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### 1 Common

### 1 Setup

- 1. Terminal: font Monospace 12
- 2. Gedit: Oblivion, font Monospace 12, auto indent, display line numbers, tab 4, highlight matching brackets, highlight current line, F9 (side panel)
- 3. /.bashrc: export CXXFLAGS='-Wall -Wshadow -Wextra Wconversion -Wno-unused-result -Wno-deprecated-declarations O2 -std=gnu++11 -g -DLOCAL'
- 4. for i in {A..K}; do mkdir \$i; cp main.cpp \$i/\$i.cpp; done

## 2 Template

```
main.cpp:
#define FNAME ""
#undef __STRICT_ANSI__
#ifdef LOCAL
 ^^I<mark>#</mark>define GLIBCXX DEBUG
#include < bits/stdc++.h>
using namespace std;
#define pb push_back
#define mp make_pair
#define fs first
#define sc second
#define fst first
#define snd second
#define sz(x) (int)((x).size())
#define forn(i,n) for (int i = 0; (i) < (n); ++i)
#define fornr(i,n) for (int i = (int)(n) - 1; (i) \geq 0; --i)
#define forab(i,a,b) for (int i = (a); (i) < (b); ++i)
\#define forba(i,a,b) for (int i = (int)(b) - 1; (i) >= (a); --i)
\#define forit(it,c) for(\_typeof((c).begin()) it = (c).begin(); it !=
\hookrightarrow (c).end(); ++it)
#define all(c) (c).begin(),(c).end()
#ifdef LOCAL
^^I#define eprintf(...) fprintf(stderr, __VA_ARGS__), fflush(stderr)
#endif
typedef long long LL;
typedef unsigned long long ULL;
typedef double dbl;
typedef long double LD;
typedef unsigned int uint;
typedef vector <int> vi;
typedef pair <int, int> pii;
int main() {
^^Ireturn 0;
}
```

### 3 Stress

#!/bin/bash

```
stress.sh:
```

```
\begin{array}{l} \text{for } ((i=0;;i++)); \ do \\ \verb|^^I./gen \$i > in \mid | \ exit \\ \verb|^^I./main < in > out 1 \mid | \ exit \\ \verb|^^I./stupid < in > out 2 \mid | \ exit \\ \verb|^^Idiff out 1 out 2 \mid | \ exit \\ \verb|^^Iecho \$i OK \\ \\ \text{done} \end{array}
```

### 4 Java

 $Java\ template:$ 

```
import java.io.BufferedReader;
import java.io.FileNotFoundException;
import java.io.FileReader;
import java.io.IOException;
import java.io.InputStreamReader;
import java.io.PrintWriter;
import java.util.*;
public class Main {
  ^IFastScanner in;
{\hat{\ }} {\hat{\ }} {\hat{\ }} {\hat{\ }} IPrintWriter\ out;
^^Ivoid solve() {
^{\hat{}}I^{\hat{}}I int a = in.nextInt();
^{\hat{}}I^{\hat{}}I int b = in.nextInt();
^\smallfrown ^\smallfrown I\}
^{^{\wedge}}I
^^Ivoid run() {
^ ^ I ^ ^ It ry {
^{\hat{I}^{\hat{I}}}I^{\hat{I}} = \text{new FastScanner}("input.txt");
\label{eq:continuity} $$ ^-\tilde{I}^-\tilde{I}^- = \operatorname{new} \operatorname{PrintWriter}("output.txt"); $$
^^I^^I^^Isolve();
^^I^^I^^Iout.flush();
^^I^^I^^Iout.close();
{^\smallfrown}{^\smallfrown}I{^\smallfrown}{^\smallfrown}I\}\ catch\ (FileNotFoundException\ e)\ \{
{^{\smallfrown}}I{^{\smallfrown}}{^{\smallfrown}}Ie.printStackTrace();
^{^{\wedge}}I^{^{\wedge}}I^{^{\wedge}}ISystem.exit(1);
^{\land}I^{\land}I
^{\smallfrown}I
^{\sim} ^{\sim} ^{\circ}
^^Iclass FastScanner \{
^^I^^IStringTokenizer st;
^^I^^Ipublic FastScanner() {
^{\hat{}}I^{\hat{}}I^{\hat{}}I new BufferedReader(new
^^I^^Ipublic FastScanner(String s) {
^\smallfrown I^\smallfrown ^\smallfrown I^\smallfrown ^\smallfrown Itry\ \{
^{\hat{I}^{\hat{I}}} I ^{\hat{I}^{\hat{I}}} I hew BufferedReader(new FileReader(s));
\label{eq:catch} \verb|^^I^^I^^I| \ catch \ (FileNotFoundException \ e) \ \{
{^\smallfrown} I{^\smallfrown} I{^\smallfrown} I{^\smallfrown} {^\smallfrown} Ie.printStackTrace();
^{\land} ^{\sqcap} I ^{\land} I
^{\  \, \widehat{}} I^{\  \, \widehat{}} \operatorname{IString} \ \operatorname{nextToken}() \ \{
^{\hat{I}^{\hat{I}}}I^{\hat{I}^{\hat{I}}} while (st == null || !st.hasMoreElements()) {
^^I^^I^^I^^I^^Ist = new StringTokenizer(br.readLine());
 \bar{\Gamma}^{-1} \bar{\Gamma}^{-1} \bar{\Gamma}^{-1}  catch (IOException e) {
{^\smallfrown} I e.printStackTrace();
```

```
\neg \neg I \neg \neg I \neg \neg I \neg \neg I
{}^\smallfrown {}^\square {}^\smallfrown {}^\square {}^\smallfrown {}^\square {}^\smallfrown {}^\square {}^\square
 ^{\land} I ^{\land} I
 ^^I^^Iint nextInt() {
\verb| ^^I ^^I ^^I return \ Integer.parseInt(nextToken()); \\
{}^\smallfrown {}^\smallfrown I {}^\smallfrown {}^\smallfrown I \}
{\hat{\ }}{\hat{\  }}{\hat{\ }}{\hat{\ }}{\hat{\ }}{\hat{\ }}{\hat{\ }}{\hat{\ }}{\hat{\ }}{\hat{\ }}{\hat{\  }}{\hat{\ }}{\hat{\ }}{\hat{\ }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}{\hat{\  }}
 {^\smallfrown} I {^\smallfrown} I {^\smallfrown} I {^\smallfrown} Urn \ Long.parseLong(nextToken());
^{\land \land}I^{\land \land}I\}
 ^^I^^I^^Ireturn Double.parseDouble(nextToken());
 ^^I^^I}
 {^\smallfrown}{^\smallfrown}{I}{^\smallfrown}{^\smallfrown}{I}{^\smallfrown}{^\smallfrown}{It}\,ry\ \big\{
 \label{eq:char} $$ ^{-1}^{-1}^{-1}^{-1}^{-1}^{-1}$ (char) (br.read());
 {^{\smallfrown}}I{^{\smallfrown}}I{^{\smallfrown}}I{^{\smallfrown}}I{^{\smallfrown}}Ie.printStackTrace();
 ^{\circ}I^{\circ}I^{\circ}I
 {^\smallfrown} I {^\smallfrown} I {^\smallfrown} I {^\smallfrown} ireturn \ 0;
~ ~ I ~ ~ I }
 ~ ~ I ~ ~ I ¸
 ^^I^^IString nextLine() {
 ^{1}^{1}^{1}^{1}^{1}^{1}^{1}^{1} Ireturn br.readLine();
 \begin{cal} \beg
 {^{\smallfrown}}I{^{\smallfrown}}I{^{\smallfrown}}I{^{\smallfrown}}I{^{\smallfrown}}Ie.printStackTrace();
 ^^I^^I^^Ireturn "";
 ^^I^^I}
 ^^Ipublic static void main(String[] args) {
 ^{\smallfrown} \, ^{\smallfrown} I \}
 }
```

