

Содержание

1 Common	10 Strings
1 Setup	63 Aho-Corasick
2 Template	64 Prefix-function
3 Stress	65 Z-function
4 Java	66 Hashes
	67 Manaker
	68 Palindromic Tree
	69 Suffix Array (+stable)
2 Big numbers	70 Suffix Automaton
5 Big Int	71 Suffix Tree
6 FFT	
7 FFT by mod and FFT with digits up to 10 ⁶	11 C++ Tricks
	72 Fast allocation
3 Data Structures	73 Hash of pair
8 Centroid Decomposition	74 Ordered Set
9 Convex Hull Trick	75 Hash Map
10 DSU	76 Fast I/O
11 Fenwick Tree	
12 Hash Table	12 Notes
13 Heavy Light Decomposition	77 Работа с деревьями
14 Next Greater in Segment Tree	78 Маски
15 Sparse Table	79 Гранди
16 Fenwick Tree 2D	80 Потoki
17 Segment Tree 2D	81 ДП
	82 Комбинаторика
4 Dynamic Programming	83 Делители
18 LIS	84 Числа Белла
19 DP tree	85 Разбиения
20 Masks tricks	86 Матричные игры
	87 Mixed
5 Flows	88 Ideas
21 Utilities	
22 Ford-Fulkerson	
23 Dinic	
24 Hungarian	
25 Min Cost Max Flow	
6 Games	
26 Retrograde Analysis	
7 Geometry	
27 ClosestPoints (SweepLine)	
28 ConvexHull	
29 GeometryBase	
30 GeometryInterTangent	
31 GeometrySimple	
8 Graphs	
32 2-SAT	
33 Bridges	
34 Cactus	
35 Cut Points	
36 Dominator Tree	
37 Eulerian Cycle	
38 Euler Tour Tree	
39 Hamilton Cycle	
40 Karp with cycle	
41 Kuhn's algorithm	
42 LCA	
43 LCA offline (Tarjan)	
44 2 Chinese	
45 Matroid Intersection	
9 Math	
46 CRT (KTO)	
47 Discrete Logarithm	
48 Discrete Root	
49 Eratosthenes	
50 Factorial	
51 Gauss	
52 Gauss binary	
53 Gcd	
54 Gray	
55 Miller-Rabin Test	
56 Phi	
57 Pollard	
58 Power And Mul	
59 Primitive Root	
60 Simpson	
61 Euclidean Burunduk-1	
62 Euclidean Burunduk-2	

Common

1 Setup

1. F9 → Commands → File Associations → Ins →

1st line: *.cpp, 3rd line: g++ -O2 -Wall -Wshadow -Wextra -Wno-unused-result -Wconversion -std=gnu++17 -g -DLOCAL !-o !.exe

2. F9 → Options → Editor settings

Auto indent, Tab size, Cursor beyond end of line, Show white space (disable).

2 Template

```
#include <bits/stdc++.h>

using namespace std;

#define pb push_back
#define mp make_pair
#define fst first
#define snd second
#define sz(x) (int) ((x).size())
#define forn(i, n) for (int i = 0; i < (n); ++i)
#define fornrr(i, n) for (int i = (n) - 1; i >= 0; --i)
#define forab(i, a, b) for (int i = (a); i < (b); ++i)
#define all(c) (c).begin(), (c).end()

using ll = long long;
using vi = vector<int>;
using pii = pair<int, int>;

#define FNAME ""

int main() {
#ifdef LOCAL
    freopen(FNAME".in", "r", stdin);
    freopen(FNAME".out", "w", stdout);
#endif
    cin.tie(0);
    ios_base::sync_with_stdio(0);

    return 0;
}
```

3 Stress

@echo off

```
for /L %i in (1,1,10000000) do (
gen.exe || exit
main.exe || exit
stupid.exe || exit
fc .out 2.out || exit
echo Test %i OK
)
```

4 Java

```
import java.io.BufferedReader;
import java.io.FileNotFoundException;
import java.io.FileReader;
import java.io.IOException;
import java.io.InputStreamReader;
import java.io.PrintWriter;
import java.util.*;

public class Main {
    FastScanner in;
    PrintWriter out;

    void solve() {
        int a = in.nextInt();
        int b = in.nextInt();
        out.print(a + b);
    }

    void run() {
        try {
```

```
        in = new FastScanner("input.txt");
        out = new PrintWriter("output.txt");
        solve();
        out.flush();
        out.close();
    } catch (FileNotFoundException e) {
        e.printStackTrace();
        System.exit(1);
    }
}

class FastScanner {
    BufferedReader br;
    StringTokenizer st;

    public FastScanner() {
        br = new BufferedReader(new InputStreamReader(System.in));
    }

    public FastScanner(String s) {
        try {
            br = new BufferedReader(new FileReader(s));
        } catch (FileNotFoundException e) {
            e.printStackTrace();
        }
    }

    String nextToken() {
        while (st == null || !st.hasMoreElements()) {
            try {
                st = new StringTokenizer(br.readLine());
            } catch (IOException e) {
                e.printStackTrace();
            }
        }
        return st.nextToken();
    }

    int nextInt() {
        return Integer.parseInt(nextToken());
    }

    long nextLong() {
        return Long.parseLong(nextToken());
    }

    double nextDouble() {
        return Double.parseDouble(nextToken());
    }

    char nextChar() {
        try {
            return (char) (br.read());
        } catch (IOException e) {
            e.printStackTrace();
        }
        return 0;
    }

    String nextLine() {
        try {
            return br.readLine();
        } catch (IOException e) {
            e.printStackTrace();
        }
        return "";
    }
}

public static void main(String[] args) {
    new Main().run();
}
```

2 Big numbers

5 Big Int

```
constexpr int BASE = 1000000000;
constexpr int BASE_DIGITS = 9;

struct BigInt {
    // value == 0 is represented by empty z
    vi z; // digits
    // sign == 1/-1 <==> value >=< 0
    int sign;
    BigInt(): sign(1) {}
    BigInt(ll v) { *this = v; }
    BigInt& operator=(ll v) {
        sign = v < 0 ? -1 : 1; v *= sign;
        z.clear(); for (; v > 0; v = v / BASE) z.pb((int) (v %
        BASE));
        return *this;
    }
    BigInt& operator+=(const BigInt& other) {
        if (sign == other.sign) {
            for (int i = 0, carry = 0; i < sz(other.z) || carry; ++i) {
                if (i == sz(z)) z.pb(0);
                z[i] += carry + (i < sz(other.z) ? other.z[i] : 0);
                carry = z[i] >= BASE;
                if (carry) z[i] -= BASE;
            }
        } else if (other != 0 /* prevent infinite loop */) {
            *this -= -other;
        }
        return *this;
    }
    friend BigInt operator+(BigInt a, const BigInt& b) { return a
    += b; }
    BigInt& operator--(const BigInt& other) {
        if (sign == other.sign) {
            if ((sign == 1 && *this >= other) || (sign == -1 && *this
            <= other)) {
                for (int i = 0, carry = 0; i < sz(other.z) || carry; ++i)
                {
                    z[i] -= carry + (i < sz(other.z) ? other.z[i] : 0);
                    carry = z[i] < 0;
                    if (carry)
                        z[i] += BASE;
                }
                trim();
            } else {
                *this = other - *this;
                this->sign = -this->sign;
            }
        } else
            *this += -other;
        return *this;
    }
    friend BigInt operator-(BigInt a, const BigInt& b) { return a
    -= b; }
    BigInt& operator*=(int v) {
        if (v < 0) sign = -sign, v = -v;
        for (int i = 0, carry = 0; i < sz(z) || carry; ++i) {
            if (i == sz(z))
                z.pb(0);
            ll cur = (ll) z[i] * v + carry;
            carry = (int) (cur / BASE);
            z[i] = (int) (cur % BASE);
        }
        trim();
        return *this;
    }
    BigInt operator*(int v) const { return BigInt(*this) *= v; }
    friend pair<BigInt, BigInt> divmod(const BigInt& a1, const
    BigInt& b1) {
        int norm = BASE / (b1.z.back() + 1);
        BigInt a = a1.abs() * norm;
        BigInt b = b1.abs() * norm;
        BigInt q, r;
        q.z.resize(sz(a.z));
        fornr (i, sz(a.z)) {
            r *= BASE, r += a.z[i];
            int s1 = sz(b.z) < sz(r.z) ? r.z[sz(b.z)] : 0;
```

```
            int s2 = sz(b.z) - 1 < sz(r.z) ? r.z[sz(b.z) - 1] : 0;
            int d = (int) (((ll) s1 * BASE + s2) / b.z.back());
            r -= b * d;
            while (r < 0) r += b, --d;
            q.z[i] = d;
        }
        q.sign = a1.sign * b1.sign, r.sign = a1.sign;
        q.trim(), r.trim();
        return {q, r / norm};
    }
    BigInt operator/(const BigInt& v) const { return divmod(*this,
    v).fst; }
    BigInt operator%(const BigInt& v) const { return divmod(*this,
    v).snd; }
    BigInt& operator/=(int v) {
        if (v < 0) sign = -sign, v = -v;
        int rem = 0;
        fornr (i, sz(z)) {
            ll cur = z[i] + rem * (ll) BASE;
            z[i] = (int) (cur / v);
            rem = (int) (cur % v);
        }
        trim();
        return *this;
    }
    BigInt operator/(int v) const { return BigInt(*this) /= v; }
    int operator%(int v) const {
        if (v < 0) v = -v;
        int m = 0;
        fornr (i, sz(z))
            m = (int) ((z[i] + m * (ll) BASE) % v);
        return m * sign;
    }
    BigInt& operator*=(const BigInt& v) { return *this = *this *
    v; }
    BigInt& operator/(const BigInt& v) { return *this = *this / v;
    }
    bool operator<(const BigInt& v) const {
        if (sign != v.sign) return sign < v.sign;
        if (sz(z) != sz(v.z)) return sz(z) * sign < sz(v.z) * v.sign;
        fornr (i, sz(z))
            if (z[i] != v.z[i])
                return z[i] * sign < v.z[i] * sign;
        return false;
    }
    bool operator>(const BigInt& v) const { return v < *this; }
    bool operator<=(const BigInt& v) const { return !(v < *this); }
    bool operator>=(const BigInt& v) const { return !(*this < v); }
    bool operator==(const BigInt& v) const { return !(*this < v)
    && !(v < *this); }
    bool operator!=(const BigInt& v) const { return *this < v || v
    < *this; }
    void trim() {
        while (!z.empty() && z.back() == 0) z.pop_back();
        if (z.empty()) sign = 1;
    }
    bool isZero() const { return z.empty(); }
    friend BigInt operator-(BigInt v) {
        if (!v.z.empty()) v.sign = -v.sign;
        return v;
    }
    BigInt abs() const {
        return sign == 1 ? *this : -*this;
    }
    void read(const string& s) {
        sign = 1, z.clear();
        int pos = 0;
        while (pos < sz(s) && (s[pos] == '-' || s[pos] == '+')) {
            if (s[pos] == '-') sign = -sign;
            ++pos;
        }
        for (int i = sz(s) - 1; i >= pos; i -= BASE_DIGITS) {
            int x = 0;
            forab (j, max(pos, i - BASE_DIGITS + 1), i)
                x = x * 10 + s[j] - '0';
            z.pb(x);
        }
        trim();
    }
    friend ostream &operator<<(ostream& stream, const BigInt& v) {
```

```

    if (v.sign == -1)
        stream << '-';
    stream << (v.z.empty() ? 0 : v.z.back());
    fornr (i, sz(v.z) - 1)
        stream << setw(BASE_DIGITS) << setfill('0') << v.z[i];
    return stream;
}

static vi convertBase(const vi& a, int oldDigits, int
→ newDigits) {
    vector<ll> p(max(oldDigits, newDigits) + 1);
    p[0] = 1;
    for (int i = 1; i < sz(p); i++)
        p[i] = p[i - 1] * 10;
    vi res;
    ll cur = 0;
    int curDigits = 0;
    for (int v : a) {
        cur += v * p[curDigits];
        curDigits += oldDigits;
        while (curDigits >= newDigits) {
            res.pb(int(cur % p[newDigits]));
            cur /= p[newDigits];
            curDigits -= newDigits;
        }
    }
    res.pb((int) cur);
    while (!res.empty() && res.back() == 0) res.pop_back();
    return res;
}

static vll karatsubaMultiply(const vll& a, const vll& b) {
    int n = sz(a);
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }

    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k), a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k), b2(b.begin() + k, b.end());
    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);
    forn (i, k) a2[i] += a1[i];
    forn (i, k) b2[i] += b1[i];

    vll r = karatsubaMultiply(a2, b2);
    forn (i, sz(a1b1)) r[i] -= a1b1[i];
    forn (i, sz(a2b2)) r[i] -= a2b2[i];
    forn (i, sz(r)) res[i + k] += r[i];
    forn (i, sz(a1b1)) res[i] += a1b1[i];
    forn (i, sz(a2b2)) res[i + n] += a2b2[i];
    return res;
}

BigInt operator*(const BigInt& v) const {
    vi a6 = convertBase(this->z, BASE_DIGITS, 6);
    vi b6 = convertBase(v.z, BASE_DIGITS, 6);
    vll a(all(a6)), b(all(b6));
    while (sz(a) < sz(b)) a.pb(0);
    while (sz(b) < sz(a)) b.pb(0);
    while (sz(a) & (sz(a) - 1)) a.pb(0), b.pb(0);
    vll c = karatsubaMultiply(a, b);
    BigInt res;
    res.sign = sign * v.sign;
    int carry = 0;
    forn (i, sz(c)) {
        ll cur = c[i] + carry;
        res.z.push_back((int) (cur % 1000000));
        carry = (int) (cur / 1000000);
    }
    res.z = convertBase(res.z, 6, BASE_DIGITS);
    res.trim();
    return res;
}
};

