Содержание

1 Common

1 Setup

1. Terminal: font Monospace 12
2. Gedit: Oblivion, font Monospace 12, auto indent, display line numbers, tab 4, highlight matching brackets, highlight current line, F9 (side panel)
3. /.bashrc: export CXXFLAGS='-Wall -Wshadow -Wextra -Wconversion -Wno-unused-result -Wno-deprecated-declarations -O2 -std=gnu++11 -g -DLOCAL'
4. `for i in {A..K}; do mkdir $i; cp main.cpp $i/$i.cpp; done`

2 Template

main.cpp:

```
#include <bits/stdc++.h>

using namespace std;

#define pb push_back
#define mp make_pair
#define fst first
#define snd second
#define sz(x) (int) ((x).size())
#define forn(i, n) for (int i = 0; i < (n); ++i)
#define fornr(i, n) for (int i = (n) - 1; i >= 0; --i)
#define forab(i, a, b) for (int i = (a); i < (b); ++i)
#define all(c) (c).begin(), (c).end()

using ll = long long;
using vi = vector<int>;
using pii = pair<int, int>;

#define FNAME ""

int main() {
#ifdef LOCAL
^^Ifreopen(FNAME".in", "r", stdin);
^^Ifreopen(FNAME".out", "w", stdout);
#endif
^^Icin.tie(0);
^^Iios_base::sync_with_stdio(0);

^^Ireturn 0;
```

3 Stress

stress.sh:

```
#!/bin/bash

for ((i = 0;; i++)); do
^^I./gen $i >in || exit
^^I./main <in >out1 || exit
^^I./stupid <in >out2 || exit
^^Idiff out1 out2 || exit
^^Iecho $i OK
done
```

4 Java

Java template:

```
import java.io.BufferedReader;
import java.io.FileNotFoundException;
import java.io.FileReader;
import java.io.IOException;
import java.io.InputStreamReader;
import java.io.PrintWriter;
import java.util.*;

public class Main {
^^IFastScanner in;
^^IPrintWriter out;

^^Ivoid solve() {
^^I^^Iint a = in.nextInt();
^^I^^Iint b = in.nextInt();
^^I^^Iout.print(a + b);
^^I}

^^Ivoid run() {
^^I^^Itry {
^^I^^I^^Iin = new FastScanner("input.txt");
^^I^^I^^Iout = new PrintWriter("output.txt");
^^I^^I^^Isolve();
^^I^^I^^Iout.flush();
^^I^^I^^Iout.close();
^^I^^I} catch (FileNotFoundException e) {
^^I^^I^^Ie.printStackTrace();
^^I^^I^^ISystem.exit(1);
^^I^^I}
^^I}

^^Iclass FastScanner {
^^I^^IBufferedReader br;
^^I^^IStringTokenizer st;

^^I^^Ipublic FastScanner() {
^^I^^I^^Ibr = new BufferedReader(new
↪  InputStreamReader(System.in));
^^I^^I}

^^I^^Ipublic FastScanner(String s) {
^^I^^I^^Itry {
^^I^^I^^I^^Ibr = new BufferedReader(new FileReader(s));
^^I^^I^^I} catch (FileNotFoundException e) {
^^I^^I^^I^^Ie.printStackTrace();
^^I^^I^^I}
^^I^^I}

^^I^^IString nextToken() {
^^I^^I^^Iwhile (st == null || !st.hasMoreElements()) {
^^I^^I^^I^^Itry {
^^I^^I^^I^^I^^Ist = new StringTokenizer(br.readLine());
^^I^^I^^I^^I} catch (IOException e) {
^^I^^I^^I^^I^^Ie.printStackTrace();
^^I^^I^^I^^I}
^^I^^I^^I}
^^I^^I^^Ireturn st.nextToken();
^^I^^I}

^^I^^Iint nextInt() {
^^I^^I^^Ireturn Integer.parseInt(nextToken());
^^I^^I}

^^I^^Ilong nextLong() {
^^I^^I^^Ireturn Long.parseLong(nextToken());
^^I^^I}

^^I^^Idouble nextDouble() {
^^I^^I^^Ireturn Double.parseDouble(nextToken());
^^I^^I}

^^I^^Ichar nextChar() {
^^I^^I^^Itry {
^^I^^I^^I^^Ireturn (char) (br.read());
```

```
^^I^^I^^I} catch (IOException e) {
^^I^^I^^I^^Ie.printStackTrace();
^^I^^I^^I}
^^I^^I^^Ireturn 0;
^^I^^I}

^^I^^IString nextLine() {
^^I^^I^^Itry {
^^I^^I^^I^^Ireturn br.readLine();
^^I^^I^^I} catch (IOException e) {
^^I^^I^^I^^Ie.printStackTrace();
^^I^^I^^I}
^^I^^I^^Ireturn "";
^^I^^I}
^^I}

^^Ipublic static void main(String[] args) {
^^I^^Inew Main().run();
^^I}
}
```

2 Big numbers

5 Big Int

```
const int BASE_LEN = 9;
const int NUM_LEN = 50000 / BASE_LEN + 2; // LEN <= NUM_LEN *
↪ BASE_LEN
const int BASE = pow(10, BASE_LEN);
const ll INF = 8e18, ADD = INF / BASE;

struct num {
  ll a[NUM_LEN];
  int len; // always > 0

  inline const ll& operator [] ( int i ) const { return a[i];
  ↪ }
  inline ll& operator [] ( int i ) { return a[i]; }

  num& operator = ( const num &x ) { // copy
    len = x.len;
    memcpy(a, x.a, sizeof(a[0]) * len);
    return *this;
  }

  num( const num &x ) { *this = x; } // copy

  num() { len = 1, a[0] = 0; } // 0
  num( ll x ) { // x
    len = 0;
    while (!len || x) {
      assert(len < NUM_LEN); // to catch overflow
      a[len++] = x % BASE, x /= BASE;
    }
  }

  num& cor() {
    while (a[len - 1] >= BASE) {
      assert(len < NUM_LEN); // to catch overflow
      if (a[len - 1] >= 2 * BASE)
        a[len] = a[len - 1] / BASE, a[len - 1] %= BASE;
      else
        a[len] = 1, a[len - 1] -= BASE;
      len++;
    }
    while (len > 1 && !a[len - 1])
      len--;
    return *this;
  }

  int length() {
    if (!len)
      return 0;
    int x = a[len - 1], res = 0;
    assert(x);
    while (x || !res)
      x /= 10, res++;
    return res + (len - 1) * BASE_LEN;
  }

  void out() const {
    int i = len - 1;
    printf("%d", (int)a[i--]);
    while (i >= 0)
      printf("%0*d", BASE_LEN, (int)a[i--]);
    puts("");
  }

  void init( const char *s ) {
    int sn = strlen(s);
    while (sn && s[sn - 1] <= 32)
      sn--;
    len = (sn + BASE_LEN - 1) / BASE_LEN;
    memset(a, 0, sizeof(a[0]) * len);
    forn(i, sn) {
      ll &r = a[(sn - i - 1) / BASE_LEN];
      r = r * 10 + (s[i] - '0');
    }
  }

  bool read() {
    static const int L = NUM_LEN * BASE_LEN + 1;
    static char s[L];
    if (!fgets(s, L, stdin))
      return 0;
    assert(!s[L - 2]);
    init(s);
    return 1;
  }

  void mul2() {
    forn(i, len)
      a[i] <<= 1;
    forn(i, len - 1)
      if (a[i] >= BASE)
        a[i + 1]++, a[i] -= BASE;
    cor();
  }

  void div2() {
    for (int i = len - 1; i >= 0; i--) {
      if (i && (a[i] & 1))
        a[i - 1] += BASE;
      a[i] >>= 1;
    }
    cor();
  }

  static ll cmp( ll *a, ll *b, int n ) {
    while (n--)
      if (a[n] != b[n])
        return a[n] - b[n];
    return 0;
  }

  int cmp( const num &b ) const {
    if (len != b.len)
      return len - b.len;
    for (int i = len - 1; i >= 0; i--)
      if (a[i] != b[i])
        return a[i] - b[i];
```

```
    return 0;
  }

  bool zero() {
    return len == 1 && !a[0];
  }

  /** c = this/b, this %= b */
  num &div( num b, num &c ) {
    c.len = len - b.len;
    for (int i = c.len; i >= 0; i--) {
      int k = (1.0L * a[len - 1] * BASE + (len >= 2 ? a[len -
      ↪  2] : 0)) / (1.0L * b[b.len - 1] * BASE + (b.len >= 2
      ↪  ? b[b.len - 2] + 1 : 0));
      c[i] = 0;
      if (k > 0) {
        c[i] += k;
        forn(j, b.len)
          a[i + j] -= (ll)b[j] * k;
        forn(j, b.len)
          if (a[i + j] < 0) {
            ll k = (-a[i + j] + BASE - 1) / BASE;
            a[i + j] += k * BASE, a[i + j + 1] -= k;
          }
      }
      if (i)
        len--, a[len - 1] += a[len] * BASE, a[len] = 0;
      else if (cmp(a, b.a, b.len) >= 0) {
        c[0]++;
        forn(j, b.len)
          if ((a[j] -= b[j]) < 0)
            a[j] += BASE, a[j + 1]--;
      }
    }
    if (c.len < 0)
      c[c.len = 0] = 0;
    forn(i, c.len)
      if (c[i] >= BASE)
        c[i + 1] += c[i] / BASE, c[i] %= BASE;
    c.len += (!c.len || c[c.len]);
    return cor();
  }
};

num& operator += ( num &a, const num &b ) {
  while (a.len < b.len)
    a[a.len++] = 0;
  forn(i, b.len)
    a[i] += b[i];
  forn(i, a.len - 1)
    if (a[i] >= BASE)
      a[i] -= BASE, a[i + 1]++;
  return a.cor();
}

num& operator -= ( num &a, const num &b ) {
  while (a.len < b.len)
    a[a.len++] = 0;
  forn(i, b.len)
    a[i] -= b[i];
  forn(i, a.len - 1)
    if (a[i] < 0)
      a[i] += BASE, a[i + 1]--;
  assert(a[a.len - 1] >= 0); // a >= b
  return a.cor();
}

num& operator *= ( num &a, int k ) {
  if (k == 1)
    return a;
  if (k == 0) {
    a.len = 0;
    return a;
  }
  forn(i, a.len)
    a[i] *= k;
  forn(i, a.len - 1)
    if (a[i] >= BASE)
      a[i + 1] += a[i] / BASE, a[i] %= BASE;
  return a.cor();
}

num& operator /= ( num &a, int k ) {
  if (k == 1)
    return a;
  assert(k != 0);
  for (int i = a.len - 1; i > 0; i--)
    a[i - 1] += (ll)(a[i] % k) * BASE, a[i] /= k;
  a[0] /= k;
  return a.cor();
}

num& mul( const num &a, const num &b, num &x ) {
  assert(a.len + b.len - 1 <= NUM_LEN);
  memset(x.a, 0, sizeof(x[0]) * (a.len + b.len - 1));
  forn(i, a.len)
    forn(j, b.len)
      if ((x[i + j] += a[i] * b[j]) >= INF)
        x[i + j + 1] += ADD, x[i + j] -= INF;
  x.len = a.len + b.len - 1;
  forn(i, x.len - 1)
    if (x[i] >= BASE)
      x[i + 1] += x[i] / BASE, x[i] %= BASE;
  return x.cor();
}

bool operator == ( const num &a, const num &b ) { return
↪  a.cmp(b) == 0; }
bool operator != ( const num &a, const num &b ) { return
↪  a.cmp(b) != 0; }
bool operator < ( const num &a, const num &b )  { return
↪  a.cmp(b) < 0; }
bool operator > ( const num &a, const num &b )  { return
↪  a.cmp(b) > 0; }
bool operator <= ( const num &a, const num &b ) { return
↪  a.cmp(b) <= 0; }
bool operator >= ( const num &a, const num &b ) { return
↪  a.cmp(b) >= 0; }

num& add( const num &a, const num &b, num &c ) { c = a; c +=
↪  b; return c; }
num& sub( const num &a, const num &b, num &c ) { c = a; c -=
↪  b; return c; }
num& mul( const num &a, int k, num &c )        { c = a; c *=
↪  k; return c; }
num& div( const num &a, int k, num &c )        { c = a; c /=
↪  k; return c; }

num& operator *= ( num &a, const num &b ) {
  static num tmp;
  mul(a, b, tmp);
  return a = tmp;
}

num operator ^ ( const num &a, int k ) {
  num res(1);
  forn(i, k)
    res *= a;
  return res;
}

num& gcd_binary( num &a, num &b ) {
```

```
  int cnt = 0;
  while (!a.zero() && !b.zero()) {
    while (!(b[0] & 1) && !(a[0] & 1))
      cnt++, a.div2(), b.div2();
    while (!(b[0] & 1))
      b.div2();
    while (!(a[0] & 1))
      a.div2();
    if (a.cmp(b) < 0)
      b -= a;
    else
      a -= b;
  }
  if (a.zero())
    std::swap(a, b);
  while (cnt)
    a.mul2(), cnt--;
  return a;
}

num& gcd( num &a, num &b ) {
  static num tmp;
  return b.zero() ? a : gcd(b, a.div(b, tmp));
}
```