# 2 Big numbers

### 5 Big uint

```
#include <cstdio>
#include <cassert>
#include < cstring>
#include < cmath>
#include <algorithm>
#define forn(i, n) for (int i = 0; i < (int)(n); i++)
typedef long long ll;
const int BASE LEN = 9;
const int NUM LEN = 50000 / BASE LEN + 2; // LEN <=
→ NUM_LEN * BASE_LEN
const int BASE = pow(10, BASE_LEN);
const ll INF = 8e18, ADD = INF / BASE;
struct num {
 ll a[NUM_LEN];
 int len; // always > 0
 inline const ll& operator [] ( int i ) const { return a[i]; }
 inline ll& operator [] ( int i ) { return a[i]; }
 num& operator = ( const num &x ) { // copy
  len = x.len;
```

```
memcpy(a, x.a, sizeof(a[0]) * len);
 return *this;
num(const num \&x) {*this = x;} // copy
num() \{ len = 1, a[0] = 0; \} // 0
num( ll x ) { // x
 len = 0;
 while (!len \mid \mid x) {
   assert(len < NUM LEN); // to catch overflow
   a[len++] = x \% \overline{BASE}, x /= BASE;
}
num& cor() {
 while (a[len - 1] >= BASE) {
   assert(len < NUM_LEN); // to catch overflow
   if (a[len - 1] >= 2 \times BASE)
    a[len] = a[len - 1] / BASE, a[len - 1] \% = BASE;
    a[len] = 1, a[len - 1] -= BASE;
   len++;
 while (len > 1 && !a[len - 1])
  len--;
 return *this;
int length() {
 if (!len)
  return 0:
 int x = a[len - 1], res = 0;
 assert(x);
 while (x | !res)
  x /= 10, res++;
 return res + (len - 1) * BASE LEN;
void out() const {
 int i = len - 1;
 printf("%d", (int)a[i--]);
 while (i >= 0)
  printf("%0*d", BASE LEN, (int)a[i--]);
 puts("");
void init( const char *s ) {
 int sn = strlen(s);
 while (sn && s[sn - 1] \leq 32)
  sn--;
 len = (sn + BASE LEN - 1) / BASE LEN;
 memset(a, 0, sizeof(a[0]) * len);
 forn(i, sn) {
  ll \&r = a[(sn - i - 1) / BASE LEN];
  r = r * 10 + (s[i] - '0');
 static const int L = NUM_LEN * BASE_LEN + 1;
 static char s[L];
 if \ (!fgets(s, \, L, \, stdin)) \\
  return 0;
 assert(!s[L - 2]);
 init(s);
 return 1;
void mul2() {
 forn(i, len)
```

```
a[i] <<= 1;
                                                                                     };
 forn(i, len - 1)
   if (a[i] >= BASE)
    a[i + 1]++, a[i] -= BASE;
                                                                                       while (a.len < b.len)
 cor();
                                                                                        \mathbf{a}[\mathbf{a}.\mathbf{len}++]=0;
                                                                                       forn(i, b.len)
                                                                                        a[i] += b[i];
void div2() {
                                                                                       forn(i, a.len - 1)
 for (int i = len - 1; i >= 0; i--) {
                                                                                        if (a[i] >= BASE)
   if (i && (a[i] & 1))
                                                                                          a[i] -= BASE, a[i + 1]++;
    a[i - 1] += BASE;
                                                                                       return a.cor();
   a[i]>>=1;
 cor();
}
                                                                                       while (a.len < b.len)
                                                                                        \mathbf{a}[\mathbf{a}.\mathbf{len}++]=0;
static ll cmp( ll *a, ll *b, int n ) {
                                                                                       forn(i, b.len)
 while (n--)
                                                                                        a[i] = b[i];
                                                                                       forn(i, a.len - 1)
   if (a[n] := b[n])
     \mathrm{return}\ a[n]\ \text{-}\ b[n];
                                                                                        if (a[i] < 0)
                                                                                          a[i] += BASE, a[i + 1]--;
 return 0;
}
                                                                                       return a.cor();
int cmp( const num &b ) const {
 if (len!= b.len)
   return len - b.len;
 for (int i = len - 1; i >= 0; i--)
                                                                                      if (k == 1)
   if (a[i] \vdash= b[i])
                                                                                        return a;
     return a[i] - b[i];
                                                                                       if (k == 0) {
                                                                                        a.len = 0;
 return 0;
}
                                                                                        return a;
bool zero() {
                                                                                       forn(i, a.len)
                                                                                        a[i] *= k;
 return len == 1 && |a[0]|;
                                                                                       forn(i, a.len - 1)
                                                                                        \mathrm{if}\ (a[i]>=\mathrm{BASE})
/** c = this/b, this %= b */
num &div( num b, num &c ) {
                                                                                       return a.cor();
 c.len = len - b.len;
 for (int i = c.len; i >= 0; i--) {
   int k = (1.0L * a[len - 1] * BASE + (len >= 2 ? a[len - 2] : 0)) / 

\leftrightarrow (1.0L * b[b.len - 1] * BASE + (b.len >= 2 ? b[b.len - 2] + 1 :
                                                                                       if (k == 1)
    \hookrightarrow 0));
                                                                                        return a:
   c[i] = 0;
                                                                                       assert(k != 0);
   if (k > 0) {
                                                                                       for (int i = a.len - 1; i > 0; i--)
     c[i] += k;
     forn(j, b.len)
                                                                                       a[0] /= k;
      a[i + j] = (ll)b[j] * k;
                                                                                       return a.cor();
     forn(j, b.len)
       if (a[i+j] < 0) {
        ll k = (-a[i + j] + BASE - 1) / BASE;
         a[i\,+\,j] \;+=\; k\;*\; BASE,\; a[i\,+\,j\,+\,1]\;-=\; k;
                                                                                       forn(i, a.len)
   if (i)
                                                                                        forn(j, b.len)
     len--, a[len - 1] += a[len] * BASE, a[len] = 0;
   else if (cmp(a, b.a, b.len) >= 0) {
     c[0]++;
                                                                                       x.len = a.len + b.len - 1;
     forn(j, b.len)
                                                                                       forn(i, x.len - 1)
       if ((a[j] -= b[j]) < 0)
                                                                                        if (x[i] >= BASE)
         a[j] += BASE, a[j + 1]--;
                                                                                       return x.cor();
 if (c.len < 0)
   c[c.len = 0] = 0;
 forn(i, c.len)
                                                                                     \Rightarrow == 0; 
   if (c[i] >= BASE)
     c[i \ + \ 1] \ + = \ c[i] \ / \ BASE, \ c[i] \ \% = \ BASE;
                                                                                     \rightarrow != 0: 
 c.len += (!c.len || c[c.len]);
                                                                                     → 0; }
 return cor();
```

```
num& operator += ( num &a, const num &b ) {
num& operator -= ( num &a, const num &b ) {
 assert(a[a.len - 1] >= 0); // a >= b
num& operator *= ( num &a, int k ) {
    a[i + 1] += a[i] / BASE, a[i] \% = BASE;
num& operator /= ( num &a, int k ) {
  a[i - 1] += (ll)(a[i] \% k) * BASE, a[i] /= k;
num& mul( const num &a, const num &b, num &x ) {
 assert(a.len + b.len - 1 \le NUM LEN);
 memset(x.a, 0, sizeof(x[0]) * (a.len + b.len - 1));
    if ((x[i + j] += a[i] * b[j]) >= INF)
     x[i + j + 1] += ADD, x[i + j] -= INF;
    x[i + 1] += x[i] / BASE, x[i] \% = BASE;
bool operator == (const num &a, const num &b) { return a.cmp(b)
bool operator != ( const num &a, const num &b ) { return a.cmp(b)
bool operator < ( const num &a, const num &b ) { return a.cmp(b) <
```

```
bool operator > (const num &a, const num &b) { return a.cmp(b) >
                                                                              inline Num operator *(dbl k) const{
bool operator <= ( const num &a, const num &b ) { return a.cmp(b)
                                                                               return Num(x*k, y*k);
\leftrightarrow \langle = 0; \}
bool operator >= ( const num &a, const num &b ) { return a.cmp(b)
                                                                              inline Num operator *(const Num &B) const {
                                                                               return Num(x*B.x - y*B.y, x*B.y + y*B.x);
\Rightarrow >= 0; }
num& add( const num &a, const num &b, num &c ) { c = a; c += b;
                                                                              inline void operator += (const Num &B){
                                                                               x\!+\!=\!B.x,\,y\!+\!=\!B.y;
num & sub( const num &a, const num &b, num &c ) { c = a; c = b;
                                                   \{ c = a; c *= k; 
                                                                              inline void operator /=(dbl k){
num& mul( const num &a, int k, num &c)
\hookrightarrow return c; }
                                                                               x/=k, y/=k;
                                                  \{c = a; c \neq k; return \}
num\&\ div(\ const\ num\ \&a,\ int\ k,\ num\ \&c )
                                                                              inline void operator *=(const Num &B){
\hookrightarrow c; }
                                                                               *this = *this * B;
num& operator *= ( num &a, const num &b ) {
 static num tmp;
                                                                            };
 mul(a, b, tmp);
                                                                            inline Num sqr(const Num &x){ return x * x; }
 return a = tmp;
                                                                            inline Num conj(const Num &x){ return Num(x.real(), -x.imag()); }
num operator ^ ( const num &a, int k ) {
                                                                            inline int getN(int n){
 num res(1);
                                                                             int k = 1;
                                                                              while(k < n)
 forn(i, k)
  res *= a;
                                                                               k << = 1;
 return res;
                                                                             return k;
num& gcd binary( num &a, num &b) {
                                                                            const int LOG = 18;
                                                                            const int MAX N = 1 \ll LOG;
 int cnt = 0:
 while (!a.zero() && !b.zero()) {
   while (!(b[0] \& 1) \& \& !(a[0] \& 1))
                                                                            Num rt[MAX N];
    cnt++, a.div2(), b.div2();
                                                                            int rev[MAX N];
   while (!(b[0] & 1))
    b.div2();
                                                                            void fft(Num *a, int n){
   while (!(a[0] \& 1))
                                                                              assert(rev[1]); // don't forget to init
    a.div2();
                                                                              \begin{array}{l} \textbf{int} \ q = MAX\_N \ / \ n; \end{array}
                                                                              forn(i, n)
   if (a.cmp(b) < 0)
    b = a;
                                                                               if(i < rev[i] / q)
                                                                                 swap(a[i], a[rev[i] / q]);
   else
    a = b;
                                                                              for(int k = 1; k < n; k <<= 1)
                                                                               rac{(int i = 0; i < n; i += 2 * k)}{}
 if (a.zero())
                                                                                 forn(j, k){
                                                                                  const Num z = a[i + j + k] * rt[j + k];
  std::swap(a, b);
                                                                                  a[i\,+\,j\,+\,k]\,=\,a[i\,+\,j]\,\,\hbox{-}\,\,z;
 while (cnt)
  a.mul2(), cnt--;
                                                                                  a[i+j] += z;
                                                                                 }
 return a:
                                                                            }
num& gcd( num &a, num &b ) {
                                                                            void fft inv(Num *a, int n){
 static num tmp;
                                                                             fft(a, n);
 return b.zero()? a : gcd(b, a.div(b, tmp));
                                                                              reverse(a + 1, a + n);
                                                                             forn(i,\,n)
                                                                               a[i] /= n;
   FFT
                                                                            void double fft(Num *a, Num *fa, Num *fb, int n){ // only if you
//typedef complex<dbl> Num;
                                                                             → need it
struct Num{
                                                                             fft(a, n);
 dbl x, y;
                                                                              const int n1 = n - 1;
 Num()\{\}
                                                                              forn(i, n){
 Num(dbl _x, dbl _y):x(_x),y(_y)\{\}
                                                                               const Num & z0 = a[i], & z1 = a[(n - i) & n1];
                                                                               fa[i] = Num(z0.real() + z1.real(), z0.imag() - z1.imag()) * 0.5;
 inline dbl real() const { return x; }
                                                                               fb[i] = Num(z0.imag() + z1.imag(), z1.real() - z0.real()) * 0.5;
 inline dbl imag() const{ return y; }
                                                                            }
 inline Num operator + (const Num &B) const {
   \texttt{return Num}(x\!+\!B.x,\,y\!+\!B.y);
                                                                            Num tmp[MAX N];
                                                                            template<class T>
 inline Num operator - (const Num &B) const {
                                                                            void mult(T *a, T *b, T *r, int n){ // n = 2^k
   return Num(x-B.x, y-B.y);
```

```
forn(i, n)
  tmp[i] = Num((dbl)a[i], (dbl)b[i]);
                                                                              mult(a, b, tc, N, MOD1);
 fft(tmp, n);
                                                                              mult(a, b, td, N, MOD2);
 const int n1 = n - 1;
 const Num c = Num(0, -0.25 / n);
 fornr(i, n / 2 + 1){
  const int j = (n - i) \& n1;
   const\ Num\ z0 = sqr(tmp[i]),\ z1 = sqr(tmp[j]);
   tmp[i] = (z1 - conj(z0)) * c;
   t\, mp[j]\, =\, (z0\, \hbox{-}\, conj(z1))\, *\, c;
                                                                                  Data Structures
 fft(tmp, n);
                                                                                 Centroid Decomposition
 forn(i, n)
   r[i] = (T)round(tmp[i].real());
                                                                            vector < int > g[MAX N];
                                                                            int d[MAX_N], par[MAX_N], centroid;
                                                                            //d par -
void init(){ // don 't forget to init
 forn(i, MAX N)
                                                                            int find(int v, int p, int total) {
   rev[i] = (rev[i >> 1] >> 1) + ((i \& 1) << (LOG - 1));
                                                                              int size = 1, ok = 1;
                                                                              for (int\ to\ :\ g[v])
 rt[1] = Num(1, 0);
                                                                               if (d[to] == -1 \&\& to != p) {
 for(int k = 1, p = 2; k < LOG; ++k, p *= 2){
                                                                                int s = find(to, v, total);
   const Num x(cos(M_PI / p), sin(M_PI / p));
                                                                                 if (s > total / 2) ok = 0;
   for(int i = p / 2; i < p; ++i)
                                                                                size += s;
    rt[2 * i] = rt[i], rt[2 * i + 1] = rt[i] * x;
                                                                              if (ok & \& size > total / 2)
}
                                                                               centroid = v;
                                                                              return size;
    FFT by mod and FFT with digits up to 10<sup>6</sup>
Num ta[MAX N], tb[MAX N], tf[MAX N], tg[MAX N];
                                                                              // do something
                                                                              for (int to: g[v])
const int HALF = 15;
                                                                               if (d[to] == -1 \&\& to != p)
void mult(int *a, int *b, int *r, int N, int mod){
                                                                                 calc_in_component(to, v, level);
 int tw = (1 << HALF) - 1;
 forn(i, N){
   int x = int(a[i] \% mod);
                                                                            //fill(d, d + n, -1)
   ta[i] = Num(x \& tw, x >> HALF);
                                                                            //decompose(0, -1, 0)
 forn(i, N){
                                                                              find(root, -1, find(root, -1, INF));
                                                                              \quad \textbf{int} \ c = \ centroid;
  int x = int(b[i] \% mod);
   tb[i] = Num(x \& tw, x >> HALF);
                                                                              par[c] = parent;
                                                                              d[c]\,=\,lev\,el;
 fft(ta, N);
 fft(t\,b,\ N);
                                                                              for(int to : g[c])
                                                                               if(d[to] == -1)
 forn(i, N){
                                                                                 decompose(to, c, level + 1);
  int j = (N - i) & (N - 1);
   Num a1 = (ta[i] + conj(ta[j])) * Num(0.5, 0);
   Num a2 = (ta[i] - conj(ta[j])) * Num(0, -0.5);
                                                                                 Convex Hull Trick
   Num b1 = (tb[i] + conj(tb[j])) * Num(0.5 / N, 0);
   Num b2 = (tb[i] - conj(tb[j]))* Num(0, -0.5 / N);
                                                                            struct Line {
   tf[j] = a1 * b1 + a2 * b2 * Num(0, 1);
                                                                              int k, b;
   tg[j]\,=\,a1\,\,{}^*\,\,b2\,+\,a2\,\,{}^*\,\,b1;
                                                                              Line() \{ \}
                                                                              Line(int kk, int bb): k(kk), b(bb) {}
                                                                              LL get(int x) \{
                                                                               return b + k * 111 * x;
 fft(tf, N);
 fft(tg, N);
                                                                              bool operator <(const Line &l) const {
                                                                               \operatorname{return} k < l.k; // >
 forn(i, N){
  LL aa = LL(tf[i].x + 0.5);
   LL bb = LL(tg[i].x + 0.5);
   LL cc = LL(tf[i].y + 0.5);
                                                                                (a,b) (a,c)
   r[i] = int((aa + ((bb \% mod) << 15) + ((cc \% mod) << 30)) \%
   \rightarrow mod);
                                                                            struct Convex {
int tc[MAX_N], td[MAX_N];
                                                                              vector <Line> st;
```

```
const int MOD1 = 1.5e9, MOD2 = MOD1 + 1;
void multLL(int *a, int *b, LL *r, int N){
   r[i] = tc[i] + (td[i] - tc[i] + (LL)MOD2) * MOD1 % MOD2 * MOD1;
void calc in component(int v, int p, int level) {
void decompose(int root, int parent, int level) {
 calc in component (centroid, -1, level);
inline bool check(Line a, Line b, Line c) {
 return (a.b - b.b) * 111 * (c.k - a.k) < (a.b - c.b) * 111 * (b.k - a.k);
```

inline void add(Line l) {

return res:

```
while(\mathbf{sz}(\mathbf{st}) >= 2 \&\& !\mathbf{check}(\mathbf{st}[\mathbf{sz}(\mathbf{st}) - 2], \, \mathbf{st}[\mathbf{sz}(\mathbf{st}) - 1], \, \mathbf{l}))
     st.pop back();
   st.pb(l);
 int get(int x) {
   int l = 0, r = sz(st);
   while (r - l > 1) {
     int m = (l + r) / 2; // >
     if (st[m-1].get(x) < st[m].get(x))
      l = m;
     else
      r = m:
   return l;
 Convex() = default;
 Convex(vector <Line> &lines) {
   st.clear();
   for(Line 1: lines)
     add(l);
 Convex(Line line) {
   st.clear();\\
   st.pb(line);
 Convex(const Convex &a, const Convex &b) {
   vector <\!Line\!> lines;
   lines.resize(sz(a.st) \,+\, sz(b.st));
   merge(all(a.st), all(b.st), lines.begin());
   st.clear();
   for(Line 1: lines)
     add(1);
};
      DSU
10
namespace DSU {
 int pr[MAX N], rank[MAX N];
  int get(int v) {
    \mathtt{return}\ v == \mathtt{pr}[v]\ ?\ \textcolor{red}{v} : \mathtt{pr}[v] = \mathtt{get}(\mathtt{pr}[v]);
  void unite(int v, int u) {
    v = get(v), u = get(u);
     if (v == u)
      return:
     if (rank[v] < rank[u])
      swap(v, u);
     pr[u] = v;
     if (rank[v] == rank[u])
       ++\operatorname{rank}[v];
  void init(int n) {
   forn (i, n)
     rank[i] = 0, pr[i] = i;
}
       Fenwick Tree
11
namespace FenwickTree {
 \quad \text{int } t[MAX\_N]; \\
 int n;
 int get(int ind) {
    int res = 0;
    for (; ind >= 0; ind &= (ind + 1), ind--)
```

res += t[ind];

```
void add(int ind, int x) {
    for (; ind < n; ind |= (ind + 1))
     t[ind] += x;
 int sum(int l, int r) \{ //[l, r) \}
   \mathtt{return}\ \mathtt{get}(\mathtt{r}\ \mathtt{-}\ 1)\ \mathtt{-}\ \mathtt{get}(\mathtt{l}\ \mathtt{-}\ 1);
      Hash Table
12
namespace HashTable {
 typedef long long H;
 {\tt const\ int\ HT\_SIZE} = 1{<<}20,\,{\tt HT\_AND} = \,{\tt HT\_SIZE} - 1,
 \rightarrow HT_SIZE_ADD = HT_SIZE / 100;
 H ht[HT_SIZE + HT_SIZE_ADD];
 int data[HT_SIZE + HT_SIZE_ADD];
 int get(const H &hash){
   ll k = ((ll) hash) \& HT AND;
   while(ht[k] && ht[k] != hash)
     ++\mathbf{k};
   return k;
 }
 void insert(const H &hash, int x){
   int k = get(hash);
   if(!ht[k])
    ht[k] = hash, data[k] = x;
 bool count (const H & hash, int x){
  int k = get(hash);
   return ht[k] != 0;
}
      Heavy Light Decomposition
namespace HeavyLightDecomposition {
 vi g[MAX_N];
 \begin{array}{ll} & \text{int size}[MAX\_N], \ comp[MAX\_N], \ num[MAX\_N], \ top[MAX\_N], \\ & \hookrightarrow \quad pr[MAX\_N], \ tin[MAX\_N], \ tout[MAX\_N]; \end{array}
 vi t[MAX_N], toPush[MAX_N], lst[MAX_N];
 int curPath = 0, curTime = 0;
 void pushST(int path, int v, int vl, int vr) {
   if (toPush[path][v] != -1) {
    if (vl != vr - 1)
      forn (j, 2)
        toPush[path][2 * v + j] = toPush[path][v];
      t[path][v] = toPush[path][v];
     toPush[path][v] = -1;
 int getST(int path, int v, int vl, int vr, int ind) {
   pushST(path, v, vl, vr);
   if (vl == vr - 1)
    return t[path][v];
   int vm = (vl + vr) / 2;
   if (ind >= vm)
     return getST(path, 2 * v + 1, vm, vr, ind);
   return getST(path, 2 * v, vl, vm, ind);
```