```

6 FFT

```

int rev[N];

//using Num = complex<dbl>;
struct Num {
    dbl x, y;
    Num() {}
    Num(dbl _x, dbl _y): x(_x), y(_y) {}
    inline dbl real() const { return x; }
    inline dbl imag() const { return y; }
    inline Num operator+(const Num &B) const { return Num(x + B.x, y
→ + B.y); }
    inline Num operator-(const Num &B) const { return Num(x - B.x, y
→ - B.y); }
    inline Num operator*(dbl k) const { return Num(x * k, y * k); }
    inline Num operator*(const Num &B) const { return Num(x * B.x -
→ y * B.y, x * B.y + y * B.x); }
    inline void operator+=(const Num &B) { x += B.x, y += B.y; }
    inline void operator/=(dbl k) { x /= k, y /= k; }
    inline void operator*=(const Num &B) { *this = *this * B; }
};

Num rt[N];

inline Num sqr(const Num &x) { return x * x; }
inline Num conj(const Num &x) { return Num(x.real(), -x.imag());
→ }

inline int getN(int n) {
    int k = 1;
    while(k < n)
        k <<= 1;
    return k;
}

void fft(Num *a, int n) {
    assert(rev[1]); // don't forget to init
    int q = N / n;
    forn (i, n)
        if(i < rev[i] / q)
            swap(a[i], a[rev[i] / q]);
    for (int k = 1; k < n; k <<= 1)
        for (int i = 0; i < n; i += 2 * k)
            forn (j, k) {
                const Num z = a[i + j + k] * rt[j + k];
                a[i + j + k] = a[i + j] - z;
                a[i + j] += z;
            }
}

void fftInv(Num *a, int n) {
    fft(a, n);
    reverse(a + 1, a + n);
    forn (i, n)
        a[i] /= n;
}

void doubleFft(Num *a, Num *fa, Num *fb, int n) { // only if you
→ need it
    fft(a, n);
    const int n1 = n - 1;
    forn (i, n) {
        const Num &z0 = a[i], &z1 = a[(n - i) & n1];
        fa[i] = Num(z0.real() + z1.real(), z0.imag() - z1.imag()) *
→ 0.5;
        fb[i] = Num(z0.imag() + z1.imag(), z1.real() - z0.real()) *
→ 0.5;
    }
}

Num tmp[N];
template<class T>
void mult(T *a, T *b, T *r, int n) { // n = 2^k
    forn (i, n)
        tmp[i] = Num((dbl) a[i], (dbl) b[i]);
    fft(tmp, n);
    const int n1 = n - 1;
    const Num c = Num(0, -0.25 / n);
    fornr (i, n / 2 + 1) {

```

```

    const int j = (n - i) & n1;
    const Num z0 = sqr(tmp[i]), z1 = sqr(tmp[j]);
    tmp[i] = (z1 - conj(z0)) * c;
    tmp[j] = (z0 - conj(z1)) * c;
}
fft(tmp, n);
forn (i, n)
    r[i] = (T) round(tmp[i].real());
}

void init() { // don't forget to init
    forn(i, N)
        rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (LOG - 1));

    rt[1] = Num(1, 0);
    for (int k = 1, p = 2; k < LOG; k++, p *= 2) {
        const Num x(cos(PI / p), sin(PI / p));
        forab (i, p / 2, p)
            rt[2 * i] = rt[i], rt[2 * i + 1] = rt[i] * x;
    }
}

```

7 FFT by mod and FFT with digits up to 10^6

Num ta[N], tb[N], tf[N], tg[N];

```

const int HALF = 15;

void mult(int *a, int *b, int *r, int n, int mod) {
    int tw = (1 << HALF) - 1;
    forn (i, n) {
        int x = int(a[i] % mod);
        ta[i] = Num(x & tw, x >> HALF);
    }
    forn (i, n) {
        int x = int(b[i] % mod);
        tb[i] = Num(x & tw, x >> HALF);
    }

    fft(ta, n), fft(tb, n);
    forn (i, n) {
        int j = (n - i) & (n - 1);
        Num a1 = (ta[i] + conj(ta[j])) * Num(0.5, 0);
        Num a2 = (ta[i] - conj(ta[j])) * Num(0, -0.5);
        Num b1 = (tb[i] + conj(tb[j])) * Num(0.5 / n, 0);
        Num b2 = (tb[i] - conj(tb[j])) * Num(0, -0.5 / n);
        tf[j] = a1 * b1 + a2 * b2 * Num(0, 1);
        tg[j] = a1 * b2 + a2 * b1;
    }

    fft(tf, n), fft(tg, n);
    forn (i, n) {
        ll aa = ll(tf[i].x + 0.5);
        ll bb = ll(tg[i].x + 0.5);
        ll cc = ll(tf[i].y + 0.5);
        r[i] = int((aa + ((bb % mod) << HALF) + ((cc % mod) << (2 *
↪ HALF))) % mod);
    }
}

int tc[N], td[N];

const int MOD1 = 1.5e9, MOD2 = MOD1 + 1;
void multLL(int *a, int *b, ll *r, int n){
    mult(a, b, tc, n, MOD1), mult(a, b, td, n, MOD2);
    forn(i, n)
        r[i] = tc[i] + (td[i] - tc[i] + (ll)MOD2) * MOD1 % MOD2 *
↪ MOD1;
}

```

3 Data Structures

8 Centroid Decomposition

```

vi g[N];
int d[N], par[N], centroid;
//d and par - in centroid tree

int find(int v, int p, int total) {

```

```

    int size = 1, ok = 1;
    for (int to : g[v])
        if (d[to] == -1 && to != p) {
            int s = find(to, v, total);
            if (s > total / 2) ok = 0;
            size += s;
        }
    if (ok && size > total / 2) centroid = v;
    return size;
}

void calcInComponent(int v, int p, int level) {
    // do something
    for (int to : g[v])
        if (d[to] == -1 && to != p)
            calcInComponent(to, v, level);
}

//fill(d, d + n, -1)
//decompose(0, -1, 0)
void decompose(int root, int parent, int level) {
    find(root, -1, find(root, -1, INF));
    int c = centroid;
    par[c] = parent, d[c] = level;
    calcInComponent(centroid, -1, level);
    for (int to : g[c])
        if (d[to] == -1)
            decompose(to, c, level + 1);
}

```

9 Convex Hull Trick

```

struct Line {
    int k, b;
    Line() {}
    Line(int _k, int _b): k(_k), b(_b) {}
    ll get(int x) { return b + k * 1ll * x; }
    bool operator<(const Line &l) const { return k < l.k; } //
↪ change to > in case of different order
};

// Checks if intersection of (a, b) is on the left from (a, c).
inline bool check(Line a, Line b, Line c) {
    return (a.b - b.b) * 1ll * (c.k - a.k) < (a.b - c.b) * 1ll *
↪ (b.k - a.k);
}

struct Convex {
    vector<Line> st;
    inline void add(Line l) {
        while (sz(st) >= 2 && !check(st[sz(st) - 2], st[sz(st) - 1],
↪ l))
            st.pop_back();
        st.pb(l);
    }
    int get(int x) {
        int l = 0, r = sz(st);
        while (r - l > 1) {
            int m = (l + r) / 2; // change to > in case of different
↪ order
            if (st[m - 1].get(x) < st[m].get(x))
                l = m;
            else
                r = m;
        }
        return l;
    }
    Convex() {}
    Convex(vector<Line> &lines) {
        st.clear();
        for (Line &l : lines)
            add(l);
    }
    Convex(Line line) { st.pb(line); }
    Convex(const Convex &a, const Convex &b) {
        vector<Line> lines;
        lines.resize(sz(a.st) + sz(b.st));
        merge(all(a.st), all(b.st), lines.begin());
        st.clear();
        for (Line &l : lines)

```

```

        add(1);
    }
};

```

10 DSU

```

int pr[N];

int get(int v) {
    return v == pr[v] ? v : pr[v] = get(pr[v]);
}

bool unite(int v, int u) {
    v = get(v), u = get(u);
    if (v == u) return 0;
    pr[u] = v;
    return 1;
}

void init(int n) {
    for (i, n) pr[i] = i;
}

```

11 Fenwick Tree

```

int t[N];

int get(int ind) {
    int res = 0;
    for (; ind >= 0; ind &= (ind + 1), ind--)
        res += t[ind];
    return res;
}

void add(int ind, int n, int val) {
    for (; ind < n; ind |= (ind + 1))
        t[ind] += val;
}

int sum(int l, int r) { // [l, r)
    return get(r - 1) - get(l - 1);
}

```

12 Hash Table

```

using H = ll;
const int HT_SIZE = 1<<20, HT_AND = HT_SIZE - 1, HT_SIZE_ADD =
    HT_SIZE / 100;
H ht[HT_SIZE + HT_SIZE_ADD];
int data[HT_SIZE + HT_SIZE_ADD];

int get(const H &hash){
    int k = ((ll) hash) & HT_AND;
    while (ht[k] && ht[k] != hash) ++k;
    return k;
}

void insert(const H &hash, int x){
    int k = get(hash);
    if (!ht[k]) ht[k] = hash, data[k] = x;
}

bool count(const H &hash){
    int k = get(hash);
    return ht[k] != 0;
}

```

13 Heavy Light Decomposition

```

vi g[N];
int size[N], comp[N], num[N], top[N], pr[N], tin[N], tout[N];
vi t[N], toPush[N], lst[N];
int curPath = 0, curTime = 0;

void pushST(int path, int v, int vl, int vr) {
    if (toPush[path][v] != -1) {
        if (vl != vr - 1)
            for (j, 2)
                toPush[path][2 * v + j] = toPush[path][v];
    }
}

```

```

        else
            t[path][v] = toPush[path][v];
            toPush[path][v] = -1;
    }
}

int getST(int path, int v, int vl, int vr, int ind) {
    pushST(path, v, vl, vr);
    if (vl == vr - 1)
        return t[path][v];
    int vm = (vl + vr) / 2;
    if (ind >= vm)
        return getST(path, 2 * v + 1, vm, vr, ind);
    return getST(path, 2 * v, vl, vm, ind);
}

void setST(int path, int v, int vl, int vr, int l, int r, int val)
    ↪ {
    if (vl >= l && vr <= r) {
        toPush[path][v] = val;
        pushST(path, v, vl, vr);
        return;
    }
    pushST(path, v, vl, vr);
    if (vl >= r || l >= vr)
        return;
    int vm = (vl + vr) / 2;
    setST(path, 2 * v, vl, vm, l, r, val);
    setST(path, 2 * v + 1, vm, vr, l, r, val);
    t[path][v] = min(t[path][2 * v], t[path][2 * v + 1]);
}

bool isUpper(int v, int u) {
    return tin[v] <= tin[u] && tout[v] >= tout[u];
}

int getHLD(int v) {
    return getST(comp[v], 1, 0, sz(t[comp[v]]) / 2, num[v]);
}

int setHLD(int v, int u, int val) {
    int ans = 0, w = 0;
    for (i, 2) {
        while (!isUpper(w = top[comp[v]], u))
            setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, 0, num[v] + 1,
                ↪ val), v = pr[w];
        swap(v, u);
    }
    setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, min(num[v], num[u]),
        ↪ max(num[v], num[u]) + 1, val);
    return ans;
}

void dfs(int v, int p) {
    tin[v] = curTime++;
    size[v] = 1;
    pr[v] = p;
    for (int u : g[v])
        if (u != p) {
            dfs(u, v);
            size[v] += size[u];
        }
    tout[v] = curTime++;
}

void build(int v) {
    if (v == 0 || size[v] * 2 < size[pr[v]])
        top[curPath] = v, comp[v] = curPath, num[v] = 0, curPath++;
    else
        comp[v] = comp[pr[v]], num[v] = num[pr[v]] + 1;
    lst[comp[v]].pb(v);
    for (int u : g[v])
        if (u != pr[v])
            build(u);
}

void initHLD() {
    dfs(0, 0);
    build(0);
}

```

```

    forn (i, curPath) {
        int curSize = 1;
        while (curSize < sz(lst[i]))
            curSize *= 2;
        t[i].resize(curSize * 2);
        toPush[i] = vi(curSize * 2, -1);
        //initialize t[i]
    }
}

```

14 Next Greater in Segment Tree

```

int t[4 * N], tSize = 1;

// Find position > pos with val > x.
int nextGreaterX(int v, int l, int r, int pos, int x) {
    if (r <= pos + 1 || t[v] <= x) return INF;
    if (v >= tSize) return v - tSize;
    int ans = nextGreaterX(2 * v, l, (l + r) / 2, pos, x);
    if (ans == INF)
        ans = nextGreaterX(2 * v + 1, (l + r) / 2, r, pos, x);
    return ans;
}

```

15 Sparse Table

```

int st[N][LOG];
int lg[N];

int get(int l, int r) { // [l, r)
    int curLog = lg[r - l];
    return min(st[l][curLog], st[r - (1 << curLog)][curLog]);
}

```

```

void initSparseTable(int *a, int n) {
    lg[1] = 0;
    forab (i, 2, n + 1) lg[i] = lg[i / 2] + 1;
    forn (i, n) st[i][0] = a[i];
    forn (j, lg[n])
        forn (i, n - (1 << (j + 1)) + 1)
            st[i][j + 1] = min(st[i][j], st[i + (1 << j)][j]);
}