6 FFT

```
int rev[MAX_N];

//typedef complex<dbl> Num;
struct Num {
  dbl x, y;
  Num() {}
  Num(dbl _x, dbl _y): x(_x), y(_y) {}
  inline dbl real() const { return x; }
  inline dbl imag() const { return y; }
  inline Num operator+(const Num &B) const { return Num(x +
  ↪  B.x, y + B.y); }
  inline Num operator-(const Num &B) const { return Num(x -
  ↪  B.x, y - B.y); }
  inline Num operator*(dbl k) const { return Num(x * k, y *
  ↪  k); }
  inline Num operator*(const Num &B) const { return Num(x *
  ↪  B.x - y * B.y, x * B.y + y * B.x); }
  inline void operator+=(const Num &B) { x += B.x, y += B.y; }
  inline void operator/=(dbl k) { x /= k, y /= k; }
  inline void operator*=(const Num &B) { *this = *this * B; }
};

Num rt[MAX_N];

inline Num sqr(const Num &x) { return x * x; }
inline Num conj(const Num &x) { return Num(x.real(),
↪  -x.imag()); }

inline int getN(int n) {
  int k = 1;
  while(k < n)
    k <<= 1;
  return k;
}

void fft(Num *a, int n) {
  assert(rev[1]); // don't forget to init
  int q = MAX_N / n;
  forn (i, n)
    if(i < rev[i] / q)
      swap(a[i], a[rev[i] / q]);
  for (int k = 1; k < n; k <<= 1)
    for (int i = 0; i < n; i += 2 * k)
      forn (j, k) {
        const Num z = a[i + j + k] * rt[j + k];
        a[i + j + k] = a[i + j] - z;
        a[i + j] += z;
      }
}

void fftInv(Num *a, int n) {
  fft(a, n);
  reverse(a + 1, a + n);
  forn (i, n)
    a[i] /= n;
}

void doubleFft(Num *a, Num *fa, Num *fb, int n) { // only if
↪  you need it
  fft(a, n);
  const int n1 = n - 1;
  forn (i, n) {
    const Num &z0 = a[i], &z1 = a[(n - i) & n1];
    fa[i] = Num(z0.real() + z1.real(), z0.imag() - z1.imag())
    ↪  * 0.5;
    fb[i] = Num(z0.imag() + z1.imag(), z1.real() - z0.real())
    ↪  * 0.5;
  }
}

Num tmp[MAX_N];
template<class T>
void mult(T *a, T *b, T *r, int n) { // n = 2^k
  forn (i, n)
    tmp[i] = Num((dbl) a[i], (dbl) b[i]);
  fft(tmp, n);
  const int n1 = n - 1;
  const Num c = Num(0, -0.25 / n);
  fornr (i, n / 2 + 1) {
    const int j = (n - i) & n1;
    const Num z0 = sqr(tmp[i]), z1 = sqr(tmp[j]);
    tmp[i] = (z1 - conj(z0)) * c;
    tmp[j] = (z0 - conj(z1)) * c;
  }
  fft(tmp, n);
  forn (i, n)
    r[i] = (T) round(tmp[i].real());
}

void init() { // don't forget to init
  forn(i, MAX_N)
    rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (LOG - 1));

  rt[1] = Num(1, 0);
  for (int k = 1, p = 2; k < LOG; k++, p *= 2) {
    const Num x(cos(PI / p), sin(PI / p));
    forab (i, p / 2, p)
      rt[2 * i] = rt[i], rt[2 * i + 1] = rt[i] * x;
  }
}
```

7 FFT by mod and FFT with digits up to 10^6

```
Num ta[MAX_N], tb[MAX_N], tf[MAX_N], tg[MAX_N];

const int HALF = 15;

void mult(int *a, int *b, int *r, int n, int mod) {
  int tw = (1 << HALF) - 1;
  forn (i, n) {
    int x = int(a[i] % mod);
    ta[i] = Num(x & tw, x >> HALF);
  }
  forn (i, n) {
    int x = int(b[i] % mod);
```

```
    tb[i] = Num(x & tw, x >> HALF);
  }

  fft(ta, n), fft(tb, n);
  forn (i, n) {
    int j = (n - i) & (n - 1);
    Num a1 = (ta[i] + conj(ta[j])) * Num(0.5, 0);
    Num a2 = (ta[i] - conj(ta[j])) * Num(0, -0.5);
    Num b1 = (tb[i] + conj(tb[j])) * Num(0.5 / n, 0);
    Num b2 = (tb[i] - conj(tb[j])) * Num(0, -0.5 / n);
    tf[j] = a1 * b1 + a2 * b2 * Num(0, 1);
    tg[j] = a1 * b2 + a2 * b1;
  }

  fft(tf, n), fft(tg, n);
  forn (i, n) {
    ll aa = ll(tf[i].x + 0.5);
    ll bb = ll(tg[i].x + 0.5);
    ll cc = ll(tf[i].y + 0.5);
    r[i] = int((aa + ((bb % mod) << HALF) + ((cc % mod) << (2
    ↪  * HALF))) % mod);
  }
}

int tc[MAX_N], td[MAX_N];

const int MOD1 = 1.5e9, MOD2 = MOD1 + 1;
void multLL(int *a, int *b, ll *r, int n){
  mult(a, b, tc, n, MOD1), mult(a, b, td, n, MOD2);
  forn(i, n)
    r[i] = tc[i] + (td[i] - tc[i] + (ll)MOD2) * MOD1 % MOD2 *
    ↪  MOD1;
}
```

3 Data Structures

8 Centroid Decomposition

```
vi g[MAX_N];
int d[MAX_N], par[MAX_N], centroid;
//d  par -

int find(int v, int p, int total) {
  int size = 1, ok = 1;
  for (int to : g[v])
    if (d[to] == -1 && to != p) {
      int s = find(to, v, total);
      if (s > total / 2) ok = 0;
      size += s;
    }
  if (ok && size > total / 2) centroid = v;
  return size;
}

void calcInComponent(int v, int p, int level) {
  // do something
  for (int to : g[v])
    if (d[to] == -1 && to != p)
      calcInComponent(to, v, level);
}

//fill(d, d + n, -1)
//decompose(0, -1, 0)
void decompose(int root, int parent, int level) {
  find(root, -1, find(root, -1, INF));
  int c = centroid;
  par[c] = parent, d[c] = level;
  calcInComponent(centroid, -1, level);
  for (int to : g[c])
    if (d[to] == -1)
      decompose(to, c, level + 1);
}
```

9 Convex Hull Trick

```
struct Line {
  int k, b;
  Line() {}
  Line(int _k, int _b): k(_k), b(_b) {}
  ll get(int x) { return b + k * 1ll * x; }
  bool operator<(const Line &l) const { return k < l.k; } //
  ↪       >
};

//     ,           (a,b)     (a,c)
inline bool check(Line a, Line b, Line c) {
  return (a.b - b.b) * 1ll * (c.k - a.k) < (a.b - c.b) * 1ll *
  ↪  (b.k - a.k);
}

struct Convex {
  vector<Line> st;
  inline void add(Line l) {
    while (sz(st) >= 2 && !check(st[sz(st) - 2], st[sz(st) -
    ↪  1], l))
      st.pop_back();
    st.pb(l);
  }
  int get(int x) {
    int l = 0, r = sz(st);
    while (r - l > 1) {
      int m = (l + r) / 2; //     >
      if (st[m - 1].get(x) < st[m].get(x))
        l = m;
      else
        r = m;
    }
    return l;
  }
  Convex() {}
  Convex(vector<Line> &lines) {
    st.clear();
    for(Line &l : lines)
      add(l);
  }
  Convex(Line line) { st.pb(line); }
  Convex(const Convex &a, const Convex &b) {
    vector<Line> lines;
    lines.resize(sz(a.st) + sz(b.st));
    merge(all(a.st), all(b.st), lines.begin());
    st.clear();
    for(Line &l : lines)
      add(l);
  }
};
```

10 DSU

```
int pr[MAX_N];

int get(int v) {
  return v == pr[v] ? v : pr[v] = get(pr[v]);
}

bool unite(int v, int u) {
  v = get(v), u = get(u);
  if (v == u) return 0;
  pr[u] = v;
  return 1;
}
```

```
void init(int n) {
  forn (i, n) pr[i] = i;
}
```

11 Fenwick Tree

```
int t[MAX_N];

int get(int ind) {
  int res = 0;
  for (; ind >= 0; ind &= (ind + 1), ind--)
    res += t[ind];
  return res;
}

void add(int ind, int n, int val) {
  for (; ind < n; ind |= (ind + 1))
    t[ind] += val;
}

int sum(int l, int r) { // [l, r)
  return get(r - 1) - get(l - 1);
}
```

12 Hash Table

```
using H = ll;
const int HT_SIZE = 1<<20, HT_AND = HT_SIZE - 1, HT_SIZE_ADD =
↪ HT_SIZE / 100;
H ht[HT_SIZE + HT_SIZE_ADD];
int data[HT_SIZE + HT_SIZE_ADD];

int get(const H &hash){
  int k = ((ll) hash) & HT_AND;
  while (ht[k] && ht[k] != hash) ++k;
  return k;
}

void insert(const H &hash, int x){
  int k = get(hash);
  if (!ht[k]) ht[k] = hash, data[k] = x;
}

bool count(const H &hash, int x){
  int k = get(hash);
  return ht[k] != 0;
}
```

13 Heavy Light Decomposition

```
vi g[MAX_N];
int size[MAX_N], comp[MAX_N], num[MAX_N], top[MAX_N],
↪ pr[MAX_N], tin[MAX_N], tout[MAX_N];
vi t[MAX_N], toPush[MAX_N], lst[MAX_N];
int curPath = 0, curTime = 0;

void pushST(int path, int v, int vl, int vr) {
  if (toPush[path][v] != -1) {
    if (vl != vr - 1)
      forn (j, 2)
        toPush[path][2 * v + j] = toPush[path][v];
    else
      t[path][v] = toPush[path][v];
    toPush[path][v] = -1;
  }
}

int getST(int path, int v, int vl, int vr, int ind) {
  pushST(path, v, vl, vr);
  if (vl == vr - 1)
    return t[path][v];
  int vm = (vl + vr) / 2;
  if (ind >= vm)
    return getST(path, 2 * v + 1, vm, vr, ind);
  return getST(path, 2 * v, vl, vm, ind);
}

void setST(int path, int v, int vl, int vr, int l, int r, int
↪ val) {
  if (vl >= l && vr <= r) {
    toPush[path][v] = val;
    pushST(path, v, vl, vr);
    return;
  }
  pushST(path, v, vl, vr);
  if (vl >= r || l >= vr)
    return;
  int vm = (vl + vr) / 2;
  setST(path, 2 * v, vl, vm, l, r, val);
  setST(path, 2 * v + 1, vm, vr, l, r, val);
  t[path][v] = min(t[path][2 * v], t[path][2 * v + 1]);
}

bool isUpper(int v, int u) {
  return tin[v] <= tin[u] && tout[v] >= tout[u];
}

int getHLD(int v) {
  return getST(comp[v], 1, 0, sz(t[comp[v]]) / 2, num[v]);
}

int setHLD(int v, int u, int val) {
  int ans = 0, w = 0;
  forn (i, 2) {
    while (!isUpper(w = top[comp[v]], u))
      setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, 0, num[v] + 1,
      ↪ val), v = pr[w];
    swap(v, u);
  }
  setST(comp[v], 1, 0, sz(t[comp[v]]) / 2, min(num[v],
  ↪ num[u]), max(num[v], num[u]) + 1, val);
  return ans;
}

void dfs(int v, int p) {
  tin[v] = curTime++;
  size[v] = 1;
  pr[v] = p;
  for (int u : g[v])
    if (u != p) {
      dfs(u, v);
      size[v] += size[u];
    }
  tout[v] = curTime++;
}

void build(int v) {
  if (v == 0 || size[v] * 2 < size[pr[v]])
    top[curPath] = v, comp[v] = curPath, num[v] = 0,
    ↪ curPath++;
  else
    comp[v] = comp[pr[v]], num[v] = num[pr[v]] + 1;
  lst[comp[v]].pb(v);
  for (int u : g[v])
    if (u != pr[v])
      build(u);
}

void initHLD() {
  dfs(0, 0);
  build(0);
```

```
  forn (i, curPath) {
    int curSize = 1;
    while (curSize < sz(lst[i]))
      curSize *= 2;
    t[i].resize(curSize * 2);
    toPush[i] = vi(curSize * 2, -1);
    //initialize t[i]
  }
}
```

