
    fornr (j, INF_LOG)
        while (bfs(n, s, t, 1 << j))
            ansFlow += dfs(s, INF, 1 << j, t);
    return ansFlow;
}

```



## 24 Hungarian

```
const int INF = 1e9;
int a[N][N];

// min = sum of a[pa[i],i]
// you may optimize speed by about 15%, just change all vectors to
↪ static arrays
vi Hungarian(int n) {
    vi pa(n + 1, -1), row(n + 1, 0), col(n + 1, 0), la(n + 1);
    forn (k, n) {
        vi u(n + 1, 0), d(n + 1, INF);
        pa[n] = k;
        int l = n, x;
        while ((x = pa[l]) != -1) {
            u[l] = 1;
            int minn = INF, tmp, l0 = 1;
            forn (j, n)
                if (!u[j]) {
                    if ((tmp = a[x][j] + row[x] + col[j]) < d[j])
                        d[j] = tmp, la[j] = l0;
                    if (d[j] < minn)
                        minn = d[j], l = j;
                }
            forn (j, n + 1)
                if (u[j])
                    col[j] += minn, row[pa[j]] -= minn;
                else
                    d[j] -= minn;
        }
        while (l != n)
            pa[l] = pa[la[l]], l = la[l];
    }
    return pa;
}
```

## 25 Min Cost Max Flow

```
int pr[N], in[N], q[N * M], used[N], d[N], pot[N];
vi g[N];

struct Edge {
    int v, u, c, f, w;
    Edge() {}
    Edge(int _v, int _u, int _c, int _w): v(_v), u(_u), c(_c),
↪ f(0), w(_w) {}
};

vector<Edge> edges;

inline void addFlow(int e, int flow) {
    edges[e].f += flow, edges[e ^ 1].f -= flow;
}

inline void addEdge(int v, int u, int c, int w) {
    g[v].pb(sz(edges)), edges.pb(Edge(v, u, c, w));
    g[u].pb(sz(edges)), edges.pb(Edge(u, v, 0, -w));
}

int dijkstra(int n, int s, int t) {
    forn (i, n) used[i] = 0, d[i] = INF;
    d[s] = 0;
    while (1) {
        int v = -1;
        forn (i, n)
            if (!used[i] && (v == -1 || d[v] > d[i]))
                v = i;
        if (v == -1 || d[v] == INF) break;
        used[v] = 1;
        for (int edge : g[v]) {
            auto &e = edges[edge];
            int w = e.w + pot[v] - pot[e.u];
            if (e.c > e.f && d[e.u] > d[v] + w)
                d[e.u] = d[v] + w, pr[e.u] = edge;
        }
    }
    if (d[t] == INF) return d[t];
    forn (i, n) pot[i] += d[i];
    return pot[t];
}
```

```
int fordBellman(int n, int s, int t) {
    forn (i, n) d[i] = INF;
    int head = 0, tail = 0;
    d[s] = 0, q[tail++] = s, in[s] = 1;
    while (tail - head > 0) {
        int v = q[head++];
        in[v] = 0;
        for (int edge : g[v]) {
            auto &e = edges[edge];
            if (e.c > e.f && d[e.u] > d[v] + e.w) {
                d[e.u] = d[v] + e.w;
                pr[e.u] = edge;
                if (!in[e.u])
                    in[e.u] = 1, q[tail++] = e.u;
            }
        }
    }
    return d[t];
}

int minCostMaxFlow(int n, int s, int t) {
    int ansFlow = 0, ansCost = 0, dist;
    while ((dist = dijkstra(n, s, t)) != INF) {
        int curFlow = INF;
        for (int cur = t; cur != s; cur = edges[pr[cur]].v)
            curFlow = min(curFlow, edges[pr[cur]].c -
↪ edges[pr[cur]].f);
        for (int cur = t; cur != s; cur = edges[pr[cur]].v)
            addFlow(pr[cur], curFlow);
        ansFlow += curFlow;
        ansCost += curFlow * dist;
    }
    return ansCost;
}
```

## 6 Games

### 26 Retrograde Analysis

```
int win[N], lose[N], outDeg[N];
vi rg[N];

void retro(int n) {
    queue<int> q;
    forn (i, n)
        if (!outDeg[i])
            lose[i] = 1, q.push(i);
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int to : rg[v])
            if (lose[v]) {
                if (!win[to])
                    win[to] = 1, q.push(to);
            } else {
                outDeg[to]--;
                if (!outDeg[to])
                    lose[to] = 1, q.push(to);
            }
    }
}
```

## 7 Geometry

### 27 ClosestPoints (SweepLine)

```
struct Pnt {
    int x, y, i;
    bool operator <(const Pnt &p) const { return mp(y, i) < mp(p.y,
↪ p.i); }
};

ll d2 = 8e18, d = (ll) sqrt(d2) + 1;
Pnt p[N];

inline ll sqr(int x){
```

```

    return (ll)x * x;
}

inline void relax(const Pnt &a, const Pnt &b){
    ll tmp = sqr(a.x - b.x) + sqr(a.y - b.y);
    if (tmp < d2)
        d2 = tmp, d = (ll)(sqrt(d2) + 1 - 1e-9); // round up
}

inline bool xless(const Pnt &a, const Pnt &b){
    return a.x < b.x;
}

int main() {
    int n;
    scanf("%d", &n);
    forn(i, n)
        scanf("%d%d", &p[i].x, &p[i].y), p[i].i = i;
    sort(p, p + n, xless);

    set<Pnt> s;
    int l = 0;
    forn(r, n){
        set<Pnt>::iterator it_r = s.lower_bound(p[r]), it_l = it_r;
        for (; it_r != s.end() && it_r->y - p[r].y < d; ++it_r)
            relax(*it_r, p[r]);
        while (it_l != s.begin() && p[r].y - (--it_l)->y < d)
            relax(*it_l, p[r]);
        s.insert(p[r]);
        while (l <= r && p[r].x - p[l].x >= d)
            s.erase(p[l++]);
    }
    printf("%.9f\n", sqrt(d2));
    return 0;
}

```

## 28 ConvexHull

```

using vpnt = vector<Pnt>;

inline bool byAngle(const Pnt& a, const Pnt& b) {
    dbl x = a % b;
    return eq(x, 0) ? a.len2() < b.len2() : x < 0;
}

vpnt convexHull(vpnt p) {
    int n = sz(p);
    assert(n > 0);
    swap(p[0], *min_element(all(p)));
    forab(i, 1, n)
        p[i] = p[i] - p[0];
    sort(p.begin() + 1, p.end(), byAngle);

    /* To keep 180 angles (1) (2)
       (1):
       int k = p.size() - 1;
       while(k > 0 && eq((p[k] - p.back()) % p.back(), 0))
           --k;
       reverse(pi.begin() + k, pi.end());*/

    int rn = 0;
    vpnt r(n);
    r[rn++] = p[0];
    forab(i, 1, n){
        Pnt q = p[i] + p[0];
        while(rn >= 2 && geq((r[rn] - r[rn - 1]) % (q - r[rn - 2]), 0)) // (2) ge
            --rn;
        r[rn++] = q;
    }
    r.resize(rn);
    return r;
}

```

## 29 GeometryBase

```

const dbl EPS = 1e-9;
const int PREC = 20;
inline bool eq(dbl a, dbl b) { return abs(a-b)<=EPS; }
inline bool gr(dbl a, dbl b) { return a>b+EPS; }

```

```

inline bool geq(dbl a, dbl b) { return a>=b-EPS; }
inline bool ls(dbl a, dbl b) { return a<b-EPS; }
inline bool leq(dbl a, dbl b) { return a<=b+EPS; }

```

```

struct Pnt {
    dbl x,y;
    Pnt(): x(0), y(0) {}
    Pnt(dbl xx, dbl yy): x(xx), y(yy) {}

    inline Pnt operator +(const Pnt &p) const { return Pnt(x +
↪ p.x, y + p.y); }
    inline Pnt operator -(const Pnt &p) const { return Pnt(x -
↪ p.x, y - p.y); }
    inline dbl operator *(const Pnt &p) const { return x * p.x + y
↪ * p.y; } // ll
    inline dbl operator %(const Pnt &p) const { return x * p.y - y
↪ * p.x; } // ll

    inline Pnt operator *(dbl k) const { return Pnt(x * k, y * k);
↪ }
    inline Pnt operator /(dbl k) const { return Pnt(x / k, y / k);
↪ }
    inline Pnt operator -() const { return Pnt(-x, -y); }

    inline void operator +=(const Pnt &p) { x += p.x, y += p.y; }
    inline void operator -=(const Pnt &p) { x -= p.x, y -= p.y; }
    inline void operator *=(dbl k) { x*=k, y*=k; }

    inline bool operator ==(const Pnt &p) const { return
↪ abs(x-p.x)<=EPS && abs(y-p.y)<=EPS; }
    inline bool operator !=(const Pnt &p) const { return
↪ abs(x-p.x)>EPS || abs(y-p.y)>EPS; }
    inline bool operator <(const Pnt &p) const { return
↪ abs(x-p.x)<=EPS ? y<p.y-EPS : x<p.x; }

    inline dbl angle() const { return atan2(y, x); } // ld
    inline dbl len2() const { return x*x+y*y; } // ll
    inline dbl len() const { return sqrt(x*x+y*y); } // ll, ld
    inline Pnt getNorm() const {
        auto l = len();
        return Pnt(x/l, y/l);
    }
    inline void normalize() {
        auto l = len();
        x/=l, y/=l;
    }

    inline Pnt getRot90() const { //counter-clockwise
        return Pnt(-y, x);
    }
    inline Pnt getRot(dbl a) const { // ld
        dbl si = sin(a), co = cos(a);
        return Pnt(x*co - y*si, x*si + y*co);
    }

    inline void read() {
        int xx, yy;
        cin >> xx >> yy;
        x = xx, y = yy;
    }
    inline void write() const{
        cout << fixed << (double)x << " " << (double)y << '\n';
    }
    Pnt bmul(const Pnt& r) const {
        return Pnt(x*r.x - y*r.y, y*r.x + x*r.y);
    }
};

struct Line{
    dbl a, b, c;
    Line(): a(0), b(0), c(0) {}
    // normalizes
    Line(dbl aa, dbl bb, dbl cc) {
        dbl norm = sqrt(aa * aa + bb * bb);
        aa /= norm, bb /= norm, cc /= norm;
        a = aa, b = bb, c = cc;
    }

    Line(const Pnt &A, const Pnt &p){ // it normalizes (a,b),
↪ important in d(), normalToP()

```