```

16 Fenwick Tree 2D

```

ll a[4][N][N];
int n, m;

inline int f(int x) { return x & ~(x - 1); }

inline void add(int k, int x, int y, ll val) {
    for (; x <= n; x += f(x))
        for (int j = y; j <= m; j += f(j))
            a[k][x][j] += val;
}

inline ll get(int k, int x, int y) {
    ll s = 0;
    for (; x > 0; x -= f(x))
        for (int j = y; j > 0; j -= f(j))
            s += a[k][x][j];
    return s;
}

inline ll get(int x, int y) {
    return ll(x + 1) * (y + 1) * get(0, x, y) - (y + 1) * get(1, x,
↪ y)
        - (x + 1) * get(2, x, y) + get(3, x, y);
}

inline void add(int x, int y, ll val) {
    add(0, x, y, val);
    add(1, x, y, val * x);
    add(2, x, y, val * y);
    add(3, x, y, val * x * y);
}

inline ll get(int x_1, int y_1, int x_2, int y_2) {
    return get(x_2, y_2) - get(x_1 - 1, y_2) - get(x_2, y_1 - 1) +
↪ get(x_1 - 1, y_1 - 1);
}

```

```

}

// Adds val to corresponding rectangle
inline void add(int x_1, int y_1, int x_2, int y_2, ll val) {
    add(x_1, y_1, val);
    if (y_2 < m) add(x_1, y_2 + 1, -val);
    if (x_2 < n) add(x_2 + 1, y_1, -val);
    if (x_2 < n && y_2 < m) add(x_2 + 1, y_2 + 1, val);
}

```

17 Segment Tree 2D

```

int tSize = (1 << 10);

struct Node1D {
    Node1D *l, *r;
    ll val, need;
    Node1D(): l(nullptr), r(nullptr), val(0), need(0) {}
    inline void norm() {
        if(!l) l = new Node1D();
        if(!r) r = new Node1D();
    }
    ll get(int ql, int qr, int vl = 0, int vr = tSize) {
        if(vl >= qr || ql >= vr)
            return 0;
        if(ql <= vl && vr <= qr)
            return val;
        int a = max(vl, ql), b = min(vr, qr), vm = (vl + vr) / 2;
        norm();
        return l->get(ql, qr, vl, vm) + r->get(ql, qr, vm, vr) + need
↪ * ll(b - a);
    }
    void add(int ql, int qr, int x, int vl = 0, int vr = tSize) {
        if (ql >= vr || vl >= qr)
            return;
        if (ql <= vl && vr <= qr){
            need += x;
            val += x * ll(vr - vl);
            return;
        }
        int vm = (vl + vr) / 2;
        norm();
        l->add(ql, qr, x, vl, vm), r->add(ql, qr, x, vm, vr);
        val = l->val + r->val + need * (vr - vl);
    }
};

```

```

struct Node2D {
    Node2D *l, *r;
    Node1D *val, *need;
    Node2D(): l(nullptr), r(nullptr), val(new Node1D()), need(new
↪ Node1D()) {}
    inline void norm() {
        if(!l) l = new Node2D();
        if(!r) r = new Node2D();
    }
    ll get(int ql0, int qr0, int ql1, int qr1, int vl = 0, int vr =
↪ tSize) {
        if(vl >= qr0 || ql0 >= vr)
            return 0;
        if(ql0 <= vl && vr <= qr0)
            return val->get(ql1, qr1);
        int a = max(vl, ql0), b = min(vr, qr0), vm = (vl + vr) / 2;
        norm();
        return l->get(ql0, qr0, ql1, qr1, vl, vm) + r->get(ql0, qr0,
↪ ql1, qr1, vm, vr) + need->get(ql1, qr1) * ll(b - a);
    }
    void add(int ql0, int qr0, int ql1, int qr1, int x, int vl = 0,
↪ int vr = tSize) {
        if (ql0 >= vr || vl >= qr0)
            return;
        if (ql0 <= vl && vr <= qr0){
            need->add(ql1, qr1, x);
            val->add(ql1, qr1, x * ll(vr - vl));
            return;
        }
        int a = max(ql0, vl), b = min(qr0, vr), vm = (vl + vr) / 2;
        norm();
        l->add(ql0, qr0, ql1, qr1, x, vl, vm), r->add(ql0, qr0, ql1,
↪ qr1, x, vm, vr);
    }
};

```



```

    val->add(ql1, qr1, x * ll(b - a));
}
};

```

4 Dynamic Programming

18 LIS

```

int longestIncreasingSubsequence(vi a) {
    int n = sz(a);
    vi d(n + 1, INF);
    d[0] = -INF;
    forn (i, n)
        *upper_bound(all(d), a[i]) = a[i];
    fornr (i, n + 1) if (d[i] != INF) return i;
    return 0;
}

```

19 DP tree

```

int dp[N][N], a[N];
vi g[N];

int dfs(int v, int n) {
    forn (i, n + 1)
        dp[v][i] = -INF;
    dp[v][1] = a[v];
    int curSz = 1;
    for (int to : g[v]) {
        int toSz = dfs(to, n);
        for (int i = curSz; i >= 1; i--)
            fornr (j, toSz + 1)
                dp[v][i + j] = max(dp[v][i + j], dp[v][i] + dp[to][j]);
        curSz += toSz;
    }
    return curSz;
}

```

20 Masks tricks

```

int dp[(1 << MASK)][MASK];

void calcDP(int n) {
    forn(mask, 1 << n) {
        dp[mask][n] = 1;
        fornr(i, n) {
            dp[mask][i] = dp[mask][i + 1];
            if ((1 << i) & mask)
                dp[mask][i] += dp[mask ^ (1 << i)][i + 1];
        }
    }
}

```

5 Flows

21 Utilities

```

vi g[N];

// for directed unweighted graph
struct Edge {
    int v, u, c, f;
    Edge() {}
    Edge(int _v, int _u, int _c): v(_v), u(_u), c(_c), f(0) {}
};

vector<Edge> edges;

inline void addFlow(int e, int flow) {
    edges[e].f += flow, edges[e ^ 1].f -= flow;
}

inline void addEdge(int v, int u, int c) {
    g[v].pb(sz(edges)), edges.pb(Edge(v, u, c));
    g[u].pb(sz(edges)), edges.pb(Edge(u, v, 0)); // for undirected 0
    ↪ should be c
}

```

22 Ford-Fulkerson

```

int used[N], pr[N];
int curTime = 1;

int dfs(int v, int can, int toPush, int t) {
    if (v == t) return can;
    used[v] = curTime;
    for (int edge : g[v]) {
        auto &e = edges[edge];
        if (used[e.u] != curTime && e.c - e.f >= toPush) {
            int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
            if (flow > 0) {
                addFlow(edge, flow), pr[e.u] = edge;
                return flow;
            }
        }
    }
    return 0;
}

```

```

int fordFulkerson(int s, int t) {
    int ansFlow = 0, flow = 0;
    // Without scaling
    while ((flow = dfs(s, INF, 1, t)) > 0)
        ansFlow += flow, curTime++;
    // With scaling
    fornr (i, INF_LOG)
        for (curTime++; (flow = dfs(s, INF, (1 << i), t)) > 0;
            ↪ curTime++)
            ansFlow += flow;
    return ansFlow;
}

```

23 Dinic

```

int pr[N], d[N], q[N], first[N];

int dfs(int v, int can, int toPush, int t) {
    if (v == t) return can;
    int sum = 0;
    for (; first[v] < (int) g[v].size(); first[v]++) {
        auto &e = edges[g[v][first[v]]];
        if (d[e.u] != d[v] + 1 || e.c - e.f < toPush) continue;
        int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
        addFlow(g[v][first[v]], flow);
        can -= flow, sum += flow;
        if (!can) return sum;
    }
    return sum;
}

bool bfs(int n, int s, int t, int curPush) {
    forn (i, n) d[i] = INF, first[i] = 0;
    int head = 0, tail = 0;
    q[tail++] = s;
    d[s] = 0;
    while (tail - head > 0) {
        int v = q[head++];
        for (int edge : g[v]) {
            auto &e = edges[edge];
            if (d[e.u] > d[v] + 1 && e.c - e.f >= curPush)
                d[e.u] = d[v] + 1, q[tail++] = e.u;
        }
    }
    return d[t] != INF;
}

```

```

int dinic(int n, int s, int t) {
    int ansFlow = 0;
    // Without scaling
    while (bfs(n, s, t, 1))
        ansFlow += dfs(s, INF, 1, t);
    // With scaling
    fornr (j, INF_LOG)
        while (bfs(n, s, t, 1 << j))
            ansFlow += dfs(s, INF, 1 << j, t);
    return ansFlow;
}

```


24 Hungarian

```
const int INF = 1e9;
int a[N][N];

// min = sum of a[pa[i],i]
// you may optimize speed by about 15%, just change all vectors to
↪ static arrays
vi Hungarian(int n) {
    vi pa(n + 1, -1), row(n + 1, 0), col(n + 1, 0), la(n + 1);
    forn (k, n) {
        vi u(n + 1, 0), d(n + 1, INF);
        pa[n] = k;
        int l = n, x;
        while ((x = pa[l]) != -1) {
            u[l] = 1;
            int minn = INF, tmp, l0 = 1;
            forn (j, n)
                if (!u[j]) {
                    if ((tmp = a[x][j] + row[x] + col[j]) < d[j])
                        d[j] = tmp, la[j] = 10;
                    if (d[j] < minn)
                        minn = d[j], l = j;
                }
            forn (j, n + 1)
                if (u[j])
                    col[j] += minn, row[pa[j]] -= minn;
                else
                    d[j] -= minn;
        }
        while (l != n)
            pa[l] = pa[la[l]], l = la[l];
    }
    return pa;
}
```

25 Min Cost Max Flow

```
int pr[N], in[N], q[N * M], used[N], d[N], pot[N];
vi g[N];

struct Edge {
    int v, u, c, f, w;
    Edge() {}
    Edge(int _v, int _u, int _c, int _w): v(_v), u(_u), c(_c),
↪ f(0), w(_w) {}
};

vector<Edge> edges;

inline void addFlow(int e, int flow) {
    edges[e].f += flow, edges[e ^ 1].f -= flow;
}

inline void addEdge(int v, int u, int c, int w) {
    g[v].pb(sz(edges)), edges.pb(Edge(v, u, c, w));
    g[u].pb(sz(edges)), edges.pb(Edge(u, v, 0, -w));
}

int dijkstra(int n, int s, int t) {
    forn (i, n) used[i] = 0, d[i] = INF;
    d[s] = 0;
    while (1) {
        int v = -1;
        forn (i, n)
            if (!used[i] && (v == -1 || d[v] > d[i]))
                v = i;
        if (v == -1 || d[v] == INF) break;
        used[v] = 1;
        for (int edge : g[v]) {
            auto &e = edges[edge];
            int w = e.w + pot[v] - pot[e.u];
            if (e.c > e.f && d[e.u] > d[v] + w)
                d[e.u] = d[v] + w, pr[e.u] = edge;
        }
    }
    if (d[t] == INF) return d[t];
    forn (i, n) pot[i] += d[i];
    return pot[t];
}
```

```
int fordBellman(int n, int s, int t) {
    forn (i, n) d[i] = INF;
    int head = 0, tail = 0;
    d[s] = 0, q[tail++] = s, in[s] = 1;
    while (tail - head > 0) {
        int v = q[head++];
        in[v] = 0;
        for (int edge : g[v]) {
            auto &e = edges[edge];
            if (e.c > e.f && d[e.u] > d[v] + e.w) {
                d[e.u] = d[v] + e.w;
                pr[e.u] = edge;
                if (!in[e.u])
                    in[e.u] = 1, q[tail++] = e.u;
            }
        }
    }
    return d[t];
}

int minCostMaxFlow(int n, int s, int t) {
    int ansFlow = 0, ansCost = 0, dist;
    while ((dist = dijkstra(n, s, t)) != INF) {
        int curFlow = INF;
        for (int cur = t; cur != s; cur = edges[pr[cur]].v)
            curFlow = min(curFlow, edges[pr[cur]].c -
↪ edges[pr[cur]].f);
        for (int cur = t; cur != s; cur = edges[pr[cur]].v)
            addFlow(pr[cur], curFlow);
        ansFlow += curFlow;
        ansCost += curFlow * dist;
    }
    return ansCost;
}
```

6 Games

26 Retrograde Analysis

```
int win[N], lose[N], outDeg[N];
vi rg[N];

void retro(int n) {
    queue<int> q;
    forn (i, n)
        if (!outDeg[i])
            lose[i] = 1, q.push(i);
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int to : rg[v])
            if (lose[v]) {
                if (!win[to])
                    win[to] = 1, q.push(to);
            } else {
                outDeg[to]--;
                if (!outDeg[to])
                    lose[to] = 1, q.push(to);
            }
    }
}
```

7 Geometry

27 ClosestPoints (SweepLine)

```
struct Pnt {
    int x, y, i;
    bool operator <(const Pnt &p) const { return mp(y, i) < mp(p.y,
↪ p.i); }
};

ll d2 = 8e18, d = (ll) sqrt(d2) + 1;
Pnt p[N];

inline ll sqr(int x){
```

```

    return (ll)x * x;
}

inline void relax(const Pnt &a, const Pnt &b){
    ll tmp = sqr(a.x - b.x) + sqr(a.y - b.y);
    if (tmp < d2)
        d2 = tmp, d = (ll)(sqrt(d2) + 1 - 1e-9); // round up
}

inline bool xless(const Pnt &a, const Pnt &b){
    return a.x < b.x;
}

int main() {
    int n;
    scanf("%d", &n);
    forn(i, n)
        scanf("%d%d", &p[i].x, &p[i].y), p[i].i = i;
    sort(p, p + n, xless);

    set<Pnt> s;
    int l = 0;
    forn(r, n){
        set<Pnt>::iterator it_r = s.lower_bound(p[r]), it_l = it_r;
        for (; it_r != s.end() && it_r->y - p[r].y < d; ++it_r)
            relax(*it_r, p[r]);
        while (it_l != s.begin() && p[r].y - (--it_l)->y < d)
            relax(*it_l, p[r]);
        s.insert(p[r]);
        while (l <= r && p[r].x - p[l].x >= d)
            s.erase(p[l++]);
    }
    printf("%.9f\n", sqrt(d2));
    return 0;
}