14 Next Greater in Segment Tree

```
int t[4 * MAX_N], tSize = 1;

//                   pos              x
int nextGreaterX(int v, int l, int r, int pos, int x) {
  if (r <= pos + 1 || t[v] <= x) return INF;
  if (v >= tSize) return v - tSize;
  int ans = nextGreaterX(2 * v, l, (l + r) / 2, pos, x);
  if (ans == INF)
    ans = nextGreaterX(2 * v + 1, (l + r) / 2, r, pos, x);
  return ans;
}
```

15 Sparse Table

```
int st[MAX_N][MAX_LOG];
int lg[MAX_N];

int get(int l, int r) { // [l, r)
  int curLog = lg[r - l];
  return min(st[l][curLog], st[r - (1 << curLog)][curLog]);
}

void initSparseTable(int *a, int n) {
  lg[1] = 0;
  forab (i, 2, n + 1) lg[i] = lg[i / 2] + 1;
  forn (i, n) st[i][0] = a[i];
  forn (j, lg[n])
    forn (i, n - (1 << (j + 1)) + 1)
      st[i][j + 1] = min(st[i][j], st[i + (1 << j)][j]);
}
```

16 Fenwick Tree 2D

```
ll a[4][MAX_N][MAX_N];
int n, m;

inline int f(int x) { return x & ~(x - 1); }

inline void add(int k, int x, int y, ll val) {
  for (; x <= n; x += f(x))
    for (int j = y; j <= m; j += f(j))
      a[k][x][j] += val;
}

inline ll get(int k, int x, int y) {
  ll s = 0;
  for (; x > 0; x -= f(x))
    for (int j = y; j > 0; j -= f(j))
      s += a[k][x][j];
  return s;
}

inline ll get(int x, int y) {
  return ll(x + 1) * (y + 1) * get(0, x, y) - (y + 1) * get(1,
  ↪  x, y)
      - (x + 1) * get(2, x, y) + get(3, x, y);
}

inline void add(int x, int y, ll val) {
  add(0, x, y, val);
  add(1, x, y, val * x);
  add(2, x, y, val * y);
  add(3, x, y, val * x * y);
}

inline ll get(int x_1, int y_1, int x_2, int y_2) {
  return get(x_2, y_2) - get(x_1 - 1, y_2) - get(x_2, y_1 - 1)
  ↪  + get(x_1 - 1, y_1 - 1);
}

// Adds val to corresponding rectangle
inline void add(int x_1, int y_1, int x_2, int y_2, ll val) {
  add(x_1, y_1, val);
  if (y_2 < m) add(x_1, y_2 + 1, -val);
  if (x_2 < n) add(x_2 + 1, y_1, -val);
  if (x_2 < n && y_2 < m) add(x_2 + 1, y_2 + 1, val);
}
```

4 Dynamic Programming

17 LIS

```
int longestIncreasingSubsequence(vi a) {
  int n = sz(a);
  vi d(n + 1, INF);
  d[0] = -INF;
  forn (i, n)
    *upper_bound(all(d), a[i]) = a[i];
  fornr (i, n + 1) if (d[i] != INF) return i;
  return 0;
}
```

18 DP tree

```
int dp[MAX_N][MAX_N], a[MAX_N];
vi g[MAX_N];

int dfs(int v, int n) {
  forn (i, n + 1)
    dp[v][i] = -INF;
  dp[v][1] = a[v];
  int curSz = 1;
  for (int to : g[v]) {
    int toSz = dfs(to, n);
    for (int i = curSz; i >= 1; i--)
      fornr (j, toSz + 1)
        dp[v][i + j] = max(dp[v][i + j], dp[v][i] +
        ↪  dp[to][j]);
    curSz += toSz;
  }
  return curSz;
}
```

19 Masks tricks

```
int dp[(1 << MAX_MASK)][MAX_MASK];

void calcDP(int n) {
  forn(mask, 1 << n) {
    dp[mask][n] = 1;
    fornr(i, n) {
      dp[mask][i] = dp[mask][i + 1];
      if ((1 << i) & mask)
        dp[mask][i] += dp[mask ^ (1 << i)][i + 1];
    }
  }
}
```

5 Flows

20 Utilities

```
vi g[MAX_N];

// for directed unweighted graph
struct Edge {
  int v, u, c, f;
  Edge() {}
  Edge(int _v, int _u, int _c): v(_v), u(_u), c(_c), f(0) {}
};

vector<Edge> edges;

inline void addFlow(int e, int flow) {
   edges[e].f += flow, edges[e ^ 1].f -= flow;
}

inline void addEdge(int v, int u, int c) {
  g[v].pb(sz(edges)), edges.pb(Edge(v, u, c));
  g[u].pb(sz(edges)), edges.pb(Edge(u, v, 0)); // for undirected 0 should be c
}
```

21 Ford-Fulkerson

```
int used[MAX_N], pr[MAX_N];
int curTime = 1;

int dfs(int v, int can, int toPush, int t) {
  if (v == t) return can;
  used[v] = curTime;
  for (int edge : g[v]) {
    auto &e = edges[edge];
    if (used[e.u] != curTime && e.c - e.f >= toPush) {
      int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
      if (flow > 0) {
        addFlow(edge, flow), pr[e.u] = edge;
        return flow;
      }
    }
  }
  return 0;
}

int fordFulkerson(int s, int t) {
  int ansFlow = 0, flow = 0;
  // Without scaling
  while ((flow = dfs(s, INF, 1, t)) > 0)
    ansFlow += flow, curTime++;
  // With scaling
  fornr (i, INF_LOG)
    for (curTime++; (flow = dfs(s, INF, (1 << i), t)) > 0; curTime++)
      ansFlow += flow;
  return ansFlow;
}
```

22 Dinic

```
int pr[MAX_N], d[MAX_N], q[MAX_N], first[MAX_N];

int dfs(int v, int can, int toPush, int t) {
  if (v == t) return can;
  int sum = 0;
  for (; first[v] < (int) g[v].size(); first[v]++) {
    auto &e = edges[g[v][first[v]]];
    if (d[e.u] != d[v] + 1 || e.c - e.f < toPush) continue;
    int flow = dfs(e.u, min(can, e.c - e.f), toPush, t);
    addFlow(g[v][first[v]], flow);
    can -= flow, sum += flow;
    if (!can)
      return sum;
  }
  return sum;
}

bool bfs(int n, int s, int t, int curPush) {
  forn (i, n) d[i] = INF, first[i] = 0;
  int head = 0, tail = 0;
  q[tail++] = s;
  d[s] = 0;
  while (tail - head > 0) {
    int v = q[head++];
    for (int edge : g[v]) {
      auto &e = edges[edge];
      if (d[e.u] > d[v] + 1 && e.c - e.f >= curPush)
        d[e.u] = d[v] + 1, q[tail++] = e.u;
    }
  }
  return d[t] != INF;
}

int dinic(int n, int s, int t) {
  int ansFlow = 0;
  // Without scaling
  while (bfs(n, s, t, 1))
    ansFlow += dfs(s, INF, 1, t);
  // With scaling
  fornr (j, INF_LOG)
    while (bfs(n, s, t, 1 << j))
      ansFlow += dfs(s, INF, 1 << j, t);
  return ansFlow;
}
```

23 Hungarian

```
const int INF = 1e9;
int a[MAX_N][MAX_N];

// min = sum of a[pa[i],i]
// you may optimize speed by about 15%, just change all vectors to static arrays
vi Hungarian(int n) {
  vi pa(n + 1, -1), row(n + 1, 0), col(n + 1, 0), la(n + 1);
  forn (k, n) {
    vi u(n + 1, 0), d(n + 1, INF);
    pa[n] = k;
    int l = n, x;
    while ((x = pa[l]) != -1) {
      u[l] = 1;
      int minn = INF, tmp, l0 = l;
      forn (j, n)
        if (!u[j]) {
          if ((tmp = a[x][j] + row[x] + col[j]) < d[j])
            d[j] = tmp, la[j] = l0;
          if (d[j] < minn)
            minn = d[j], l = j;
        }
      forn (j, n + 1)
        if (u[j])
          col[j] += minn, row[pa[j]] -= minn;
        else
          d[j] -= minn;
    }
    while (l != n)
      pa[l] = pa[la[l]], l = la[l];
  }
  return pa;
}
```

24 Min Cost Max Flow

```
const int MAX_M = 1e4;
int pr[MAX_N], in[MAX_N], q[MAX_N * MAX_M], used[MAX_N],
↪ d[MAX_N], pot[MAX_N];
vi g[MAX_N];

struct Edge {
  int v, u, c, f, w;
  Edge() {}
  Edge(int _v, int _u, int _c, int _w): v(_v), u(_u), c(_c),
  ↪ f(0), w(_w) {}
};

vector<Edge> edges;

inline void addFlow(int e, int flow) {
  edges[e].f += flow, edges[e ^ 1].f -= flow;
}

inline void addEdge(int v, int u, int c, int w) {
  g[v].pb(sz(edges)), edges.pb(Edge(v, u, c, w));
  g[u].pb(sz(edges)), edges.pb(Edge(u, v, 0, -w));
}

int dijkstra(int n, int s, int t) {
  forn (i, n) used[i] = 0, d[i] = INF;
  d[s] = 0;
  while (1) {
    int v = -1;
    forn (i, n)
      if (!used[i] && (v == -1 || d[v] > d[i]))
        v = i;
    if (v == -1 || d[v] == INF) break;
    used[v] = 1;
    for (int edge : g[v]) {
      auto &e = edges[edge];
      int w = e.w + pot[v] - pot[e.u];
      if (e.c > e.f && d[e.u] > d[v] + w)
        d[e.u] = d[v] + w, pr[e.u] = edge;
    }
  }
  if (d[t] == INF) return d[t];
  forn (i, n) pot[i] += d[i];
  return pot[t];
}

int fordBellman(int n, int s, int t) {
  forn (i, n) d[i] = INF;
  int head = 0, tail = 0;
  d[s] = 0, q[tail++] = s, in[s] = 1;
  while (tail - head > 0) {
    int v = q[head++];
    in[v] = 0;
    for (int edge : g[v]) {
      auto &e = edges[edge];
      if (e.c > e.f && d[e.u] > d[v] + e.w) {
        d[e.u] = d[v] + e.w;
        pr[e.u] = edge;
        if (!in[e.u])
          in[e.u] = 1, q[tail++] = e.u;
      }
    }
  }
  return d[t];
}

int minCostMaxFlow(int n, int s, int t) {
  int ansFlow = 0, ansCost = 0, dist;
  while ((dist = dijkstra(n, s, t)) != INF) {
    int curFlow = INF;
    for (int cur = t; cur != s; cur = edges[pr[cur]].v)
      curFlow = min(curFlow, edges[pr[cur]].c -
      ↪ edges[pr[cur]].f);
    for (int cur = t; cur != s; cur = edges[pr[cur]].v)
      addFlow(pr[cur], curFlow);
    ansFlow += curFlow;
    ansCost += curFlow * dist;
  }
  return ansCost;
}
```

6 Games

25 Retrograde Analysis

```
int win[MAX_N], lose[MAX_N], outDeg[MAX_N];
vi rg[MAX_N];

void retro(int n) {
  queue<int> q;
  forn (i, n)
    if (!outDeg[i])
      lose[i] = 1, q.push(i);
  while (!q.empty()) {
    int v = q.front();
    q.pop();
    for (int to : rg[v])
      if (lose[v]) {
        if (!win[to])
          win[to] = 1, q.push(to);
      } else {
        outDeg[to]--;
        if (!outDeg[to])
          lose[to] = 1, q.push(to);
      }
  }
}
```

7 Geometry

26 ClosestPoints (SweepLine)

```
#include "header.h"

const int N = 2e5;

struct Pnt {
  int x, y, i;
  bool operator <(const Pnt &p) const { return mp(y, i) <
  ↪ mp(p.y, p.i); }
};

ll d2 = 8e18, d = (ll) sqrt(d2) + 1;
Pnt p[N];

inline ll sqr(int x){
  return (LL)x * x;
}

inline void relax(const Pnt &a, const Pnt &b){
  ll tmp = sqr(a.x - b.x) + sqr(a.y - b.y);
  if (tmp < d2)
    d2 = tmp, d = (LL)(sqrt(d2) + 1 - 1e-9); // round up
}

inline bool xless(const Pnt &a, const Pnt &b){
  return a.x < b.x;
}

int main() {
```

```
  int n;
  scanf("%d", &n);
  forn(i, n)
    scanf("%d%d", &p[i].x, &p[i].y), p[i].i = i;
  sort(p, p + n, xless);

  set <Pnt> s;
  int l = 0;
  forn(r, n){
    set<Pnt>::iterator it_r = s.lower_bound(p[r]), it_l = it_r;
    for (; it_r != s.end() && it_r->y - p[r].y < d; ++it_r)
      relax(*it_r, p[r]);
    while (it_l != s.begin() && p[r].y - (--it_l)->y < d)
      relax(*it_l, p[r]);
    s.insert(p[r]);
    while (l <= r && p[r].x - p[l].x >= d)
      s.erase(p[l++]);
  }
  printf("%.9f\n", sqrt(d2));
  return 0;
}
```