```

    Pnt n = (p-A).getRot90().getNorm();
    a = n.x, b = n.y, c = -(a * A.x + b * A.y);
}

inline dbl d(const Pnt &p) const { return a*p.x + b*p.y + c; }
inline Pnt no() const {return Pnt(a, b);}
inline Pnt normalToP(const Pnt &p) const { return Pnt(a,b) *
↪ (a*p.x + b*p.y + c); }

inline void write() const{
    cout << fixed << (double)a << " " << (double)b << " " <<
↪ (double)c << '\n';
}
};

```

### 30 GeometryInterTangent

```

inline dbl sqr(dbl x) { return x * x; }

struct Circle {
    Pnt p;
    dbl r;
};

Pnt tangent(Pnt x, Circle y, int t = 0) {
    y.r = abs(y.r); // abs needed because internal calls y.s < 0
    if (y.r == 0) return y.p;
    dbl d = (x - y.p).len();
    Pnt a = (x - y.p) * pow(y.r / d, 2) + y.p;
    Pnt b = ((x - y.p).getNorm() * sqrt(d * d - y.r * y.r) / d *
↪ y.r).bmul(Pnt(0, 1));
    return t == 0 ? a+b : a-b;
}

vector<pair<Pnt,Pnt>> external(const Circle &x, const Circle &y)
↪ {
    vector<pair<Pnt,Pnt>> v;
    if (x.r == y.r) {
        Pnt tmp = ((x.p-y.p).getNorm()*x.r).bmul(Pnt(0,1));
        v.pb(mp(x.p+tmp,y.p+tmp));
        v.pb(mp(x.p-tmp,y.p-tmp));
    } else {
        Pnt p = (x.p*y.r-y.p*x.r)/(y.r-x.r);
        forn(i,2) v.pb(mp(tangent(p,x,i),tangent(p,y,i)));
    }
    return v;
}

vector<pair<Pnt,Pnt>> internal(const Circle &x, const Circle &y)
↪ {
    return external({x.p,-x.r},y); }

```

```

vector<Pnt> line_line(const Line &l, const Line &m){
    dbl z = m.a * l.b - l.a * m.b;
    dbl x = m.c * l.b - l.c * m.b;
    dbl y = m.c * l.a - l.c * m.a;
    if(fabs(z) > EPS)
        return {Pnt(-x/z, y/z)};
    else if(fabs(x) > EPS || fabs(y) > EPS)
        return {}; // parallel lines
    else
        return {Pnt(0, 0), Pnt(0, 0)}; // same lines
}

vector<Pnt> circle_line(const Circle &c, const Line &l){
    dbl d = l.d(c.p);
    if(fabs(d) > c.r + EPS)
        return {};
    if(fabs(fabs(d) / c.r - 1) < EPS) {
        return {c.p - l.no() * d};
    } else {
        dbl s = sqrt(fabs(sqr(c.r) - sqr(d)));
        return {c.p - l.no() * d + l.no().getRot90() * s,
            c.p - l.no() * d - l.no().getRot90() * s};
    }
}

vector<Pnt> circle_circle(const Circle &x, const Circle &y) {
    dbl d = (x.p-y.p).len(), a = x.r, b = y.r;
    if (eq(d, 0)) { assert(a != b); return {}; }

```

```

    dbl C = (a*a+d*d-b*b)/(2*a*d);
    if (abs(C) > 1+EPS) return {};
    dbl S = sqrt(max(1-C*C,(dbl)0)); Pnt tmp = (y.p-x.p)/d*x.r;
    if (eq(S, 0)) return {x.p+tmp.bmul(Pnt(C,0))};
    return {x.p+tmp.bmul(Pnt(C,S)),x.p+tmp.bmul(Pnt(C,-S))};
}

dbl circle_isect_area(const Circle &x, const Circle &y) {
    dbl d = (x.p-y.p).len(), a = x.r, b = y.r; if (a < b)
↪ swap(a,b);
    if (geq(d, a+b)) return 0;
    if (leq(d, a-b)) return PI*b*b;
    dbl ca = acos((a*a+d*d-b*b)/(2*a*d)), cb =
↪ acos((b*b+d*d-a*a)/(2*b*d));
    return (ca*a*a-0.5*a*a*sin(ca*2))+(cb*b*b-0.5*b*b*sin(cb*2));
}

// Squared distance between point p and segment [a..b]
dbl dist2(Pnt p, Pnt a, Pnt b){
    if ((p - a) * (b - a) < 0) return (p - a).len2();
    if ((p - b) * (a - b) < 0) return (p - b).len2();
    dbl d = fabs((p - a) % (b - a));
    return d * d / (b - a).len2();
}

```

### 31 GeometrySimple

```

int sign(dbl a) { return (a > EPS) - (a < -EPS); }

// Checks, if point is inside the segment
inline bool inSeg(const Pnt &p, const Pnt &a, const Pnt &b) {
    return eq((p - a) % (p - b), 0) && leq((p - a) * (p - b), 0);
}

// Checks, if two intervals (segments without ends) intersect AND
↪ do not lie on the same line
inline bool subIntr(const Pnt &a, const Pnt &b, const Pnt &c,
↪ const Pnt &d){
    return
        sign((b - a) % (c - a)) * sign((b - a) % (d - a)) ==
↪ -1 &&
        sign((d - c) % (a - c)) * sign((d - c) % (b - c)) ==
↪ -1;
}

// Checks, if two segments (ends are included) has an intersection
inline bool checkSegInter(const Pnt &a, const Pnt &b, const Pnt
↪ &c, const Pnt &d){
    return inSeg(c, a, b) || inSeg(d, a, b) || inSeg(a, c, d) ||
↪ inSeg(b, c, d) || subIntr(a, b, c, d);
}

inline dbl area(vector<Pnt> p){
    dbl s = 0;
    int n = sz(p);
    p.pb(p[0]);
    forn(i, n)
        s += p[i + 1] % p[i];
    p.pop_back();
    return abs(s) / 2;
}

// Check if point p is inside polygon <n, q[]>
int containsSlow(Pnt p, Pnt *z, int n){
    int cnt = 0;
    forn(j, n){
        Pnt a = z[j], b = z[(j + 1) % n];
        if (inSeg(p, a, b))
            return -1; // border
        if (min(a.y, b.y) - EPS <= p.y && p.y < max(a.y, b.y) -
↪ EPS)
            cnt += (p.x < a.x + (p.y - a.y) * (b.x - a.x) / (b.y
↪ - a.y));
    }
    return cnt & 1; // 0 = outside, 1 = inside
}

//for convex polygon
//assume polygon is counterclockwise-ordered

```

```

bool containsFast(Pnt p, Pnt *z, int n) {
    Pnt o = z[0];
    if(gr((p - o) % (z[1] - o), 0) || ls((p - o) % (z[n - 1] -
↪ o), 0))
        return 0;
    int l = 0, r = n - 1;
    while(r - l > 1){
        int m = (l + r) / 2;
        if(gr((p - o) % (z[m] - o), 0))
            r = m;
        else
            l = m;
    }
    return leq((p - z[l]) % (z[r] - z[l]), 0);
}

// Checks, if point "p" is in the triangle "abc" IFF triangle in
↪ CCW order
inline int isInTr(const Pnt &p, const Pnt &a, const Pnt &b, const
↪ Pnt &c){
    return
        gr((b - a) % (p - a), 0) &&
        gr((c - b) % (p - b), 0) &&
        gr((a - c) % (p - c), 0);
}

```

## 32 Halfplanes Intersection

```

namespace halfplanes {
    Pnt st, v, p[N];
    int n, sp, ss[N], ind[N], no[N], cnt[N], k = 0, a[N], b[N];
    dbl ang[N];

    Pnt Norm(int j) { return (p[a[j]] - p[b[j]]).getRot90(); }

    void AddPlane( int i, int j ){
        a[k] = i, b[k] = j, ind[k] = k;
        ang[k] = Norm(k).angle();
        k++;
    }

    bool angLess(int i, int j) { return ang[i] < ang[j]; }

    void Unique() {
        int i = 0, k2 = 0;
        while (i < k)
        {
            int ma = ind[i], st_ = i;
            Pnt no_ = Norm(ma);

            for (i++; i < k && fabs(ang[ind[st_]] - ang[ind[i]]) < EPS;
↪ i++)
                if ((no_ * p[a[ma]]) < (no_ * p[a[ind[i]]]))
                    ma = ind[i];
            ind[k2++] = ma;
        }
        k = k2;
    }

    dbl xx, yy, tmp;

#define BUILD(a1, b1, c1, i) \
    dbl a1 = Norm(i).x; \
    dbl b1 = Norm(i).y; \
    tmp = sqrt(a1 * a1 + b1 * b1); \
    a1 /= tmp, b1 /= tmp; \
    dbl c1 = -(a1 * p[a[i]].x + b1 * p[a[i]].y);

    void FindPoint(int i, int j, dbl step = 0.0) {
        BUILD(a1, b1, c1, i);
        BUILD(a2, b2, c2, j);

        xx = -(c1 * b2 - c2 * b1) / (a1 * b2 - a2 * b1);
        yy = (c1 * a2 - c2 * a1) / (a1 * b2 - a2 * b1);

        dbl no_ = sqrt(sqr(a1 + a2) + sqr(b1 + b2));
        xx += (a1 + a2) * step / no_;
        yy += (b1 + b2) * step / no_;
    }
}

```

```

void TryShiftPoint(int i, int j, dbl step) {
    FindPoint(i, j, step);

    forn (g, k) {
        BUILD(a1, b1, c1, ind[g]);
        if (a1 * xx + b1 * yy + c1 < EPS)
            return;
    }

    puts("Possible");
    printf("%.20lf %.20lf\n", (double)xx, (double)yy);
    exit(0);
}

void PushPlaneIntoStack(int i) {
    while (sp >= 2 && ang[i] - ang[ss[sp - 2]] + EPS < M_PI){
        FindPoint(i, ss[sp - 2]);

        BUILD(a1, b1, c1, ss[sp - 1]);
        if ((a1 * xx + b1 * yy + c1) < -EPS)
            break;

        sp--;
    }
    ss[sp++] = i;
}

void solve() {
    cin >> n;
    forn (i, n)
        cin >> p[i].x >> p[i].y;
    p[n] = p[0];

    // Find set of planes
    forn (i, sp)
        AddPlane(max(ss[i], ss[i + 1]), min(ss[i], ss[i + 1]));
    forn (i, n - 1)
        AddPlane(i + 1, i);
    sort(ind, ind + k, angLess);

    int oldK = k;
    Unique();

    forn (i, oldK)
        no[i] = i;
    forn (i, k){
        int j = oldK + i, x = ind[i];
        ang[j] = ang[x] + 2 * M_PI;
        a[j] = a[x];
        b[j] = b[x];
        ind[i + k] = j, no[j] = x;
    }

    sp = 0;
    forn (i, 2 * k)
        PushPlaneIntoStack(ind[i]);
    forn (t, sp)
        if (++cnt[no[ss[t]]] > 1){
            TryShiftPoint(ss[t], ss[t - 1], 1e-5);
            break;
        }
    }
}

```

## 8 Graphs

### 33 2-SAT