```

28 ConvexHull

```

using vpnt = vector<Pnt>;

inline bool byAngle(const Pnt& a, const Pnt& b) {
    dbl x = a % b;
    return eq(x, 0) ? a.len2() < b.len2() : x < 0;
}

vpnt convexHull(vpnt p) {
    int n = sz(p);
    assert(n > 0);
    swap(p[0], *min_element(all(p)));
    forab(i, 1, n)
        p[i] = p[i] - p[0];
    sort(p.begin() + 1, p.end(), byAngle);

    /* To keep 180 angles (1) (2)
       (1):
       int k = p.size() - 1;
       while(k > 0 && eq((p[k] - p.back()) % p.back(), 0))
           --k;
       reverse(pi.begin() + k, pi.end());*/

    int rn = 0;
    vpnt r(n);
    r[rn++] = p[0];
    forab(i, 1, n){
        Pnt q = p[i] + p[0];
        while(rn >= 2 && geq((r[rn] - r[rn - 1]) % (q - r[rn - 2]), 0)) // (2) ge
            --rn;
        r[rn++] = q;
    }
    r.resize(rn);
    return r;
}

```

29 GeometryBase

```

const dbl EPS = 1e-9;
const int PREC = 20;
inline bool eq(dbl a, dbl b) { return abs(a-b)<=EPS; }
inline bool gr(dbl a, dbl b) { return a>b+EPS; }

```

```

inline bool geq(dbl a, dbl b) { return a>=b-EPS; }
inline bool ls(dbl a, dbl b) { return a<b-EPS; }
inline bool leq(dbl a, dbl b) { return a<=b+EPS; }

struct Pnt {
    dbl x,y;
    Pnt(): x(0), y(0) {}
    Pnt(dbl xx, dbl yy): x(xx), y(yy) {}

    inline Pnt operator +(const Pnt &p) const { return Pnt(x +
↪ p.x, y + p.y); }
    inline Pnt operator -(const Pnt &p) const { return Pnt(x -
↪ p.x, y - p.y); }
    inline dbl operator *(const Pnt &p) const { return x * p.x + y
↪ * p.y; } // ll
    inline dbl operator %(const Pnt &p) const { return x * p.y - y
↪ * p.x; } // ll

    inline Pnt operator *(dbl k) const { return Pnt(x * k, y * k);
↪ }
    inline Pnt operator /(dbl k) const { return Pnt(x / k, y / k);
↪ }
    inline Pnt operator -() const { return Pnt(-x, -y); }

    inline void operator +=(const Pnt &p) { x += p.x, y += p.y; }
    inline void operator -=(const Pnt &p) { x -= p.x, y -= p.y; }
    inline void operator *=(dbl k) { x*=k, y*=k; }

    inline bool operator ==(const Pnt &p) const { return
↪ abs(x-p.x)<=EPS && abs(y-p.y)<=EPS; }
    inline bool operator !=(const Pnt &p) const { return
↪ abs(x-p.x)>EPS || abs(y-p.y)>EPS; }
    inline bool operator <(const Pnt &p) const { return
↪ abs(x-p.x)<=EPS ? y<p.y-EPS : x<p.x; }

    inline dbl angle() const { return atan2(y, x); } // ld
    inline dbl len2() const { return x*x+y*y; } // ll
    inline dbl len() const { return sqrt(x*x+y*y); } // ll, ld
    inline Pnt getNorm() const {
        auto l = len();
        return Pnt(x/l, y/l);
    }
    inline void normalize() {
        auto l = len();
        x/=l, y/=l;
    }

    inline Pnt getRot90() const { //counter-clockwise
        return Pnt(-y, x);
    }
    inline Pnt getRot(dbl a) const { // ld
        dbl si = sin(a), co = cos(a);
        return Pnt(x*co - y*si, x*si + y*co);
    }

    inline void read() {
        int xx, yy;
        cin >> xx >> yy;
        x = xx, y = yy;
    }
    inline void write() const{
        cout << fixed << (double)x << " " << (double)y << '\n';
    }
    Pnt multBenq(const Pnt& r) const {
        return Pnt(x*r.x - y*r.y, y*r.x + x*r.y);
    }
};

struct Line{
    dbl a, b, c;
    Line(): a(0), b(0), c(0) {}
    // normalizes
    Line(dbl aa, dbl bb, dbl cc) {
        dbl norm = sqrt(aa * aa + bb * bb);
        aa /= norm, bb /= norm, cc /= norm;
        a = aa, b = bb, c = cc;
    }

    Line(const Pnt &A, const Pnt &p){ // it normalizes (a,b),
↪ important in d(), normalToP()

```

```

    Pnt n = (p-A).getRot90().getNorm();
    a = n.x, b = n.y, c = -(a * A.x + b * A.y);
}

inline dbl d(const Pnt &p) const { return a*p.x + b*p.y + c; }
inline Pnt no() const {return Pnt(a, b);}
inline Pnt normalToP(const Pnt &p) const { return Pnt(a,b) *
↪ (a*p.x + b*p.y + c); }

inline void write() const{
    cout << fixed << (double)a << " " << (double)b << " " <<
↪ (double)c << '\n';
}
};

```

30 GeometryInterTangent

```

inline dbl sqr(dbl x) { return x * x; }

struct Circle {
    Pnt p;
    dbl r;
};

Pnt tangent(Pnt x, Circle y, int t = 0) {
    y.r = abs(y.r); // abs needed because internal calls y.s < 0
    if (y.r == 0) return y.p;
    dbl d = (x - y.p).len();
    Pnt a = (x - y.p) * pow(y.r / d, 2) + y.p;
    Pnt b = multBenq((x - y.p).unit() * sqrtl(d * d - y.r * y.r) /
↪ d * y.r, Pnt(0, 1));
    return t == 0 ? a+b : a-b;
}

vector<pair<Pnt,Pnt>> external(const Circle &x, const Circle &y)
↪ {
    vector<pair<Pnt,Pnt>> v;
    if (x.r == y.r) {
        Pnt tmp = ((x.p-y.p).getNorm()*x.r).bmul(Pnt(0,1));
        v.pb(mp(x.p+tmp,y.p+tmp));
        v.pb(mp(x.p-tmp,y.p-tmp));
    } else {
        Pnt p = (x.p*y.r-y.p*x.r)/(y.r-x.r);
        forn(i,2) v.pb(mp(tangent(p,x,i),tangent(p,y,i)));
    }
    return v;
}

vector<pair<Pnt,Pnt>> internal(const Circle &x, const Circle &y)
↪ {
    return external({x.p,-x.r},y); }

```

```

vector<Pnt> line_line(const Line &l, const Line &m){
    dbl z = m.a * l.b - l.a * m.b;
    dbl x = m.c * l.b - l.c * m.b;
    dbl y = m.c * l.a - l.c * m.a;
    if(fabs(z) > EPS)
        return {Pnt(-x/z, y/z)};
    else if(fabs(x) > EPS || fabs(y) > EPS)
        return {}; // parallel lines
    else
        return {Pnt(0, 0), Pnt(0, 0)}; // same lines
}

vector<Pnt> circle_line(const Circle &c, const Line &l){
    dbl d = l.d(c.p);
    if(fabs(d) > c.r + EPS)
        return {};
    if(fabs(fabs(d) / c.r - 1) < EPS) {
        return {c.p - l.no() * d};
    } else {
        dbl s = sqrt(fabs(sqr(c.r) - sqr(d)));
        return {c.p - l.no() * d + l.no().getRot90() * s,
            c.p - l.no() * d - l.no().getRot90() * s};
    }
}

vector<Pnt> circle_circle(const Circle &x, const Circle &y) {
    dbl d = (x.p-y.p).len(), a = x.r, b = y.r;
    if (eq(d, 0)) { assert(a != b); return {}; }
}

```

```

    dbl C = (a*a+d*d-b*b)/(2*a*d);
    if (abs(C) > 1+EPS) return {};
    dbl S = sqrtl(max(1-C*C,(dbl)0)); Pnt tmp = (y.p-x.p)/d*x.r;
    if (eq(S, 0)) return {x.p+tmp.bmul(Pnt(C,0))};
    return {x.p+tmp.bmul(Pnt(C,S)),x.p+tmp.bmul(Pnt(C,-S))};
}

dbl circle_isect_area(const Circle &x, const Circle &y) {
    dbl d = (x.p-y.p).len(), a = x.r, b = y.r; if (a < b)
↪ swap(a,b);
    if (geq(d, a+b)) return 0;
    if (leq(d, a-b)) return PI*b*b;
    dbl ca = acosl((a*a+d*d-b*b)/(2*a*d)), cb =
↪ acosl((b*b+d*d-a*a)/(2*b*d));
    return (ca*a*a-0.5*a*a*sin(ca*2))+(cb*b*b-0.5*b*b*sin(cb*2));
}

// Squared distance between point p and segment [a..b]
dbl dist2(Pnt p, Pnt a, Pnt b){
    if ((p - a) * (b - a) < 0) return (p - a).len2();
    if ((p - b) * (a - b) < 0) return (p - b).len2();
    dbl d = fabs((p - a) % (b - a));
    return d * d / (b - a).len2();
}

```

31 GeometrySimple

```

int sign(dbl a) { return (a > EPS) - (a < -EPS); }

// Checks, if point is inside the segment
inline bool inSeg(const Pnt &p, const Pnt &a, const Pnt &b) {
    return eq((p - a) % (p - b), 0) && leq((p - a) * (p - b), 0);
}

// Checks, if two intervals (segments without ends) intersect AND
↪ do not lie on the same line
inline bool subIntr(const Pnt &a, const Pnt &b, const Pnt &c,
↪ const Pnt &d){
    return
        sign((b - a) % (c - a)) * sign((b - a) % (d - a)) ==
↪ -1 &&
        sign((d - c) % (a - c)) * sign((d - c) % (b - c)) ==
↪ -1;
}

// Checks, if two segments (ends are included) has an intersection
inline bool checkSegInter(const Pnt &a, const Pnt &b, const Pnt
↪ &c, const Pnt &d){
    return inSeg(c, a, b) || inSeg(d, a, b) || inSeg(a, c, d) ||
↪ inSeg(b, c, d) || subIntr(a, b, c, d);
}

inline dbl area(vector<Pnt> p){
    dbl s = 0;
    int n = sz(p);
    p.pb(p[0]);
    forn(i, n)
        s += p[i + 1] % p[i];
    p.pop_back();
    return abs(s) / 2;
}

// Check if point p is inside polygon <n, q[]>
int containsSlow(Pnt p, Pnt *z, int n){
    int cnt = 0;
    forn(j, n){
        Pnt a = z[j], b = z[(j + 1) % n];
        if (inSeg(p, a, b))
            return -1; // border
        if (min(a.y, b.y) - EPS <= p.y && p.y < max(a.y, b.y) -
↪ EPS)
            cnt += (p.x < a.x + (p.y - a.y) * (b.x - a.x) / (b.y
↪ - a.y));
    }
    return cnt & 1; // 0 = outside, 1 = inside
}

//for convex polygon
//assume polygon is counterclockwise-ordered

```

```

bool containsFast(Pnt p, Pnt *z, int n) {
    Pnt o = z[0];
    if(gr((p - o) % (z[1] - o), 0) || ls((p - o) % (z[n - 1] -
↪ o), 0))
        return 0;
    int l = 0, r = n - 1;
    while(r - l > 1){
        int m = (l + r) / 2;
        if(gr((p - o) % (z[m] - o), 0))
            r = m;
        else
            l = m;
    }
    return leq((p - z[l]) % (z[r] - z[l]), 0);
}

// Checks, if point "p" is in the triangle "abc" IFF triangle in
↪ CCW order
inline int isInTr(const Pnt &p, const Pnt &a, const Pnt &b, const
↪ Pnt &c){
    return
        gr((b - a) % (p - a), 0) &&
        gr((c - b) % (p - b), 0) &&
        gr((a - c) % (p - c), 0);
}

```

8 Graphs

32 2-SAT

```

// VAR - 2 * vars
int cntVar = 0, val[VAR], usedSat[VAR], comp[VAR];
vi topsortSat;

vi g[VAR], rg[VAR];

inline int newVar() {
    cntVar++;
    return (cntVar - 1) * 2;
}

inline int Not(int v) { return v ^ 1; }

inline void Implies(int v1, int v2) { g[v1].pb(v2),
↪ rg[v2].pb(v1); }

inline void Or(int v1, int v2) { Implies(Not(v1), v2),
↪ Implies(Not(v2), v1); }

inline void Nand(int v1, int v2) { Or(Not(v1), Not(v2)); }

inline void setTrue(int v) { Implies(Not(v), v); }

void dfs1(int v) {
    usedSat[v] = 1;
    for (int to : g[v])
        if (!usedSat[to]) dfs1(to);
    topsortSat.pb(v);
}

void dfs2(int v, int c) {
    comp[v] = c;
    for (int to : rg[v])
        if (!comp[to]) dfs2(to, c);
}

int getVal(int v) { return val[v]; }

// cntVar
bool solveSat() {
    forn(i, 2 * cntVar) usedSat[i] = 0;
    forn(i, 2 * cntVar)
        if (!usedSat[i]) dfs1(i);
    reverse(all(topsortSat));
    int c = 0;
    for (int v : topsortSat)
        if (!comp[v]) dfs2(v, ++c);
    forn(i, cntVar) {
        if (comp[2 * i] == comp[2 * i + 1]) return false;