27 ConvexHull

```
typedef vector<Pnt> vpnt;

inline bool byAngle(const Pnt &a, const Pnt &b) {
  dbl x = a % b;
  return eq(x, 0) ? a.len2() < b.len2() : x < 0;
}

vpnt convexHull(vpnt p) {
  int n = sz(p);
  assert(n > 0);
  swap(p[0], *min_element(all(p)));
  forab(i, 1, n)
  p[i] = p[i] - p[0];
  sort(p.begin() + 1, p.end(), byAngle);

/*                          ,     (1)   (2)
  (1):
  int k = p.size() - 1;
  while(k > 0 && eq((p[k - 1] - p.back()) % p.back(), 0))
    --k;
  reverse(pi.begin() + k, pi.end());*/

  int rn = 0;
  vpnt r(n);
  r[rn++] = p[0];
  forab(i, 1, n){
    Pnt q = p[i] + p[0];
    while(rn >= 2 && geq((r[rn - 1] - r[rn - 2]) % (q - r[rn - 2]), 0)) // (2) ge
      --rn;
    r[rn++] = q;
  }
  r.resize(rn);
  return r;
}
```

28 GeometryBase

```
#include<bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef long double ld;
typedef double dbl;

const dbl EPS = 1e-9;
const int PREC = 20;
inline bool eq(dbl a, dbl b) { return abs(a-b)<=EPS; }
inline bool gr(dbl a, dbl b) { return a>b+EPS; }
inline bool geq(dbl a, dbl b) { return a>=b-EPS; }
inline bool ls(dbl a, dbl b) { return a<b-EPS; }
inline bool leq(dbl a, dbl b) { return a<=b+EPS; }

struct Pnt {
    dbl x,y;
    Pnt(): x(0), y(0) {}
    Pnt(dbl xx, dbl yy): x(xx), y(yy) {}

    inline Pnt operator +(const Pnt &p) const { return Pnt(x + p.x, y + p.y); }
    inline Pnt operator -(const Pnt &p) const { return Pnt(x - p.x, y - p.y); }
    inline dbl operator *(const Pnt &p) const { return x * p.x + y * p.y; } // ll
    inline dbl operator %(const Pnt &p) const { return x * p.y - y * p.x; } // ll

    inline Pnt operator *(dbl k) const { return Pnt(x * k, y * k); }
    inline Pnt operator /(dbl k) const { return Pnt(x / k, y / k); }
    inline Pnt operator -() const { return Pnt(-x, -y); }

    inline void operator +=(const Pnt &p) { x += p.x, y += p.y; }
    inline void operator -=(const Pnt &p) { x -= p.x, y -= p.y; }
    inline void operator *=(dbl k) { x*=k, y*=k; }

    inline bool operator ==(const Pnt &p) { return abs(x-p.x)<=EPS && abs(y-p.y)<=EPS; }
    inline bool operator !=(const Pnt &p) { return abs(x-p.x)>EPS || abs(y-p.y)>EPS; }
    inline bool operator <(const Pnt &p) { return abs(x-p.x)<=EPS ? y<p.y-EPS : x<p.x; }

    inline dbl angle() const { return atan2(y, x); } // ld
    inline dbl len2() const { return x*x+y*y; } // ll
    inline dbl len() const { return sqrt(x*x+y*y); } // ll, ld
    inline Pnt getNorm() const {
        auto l = len();
        return Pnt(x/l, y/l);
    }
    inline void normalize() {
        auto l = len();
        x/=l, y/=l;
    }

    inline Pnt getRot90() const { //counter-clockwise
        return Pnt(-y, x);
    }
    inline Pnt getRot(dbl a) const { // ld
        dbl si = sin(a), co = cos(a);
        return Pnt(x*co - y*si, x*si + y*co);
    }

    inline void read() {
        int xx, yy;
    cin >> xx >> yy;
        x = xx, y = yy;
    }
    inline void write() const{
        cout << fixed << (double)x << (double)y;
    }
};

struct Line{
```

```
    dbl a, b, c;
    Line(): a(0), b(0), c(0) {}
    Line(dbl aa, dbl bb, dbl cc): a(aa), b(bb), c(cc) {}

    Line(const Pnt &A, const Pnt &p){ // it normalizes (a,b),
    ↪  important in d(), normalToP()
        Pnt n = (p-A).getRot90().getNorm();
        a = n.x, b = n.y, c = -(a * A.x + b * A.y);
    }

    inline dbl d(const Pnt &p) const { return a*p.x + b*p.y +
    ↪  c; }
    inline Pnt no() const {return Pnt(a, b);}
    inline Pnt normalToP(const Pnt &p) const { return Pnt(a,b)
    ↪  * (a*p.x + b*p.y + c); }

    inline void write() const{
      cout << fixed << (double)a << " " << (double)b << " " <<
      ↪  (double)c << '\n';
    }
};
```

29 GeometryInterTangent

```
void buildTangent(Pnt p1, dbl r1, Pnt p2, dbl r2, Line &l) {
↪  // r1, r2 = radius with sign
    Pnt p = p2 - p1;
    l.c = r1;
    dbl c2 = p.len2(), c1 = sqrt(c2 - sqr(r2));
    l.a = (-p.x * (r1 - r2) + p.y * c1) / c2;
    l.b = (-p.y * (r1 - r2) - p.x * c1) / c2;
    l.c -= l.no() * p1;
    assert(eq(l.d(p1), r1));
    assert(eq(l.d(p2), r2));
}


struct Circle {
    Pnt p;
    dbl r;
};

vector<Pnt> v; // to store intersection

// Intersection of two lines
int line_line(const Line &l, const Line &m){
    dbl z = m.a * l.b - l.a * m.b;
  dbl x = m.c * l.b - l.c * m.b;
  dbl y = m.c * l.a - l.c * m.a;
    if(fabs(z) > EPS){
        v.pb(Pnt(-x/z, y/z));
        return 1;
    }else if(fabs(x) > EPS || fabs(y) > EPS)
        return 0; // parallel lines
    else
        return 2; // same lines
}

// Intersection of Circle and line
void circle_line(const Circle &c, const Line &l){
    dbl d = l.d(c.p);
    if(fabs(d) > c.r + EPS)
        return;
    if(fabs(fabs(d) / c.r - 1) < EPS)
        v.pb(c.p - l.no() * d);
    else{
        dbl s = sqrt(fabs(sqr(c.r) - sqr(d)));
        v.pb(c.p - l.no() * d + l.no().getRot90() * s);
        v.pb(c.p - l.no() * d - l.no().getRot90() * s);
    }
}

// Intersection of two circles
void circle_circle(const Circle &a, const Circle &b){
    circle_line(a, Line((b.p - a.p) * 2, a.p.len2() -
    ↪  b.p.len2() + sqr(b.r) - sqr(a.r)));
}

// Squared distance between point p and segment [a..b]
dbl dist2(Pnt p, Pnt a, Pnt b){
    if ((p - a) * (b - a) < 0) return (p - a).len2();
    if ((p - b) * (a - b) < 0) return (p - b).len2();
    dbl d = fabs((p - a) % (b - a));
    return d * d / (b - a).len2();
}
```

30 GeometrySimple

```
int sign(dbl a) { return (a > EPS) - (a < -EPS); }

// Checks, if point is inside the segment
inline bool inSeg(const Pnt &p, const Pnt &a, const Pnt &b) {
    return eq((p - a) % (p - b), 0) && leq((p - a) * (p - b),
    ↪  0);
}

// Checks, if two intervals (segments without ends) intersect
↪  AND do not lie on the same line
inline bool subIntr(const Pnt &a, const Pnt &b, const Pnt &c,
↪  const Pnt &d){
    return
            sign((b - a) % (c - a)) * sign((b - a) % (d - a))
            ↪  == -1 &&
            sign((d - c) % (a - c)) * sign((d - c) % (b - c))
            ↪  == -1;
}

// Checks, if two segments (ends are included) has an
↪  intersection
inline bool checkSegInter(const Pnt &a, const Pnt &b, const
↪  Pnt &c, const Pnt &d){
    return inSeg(c, a, b) || inSeg(d, a, b) || inSeg(a, c, d)
    ↪  || inSeg(b, c, d) || subIntr(a, b, c, d);
}

inline dbl area(vector<Pnt> p){
    dbl s = 0;
    int n = sz(p);
    p.pb(p[0]);
    forn(i, n)
        s += p[i + 1] % p[i];
    p.pop_back();
    return abs(s) / 2;
}

// Check if point p is inside polygon <n, q[]>
int containsSlow(Pnt p, Pnt *z, int n){
    int cnt = 0;
    forn(j, n){
        Pnt a = z[j], b = z[(j + 1) % n];
        if (inSeg(p, a, b))
            return -1; // border
        if (min(a.y, b.y) - EPS <= p.y && p.y < max(a.y, b.y)
        ↪  - EPS)
            cnt += (p.x < a.x + (p.y - a.y) * (b.x - a.x) /
            ↪  (b.y - a.y));
    }
    return cnt & 1; // 0 = outside, 1 = inside
}


//for convex polygon
```

```
//assume polygon is counterclockwise-ordered
bool containsFast(Pnt p, Pnt *z, int n) {
    Pnt o = z[0];
    if(gr((p - o) % (z[1] - o), 0) || ls((p - o) % (z[n - 1] -
    ↪  o), 0))
        return 0;
    int l = 0, r = n - 1;
    while(r - l > 1){
        int m = (l + r) / 2;
        if(gr((p - o) % (z[m] - o), 0))
            r = m;
        else
            l = m;
    }
    return leq((p - z[l]) % (z[r] - z[l]), 0);
}

// Checks, if point "i" is in the triangle "abc" IFF triangle
↪  in CCW order
inline int isInTr(int i, int a, int b, int c){
    return
            gr((p[b] - p[a]) % (p[i] - p[a]), 0) &&
            gr((p[c] - p[b]) % (p[i] - p[b]), 0) &&
            gr((p[a] - p[c]) % (p[i] - p[c]), 0);
}
```

31 Halfplanes Intersection

```
const int maxn = (int)4e5 + 9;
const dbl eps = 1e-12;

dbl sqr( dbl x ) { return x * x; }

struct pnt{
  LL operator * ( pnt p ) { return (LL)x * p.y - (LL)y * p.x;
  ↪  }
  LL operator ^ ( pnt p ) { return (LL)x * p.x + (LL)y * p.y;
  ↪  }
  pnt ort() { return pnt(-y, x); }
  dbl ang() { return atan2(y, x); }
  LL d2() { return x * x + y * y; }
};

pnt st, v, p[maxn];
int n, sp, ss[maxn], ind[maxn], no[maxn], cnt[maxn], k = 0,
↪  a[maxn], b[maxn];
dbl ang[maxn];

pnt Norm( int k ){  return (p[a[k]] - p[b[k]]).ort();}

void AddPlane( int i, int j ){
  a[k] = i, b[k] = j, ind[k] = k;
  ang[k] = Norm(k).ang();
  k++;
}

bool angLess( int i, int j ){ return ang[i] < ang[j];}

void Unique()
{
  int i = 0, k2 = 0;
  while (i < k)
  {
    int ma = ind[i], st = i;
    pnt no = Norm(ma);

    for (i++; i < k && fabs(ang[ind[st]] - ang[ind[i]]) < eps;
    ↪  i++)
      if ((no ^ p[a[ma]]) < (no ^ p[a[ind[i]]]))
        ma = ind[i];
    ind[k2++] = ma;
  }
  k = k2;
}

dbl xx, yy, tmp;

#define BUILD(a1, b1, c1, i) \
  dbl a1 = Norm(i).x; \
  dbl b1 = Norm(i).y; \
  tmp = sqrt(a1 * a1 + b1 * b1); \
  a1 /= tmp, b1 /= tmp; \
  dbl c1 = -(a1 * p[a[i]].x + b1 * p[a[i]].y);

void FindPoint( int i, int j, dbl step = 0.0 ){
  BUILD(a1, b1, c1, i);
  BUILD(a2, b2, c2, j);

  xx = -(c1 * b2 - c2 * b1) / (a1 * b2 - a2 * b1);
  yy = (c1 * a2 - c2 * a1) / (a1 * b2 - a2 * b1);

  dbl no = sqrt(sqr(a1 + a2) + sqr(b1 + b2));
  xx += (a1 + a2) * step / no;
  yy += (b1 + b2) * step / no;
}

void TryShiftPoint( int i, int j, dbl step )
{
  FindPoint(i, j, step);

  forn(i, k){
    BUILD(a1, b1, c1, ind[i]);
    if (a1 * xx + b1 * yy + c1 < eps)
      return;
  }

  puts("Possible");
  printf("%.20lf %.20lf\n", (double)xx, (double)yy);
  exit(0);
}

void PushPlaneIntoStack( int i )
{
  while (sp >= 2 && ang[i] - ang[ss[sp - 2]] + eps < M_PI){
    FindPoint(i, ss[sp - 2]);

    BUILD(a1, b1, c1, ss[sp - 1]);
    if ((a1 * xx + b1 * yy + c1) < -eps)
      break;

    sp--;
  }
  ss[sp++] = i;
}

int main()
{
  scanf("%d", &n);
  forn(i, n)
    scanf("%d%d", &p[i].x, &p[i].y);
  p[n] = p[0];

  // Find set of planes
  forn(i, sp)
    AddPlane(max(ss[i], ss[i + 1]), min(ss[i], ss[i + 1]));
  forn(i, n - 1)
    AddPlane(i + 1, i);
  sort(ind, ind + k, angLess);

  int oldK = k;
  Unique();
```

```
  forn(i, oldK)
    no[i] = i;
  forn(i, k){
    int j = oldK + i, x = ind[i];
    ang[j] = ang[x] + 2 * M_PI;
    a[j] = a[x];
    b[j] = b[x];
    ind[i + k] = j, no[j] = x;
  }

  sp = 0;
  forn(i, 2 * k)
    PushPlaneIntoStack(ind[i]);
  forn(t, sp)
    if (++cnt[no[ss[t]]] > 1){
      TryShiftPoint(ss[t], ss[t - 1], 1e-5);
      break;
    }
  return 0;
}
```