int curSize = 1;

```
void setST(int path, int v, int vl, int vr, int l, int r, int val) {
                                                                                    while (curSize < sz(lst[i]))
 if (vl >= l && vr <= r) {
                                                                                     curSize *= 2;
   toPush[path][v] = val;
                                                                                    t[i].resize(curSize * 2);
                                                                                    toPush[i] = vi(curSize * 2, -1);
   pushST(path, v, vl, vr);
   return:
                                                                                    //initialize t[i]
 pushST(path, v, vl, vr);
                                                                                 }
                                                                               }
 if (vl >= r || l >= vr)
  return;
 int vm = (vl + vr) / 2;
                                                                                     Next Greater in Segment Tree
 \mathbf{setST}(\mathbf{path},\; 2\; *\; \mathbf{v},\, \mathbf{vl},\, \mathbf{vm},\; \mathbf{l},\; \mathbf{r},\, \mathbf{val});
 setST(path, 2 * v + 1, vm, vr, l, r, val);
 t[path][v] = \min(t[path][2 * v], \, t[path][2 * v + 1]);
                                                                               int nextGreaterX(int v, int l, int r, int pos, int x) {
                                                                                 if (r \le pos + 1 || tree[v] \le x)
                                                                                  return INF;
bool isUpper(int v, int u) {
                                                                                 if (v >= tSize)
  \mathtt{return}\ tin[v] <= tin[u]\ \&\&\ tout[v] >= tout[u];
                                                                                  return v - tSize;
                                                                                 int ans = nextGreaterX(2 * v, l, (l + r) / 2, pos, x);
                                                                                 if (ans == INF)
int getHLD(int v) {
                                                                                  ans = nextGreaterX(2 * v + 1, (l + r) / 2, r, pos, x);
 return getST(comp[v], 1, 0, sz(t[comp[v]]) / 2, num[v]);
                                                                                 return ans;
int setHLD(int v, int u, int val) {
                                                                               15
                                                                                     Sparse Table
 int ans = 0, w = 0;
 forn (i, 2) {
                                                                               namespace SparseTable {
    while (!isUpper(w = top[comp[v]], u))
                                                                                 int st[MAX_N][MAX_LOG];
int lg[MAX_N];
     setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, 0, num[v] + 1, val), v =
      \hookrightarrow pr[w];
   swap(v, u);
                                                                                 int get(int l, int r) { //[l, r)
                                                                                  int curLog = lg[r - l];
 setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, min(num[v], num[u]),
                                                                                  return \ min(st[l][curLog], \ st[r - (1 << curLog)][curLog]);
 \rightarrow max(num[v], num[u]) + 1, val);
 return ans;
                                                                                 void initSparseTable(int *a, int n) {
                                                                                  \lg[1] = 0;
void dfs(int v, int p) {
                                                                                  forab (i, 2, n + 1)
 tin[v] = curTime++;
                                                                                    \lg[i] = \lg[i / 2] + 1;
 size[v] = 1;
                                                                                  forn (i, n)
 pr[v] = p;
                                                                                    st[i][0] = a[i];
 for (int \ u : g[v])
                                                                                  forn (i, lg[n])
   if (u != p) {
                                                                                    forn (j, n - (1 << (i + 1)) + 1)
     dfs(u, v);
                                                                                      st[j][i + 1] = min(st[j][i], st[j + (1 << i)][i]);
     size[v] += size[u];
                                                                               }
 tout[v] = curTime++;
                                                                                      Treap (Rope)
void build(int v) {
                                                                               #include < bits/stdc++.h>
 if (v == 0 || size[v] * 2 < size[pr[v]]) {
                                                                               const int INF=1e9;
   top[curPath] = v;
                                                                               using namespace std;
   comp[v] = curPath;
   num[v]\,=\,0;
                                                                               struct Node;
   curPath++;
                                                                               typedef Node *pNode;
 else {
                                                                               struct Node{
   comp[v] = comp[pr[v]];
                                                                                  static pNode null;
   num[v] = num[pr[v]] + 1;
                                                                                  pNode l, r;
                                                                                  int y, val, size, m;
 lst[comp[v]].pb(v);
 for\ (\textbf{int}\ \textbf{u}:g[v])
                                                                                  Node(): l(this), r(this), y(-1), val(INF), size(0), m(INF){}
     if (u != pr[v])
                                                                                  Node(int \ v): l(null), r(null), y(rand()), val(v), size(1), m(v) \{ \}
     build(u);
                                                                                  void calc(){
                                                                                      size = 1 + l-> size + r-> size;
void initHLD() {
                                                                                      m = min(val, min(l->m, r->m));
 dfs(0, 0);
 build(0);
                                                                               };
 forn (i, curPath) {
                                                                               pNode\ Node::null = new\ Node();
```

```
void merge(pNode &t, pNode l, pNode r){
   if (l == Node::null)
      t = r;
   else if(r == Node::null)
     t = 1;
   else if (l-y < r-y)
     merge(l->r, l->r, r), (t = l)->calc();
      merge(r\!-\!>\!l,\;l,\;r\!-\!>\!l),\,(t\;=\;r)\!-\!>\!calc();
void split(pNode t, pNode &l, pNode &r, int k){
   if(t == Node::null)\\
     l = r = t;
   else if(t->l->size>= k)
     {
m split}(t->l,\,l,\,t->l,\,k),\,(r=t)->{
m calc}();
      split(t->r, t->r, r, k-t->l->size-1), (l = t)->calc();
}
void insert(pNode &root, int k, int x){
 pNode r, n = new Node(x);
 split(root, root, r, k);
 merge(root, root, n);
 merge(root, root, r);
void erase(pNode &root, int k){
 pNode l. n:
 split(root, l, root, k);
 split(root, n, root, 1);
 merge(root, l, root);
pNode build(int k){
   if(k == 1)
      return new Node(0);
   pNode root:
   merge(root, build(k / 2), build((k + 1) / 2));
   return root;
void print(pNode t, bool root = 1){
   if(t != Node::null){
      print(t->l, 0);
      printf("%d ", t->val);
      print(t->r,0);
   if (root)
    puts("");
int main(){
 pNode r=Node::null; // r is an empty tree
    work with r
      Segtree 2D
17
const int ST SZ=1<<10, ST SZ2=2*ST SZ;
struct Node 1d{
 Node 1d *l,*r;
 LL val, need;
 Node 1d():l(NULL),r(NULL),val(0),need(0){}
 inline void norm(){
   if (l==NULL)
    l=new Node_1d();
   if (r==NULL)
    r=new Node_1d();
```

```
LL Get(int L0,int R0,int L=0,int R=ST SZ){
  if(L0>=R \parallel L>=R0)
    return 0;
  if(L0 <= L \&\& R <= R0)
    return val;
  int a=\max(L0,L), b=\min(R0,R), M=(L+R)>>1;
  norm();
  return l->Get(L0,R0,L,M)+r->Get(L0,R0,M,R)+need*LL(b-a);
 void Add(int L0,int R0,int x,int L=0,int R=ST SZ){
  if(L0>=R \parallel L>=R0)
    return;
  _{\rm if(L0<=L~\&\&~R<=R0)\{}
    need+=x;
    val += x*LL(R-L);
    return;
  int M = (L+R) > 1;
  1 - Add(L0,R0,x,L,M), r - Add(L0,R0,x,M,R);
  val=l->val+r->val+need*(R-L);
 }
};
struct Node 2d{
 Node_2d *l,*r;
 Node\_1d *val,*need;
 Node_2d():l(NULL),r(NULL),val(new Node_1d()),need(new
 \rightarrow Node 1d()){}
 inline void norm(){
  if(l==NULL)
    l=new Node 2d();
  if(r==NULL)
    r=new Node 2d();
 LL Get(int L0,int R0,int L1,int R1,int L=0,int R=ST SZ){
  if(L0>=R \mid\mid L>=R0)
    return 0;
  if(L0<=L && R<=R0)
    return val->Get(L1,R1);
  int a=max(L0,L), b=min(R0,R), M=(L+R)>>1;
  norm();
  return l-> Get(L0,R0,L1,R1,L,M)+r-
      > Get(L0,R0,L1,R1,M,R) + need-> Get(L1,R1)*LL(b-a);
 void Add(int L0,int R0,int L1,int R1,int x,int L=0,int R=ST SZ){
  if(L0>=R \parallel L>=R0)
    return;
  if(L0<=L && R<=R0){
    need->Add(L1,R1,x);
    val->Add(L1,R1,x*LL(R-L));
    return:
  int a=\max(L0,L), b=\min(R0,R), M=(L+R)>>1;
  l->Add(L0,R0,L1,R1,x,L,M), r->Add(L0,R0,L1,R1,x,M,R);
  val->Add(L1,R1,x*LL(b-a));
};
int main(){
 Node 2d *Z=new Node 2d();
 int x,y,Q;
 \operatorname{scanf}("%d%d%d",\&x,\&y,\&Q);
 forn(i,Q){
  int type,a,b,c,d;
  scanf("\%d\%d\%d\%d\%d",\&type,\&a,\&b,\&c,\&d);
  --a, --b;
```

```
if(type==1){
     int w;
    scanf("%d",&w);
     Z->Add(a,c,b,d,w);
     printf(lld'' \setminus n'', Z - Set(a, c, b, d));
}
18
      Segtree 2D — Fenwick
const int MAX N=1002;
LL F[4][MAX_N][MAX_N];
int N,M;
inline int Z(int a){
 return a\&^{\sim}(a-1);
inline void add(int k,int x,int y,LL a){
 for(;x <= N;x+=Z(x))
   for(int j=y;j<=M;j+=Z(j))
     F[k][x][j] \! + \! = \! a;
inline LL get(int k,int x,int y){
 LL s=0;
 for(x>0;x=Z(x))
   for(int j=y;j>0;j-=Z(j))
    s+\!=\!F[k][x][j];
 return s;
inline LL Get(int a,int b){
 return LL(a+1)*(b+1)*get(0,a,b)-(b+1)*get(1,a,b)
     -(a+1)*get(2,a,b)+get(3,a,b);
inline void Add(int a,int b,LL w){
 add(0,a,b,w);
 add(1,a,b,w*a);
 add(2,a,b,w*b);
 add(3,a,b,w*a*b);
inline LL Get(int a,int b,int c,int d){
 \operatorname{return} \operatorname{Get}(c,d)\operatorname{-Get}(a\text{-}1,d)\operatorname{-Get}(c,b\text{-}1)+\operatorname{Get}(a\text{-}1,b\text{-}1);
inline void Add(int a,int b,int c,int d,LL w){
  Add(a,b,w);
 if(d < M)
   Add(a,d+1,-w);
 if(c < N)
   Add(c+1,b,-w);
 if(c < N \&\& d < M)
   Add(c+1,d+1,w);\\
int main(){
 int Q:
 scanf("%d%d%d",&N,&M,&Q);
 forn(i,Q){
   int type,a,b,c,d;
   scanf("%d%d%d%d%d",\&type,\&a,\&b,\&c,\&d);
   if(type==1){
    int w;
    scanf("%d",&w);
     Add(a,b,c,d,w);\\
   }else
     printf(lld"\n",Get(a,b,c,d));
```

```
}
4
     Dynamic Programming
19
     LIS
namespace DP {
 int longestIncreasingSubsequence(vi a) {
   int n = sz(a);
   vi d(n + 1, INF);
   d[0] = -INF;
   forn (i, n)
    *upper\_bound(all(d), a[i]) = a[i];
   fornr (i, n + 1)
    if (d[i] := INF)
     return i;
  assert(0);
5
     Flows
20
     Utilities
//for directed unweighted graph
struct Edge {
 \quad \text{int}\ v,\ u,\ c,\ f;
 Edge() {}
 Edge(int v, int u, int c): v(v), u(u), c(c), f(0) {}
vector < Edge> edges;
inline void addFlow(int e, int flow) {
  edges[e].f += flow;
  edges[e ^1].f = flow;
inline void addEdge(int v, int u, int c) {
 g[v].pb(sz(edges));
 edges.pb(Edge(v, u, c));
 g[u].pb(sz(edges));
 edges.pb(Edge(u, v, 0)); //for undirected 0 should be c
void read(int m) {
  forn (i, m) {
    int v, u, c;
    scanf("%d%d%d", &v, &u, &c);
    addEdge(v - 1, u - 1, c);
}
      Ford-Fulkerson
namespace FordFulkerson {
 int used[MAX_N], pr[MAX_N];
 vi g[MAX_N];
 int curTime = 1;
 #include "Utilities.cpp"
 int dfs(int v, int can, int toPush, int t) {
  if (v == t)
    return can;
   used[v] = curTime;
   for (int edge : g[v]) {
     auto \&e = edges[edge];
     if (used[e.u] := curTime \&\& e.c - e.f >= toPush) {
```

 $\quad \textbf{int} \ flow = \ dfs(e.u, \ min(can, \ e.c \ \text{-} \ e.f), \ toPush, \ t); \\$ 

```
if (flow > 0) {
       addFlow(edge, flow);
       pr[e.u] = edge;
       return flow;
   }
  return 0;
int fordFulkerson(int n, int m, int s, int t) {
 read(m):
 int ansFlow = 0, flow = 0;
 //Without scaling
 while ((flow = dfs(s, INF, 1, t)) > 0) {
  ansFlow \mathrel{+}= flow;
  curTime++;\\
 //With scaling
 fornr (i, INF LOG)
  for (curTime++; (flow = dfs(s, INF, (1 << i), t)) > 0;
  curTime++)
    ansFlow += flow;
 return ansFlow;
```