```

// VAR - 2 * vars
int cntVar = 0, val[VAR], usedSat[VAR], comp[VAR];
vi topsortSat;

vi g[VAR], rg[VAR];

inline int newVar() {
    cntVar++;
    return (cntVar - 1) * 2;
}

```

```

inline int Not(int v) { return v ^ 1; }

inline void Implies(int v1, int v2) { g[v1].pb(v2),
↪ rg[v2].pb(v1); }

inline void Or(int v1, int v2) { Implies(Not(v1), v2),
↪ Implies(Not(v2), v1); }

inline void Nand(int v1, int v2) { Or(Not(v1), Not(v2)); }

inline void setTrue(int v) { Implies(Not(v), v); }

void dfs1(int v) {
    usedSat[v] = 1;
    for (int to : g[v])
        if (!usedSat[to]) dfs1(to);
    topsortSat.pb(v);
}

void dfs2(int v, int c) {
    comp[v] = c;
    for (int to : rg[v])
        if (!comp[to]) dfs2(to, c);
}

int getVal(int v) { return val[v]; }

// cntVar
bool solveSat() {
    forn(i, 2 * cntVar) usedSat[i] = 0;
    forn(i, 2 * cntVar)
        if (!usedSat[i]) dfs1(i);
    reverse(all(topsortSat));
    int c = 0;
    for (int v : topsortSat)
        if (!comp[v]) dfs2(v, ++c);
    forn(i, cntVar) {
        if (comp[2 * i] == comp[2 * i + 1]) return false;
        if (comp[2 * i] < comp[2 * i + 1]) val[2 * i + 1] = 1;
        else val[2 * i] = 1;
    }
    return true;
}

```

## 34 Bridges

```

int up[N], tIn[N], timer;
vector<vi> comps;
vi st;

```

```

struct Edge {
    int to, id;
    Edge(int _to, int _id) : to(_to), id(_id) {}
};

```

```
vector<Edge> g[N];
```

```

void newComp(int size = 0) {
    comps.emplace_back(); // new empty
    while (sz(st) > size) {
        comps.back().pb(st.back());
        st.pop_back();
    }
}

```

```

void findBridges(int v, int parentEdge = -1) {
    if (up[v]) // visited
        return;
    up[v] = tIn[v] = ++timer;
    st.pb(v);
    for (Edge e : g[v]) {
        if (e.id == parentEdge)
            continue;
        int u = e.to;
        if (!tIn[u]) {
            int size = sz(st);
            findBridges(u, e.id);
            if (up[u] > tIn[v])
                newComp(size);
        }
    }
}

```

```

        up[v] = min(up[v], up[u]);
    }
}

```

```

// after find_bridges newComp() for root
void run(int n) {
    forn(i, n)
        if (!up[i]) {
            findBridges(i);
            newComp();
        }
}

```

## 35 Cactus

```
int used[N];
```

```

struct Edge {
    ll l;
    Edge() {}
    Edge(int _l) : l(_l) {}
};

```

```
vector<pair<int, Edge>> g[N], rev[N], path;
pair<int, Edge> pr[N];
```

```

void dfsInit(int v, int p, Edge prE) {
    used[v] = 1;
    pr[v] = mp(p, prE);
    for (auto e : g[v]) {
        int u = e.fst;
        if (u == p)
            continue;
        if (used[u] == 1)
            rev[u].pb(mp(v, e.snd));
        else if (used[u] != 2)
            dfsInit(u, v, e.snd);
    }
    used[v] = 2;
}

```

```

void calc(int v) {
    used[v] = 1;
    for (auto e : rev[v]) {
        path.clear();
        int u = e.fst;
        while (u != v) {
            calc(u);
            path.pb(mp(u, pr[u].snd));
            u = pr[u].fst;
        }
        // Calculate answer for cycle -- path and vertex v
    }
    for (auto e : g[v])
        if (!used[e.fst] && e.fst != pr[v].fst) {
            calc(e.fst);
            // Update answer for tree edges
        }
}

```

## 36 Cut Points

```

bool used[M];
int tIn[N], timer, isCut[N], color[M], compCnt;
vi st;

```

```

struct Edge {
    int to, id;
    Edge(int _to, int _id) : to(_to), id(_id) {}
};

```

```
vector<Edge> g[N];
```

```

int dfs(int v, int parent = -1) {
    tIn[v] = ++timer;
    int up = tIn[v], x = 0, y = (parent != -1);
    for (Edge p : g[v]) {
        int u = p.to, id = p.id;
        if (id != parent) {
            int t, size = sz(st);

```

```

    if (!used[id])
        used[id] = 1, st.push_back(id);
    if (!tIn[u]) { // not visited yet
        t = dfs(u, id);
        if (t >= tIn[v]) {
            ++x, ++compCnt;
            while (sz(st) != size) {
                color[st.back()] = compCnt;
                st.pop_back();
            }
        }
    } else
        t = tIn[u];
    up = min(up, t);
}
}
if (x + y >= 2)
    isCut[v] = 1; // v is cut vertex
return up;
}

```

## 37 Dominator Tree

```

// clean: forn(i, n+1)!!!
vi adj[N], ans[N]; // input edges, edges of dominator tree
vi radj[N], child[N], sdomChild[N];
int label[N], rlabel[N], sdom[N], dom[N], co = 0;
int par[N], bes[N];
int get(int x) { // DSU with path compression
    // get vertex with smallest sdom on path to root
    if (par[x] != x) {
        int t = get(par[x]); par[x] = par[par[x]];
        if (sdom[t] < sdom[bes[x]]) bes[x] = t;
    }
    return bes[x];
}
void dfs(int x) { // create DFS tree
    label[x] = ++co; rlabel[co] = x;
    sdom[co] = par[co] = bes[co] = co;
    for(auto y : adj[x]) {
        if (!label[y]) {
            dfs(y); child[label[x]].pb(label[y]); }
        radj[label[y]].pb(label[x]);
    }
}
void init(int root) {
    dfs(root);
    for(int i = co; i >= 1; i--) {
        for(auto j : radj[i]) sdom[i] = min(sdom[i], sdom[get(j)]);
        if (i > 1) sdomChild[sdom[i]].pb(i);
        for(auto j : sdomChild[i]) {
            int k = get(j);
            if (sdom[j] == sdom[k]) dom[j] = sdom[j];
            else dom[j] = k;
        }
        for(auto j : child[i]) par[j] = i;
    }
    forab(i, 2, co+1) {
        if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
        ans[rlabel[dom[i]]].pb(rlabel[i]);
    }
}

```

## 38 Eulerian Cycle

```

struct Edge {
    int to, used;
    Edge(): to(-1), used(0) {}
    Edge(int v): to(v), used(0) {}
};

vector<Edge> edges;
vi g[N], res, ptr;
// don't forget to clear ptr!

void dfs(int v) {
    for(; ptr[v] < sz(g[v]);) {
        int id = g[v][ptr[v]++];
        if (!edges[id].used) {
            edges[id].used = edges[id ^ 1].used = 1;

```

```

            dfs(edges[id].to);
            res.pb(id); // edges
        }
    }
    res.pb(v); // res contains vertices
}

```

## 39 Euler Tour Tree

```

mt19937 rng(239);

struct Edge {
    int v, u;
    Edge(int _v, int _u): v(_v), u(_u) {}
};

struct Node {
    Node *l, *r, *p;
    Edge e;
    int y, size;
    Node(Edge _e): l(nullptr), r(nullptr), p(this), e(_e), y(rng()),
    ↪ size(1) {}
};

inline int getSize(Node* root) { return root ? root->size : 0; }

inline void recalc(Node* root) { root->size = getSize(root->l) +
    ↪ getSize(root->r) + 1; }

set<pair<int, Node*>> edges[N];

Node* merge(Node *a, Node *b) {
    if (!a) return b;
    if (!b) return a;
    if (a->y < b->y) {
        a->r = merge(a->r, b);
        if (a->r) a->r->p = a;
        recalc(a);
        return a;
    }
    b->l = merge(a, b->l);
    if (b->l) b->l->p = b;
    recalc(b);
    return b;
}

void split(Node *root, Node *&a, Node *&b, int size) {
    if (!root) {
        a = b = nullptr;
        return;
    }
    int lSize = getSize(root->l);
    if (lSize >= size) {
        split(root->l, a, root->l, size);
        if (root->l) root->l->p = root;
        b = root, b->p = b;
    } else {
        split(root->r, root->r, b, size - lSize - 1);
        if (root->r) root->r->p = root;
        a = root, a->p = a;
        a->p = a;
    }
    recalc(root);
}

```

```

inline Node* rotate(Node* root, int k) {
    if (k == 0) return root;
    Node *l, *r;
    split(root, l, r, k);
    return merge(r, l);
}

inline pair<Node*, int> goUp(Node* root) {
    int pos = getSize(root->l);
    while (root->p != root)
        pos += (root->p->r == root ? getSize(root->p->r->l) + 1 : 0),
    ↪ root = root->p;
    return mp(root, pos);
}

```

```

inline Node* deleteFirst(Node* root) {
    Node* a;
    split(root, a, root, 1);
    edges[a->e.v].erase(mp(a->e.u, a));
    return root;
}

inline Node* getNode(int v, int u) {
    return edges[v].lower_bound(mp(u, nullptr))->snd;
}

inline void cut(int v, int u) {
    auto pV = goUp(getNode(v, u));
    auto pU = goUp(getNode(u, v));
    int l = min(pV.snd, pU.snd), r = max(pV.snd, pU.snd);
    Node *a, *b, *c;
    split(pV.fst, a, b, l);
    split(b, b, c, r - l);
    deleteFirst(b);
    merge(a, deleteFirst(c));
}

inline pair<Node*, int> getRoot(int v) {
    return !sz(edges[v]) ? mp(nullptr, 0) :
    ↪ goUp(edges[v].begin()->snd);
}

inline Node* makeRoot(int v) {
    auto root = getRoot(v);
    return rotate(root.fst, root.snd);
}

inline Node* makeEdge(int v, int u) {
    Node* e = new Node(Edge(v, u));
    edges[v].insert(mp(u, e));
    return e;
}

inline void link(int v, int u) {
    Node *vN = makeRoot(v), *uN = makeRoot(u);
    merge(merge(merge(vN, makeEdge(v, u)), uN), makeEdge(u, v));
}

```

## 40 Hamilton Cycle