```

```

        if (comp[2 * i] < comp[2 * i + 1]) val[2 * i + 1] = 1;
        else val[2 * i] = 1;
    }
    return true;
}

```

33 Bridges

```

int up[N], tIn[N], timer;
vector<vi> comps;
vi st;

struct Edge {
    int to, id;
    Edge(int _to, int _id) : to(_to), id(_id) {}
};

vector<Edge> g[N];

void newComp(int size = 0) {
    comps.emplace_back(); // new empty
    while (sz(st) > size) {
        comps.back().pb(st.back());
        st.pop_back();
    }
}

void findBridges(int v, int parentEdge = -1) {
    if (up[v]) // visited
        return;
    up[v] = tIn[v] = ++timer;
    st.pb(v);
    for (Edge e : g[v]) {
        if (e.id == parentEdge)
            continue;
        int u = e.to;
        if (!tIn[u]) {
            int size = sz(st);
            findBridges(u, e.id);
            if (up[u] > tIn[v])
                newComp(size);
        }
        up[v] = min(up[v], up[u]);
    }
}

// after find_bridges newComp() for root
void run(int n) {
    forn(i, n)
        if (!up[i]) {
            findBridges(i);
            newComp();
        }
}

```

34 Cactus

```

int used[N];

struct Edge {
    ll l;
    Edge() {}
    Edge(int _l): l(_l) {}
};

vector<pair<int, Edge>> g[N], rev[N], path;
pair<int, Edge> pr[N];

void dfsInit(int v, int p, Edge prE) {
    used[v] = 1;
    pr[v] = mp(p, prE);
    for (auto e : g[v]) {
        int u = e.fst;
        if (u == p)
            continue;
        if (used[u] == 1)
            rev[u].pb(mp(v, e.snd));
        else if (used[u] != 2)
            dfsInit(u, v, e.snd);
    }
}

```

```

    used[v] = 2;
}

void calc(int v) {
    used[v] = 1;
    for (auto e : rev[v]) {
        path.clear();
        int u = e.fst;
        while (u != v) {
            calc(u);
            path.pb(mp(u, pr[u].snd));
            u = pr[u].fst;
        }
        // Calculate answer for cycle -- path and vertex v
    }
    for (auto e : g[v])
        if (!used[e.fst] && e.fst != pr[v].fst) {
            calc(e.fst);
            // Update answer for tree edges
        }
}

```

35 Cut Points

```

bool used[M];
int tIn[N], timer, isCut[N], color[M], compCnt;
vi st;

```

```

struct Edge {
    int to, id;
    Edge(int _to, int _id) : to(_to), id(_id) {}
};

```

```

vector<Edge> g[N];

```

```

int dfs(int v, int parent = -1) {
    tIn[v] = ++timer;
    int up = tIn[v], x = 0, y = (parent != -1);
    for (Edge p : g[v]) {
        int u = p.to, id = p.id;
        if (id != parent) {
            int t, size = sz(st);
            if (!used[id])
                used[id] = 1, st.push_back(id);
            if (!tIn[u]) { // not visited yet
                t = dfs(u, id);
                if (t >= tIn[v]) {
                    ++x, ++compCnt;
                    while (sz(st) != size) {
                        color[st.back()] = compCnt;
                        st.pop_back();
                    }
                }
            } else
                t = tIn[u];
            up = min(up, t);
        }
    }
    if (x + y >= 2)
        isCut[v] = 1; // v is cut vertex
    return up;
}

```

36 Dominator Tree

```

// clean: forn(i, n+1)!!!
vi adj[N], ans[N]; // input edges, edges of dominator tree
vi radj[N], child[N], sdomChild[N];
int label[N], rlabel[N], sdom[N], dom[N], co = 0;
int par[N], bes[N];
int get(int x) { // DSU with path compression
    // get vertex with smallest sdom on path to root
    if (par[x] != x) {
        int t = get(par[x]); par[x] = par[par[x]];
        if (sdom[t] < sdom[bes[x]]) bes[x] = t;
    }
    return bes[x];
}

void dfs(int x) { // create DFS tree
    label[x] = ++co; rlabel[co] = x;
}

```

```

sdom[co] = par[co] = bes[co] = co;
for(auto y : adj[x]) {
    if (!label[y]) {
        dfs(y); child[label[x]].pb(label[y]);
        radj[label[y]].pb(label[x]);
    }
}

void init(int root) {
    dfs(root);
    for(int i = co; i >= 1; i--) {
        for(auto j : radj[i]) sdom[i] = min(sdom[i], sdom[get(j)]);
        if (i > 1) sdomChild[sdom[i]].pb(i);
        for(auto j : sdomChild[i]) {
            int k = get(j);
            if (sdom[j] == sdom[k]) dom[j] = sdom[j];
            else dom[j] = k;
        }
        for(auto j : child[i]) par[j] = i;
    }
    forab(i, 2, co+1) {
        if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
        ans[rlabel[dom[i]]].pb(rlabel[i]);
    }
}

```

37 Eulerian Cycle

```

struct Edge {
    int to, used;
    Edge(): to(-1), used(0) {}
    Edge(int v): to(v), used(0) {}
};

```

```

vector<Edge> edges;
vi g[N], res, ptr;

```

// don't forget to clear ptr!

```

void dfs(int v) {
    for(; ptr[v] < sz(g[v]);) {
        int id = g[v][ptr[v]++];
        if (!edges[id].used) {
            edges[id].used = edges[id ^ 1].used = 1;
            dfs(edges[id].to);
            res.pb(id); // edges
        }
    }
    res.pb(v); // res contains vertices
}

```

38 Euler Tour Tree

```

mt19937 rng(239);

```

```

struct Edge {
    int v, u;
    Edge(int _v, int _u): v(_v), u(_u) {}
};

```

```

struct Node {
    Node *l, *r, *p;
    Edge e;
    int y, size;
    Node(Edge _e): l(nullptr), r(nullptr), p(this), e(_e), y(rng()),
        size(1) {}
};

```

```

inline int getSize(Node* root) { return root ? root->size : 0; }

inline void recalc(Node* root) { root->size = getSize(root->l) +
    ↪ getSize(root->r) + 1; }

```

```

set<pair<int, Node*>> edges[N];

```

```

Node* merge(Node *a, Node *b) {
    if (!a) return b;
    if (!b) return a;
    if (a->y < b->y) {
        a->r = merge(a->r, b);
        if (a->r) a->r->p = a;
        recalc(a);
    }
}

```

```

    return a;
}
b->l = merge(a, b->l);
if (b->l) b->l->p = b;
recalc(b);
return b;
}

void split(Node *root, Node *&a, Node *&b, int size) {
    if (!root) {
        a = b = nullptr;
        return;
    }
    int lSize = getSize(root->l);
    if (lSize >= size) {
        split(root->l, a, root->l, size);
        if (root->l) root->l->p = root;
        b = root, b->p = b;
    } else {
        split(root->r, root->r, b, size - lSize - 1);
        if (root->r) root->r->p = root;
        a = root, a->p = a;
        a->p = a;
    }
    recalc(root);
}

inline Node* rotate(Node* root, int k) {
    if (k == 0) return root;
    Node *l, *r;
    split(root, l, r, k);
    return merge(r, l);
}

inline pair<Node*, int> goUp(Node* root) {
    int pos = getSize(root->l);
    while (root->p != root)
        pos += (root->p->r == root ? getSize(root->p->l) + 1 : 0),
        ↪ root = root->p;
    return mp(root, pos);
}

inline Node* deleteFirst(Node* root) {
    Node* a;
    split(root, a, root, 1);
    edges[a->e.v].erase(mp(a->e.u, a));
    return root;
}

inline Node* getNode(int v, int u) {
    return edges[v].lower_bound(mp(u, nullptr))->snd;
}

inline void cut(int v, int u) {
    auto pV = goUp(getNode(v, u));
    auto pU = goUp(getNode(u, v));
    int l = min(pV.snd, pU.snd), r = max(pV.snd, pU.snd);
    Node *a, *b, *c;
    split(pV.fst, a, b, l);
    split(b, b, c, r - l);
    deleteFirst(b);
    merge(a, deleteFirst(c));
}

inline pair<Node*, int> getRoot(int v) {
    return !sz(edges[v]) ? mp(nullptr, 0) :
    ↪ goUp(edges[v].begin()->snd);
}

inline Node* makeRoot(int v) {
    auto root = getRoot(v);
    return rotate(root.fst, root.snd);
}

inline Node* makeEdge(int v, int u) {
    Node* e = new Node(Edge(v, u));
    edges[v].insert(mp(u, e));
    return e;
}

```

```

inline void link(int v, int u) {
    Node *vN = makeRoot(v), *uN = makeRoot(u);
    merge(merge(merge(vN, makeEdge(v, u)), uN), makeEdge(u, v));
}

```

39 Hamilton Cycle

```

// DP in  $O(n \cdot 2^n)$  for Ham cycle
vi g[MASK];
int adj[MASK], dp[1 << MASK];

vi hamiltonCycle(int n) {
    fill(dp, dp + (1 << n), 0);
    forn (v, n) {
        adj[v] = 0;
        for (int to : g[v])
            adj[v] |= (1 << to);
    }
    dp[1] = 1;
    forn (mask, (1 << n))
        forn(v, n)
            if (mask & (1 << v) && dp[mask ^ (1 << v)] & adj[v])
                dp[mask] |= (1 << v);
    vi ans;
    int mask = (1 << n) - 1, v;
    if (dp[mask] & adj[0]) {
        forab (i, 1, n)
            if ((1 << i) & (mask & adj[0]))
                v = i;
        ans.pb(v);
        mask ^= (1 << v);
        while(v) {
            forn(i, n)
                if ((dp[mask] & (1 << i)) && (adj[i] & (1 << v))) {
                    v = i;
                    break;
                }
            mask ^= (1 << v);
            ans.pb(v);
        }
        return ans;
    }
}

```

40 Karp with cycle

```

int d[N][N], p[N][N];
vi g[N], ans;

struct Edge {
    int a, b, w;
    Edge(int _a, int _b, int _w): a(_a), b(_b), w(_w) {}
};

vector<Edge> edges;

void fordBellman(int s, int n) {
    forn (i, n + 1)
        forn (j, n + 1)
            d[i][j] = INF;
    d[0][s] = 0;
    forab (i, 1, n + 1)
        for (auto &e : edges)
            if (d[i - 1][e.a] < INF && d[i][e.b] > d[i - 1][e.a] + e.w)
                ↪ d[i][e.b] = d[i - 1][e.a] + e.w, p[i][e.b] = e.a;
}

ld karp(int n) {
    int s = n++;
    forn (i, n - 1)
        g[s].pb(sz(edges)), edges.pb(Edge(s, i, 0));
    fordBellman(s, n);
    ld ansValue = INF;
    int curV = -1, dist = -1;
    forn (v, n - 1)
        if (d[n][v] != INF) {
            ld curAns = -INF;
            int curPos = -1;
            forn(k, n)

```

```

        if (curAns <= (d[n][v] - d[k][v]) * (1d) (1) / (n - k))
            curAns = (d[n][v] - d[k][v]) * (1d) (1) / (n - k),
↪   curPos = k;
        if (ansValue > curAns)
            ansValue = curAns, dist = curPos, curV = v;
    }
    if (curV == -1) return ansValue;
    for (int iter = n; iter != dist; iter--)
        ans.pb(curV), curV = p[iter][curV];
    reverse(all(ans));
    return ansValue;
}

```

41 Kuhn's algorithm

```

// sz(LEFT) = n, sz(RIGHT) = m
// numbered consequently
int n, m, paired[2 * N], used[2 * N];
vi g[N];

bool dfs(int v) {
    if (used[v]) return false;
    used[v] = 1;
    for (int to : g[v])
        if (paired[to] == -1 || dfs(paired[to])) {
            paired[to] = v, paired[v] = to;
            return true;
        }
    return false;
}

int kuhn() {
    int ans = 0;
    for (i, n + m) paired[i] = -1;
    for (int run = 1; run;) {
        run = 0;
        fill(used, used + n + m, 0);
        for (i, n)
            if (!used[i] && paired[i] == -1 && dfs(i))
                ans++, run = 1;
    }
    return ans;
}

// Start from unpaired vertex in Left part, go from Left anywhere,
↪   from Right only to pair
// Max Independent -- A+, B-
// Min Cover      -- A-, B+

```

```
vi minCover, maxIndependent;
```

```

void dfsCoverIndependent(int v) {
    if (used[v]) return;
    used[v] = 1;
    for (int to : g[v])
        if (!used[to])
            used[to] = 1, dfsCoverIndependent(paired[to]);
}

```

```
// Kuhn first!
```

```

void findCoverIndependent() {
    fill(used, used + n + m, 0);
    for (i, n)
        if (paired[i] == -1)
            dfsCoverIndependent(i);
    for (i, n)
        if (used[i]) maxIndependent.pb(i);
        else minCover.pb(i);
    for (i, n, n + m)
        if (used[i]) minCover.pb(i);
        else maxIndependent.pb(i);
}

```

42 LCA

```
int tin[N], tout[N], up[N][LOG], curTime = 0;
vi g[N];
```

```

void dfs(int v, int p) {
    up[v][0] = p;

```

```

    for (i, LOG - 1)
        up[v][i + 1] = up[up[v][i]][i];
    tin[v] = curTime++;
    for (int u : g[v])
        if (u != p)
            dfs(u, v);
    tout[v] = curTime++;
}