8 Graphs

32 2-SAT

```
// MAXVAR - 2 * vars
int cntVar = 0, val[MAXVAR], usedSat[MAXVAR], comp[MAXVAR];
vi topsortSat;

vi g[MAXVAR], rg[MAXVAR];

inline int newVar() {
  cntVar++;
  return (cntVar - 1) * 2;
}

inline int Not(int v) { return v ^ 1; }

inline void Implies(int v1, int v2) { g[v1].pb(v2),
↪  rg[v2].pb(v1); }

inline void Or(int v1, int v2) { Implies(Not(v1), v2),
↪  Implies(Not(v2), v1); }

inline void Nand(int v1, int v2) { Or(Not(v1), Not(v2)); }

inline void setTrue(int v) { Implies(Not(v), v); }

void dfs1(int v) {
  usedSat[v] = 1;
  for (int to : g[v])
    if (!usedSat[to]) dfs1(to);
  topsortSat.pb(v);
}

void dfs2(int v, int c) {
  comp[v] = c;
  for (int to : rg[v])
    if (!comp[to]) dfs2(to, c);
}

int getVal(int v) { return val[v]; }

// cntVar
bool solveSat() {
  forn(i, 2 * cntVar) usedSat[i] = 0;
  forn(i, 2 * cntVar)
    if (!usedSat[i]) dfs1(i);
  reverse(all(topsortSat));
  int c = 0;
  for (int v : topsortSat)
    if (!comp[v]) dfs2(v, ++c);
  forn(i, cntVar) {
    if (comp[2 * i] == comp[2 * i + 1]) return false;
    if (comp[2 * i] < comp[2 * i + 1]) val[2 * i + 1] = 1;
    else val[2 * i] = 1;
  }
  return true;
}
```

33 Bridges

```
int up[MAX_N], tIn[MAX_N], timer;
vector<vi> comps;
vi st;

struct Edge {
  int to, id;
  Edge(int _to, int _id) : to(_to), id(_id) {}
};

vector<Edge> g[MAX_N];

void newComp(int size = 0) {
  comps.emplace_back(); //
  while (sz(st) > size) {
    comps.back().pb(st.back());
    st.pop_back();
  }
}

void findBridges(int v, int parentEdge = -1) {
  if (up[v]) //
    return;
  up[v] = tIn[v] = ++timer;
  st.pb(v);
  for (Edge e : g[v]) {
    if (e.id == parentEdge)
      continue;
    int u = e.to;
    if (!tIn[u]) {
      int size = sz(st);
      findBridges(u, e.id);
      if (up[u] > tIn[v])
        newComp(size);
    }
    up[v] = min(up[v], up[u]);
  }
}

//          find_bridges newComp()
void run(int n) {
  forn (i, n)
    if (!up[i]) {
      findBridges(i);
      newComp();
    }
}
```

34 Cut Points

```
bool used[MAX_M];
int tIn[MAX_N], timer, isCut[MAX_N], color[MAX_M], compCnt;
vi st;

struct Edge {
  int to, id;
  Edge(int _to, int _id) : to(_to), id(_id) {}
};

vector<Edge> g[MAX_N];
```

```
int dfs(int v, int parent = -1) {
  tIn[v] = ++timer;
  int up = tIn[v], x = 0, y = (parent != -1);
  for (Edge p : g[v]) {
    int u = p.to, id = p.id;
    if (id != parent) {
      int t, size = sz(st);
      if (!used[id])
        used[id] = 1, st.push_back(id);
      if (!tIn[u]) { // not visited yet
        t = dfs(u, id);
        if (t >= tIn[v]) {
          ++x, ++compCnt;
          while (sz(st) != size) {
            color[st.back()] = compCnt;
            st.pop_back();
          }
        }
      } else
        t = tIn[u];
      up = min(up, t);
    }
  }
  if (x + y >= 2)
    isCut[v] = 1; // v is cut vertex
  return up;
}
```

35 Eulerian Cycle

```
struct Edge {
  int to, used;
  Edge(): to(-1), used(0) {}
  Edge(int v): to(v), used(0) {}
};

vector<Edge> edges;
vi g[MAX_N], res, ptr;
//             ptr

void dfs(int v) {
  for(; ptr[v] < sz(g[v]);) {
    int id = g[v][ptr[v]++];
    if (!edges[id].used) {
      edges[id].used = edges[id ^ 1].used = 1;
      dfs(edges[id].to);
      res.pb(id); //
    }
  }
  res.pb(v); //   res
}
```

36 Euler Tour Tree

```
mt19937 rng(239);

struct Edge {
   int v, u;
   Edge(int _v, int _u): v(_v), u(_u) {}
};

struct Node {
  Node *l, *r, *p;
  Edge e;
  int y, size;
  Node(Edge _e): l(nullptr), r(nullptr), p(this), e(_e),
  ↪ y(rng()), size(1) {}
};

inline int getSize(Node* root) { return root ? root->size : 0;
↪ }

inline void recalc(Node* root) { root->size = getSize(root->l)
↪ + getSize(root->r) + 1; }

set<pair<int, Node*>> edges[MAX_N];

Node* merge(Node *a, Node *b) {
  if (!a) return b;
  if (!b) return a;
  if (a->y < b->y) {
    a->r = merge(a->r, b);
    if (a->r) a->r->p = a;
    recalc(a);
    return a;
  }
  b->l = merge(a, b->l);
  if (b->l) b->l->p = b;
  recalc(b);
  return b;
}

void split(Node *root, Node *&a, Node *&b, int size) {
  if (!root) {
    a = b = nullptr;
    return;
  }
  int lSize = getSize(root->l);
  if (lSize >= size) {
    split(root->l, a, root->l, size);
    if (root->l) root->l->p = root;
    b = root, b->p = b;
  } else {
    split(root->r, root->r, b, size - lSize - 1);
    if (root->r) root->r->p = root;
    a = root, a->p = a;
    a->p = a;
  }
  recalc(root);
}

inline Node* rotate(Node* root, int k) {
  if (k == 0) return root;
  Node *l, *r;
  split(root, l, r, k);
  return merge(r, l);
}

inline pair<Node*, int> goUp(Node* root) {
  int pos = getSize(root->l);
  while (root->p != root)
    pos += (root->p->r == root ? getSize(root->p->l) + 1 : 0),
    ↪ root = root->p;
  return mp(root, pos);
}

inline Node* deleteFirst(Node* root) {
  Node* a;
  split(root, a, root, 1);
  edges[a->e.v].erase(mp(a->e.u, a));
  return root;
}

inline Node* getNode(int v, int u) {
  return edges[v].lower_bound(mp(u, nullptr))->snd;
}

inline void cut(int v, int u) {
  auto pV = goUp(getNode(v, u));
  auto pU = goUp(getNode(u, v));
```

```
  int l = min(pV.snd, pU.snd), r = max(pV.snd, pU.snd);
  Node *a, *b, *c;
  split(pV.fst, a, b, l);
  split(b, b, c, r - l);
  deleteFirst(b);
  merge(a, deleteFirst(c));
}

inline pair<Node*, int> getRoot(int v) {
  return !sz(edges[v]) ? mp(nullptr, 0) :
  ↪  goUp(edges[v].begin()->snd);
}

inline Node* makeRoot(int v) {
  auto root = getRoot(v);
  return rotate(root.fst, root.snd);
}

inline Node* makeEdge(int v, int u) {
  Node* e = new Node(Edge(v, u));
  edges[v].insert(mp(u, e));
  return e;
}

inline void link(int v, int u) {
  Node *vN = makeRoot(v), *uN = makeRoot(u);
  merge(merge(merge(vN, makeEdge(v, u)), uN), makeEdge(u, v));
}
```

37 Hamilton Cycle

```
//          -         n*2^n
vi g[MAX_MASK];
int adj[MAX_MASK], dp[1 << MAX_MASK];

vi hamiltonCycle(int n) {
  fill(dp, dp + (1 << n), 0);
  forn (v, n) {
    adj[v] = 0;
    for (int to : g[v])
      adj[v] |= (1 << to);
  }
  dp[1] = 1;
  forn (mask, (1 << n))
    forn(v, n)
      if (mask & (1 << v) && dp[mask ^ (1 << v)] & adj[v])
        dp[mask] |= (1 << v);
  vi ans;
  int mask = (1 << n) - 1, v;
  if (dp[mask] & adj[0]) {
    forab (i, 1, n)
      if ((1 << i) & (mask & adj[0]))
        v = i;
    ans.pb(v);
    mask ^= (1 << v);
    while(v) {
      forn(i, n)
        if ((dp[mask] & (1 << i)) && (adj[i] & (1 << v))) {
          v = i;
          break;
        }
      mask ^= (1 << v);
      ans.pb(v);
    }
  }
  return ans;
}
```

38 Karp with cycle

```
int d[MAX_N][MAX_N], p[MAX_N][MAX_N];
vi g[MAX_N], ans;

struct Edge {
  int a, b, w;
  Edge(int _a, int _b, int _w): a(_a), b(_b), w(_w) {}
};

vector<Edge> edges;

void fordBellman(int s, int n) {
  forn (i, n + 1)
    forn (j, n + 1)
      d[i][j] = INF;
  d[0][s] = 0;
  forab (i, 1, n + 1)
    for (auto &e : edges)
      if (d[i - 1][e.a] < INF && d[i][e.b] > d[i - 1][e.a] +
      ↪  e.w)
        d[i][e.b] = d[i - 1][e.a] + e.w, p[i][e.b] = e.a;
}

ld karp(int n) {
  int s = n++;
  forn (i, n - 1)
    g[s].pb(sz(edges)), edges.pb(Edge(s, i, 0));
  fordBellman(s, n);
  ld ansValue = INF;
  int curV = -1, dist = -1;
  forn (v, n - 1)
    if (d[n][v] != INF) {
      ld curAns = -INF;
      int curPos = -1;
      forn(k, n)
        if (curAns <= (d[n][v] - d[k][v]) * (ld) (1) / (n -
        ↪  k))
          curAns = (d[n][v] - d[k][v]) * (ld) (1) / (n - k),
          ↪  curPos = k;
      if (ansValue > curAns)
        ansValue = curAns, dist = curPos, curV = v;
    }
  if (curV == -1) return ansValue;
  for (int iter = n; iter != dist; iter--)
    ans.pb(curV), curV = p[iter][curV];
  reverse(all(ans));
  return ansValue;
}
```

39 Kuhn's algorithm

```
//          - n    ,          - m
//
int n, m, paired[2 * MAX_N], used[2 * MAX_N];
vi g[MAX_N];

bool dfs(int v) {
  if (used[v]) return false;
  used[v] = 1;
  for (int to : g[v])
    if (paired[to] == -1 || dfs(paired[to])) {
      paired[to] = v, paired[v] = to;
      return true;
    }
  return false;
}

int kuhn() {
  int ans = 0;
  forn (i, n + m) paired[i] = -1;
  for (int run = 1; run;) {
```

```
    run = 0;
    fill(used, used + n + m, 0);
    forn(i, n)
      if (!used[i] && paired[i] == -1 && dfs(i))
        ans++, run = 1;
  }
  return ans;
}

//                             ,           ,     -
↪       .
// Max        -- A+, B-
// Min          -- A-, B+

vi minCover, maxIndependent;

void dfsCoverIndependent(int v) {
  if (used[v]) return;
  used[v] = 1;
  for (int to : g[v])
    if (!used[to])
      used[to] = 1, dfsCoverIndependent(paired[to]);
}

//
void findCoverIndependent() {
  fill(used, used + n + m, 0);
  forn (i, n)
    if (paired[i] == -1)
      dfsCoverIndependent(i);
  forn (i, n)
    if (used[i]) maxIndependent.pb(i);
    else minCover.pb(i);
  forab (i, n, n + m)
    if (used[i]) minCover.pb(i);
    else maxIndependent.pb(i);
}
```