```

// DP in  $O(n \cdot 2^n)$  for Ham cycle
vi g[MASK];
int adj[MASK], dp[1 << MASK];

vi hamiltonCycle(int n) {
    fill(dp, dp + (1 << n), 0);
    forn (v, n) {
        adj[v] = 0;
        for (int to : g[v])
            adj[v] |= (1 << to);
    }
    dp[1] = 1;
    forn (mask, (1 << n))
        forn (v, n)
            if (mask & (1 << v) && dp[mask ^ (1 << v)] & adj[v])
                dp[mask] |= (1 << v);
    vi ans;
    int mask = (1 << n) - 1, v;
    if (dp[mask] & adj[0]) {
        forab (i, 1, n)
            if ((1 << i) & (mask & adj[0]))
                v = i;
        ans.pb(v);
        mask ^= (1 << v);
        while(v) {
            forn(i, n)
                if ((dp[mask] & (1 << i)) && (adj[i] & (1 << v))) {
                    v = i;
                    break;
                }
            mask ^= (1 << v);
            ans.pb(v);
        }
    }
    return ans;
}

```

```

}

```

## 41 Karp with cycle

```

int d[N][N], p[N][N];
vi g[N], ans;

struct Edge {
    int a, b, w;
    Edge(int _a, int _b, int _w) : a(_a), b(_b), w(_w) {}
};

vector<Edge> edges;

void fordBellman(int s, int n) {
    forn (i, n + 1)
        forn (j, n + 1)
            d[i][j] = INF;
    d[0][s] = 0;
    forab (i, 1, n + 1)
        for (auto &e : edges)
            if (d[i - 1][e.a] < INF && d[i][e.b] > d[i - 1][e.a] + e.w)
                ↪ d[i][e.b] = d[i - 1][e.a] + e.w, p[i][e.b] = e.a;
}

ld karp(int n) {
    int s = n++;
    forn (i, n - 1)
        g[s].pb(sz(edges)), edges.pb(Edge(s, i, 0));
    fordBellman(s, n);
    ld ansValue = INF;
    int curV = -1, dist = -1;
    forn (v, n - 1)
        if (d[n][v] != INF) {
            ld curAns = -INF;
            int curPos = -1;
            forn(k, n)
                if (curAns <= (d[n][v] - d[k][v]) * (ld) (1) / (n - k))
                    curAns = (d[n][v] - d[k][v]) * (ld) (1) / (n - k),
                    ↪ curPos = k;
            if (ansValue > curAns)
                ansValue = curAns, dist = curPos, curV = v;
        }
    if (curV == -1) return ansValue;
    for (int iter = n; iter != dist; iter--)
        ans.pb(curV), curV = p[iter][curV];
    reverse(all(ans));
    return ansValue;
}

```

## 42 Kuhn's algorithm

```

// sz(LEFT) = n, sz(RIGHT) = m
// numbered consequently
int n, m, paired[2 * N], used[2 * N];
vi g[N];

bool dfs(int v) {
    if (used[v]) return false;
    used[v] = 1;
    for (int to : g[v])
        if (paired[to] == -1 || dfs(paired[to])) {
            paired[to] = v, paired[v] = to;
            return true;
        }
    return false;
}

int kuhn() {
    int ans = 0;
    forn (i, n + m) paired[i] = -1;
    for (int run = 1; run;) {
        run = 0;
        fill(used, used + n + m, 0);
        forn(i, n)
            if (!used[i] && paired[i] == -1 && dfs(i))
                ans++, run = 1;
    }
    return ans;
}

```



```

}

// Start from unpaired vertex in Left part, go from Left anywhere,
↪ from Right only to pair
// Max Independent -- A+, B-
// Min Cover      -- A-, B+

vi minCover, maxIndependent;

void dfsCoverIndependent(int v) {
    if (used[v]) return;
    used[v] = 1;
    for (int to : g[v])
        if (!used[to])
            used[to] = 1, dfsCoverIndependent(paired[to]);
}

// Kuhn first!
void findCoverIndependent() {
    fill(used, used + n + m, 0);
    forn (i, n)
        if (paired[i] == -1)
            dfsCoverIndependent(i);
    forn (i, n)
        if (used[i]) maxIndependent.pb(i);
        else minCover.pb(i);
    forab (i, n, n + m)
        if (used[i]) minCover.pb(i);
        else maxIndependent.pb(i);
}

```

## 43 Blossom algorithm

```

mt19937 rng(239017);
template<int SZ> struct UnweightedMatch {
    int match[SZ], N;
    vi adj[SZ];

    void ae(int u, int v) {
        adj[u].pb(v);
        adj[v].pb(u);
    }

    queue<int> q;
    int par[SZ], vis[SZ], orig[SZ], aux[SZ];

    void augment(int u, int v) { // toggle edges on u-v path
        while (1) { // one more matched pair
            int pv = par[v], nv = match[pv];
            match[v] = pv; match[pv] = v;
            v = nv; if (u == pv) return;
        }
    }

    int lca(int u, int v) { // find LCA of supernodes in O(dist)
        static int t = 0;
        for (++t; swap(u, v)) {
            if (!u) continue;
            if (aux[u] == t) return u; // found LCA
            aux[u] = t; u = orig[par[match[u]]];
        }
    }

    void blossom(int u, int v, int a) { // go other way
        for (; orig[u] != a; u = par[v]) { // around cycle
            par[u] = v; v = match[u]; // treat u as if vis[u] = 1
            if (vis[v] == 1) vis[v] = 0, q.push(v);
            orig[u] = orig[v] = a; // merge into supernode
        }
    }
}

```

```

bool bfs(int u) { // u is initially unmatched
    forn (i, N+1)
        par[i] = 0, vis[i] = -1, orig[i] = i;
    q = queue<int>();
    vis[u] = 0;
    q.push(u);
    while (sz(q)) { // each node is pushed to q at most once
        int v = q.front(); q.pop(); // 0 -> unmatched vertex
        for (int x : adj[v]) {

```

```

            if (vis[x] == -1) { // neither of x, match[x] visited
                vis[x] = 1; par[x] = v;
                if (!match[x])
                    return augment(u, x), 1;
                vis[match[x]] = 0;
                q.push(match[x]);
            } else if (vis[x] == 0 && orig[v] != orig[x]) {
                int a = lca(orig[v], orig[x]); // odd cycle
                blossom(x, v, a), blossom(v, x, a);
            } // contract O(n) times
        }
    }
    return 0;
}

int calc(int _N) { // rand matching -> constant improvement
    N = _N;
    forn (i, N+1)
        match[i] = aux[i] = 0;
    int ans = 0; vi V(N); iota(all(V), 1); shuffle(all(V), rng); //
↪ find rand matching
    for (int x : V) {
        if (!match[x]) {
            for (int y : adj[x]) {
                if (!match[y]) {
                    match[x] = y, match[y] = x; ++ans;
                    break;
                }
            }
        }
    }
    forab (i, 1, N+1)
        if (!match[i] && bfs(i))
            ++ans;
    return ans;
}
};

```

## 44 LCA

```

int tin[N], tout[N], up[N][LOG], curTime = 0;;
vi g[N];

```

```

void dfs(int v, int p) {
    up[v][0] = p;
    forn (i, LOG - 1)
        up[v][i + 1] = up[up[v][i]][i];
    tin[v] = curTime++;
    for (int u : g[v])
        if (u != p)
            dfs(u, v);
    tout[v] = curTime++;
}

int isUpper(int v, int u) {
    return tin[v] <= tin[u] && tout[v] >= tout[u];
}

int lca(int v, int u) {
    if (isUpper(u, v)) return u;
    fornr (i, LOG)
        if (!isUpper(up[u][i], v))
            u = up[u][i];
    return up[u][0];
}

void init() {
    dfs(0, 0);
}

```

## 45 LCA offline (Tarjan)

```

vi g[N], q[N];
int pr[N], ancestor[N], used[N];

int get(int v) {
    return v == pr[v] ? v : pr[v] = get(pr[v]);
}

void unite(int v, int u, int anc) {

```

```

    v = get(v), u = get(u);
    pr[u] = v, ancestor[v] = anc;
}

void dfs(int v) {
    used[v] = 1;
    for (int u : g[v])
        if (!used[u])
            dfs(u), unite(v, u, v);
    for (int u : q[v])
        if (used[u])
            ancestor[get(u)]; // handle answer somehow
}

void init(int n) {
    forn (i, n) pr[i] = i, ancestor[i] = i;
    dfs(0);
}

```

## 46 2 Chinese

```

struct Edge {
    int fr, to, w, id;
    bool operator<(const Edge& o) const { return w < o.w; }
};

// find oriented mst (tree)
// there are no edge --> root (root is 0)
// 0 .. n - 1, weights and vertices will be changed, but ids are
// ok
vector<Edge> work(const vector<vector<Edge>>& graph) {
    int n = sz(graph);
    vi color(n), used(n, -1);
    forn (i, n)
        color[i] = i;
    vector<Edge> e(n);
    forn (i, n) {
        if (graph[i].empty())
            e[i] = {-1, -1, -1, -1};
        else
            e[i] = *min_element(graph[i].begin(),
                graph[i].end());
    }
    vector<vi> cycles;
    used[0] = -2;
    forn (s, n) {
        if (used[s] != -1)
            continue;
        int x = s;
        while (used[x] == -1) {
            used[x] = s;
            x = e[x].fr;
        }
        if (used[x] != s)
            continue;
        vi cycle = {x};
        for (int y = e[x].fr; y != x; y = e[y].fr)
            cycle.push_back(y), color[y] = x;
        cycles.push_back(cycle);
    }
    if (cycles.empty())
        return e;
    vector<vector<Edge>> next_graph(n);
    forn (s, n) {
        for (const Edge& edge : graph[s]) {
            if (color[edge.fr] != color[s])
                next_graph[color[s]].push_back({
                    color[edge.fr], color[s], edge.w - e[s].w,
                    edge.id
                });
        }
    }
    vector<Edge> tree = work(next_graph);
    for (const auto& cycle : cycles) {
        int c1 = color[cycle[0]];
        Edge next_out = tree[c1], out{};
        int from = -1;
        for (int v : cycle) {
            tree[v] = e[v];

```

```

                for (const Edge& edge : graph[v])
                    if (edge.id == next_out.id)
                        from = v, out = edge;
            }
            tree[from] = out;
        }
        return tree;
    }
}

```

## 47 Matroid Intersection