## Edmonds-Karp

```
namespace EdmondsKarp {
 \mathbf{int}\ \mathbf{used}[\mathbf{MAX\_N}],\ \mathbf{pr}[\mathbf{MAX\_N}],\ \mathbf{d}[\mathbf{MAX\_N}],\ \mathbf{q}[\mathbf{MAX\_N}],
 \hookrightarrow maxFlow[MAX N];
 vi g[MAX_N];
 #include "Utilities.cpp"
 int edmondsKarp(int n, int m, int s, int t) {
   read(m);
   int ansFlow = 0;
   while(1) {
    for (int i = 0; i < n; i++)
      d[i] = INF, maxFlow[i] = 0;
    int head = 0, tail = 0;
    q[tail++] = s;
    d[s] = 0;
    maxFlow[s] = INF;
    while (tail - head > 0) {
      int v = q[head++];
      for (int edge : g[v]) {
        auto &e = edges[edge];
        if (d[e.u] > d[v] + 1 \&\& e.c - e.f > 0) {
         d[e.u] = d[v] + 1;
         maxFlow[e.u] = min(maxFlow[e.v], e.c - e.f);
         q[tail++] = e.u;
         pr[e.u] = edge;
    if (d[t] == INF)
      break:
    for (int u = t; u != s; u = edges[pr[u]].v)
      addFlow(pr[u], maxFlow[t]);
    ansFlow += maxFlow[t];
   return ansFlow;
}
```

```
23 Dinic
namespace Dinic {
 int \ pr[MAX\_N], \ d[MAX\_N], \ q[MAX\_N], \ first[MAX\_N];
 vector < int > g[MAX_N];
 #include "Utilities.cpp"
 int dfs(int v, int can, int toPush, int t) {
  if (v == t)
    return can;
   int sum = 0;
  for (; first[v] < (int) g[v].size(); first[v] ++) \{
    auto \&e = edges[g[v][first[v]]];
    if (d[e.u] != d[v] + 1 || e.c - e.f < toPush)
      continue;
    int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
    addFlow(g[v][first[v]], flow);
    can = flow, sum += flow;
    if (!can)
      return sum;
   return sum;
 bool bfs(int n, int s, int t, int curPush) {
   for (int i = 0; i < n; i++)
    d[i] = INF, first[i] = 0;
   int head = 0, tail = 0;
   q[tail++]\,=\,s;
  d[s] = 0:
   while (tail - head > 0) {
    \quad \text{int } v \, = \, q[head + +]; \\
    for (int edge : g[v]) {
      auto \ \&e = edges[edge]; \\
      if (d[e.u] > d[v] + 1 \&\& e.c - e.f >= curPush) {
       d[e.u] = d[v] + 1;
       q[tail++] = e.u;
    }
  return d[t] != INF;
 int dinic(int n, int m, int s, int t) {
  read(m);
  int ansFlow = 0;
   //Without scaling
  while(bfs(n, s, t, 1))
    ansFlow += dfs(s, INF, 1, t);
   //With scaling
  fornr (j, INF LOG)
    while (bfs(n, s, t, 1 << j))
      ansFlow += dfs(s, INF, 1 << j, t);
   return ansFlow;
}
24
      Hungarian
const int INF = 1e9;
int a[MAX_N][MAX_N];
// \min = \text{sum of a}[pa[i],i]
// you may optimize speed by about 15%, just change all vectors to

→ static arrays

vi Hungrian(int n) {
 vi\ pa(n+1,-1),\ row(n+1,\,0),\ col(n+1,\,0),\ la(n+1);
 forn(k, n) {
```

vi u(n + 1, 0), d(n + 1, INF);

```
pa[n] = k;
   int l = n, x;
   while ((x = pa[l]) != -1) {
     u[l] = 1;
     \quad \text{int minn} = \text{INF}, \, \text{tmp}, \, \text{l0} = \text{l}; \\
     forn(j, n)
      if (!u[j]) {
        \mathrm{if} \ ((t\, mp \,=\, a[x][j] \,+\, row[x] \,+\, col[j]) \,<\, d[j])
          d[j] = tmp, la[j] = l0;
        \mathrm{if}\ (d[j]\,<\,minn)
          minn = d[j], l = j;
     forn(j, n + 1)
      if (u[j])
        col[j] += minn, row[pa[j]] -= minn;
        d[j] \mathrel{-}= minn;
   while (1 != n)
     pa[l] \, = \, pa[la[l]], \, l = \, la[l];
 return pa;
}
       Min Cost Max Flow
25
namespace MinCostMaxFlow {
 const int MAX_M = 1e4;
 \mathbf{int}\ \operatorname{pr}[MAX\_N],\ \operatorname{in}[MAX\_N],\ \operatorname{q}[MAX\_N\ *\ MAX\_M],
      used[MAX N], d[MAX N], pot[MAX N];
 vi g[MAX_N];
 struct Edge {
   int v, u, c, f, w;
   Edge() {}
   Edge(int v, int u, int c, int w): v(v), u(u), c(c), f(0), w(w) {}
 vector < Edge> edges;
 inline void addFlow(int e, int flow) {
    edges[e].f \mathrel{+}= flow;
    edges[e ^ 1].f -= flow;
 inline void addEdge(int v, int u, int c, int w) {
   g[v].pb(sz(edges));
   edges.pb(Edge(v, u, c, w));
   g[u].pb(sz(edges));
   edges.pb(Edge(u, v, 0, -w));
                                                                                       }
 void read(int m) {
    forn (i, m) {
      int v, u, c, w;
       scanf("%d%d%d%d", &v, &u, &c, &w);
       addEdge(v - 1, u - 1, c, w);
 int dijkstra(int n, int s, int t) {
   forn (i, n)
     used[i] = 0, d[i] = INF;
   \mathbf{d}[\mathbf{s}] = 0;
   while (1) {
     int v = -1;
      forn (i, n)
       if (!used[i] && (v == -1 \parallel d[v] > d[i]))
         v = i;
     if (v == -1 || d[v] == INF)
```

break:

```
used[v] = 1;
    for (int edge : g[v]) {
      auto \&e = edges[edge];
      ll w = e.w + pot[v] - pot[e.u];
      if (e.c > e.f \&\& d[e.u] > d[v] + w)
        d[e.u] = d[v] + w, pr[e.u] = edge;
 if (d[t] == INF)
   return d[t];
 forn (i, n)
  pot[i] += d[i];
 return pot[t];
int fordBellman(int n, int s, int t) {
 forn (i, n)
   d[i] = INF;
 int head = 0, tail = 0;
 \mathbf{d}[\mathbf{s}] = 0;
 q[tail++] = s;
 in[s] = 1;
 while (tail - head > 0) {
  int v = q[head++];
   in[v] = 0;
   for (int edge : g[v]) {
    auto &e = edges[edge];
    if (e.c > e.f && d[e.u] > d[v] + e.w) {
      d[e.u] = d[v] + e.w;
      pr[e.u] = edge;
      if (!in[e.u])
       in[e.u] = 1, q[tail++] = e.u;
 return d[t];
int minCostMaxFlow(int n, int m, int s, int t) {
 read(m);
 int ansFlow = 0, ansCost = 0, dist;
 while((dist = dijkstra(n, s, t)) != INF) {
  int curFlow = INF;
   for (int cur = t; cur != s; cur = edges[pr[cur]].v)
    curFlow = min(curFlow, edges[pr[cur]].c - edges[pr[cur]].f);
   for (int cur = t; cur != s; cur = edges[pr[cur]].v)
    addFlow(pr[cur], curFlow);
   ansFlow += curFlow:
   ansCost += curFlow * dist;
 return ansCost;
```

#### 6 Games

#### 26 Retrograde Analysis

```
namespace Games {
 vi g[MAX N]; //reversed edges
 int win[MAX_N], lose[MAX_N], used[MAX_N], deg[MAX_N];
 void dfs(int v) {
  used[v] = 1;
  for (int u : g[v])
    if (!used[u]) {
      if \ (lose[v])
       win[u]=1;
      else if (--deg[u] == 0)
       lose[u] = 1;
```

# 7 Geometry

#include "header.h"

### 27 ClosestPoints (SweepLine)

```
const int N = 2e5;
struct Pnt {
 int x, y, i;
 bool operator <(const Pnt &p) const{
   return mp(y, i) < mp(p.y, p.i);
LL d2 = 8e18, d = (LL)sqrt(d2) + 1;
Pnt p[N];
inline LL sqr(int x){
 return (LL)x * x;
inline void relax(const Pnt &a, const Pnt &b){
 LL \; tmp = sqr(a.x \; \text{-} \; b.x) \; + \; sqr(a.y \; \text{-} \; b.y);
 if (tmp < d2)
   d2 = tmp, d = (LL)(sqrt(d2) + 1 - 1e-9); // round up
inline bool xless(const Pnt &a, const Pnt &b){
 return a.x < b.x;
int main() {
 int n:
 scanf("%d", &n);
 forn(i, n)
   scanf("%d%d", &p[i].x, &p[i].y), p[i].i = i;
 sort(p, p + n, xless);
 set <Pnt> s;
 int l = 0;
 forn(r, n){
   set < Pnt > :: iterator it r = s.lower bound(p[r]), it l = it r;
   for (; it_r != s.end() && it_r->y - p[r].y < d; ++it_r)
    relax(*it r, p[r]);
   while (it _l != s.begin() && p[r].y - (--it _l)->y < d)
    relax(*it_l, p[r]);
   s.insert(p[r]);
   while (l \le r \&\& p[r].x - p[l].x >= d)
    s.erase(p[l++]);
 printf("\%.9f\n", sqrt(d2));
 return 0;
```

```
}
28
       ConvexHull
typedef vector<Pnt> vpnt;
inline bool by Angle (const Pnt &a, const Pnt &b) {
 dbl x = a \% b:
 return eq(x, 0) ? a.len2() < b.len2() : x < 0;
vpnt convexHull(vpnt p){
 int n = sz(p);
 assert(n > 0);
 swap(p[0], *min element(all(p)));
 forab(i, 1, n)
 \mathbf{p}[\mathbf{i}] = \mathbf{p}[\mathbf{i}] - \mathbf{p}[0];
 sort(p.begin() + 1, p.end(), byAngle);
       , (1) (2)
 (1):
 int k = p.size() - 1;
 while(k > 0 \&\& eq((p[k-1] - p.back()) \% p.back(), 0))
 reverse(pi.begin() + k, pi.end());*/
 int rn = 0;
 vpnt r(n);
 r[rn++] = p[0];
 forab(i, 1, n){
   Pnt \ q = \ p[i] + \ p[0];
   while(rn \geq 2 \&\& geq((r[rn - 1] - r[rn - 2]) \% (q - r[rn - 2]), 0)) //
   \hookrightarrow (2) ge
    -- rn ;
   \mathbf{r}[\mathbf{r}\mathbf{n}++] = \mathbf{q};
 r.resize(rn);
 return r;
29
       GeometryBase
#include < bits/stdc++.h>
using namespace std;
typedef long long LL;
typedef long double LD;
typedef double dbl;
const dbl EPS = 1e-9;
const int PREC = 20:
inline bool eq(dbl a, dbl b) { return abs(a-b) <= EPS; }
inline bool gr(dbl a, dbl b) { return a>b+EPS; }
inline bool geq(dbl a, dbl b) { return a>=b-EPS; }
inline bool ls(dbl a, dbl b){ return a < b-EPS; }
inline bool leq(dbl a, dbl b) { return a <= b + EPS; }
struct Pnt{
   dbl\;x,\!y;
   Pnt():x(0),y(0)\{\}
   Pnt(dbl \_x,dbl \_y):x(\_x),y(\_y)\{\}
   inline Pnt operator + (const Pnt &B) const{ return Pnt(x+B.x,
   \hookrightarrow y+B.y); }
   inline Pnt operator - (const Pnt &B) const { return Pnt(x-B.x,
    \hookrightarrow y-B.y); }
   inline dbl operator *(const Pnt &B) const{ return x*B.x + y*B.y; }
   inline dbl operator %(const Pnt &B) const{ return x*B.y - y*B.x; }
    \rightarrow // LL
```

```
inline Pnt operator *(dbl k) const { return Pnt(x*k, y*k); }
   inline Pnt operator /(dbl k) const{ return Pnt(x/k, y/k); }
   inline Pnt operator -() const{ return Pnt(-x, -y); }
   inline void operator += (const Pnt &B){ x+=B.x, y+=B.y; }
   inline void operator -= (const Pnt &B) { x-=B.x, y-=B.y; }
   inline void operator *=(dbl k)\{x^*=k, y^*=k;\}
   inline bool operator == (const Pnt \&B) \{ return abs(x-B.x) \le EPS \}
   \rightarrow && abs(y-B.y)<=EPS; }
   inline bool operator !=(const Pnt &B){ return abs(x-B.x)>EPS ||
   \hookrightarrow \quad abs(y\text{-}B.y) {>} EPS; \ \}
   inline bool operator < (const Pnt &B) { return abs(x-B.x) <= EPS ?
   \rightarrow y<B.y-EPS: x<B.x; }
   inline dbl angle() const { return atan2(y, x); } // LD
   inline dbl len2() const{ return x*x+y*y; } // LL
   inline dbl len() const { return sqrt(x*x+y*y); } // LL, LD
   inline Pnt getNorm() const{
      auto l = len();
      return Pnt(x/l, y/l);
   inline void normalize(){
      auto l = len();
      x/=l, y/=l;
   inline Pnt getRot90() const{ //counter-clockwise
      return Pnt(-y, x);
   inline Pnt getRot(dbl a) const{ // LD
      dbl si = sin(a), co = cos(a);
      return Pnt(x*co - y*si, x*si + y*co);
   inline void read(){
      \quad \text{int } \_x, \_y;
      scanf("%d%d",\&\_x,\&\_y);
      x\!=\!\_x,\;y\!=\!\_y;
   inline void write() const {
      printf("%.*f %.*f ", PREC, (double)x, PREC, (double)y);
};
struct Line{
   dbl a, b, c;
   Line():a(0),b(0),c(0){}
   \operatorname{Line}(\operatorname{dbl} \_a, \operatorname{dbl} \_b, \operatorname{dbl} \_c) : a(\_a), b(\_b), c(\_c)\{\}
   Line(const Pnt &A, const Pnt &B){ // it normalizes (a,b),

→ important in d(), normalToP()

      Pnt n = (B-A).getRot90().getNorm();
      a = n.x, b = n.y, c = -(a*A.x + b*A.y);
   inline dbl d(const Pnt &p) const{ return a*p.x + b*p.y + c; }
   inline Pnt no() const {return Pnt(a, b);}
   inline Pnt normalToP(const Pnt &p) const{ return Pnt(a,b) *
   \rightarrow (a*p.x + b*p.y + c); }
   inline void write() const {
      printf("%.*f %.*f %.*f ", PREC, (double)a, PREC, (double)b,
          PREC, (double)c);
};
```