40 LCA

```
int tin[MAX_N], tout[MAX_N], up[MAX_N][MAX_LOG];
vi g[MAX_N];
int curTime = 0;

void dfs(int v, int p) {
  up[v][0] = p;
  forn (i, MAX_LOG - 1)
    up[v][i + 1] = up[up[v][i]][i];
  tin[v] = curTime++;
  for (int u : g[v])
    if (u != p)
      dfs(u, v);
  tout[v] = curTime++;
}

int isUpper(int v, int u) {
  return tin[v] <= tin[u] && tout[v] >= tout[u];
}

int lca(int v, int u) {
  if (isUpper(u, v)) return u;
  fornr (i, MAX_LOG)
    if (!isUpper(up[u][i], v))
      u = up[u][i];
  return up[u][0];
}

void init() {
  dfs(0, 0);
}
```

41 LCA offline (Tarjan)

```
vi g[MAX_N], q[MAX_N];
int pr[MAX_N], ancestor[MAX_N], used[MAX_N];

int get(int v) {
  return v == pr[v] ? v : pr[v] = get(pr[v]);
}

void unite(int v, int u, int anc) {
  v = get(v), u = get(u);
  pr[u] = v, ancestor[v] = anc;
}

void dfs(int v) {
  used[v] = 1;
  for (int u : g[v])
    if (!used[u])
      dfs(u), unite(v, u, v);
  for (int u : q[v])
    if (used[u])
      ancestor[get(u)]; // handle answer somehow
}

void init(int n) {
  forn (i, n) pr[i] = i, ancestor[i] = i;
  dfs(0);
}
```

9 Math

42 CRT (KTO)

```
vi crt(vi a, vi mod) {
  int n = sz(a);
  vi x(n);
  forn (i, n) {
    x[i] = a[i];
    forn (j, i) {
      x[i] = inverse(mod[j], mod[i]) * (x[i] - x[j]) % mod[i];
      if (x[i] < 0) x[i] += mod[i];
    }
  }
  return x;
}
```

43 Discrete Logarithm

```
// Returns x: a^x = b (mod mod) or -1, if no such x exists
int discreteLogarithm(int a, int b, int mod) {
  int sq = sqrt(mod);
  int sq2 = mod / sq + (mod % sq ? 1 : 0);
  vector<pii> powers(sq2);
  forn (i, sq2)
    powers[i] = mp(power(a, (i + 1) * sq, mod), i + 1);
  sort(all(powers));
  forn (i, sq + 1) {
    int cur = power(a, i, mod);
    cur = (cur * 1ll * b) % mod;
    auto it = lower_bound(all(powers), mp(cur, 0));
    if (it != powers.end() && it->fst == cur)
      return it->snd * sq - i;
  }
  return -1;
}
```

44 Discrete Root

```
// Returns x: x^k = a mod mod, mod is prime
int discreteRoot(int a, int k, int mod) {
  if (a == 0)
```

```
    return 0;
  int g = primitiveRoot(mod);
  int y = discreteLogarithm(power(g, k, mod), a, mod);
  return power(g, y, mod);
}
```

45 Eratosthenes

```
vi eratosthenes(int n) {
  vi minDiv(n + 1, 0);
  minDiv[1] = 1;
  forab (i, 2, n + 1)
    if (minDiv[i] == 0)
      for (int j = i; j <= n; j += i)
        if (minDiv[j] == 0) minDiv[j] = i;
  return minDiv;
}

vi eratosthenesLinear(int n) {
  vi minDiv(n + 1, 0), primes;
  minDiv[1] = 1;
  forab (i, 2, n + 1) {
    if (minDiv[i] == 0)
      minDiv[i] = i, primes.pb(i);
    for (int j = 0; j < sz(primes) && primes[j] <= minDiv[i]
    ↪   && i * primes[j] <= n; j++)
      minDiv[i * primes[j]] = primes[j];
  }
  return minDiv;
}
```

46 Factorial

```
// Returns pair (rem, deg), where rem = n! % mod,
// deg = k: mod^k | n!, mod is prime, O(mod log mod)
pii fact(int n, int mod) {
  int rem = 1, deg = 0, nCopy = n;
  while (nCopy) nCopy /= mod, deg += nCopy;
  while (n > 1) {
    rem = (rem * ((n / mod) % 2 ? -1 : 1) + mod) % mod;
    for (int i = 2; i <= n % mod; i++)
      rem = (rem * 1ll * i) % mod;
    n /= mod;
  }
  return mp(rem % mod, deg);
}
```

47 Gauss

```
const double EPS = 1e-9;

int gauss(double **a, int n, int m) { // n is number of
↪  equations, m is number of variables
  int row = 0, col = 0;
  vi par(m, -1);
  vector<double> ans(m, 0);
  for (col = 0; col < m && row < n; col++) {
    int best = row;
    for (int i = row; i < n; i++)
      if (abs(a[i][col]) > abs(a[best][col]))
        best = i;
    if (abs(a[best][col]) < EPS) continue;
    par[col] = row;
    forn (i, m + 1) swap(a[row][i], a[best][i]);
    forn (i, n)
      if (i != row) {
        double k = a[i][col] / a[row][col];
        for (int j = col; j <= m; j++)
          a[i][j] -= k * a[row][j];
      }
    row++;
  }
  int single = 1;
  forn (i, m)
    if (par[i] != -1) ans[i] = a[par[i]][m] / a[par[i]][i];
    else single = 0;
  forn (i, n) {
    double cur = 0;
    for (int j = 0; j < m; j++)
      cur += ans[j] * a[i][j];
    if (abs(cur - a[i][m]) > EPS)
      return 0;
  }
  if (!single)
    return 2;
  return 1;
}
```

48 Gauss binary

```
const int MAX = 1024;

int gaussBinary(vector<bitset<MAX>> a, int n, int m) {
  int row = 0, col = 0;
  vi par(m, -1);
  for (col = 0; col < m && row < n; col++) {
    int best = row;
    for (int i = row; i < n; i++)
      if (a[i][col] > a[best][col])
        best = i;
    if (a[best][col] == 0)
      continue;
    par[col] = row;
    swap(a[row], a[best]);
    forn (i, n)
      if (i != row && a[i][col])
          a[i] ^= a[row];
    row++;
  }
  vi ans(m, 0);
  forn (i, m)
    if (par[i] != -1)
      ans[i] = a[par[i]][n] / a[par[i]][i];
  bool ok = 1;
  forn (i, n) {
    int cur = 0;
    forn (j, m) cur ^= (ans[j] & a[i][j]);
    if (cur != a[i][n]) ok = 0;
  }
  return ok;
}
```

49 Gcd

```
int gcd(int a, int b) {
  return b ? gcd(b, a % b) : a;
}

int gcd(int a, int b, int &x, int &y) {
  if (b == 0) {
    x = 1, y = 0;
    return a;
  }
  int g = gcd(b, a % b, x, y), newX = y;
  y = x - a / b * y;
  x = newX;
  return g;
}

void diophant(int a, int b, int c, int &x, int &y) {
  int g = gcd(a, b, x, y);
  if (c % g != 0) return;
```

```
  x *= c / g, y *= c / g;
  // next solutions: x += b / g, y -= a / g
}

int inverse(int a, int mod) { // Returns -1, if a and mod are
↪   not coprime
  int x, y;
  int g = gcd(a, mod, x, y);
  return g == 1 ? (x % mod + mod) % mod : -1;
}

vi inverseForAll(int mod) {
  vi r(mod, 0);
  r[1] = 1;
  for (int i = 2; i < mod; i++)
    r[i] = (mod - r[mod % i]) * (mod / i) % mod;
  return r;
}
```

50 Gray

```
int gray(int n) {
  return n ^ (n >> 1);
}

int revGray(int n) {
  int k = 0;
  for (; n; n >>= 1) k ^= n;
  return k;
}
```

51 Miller-Rabin Test

```
vector <int> primes = {2, 3, 5, 7, 11, 13, 17, 19, 23};

bool isPrimeMillerRabin(ll n) {
  int k = 0;
  ll t = n - 1;
  while (t % 2 == 0) k++, t /= 2;
  for (auto p : primes) {
    ll g = __gcd(n, (ll) p);
    if (g > 1 && g < n) return 0;
    if (g == n) return 1;
    ll b = power(p, t, n), last = n - 1;
    bool was = 0;
    forn (i, k + 1) {
      if (b == 1 && last != n - 1)
        return 0;
      if (b == 1) {
        was = 1;
        break;
      }
      last = b, b = mul(b, b, n);
    }
    if (!was) return 0;
  }
  return 1;
}
```

52 Phi

```
int phi(int n) {
  int result = n;
  for (int i = 2; i * i <= n; i++)
    if (n % i == 0) {
      while (n % i == 0) n /= i;
      result -= result / i;
    }
  if (n > 1) result -= result / n;
  return result;
}

int inversePhi(int a, int mod) {
  return power(a, phi(mod) - 1, mod);
}
```

53 Pollard

```
inline void pollardFoo(ll& x, ll mod) {
  x = (mul(x, x, mod) + 1) % mod;
}

vector<pair<ll, int>> factorize(ll n) {
  if (n == 1) return {};
  if (isPrimeMillerRabin(n)) return {mp(n, 1)};
  if (n <= 100) {
    vector<pair<ll, int>> ans;
    for (int i = 2; i * i <= n; i++)
      if (n % i == 0) {
        int cnt = 0;
        while (n % i == 0) n /= i, cnt++;
        ans.pb(mp(i, cnt));
      }
    if (n != 1) ans.pb(mp(n, 1));
    sort(all(ans));
    return ans;
  }
  while (1) {
    ll a = rand() % n, b = a;
    while (1) {
      pollardFoo(a, n), pollardFoo(b, n), pollardFoo(b, n);
      ll g = __gcd(abs(a-b), n);
      if (g != 1) {
        if (g == n)
          break;
        auto ans1 = factorize(g);
        auto ans2 = factorize(n / g);
        vector<pair<ll, int>> ans;
        ans1.insert(ans1.end(), all(ans2));
        sort(all(ans1));
        for (auto np : ans1)
          if (sz(ans) == 0 || np.fst != ans.back().fst)
            ans.pb(np);
          else
            ans.back().snd += np.snd;
        return ans;
      }
    }
  }
  assert(0);
}
```

54 Power And Mul

```
inline ll fix(ll a, ll mod) { // a in [0, 2 * mod)
  if (a >= mod) a -= mod;
  return a;
}

// Returns (a * b) % mod, 0 <= a < mod, 0 <= b < mod
ll mulSlow(ll a, ll b, ll mod) {
  if (!b) return 0;
  ll c = fix(mulSlow(a, b / 2, mod) * 2, mod);
  return b & 1 ? fix(c + a, mod) : c;
}

ll mul(ll a, ll b, ll mod) {
  ll q = (ld) a * b / mod;
  ll r = a * b - mod * q;
  while (r < 0) r += mod;
  while (r >= mod) r -= mod;
  return r;
```

```
}

int power(int a, int n, int mod) {
  if (!n) return 1;
  int b = power(a, n / 2, mod);
  b = (b * 1ll * b) % mod;
  return n & 1 ? (a * 1ll * b) % mod : b;
}

ll powerLL(ll a, ll n, ll mod) {
  if (!n) return 1;
  ll b = power(a, n / 2, mod);
  b = mul(b, b, mod);
  return n & 1 ? mul(a, b, mod) : b;
}

int powerFast(int a, int n, int mod) {
  int res = 1;
  while (n) {
    if (n & 1)
      res = (res * 1ll * a) % mod;
    a = (a * 1ll * a) % mod;
    n /= 2;
  }
  return res;
}
```

55 Primitive Root

```
int primitiveRoot(int mod) { // Returns -1 if no primitive
↪   root exists
  vi fact;
  int ph = phi(mod);
  int n = mod;
  for (int i = 2; i * i <= n; i++) {
    if (n % i == 0) {
      fact.pb(i);
      while (n % i == 0) n /= i;
    }
  }
  if (n > 1) fact.pb(n);
  forab (i, 2, mod + 1) {
    bool ok = 1;
    for (int j = 0; j < sz(fact) && ok; j++)
      ok &= power(i, ph / fact[j], mod) != 1;
    if (ok) return i;
  }
  return -1;
}
```

56 Simpson

```
double f(double x) { return x; }

double simpson(double a, double b, int iterNumber) {
  double res = 0, h = (b - a) / iterNumber;
  forn (i, iterNumber + 1)
    res += f(a + h * i) * ((i == 0) || (i == iterNumber) ? 1 :
    ↪  ((i & 1) == 0) ? 2 : 4);
  return res * h / 3;
}
```