```

```

int isUpper(int v, int u) {
    return tin[v] <= tin[u] && tout[v] >= tout[u];
}

```

```

int lca(int v, int u) {
    if (isUpper(u, v)) return u;
    for (i, LOG)
        if (!isUpper(up[u][i], v))
            u = up[u][i];
    return up[u][0];
}

```

```

void init() {
    dfs(0, 0);
}

```

43 LCA offline (Tarjan)

```
vi g[N], q[N];
int pr[N], ancestor[N], used[N];
```

```

int get(int v) {
    return v == pr[v] ? v : pr[v] = get(pr[v]);
}

```

```

void unite(int v, int u, int anc) {
    v = get(v), u = get(u);
    pr[u] = v, ancestor[v] = anc;
}

```

```

void dfs(int v) {
    used[v] = 1;
    for (int u : g[v])
        if (!used[u])
            dfs(u), unite(v, u, v);
    for (int u : q[v])
        if (used[u])
            ancestor[get(u)]; // handle answer somehow
}

```

```

void init(int n) {
    for (i, n) pr[i] = i, ancestor[i] = i;
    dfs(0);
}

```

44 2 Chinese

```

struct Edge {
    int fr, to, w, id;
    bool operator<(const Edge& o) const { return w < o.w; }
};

```

```

// find oriented mst (tree)
// there are no edge --> root (root is 0)
// 0 .. n - 1, weights and vertices will be changed, but ids are
↪   ok

```

```

vector<Edge> work(const vector<vector<Edge>>& graph) {
    int n = sz(graph);
    vi color(n), used(n, -1);
    for (i, n)
        color[i] = i;
    vector<Edge> e(n);
    for (i, n) {
        if (graph[i].empty())
            e[i] = {-1, -1, -1, -1};
        else
            e[i] = *min_element(graph[i].begin(),
↪   graph[i].end());
    }
    vector<vi> cycles;

```



```

used[0] = -2;
for (s, n) {
    if (used[s] != -1)
        continue;
    int x = s;
    while (used[x] == -1) {
        used[x] = s;
        x = e[x].fr;
    }
    if (used[x] != s)
        continue;
    vi cycle = {x};
    for (int y = e[x].fr; y != x; y = e[y].fr)
        cycle.push_back(y), color[y] = x;
    cycles.push_back(cycle);
}
if (cycles.empty())
    return e;
vector<vector<Edge>> next_graph(n);
for (s, n) {
    for (const Edge& edge : graph[s]) {
        if (color[edge.fr] != color[s])
            next_graph[color[s]].push_back({
                color[edge.fr], color[s], edge.w - e[s].w,
});
        edge.id
    }
}
vector<Edge> tree = work(next_graph);
for (const auto& cycle : cycles) {
    int c1 = color[cycle[0]];
    Edge next_out = tree[c1], out{};
    int from = -1;
    for (int v : cycle) {
        tree[v] = e[v];
        for (const Edge& edge : graph[v])
            if (edge.id == next_out.id)
                from = v, out = edge;
    }
    tree[from] = out;
}
return tree;
}
}

```

45 Matroid Intersection

```

struct Gmat { // graphic matroid
    int V = 0; vector<pii> ed; vi par;
    Gmat(vector<pii> _ed):ed(_ed) {
        map<int,int> m;
        for(auto &t : ed) m[t.fst] = m[t.snd] = 0;
        for(auto &t : m) t.snd = V++;
        for(auto &t : ed) t.fst = m[t.fst], t.snd = m[t.snd];
    }
    int p(int v) {
        return par[v] == v ? v : par[v] = p(par[v]);
    }
    bool unite(int v, int u) {
        v = p(v), u = p(u);
        if (v != u) { par[v] = u; return true; }
        return false;
    }
    void clear() {
        par.resize(V);
        forn(i,V) par[i] = i;
    }
    void ins(int i) { assert(unite(ed[i].fst,ed[i].snd)); }
    bool indep(int i) { return p(ed[i].fst) != p(ed[i].snd); }
};

struct Cmat { // colorful matroid
    int C = 0; vi col; vi used;
    Cmat(vi _col):col(_col) {for(auto t : col) C = max(C, t+1);}
    void clear() { used.assign(C,0); }
    void ins(int i) { used[col[i]] = 1; }
    bool indep(int i) { return !used[col[i]]; }
};

template<class M1, class M2> struct MatroidIsect {
    int n; vi iset; M1 m1; M2 m2;
    bool augment() {
        vi pre(n+1,-1); queue<int> q({n});

```

```

        while (sz(q)) {
            int x = q.front(); q.pop();
            if (iset[x]) {
                m1.clear(); forn(i,n) if (iset[i] && i != x) m1.ins(i);
                forn(i,n) if (!iset[i] && pre[i] == -1 && m1.indep(i))
                    pre[i] = x, q.push(i);
            } else {
                auto backE = [&]() { // back edge
                    m2.clear();
                    forn(c,2)forn(i,n)
}
                if((x==i||iset[i])&&(pre[i]==-1)==c){
                    if (!m2.indep(i))return c?pre[i]=x,q.push(i),i:-1;
                    m2.ins(i); }
                return n;
            };
            for (int y; (y = backE()) != -1;) if (y == n) {
                for(; x != n; x = pre[x]) iset[x] = !iset[x];
                return 1; }
        }
    }
    return 0;
}

MatroidIsect(int _n, M1 _m1, M2 _m2):n(_n), m1(_m1), m2(_m2) {
    iset.assign(n+1,0); iset[n] = 1;
    m1.clear(); m2.clear(); // greedily add to basis
    forn(r,i,n) if (m1.indep(i) && m2.indep(i))
        iset[i] = 1, m1.ins(i), m2.ins(i);
    while (augment());
}
};

```

9 Math

46 CRT (KTO)

```

vi crt(vi a, vi mod) {
    int n = sz(a);
    vi x(n);
    forn (i, n) {
        x[i] = a[i];
        forn (j, i) {
            x[i] = inverse(mod[j], mod[i]) * (x[i] - x[j]) % mod[i];
            if (x[i] < 0) x[i] += mod[i];
        }
    }
    return x;
}

```

47 Discrete Logarithm

```

// Returns x: a^x = b (mod mod) or -1, if no such x exists
int discreteLogarithm(int a, int b, int mod) {
    int sq = (int) sqrt(mod);
    int sq2 = mod / sq + (mod % sq ? 1 : 0);
    vector<pii> powers(sq2);
    forn (i, sq2)
        powers[i] = mp(power(a, (i + 1) * sq, mod), i + 1);
    sort(all(powers));
    forn (i, sq + 1) {
        int cur = power(a, i, mod);
        cur = mul(cur, b, mod);
        auto it = lower_bound(all(powers), mp(cur, 0));
        if (it != powers.end() && it->fst == cur)
            return it->snd * sq - i;
    }
    return -1;
}

```

48 Discrete Root

```

// Returns x: x^k = a mod mod, mod is prime
int discreteRoot(int a, int k, int mod) {
    if (a == 0)
        return 0;
    int g = primitiveRoot(mod);
    int y = discreteLogarithm(power(g, k, mod), a, mod);
    return power(g, y, mod);
}

```

49 Eratosthenes

```

vi eratosthenes(int n) {
    vi minDiv(n + 1, 0);
    minDiv[1] = 1;
    forab (i, 2, n + 1)
        if (minDiv[i] == 0)
            for (int j = i; j <= n; j += i)
                if (minDiv[j] == 0) minDiv[j] = i;
    return minDiv;
}

vi eratosthenesLinear(int n) {
    vi minDiv(n + 1, 0), primes;
    minDiv[1] = 1;
    forab (i, 2, n + 1) {
        if (minDiv[i] == 0)
            minDiv[i] = i, primes.pb(i);
        for (int j = 0; j < sz(primes) && primes[j] <= minDiv[i] && i
↪ * primes[j] <= n; j++)
            minDiv[i * primes[j]] = primes[j];
    }
    return minDiv;
}

```

50 Factorial

```

// Returns pair (rem, power), where rem = n! % mod,
// power = k: mod^k | n!, mod is prime, 0(mod log mod)
pii fact(int n, int mod) {
    int rem = 1, power = 0, nCopy = n;
    while (nCopy) nCopy /= mod, power += nCopy;
    while (n > 1) {
        rem = (rem * ((n / mod) % 2 ? -1 : 1) + mod) % mod;
        for (int i = 2; i <= n % mod; i++)
            rem = mul(rem, i, mod);
        n /= mod;
    }
    return mp(rem % mod, power);
}

```

51 Gauss

```

const double EPS = 1e-9;

int gauss(double **a, int n, int m) { // n is number of equations,
↪ m is number of variables
    int row = 0, col = 0;
    vi par(m, -1);
    vector<double> ans(m, 0);
    for (col = 0; col < m && row < n; col++) {
        int best = row;
        for (int i = row; i < n; i++)
            if (abs(a[i][col]) > abs(a[best][col]))
                best = i;
        if (abs(a[best][col]) < EPS) continue;
        par[col] = row;
        forn (i, m + 1) swap(a[row][i], a[best][i]);
        forn (i, n)
            if (i != row) {
                double k = a[i][col] / a[row][col];
                for (int j = col; j <= m; j++)
                    a[i][j] -= k * a[row][j];
            }
        row++;
    }
    int single = 1;
    forn (i, m)
        if (par[i] != -1) ans[i] = a[par[i]][m] / a[par[i]][i];
        else single = 0;
    forn (i, n) {
        double cur = 0;
        for (int j = 0; j < m; j++)
            cur += ans[j] * a[i][j];
        if (abs(cur - a[i][m]) > EPS)
            return 0;
    }
    if (!single)
        return 2;
}

```

```

    return 1;
}

```

52 Gauss binary

```

const int MAX = 1024;

int gaussBinary(vector<bitset<MAX>> a, int n, int m) {
    int row = 0, col = 0;
    vi par(m, -1);
    for (col = 0; col < m && row < n; col++) {
        int best = row;
        for (int i = row; i < n; i++)
            if (a[i][col] > a[best][col])
                best = i;
        if (a[best][col] == 0)
            continue;
        par[col] = row;
        swap(a[row], a[best]);
        forn (i, n)
            if (i != row && a[i][col])
                a[i] ^= a[row];
        row++;
    }
    vi ans(m, 0);
    forn (i, m)
        if (par[i] != -1)
            ans[i] = a[par[i]][n] / a[par[i]][i];
    bool ok = 1;
    forn (i, n) {
        int cur = 0;
        forn (j, m) cur ^= (ans[j] & a[i][j]);
        if (cur != a[i][n]) ok = 0;
    }
    return ok;
}

```

53 Gcd

```

int gcd(int a, int b) {
    return b ? gcd(b, a % b) : a;
}

int gcd(int a, int b, int &x, int &y) {
    if (b == 0) {
        x = 1, y = 0;
        return a;
    }
    int g = gcd(b, a % b, x, y), newX = y;
    y = x - a / b * y;
    x = newX;
    return g;
}

void diophant(int a, int b, int c, int &x, int &y) {
    int g = gcd(a, b, x, y);
    if (c % g != 0) return;
    x *= c / g, y *= c / g;
    // next solutions: x += b / g, y -= a / g
}

int inverse(int a, int mod) { // Returns -1, if a and mod are not
↪ coprime
    int x, y;
    int g = gcd(a, mod, x, y);
    return g == 1 ? (x % mod + mod) % mod : -1;
}

vi inverseForAll(int mod) {
    vi r(mod, 0);
    r[1] = 1;
    for (int i = 2; i < mod; i++)
        r[i] = (mod - r[mod % i]) * (mod / i) % mod;
    return r;
}

```

54 Gray

```

int gray(int n) {
    return n ^ (n >> 1);
}

```

55 Miller-Rabin Test

```
bool isPrimeMillerRabin(ull n) { // not ll!
    if (n < 2 || n % 6 % 4 != 1)
        return n - 2 < 2;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
    ull s = __builtin_ctzll(n - 1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = power(a, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = mul(p, p, n);
        if (p != n - 1 && i != s) return 0;
    }
    return 1;
}
```

56 Phi

```
int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; i++)
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    if (n > 1) result -= result / n;
    return result;
}
```

```
int inversePhi(int a, int mod) {
    return power(a, phi(mod) - 1, mod);
}
```

57 Pollard

```
ull pollard(ull n) { // return some nontrivial factor of n
    auto f = [n](ull x) { return mul(x, x, n) + 1; };
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) { /// speedup: don't take
        ↪ gcd every it
        if (x == y) x = ++i, y = f(x);
        if ((q = mul(prd, max(x, y) - min(x, y), n))) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}
```

```
void factorize(ull n, map<ull,int>& cnt) {
    if (n == 1) return;
    if (isPrimeMillerRabin(n)) {
        ++cnt[n];
        return;
    }
    ull u = pollard(n);
    factorize(u, cnt), factorize(n / u, cnt);
}
```

58 Power And Mul

```
template <typename T>
inline T add(T a, T b, T mod) {
    a += b;
    return a >= mod ? a - mod : a;
}
```

```
template <typename T>
inline T sub(T a, T b, T mod) {
    a -= b;
    return a < 0 ? a + mod : a;
}
```

```
template <typename T>
T mul(T a, T b, T mod) {
    return T((a * 1ll * b) % mod);
}
```

```
template <>
ll mul<ll>(ll a, ll b, ll mod) {
    ll q = 1ll * a * b / mod;
    ll r = a * b - mod * q;
    while (r < 0) r += mod;
    while (r >= mod) r -= mod;
    return r;
}
```

```
template <typename T>
T power(T a, T n, T mod) {
    if (!n) return 1;
    T b = power(a, n / 2, mod);
    b = mul(b, b, mod);
    return n & 1 ? mul<T>(a, b, mod) : b;
}
```

```
int powerFast(int a, int n, int mod) {
    int res = 1;
    while (n) {
        if (n & 1)
            res = mul(res, a, mod);
        a = mul(a, a, mod);
        n /= 2;
    }
    return res;
}
```