### 30 GeometryInterTangent

```
void buildTangent(Pnt p1, dbl r1, Pnt p2, dbl r2, Line &1) { // r1, r2
     = radius with sign
   Pnt p = p2 - p1;
   l.c = r1;
   dbl c2 = p.len2(), c1 = sqrt(c2 - sqr(r2));
   \begin{array}{l} l.a = \left( -p.x \ ^* \left( r1 \ ^- \ r2 \right) \ + \ p.y \ ^* \ c1 \right) \ / \ c2; \\ l.b = \left( -p.y \ ^* \left( r1 \ ^- \ r2 \right) \ - \ p.x \ ^* \ c1 \right) \ / \ c2; \end{array}
   l.c = l.no() * p1;
   assert(eq({\rm l.d(p1)},\ r1));
   assert(eq(l.d(p2), r2));
struct Circle {
   Pnt p;
   dbl r;
};
vector<Pnt> v; // to store intersection
// Intersection of two lines
int line line(const Line &l, const Line &m){
   dbl z = m.a * l.b - l.a * m.b,
          x = m.c * l.b - l.c * m.b,
          y = m.c * l.a - l.c * m.a;
   if(fabs(z) > EPS){
       v.pb(Pnt(-x/z, y/z));
       return 1;
   else if(fabs(x) > EPS || fabs(y) > EPS)
       return 0; // parallel lines
       return 2; // same lines
// Intersection of Circle and line
void circle line(const Circle &c, const Line &l){
   dbl d = l.d(c.p);
   if(fabs(d) > c.r + EPS)
       return;
   if(fabs(fabs(d) / c.r - 1) < EPS)
       v.pb(c.p - l.no() * d);
   else{
       dbl \ s = sqrt(fabs(sqr(c.r) - sqr(d)));
       v.pb(c.p - l.no() * d + l.no().getRot90() * s);
       v.pb(c.p - l.no() * d - l.no().getRot90() * s);
}
// Intersection of two circles
void circle circle(const Circle &a, const Circle &b){
   circle_{line}(a, Line((b.p - a.p) * 2, a.p.len2() - b.p.len2() + sqr(b.r) -
   \rightarrow sqr(a.r)));
// Squared distance between point p and segment [a..b]
dbl dist2(Pnt p, Pnt a, Pnt b){
   if ((p - a) * (b - a) < 0) return (p - a).len2();
   if ((p - b) * (a - b) < 0) return (p - b).len2();
   dbl d = fabs((p - a) \% (b - a));
   return d * d / (b - a).len2();
}
       GeometrySimple
int sign(dbl a) \{ return (a > EPS) - (a < -EPS); \}
// Checks, if point is inside the segment
inline bool inSeg(const Pnt &p, const Pnt &a, const Pnt &b) {
```

return eq((p - a) % (p - b), 0) && leq((p - a) \* (p - b), 0);

const dbl eps = 1e-12;

```
// Checks, if two intervals (segments without ends) intersect AND do
                                                                              dbl sqr(dbl x) { return x * x; }
→ not lie on the same line
inline bool subIntr(const Pnt &a, const Pnt &b, const Pnt &c, const
                                                                              struct pnt {
                                                                                LL operator * (pnt p) { return (LL)x * p.y - (LL)y * p.x; }
→ Pnt &d){
                                                                                LL operator \hat{\ } ( pnt p ) { return (LL)x * p.x + (LL)y * p.y; }
   return
         sign((b - a) \% (c - a)) * sign((b - a) \% (d - a)) == -1 \&\&
                                                                                pnt ort() { return pnt(-y, x); }
         sign((d - c) \% (a - c)) * sign((d - c) \% (b - c)) == -1;
                                                                                dbl ang() { return atan2(y, x); }
}
                                                                                LL d2() { return x * x + y * y; }
// Checks, if two segments (ends are included) has an intersection
inline bool checkSegInter(const Pnt &a, const Pnt &b, const Pnt &c,
                                                                               pnt st, v, p[maxn];
int n, sp, ss[maxn], ind[maxn], no[maxn], cnt[maxn], k = 0, a[maxn],
  return inSeg(c, a, b) || inSeg(d, a, b) || inSeg(a, c, d) || inSeg(b, c, d)
                                                                               \hookrightarrow b[maxn];
   \rightarrow || subIntr(a, b, c, d);
                                                                               dbl ang[maxn];
                                                                               pnt Norm(int k) \{ return (p[a[k]] - p[b[k]]).ort(); \}
inline dbl area(vector<Pnt>p){
                                                                              void AddPlane( int i, int j ){
   dbl s = 0;
   int n = sz(p);
                                                                                a[k] = i, b[k] = j, ind[k] = k;
   p.pb(p[0]);\\
                                                                                ang[k] = Norm(k).ang();
   forn(i, n)
                                                                                k++;
                                                                               }
     s += p[i + 1] \% p[i];
   p.pop back();
   return abs(s) / 2;
                                                                               bool angLess(int i, int j) { return ang[i] < ang[j];}
                                                                               void Unique()
// Check if point p is inside polygon <n, q[]>
int contains_slow(Pnt p, Pnt *z, int n){
                                                                                int i = 0, k2 = 0;
   int cnt = 0:
                                                                                while (i < k)
   forn(j, n)
      Pnt a = z[j], b = z[(j + 1) \% n];
                                                                                 int ma = ind[i], st = i;
      if (inSeg(p, a, b))
                                                                                  pnt no = Norm(ma);
         return -1; // border
                                                                                  for (i++; i < k \&\& fabs(ang[ind[st]] - ang[ind[i]]) < eps; i++)
      if (\min(a.y, b.y) - EPS \le p.y \&\& p.y \le \max(a.y, b.y) - EPS)
         cnt += (p.x < a.x + (p.y - a.y) * (b.x - a.x) / (b.y - a.y));
                                                                                   if ((no ^p[a[ma]]) < (no ^p[a[ind[i]]]))
                                                                                     ma = ind[i];
   return cnt & 1; // 0 = outside, 1 = inside
                                                                                  ind[k2++]\,=\,ma;
}
                                                                                k
                                                                                   = k2;
//for convex polygon
 //assume polygon is counterclockwise-ordered
                                                                               dbl xx, yy, tmp;
bool contains fast(Pnt p, Pnt *z, int n) {
   Pnt \mathbf{o} = \mathbf{z}[0];
                                                                               #define BUILD(a1, b1, c1, i) \
   if (gr((p-o)\%(z[1]-o), 0) || ls((p-o)\%(z[n-1]-o), 0))
                                                                                dbl a1 = Norm(i).x; \
      return 0;
                                                                                dbl b1 = Norm(i).y;
   int l = 0, r = n - 1;
                                                                                tmp = sqrt(a1 * a1 + b1 * b1); \
                                                                                a1 /= tmp, b1 /= tmp; \setminus
   \mathrm{while}(\mathbf{r} - \mathbf{l} > 1)\{
      \quad \quad \mathbf{int} \ m = (l + r) \ / \ 2;
                                                                                d\,bl\,\,c1 = \hbox{-}(a1 * p[a[i]].x + b1 * p[a[i]].y);
      if(gr((p - o) \% (z[m] - o), 0))
                                                                               void FindPoint( int i, int j, dbl step = 0.0 ){
         r = m;
                                                                                BUILD(a1, b1, c1, i);
      else
                                                                                BUILD(a2,\ b2,\ c2,\ j);
         l = m:
   return leq((p - \mathbf{z}[l]) % (\mathbf{z}[r] - \mathbf{z}[l]), 0);
                                                                                xx = -(c1 * b2 - c2 * b1) / (a1 * b2 - a2 * b1);
                                                                                yy = (c1 * a2 - c2 * a1) / (a1 * b2 - a2 * b1);
}
// Checks, if point "i" is in the triangle "abc" IFF triangle in CCW
                                                                                dbl no = sqrt(sqr(a1 + a2) + sqr(b1 + b2));
                                                                                xx += (a1 + a2) * step / no;
                                                                                yy += (b1 + b2) * step / no;
inline int isInTr(int i, int a, int b, int c){
         gr((p[b] - p[a]) \% (p[i] - p[a]), 0) \&\&
         gr((p[c] - p[b]) \% (p[i] - p[b]), 0) \&\&
                                                                               void TryShiftPoint( int i, int j, dbl step )
         gr((p[a] - p[c]) \% (p[i] - p[c]), 0);
                                                                                FindPoint(i, j, step);
}
                                                                                forn(i, k){
      Halfplanes Intersection
32
                                                                                 BUILD(a1, b1, c1, ind[i]);
                                                                                 if (a1 * xx + b1 * yy + c1 < eps)
const int \max n = (int)4e5 + 9;
                                                                                   return;
```

```
}
                                                                                     if (!used[to])
                                                                                      dfs(to);
 puts("Possible");
                                                                                   tsort.pb(v);
 printf("%.20lf %.20lf\n", (double)xx, (double)yy);
                                                                                  void rdfs(int v, int num) {
                                                                                   used[v] = 1;
                                                                                   comp[v] = num;
void PushPlaneIntoStack(int i)
                                                                                   for(int to: rg[v])
  while (sp \ge 2 \&\& ang[i] - ang[ss[sp - 2]] + eps < M PI)
                                                                                     if (!used[to])
   FindPoint(i, ss[sp - 2]);
                                                                                       rdfs(to, num);
   BUILD(a1, b1, c1, ss[sp - 1]);
                                                                                  void addEdge(int a, int b) {
   if ((a1 * xx + b1 * yy + c1) < -eps)
                                                                                   g[a ^ 1].pb(b);
g[b ^ 1].pb(a);
    break;
                                                                                   rg[b].pb(a ^ 1);
   sp--;
                                                                                   rg[a].pb(b ^ 1);
 ss[sp++]\,=\,i;
                                                                                  //n -
                                                                                  bool sat2(const vector <pii> &v, int n) {
int main()
                                                                                   forn(i, sz(v)) {
                                                                                     addEdge(v[i].fst, v[i].snd);
 scanf("%d", &n);
                                                                                   memset(used, 0, sizeof(used));
 forn(i, n)
   scanf("%d%d", &p[i].x, &p[i].y);
                                                                                   forn(i, n)
 \mathbf{p}[\mathbf{n}] = \mathbf{p}[0];
                                                                                     if (!used[i])
                                                                                      dfs(i);
  // Find set of planes
                                                                                   memset(used, 0, sizeof(used));
 forn(i, sp)
                                                                                   int num = 0:
   AddPlane(max(ss[i], ss[i+1]), min(ss[i], ss[i+1]));
                                                                                   fornr(i, n) {
                                                                                     \quad \text{int} \ u = tsort[i]; \\
 forn(i, n - 1)
  AddPlane(i + 1, i);
                                                                                     if (!used[u])
 sort(ind, ind + k, angLess);
                                                                                       rdfs(u, num), num++;
 int oldK = k;
                                                                                   values.resize(n);
 Unique();
                                                                                   for(int i = 0; i < n; i += 2)
                                                                                     if (comp[i] == comp[i ^ 1])
 forn(i, oldK)
                                                                                      return 0:
   no[i] = i;
                                                                                     else if (comp[i] > comp[i ^ 1])
                                                                                      values[i] = 1, values[i ^ 1] = 0;
 forn(i, k){
   int j = oldK + i, x = ind[i];
   ang[j] = ang[x] + 2 * M_PI;
                                                                                      values[i] = 0, values[i ^ 1] = 1;
   a[j] = a[x];
                                                                                   return 1;
   b[j] = b[x];
                                                                                  }
   ind[i + k] = j, no[j] = x;
                                                                                  34 Bridges
 sp = 0;
                                                                                  struct Edge {
 forn(i, 2 * k)
                                                                                   int to, id;
   PushPlaneIntoStack(ind[i]);
                                                                                   Edge(int aa, int bb) : to(aa), id(bb) {}
 forn(t, sp)
   if (++cnt[no[ss[t]]] > 1){
    TryShiftPoint(ss[t], ss[t - 1], 1e-5);
                                                                                  int up[MAX N], tin[MAX N], timer;
     break;
                                                                                  vector < Edge > g[MAX\_N];
                                                                                  vector <vector <int>> comp;
 return 0;
                                                                                  vector < \underbrace{int} > st;
}
                                                                                  void newComp(int size = 0) {
      Graphs
                                                                                   comp.emplace_back(); //
8
                                                                                   while (sz(st) > size) {
                                                                                     comp.back().pb(st.back());
     2-SAT
33
                                                                                     st.pop_back();
//MAX N - 2 * vars
vector < int > g[MAX N], rg[MAX N], tsort;
vector <bool> values;
\mathbf{int}\ \mathbf{used}[\mathbf{MAX}_{-}\mathbf{N}],\ \mathbf{comp}[\mathbf{MAX}_{-}\mathbf{N}];
                                                                                  void find bridges(int v, int parentEdge = -1) {
                                                                                   if (up[v]) //
void dfs(int v) {
                                                                                     return;
 used[v] = 1;
                                                                                   up[v] = tin[v] = ++timer;
 for( \textbf{int to} : g[v])
                                                                                   \operatorname{st.pb}(v); // \operatorname{st} - \operatorname{stack}
```

```
int c = inCycle[v];
 for (Edge e : g[v]) \{
   if (e.id == parentEdge)
                                                                                   for (int u : cycles[c])
                                                                                     inProcess[u] = 1;
    continue:
                                                                                   for (int u : cycles[c])
   \quad \quad \textbf{int} \ u = e.to; \\
                                                                                     for (int w : g[u])
   if (!tin[u]) {
                                                                                       if (w != p \&\& inCycle[w] != c)
    int size = sz(st);
    find_bridges(u, e.id);
                                                                                        dfs(w, u), sons[u].pb(w);
    \mathrm{if}\ (up[u] > \mathrm{tin}[v])
                                                                                    //calc dp on cactus
      newComp(size);
                                                                                   for (int u : cycles[c])
                                                                                     inProcess[u] = 0, used[u] = 1;
   up[v] = min(up[v], up[u]);
                                                                                  void dfs(int v, int p) {
  find_bridges newComp()
                                                                                   if (used[v])
void run(int n) {
                                                                                     return;
                                                                                   if (!inProcess[v] \&\& inCycle[v] != -1)
 forn(i, n) {
   if (!up[i]) {
                                                                                     calcCycle(v, p);
    find_bridges(i);
                                                                                   else
                                                                                     calcTree(v,\;p);
    newComp();
                                                                                  int init(int n) {
                                                                                   forn (i, n)
                                                                                     inCycle[i] = -1;
35
      Cactuses
                                                                                   getCycles(0, -1);
                                                                                   forn (i, n)
namespace Cactus {
                                                                                     used[i] = 0;
 int used[MAX N], inCycle[MAX N], dp[MAX N],
                                                                                   dfs(0, -1);
 \hookrightarrow inProcess[MAX N];
                                                                                   return dp[0];
 vi g[MAX_N], sons[MAX_N], st, cycle;
 set < pii > forbidden;
 vector<vi> cycles;
 int curCycle = 0;
                                                                                       Cut Points
                                                                                36
 void getCycles(int v, int p) {
   used[v] = 1;
                                                                                struct Edge {
   st.pb(v);
                                                                                 int to, id;
   for (int u : g[v])
                                                                                 Edge(int aa, int bb) : to(aa), id(bb) \{\}
    if (u != p \&\& used[u] == 1) {
      cycle.clear();
      fornr (i, sz(st)) {
                                                                                vector{<}Edge{>}\ g[MAX\_N];\ //\ (to,\ id)
       cycle.pb(st[i]);
                                                                                vector<int> st; // stack
                                                                                \color{red}\textbf{bool}\ used[MAX\_M];\\
       inCycle[st[i]] = curCycle;
       if (st[i] == u)
                                                                                int tin[MAX N], timer, is cut[MAX N], color[MAX M], compCnt;
         break;
                                                                                int dfs(int v, int parent = -1) {
                                                                                  tin[v] = ++timer;
      curCycle++;\\
      reverse(all(cycle));
                                                                                  int up = tin[v], x = 0, y = (parent != -1);
                                                                                  for (Edge p : g[v]){
      cycles.pb(cycle);
                                                                                   int u = p.to, id = p.id;
    else if (u != p \&\& !used[u])
                                                                                   if (id != parent) {
                                                                                     int t, size = sz(st);
      getCycles(u, v);
  st.pop_back();
used[v] = 2;
                                                                                     if (!used[id]){
                                                                                       st.push_back(id);
                                                                                       used[id] = 1;
 bool isForbidden(int v, int u) {
                                                                                     if (!tin[u]) { // not visited yet
   \mathrm{return}\ forbidden.count(mp(v,\,u))\ ||\ forbidden.count(mp(u,\,v));
                                                                                      t = dfs(u, id);
                                                                                       \mathrm{if}\ (t\,>=\,\mathrm{tin}[v])\{
                                                                                         ++x, ++compCnt;
 void dfs(int v, int p);
                                                                                        while(sz(st) != size){}
                                                                                          color[st.back()] = compCnt;
 void calcTree(int v, int p) {
                                                                                          st.pop_back();
   used[v] = 1;
   for (int \ u : g[v])
    if (u != p && !isForbidden(v, u)) {
                                                                                     } else
      dfs(u, v);
                                                                                       t = tin[u];
      //calc dp
                                                                                     up = min(up, t);
 }
                                                                                  if(x + y >= 2)
                                                                                   is\_cut[v] = 1; // v is cut vertex
 void calcCycle(int v, int p) {
```

```
return up;
     DP tree
int dfs(int v) {
 forn(i, n + 1)
  dp[v][i] = -INF;
 dp[v][1] = num[v];
 int mxsz = 1;
 for (int to : g[v]) {
  int size = dfs(to);
   forba(i, 1, mxsz + 1)
    fornr(j, size + 1)
      dp[v][i + j] = max(dp[v][i + j], dp[v][i] + dp[to][j]);
   mxsz += size;
 return mxsz;
38
     Eulerian Cycle
struct Edge {
 int to, used;
 Edge(): to(-1), used(0) {}
 Edge(int v): to(v), used(0) \{\}
vector <Edge> edges;
vector < int > g[MAX N], res, ptr;
// ptr
void dfs(int v) {
 for(; ptr[v] < sz(g[v]);)  {
   int id = g[v][ptr[v]++];
   if (!edges[id].used) {
    edges[id].used = edges[id ^ 1].used = 1;
    dfs(edges[id].to);
      res.pb(id); //
 res.pb(v); // res
39
     Hamilton Cycle
// - n2^n
const int MAX N = 20;
vector < int > g[\overline{M}AX\_N];
int adj[MAX N], dp[1 << MAX N];
vector < int > hamiltonCycle(int n) {
 memset(dp, 0, sizeof(dp));
 forn(v, n) {
   adj[v] = 0;
   for (int to : g[v])
    adj[v] = (1 << to);
 dp[1] = 1;
 forn(mask, (1 << n)) {
    if (mask \& (1 << v) \&\& dp[mask ^ (1 << v)] \& adj[v])
      dp[mask] = (1 << v);
 vector < int > ans;
 int mask = (1 << n) - 1, v;
 if (dp[mask] \& adj[0]) \{
   forab(i, 1, n)
    if ((1 << i) & (mask & adj[0]))
     v = i;
   ans.pb(v);
```

```
mask ^= (1 << v);
   while(v) {
     forn(i, n)
       if ((dp[mask] \& (1 << i)) \&\& (adj[i] \& (1 << v))) {
        break;
     mask \hat{} = (1 << v);
     ans.pb(v);
 return ans;
       Karp with cycle
40
struct Edge {
 int a, b, w;
 Edge(int aa, int bb, int ww): a(aa), b(bb), w(ww) {}
\mathbf{int}\ d[MAX\_N][MAX\_N],\ p[MAX\_N][MAX\_N];
vector < int > g[MAX_N], resultingCycle;
vector < Edge> edges;
void ford_bellman(int s, int n) {
 forn(i, n + 1)
   forn(j, n + 1)
     d[i][j] = INF;
  d[0][s] = 0;
 forab(i, 1, n + 1)
   for (auto e: edges)
     \mathrm{if} \ (d[i-1][e.a] < \mathrm{INF} \ \&\& \ d[i][e.b] > d[i-1][e.a] + e.w) \ \{
       d[i][e.b] = d[i-1][e.a] + e.w;
       p[i][e.b] = e.a;
LD karp(int n) {
 \quad \text{int } s = n + +; \\
 forn(i, n-1)
   g[s].pb(sz(edges)), edges.pb(Edge(s, i, 0));
  ford bellman(s, n);
 LD\ ans=INF;
 \quad \textbf{int} \ \ \textbf{pos} = \textbf{-1}, \ \textbf{local\_pos} = \textbf{-1}, \ \textbf{dist} = \textbf{-1};
 forn(v, n - 1)
   if (d[n][v] := INF) {
     LD\ local\ \ ans = -INF;
     local\_pos = -1;
        \text{if } (local\_ans <= (d[n][v] - d[k][v]) * (LD)(1) / (n - k)) \ \{ \\
         local ans = (d[n][v] - d[k][v]) * (LD)(1) / (n - k);
        local \ pos = k;
       }
     if (ans > local_ans) {
      ans = local ans;
       dist = local\_pos;
       pos = v;
  if (pos == -1)
   return ans:
  for (int iter = n; iter != dist; iter--) {
   resultingCycle.pb(pos);
   pos = p[iter][pos];
 reverse(all(resultingCycle));
 return ans;
```