```

struct Gmat { // graphic matroid
    int V = 0; vector<pii> ed; vi par;
    Gmat(vector<pii> _ed):ed(_ed) {
        map<int,int> m;
        for(auto &t : ed) m[t.fst] = m[t.snd] = 0;
        for(auto &t : m) t.snd = V++;
        for(auto &t : ed) t.fst = m[t.fst], t.snd = m[t.snd];
    }
    int p(int v) {
        return par[v] == v ? v : par[v] = p(par[v]);
    }
    bool unite(int v, int u) {
        v = p(v), u = p(u);
        if (v != u) { par[v] = u; return true; }
        return false;
    }
    void clear() {
        par.resize(V);
        forn(i,V) par[i] = i;
    }
    void ins(int i) { assert(unite(ed[i].fst,ed[i].snd)); }
    bool indep(int i) { return p(ed[i].fst) != p(ed[i].snd); }
};

struct Cmat { // colorful matroid
    int C = 0; vi col; vi used;
    Cmat(vi _col):col(_col) {for(auto t : col) C = max(C, t+1);}
    void clear() { used.assign(C,0); }
    void ins(int i) { used[col[i]] = 1; }
    bool indep(int i) { return !used[col[i]]; }
};

template<class M1, class M2> struct MatroidIsect {
    int n; vi iset; M1 m1; M2 m2;
    bool augment() {
        vi pre(n+1,-1); queue<int> q({n});
        while (sz(q)) {
            int x = q.front(); q.pop();
            if (iset[x]) {
                m1.clear(); forn(i,n) if (iset[i] && i != x) m1.ins(i);
                forn(i,n) if (!iset[i] && pre[i] == -1 && m1.indep(i))
                    pre[i] = x, q.push(i);
            } else {
                auto backE = [&]() { // back edge
                    m2.clear();
                    forn(c,2)forn(i,n)
                        if ((x==i||iset[i]&&(pre[i]==-1)==c){
                            if (!m2.indep(i))return c?pre[i]=x,q.push(i),i:-1;
                            m2.ins(i); }
                    return n;
                };
                for (int y; (y = backE()) != -1; y = n) {
                    for(; x != n; x = pre[x]) iset[x] = !iset[x];
                    return 1; }
            }
        }
        return 0;
    }
};

MatroidIsect(int _n, M1 _m1, M2 _m2):n(_n), m1(_m1), m2(_m2) {
    iset.assign(n+1,0); iset[n] = 1;
    m1.clear(); m2.clear(); // greedily add to basis
    forn(i,n) if (m1.indep(i) && m2.indep(i))
        iset[i] = 1, m1.ins(i), m2.ins(i);
    while (augment());
}

```

## 9 Math

### 48 Berlekamp

```
using T = int;
using poly = vector<int>;

void remz(poly& p) { while (sz(p)&& p.back() == T(0)) p.pop_back();
↪ }

poly operator*(const poly& l, const poly& r) {
    if (!min(sz(l), sz(r))) return {};
    poly x(sz(l)+sz(r)-1);
    forn(i, sz(l)) forn(j, sz(r)) x[i+j] += l[i]*r[j];
    return x;
}

pair<poly, poly> quoRem(poly a, poly b) {
    remz(a); remz(b); assert(sz(b));
    T lst = b.back(), B = T(1)/lst; for(auto &t : a) t *= B;
    for(auto &t : b) t *= B;
    poly q(max(sz(a)-sz(b)+1, 0));
    for(int dif; (dif=sz(a)-sz(b)) >= 0; remz(a)) {
        q[dif] = a.back(); forn(i, sz(b)) a[i+dif] -= q[dif]*b[i]; }
    for(auto &t : a) t *= lst;
    return {q, a}; // quotient, remainder
}

poly operator%(const poly& a, const poly& b) {
    return quoRem(a, b).snd; }

struct LinRec {
    poly s, C, rC;
    void BM() { // find smallest C such that C[0]=1 and
        // for all i >= sz(C)-1, sum_{j=0}^{sz(C)-1} C[j]*s[i-j]=0
        // If we treat C and s as polynomials in D, then
        // for all i >= sz(C)-1, [D^i]C*s=0
        int x = 0; T b = 1;
        poly B; B = C = {1}; // B is fail vector
        /// for all sz(B)+x-1 <= j < i, [D^j](B<<x)*s=0
        /// but [D^i](B<<x)*s=b
        /// invariant: sz(B)+x = M
        forn(i, sz(s)) { // update C after adding a term of s
            ++x; int L = sz(C), M = i+3-L;
            T d = 0; forn(j, L) d += C[j]*s[i-j]; // [D^i]C*s
            if (d == 0) continue; // [D^i]C*s=0
            poly _C = C; T coef = d/b; /// d-coef*b = 0
            /// set C := C-coef*(B<<x) to satisfy condition
            C.resize(max(L, M)); forn(j, sz(B)) C[j+x] -= coef*B[j];
            if (L < M) B = _C, b = d, x = 0;
        } /// replace B<<x with C<<0
    }

    void init(const poly& _s) {
        s = _s; BM();
        rC = C; reverse(all(rC)); // poly for getPow
        C.erase(begin(C)); for(auto &t : C) t *= -1;
    } // now s[i]=sum_{j=0}^{sz(C)-1} C[j]*s[i-j-1]
    poly getPow(ll p) { // get x^p mod rC
        if (p == 0) return {1};
        poly r = getPow(p/2); r = (r*r)%rC;
        return p&1?(r*poly{0,1})%rC:r;
    }

    T dot(poly v) { // dot product with seq
        T ans = 0; forn(i, sz(v)) ans += v[i]*s[i];
        return ans; } // get p-th term of rec
    T eval(ll p) { assert(p >= 0); return dot(getPow(p)); }
};
```

### 49 CRT (KTO)

```
vi crt(vi a, vi mod) {
    int n = sz(a);
    vi x(n);
    forn(i, n) {
        x[i] = a[i];
        forn(j, i) {
            x[i] = inverse(mod[j], mod[i]) * (x[i] - x[j]) % mod[i];
            if (x[i] < 0) x[i] += mod[i];
        }
    }
    return x;
}
```

## 50 Discrete Logarithm

```
// Returns x: a^x = b (mod mod) or -1, if no such x exists
int discreteLogarithm(int a, int b, int mod) {
    int sq = (int) sqrt(mod);
    int sq2 = mod / sq + (mod % sq ? 1 : 0);
    vector<pii> powers(sq2);
    forn(i, sq2)
        powers[i] = mp(power(a, (i + 1) * sq, mod), i + 1);
    sort(all(powers));
    forn(i, sq + 1) {
        int cur = power(a, i, mod);
        cur = mul(cur, b, mod);
        auto it = lower_bound(all(powers), mp(cur, 0));
        if (it != powers.end() && it->fst == cur)
            return it->snd * sq - i;
    }
    return -1;
}
```

## 51 Discrete Root

```
// Returns x: x^k = a mod mod, mod is prime
int discreteRoot(int a, int k, int mod) {
    if (a == 0)
        return 0;
    int g = primitiveRoot(mod);
    int y = discreteLogarithm(power(g, k, mod), a, mod);
    return power(g, y, mod);
}
```

## 52 Eratosthenes

```
vi eratosthenes(int n) {
    vi minDiv(n + 1, 0);
    minDiv[1] = 1;
    forab(i, 2, n + 1)
        if (minDiv[i] == 0)
            for(int j = i; j <= n; j += i)
                if (minDiv[j] == 0) minDiv[j] = i;
    return minDiv;
}

vi eratosthenesLinear(int n) {
    vi minDiv(n + 1, 0), primes;
    minDiv[1] = 1;
    forab(i, 2, n + 1) {
        if (minDiv[i] == 0)
            minDiv[i] = i, primes.pb(i);
        for(int j = 0; j < sz(primes) && primes[j] <= minDiv[i] && i
↪ * primes[j] <= n; j++)
            minDiv[i * primes[j]] = primes[j];
    }
    return minDiv;
}
```

## 53 Factorial

```
// Returns pair (rem, power), where rem = n! % mod,
// power = k: mod^k | n!, mod is prime, 0(mod log mod)
pii fact(int n, int mod) {
    int rem = 1, power = 0, nCopy = n;
    while (nCopy != 0) power += nCopy;
    while (n > 1) {
        rem = (rem * ((n / mod) % 2 ? -1 : 1) + mod) % mod;
        for(int i = 2; i <= n % mod; i++)
            rem = mul(rem, i, mod);
        n /= mod;
    }
    return mp(rem % mod, power);
}
```

## 54 Gauss

```
const double EPS = 1e-9;

int gauss(double **a, int n, int m) { // n is number of equations,
↪ m is number of variables
    int row = 0, col = 0;
    vi par(m, -1);
```

```

vector<double> ans(m, 0);
for (col = 0; col < m && row < n; col++) {
    int best = row;
    for (int i = row; i < n; i++)
        if (abs(a[i][col]) > abs(a[best][col]))
            best = i;
    if (abs(a[best][col]) < EPS) continue;
    par[col] = row;
    forn (i, m + 1) swap(a[row][i], a[best][i]);
    forn (i, n)
        if (i != row) {
            double k = a[i][col] / a[row][col];
            for (int j = col; j <= m; j++)
                a[i][j] -= k * a[row][j];
        }
    row++;
}
int single = 1;
forn (i, m)
    if (par[i] != -1) ans[i] = a[par[i]][m] / a[par[i]][i];
    else single = 0;
forn (i, n) {
    double cur = 0;
    for (int j = 0; j < m; j++)
        cur += ans[j] * a[i][j];
    if (abs(cur - a[i][m]) > EPS)
        return 0;
}
if (!single)
    return 2;
return 1;
}

```

## 55 Gauss binary

```
const int MAX = 1024;
```

```

int gaussBinary(vector<bitset<MAX>> a, int n, int m) {
    int row = 0, col = 0;
    vi par(m, -1);
    for (col = 0; col < m && row < n; col++) {
        int best = row;
        for (int i = row; i < n; i++)
            if (a[i][col] > a[best][col])
                best = i;
        if (a[best][col] == 0)
            continue;
        par[col] = row;
        swap(a[row], a[best]);
        forn (i, n)
            if (i != row && a[i][col])
                a[i] ^= a[row];
        row++;
    }
    vi ans(m, 0);
    forn (i, m)
        if (par[i] != -1)
            ans[i] = a[par[i]][n] / a[par[i]][i];
    bool ok = 1;
    forn (i, n) {
        int cur = 0;
        forn (j, m) cur ^= (ans[j] & a[i][j]);
        if (cur != a[i][n]) ok = 0;
    }
    return ok;
}

```

## 56 Gcd

```

int gcd(int a, int b) {
    return b ? gcd(b, a % b) : a;
}

```

```

int gcd(int a, int b, int &x, int &y) {
    if (b == 0) {
        x = 1, y = 0;
        return a;
    }
    int g = gcd(b, a % b, x, y), newX = y;
    y = x - a / b * y;
}

```