59 Primitive Root

```
int primitiveRoot(int mod) { // Returns -1 if no primitive root
    ↪ exists
    vi fact;
    int ph = phi(mod);
    int n = mod;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            fact.pb(i);
            while (n % i == 0) n /= i;
        }
    }
    if (n > 1) fact.pb(n);
    forab (i, 2, mod + 1) {
        bool ok = 1;
        for (int j = 0; j < sz(fact) && ok; j++)
            ok &= power(i, ph / fact[j], mod) != 1;
        if (ok) return i;
    }
    return -1;
}
```

60 Simpson

```
double f(double x) { return x; }
```

```
double simpson(double a, double b, int iterNumber) {
    double res = 0, h = (b - a) / iterNumber;
    forn (i, iterNumber + 1)
        res += f(a + h * i) * ((i == 0) || (i == iterNumber) ? 1 :
        ↪ ((i & 1) == 0) ? 2 : 4);
    return res * h / 3;
}
```

61 Euclidean Burunduk-1

```
/**
 * Sergey Kopeliovich (burunduk30@gmail.com)
 */
```

```
#include <iostream>
```

```
using namespace std;
```

```
// finds x:
```

```
//      a+k*x mod m --> min, 0 <= x <= r (0 <= a, k < m, 0 <= r)
//      +k costs pk, -m costs pm
//      return r-x
int go(int a, int k, int m, int pk, int pm, int r) {
    if (!k) return r;
    if (a >= k) { // make a: 0 <= a < k
        int add = (m - a + k - 1) / k;
        if (((int64_t)add * pk + pm > r) return r;
        a += (int64_t)add * k - m, r -= add * pk + pm;
    }
    int m1 = m % k, pm1 = (m / k) * pk + pm;
    if (!m1) return r;
    int k1 = k % m1, pk1 = (k / m1) * pm1 + pk;
    if (pm1 * (a / m1) > r) return r % pm1;
    return go(a % m1, k1, m1, pk1, pm1, r - (a / m1) * pm1);
}

// finds x: a+k*x mod m --> min, 0 <= a, k < m, 0 <= r
int go(int a, int k, int m, int r) {
    return r - go(a, k, m, 1, 0, r);
}

int main() {
    ios_base::sync_with_stdio(false), cin.tie(0);

    int a, k, m, r;
    while (cin >> a >> k >> m >> r) {
        int x = go(a, k, m, r);
        cout << ((int64_t)x * k + a) % m << ' ' << x << '\n';
    }
}
```

62 Euclidean Burunduk-2

```
/**
 * Sergey Kopeliovich (burunduk30@gmail.com)
 */

#include <iostream>

using namespace std;

// finds min x:
//      a+k*x mod m \in [l..r]
//      +k costs pk, -m costs pm
//      l <= r < a, first tries -m then +k
int go(int a, int k, int m, int pk, int pm, int l, int r) {
    int ans = 0, steps;
    while (1) {
        steps = (a - r + m - 1) / m;
        ans += steps * pm, a -= steps * m;
        if (l <= a) return ans;
        if (!k) return -1;
        steps = (l - a + k - 1) / k;
        ans += steps * pk, a += steps * k;
        if (a <= r) return ans;
        int m1 = m % k, pm1 = (m / k) * pk + pm;
        if (!m1) return -1;
        int k1 = k % m1, pk1 = (k / m1) * pm1 + pk;
        k = k1, m = m1, pk = pk1, pm = pm1; // recursion =)
    }
}

int go(int a, int k, int m, int l, int r) {
    if (a < r)
        a += ((r - a) / m + 1) * m;
    return go(a, k, m, 1, 0, l, r);
}

int main() {
    ios_base::sync_with_stdio(false), cin.tie(0);

    int a, k, m, l, r;
    while (cin >> a >> k >> m >> l >> r)
        cout << go(a, k, m, l, r) << '\n';
}
```

10 Strings

63 Aho-Corasick

```
struct Node {
    int next[ALPHA], term; //
    int go[ALPHA], suf, p, pCh; //
    Node(): term(0), suf(-1), p(-1) {
        fill(next, next + ALPHA, -1);
        fill(go, go + ALPHA, -1);
    }
};

Node g[N];
int last;

void add(const string &s) {
    int now = 0;
    for(char x : s) {
        if (g[now].next[x - 'a'] == -1) {
            g[now].next[x - 'a'] = ++last;
            g[last].p = now, g[last].pCh = x;
        }
        now = g[now].next[x - 'a'];
    }
    g[now].term = 1;
}

int go(int v, int c);

int getLink(int v) {
    if (g[v].suf == -1) {
        if (!v || !g[v].p) g[v].suf = 0;
        else g[v].suf = go(getLink(g[v].p), g[v].pCh);
    }
    return g[v].suf;
}

int go(int v, int c) {
    if (g[v].go[c] == -1) {
        if (g[v].next[c] != -1) g[v].go[c] = g[v].next[c];
        else g[v].go[c] = !v ? 0 : go(getLink(v), c);
    }
    return g[v].go[c];
}
```

64 Prefix-function

```
vi prefix(const string &s) {
    int n = sz(s);
    vi pr(n);
    forab (i, 1, n + 1) {
        int j = pr[i - 1];
        while (j > 0 && s[i] != s[j]) j = pr[j - 1];
        if (s[i] == s[j]) j++;
        pr[i] = j;
    }
    return pr;
}
```

65 Z-function

```
vi z(const string& s) {
    int n = sz(s);
    vi z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}
```

66 Hashes

```
#include "../math/PowerAndMul.cpp"
const int P = 239017, MOD_X = 1e9 + 7, MOD_Y = 1e9 + 9;

// using H = unsigned long long;
struct H {
```

```

    int x, y;
    H() = default;
    H(int _x): x(_x), y(_x) {}
    H(int _x, int _y): x(_x), y(_y) {}
    inline H operator+(const H& h) const { return H(add(x, h.x,
↪ MOD_X), add(y, h.y, MOD_Y)); }
    inline H operator-(const H& h) const { return H(sub(x, h.x,
↪ MOD_X), sub(y, h.y, MOD_Y)); }
    inline H operator*(const H& h) const { return H(mul(x, h.x,
↪ MOD_X), mul(y, h.y, MOD_Y)); }
    inline bool operator==(const H& h) const { return x == h.x && y
↪ == h.y; }
};

H p[N], h[N];

inline H get(int l, int r) { return h[r] - h[l] * p[r - l]; }

void init(const string& s) {
    int n = sz(s);
    p[0] = 1;
    forn (i, n)
        h[i + 1] = h[i] * P + s[i], p[i + 1] = p[i] * P;
}

```

67 Manaker

```

void manaker(const string& s, int *z0, int *z1) {
    int n = sz(s);
    forn (t, 2) {
        int *z = t ? z1 : z0, l = -1, r = -1; // [l..r]
        forn (i, n - t) {
            int k = 0;
            if (r > i + t) {
                int j = l + (r - i - t);
                k = min(z[j], j - l);
            }
            while (i - k >= 0 && i + k + t < n && s[i - k] == s[i + k +
↪ t])
                k++;
            z[i] = k;
            if (k && i + k + t > r)
                l = i - k + 1, r = i + k + t - 1;
        }
    }
}

```

68 Palindromic Tree

```

struct Vertex {
    int suf, len, next[ALPHA];
    Vertex() { fill(next, next + ALPHA, 0); }
};

int vn, v;
Vertex t[N + 2];
int n, s[N];

int get(int i) { return i < 0 ? -1 : s[i]; }

void init() {
    t[0].len = -1, vn = 2, v = 0, n = 0;
}

void add(int ch) {
    s[n++] = ch;
    while (v != 0 && ch != get(n - t[v].len - 2))
        v = t[v].suf;
    int& r = t[v].next[ch];
    if (!r) {
        t[vn].len = t[v].len + 2;
        if (!v) t[vn].suf = 1;
        else {
            v = t[v].suf;
            while (v != 0 && ch != get(n - t[v].len - 2))
                v = t[v].suf;
            t[vn].suf = t[v].next[ch];
        }
        r = vn++;
    }
}

```

```

    v = r;
}

```

69 Suffix Array (+stable)

```

int sLen, num[N + 1], p[N], col[N], inv[N], lcp[N];
char s[N + 1];

inline int add(int a, int b) {
    a += b;
    return a >= sLen ? a - sLen : a;
}

inline int sub(int a, int b) {
    a -= b;
    return a < 0 ? a + sLen : a;
}

void buildArray(int n) {
    sLen = n;
    int ma = max(n, 256);
    forn (i, n)
        col[i] = s[i], p[i] = i;

    for (int k2 = 1; k2 / 2 < n; k2 *= 2) {
        int k = k2 / 2;
        memset(num, 0, sizeof(num));
        forn (i, n) num[col[i] + 1]++;
        forn (i, ma) num[i + 1] += num[i];
        forn (i, n)
            inv[num[col[sub(p[i], k)]]++] = sub(p[i], k);
        int cc = 0;
        forn (i, n) {
            bool flag = col[inv[i]] != col[inv[i - 1]];
            flag |= col[add(inv[i], k)] != col[add(inv[i - 1], k)];
            if (i && flag) cc++;
            num[inv[i]] = cc;
        }
        forn (i, n) p[i] = inv[i], col[i] = num[i];
    }

    memset(num, 0, sizeof(num));
    forn (i, n) num[col[i] + 1]++;
    forn (i, ma) num[i + 1] += num[i];
    forn (i, n) inv[num[col[i]]++] = i;
    forn (i, n) p[i] = inv[i];
    forn (i, n) inv[p[i]] = i;
}

```

```

void buildLCP(int n) {
    int len = 0;
    forn (ind, n){
        int i = inv[ind];
        len = max(0, len - 1);
        if (i != n - 1)
            while (len < n && s[add(p[i], len)] == s[add(p[i + 1],
↪ len)])
                len++;
        lcp[i] = len;
        if (i != n - 1 && p[i + 1] == n - 1) len = 0;
    }
}

```

70 Suffix Automaton

```

struct Vx {
    int len, suf;
    int next[ALPHA];
    Vx() {}
    Vx(int l, int s): len(l), suf(s) {}
};

struct SA {
    static const int V = 2 * LEN;
    int last, vcnt;
    Vx v[V];

    SA() { vcnt = 1, last = newV(0, 0); } // root = vertex with
↪ number 1
    int newV(int len, int suf){

```

```

    v[vcnt] = Vx(len, suf);
    return vcnt++;
}
int add(char ch) {
    int p = last, c = ch - 'a';
    last = newV(v[last].len + 1, 0);
    while (p && !v[p].next[c]) // added p &&
        v[p].next[c] = last, p = v[p].suf;
    if (!p)
        v[last].suf = 1;
    else {
        int q = v[p].next[c];
        if (v[q].len == v[p].len + 1) v[last].suf = q;
        else {
            int r = newV(v[p].len + 1, v[q].suf);
            v[last].suf = v[q].suf = r;
            memcpy(v[r].next, v[q].next, sizeof(v[r].next));
            while (p && v[p].next[c] == q)
                v[p].next[c] = r, p = v[p].suf;
        }
    }
    return last;
}
};

```

71 Suffix Tree

```

char S[N];
int L;

struct Node;
struct Pos;
typedef Node *pNode;
typedef map<char,pNode> mapt;

struct Node{
    pNode P, link;
    int L, R;
    mapt next;

    Node(): P(NULL), link(this), L(0), R(0) {}
    Node(pNode P_, int L_, int R_): P(P_), link(NULL), L(L_), R(R_)
    ↪ {}

    inline int elen() const { return R - L; }
    inline pNode addEdge(int L_, int R_) { return next[S[L_]] = new
    ↪ Node(this, L_, R_); }
};

struct Pos{
    pNode V;
    int up;
    Pos(): V(NULL), up(0) {}
    Pos(pNode V_, int up_): V(V_), up(up_) {}

    pNode splitEdge() const {
        if(!up)
            return V;
        int L_ = V->L, M = V->R - up;
        pNode P = V->P, n = new Node(P, L_, M);
        P->next[S[L_]] = n;
        n->next[S[M]] = V;
        V->P = n, V->L = M;
        return n;
    }
    Pos nextChar(char c) const {
        if(up)
            return S[V->R - up] == c ? Pos(V, up - 1) : Pos();
        else{
            mapt::iterator it = V->next.find(c);
            return it == V->next.end() ? Pos() : Pos(it->snd,
    ↪ it->snd->elen() - 1);
        }
    }
};

Pos goDown(pNode V, int L_, int R_) {
    if(L_ == R_)
        return Pos(V, 0);
    while (1) {

```

```

        V = V->next[S[L_]];
        L_ += V->elen();
        if(L_ >= R_)
            return Pos(V, L_ - R_);
    }
}

inline pNode calcLink(pNode &V) {
    if (!V->link)
        V->link = goDown(V->P->link, V->L + !V->P->P,
    ↪ V->R).splitEdge();
    return V->link;
}

Pos addChar(Pos P, int k) {
    while(1) {
        Pos p = P.nextChar(S[k]);
        if (p.V)
            return p;
        pNode n = P.splitEdge();
        n->addEdge(k, LEN);
        if (!n->P)
            return Pos(n, 0);
        P = Pos(calcLink(n), 0);
    }
}

```

```

pNode Root;
void make_tree(){
    Root = new Node();
    Pos P(Root, 0);
    forn (i, L)
        P = addChar(P, i);
}