### 41 Kuhn's algorithm

```
const int MAX N = 1e5 + 100;
int n, m, paired[2 * MAX_N], used[2 * MAX_N];
vector < int > g[MAX N];
bool dfs(int v) {
 if (used[v])
  return false;
 used[v] = 1;
 for(int to : g[v])
   if (paired[to] == -1 || dfs(paired[to])) {
    paired[to] = v;
    paired[v] = to;
    return true:
 return false;
int kuhn() {
 int ans = 0;
 forn(i, n + m)
   paired[i] = -1;
 for (int run = 1; run;) \{
   run = 0;
   memset(used, 0, sizeof(used));
    if (!used[i] && paired[i] == -1 && dfs(i)) {
      ans++;
     run = 1;
    }
 return ans;
//Max - A+, B-
//Min
        - A-, B+
vector < int > minCover, maxIndependent;
void dfsCoverIndependent(int v) {
 if (used[v])
  return;
 used[v] = 1;
 for(int to:g[v])
   if (!used[to])
    used[to] = 1, dfsCoverIndependent(paired[to]);
}
void findCoverIndependent() {
 memset(used, 0, sizeof(used));
 forn(i, n)
   if (paired[i] == -1)
    dfsCoverIndependent(i);
 forn(i, n)
   if (used[i])
    maxIndependent.pb(i);
    minCover.pb(i);
 forab(i, n, n + m)
   if (used[i])
    minCover.pb(i);
   else
    maxIndependent.pb(i);
}
```

```
42 LCA
namespace Lca {
 int tin[MAX_N], tout[MAX_N], up[MAX_N][MAX_LOG];
 vi g[MAX_N];
 int curTime = 0;
 void dfs(int v, int p) {
   up[v][0] = p;
   forn (i, MAX_LOG - 1)
    up[v][i+1] = up[up[v][i]][i];
    tin[v] = curTime++;
   \quad \text{for } (\text{int } u:g[v])
    if (u != p)
       dfs(u, v);
    tout[v] = curTime++;
 int isUpper(int v, int u) {
    \texttt{return tin}[v] <= \texttt{tin}[u] \&\& \texttt{tout}[v] >= \texttt{tout}[u];
 int lca(int v, int u) {
    if (isUpper(u, v))
     return u;
    fornr (i, MAX LOG)
      if \ (!isUpper(up[u][i], \, v)) \\
       \mathbf{u} = \mathbf{u}\mathbf{p}[\mathbf{u}][\mathbf{i}];
    {\tt return}\ {\tt up}[{\tt u}][0];
 void init() {
    dfs(0, 0);
     LCA offline (Tarjan)
43
namespace LcaTarjan {
 vi g[MAX_N], q[MAX_N];
 int \ \operatorname{pr}[MAX\_N], \ \operatorname{rank}[MAX\_N], \ \operatorname{ancestor}[MAX\_N], \ \operatorname{used}[MAX\_N];
 int get(int v) {
    return v == pr[v] ? v : pr[v] = get(pr[v]);
 void unite(int v, int u, int anc) {
   v = get(v), u = get(u);
   \quad \text{if } (\operatorname{rank}[v] < \operatorname{rank}[u]) \\
    swap(v, u);
   if (rank[v] == rank[u])
    rank[v]++;
   pr[u] = v;
   ancestor[v] = anc;
 void dfs(int v) {
   used[v] = 1;
   for (int u : g[v])
     if (!used[u])
       dfs(u), unite(v, u, v);
   for (int u : q[v])
     if (used[u])
       ancestor[get(u)]; //handle answer somehow
 }
   void init(int n) {
   forn (i, n)
    pr[i] = i, ancestor[i] = i, rank[i] = 0;
   dfs(0);
   }
```

}

#### 9 Math

```
CRT (KTO)
namespace Math {
 vi crt(vi a, vi mod) {
  int n = sz(a);
  vi x(n);
  forn\ (i,\ n)\ \{
    x[i] = a[i];
    forn (j, i) {
      x[i] = inverse(mod[j], mod[i]) * (x[i] - x[j]) % mod[i];
      if (x[i] < 0)
        x[i] += mod[i];
  }
  return x;
```

### Discrete Logariphm

```
namespace Math {
 int discreteLogariphm(int a, int b, int mod) { //returns x: a^x = b
 → (mod mod) or -1, if no such x exists
  int sq = sqrt(mod);
  int sq2 = mod / sq + (mod \% sq ? 1 : 0);
  vector<pii> powers(sq2);
  forn (i, sq2)
    powers[i] = mp(power(a, (i + 1) * sq, mod), i + 1);
  sort(all(powers));
  forn (i, sq + 1) {
    int cur = power(a, i, mod);
     cur = (cur * 1 ll * b) \% mod;
     auto it = lower bound(all(powers), mp(cur, 0));
    if (it != powers.end() && it->fs == cur)
      return it->sc * sq - i;
  }
  return -1;
```

#### 46 Discrete Root

```
namespace Math {
 //\text{returns } x: x^k = a \mod mod, \mod is prime
 int discreteRoot(int a, int k, int mod) {
  if (a == 0)
    return 0;
  int g = primitiveRoot(mod);
  int \ y = discreteLogariphm(power(g, \ k, \ mod), \ a, \ mod);
  return power(g, y, mod);
```

#### 47 Eratosthenes

```
namespace Math {
 vi eratosthenes(int n) {
  vi\ minDiv(n\,+\,1,\,0);
  \min Div[1] = 1;
  for (int i = 2; i <= n; i++)
    if (\min Div[i] == 0)
     for (int j = i; j \le n; j += i)
       if (\min Div[j] == 0)
         \min Div[j] = i;
  return minDiv;
 vi eratosthenesFast(int n) {
  vi minDiv(n + 1, 0);
```

```
vi primes;
   \min Div[1] = 1;
   for (int i = 2; i \le n; i++) {
    if (\min Div[i] == 0) {
      \min Div[i] = i;
      primes.pb(i);
    for (int j = 0; j < sz(primes) && primes[j] <= minDiv[i] && i *
     \rightarrow primes[j] \langle = n; j++ \rangle
      minDiv[i * primes[j]] = primes[j];
   return minDiv;
48
      Factorial
namespace Math {
 //returns pair (rem, deg), where rem = n! \% mod,
  //deg = k: mod^k | n!, mod is prime, O(mod log mod)
 pii fact(int n, int mod) {
  int rem = 1, deg = 0;
  int n2 = n;
   while (n2)
    n2 /= mod, deg += n2;
   while (n > 1) {
    rem = (rem * ((n / mod) \% 2 ? -1 : 1) + mod) \% mod;
    for (int i = 2; i <= n \% \mod; i++)
      rem = (rem * 111 * i) \% mod;
    n /= mod;
   }
   return mp(rem % p, deg);
}
49
      Gauss
namespace Math {
 const double EPS = 1e-9;
 int gauss(double **a, int n, int m) { //n is number of equations, m is
 int row = 0, col = 0;
  vi par(m, -1);
   vector < double > ans(m, 0);
  for (col = 0; col < m \&\& row < n; col++) {
    int best = row;
    for (int i = row; i < n; i++)
      \quad \text{if } (abs(a[i][col]) > abs(a[best][col])) \\
    if (abs(a[best][col]) < EPS)
      continue;
    par[col] = row;
    for (int i = 0; i <= m; i++)
      swap(a[row][i], a[best][i]);
    for (int i = 0; i < n; i++)
      if (i != row) {
       \label{eq:collinear} \mbox{long double } k = a[i][col] \ / \ a[row][col];
       for (int j = col; j \le m; j++)
         a[i][j] \mathrel{-}= k \, * \, a[row][j];
    row++;
   int single = 1;
   for (int i = 0; i < m; i++)
    if (par[i] != -1)
      ans[i] = a[par[i]][m] / a[par[i]][i];
    _{
m else}
```

single = 0;