```

x = newX;
return g;
}

```

```

void diophant(int a, int b, int c, int &x, int &y) {
    int g = gcd(a, b, x, y);
    if (c % g != 0) return;
    x *= c / g, y *= c / g;
    // next solutions: x += b / g, y -= a / g
}

```

```

int inverse(int a, int mod) { // Returns -1, if a and mod are not
    ↪ coprime
    int x, y;
    int g = gcd(a, mod, x, y);
    return g == 1 ? (x % mod + mod) % mod : -1;
}

```

```

vi inverseForAll(int mod) {
    vi r(mod, 0);
    r[1] = 1;
    for (int i = 2; i < mod; i++)
        r[i] = (mod - r[mod % i]) * (mod / i) % mod;
    return r;
}

```

## 57 Gray

```

int gray(int n) {
    return n ^ (n >> 1);
}

```

```

int revGray(int n) {
    int k = 0;
    for (; n; n >>= 1) k ^= n;
    return k;
}

```

## 58 Miller-Rabin Test

```

bool isPrimeMillerRabin(ull n) { // not ll!
    if (n < 2 || n % 6 % 4 != 1)
        return n - 2 < 2;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
    ull s = __builtin_ctzll(n - 1), d = n >> s;
    for (ull a : A) { // ^count trailing zeroes
        ull p = power(a, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = mul(p, p, n);
        if (p != n - 1 && i != s) return 0;
    }
    return 1;
}

```

## 59 Phi

```

int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; i++)
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    if (n > 1) result -= result / n;
    return result;
}

```

```

int inversePhi(int a, int mod) {
    return power(a, phi(mod) - 1, mod);
}

```

## 60 Pollard

```

ull pollard(ull n) { // return some nontrivial factor of n
    auto f = [n](ull x) { return mul(x, x, n) + 1; };
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) { /// speedup: don't take
    ↪ gcd every it
        if (x == y) x = ++i, y = f(x);
    }
}

```

```

    if ((q = mul(prd, max(x, y) - min(x, y), n))) prd = q;
    x = f(x), y = f(f(y));
}
return __gcd(prd, n);
}

```

```

void factorize(ull n, map<ull,int>& cnt) {
    if (n == 1) return;
    if (isPrimeMillerRabin(n)) {
        ++cnt[n];
        return;
    }
    ull u = pollard(n);
    factorize(u, cnt), factorize(n / u, cnt);
}

```

## 61 Power And Mul

```

template <typename T>
inline T add(T a, T b, T mod) {
    a += b;
    return a >= mod ? a - mod : a;
}

```

```

template <typename T>
inline T sub(T a, T b, T mod) {
    a -= b;
    return a < 0 ? a + mod : a;
}

```

```

template <typename T>
T mul(T a, T b, T mod) {
    return T((a * 1ll * b) % mod);
}

```

```

template <>
ll mul<ll>(ll a, ll b, ll mod) {
    ll q = 1ll * a * b / mod;
    ll r = a * b - mod * q;
    while (r < 0) r += mod;
    while (r >= mod) r -= mod;
    return r;
}

```

```

template <typename T>
T power(T a, T n, T mod) {
    if (!n) return 1;
    T b = power(a, n / 2, mod);
    b = mul(b, b, mod);
    return n & 1 ? mul<T>(a, b, mod) : b;
}

```

```

int powerFast(int a, int n, int mod) {
    int res = 1;
    while (n) {
        if (n & 1)
            res = mul(res, a, mod);
        a = mul(a, a, mod);
        n /= 2;
    }
    return res;
}

```

## 62 Primitive Root

```

int primitiveRoot(int mod) { // Returns -1 if no primitive root
    ↪ exists
    vi fact;
    int ph = phi(mod);
    int n = mod;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            fact.pb(i);
            while (n % i == 0) n /= i;
        }
    }
    if (n > 1) fact.pb(n);
    forab (i, 2, mod + 1) {
        bool ok = 1;

```

```

        for (int j = 0; j < sz(fact) && ok; j++)
            ok &= power(i, ph / fact[j], mod) != 1;
        if (ok) return i;
    }
    return -1;
}

```

## 63 Simpson

```
double f(double x) { return x; }
```

```

double simpson(double a, double b, int iterNumber) {
    double res = 0, h = (b - a) / iterNumber;
    forn (i, iterNumber + 1)
        res += f(a + h * i) * ((i == 0) || (i == iterNumber) ? 1 :
    ↪ ((i & 1) == 0) ? 2 : 4);
    return res * h / 3;
}

```

## 64 Simplex

```

/**
 * maximize  $c^T x$  subject to  $Ax \leq b, x \geq 0$ .
 *  $-\text{inf} / \text{inf} / \text{max } c^T x$ 
 * define variables such that  $x = 0$  is viable.
 * vvd  $A = \{\{1, -1\}, \{-1, 1\}, \{-1, -2\}\}$ ;
 * vd  $b = \{1, 1, -4\}$ ,  $c = \{-1, -1\}$ ,  $x$ ;
 *  $T \text{ val} = \text{LPSolver}(A, b, c).solve(x)$ ;
 * Time:  $O(NM \cdot \#\text{pivots})$ , where a pivot may be e.g. an edge
    ↪ relaxation.  $O(2^N)$  in the general case.
 */

```

```

using vi = vector<int>;
using dbl = double;
using vd = vector<dbl>;
using vvd = vector<vd>;
const dbl eps = 1e-8, inf = 1/.0;

```

```

#define ltj(X) if (s== -1 || mp(X[j],N[j])<mp(X[s],N[s])) s=j
struct LPSolver {
    int m, n; vi N, B; vvd D; // # constraints, # variables
    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        forn(i,m) forn(j,n) D[i][j] = A[i][j];
        forn(i,m) { // B[i]: add basic variable for each constraint,
            B[i] = n+i, D[i][n] = -1, D[i][n+1] = b[i];
            // convert ineqs to eqs
        } // D[i][n]: artificial variable for testing feasibility
        forn(j,n) {
            N[j] = j; // non-basic variables, all zero
            D[m][j] = -c[j]; // minimize  $-c^T x$ 
        }
        N[n] = -1; D[m+1][n] = 1;
    }
    void pivot(int r, int s) { // r = row, c = column
        dbl *a = D[r].data(), inv = 1/a[s];
        forn(i,m+2) if (i != r && abs(D[i][s]) > eps) {
            dbl *b = D[i].data(), binv = b[s]*inv;
            forn(j,n+2) b[j] -= a[j]*binv;
            // make column corresponding to s all 0s
            b[s] = a[s]*binv; // swap N[s] with B[r]
        }
        // equation for r scaled so  $x_r$  coefficient equals 1
        forn(j,n+2) if (j != s) D[r][j] *= inv;
        forn(i,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv; swap(B[r], N[s]); // swap basic w/ non-basic
    }
}

```

```

bool simplex(int phase) {
    int x = m+phase-1;
    while (1) {
        int s = -1; forn(j,n+1) if (N[j] != -phase) ltj(D[x]);
        // find most negative col for nonbasic (nb) variable
        if (D[x][s] >= -eps) return 1;
        // can't get better sol by increasing nb variable
        int r = -1;
        forn(i,m) {
            if (D[i][s] <= eps) continue;
            if (r == -1 || mp(D[i][n+1] / D[i][s], B[i])
                < mp(D[r][n+1] / D[r][s], B[r])) r = i;
            // find smallest positive ratio

```

```

    } // -> max increase in nonbasic variable
    if (r == -1) return 0; // unbounded
    pivot(r,s);
}
}
dbl solve(vd& x) {
    int r = 0; forab(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) { // run simplex, find feasible x!=0
        pivot(r, n); // N[n] = -1 is artificial variable
        // initially set to smth large
        if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
        // D[m+1][n+1] is max possible value of the negation of
        // artificial variable, optimal value should be zero
        // if exists feasible solution
        forn(i,m) if (B[i] == -1) { // ?
            int s = 0; forab(j,1,n+1) ltj(D[i]);
            pivot(i,s);
        }
    }
    bool ok = simplex(1); x = vd(n);
    forn(i,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
}
};

```

## 65 Euclidean Burunduk-1

```

/**
 * Sergey Kopeliovich (burunduk30@gmail.com)
 */

#include <iostream>

using namespace std;

// finds x:
// a+k*x mod m --> min, 0 <= x <= r (0 <= a, k < m, 0 <= r)
// +k costs pk, -m costs pm
// return r-x
int go(int a, int k, int m, int pk, int pm, int r) {
    if (!k) return r;
    if (a >= k) { // make a: 0 <= a < k
        int add = (m - a + k - 1) / k;
        if (((int64_t)add * pk + pm > r) return r;
        a += (int64_t)add * k - m, r -= add * pk + pm;
    }
    int m1 = m % k, pm1 = (m / k) * pk + pm;
    if (!m1) return r;
    int k1 = k % m1, pk1 = (k / m1) * pm1 + pk;
    if (pm1 * (a / m1) > r) return r % pm1;
    return go(a % m1, k1, m1, pk1, pm1, r - (a / m1) * pm1);
}

// finds x: a+k*x mod m --> min, 0 <= a, k < m, 0 <= r
int go(int a, int k, int m, int r) {
    return r - go(a, k, m, 1, 0, r);
}

int main() {
    ios_base::sync_with_stdio(false), cin.tie(0);

    int a, k, m, r;
    while (cin >> a >> k >> m >> r) {
        int x = go(a, k, m, r);
        cout << ((int64_t)x * k + a) % m << ' ' << x << '\n';
    }
}

```

## 66 Euclidean Burunduk-2

```

/**
 * Sergey Kopeliovich (burunduk30@gmail.com)
 */

#include <iostream>

using namespace std;

// finds min x:
// a+k*x mod m \in [l..r]

```

```

// +k costs pk, -m costs pm
// l <= r < a, first tries -m then +k
int go(int a, int k, int m, int pk, int pm, int l, int r) {
    int ans = 0, steps;
    while (1) {
        steps = (a - r + m - 1) / m;
        ans += steps * pm, a -= steps * m;
        if (l <= a) return ans;
        if (!k) return -1;
        steps = (l - a + k - 1) / k;
        ans += steps * pk, a += steps * k;
        if (a <= r) return ans;
        int m1 = m % k, pm1 = (m / k) * pk + pm;
        if (!m1) return -1;
        int k1 = k % m1, pk1 = (k / m1) * pm1 + pk;
        k = k1, m = m1, pk = pk1, pm = pm1; // recursion =)
    }
}

int go(int a, int k, int m, int l, int r) {
    if (a < r)
        a += ((r - a) / m + 1) * m;
    return go(a, k, m, l, 0, l, r);
}

int main() {
    ios_base::sync_with_stdio(false), cin.tie(0);

    int a, k, m, l, r;
    while (cin >> a >> k >> m >> l >> r)
        cout << go(a, k, m, l, r) << '\n';
}

```

## 10 Strings

### 67 Aho-Corasick