10 Strings

57 Aho-Corasick

```
const int ALPHA = 26;
const int MAX_N = 1e5;

struct Node {
  int next[ALPHA], term; //
  int go[ALPHA], suf, p, pCh; //
  Node(): term(0), suf(-1), p(-1) {
    fill(next, next + ALPHA, -1);
    fill(go, go + ALPHA, -1);
  }
};

Node g[MAX_N];
int last;

void add(const string &s) {
  int now = 0;
  for(char x : s) {
    if (g[now].next[x - 'a'] == -1) {
      g[now].next[x - 'a'] = ++last;
      g[last].p = now, g[last].pCh = x;
    }
    now = g[now].next[x - 'a'];
  }
  g[now].term = 1;
}

int go(int v, int c);

int getLink(int v) {
  if (g[v].suf == -1) {
    if (!v || !g[v].p) g[v].suf = 0;
    else g[v].suf = go(getLink(g[v].p), g[v].pCh);
  }
  return g[v].suf;
}

int go(int v, int c) {
  if (g[v].go[c] == -1) {
    if (g[v].next[c] != -1) g[v].go[c] = g[v].next[c];
    else g[v].go[c] = !v ? 0 : go(getLink(v), c);
  }
  return g[v].go[c];
}
```

58 Prefix-function

```
vi prefix(const string &s) {
  int n = sz(s);
  vi pr(n);
  forab (i, 1, n + 1) {
    int j = pr[i - 1];
    while (j > 0 && s[i] != s[j]) j = pr[j - 1];
    if (s[i] == s[j]) j++;
    pr[i] = j;
  }
  return pr;
}
```

59 Z-function

```
vi z(const string& s) {
   int n = sz(s);
   vi z(n);
  for (int i = 1, l = 0, r = 0; i < n; i++) {
    if (i <= r) z[i] = min(r - i + 1, z[i - l]);
    while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
  return z;
}
```

60 Hashes

```
const int P = 239017;

inline int add(int a, int b, int m) {
  a += b;
  return a >= m ? a - m : a;
}

inline int sub(int a, int b, int m) {
  a -= b;
  return a < 0 ? a + m : a;
}

const int MOD_X = 1e9 + 9, MOD_Y = 1e9 + 7;

// using H = unsigned long long;
struct H {
  int x, y;
  H(): x(0), y(0) {}
  H(int _x): x(_x), y(_x) {}
  H(int _x, int _y): x(_x), y(_y) {}
  inline H operator+(const H& h) const { return H(add(x, h.x,
  ↪  MOD_X), add(y, h.y, MOD_Y)); }
  inline H operator-(const H& h) const { return H(sub(x, h.x,
  ↪  MOD_X), sub(y, h.y, MOD_Y)); }
  inline H operator*(ll k) const { return H(int((x * k) %
  ↪  MOD_X), int((y * k) % MOD_Y)); }
  inline H operator*(const H& h) const{ return H(int((ll(x) *
  ↪  h.x) % MOD_X), int((ll(y) * h.y) % MOD_Y)); }
  inline bool operator==(const H& h) const { return x == h.x
  ↪  && y == h.y; }
  inline bool operator!=(const H& h) const { return x != h.x
  ↪  || y != h.y; }
  inline bool operator<(const H& h) const { return x < h.x ||
  ↪  (x == h.x && y < h.y); }
  explicit inline operator ll() const { return ll(x) * MOD_Y +
  ↪  y + 1; } // > 0
};

H deg[MAX_N], h[MAX_N];

inline H get(int l, int r) { return h[r] - h[l] * deg[r - l];
↪  }

void init(const string& s) {
  int n = sz(s);
  deg[0] = 1;
  forn (i, n)
    h[i + 1] = h[i] * P + s[i], deg[i + 1] = deg[i] * P;
}
```

61 Manaker

```
void manaker(const string& s, int *z0, int *z1) {
  int n = sz(s);
  forn (t, 2) {
    int *z = t ? z1 : z0, l = -1, r = -1; // [l..r]
    forn (i, n - t) {
      int k = 0;
      if (r > i + t) {
        int j = l + (r - i - t);
        k = min(z[j], j - l);
      }
      while (i - k >= 0 && i + k + t < n && s[i - k] == s[i +
      ↪  k + t])
        k++;
      z[i] = k;
      if (k && i + k + t > r)
        l = i - k + 1, r = i + k + t - 1;
    }
  }
}
```

62 Palindromic Tree

```
const int ALPHA = 26;

struct Vertex {
  int suf, len, next[ALPHA];
  Vertex() { fill(next, next + ALPHA, 0); }
};

int vn, v;
Vertex t[MAX_N + 2];
int n, s[MAX_N];

int get(int i) { return i < 0 ? -1 : s[i]; }

void init() {
  t[0].len = -1, vn = 2, v = 0, n = 0;
}

void add(int ch) {
  s[n++] = ch;
  while (v != 0 && ch != get(n - t[v].len - 2))
    v = t[v].suf;
  int& r = t[v].next[ch];
  if (!r) {
    t[vn].len = t[v].len + 2;
    if (!v) t[vn].suf = 1;
    else {
      v = t[v].suf;
      while (v != 0 && ch != get(n - t[v].len - 2))
        v = t[v].suf;
      t[vn].suf = t[v].next[ch];
    }
    r = vn++;
  }
  v = r;
}
```

63 Suffix Array (+stable)

```
int sLen, num[MAX_N + 1];
char s[MAX_N + 1];
int p[MAX_N], col[MAX_N], inv[MAX_N], lcp[MAX_N];

inline int mod(int x) {
  return (x + sLen) % sLen;
}

void buildArray(int n) {
  sLen = n;
  int ma = max(n, 256);
  forn (i, n)
    col[i] = s[i], p[i] = i;

  for (int k2 = 1; k2 / 2 < n; k2 *= 2) {
    int k = k2 / 2;
    memset(num, 0, sizeof(num));
    forn (i, n) num[col[i] + 1]++;
    forn (i, ma) num[i + 1] += num[i];
    forn (i, n)
      inv[num[col[mod(p[i] - k)]]++] = mod(p[i] - k);
    int cc = 0;
    forn (i, n) {
      bool add = col[inv[i]] != col[inv[i - 1]];
      add |= col[mod(inv[i] + k)] != col[mod(inv[i - 1] + k)];
      if (i && add) cc++;
      num[inv[i]] = cc;
    }
    forn (i, n) p[i] = inv[i], col[i] = num[i];
```

```
  }

  memset(num, 0, sizeof(num));
  forn (i, n) num[col[i] + 1]++;
  forn (i, ma) num[i + 1] += num[i];
  forn (i, n) inv[num[col[i]]++] = i;
  forn (i, n) p[i] = inv[i];
  forn (i, n) inv[p[i]] = i;
}

void buildLCP(int n) {
  int len = 0;
  forn (ind, n){
    int i = inv[ind];
    len = max(0, len - 1);
    if (i != n - 1)
      while (len < n && s[mod(p[i] + len)] == s[mod(p[i + 1] +
      ↪  len)])
        len++;
    lcp[i] = len;
    if (i != n - 1 && p[i + 1] == n - 1) len = 0;
  }
}
```

64 Suffix Automaton

```
struct Vx {
    static const int AL = 26;
    int len, suf;
    int next[AL];
    Vx() {}
    Vx(int l, int s): len(l), suf(s) {}
};

struct SA {
    static const int MAX_LEN = 1e5 + 100, MAX_V = 2 * MAX_LEN;
    int last, vcnt;
    Vx v[MAX_V];

    SA() { vcnt = 1, last = newV(0, 0); } // root = vertex
    ↪  with number 1
    int newV(int len, int suf){
        v[vcnt] = Vx(len, suf);
        return vcnt++;
    }
    int add(char ch) {
        int p = last, c = ch - 'a';
        last = newV(v[last].len + 1, 0);
        while (p && !v[p].next[c]) // added p &&
            v[p].next[c] = last, p = v[p].suf;
        if (!p)
            v[last].suf = 1;
        else {
            int q = v[p].next[c];
            if (v[q].len == v[p].len + 1) v[last].suf = q;
            else {
                int r = newV(v[p].len + 1, v[q].suf);
                v[last].suf = v[q].suf = r;
                memcpy(v[r].next, v[q].next,
                ↪  sizeof(v[r].next));
                while (p && v[p].next[c] == q)
                    v[p].next[c] = r, p = v[p].suf;
            }
        }
        return last;
    }
};
```

65 Suffix Tree

```
const int MAX_L=1e5+10;
char S[MAX_L];
int L;

struct Node;
struct Pos;
typedef Node *pNode;
typedef map<char,pNode> mapt;

struct Node{
  pNode P,link;
  int L,R;
  mapt next;

  Node():P(NULL),link(this),L(0),R(0){}
  Node(pNode P,int L,int R):P(P),link(NULL),L(L),R(R){}

  inline int elen() const{return R-L;}
  inline pNode add_edge(int L,int R){return next[S[L]]=new
  ↪  Node(this,L,R);}
};

struct Pos{
  pNode V;
  int up;
  Pos():V(NULL),up(0){}
  Pos(pNode V,int up):V(V),up(up){}

  pNode split_edge() const{
    if(!up)
      return V;
    int L=V->L, M=V->R-up;
    pNode P=V->P, n=new Node(P,L,M);
    P->next[S[L]]=n;
    n->next[S[M]]=V;
    V->P=n, V->L=M;
    return n;
  }
  Pos next_char(char c) const{
    if(up)
      return S[V->R-up]==c ? Pos(V,up-1) : Pos();
    else{
      mapt::iterator it=V->next.find(c);
      return it==V->next.end() ? Pos() :
      ↪  Pos(it->snd,it->snd->elen()-1);
    }
  }
};

Pos go_down(pNode V,int L,int R){
  if(L==R)
    return Pos(V,0);
  while(1){
    V=V->next[S[L]];
    L+=V->elen();
    if(L>=R)
      return Pos(V,L-R);
  }
}

inline pNode calc_link(pNode &V){
  if(!V->link)
    V->link=go_down(V->P->link,V->L+!V->P->P,V->R).split_edge();
  return V->link;
}

Pos add_char(Pos P,int k){
  while(1){
    Pos p=P.next_char(S[k]);
    if(p.V)
```

```
      return p;
    pNode n=P.split_edge();
    n->add_edge(k,MAX_L);
    if(!n->P)
      return Pos(n,0);
    P=Pos(calc_link(n),0);
  }
}

pNode Root;
void make_tree(){
  Root=new Node();
  Pos P(Root,0);
  forn(i,L)
    P=add_char(P,i);
}
```

11 C++ Tricks

66 Fast allocation

```
const int MAX_MEM = 1e8;

int mpos = 0;
char mem[MAX_MEM];
inline void* operator new(size_t n) {
  char *res = mem + mpos;
  mpos += n;
  assert(mpos <= MAX_MEM);
  return (void*) res;
}
inline void operator delete(void*) {}

inline void* operator new[](size_t) { assert(0); }
inline void operator delete[](void*) { assert(0); }
```

67 Hash of pair

```
struct PairHasher {
  size_t operator()(const pair<int, int>& p) const { return
  ↪  p.fst * 239017 + p.snd; }
};
```

68 Ordered Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>, rb_tree_tag,
↪  tree_order_statistics_node_update> ordered_set;

void example() {
  ordered_set X;
  X.insert(1);
  cout << *X.find_by_order(1) << " " << X.order_of_key(1) <<
  ↪  "\n";
}
```

69 Hash Map

```
#include <ext/pb_ds/assoc_container.hpp>

using namespace __gnu_pbds;

struct chash { // To use most bits rather than just the lowest
↪  ones:
  const uint64_t C = ll(2e18 * PI) + 71; // large odd number
  const int RANDOM = 912387491;
  ll operator()(ll x) const { return __builtin_bswap64((x ^
  ↪  RANDOM) * C); }
};
template<class K, class V> using ht = gp_hash_table<K, V,
↪  chash>;
template<class K, class V> V get(ht<K, V>& u, K x) {
  return u.find(x) == end(u) ? 0 : u[x];
}

ht<ll, int> h({}, {}, {}, {}, {1<<20});
```

70 Fast I/O (short)

```
inline int readChar();
inline int readInt();
template <class T> inline void writeInt(T x);

inline int readChar() {
  int c = getchar();
  while (c <= 32)
    c = getchar();
  return c;
}

inline int readInt() {
  int s = 0, c = readChar(), x = 0;
  if (c == '-')
    s = 1, c = readChar();
  while ('0' <= c && c <= '9')
    x = x * 10 + c - '0', c = readChar();
  return s ? -x : x;
}

template <class T> inline void writeInt(T x) {
  if (x < 0)
    putchar('-'), x = -x;
  char s[24];
  int n = 0;
  while (x || !n)
    s[n++] = '0' + x % 10, x /= 10;
  while (n--)
    putchar(s[n]);
}
```

71 Fast I/O (long)