for (int i = 0; i < n; i++) {

long double cur = 0;

```
\begin{array}{l} {\rm for}\; ({\rm int}\; j=0;\, j< m;\, j++) \\ {\rm cur}\; +=\, ans[j]\, *\, a[i][j]; \end{array}
     if (abs(cur - a[i][m]) > EPS)
       return 0;
   if (!single)
     return 2;
   return 1:
50
       Gauss mod 2
namespace Math {
 const int MAX = 1024;
 int gaussMod2(vector<br/>bitset<MAX>> a, int n, int m) {
   \quad \textbf{int row} = 0,\, \textbf{col} = 0;
   vi par(m, -1);
   for (col = 0; col < m \&\& row < n; col++) {
     int best = row;
     for (int i = row; i < n; i++)
       if (a[i][col] > a[best][col])
         best = i;
     if (a[best][col] == 0)
       continue;
     par[col] = row;
     swap(a[row], a[best]);
     for (int i = 0; i < n; i++)
       if (i != row) \{
         \mathrm{if}\ (a[\mathrm{i}][\mathrm{col}])
           a[i] \hat{} = a[row];
       }
     row++;
   vi ans(m, 0);
   for (int i = 0; i < m; i++)
     if (par[i] != -1)
       ans[i] = a[par[i]][n] / a[par[i]][i];
   bool ok = 1;
   for (int i = 0; i < n; i++) {
     int cur = 0;
     for (int j = 0; j < m; j++)
      \operatorname{cur} \hat{} = (\operatorname{ans}[j] \& \operatorname{a}[i][j]);
     if (\operatorname{cur} := a[i][n])
       ok = 0;
   }
   return ok:
       \operatorname{Gcd}
namespace Math {
 int gcd(int a, int b) {
   return b ? gcd(b, a % b) : a;
 int gcd(int a, int b, int &x, int &y) {
    if (b == 0) {
      x = 1, y = 0;
      return a;
    else {
     int g = gcd(b, a \% b, x, y);
     int new X = y;
     y = x - a / b * y;
     x\,=\,newX;
     return g;
 }
```

```
void diophant(int a, int b, int c, int &x, int &y) {
   int g = \gcd(a, b, x, y);
   if (\bar{c}\% g != 0)
    return;
  x \ *= c \ / \ g, \, y \ *= c \ / \ g;
   //next solutions: x += b / g, y -= a / g
 int inverse(int a, int mod) { //returns -1, if a and mod are not
  int x, y;
  int g = \gcd(a, \mod, x, y);
   return g == 1 ? (x \% mod + mod) \% mod : -1;
 vi inverseForAll(int mod) {
   vi r(mod, 0);
    \mathbf{r}[1] \, = \, 1;
    for (int i = 2; i < mod; i++)
     r[i] = (mod - r[mod \% i]) * (mod / i) \% mod;
 }
}
52
       Gray
namespace Math {
 int gray(int n) {
   return n \hat{} (n >> 1);
 int revGray(int n) {
    int k = 0;
    for (; n; n >>= 1)
     k ´
    return k:
}
53
       Miller-Rabin Test
namespace Math {
 vector \langle int \rangle primes = \{2,3,5,7,11,13,17,19,23\};
 bool isPrimeMillerRabin(ll n) {
   int k = 0;
   ll t = n - 1;
   while (t \% 2 == 0)
    k++, t /= 2;
   for (auto p : primes) {
    \begin{array}{l} \text{ll } g = \_\_gcd(n, (ll) p); \\ \text{if } (g > 1 \&\& g < n) \end{array}
      return 0;
    if (g == n)
      return 1;
    ll b = power(p, t, n);
    ll\ last = n-1;
    bool was = 0;
    forn~(i,~k~+~1)~\{
      if (b == 1 \&\& last != n - 1)
        return 0:
      if (b == 1) {
        was = 1;
        break;
      last = b;
      b = mul(b, b, n);
    if (!was)
      return 0;
```

```
return ans:
  return 1;
                                                                                 }
                                                                               assert(0);
     Phi
54
namespace Math {
 int phi(int n) {
  int result = n;
                                                                                  Power And Mul
                                                                            56
  for (int i = 2; i * i <= n; i++)
    if (n \% i == 0) \{
                                                                            namespace Math {
                                                                             inline ll fix(ll a, il mod) { //a in [0, 2 * mod)
     while (n \% i == 0)
      n /= i;
                                                                                if (a > = mod)
      result -= result / i;
                                                                                 a = mod:
                                                                                return a;
  if (n > 1)
                                                                              }
    result -= result / n;
                                                                              ll mulSlow(ll a, ll b, ll mod) { //returns (a * b) % mod, 0 <= a <
  return result;
                                                                              \,\,\hookrightarrow\,\, \mod,\, \overset{\cdot}{0} < \equiv \, b < \, \mod
 }
                                                                               if (!b)
 int inversePhi(int a, int mod) {
                                                                                 return 0;
                                                                               ll c = fix(mulSlow(a, b / 2, mod) * 2, mod);
  return power(a, phi(mod) - 1, mod);
                                                                               return b & 1 ? fix(c + a, mod) : c;
                                                                              ll mul(ll a, ll b, ll mod) {
55
     Pollard
                                                                               ll q = (ld) a * b / mod;
                                                                               ll r = a * b - mod * q;
namespace Math {
                                                                               while (r < 0)
 inline void pollardFoo(ll &x, ll mod) {
                                                                                r += mod;
  x = (mul(x, x, mod) + 1) \% mod;
                                                                               while (r >= mod)
                                                                                r = mod;
                                                                               return r;
 vector <pair <ll, int> > factorize(ll n) {
  if (n == 1)
    return {};
                                                                              int power(int a, int n, int mod) {
  if (isPrimeMillerRabin(n))
    return \{mp(n, 1)\};
                                                                                  return 1;
  if (n <= 100) {
                                                                               int b = power(a, n / 2, mod);
    vector <\! pair <\! ll, \ int >\! > ans;
                                                                               b = (b * 111 * b) \% mod;
    for (int i = 2; i * i <= n; i++)
                                                                               return n & 1? (a * 111 * b) % mod : b;
      if (n % i == 0) {
       int cnt = 0;
       while (n \% i == 0)
                                                                              ll powerLL(ll a, ll n, ll mod) {
        n /= i, cnt++;
                                                                                if (!n)
       ans.pb(mp(i, cnt));
                                                                                 return 1;
                                                                               ll\ b = power(a,\ n\ /\ 2,\ mod);
    if (n!= 1)
                                                                               b = mul(b, b, mod);
     ans.pb(mp(n, 1));
                                                                               return n & 1? mul(a, b, mod) : b;
    sort(all(ans));
                                                                              }
    return ans;
                                                                              int powerFast(int a, int n, int mod) {
  while (1) {
                                                                               int res = 1;
    ll a = rand() \% n, b = a;
                                                                               while (n) {
    while (1) {
                                                                                 if (n & 1)
      pollardFoo(a, n), pollardFoo(b, n), pollardFoo(b, n);
                                                                                  res = (res * 111 * a) \% mod;
      ll\ g = \_\_gcd(abs(a-b),\ n);
                                                                                 a = (a * 111 * a) \% mod;
      if (g := 1) {
                                                                                 n /= 2;
       if (g == n)
                                                                               }
        break;
                                                                               return res;
         auto ans1 = factorize(g);
         auto ans2 = factorize(n / g);
         vector <pair <ll, int> > ans;
         ans1.insert(ans1.end(), all(ans2));
         sort(all(ans1));
                                                                                   Primitive Root
         for (auto np : ans1)
          if (sz(ans) == 0 \mid\mid np.fs != ans.back().fs)
                                                                            namespace Math {
                                                                             int primitiveRoot(int mod) { //returns -1 if no primitive root exists
            ans.pb(np);
            ans.back().sc += np.sc;
                                                                                int ph = phi(mod);
```

}

```
int n = mod;
    for (int i = 2; i * i <= n; i++) {
                                                                             inline int readInt() {
      if (n % i == 0) {
                                                                               int s = 0, c = readChar(), x = 0;
                                                                               if (c == '-')
       fact.pb(i);
        while (n \% i == 0)
                                                                                s = 1, c = readChar();
                                                                               while ('0' <= c \&\& c <= '9')
          n /= i;
                                                                                x = x * 10 + c - '0', c = readChar();
                                                                               \mathtt{return}\ \mathbf{s}\ ?\ \textbf{-x}\ :\ x;
   if (n > 1)
     fact.pb(n);
    forab (i, 2, mod + 1) {
                                                                             template < class T > inline void writeInt( T x ) {
      bool ok = 1:
                                                                               if (x < 0)
      for (int j = 0; j < sz(fact) \&\& ok; j++)
                                                                                putchar('-'), x = -x;
       ok &= power(i, ph / fact[j], mod) != 1;
                                                                               char s[24];
      if (ok)
                                                                               int n = 0;
        return i;
                                                                               while (x || !n)
                                                                                s[n++] = {}^{\bullet}0 + x \% 10, x /= 10;
    }
    return -1;
                                                                               while (n--)
                                                                                putchar(s[n]);
      Simpson
58
                                                                                   Fast I/O (long)
namespace Math {
                                                                              #include <cstdio>
 double f(double x) {
                                                                              #include <algorithm>
   return x;
                                                                              /** Interface */
 double simpson(double a, double b, int iterNumber) {
                                                                             template < class T = int > inline T readInt();
   \label{eq:double_res} \begin{array}{l} \textbf{double} \ res = \, 0, \ h = \, (b \, \text{-} \, a) \, \, / \, \, iterNumber; \end{array}
                                                                              inline double readDouble();
   forn (i, iterNumber + 1)
                                                                             inline int readUInt();
    res += f(a + h * i) * ((i == 0) || (i == iterNumber) ? 1 : ((i & 1))
                                                                             inline int readChar();
     \Rightarrow == 0) ? 2 : 4);
                                                                              inline void readWord( char *s);
   return res * h / 3;
                                                                              inline bool readLine( char *s ); // do not save '\n'
                                                                             inline bool isEof();
                                                                             inline int peekChar();
                                                                             inline bool seekEof();
10
       Mix
                                                                              template < class T> inline void writeInt( T x, int len );
                                                                             template < class T > inline void writeUInt( T x, int len );
59
      Fast allocation (operator new)
                                                                             template < class T > inline void writeInt( T x ) { writeInt(x, -1); };
#include <cassert>
                                                                             template <class T> inline void writeUInt( T x ) { writeUInt(x, -1); };
                                                                              inline void writeChar( int x );
/** Begin fast allocation */
                                                                             inline void writeWord( const char *s );
const int MAX MEM = 1e8;
                                                                             inline void writeDouble( double x, int len = 0 );
int mpos = 0;
                                                                             inline void flush();
char mem[MAX MEM];
inline void * operator new ( size_t n ) {
                                                                              /** Read */
 char *res = mem + mpos;
 mpos += n;
                                                                             static const int buf size = 4096;
 assert(mpos <= MAX MEM);
 return (void *)res;
                                                                             static char buf[buf size];
                                                                             static int buf_len = 0, pos = 0;
inline void operator delete (void *) {}
/** End fast allocation */
                                                                             inline bool isEof() {
                                                                               if (pos == buf_len) {
inline void * operator new [] ( size_t ) { assert(0); }
                                                                                 pos = 0, buf_len = fread(buf, 1, buf_size, stdin);
inline void operator delete [] ( void * ) { assert(0); }
                                                                                if (pos == buf_len)
                                                                                  return 1;
                                                                               }
      Fast I/O (short)
                                                                               return 0;
inline int readChar();
                                                                              }
inline int readInt();
template <class T> inline void writeInt( T x );
                                                                             inline int getChar() {
                                                                               return isEof()? -1: buf[pos++];
inline int readChar() {
 int c = getchar();
 while (c <= 32)
                                                                             inline int peekChar() {
                                                                               return isEof() ? -1 : buf[pos];
  c = getchar();
 return c;
```

```
inline bool seekEof() {
 while ((c = peekChar()) != -1 \&\& c <= 32)
  pos++;
 \operatorname{return}\; c == \text{-1};
inline int readChar() {
 int c = getChar();
 while (c != -1 && c <= 32)
  c = getChar();
 return c:
}
inline int readUInt() {
 int c = readChar(), x = 0;
 while ('0' \le c \&\& c \le '9')
  x = x * 10 + c - 0', c = getChar();
 return x:
template <class T>
inline T readInt() {
 int s = 1, c = readChar();
 T \mathbf{x} = 0;
 if (c == '-')
  s = -1, c = getChar();
 while ('0' <= c \&\& c <= '9')
  x = x * 10 + c - 0', c = getChar();
 return s == 1 ? x : -x;
inline double readDouble() {
 int s = 1, c = readChar();
 double x = 0;
 if (c == '-')
  s = -1, c = getChar();
 while ('0' <= c \&\& c <= '9')
  x = x * 10 + c - 0', c = getChar();
 if (c == '.') {
  c = getChar();
   double coef = 1;
   while ('0' <= c && c <= '9')
    x += (c - '0') * (coef *= 1e-1), c = getChar();
 return \mathbf{s} == 1 ? \mathbf{x} : -\mathbf{x};
}
inline void readWord( char *s ) {
 int c = readChar();
 while (c > 32)
   *s++ = c, c = getChar();
 *s = 0;
}
inline bool readLine( char *s ) {
 int c = getChar();
 while (c != '\n' && c != -1)
   *s{++} = c,\, c = getChar();
 *s = 0;
 return c != -1;
/** Write */
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar( int x ) {
 if (write\_pos == buf\_size)
   fwrite(write_buf, 1, buf_size, stdout), write_pos = 0;
```

```
write\_buf[write\_pos++] = x;
inline void flush() {
 if (write_pos)
   fwrite(write buf, 1, write pos, stdout), write pos = 0;
template < class T >
inline void writeInt( T x, int output len ) {
 if (x < 0)
   writeChar('-'), x = -x;
 char s[24];
 int n = 0;
 while (x || !n)
  s[n++] = {}^{\bullet}0^{\bullet} + x \% 10, x /= 10;
 while (n < output_len)
  s[n++] = '0';
 while (n--)
   writeChar(s[n]);
template < class T>
inline void writeUInt( T x, int output len ) {
 char\ s[24];
 int n = 0;
 while (x \mid \mid !n)
  s[n++] = '0' + x \% 10, x /= 10;
 while (n < output_len)
  s[n++] = '0';
 while (n--)
   writeChar(s[n]);
inline void writeWord( const char *s ) {
 while (*s)
   writeChar(*s++);
inline void writeDouble( double x, int output len ) {
 if (x < 0)
  writeChar('-'), x = -x;
 int t = (int)x;
 writeUInt(t), x = t;
 writeChar('.');
 for (int i = output len - 1; i > 0; i--) {
  x *= 10;
  t = std::min(9, (int)x);
   writeChar("0" + t), x = t;
 }
 x *= 10;
 t = std::min(9, (int)(x + 0.5));
 writeChar('0' + t);
62
      Masks tricks
forn(mask, 1 << d) {
 dp[mask][d] = 1;
 fornr(i, d) {
   dp[mask][i] = dp[mask][i\,+\,1];
   if ((1 << i) & mask)
    dp[mask][i] += dp[mask ^ (1 << i)][i + 1];
 cout << mask << " -> " << dp[mask][0] << '\n';
int num[64];
for (ULL i = 0; i < 64; ++i) {
```