```

struct Node {
    int next[ALPHA], term; //
    int go[ALPHA], suf, p, pCh; //
    Node(): term(0), suf(-1), p(-1) {
        fill(next, next + ALPHA, -1);
        fill(go, go + ALPHA, -1);
    }
};

Node g[N];
int last;

void add(const string &s) {
    int now = 0;
    for(char x : s) {
        if (g[now].next[x - 'a'] == -1) {
            g[now].next[x - 'a'] = ++last;
            g[last].p = now, g[last].pCh = x;
        }
        now = g[now].next[x - 'a'];
    }
    g[now].term = 1;
}

int go(int v, int c);

int getLink(int v) {
    if (g[v].suf == -1) {
        if (!v || !g[v].p) g[v].suf = 0;
        else g[v].suf = go(getLink(g[v].p), g[v].pCh);
    }
    return g[v].suf;
}

int go(int v, int c) {
    if (g[v].go[c] == -1) {
        if (g[v].next[c] != -1) g[v].go[c] = g[v].next[c];
        else g[v].go[c] = !v ? 0 : go(getLink(v), c);
    }
    return g[v].go[c];
}

```



## 68 Prefix-function

```
vi prefix(const string &s) {
    int n = sz(s);
    vi pr(n);
    forab (i, 1, n + 1) {
        int j = pr[i - 1];
        while (j > 0 && s[i] != s[j]) j = pr[j - 1];
        if (s[i] == s[j]) j++;
        pr[i] = j;
    }
    return pr;
}
```

## 69 Z-function

```
vi z(const string& s) {
    int n = sz(s);
    vi z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}
```

## 70 Hashes

```
#include "../math/PowerAndMul.cpp"
const int P = 239017, MOD_X = 1e9 + 7, MOD_Y = 1e9 + 9;

// using H = unsigned long long;
struct H {
    int x, y;
    H() = default;
    H(int _x): x(_x), y(_x) {}
    H(int _x, int _y): x(_x), y(_y) {}
    inline H operator+(const H& h) const { return H(add(x, h.x,
→ MOD_X), add(y, h.y, MOD_Y)); }
    inline H operator-(const H& h) const { return H(sub(x, h.x,
→ MOD_X), sub(y, h.y, MOD_Y)); }
    inline H operator*(const H& h) const { return H(mul(x, h.x,
→ MOD_X), mul(y, h.y, MOD_Y)); }
    inline bool operator==(const H& h) const { return x == h.x && y
→ == h.y; }
};

H p[N], h[N];

inline H get(int l, int r) { return h[r] - h[l] * p[r - l]; }
```

```
void init(const string& s) {
```

```
    int n = sz(s);
    p[0] = 1;
    forn (i, n)
        h[i + 1] = h[i] * P + s[i], p[i + 1] = p[i] * P;
}
```

## 71 Manaker

```
void manaker(const string& s, int *z0, int *z1) {
    int n = sz(s);
    forn (t, 2) {
        int *z = t ? z1 : z0, l = -1, r = -1; // [l..r]
        forn (i, n - t) {
            int k = 0;
            if (r > i + t) {
                int j = l + (r - i - t);
                k = min(z[j], j - l);
            }
            while (i - k >= 0 && i + k + t < n && s[i - k] == s[i + k +
→ t])
                k++;
            z[i] = k;
            if (k && i + k + t > r)
                l = i - k + 1, r = i + k + t - 1;
        }
    }
}
```

## 72 Palindromic Tree

```
struct Vertex {
    int suf, len, next[ALPHA];
    Vertex() { fill(next, next + ALPHA, 0); }
};

int vn, v;
Vertex t[N + 2];
int n, s[N];

int get(int i) { return i < 0 ? -1 : s[i]; }

void init() {
    t[0].len = -1, vn = 2, v = 0, n = 0;
}

void add(int ch) {
    s[n++] = ch;
    while (v != 0 && ch != get(n - t[v].len - 2))
        v = t[v].suf;
    int& r = t[v].next[ch];
    if (!r) {
        t[vn].len = t[v].len + 2;
        if (!v) t[vn].suf = 1;
        else {
            v = t[v].suf;
            while (v != 0 && ch != get(n - t[v].len - 2))
                v = t[v].suf;
            t[vn].suf = t[v].next[ch];
        }
        r = vn++;
    }
    v = r;
}
```

## 73 Suffix Array (+stable)

```
int sLen, num[N + 1], p[N], col[N], inv[N], lcp[N];
char s[N + 1];
```

```
inline int add(int a, int b) {
    a += b;
    return a >= sLen ? a - sLen : a;
}
```

```
inline int sub(int a, int b) {
    a -= b;
    return a < 0 ? a + sLen : a;
}
```

```
void buildArray(int n) {
    sLen = n;
    int ma = max(n, 256);
    forn (i, n)
        col[i] = s[i], p[i] = i;
```

```
    for (int k2 = 1; k2 / 2 < n; k2 *= 2) {
        int k = k2 / 2;
        memset(num, 0, sizeof(num));
        forn (i, n) num[col[i] + 1]++;
        forn (i, ma) num[i + 1] += num[i];
        forn (i, n)
            inv[num[col[sub(p[i], k)]]++] = sub(p[i], k);
        int cc = 0;
        forn (i, n) {
            bool flag = col[inv[i]] != col[inv[i - 1]];
            flag |= col[add(inv[i], k)] != col[add(inv[i - 1], k)];
            if (i && flag) cc++;
            num[inv[i]] = cc;
        }
        forn (i, n) p[i] = inv[i], col[i] = num[i];
    }
```

```
    memset(num, 0, sizeof(num));
    forn (i, n) num[col[i] + 1]++;
    forn (i, ma) num[i + 1] += num[i];
    forn (i, n) inv[num[col[i]]++] = i;
    forn (i, n) p[i] = inv[i];
    forn (i, n) inv[p[i]] = i;
```



```

}

void buildLCP(int n) {
    int len = 0;
    forn (ind, n){
        int i = inv[ind];
        len = max(0, len - 1);
        if (i != n - 1)
            while (len < n && s[add(p[i], len)] == s[add(p[i + 1],
↪ len)])
                len++;
        lcp[i] = len;
        if (i != n - 1 && p[i + 1] == n - 1) len = 0;
    }
}

```

## 74 Suffix Automaton

```

struct Vx {
    int len, suf;
    int next[ALPHA];
    Vx() {}
    Vx(int l, int s): len(l), suf(s) {}
};

struct SA {
    static const int V = 2 * LEN;
    int last, vcnt;
    Vx v[V];

    SA() { vcnt = 1, last = newV(0, 0); } // root = vertex with
↪ number 1
    int newV(int len, int suf){
        v[vcnt] = Vx(len, suf);
        return vcnt++;
    }
    int add(char ch) {
        int p = last, c = ch - 'a';
        last = newV(v[last].len + 1, 0);
        while (p && !v[p].next[c]) // added p &&
            v[p].next[c] = last, p = v[p].suf;
        if (!p)
            v[last].suf = 1;
        else {
            int q = v[p].next[c];
            if (v[q].len == v[p].len + 1) v[last].suf = q;
            else {
                int r = newV(v[p].len + 1, v[q].suf);
                v[last].suf = v[q].suf = r;
                memcpy(v[r].next, v[q].next, sizeof(v[r].next));
                while (p && v[p].next[c] == q)
                    v[p].next[c] = r, p = v[p].suf;
            }
        }
        return last;
    }
};

```

## 11 C++ Tricks

### 75 Fast allocation

```

const int MEM = 100 << 20;
static char buf[MEM];
inline void* operator new(size_t n) {
    static size_t i = sizeof buf;
    assert(n < i);
    return (void*) &buf[i -= n];
}
inline void operator delete(void*) {}
inline void* operator new[](size_t) { assert(0); }
inline void operator delete[](void*) { assert(0); }

```

### 76 Hash of pair

```

struct PairHasher {
    size_t operator()(const pair<int, int>& p) const { return p.fst
↪ * 239017 + p.snd; }
};

```

## 77 Ordered Set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

template <class T> using ordered_set = tree<T, null_type, less<T>,
↪ rb_tree_tag, tree_order_statistics_node_update>;

void example() {
    ordered_set<int> s;
    s.insert(1), s.insert(3);
    assert(s.order_of_key(3) == 1 && s.order_of_key(4) == 2 &&
↪ *s.find_by_order(0) == 1);
}

```

## 78 Hash Map

```

#include <ext/pb_ds/assoc_container.hpp>

using namespace __gnu_pbds;

struct chash { // To use most bits rather than just the lowest
↪ ones:
    const uint64_t C = 11(2e18 * PI) + 71; // large odd number
    const int RANDOM = 912387491;
    ll operator()(ll x) const { return __builtin_bswap64((x ^
↪ RANDOM) * C); }
};

template<class K, class V> using ht = gp_hash_table<K, V, chash>;
template<class K, class V> V get(ht<K, V>& u, K x) {
    auto it = u.find(x); return it == end(u) ? 0 : it->snd;
}

ht<ll, int> h({}, {}, {}, {}, {1<<20});

```

## 79 Fast I/O

```

const int BUF_SIZE = 4096;

char buf[BUF_SIZE];
int bufLen = 0, pos = 0;

inline int getChar() {
    if (pos == bufLen) {
        pos = 0, bufLen = (int) fread(buf, 1, BUF_SIZE, stdin);
        if (!bufLen)
            return -1;
        return buf[pos++];
    }
}

inline int readChar() {
    int c = getChar();
    while (c != -1 && c <= 32)
        c = getChar();
    return c;
}

template <class T>
inline T readInt() {
    int s = 1, c = readChar();
    T x = 0;
    if (c == '-')
        s = -1, c = getChar();
    while ('0' <= c && c <= '9')
        x = x * 10 + c - '0', c = getChar();
    return s == 1 ? x : -x;
}

inline void readWord(char *s) {
    int c = readChar();
    while (c > 32)
        *s++ = (char) c, c = getChar();
    *s = 0;
}

int writePos = 0;
char writeBuf[BUF_SIZE];

```

```
inline void flush() {
    if (writePos)
        fwrite(writeBuf, 1, writePos, stdout), writePos = 0;
}

inline void writeChar(int x) {
    if (writePos == BUF_SIZE)
        flush();
    writeBuf[writePos++] = (char) x;
}

template <class T>
inline void writeInt(T x, char after = '\\0') {
    if (x < 0)
        writeChar('-'), x = -x;

    char s[24];
    int n = 0;
    while (x || !n)
        s[n++] = '0' + x % 10, x /= 10;
    while (n--)
        writeChar(s[n]);
    if (after)
        writeChar(after);
}

inline void writeWord(const char *s) {
    while (*s)
        writeChar(*s++);
}
```