```
template <class T = int> inline T readInt();
inline double readDouble();
inline int readUInt();
inline int readChar();
inline void readWord(char *s);
inline bool readLine(char *s); // do not save '\n'
inline bool isEof();
inline int peekChar();
inline bool seekEof();

template <class T> inline void writeInt(T x, int len);
template <class T> inline void writeUInt(T x, int len);
template <class T> inline void writeInt(T x) { writeInt(x,
↪  -1); };
template <class T> inline void writeUInt(T x) { writeUInt(x,
↪  -1); };
inline void writeChar(int x);
inline void writeWord(const char *s);
inline void writeDouble(double x, int len = 0);
inline void flush();

const int BUF_SIZE = 4096;

char buf[BUF_SIZE];
int bufLen = 0, pos = 0;
```

```
inline bool isEof() {
  if (pos == bufLen) {
    pos = 0, bufLen = fread(buf, 1, BUF_SIZE, stdin);
    if (pos == bufLen)
      return 1;
  }
  return 0;
}

inline int getChar() {
  return isEof() ? -1 : buf[pos++];
}

inline int peekChar() {
  return isEof() ? -1 : buf[pos];
}

inline bool seekEof() {
  int c;
  while ((c = peekChar()) != -1 && c <= 32)
    pos++;
  return c == -1;
}

inline int readChar() {
  int c = getChar();
  while (c != -1 && c <= 32)
    c = getChar();
  return c;
}

inline int readUInt() {
  int c = readChar(), x = 0;
  while ('0' <= c && c <= '9')
    x = x * 10 + c - '0', c = getChar();
  return x;
}

template <class T>
inline T readInt() {
  int s = 1, c = readChar();
  T x = 0;
  if (c == '-')
    s = -1, c = getChar();
  while ('0' <= c && c <= '9')
    x = x * 10 + c - '0', c = getChar();
  return s == 1 ? x : -x;
}

inline double readDouble() {
  int s = 1, c = readChar();
  double x = 0;
  if (c == '-')
    s = -1, c = getChar();
  while ('0' <= c && c <= '9')
  x = x * 10 + c - '0', c = getChar();
  if (c == '.') {
    c = getChar();
    double coef = 1;
    while ('0' <= c && c <= '9')
      x += (c - '0') * (coef *= 1e-1), c = getChar();
  }
  return s == 1 ? x : -x;
}

inline void readWord(char *s) {
  int c = readChar();
  while (c > 32)
    *s++ = c, c = getChar();
  *s = 0;
}

inline bool readLine(char *s) {
  int c = getChar();
  while (c != '\n' && c != -1)
    *s++ = c, c = getChar();
  *s = 0;
  return c != -1;
}


int writePos = 0;
char writeBuf[BUF_SIZE];

inline void writeChar(int x) {
  if (writePos == BUF_SIZE)
    fwrite(writeBuf, 1, BUF_SIZE, stdout), writePos = 0;
  writeBuf[writePos++] = x;
}

inline void flush() {
  if (writePos)
    fwrite(writeBuf, 1, writePos, stdout), writePos = 0;
}

template <class T>
inline void writeInt(T x, int outputLen) {
  if (x < 0)
    writeChar('-'), x = -x;

  char s[24];
  int n = 0;
  while (x || !n)
    s[n++] = '0' + x % 10, x /= 10;
  while (n < outputLen)
    s[n++] = '0';
  while (n--)
    writeChar(s[n]);
}

template <class T>
inline void writeUInt(T x, int outputLen) {
  char s[24];
  int n = 0;
  while (x || !n)
    s[n++] = '0' + char(x % 10), x /= 10;
  while (n < outputLen)
    s[n++] = '0';
  while (n--)
    writeChar(s[n]);
}

inline void writeWord(const char *s) {
  while (*s)
    writeChar(*s++);
}

inline void writeDouble(double x, int outputLen) {
  if (x < 0)
    writeChar('-'), x = -x;
  int t = (int) x;
  writeUInt(t), x -= t;
  writeChar('.');
  for (int i = outputLen - 1; i > 0; i--) {
    x *= 10;
    t = std::min(9, (int) x);
    writeChar('0' + t), x -= t;
  }
  x *= 10;
  t = std::min(9, (int)(x + 0.5));
  writeChar('0' + t);
```

```
}
```

12 Notes

72 Работа с деревьями

Приемы для работы с деревьями:

1. Двоичные подъемы
2. Поддеревья как отрезки Эйлерова обхода
3. Вертикальные пути в Эйлеровом обходе (на ребрах вниз $+k$, на ребрах вверх $-k$).
4. Храним в вершине значение функции на пути от корня до нее, дальше LCA.
5. Спуск с DFS, поддерживаем ДО на пути до текущей вершины.
6. Heavy-light decomposition
7. Centroid decomposition
8. Корневая по запросам
9. Тяжелые/легкие вершины
10. DFS $\to$ дерево блоков, размеры $\in [K..2K]$
11. У вершины не более $O(\sqrt{N})$ разных поддеревьев
12. Сумма размеров поддеревьев без тяжелого ребенка $O(n \log n)$
13. Сумма глубин поддеревьев без глубокого ребенка $O(n)$

73 Маски

Считаем динамику по маскам за $O(2^n \cdot n)$ $f[mask] = sum$ по $submask$ $g[submask]$. $dp[mask][i]$ — значение динамики для маски $mask$, если младшие i бит в ней зафиксированы (то есть мы не можем удалять оттуда). Ответ в $dp[mask][0]$. $dp[mask][len] = g[mask]$. Если i-ый бит 0, то $dp[mask][i] = dp[mask][i+1]$, иначе $dp[mask][i] = dp[mask][i+1] + dp[mask^(1 << i)][i+1]$.

Старший бит: предподсчет.

Младший бит: $x \& \sim (-x)$

Чтобы по степени двойки получить логарифм, можно воспользоваться тем, что все степени двойки имеют разный остаток по модулю 67.

```
for (int mask = 0; mask < (1 << n); mask++)
^^Isubmask : for (int s = mask; s; s = (s - 1) & mask)
^^Isupmask : for (int s = mask; s < (1 << n); s = (s + 1) |
↪  mask)
```

74 Гранди

Теорема Шпрага-Гранди: берем mex всех значений функции Гранди по состояниям, в которые можем перейти из данного. Если сумма независимых игр, то значение функции Гранди равно xor значений функций Гранди по всем играм. Бывает полезно вывести первые n значений и поискать закономерность. Часто сводится к xor по чему-нибудь.

75 Потоки

Потоки:

Name	Asympthotic
Ford-Fulkerson	$O(\lvert f\rvert \cdot E)$
Ford-Fulkerson with scaling	$O(\log \lvert f\rvert \cdot E^2)$
Edmonds-Karp	$O(V \cdot E^2)$
Dinic	$O(V^2 \cdot E)$
Dinic with scaling	$O(V \cdot E \cdot \log C)$
Dinic on bipartite graph	$O(E\sqrt{V})$
Dinic on unit network	$O(E\sqrt{E})$

L-R потоки: Есть граф с недостатками или избытками в каждой вершине. Создаем фиктивные исток и сток (из истока все ребра в избытки, из недостатков все ребра в сток). Теперь пусть у нас есть L-R граф, для каждого ребра e $(v \to u)$ известны L_e и R_e. Добавим в v избыток L_e, в u недостаток L_e, а пропускную способность сделаем $R_e - L_e$. Получили решение задачи о LR-циркуляции. Если у нас обычный граф с истоком и стоком, то добавляем бесконечное ребро из стока в сток и ищем циркуляцию. Таким образом нашли удовлетворяющий условиям LR-поток. Если хотим максимальный поток, то на остаточной сети запускаем поиск максимального потока. В новом графе в прямую сторону пропускная способность равна $R_e - f_e$, в обратную $f_e - L_e$.

MinCostCirculation: Пока есть цикл отрицательного веса, запускаем алгоритм Карпа и пускаем максимальный поток по найденному циклу.

76 ДП

Табличка с оптимизациями для динамики:

Name	Original recurrence Sufficient Condition	From To
CHT1	$dp[i] = \min\limits_{j<i} dp[j] + b[j] \cdot a[i]$ $b[j] \geqslant b[j+1] \mid\mid a[i] \leqslant a[i+1]$	$O(n^2)$ $O(n)$
CHT2	$dp[i][j] = \min\limits_{k<j} dp[i-1][k] + b[k] \cdot a[j]$ $b[k] \geqslant b[k+1] \mid\mid a[j] \leqslant a[j+1]$	$O(kn^2)$ $O(kn)$
D&C	$dp[i][j] = \min\limits_{k<j} dp[i-1][k] + c[k][j]$ $p[i,j] \leqslant p[i,j+1]$	$O(kn^2)$ $O(kn \log n)$
Knuth	$dp[i][j] = \min\limits_{i<k<j} dp[i][k] + dp[k][j] + c[i][j]$ $p[i,j-1] \leqslant p[i,j] \leqslant p[i+1,j]$	$O(n^3)$ $O(n^2)$
IOI	$f_n(k)$ — best for fixed k f_n — convex, add penalty $\lambda \cdot k$	$O(k^{(2)}n)$ $O(n \log C)$

77 Комбинаторика

Биномиальные коэффициенты:

Теорема Люка для биномиальных коэффициентов: Хотим посчитать C_n^k, разложим в p-ичной системе счисления, $n = (n_0, n_1, \dots)$, $k = (k_0, k_1, \dots)$. $ans = C_{n_0}^{k_0} \cdot C_{n_1}^{k_1} \cdot \dots$

Способы вычисления C_n^k:

1. $C_n^k = C_{n-1}^k + C_{n-1}^{k-1}$

   precalc: $O(n^2)$, query: $O(1)$.

2. $C_n^k = \frac{n!}{k!(n-k)!}$, предподсчитываем факториалы

   precalc: $O(n \log n)$, query: $O(\log n)$

3. Теорема Люка

   precalc: $O(p \log p)$, query: $O(logp)$.

4. $C_n^k = C_n^{k-1} \cdot \frac{n-k+1}{k}$

5. $C_n^k = \frac{n!}{k!(n-k)!}$, для каждого факториала считаем степень вхождения и остаток

   precalc: $O(p \log p)$, query: $O(logp)$.

$C_n^{\frac{n}{2}} = \frac{2^n}{\sqrt{\frac{\pi n}{2}}}$

78 Делители

- $\leq 20 : d(12) = 6$
- $\leq 50 : d(48) = 10$
- $\leq 100 : d(60) = 12$
- $\leq 1000 : d(840) = 32$

- $\leq 10^4$: $d(9\,240) = 64$
- $\leq 10^5$: $d(83\,160) = 128$
- $\leq 10^6$: $d(720\,720) = 240$
- $\leq 10^7$: $d(8\,648\,640) = 338$
- $\leq 10^8$: $d(91\,891\,800) = 768$
- $\leq 10^9$: $d(931\,170\,240) = 1344$
- $\leq 10^{11}$: $d(97\,772\,875\,200) = 4032$
- $\leq 10^{12}$: $d(963\,761\,198\,400) = 6720$
- $\leq 10^{15}$: $d(866\,421\,317\,361\,600) = 15360$
- $\leq 10^{18}$: $d(897\,612\,484\,786\,617\,600) = 103680$

79 Числа Белла

i	B_i	i	B_i
0	1	12	4,213,597
1	1	13	27,644,437
2	2	14	190,899,322
3	5	15	1,382,958,545
4	15	16	10,480,142,147
5	52	17	82,864,869,804
6	203	18	682,076,806,159
7	877	19	5,832,742,205,057
8	4,140	20	51,724,158,235,372
9	21,147	21	474,869,816,156,751
10	115,975	22	4,506,715,738,447,323
11	678,570	23	44,152,005,855,084,346

80 Разбиения

Число неупорядоченных разбиений n на положительные слагаемые.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

81 Матричные игры

Пишем матрицу стратегий $A_{i,j}$ это выигрыш первого и проигрыш второго, i стратегия 1-го. Седловая точка есть для несмешанной стратегии если $\max_i \min A_{i,*} = \min_j \max A_{*,j}$. Иначе:

$$f(x) = sum(x_i) \to max,\ \ Ans = 1/f(x)$$

$$Ax \leq 1_n,\ \ x_i \geq 0$$

Для 2×2, p первый игрок, q — второй:

$$p^* = \left(\frac{a_{22} - a_{21}}{a_{22} - a_{12} + a_{11} - a_{21}}; \frac{a_{11} - a_{12}}{a_{22} - a12 + a_{11} - a_{21}}\right)$$

$$q^* = \left(\frac{a_{22} - a_{12}}{a_{22} - a_{12} + a_{11} - a_{21}}; \frac{a_{11} - a_{21}}{a_{22} - a12 + a_{11} - a_{21}}\right)$$

$$Ans = \frac{a_{22}a_{11} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

82 Mixed

- Формула Пика: $S = Inside + Edge/2 - 1$
- Теорема Люка: $0 \leq n, m \in \mathbb{Z}$, p простое. $n = n_k p^k + ... + n_1 p + n_0$ и $m = m_k p^k + ... + m_1 p + m_0$. Тогда $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.
- Лемма Бернсайда: $|X/G|$ число орбит G. $X^g = \{x \in X | gx = x\}$

$$|X/G| = \frac{1}{|G|} \sum_{g\in G} |X^g|$$