 $\mathbf{pr}[0] = \mathbf{pr}[1] = 0;$ 

```
num[(1ull << i) \% 67] = i;
      Hash of pair
63
struct MyHash {
 size t operator()(const pair<int,int> &t) const {
   return t.first * 239017 + t.second;
};
11
       Strings
64
      Aho-Corasick
const int ALPHA = 26;
const int MAX N = 1e5;
struct Node {
 int next[ALPHA], term; //
 int go[ALPHA], suf, p, pch; //
 Node() {
  memset(next, -1, sizeof(next));
  term = 0;
   memset(go, -1, sizeof(go));
   suf = p = -1;
};
Node g[MAX N];
int last;
void add(const string &s) {
 int now = 0;
 for(char x : s) { // x -= 'a'}
   if (g[now].next[x] == -1) {
    g[now].next[x] = ++last;
    g[last].p = now;
    g[last].pch = x;
   now = g[now].next[x];
 g[now].term = 1;
int go(int v, int c);
int get link(int v) {
 if (g[v].suf == -1) {
   if (!v || !g[v].p)
    g[v].suf = 0;
    g[v].suf = go(get\_link(g[v].p), g[v].pch);
 return g[v].suf;
int go(int v, int c) {
 if (g[v].go[c] == -1) {
   if \ (g[v].next[c] \mathrel{\mathop:}= \text{-}1)
    g[v].go[c] = g[v].next[c];
    g[v].go[c] = !v ? \ 0 : go(get\_link(v), \ c);
 return g[v].go[c];
     Prefix-function
// pr[len] -
int k = 0;
```

```
for (int i = 2; i <= n; i++) {
 k = pr[i - 1];
 while (k \&\& s[k] != s[i - 1])
   k = pr[k];
 if (s[k] == s[i - 1])
  k++;
 pr[i] = k;
66
      Z-function
\begin{array}{ll} //\mathbf{z}[i] \text{ - } i \\ \text{int } l = \text{-1, } \mathbf{r} = \text{-1;} \end{array}
\mathbf{z}[0] = 0;
for (int i = 1; i < n; i++) {
 int k = 0;
 if (r >= i)
  k = \min(z[i-l], r-i);
 while (i + k < n \&\& s[i + k] == s[k])
   k++;
 z[i] = k;
 if (i + z[i] > r)
   l = i, r = i + z[i];
67 Hash
#include < bits/stdc++.h>
typedef long long LL;
inline int by Mod(int a, int m){
   return a >= m ? a - m : a;
const int MX = 1e9 + 9, MY = 1e9 + 7;
//typedef unsigned long long H;
struct H{
   int x, y;
   H(): x(0), y(0)\{\}
   \begin{array}{ll} H(int \ \_x)\colon x(\_x), \, y(\_x)\{\} \\ H(int \ \_x, \, int \ \_y)\colon x(\_x), \, y(\_y)\{\} \end{array}
   inline H operator + (const H &B) const {return H(byMod(x + B.x,
    \hookrightarrow MX), byMod(y + B.y, MY));}
   inline H operator -(const H &B) const{return H(byMod(x + MX - \rightarrow B.x, MX), byMod(y + MY - B.y, MY));}
   inline H operator *(LL\ k) const{return H(int((x\ *\ k)\ \%\ MX),\ int((y\ k)\ MX),\ int((y\ k)\ MX))

→ * k) % MY));}

   inline H operator *(const H &B) const{return H(int((LL(x) * B.x)
    \rightarrow % MX), int((LL(y) * B.y) % MY));}
   inline bool operator == (const H &B) const {return x == B.x && y
    \Rightarrow == B.y;
   inline bool operator != (const H &B) const {return x != B.x || y !=
    \hookrightarrow B.y;}
   inline bool operator < (const H &B) const{return x < B.x || (x =
   \rightarrow B.x && y < B.y);}
   explicit inline operator LL() const{return (LL)x * MY + y + 1;} //
    \rightarrow > 0
const int P = 239017, MAX N = 1e6 + 10;
H deg[MAX_N], h[MAX_N];
char s[MAX N];
inline H Get(int a, int l){
   \operatorname{return}\ h[a\ +\ l]\ \text{-}\ h[a]\ *\ deg[l];
int main(){
```

```
#ifdef LOCAL
                                                                                 int get(int i) { return i < 0 ? -1 : s[i]; }
   assert(freopen("test.in", "r", stdin));
   assert(freopen("test.out", "w", stdout));
                                                                                 void init() {
                                                                                  fill(t, 0);
                                                                                  t[0].len = -1, vn = 2, v = 0, n = 0;
   gets(s);
   int L = strlen(s);
   \deg[0] = 1;
                                                                                 void add( int ch ) {
   for(int i = 0; i < L; ++i)
                                                                                   s[n++] = ch;
                                                                                   while (v != 0 && ch != get(n - t[v].len - 2))
      h[i+1] = h[i] * P + s[i], \, deg[i+1] = deg[i] * P;
                                                                                    v = t[v].suf;
                                                                                   \quad \text{int } \&r = t[v].next[ch]; \\
   return 0:
}
                                                                                   if (!r) {
                                                                                    t[vn].len = t[v].len + 2;
                                                                                    if (!v)
68
      Manaker
                                                                                      t[vn].suf = 1;
                                                                                    else {
#include <bits/stdc++.h>
                                                                                      v = t[v].suf;
                                                                                      while (v != 0 \&\& ch != get(n - t[v].len - 2))
using namespace std;
                                                                                        v = t[v].suf;
                                                                                      t[vn].suf = t[v].next[ch];
#define forn(i, n) for (int i = 0; i < (int)(n); i++)
                                                                                    }
                                                                                    r = vn++;
void\ manaker(int\ n,\ char\ *s,\ int\ *z0,\ int\ *z1\ )\ \{
 forn(t, 2) {
                                                                                   v = r;
   int *z = t ? z1 : z0, l = -1, r = -1; // [l..r]
                                                                                 }
   forn(i, n - t) {
                                                                                };
    int k = 0;
    if (r > i + t) {
                                                                                const int N = 1e5;
      int j = l + (r - i - t);
      k = \min(z[j], j - l);
                                                                                PalindromeTree<N> pt;
     while (i - k) = 0 \&\& i + k + t < n \&\& s[i - k] == s[i + k + t])
                                                                               char s[N + 1];
     k++;
     z[i] = k;
                                                                               int main() {
    if (k \&\& i + k + t > r)
                                                                                 gets(s);
      l = i - k + 1, r = i + k + t - 1;
                                                                                 int n = strlen(s);
                                                                                 pt.init();
                                                                                 forn(i, n) {
                                                                                   \operatorname{pt.add}(s[i] - 'a');
                                                                                   printf("%d ", pt.vn - 2);
const int N = 1e5;
                                                                                 return 0;
int n, r0[N], r1[N];
char s[N + 1];
int main() {
                                                                                      Suffix Array (+stable)
 assert(freopen("palindrome.in", "rt", stdin));
 assert(freopen("palindrome.out", "wt", stdout));
                                                                                const int MAX N = 250000;
 gets(s);
                                                                                int n, num[MAX_N + 1];
 n = strlen(s);
                                                                                \frac{\text{char s}[\text{MAX}_{\text{N}} + 1];}{\text{char s}[\text{MAX}_{\text{N}} + 1];}
 manaker(n,\;s,\;r0,\;r1);
                                                                                int p[MAX N], col[MAX N], p2[MAX N], len[MAX N];
 cout << accumulate(r0, r0 + n, 0LL) + accumulate(r1, r1 + n, 0LL)
 \hookrightarrow - n << endl;
                                                                                void BuildArray(){
 return 0;
                                                                                 int ma = \max(n, 256);
                                                                                 forn(i, n)
                                                                                   col[i] = s[i], p[i] = i;
      Palindromic Tree
                                                                                 for (int k2 = 1; k2 / 2 < n; k2 *= 2){
\#define fill(a, x) memset(a, x, sizeof(a))
                                                                                   int k = k2 / 2;
                                                                                   memset(num, 0, sizeof(num));
                                                                                   forn(i, n)
template<const int N>
struct PalindromeTree {
                                                                                    num[col[i] + 1] + +;
                                                                                   forn(i, ma)
 struct Vertex {
                                                                                    num[i + 1] += num[i];
  int suf, len, next[26];
 };
                                                                                    p2[num[col[(p[i] - k + n) \% n]]++] = (p[i] - k + n) \% n;
 int vn, v;
 Vertex t[N + 2];
                                                                                   int cc = 0;
 int n, s[N];
                                                                                   forn(i, n){
                                                                                    if (i && (col[p2[i]] != col[p2[i - 1]] ||
```

```
col[(p2[i] \ + \ k) \ \% \ n] \ != \ col[(p2[i \ - \ 1] \ + \ k) \ \% \ n]))
                                                                                  \mathrm{SA}()\{
                                                                                      vcnt = 1;
    num[p2[i]] = cc;
                                                                                      last = newV(0, 0); // root = vertex with number 1
                                                                                  int newV(int len, int suf){
   forn(i, n)
    p[i] = p2[i], col[i] = num[i];
                                                                                      v[vcnt] = Vx(len, suf);
                                                                                      return vcnt++;
 // make it stable
 memset(num, 0, sizeof(num));
                                                                                  int add(char ch){
 forn(i, n)
                                                                                      int p = last, c = ch - 'a';
   num[col[i] + 1] + +;
                                                                                      last = newV(v[last].len + 1, 0);
 forn(i, ma)
                                                                                      while(p && !v[p].next[c]) //added p &&
  num[i + 1] += num[i];
                                                                                         v[p].next[c] = last, p = v[p].suf;
 forn(i, n)
                                                                                      if(!p)
   p2[num[col[i]]\!+\!+]\,=\,i;
                                                                                         v[last].suf = 1;
 forn(i, n)
                                                                                      else{
   p[i] = p2[i];
                                                                                         int q = v[p].next[c];
                                                                                         if (v[q].len == v[p].len + 1)
   / calc inverse permutation
                                                                                            v[last].suf = q;
 forn(i, n)
                                                                                         else{
   p2[p[i]] = i;
                                                                                            int r = \text{newV}(v[p].\text{len} + 1, v[q].\text{suf});
                                                                                            v[last].suf = v[q].suf = r;
                                                                                            memcpy(v[r].next, v[q].next, sizeof(v[r].next));
void BuildLCP(){
                                                                                            while(p \&\& v[p].next[c] == q)
 int lcp = 0;
                                                                                               v[p].next[c] = r, p = v[p].suf;
 forn(i, n){
   int j = p2[i];
   lcp = max(0, lcp - 1);
                                                                                      return last;
   if (j != n - 1)
    while (lcp < n \&\& s[(p[j] + lcp) \% n] == s[(p[j + 1] + lcp) \% n])
                                                                               };
   len[j] = lcp;
                                                                               72
                                                                                      Suffix Tree
   if (j!= n - 1 \&\& p[j + 1] == n - 1)
    lcp = 0;
                                                                               const int MAX L=1e5+10;
                                                                               char S[MAX_L];
                                                                               int L;
int main()
                                                                               struct Node;
                                                                               struct Pos:
 scanf("%d%s", &n, s);
                                                                               typedef Node *pNode;
                                                                               typedef map < char, pNode > mapt;
 BuildArray();
 BuildLCP();
                                                                               struct Node{
                                                                                 pNode P,link;
 // \text{ res} = \text{sum of all LCP}[i,i+1]
                                                                                 int L,R;
 LL res = 0;
                                                                                 mapt next;
 forn(i, n)
  res += len[i];
                                                                                 Node():P(NULL),link(this),L(0),R(0)\{\}
 printf("\%.3f\n", (double)res / (n-1));
                                                                                 Node(pNode P,int L,int R):P(P),link(NULL),L(L),R(R){}
 return 0;
                                                                                 inline int elen() const{return R-L;}
                                                                                 in line\ pNode\ add\_edge(int\ L,int\ \mathring{R})\{return\ next[S[L]]=new
                                                                                 → Node(this,L,R);}
      Suffix Automaton
                                                                               };
#include < bits/stdc++.h>
                                                                               struct Pos{
                                                                                pNode V;
struct Vx{
   static const int AL = 26;
                                                                                 int up;
                                                                                 Pos():V(NULL),up(0)\{\}
   int len, suf;
   int next[AL];
                                                                                 Pos(pNode V,int up):V(V),up(up){}
   Vx()\{\}
   V_X(int | l, int | s):len(l), suf(s){}
                                                                                 pNode split edge() const {
};
                                                                                  if(!up)
                                                                                    return V;
                                                                                  _{\text{int }L=V\text{--}>L,\;M=V\text{--}>R\text{-up};}
   static const int MAX LEN = 1e5 + 100, MAX V = 2 *
                                                                                  pNode P=V->P, n=new Node(P,L,M);
   \hookrightarrow MAX_LEN;
                                                                                  P->next[S[L]]=n;
                                                                                  n\text{-}\!>\!n\!\exp\!\left[S[M]\right]\!=\!V;
   int last, vcnt;
   Vx v[MAX_V];
                                                                                  V->P=n, V->L=M;
```

return n;

```
Pos next char(char c) const{
  if (up)
    return S[V->R-up]==c ? Pos(V,up-1): Pos();
    mapt::iterator it=V->next.find(c);
    return it == V-> next.end() ? Pos() :
    \rightarrow Pos(it->snd,it->snd->elen()-1);
};
Pos go_down(pNode V,int L,int R){
 if(L \!=\! -R)
  return Pos(V,0);
 while(1){
  V = V - \sum_{n \in X} [S[L]];
  L+=V->elen();
  if(L>=R)
    return Pos(V,L-R);
inline pNode calc link(pNode &V){
 if(!V-> link)
  \rightarrow >R).split_edge();
 {\rm return}\ V\text{-}\!>\!link;
Pos add char(Pos P,int k){
 while(1){
  Pos p=P.next_char(S[k]);
  if (p.V)
   return p;
  pNode n=P.split edge();
  n->add_edge(k,MAX_L);
  if(!n->P)
   return Pos(n,0);
  P = Pos(calc_link(n), 0);
pNode Root;
void make_tree(){
 Root=new Node();
 Pos P(Root,0);
 forn(i,L)
  P = add \_char(P,i);
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       C++ Tricks
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      Tree
#include < ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int, null type, less<int>, rb tree tag,
\ \hookrightarrow \ tree\_order\_statistics\_node\_up\,date > ordered\_set;
void example() {
 ordered set X;
 X.insert(1);
 cout << *X.find\_by\_order(1) << endl;
 cout << X.order_of_key(1) <<endl;
```