```

11 C++ Tricks

72 Fast allocation

```

const int MEM = 100 << 20;
static char buf[MEM];
inline void* operator new(size_t n) {
    static size_t i = sizeof buf;
    assert(n < i);
    return (void*) &buf[i -= n];
}
inline void operator delete(void*) {}
inline void* operator new[](size_t) { assert(0); }
inline void operator delete[](void*) { assert(0); }

```

73 Hash of pair

```

struct PairHasher {
    size_t operator()(const pair<int, int>& p) const { return p.fst
    ↪ * 239017 + p.snd; }
};

```

74 Ordered Set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

```

```
using namespace __gnu_pbds;
```

```
template <class T> using ordered_set = tree<T, null_type, less<T>,
    ↪ rb_tree_tag, tree_order_statistics_node_update>;

```

```

void example() {
    ordered_set<int> s;
    s.insert(1), s.insert(3);
    assert(s.order_of_key(3) == 1 && s.order_of_key(4) == 2 &&
    ↪ *s.find_by_order(0) == 1);
}

```

75 Hash Map

```
#include <ext/pb_ds/assoc_container.hpp>
```

```
using namespace __gnu_pbds;
```

```
struct chash { // To use most bits rather than just the lowest
↪ ones:
    const uint64_t C = 11(2e18 * PI) + 71; // large odd number
    const int RANDOM = 912387491;
    ll operator()(ll x) const { return __builtin_bswap64((x ^
↪ RANDOM) * C); }
};
template<class K, class V> using ht = gp_hash_table<K, V, chash>;
template<class K, class V> V get(ht<K, V>& u, K x) {
    auto it = u.find(x); return it == end(u) ? 0 : it->snd;
}

ht<ll, int> h({}, {}, {}, {}, {1<<20});
```

76 Fast I/O

```
const int BUF_SIZE = 4096;

char buf[BUF_SIZE];
int bufLen = 0, pos = 0;

inline int getChar() {
    if (pos == bufLen) {
        pos = 0, bufLen = (int) fread(buf, 1, BUF_SIZE, stdin);
        if (!bufLen)
            return -1;
    }
    return buf[pos++];
}

inline int readChar() {
    int c = getChar();
    while (c != -1 && c <= 32)
        c = getChar();
    return c;
}

template <class T>
inline T readInt() {
    int s = 1, c = readChar();
    T x = 0;
    if (c == '-')
        s = -1, c = getChar();
    while ('0' <= c && c <= '9')
        x = x * 10 + c - '0', c = getChar();
    return s == 1 ? x : -x;
}

inline void readWord(char *s) {
    int c = readChar();
    while (c > 32)
        *s++ = (char) c, c = getChar();
    *s = 0;
}

int writePos = 0;
char writeBuf[BUF_SIZE];

inline void flush() {
    if (writePos)
        fwrite(writeBuf, 1, writePos, stdout), writePos = 0;
}

inline void writeChar(int x) {
    if (writePos == BUF_SIZE)
        flush();
    writeBuf[writePos++] = (char) x;
}

template <class T>
inline void writeInt(T x, char after = '\0') {
    if (x < 0)
        writeChar('-'), x = -x;

    char s[24];
    int n = 0;
    while (x || !n)
        s[n++] = '0' + x % 10, x /= 10;
    while (n--)
```

```
        writeChar(s[n]);
    if (after)
        writeChar(after);
}

inline void writeWord(const char *s) {
    while (*s)
        writeChar(*s++);
}
```

12 Notes

77 Работа с деревьями

Приемы для работы с деревьями:

- 1. Двоичные подьемы
- 2. Поддеревья как отрезки Эйлераова обхода
- 3. Вертикальные пути в Эйлеровом обходе (на ребрах вниз $+k$, на ребрах вверх $-k$).
- 4. Храним в вершине значение функции на пути от корня до нее, дальше LCA.
- 5. Спуск с DFS, поддерживаем ДО на пути до текущей вершины.
- 6. Heavy-light decomposition
- 7. Centroid decomposition
- 8. Корневая по запросам
- 9. Тяжелые/легкие вершины
- 10. DFS \rightarrow дерево блоков, размеры $\in [K..2K]$
- 11. У вершины не более $O(\sqrt{N})$ разных поддеревьев
- 12. Сумма размеров поддеревьев без тяжелого ребенка $O(n \log n)$
- 13. Сумма глубин поддеревьев без глубокого ребенка $O(n)$

78 Маски

Считаем динамику по маскам за $O(2^n \cdot n)$ $f[mask] = \text{sum по } submask$ $g[submask]$.
 $dp[mask][i]$ — значение динамики для маски $mask$, если младшие i бит в ней зафиксированы (то есть мы не можем удалять оттуда).
 Ответ в $dp[mask][0]$.
 $dp[mask][len] = g[mask]$. Если i -ый бит 0, то $dp[mask][i] = dp[mask][i + 1]$, иначе $dp[mask][i] = dp[mask][i + 1] + dp[mask^{1 \ll i}][i + 1]$.
 Старший бит: предподсчет.
 Младший бит: $x \& \sim (-x)$
 Чтобы по степени двойки получить логарифм, можно воспользоваться тем, что все степени двойки имеют разный остаток по модулю 67.

```
for (int mask = 0; mask < (1 << n); mask++)
    ^^Isubmask : for (int s = mask; s; s = (s - 1) & mask)
    ^^Isupmask : for (int s = mask; s < (1 << n); s = (s + 1) | mask)
```

79 Гранди

Теорема Шпрага-Гранди: берем mex всех значений функции Гранди по состояни-ям, в которые можем перейти из данного.
 Если сумма независимых игр, то значение функции Гранди равно хог значений функций Гранди по всем играм.
 Бывает полезно вывести первые n значений и поискать закономерность.
 Часто сводится к xor по чему-нибудь.

80 Потоки

Потоки:

Name	Asympthotic
Ford-Fulkerson	$O(f \cdot E)$
Ford-Fulkerson with scaling	$O(\log f \cdot E^2)$
Edmonds-Karp	$O(V \cdot E^2)$
Dinic	$O(V^2 \cdot E)$
Dinic with scaling	$O(V \cdot E \cdot \log C)$
Dinic on bipartite graph	$O(E\sqrt{V})$
Dinic on unit network	$O(E\sqrt{E})$

L—R потоки:
Есть граф с недостатками или избытками в каждой вершине. Создаем фиктивные исток и сток (из истока все ребра в избытки, из недостатков все ребра в сток).
Теперь пусть у нас есть L-R граф, для каждого ребра $e\ (v \rightarrow u)$ известны L_e и R_e . Добавим в v избыток L_e , в u недостаток L_e , а пропускную способность сделаем $R_e - L_e$.
Получили решение задачи о LR-циркуляции.
Если у нас обычный граф с истоком и стоком, то добавляем бесконечное ребро из стока в сток и ищем циркуляцию.
Таким образом нашли удовлетворяющий условиям LR-поток. Если хотим максимальный поток, то на остаточной сети запускаем поиск максимального потока.
В новом графе в прямую сторону пропускная способность равна $R_e - f_e$, в обратную $f_e - L_e$.
MinCostCirculation:
Пока есть цикл отрицательного веса, запускаем алгоритм Карпа и пускаем максимальный поток по найденному циклу.

81 ДП

Табличка с оптимизациями для динамики:

Name	Original recurrence	From To
CHT1	$dp[i] = \min_{j < i} dp[j] + b[j] \cdot a[i]$	$O(n^2)$
	$b[j] \geq b[j + 1] \ \ a[i] \leq a[i + 1]$	$O(n)$
CHT2	$dp[i][j] = \min_{k < j} dp[i - 1][k] + b[k] \cdot a[j]$	$O(kn^2)$
	$b[k] \geq b[k + 1] \ \ a[j] \leq a[j + 1]$	$O(kn)$
D&C	$dp[i][j] = \min_{k < j} dp[i - 1][k] + c[k][j]$	$O(kn^2)$
	$p[i, j] \leq p[i, j + 1]$	$O(kn \log n)$
Knuth	$dp[i][j] = \min_{i < k < j} dp[i][k] + dp[k][j] + c[i][j]$	$O(n^3)$
	$p[i, j - 1] \leq p[i, j] \leq p[i + 1, j]$	$O(n^2)$
IOI	$f_n(k)$ — best for fixed k f_n — convex, add penalty $\lambda \cdot k$	$O(k^{(2)}n)$ $O(n \log C)$

82 Комбинаторика

Биномиальные коэффициенты:
Теорема Люка для биномиальных коэффициентов: Хотим посчитать C_n^k , разложим в p-ичной системе счисления, $n = (n_0, n_1, \dots), k = (k_0, k_1, \dots)$. $ans = C_{n_0}^{k_0} \cdot C_{n_1}^{k_1} \cdot \dots$
Способы вычисления C_n^k :

1.

$C_n^k = C_{n-1}^k + C_{n-1}^{k-1}$
precalc: $O(n^2)$, query: $O(1)$.
2.

$C_n^k = \frac{n!}{k!(n-k)!}$, предподсчитываем факториалы
precalc: $O(n \log n)$, query: $O(\log n)$
3.

Теорема Люка
precalc: $O(p \log p)$, query: $O(\log p)$.
4.

$C_n^k = C_n^{k-1} \cdot \frac{n-k+1}{k}$
5.

$C_n^k = \frac{n!}{k!(n-k)!}$, для каждого факториала считаем степень вхождения и остаток
precalc: $O(p \log p)$, query: $O(\log p)$.
- $C_n^{\frac{n}{2}} = \frac{2^n}{\sqrt{\frac{\pi n}{2}}}$

83 Делители

- $\leq 20 : d(12) = 6$
- $\leq 50 : d(48) = 10$
- $\leq 100 : d(60) = 12$
- $\leq 1000 : d(840) = 32$
- $\leq 10^4 : d(9\ 240) = 64$
- $\leq 10^5 : d(83\ 160) = 128$

- $\leq 10^6 : d(720\ 720) = 240$
- $\leq 10^7 : d(8\ 648\ 640) = 338$
- $\leq 10^8 : d(91\ 891\ 800) = 768$
- $\leq 10^9 : d(931\ 170\ 240) = 1344$
- $\leq 10^{11} : d(97\ 772\ 875\ 200) = 4032$
- $\leq 10^{12} : d(963\ 761\ 198\ 400) = 6720$
- $\leq 10^{15} : d(866\ 421\ 317\ 361\ 600) = 15360$
- $\leq 10^{18} : d(897\ 612\ 484\ 786\ 617\ 600) = 103680$

84 Числа Белла

i	B_i	i	B_i
0	1	12	4,213,597
1	1	13	27,644,437
2	2	14	190,899,322
3	5	15	1,382,958,545
4	15	16	10,480,142,147
5	52	17	82,864,869,804
6	203	18	682,076,806,159
7	877	19	5,832,742,205,057
8	4,140	20	51,724,158,235,372
9	21,147	21	474,869,816,156,751
10	115,975	22	4,506,715,738,447,323
11	678,570	23	44,152,005,855,084,346

85 Разбиения

Число неупорядоченных разбиений n на положительные слагаемые.

$$p(0) = 1, \ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

86 Матричные игры

Пишем матрицу стратегий $A_{i,j}$ это выигрыш первого и проигрыш второго, i стратегия 1-го. Седловая точка есть для несмешанной стратегии если $\max_i \min A_{i,*} = \min_j \max A_{*,j}$. Иначе:

$$f(x) = \text{sum}(x_i) \rightarrow \max, \ \text{Ans} = 1/f(x)$$

$$Ax \leq 1_n, \ x_i \geq 0$$

Для 2×2 , p первый игрок, q — второй:

$$p^* = \left(\frac{a_{22} - a_{21}}{a_{22} - a_{12} + a_{11} - a_{21}}; \frac{a_{11} - a_{12}}{a_{22} - a_{12} + a_{11} - a_{21}} \right)$$

$$q^* = \left(\frac{a_{22} - a_{12}}{a_{22} - a_{12} + a_{11} - a_{21}}; \frac{a_{11} - a_{21}}{a_{22} - a_{12} + a_{11} - a_{21}} \right)$$

$$\text{Ans} = \frac{a_{22}a_{11} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

87 Mixed

- Формула Пика: $S = \text{Inside} + \text{Edge}/2 - 1$
- Теорема Люка: $0 \leq n, m \in \mathbb{Z}, p$ простое. $n = n_k p^k + \dots + n_1 p + n_0$ и $m = m_k p^k + \dots + m_1 p + m_0$. Тогда $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod p$.
- Лемма Бернсайда: $|X/G|$ число орбит G . $X^g = \{x \in X | gx = x\}$

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

88 Ideas

- **Generic:** binary search, ternary search, sort, dp, meet-in-the-middle, divide&conquer, greedy, sqrt-decomposition, matroids, Gauss, FFT, suffix array, suffix automaton, DSU;
- **Graphs:** build graph, add vertices / edges, 2-SAT, flows / cut, matching, Hall's theorem, topsort, HLD, centroid decomposition, MST, Euler cycle, Binary lifting, LCA;
- **Tricks:** consider the process from the end / from the middle, try any one, draw on 2D plane, simplify the problem / consider special case / consider more general case, simplify solution, prefix sums, differences of adjacent elements, consider min/max, analyze why a straightforward solution doesn't work, check limitations, consider contribution of separate element, small answer, different solutions for different limitations, consider complement set, maintain sum / sum of squares, convex function, store $O(1)$ top candidates, inversions, inclusion-exclusion formula, bounding box, angle sort, Grundy function, Eucklid, Mo's algorithm, iterate over divisors, matrix exponentiation;