12 Notes

80 Работа с деревьями

Приемы для работы с деревьями:

- 1. Двоичные подъемы
- 2. Поддеревья как отрезки Эйлераова обхода
- 3. Вертикальные пути в Эйлеровом обходе (на ребрах вниз +k, на ребрах вверх -k).
- 4. Храним в вершине значение функции на пути от корня до нее, дальше LCA.
- 5. Спуск с DFS, поддерживаем ДО на пути до текущей вершины.
- 6. Heavy-light decomposition
- 7. Centroid decomposition
- 8. Корневая по запросам
- 9. Тяжелые/легкие вершины
- 10. DFS → дерево блоков, размеры ∈ [K..2K]
- 11. У вершины не более O(√N) разных поддеревьев
- 12. Сумма размеров поддеревьев без тяжелого ребенка O(n log n)
- 13. Сумма глубин поддеревьев без глубокого ребенка O(n)

81 Маски

Считаем динамику по маскам за  $O(2^n \cdot n)$   $f[mask] = \text{sum по submask } g[submask]$ .  
 $dp[mask][i]$  — значение динамики для маски  $mask$ , если младшие  $i$  бит в ней зафиксированы (то есть мы не можем удалять оттуда).  
Ответ в  $dp[mask][0]$ .  
 $dp[mask][len] = g[mask]$ . Если  $i$ -ый бит 0, то  $dp[mask][i] = dp[mask][i + 1]$ , иначе  $dp[mask][i] = dp[mask][i + 1] + dp[mask \setminus 1 << i][i + 1]$ .  
Старший бит: предподсчет.  
Младший бит:  $x \& \sim (-x)$   
Чтобы по степени двойки получить логарифм, можно воспользоваться тем, что все степени двойки имеют разный остаток по модулю 67.

```
for (int mask = 0; mask < (1 << n); mask++)
    ^^Isubmask : for (int s = mask; s; s = (s - 1) & mask)
    ^^Isupmask : for (int s = mask; s < (1 << n); s = (s + 1) | mask)
```

82 Гранди

Теорема Шпрага-Гранди: берем тех всех значений функции Гранди по состояни-ям, в которые можем перейти из данного.  
Если сумма независимых игр, то значение функции Гранди равно хог значений функций Гранди по всем играм.  
Бывает полезно вывести первые n значений и поискать закономерность.  
Часто сводится к xor по чему-нибудь.

83 Потoki

Потоки:

Name	Asympthotic
Ford-Fulkerson	$O( f  \cdot E)$
Ford-Fulkerson with scaling	$O(\log  f  \cdot E^2)$
Edmonds-Karp	$O(V \cdot E^2)$
Dinic	$O(V^2 \cdot E)$
Dinic with scaling	$O(V \cdot E \cdot \log C)$
Dinic on bipartite graph	$O(E\sqrt{V})$
Dinic on unit network	$O(E\sqrt{E})$

L—R потоки:  
Есть граф с недостатками или избытками в каждой вершине. Создаем фиктив-ные исток и сток (из истока все ребра в избытки, из недостатков все ребра в сток).  
Теперь пусть у нас есть L-R граф, для каждого ребра  $e (v \rightarrow u)$  известны  $L_e$  и  $R_e$ . Добавим в  $v$  избыток  $L_e$ , в  $u$  недостаток  $L_e$ , а пропускную способность сделаем  $R_e - L_e$ .  
Получили решение задачи о LR-циркуляции.  
Если у нас обычный граф с истоком и стоком, то добавляем бесконечное ребро из стока в сток и ищем циркуляцию.  
Таким образом нашли удовлетворяющий условиям LR-поток. Если хотим мак-симальный поток, то на остаточной сети запускаем поиск максимального потока.  
В новом графе в прямую сторону пропускная способность равна  $R_e - f_e$ , в обратную  $f_e - L_e$ .  
MinCostCirculation:  
Пока есть цикл отрицательного веса, запускаем алгоритм Карпа и пускаем мак-симальный поток по найденному циклу.

84 ДП

Табличка с оптимизациями для динамики:

Name	Original recurrence Sufficient Condition	From To
CHT1	$dp[i] = \min_{j < i} dp[j] + b[j] \cdot a[i]$ $b[j] \geq b[j + 1] \parallel a[i] \leq a[i + 1]$	$O(n^2)$ $O(n)$
CHT2	$dp[i][j] = \min_{k < j} dp[i - 1][k] + b[k] \cdot a[j]$ $b[k] \geq b[k + 1] \parallel a[j] \leq a[j + 1]$	$O(kn^2)$ $O(kn)$
D&C	$dp[i][j] = \min_{k < j} dp[i - 1][k] + c[k][j]$ $p[i, j] \leq p[i, j + 1]$	$O(kn^2)$ $O(kn \log n)$
Knuth	$dp[i][j] = \min_{i < k < j} dp[i][k] + dp[k][j] + c[i][j]$ $p[i, j - 1] \leq p[i, j] \leq p[i + 1, j]$	$O(n^3)$ $O(n^2)$
IOI	$f_n(k)$ — best for fixed k $f_n$ — convex, add penalty $\lambda \cdot k$	$O(k^{(2)}n)$ $O(n \log C)$

85 Комбинаторика

Биномиальные коэффициенты:  
Теорема Люка для биномиальных коэффициентов: Хотим посчитать  $C_n^k$ , раз-ложим в p-ичной системе счисления,  $n = (n_0, n_1, \dots)$ ,  $k = (k_0, k_1, \dots)$ .  $ans = C_{n_0}^{k_0} \cdot C_{n_1}^{k_1} \cdot \dots$   
Способы вычисления  $C_n^k$ :  
1.  $C_n^k = C_{n-1}^k + C_{n-1}^{k-1}$   
precalc:  $O(n^2)$ , query:  $O(1)$ .  
2.  $C_n^k = \frac{n!}{k!(n-k)!}$ , предподсчитываем факториалы  
precalc:  $O(n \log n)$ , query:  $O(\log n)$

3. Теорема Люка  
  
precalc:  $O(p \log p)$ , query:  $O(\log p)$ .
4.  $C_n^k = C_n^{k-1} \cdot \frac{n-k+1}{k}$
5.  $C_n^k = \frac{n!}{k!(n-k)!}$ , для каждого факториала считаем степень вхождения и остаток  
  
precalc:  $O(p \log p)$ , query:  $O(\log p)$ .
- $C_n^{\frac{n}{2}} = \frac{2^n}{\sqrt{\frac{\pi n}{2}}}$

86 Делители

- $\leq 20 : d(12) = 6$
- $\leq 50 : d(48) = 10$
- $\leq 100 : d(60) = 12$
- $\leq 1000 : d(840) = 32$
- $\leq 10^4 : d(9\,240) = 64$
- $\leq 10^5 : d(83\,160) = 128$
- $\leq 10^6 : d(720\,720) = 240$
- $\leq 10^7 : d(8\,648\,640) = 338$
- $\leq 10^8 : d(91\,891\,800) = 768$
- $\leq 10^9 : d(931\,170\,240) = 1344$
- $\leq 10^{11} : d(97\,772\,875\,200) = 4032$
- $\leq 10^{12} : d(963\,761\,198\,400) = 6720$
- $\leq 10^{15} : d(866\,421\,317\,361\,600) = 15360$
- $\leq 10^{18} : d(897\,612\,484\,786\,617\,600) = 103680$

87 Числа Белла

$i$	$B_i$	$i$	$B_i$
0	1	12	4,213,597
1	1	13	27,644,437
2	2	14	190,899,322
3	5	15	1,382,958,545
4	15	16	10,480,142,147
5	52	17	82,864,869,804
6	203	18	682,076,806,159
7	877	19	5,832,742,205,057
8	4,140	20	51,724,158,235,372
9	21,147	21	474,869,816,156,751
10	115,975	22	4,506,715,738,447,323
11	678,570	23	44,152,005,855,084,346

88 Разбиения

Число неупорядоченных разбиений  $n$  на положительные слагаемые.

$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$

$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

89 Матричные игры

Пишем матрицу стратегий  $A_{i,j}$  это выигрыш первого и проигрыш второго,  $i$  стратегия 1-го. Седловая точка есть для несмешанной стратегии если  $\max_i \min_j A_{i,*} = \min_j \max_i A_{*,j}$ . Иначе:

$f(x) = \sum (x_i) \rightarrow \max, Ans = 1/f(x)$

$Ax \leq 1_n, x_i \geq 0$

Для  $2 \times 2, p$  первый игрок,  $q$  — второй:

$$p^* = \left( \frac{a_{22} - a_{21}}{a_{22} - a_{12} + a_{11} - a_{21}}; \frac{a_{11} - a_{12}}{a_{22} - a_{12} + a_{11} - a_{21}} \right)$$
$$q^* = \left( \frac{a_{22} - a_{12}}{a_{22} - a_{12} + a_{11} - a_{21}}; \frac{a_{11} - a_{21}}{a_{22} - a_{12} + a_{11} - a_{21}} \right)$$
$$Ans = \frac{a_{22}a_{11} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

90 Mixed

- Формула Пика:  $S = Inside + Edge/2 - 1$
- Теорема Люка:  $0 \leq n, m \in \mathbb{Z}, p$  простое.  $n = n_k p^k + \dots + n_1 p + n_0$  и  $m = m_k p^k + \dots + m_1 p + m_0$ . Тогда  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod p$ .
- Лемма Бернсайда:  $|X/G|$  число орбит  $G. X^g = \{x \in X | gx = x\}$

$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$

91 Ideas

- Generic:** binary search, ternary search, sort, dp, meet-in-the-middle, divide&conquer, greedy, sqrt-decomposition, matroids, Gauss, FFT, suffix array, suffix automaton, DSU;
- Graphs:** build graph, add vertices / edges, 2-SAT, flows / cut, matching, Hall’s theorem, topsort, HLD, centroid decomposition, MST, Euler cycle, Binary lifting, LCA;
- Tricks:** consider the process from the end / from the middle, try any one, draw on 2D plane, simplify the problem / consider special case / consider more general case, simplify solution, prefix sums, differences of adjacent elements, consider min/max, analyze why a straightforward solution doesn’t work, check limitations, consider contribution of separate element, small answer, different solutions for different limitations, consider complement set, maintain sum / sum of squares, convex function, store O(1) top candidates, inversions, inclusion-exclusion formula, bounding box, angle sort, Grundy function, Eucklid, Mo’s algorithm, iterate over divisors, matrix exponentiation;