Решение транспортной задачи:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | Запасы | | *B*1 | *B*2 | *B*3 | *B*4 | | *A*1 | |  | | --- | | 12 | | |  | | --- | | 15 | | |  | | --- | | 21 | | |  | | --- | | 11 | | |  | | --- | | 240 | | | *A*2 | |  | | --- | | 14 | | |  | | --- | | 8 | | |  | | --- | | 15 | | |  | | --- | | 20 | | |  | | --- | | 190 | | | *A*3 | |  | | --- | | 19 | | |  | | --- | | 16 | | |  | | --- | | 26 | | |  | | --- | | 19 | | |  | | --- | | 190 | | | Потребности | |  | | --- | | 140 | | |  | | --- | | 190 | | |  | | --- | | 170 | | |  | | --- | | 120 | | |  | | --- | | 0 | | |

Число пунктов отправления *m=*3, а число пунктов назначения *n*=4. Следовательно опорный план задачи определяется числами, стоящими в *m+n*−1=3+4−1=6 заполненых клетках таблицы. Тарифы перевозок единицы груза из каждого пункта отправления во все пункты назначения задаются матрицей

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *C=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 12 | | |  | | --- | | 15 | | |  | | --- | | 21 | | |  | | --- | | 11 | | | |  | | --- | | 14 | | |  | | --- | | 8 | | |  | | --- | | 15 | | |  | | --- | | 20 | | | |  | | --- | | 19 | | |  | | --- | | 16 | | |  | | --- | | 26 | | |  | | --- | | 19 | | |  | |

Наличие груза у поставщиков равно:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ∑ *A*i= | |  | | --- | | 240 | | + | |  | | --- | | 190 | | + | |  | | --- | | 190 | | = | |  | | --- | | 620 | |  |

Общая потребность в грузе в пунктах назначения равна:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ∑ *B*i= | |  | | --- | | 140 | | + | |  | | --- | | 190 | | + | |  | | --- | | 170 | | + | |  | | --- | | 120 | | = | |  | | --- | | 620 | |  |

∑ *A*i=∑ *B*i. Модель транспортной задачи является закрытой. Следовательно она разрешима.

**Этап I. Нахождение первого опорного плана**

Найдем опорный план задачи *методом северно-западного* угла.

*A*1>*B*1. Следовательно в клетку (*A*1, *B*1) помещаем число *min*(*A*1, *B*1)=140. Потребности пункта *B*1 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*1 и будем считать запасы пункта *A*1 равными 240−140=100.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 100 | | |  | |  | | --- | | 140 | |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 240 | |  |  | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 140 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 170 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 120 | |  |  | | | |

*A*1≤*B*2. Следовательно в клетку (*A*1, *B*2 ) помещаем число *min*(*A*1, *B*2 )=100. Запасы пункта *A*1 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*1 и будем считать потребности пункта *B*2 равными 190−100=90.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 140 | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 240 | |  |  | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 90 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 140 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 170 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 120 | |  |  | | | |

*A*2>*B*2. Следовательно в клетку (*A*2, *B*2) помещаем число *min*(*A*2, *B*2)=90. Потребности пункта *B*2 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*2 и будем считать запасы пункта *A*2 равными 190−90=100.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 140 | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 240 | |  |  | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 100 | | |  |  |  | |  | | --- | | 90 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 140 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 170 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 120 | |  |  | | | |

*A*2≤*B*3. Следовательно в клетку (*A*2, *B*3 ) помещаем число *min*(*A*2, *B*3 )=100. Запасы пункта *A*2 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*2 и будем считать потребности пункта *B*3 равными 170−100=70.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 140 | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 240 | |  |  | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 0 | | |  |  |  | |  | | --- | | 90 | |  | |  | | --- | | 100 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 70 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 140 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 170 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 120 | |  |  | | | |

*A*3>*B*3. Следовательно в клетку (*A*3, *B*3) помещаем число *min*(*A*3, *B*3)=70. Потребности пункта *B*3 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*3 и будем считать запасы пункта *A*3 равными 190−70=120.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 140 | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 240 | |  |  | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 0 | | |  |  |  | |  | | --- | | 90 | |  | |  | | --- | | 100 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 120 | | |  |  |  |  |  | |  | | --- | | 70 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 140 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 170 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 120 | |  |  | | | |

*A*3≤*B*4. Следовательно в клетку (*A*3, *B*4 ) помещаем число *min*(*A*3, *B*4 )=120. Запасы пункта *A*3 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*3 и будем считать потребности пункта *B*4 равными 120−120=0.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 140 | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 240 | |  |  | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 0 | | |  |  |  | |  | | --- | | 90 | |  | |  | | --- | | 100 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 0 | | |  |  |  |  |  | |  | | --- | | 70 | |  | |  | | --- | | 120 | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 620 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 140 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 190 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 170 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 120 | |  |  | | | |

**Этап II. Улучшение опорного плана**

Найдем оптимальный план транспортной задачи *методом потенциалов*.

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 140 | | |  | | --- | | 100 | | |  | | --- | | 0 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 90 | | |  | | --- | | 100 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 70 | | |  | | --- | | 120 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 12 | · | 140 | + | 15 | · | 100 | + | 8 | · | 90 | + | 15 | · | 100 | + | 26 | · | 70 | + | 19 | · | 120 | = | 9500 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=12
* β2−α1=15
* β2−α2=8
* β3−α2=15
* β3−α3=26
* β4−α3=19

Полагая α1=0, находим β1=12 β2=15 α2=7 β3=22 α3=-4 β4=15 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α13=1, α14=4, α21=-9, α24=-12, α31=-3, α32=3.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  | |  | | --- | | 100 | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −9 | | |  | |  | | --- | | 90 | |  | |  | | --- | | 100 | |  | |  |  | | --- | --- | | |  | | --- | | −12 | | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  |  | | --- | --- | | |  | | --- | | 3 | | |  | |  | | --- | | 70 | |  | |  | | --- | | 120 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 4 находится в пересечении строки *A*1 и столбца *B*4. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | | − | |  | | --- | | 21 | |  | |  | | --- | | 11 | | + | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  | |  | | --- | | 100 | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | | + | |  | | --- | | 15 | | − | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −9 | | |  | |  | | --- | | 90 | |  | |  | | --- | | 100 | |  | |  |  | | --- | --- | | |  | | --- | | −12 | | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | | + | |  | | --- | | 19 | | − | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  |  | | --- | --- | | |  | | --- | | 3 | | |  | |  | | --- | | 70 | |  | |  | | --- | | 120 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Наименьшее из чисел в минусовых клетках равно 100. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 100, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  |  |  |  |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  |  |  | |  | | --- | | 190 | |  | |  | | --- | | 0 | |  |  | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  |  |  |  |  | |  | | --- | | 170 | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 140 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 100 | | | |  | | --- | | 0 | | |  | | --- | | 190 | | |  | | --- | | 0 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 170 | | |  | | --- | | 20 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 12 | · | 140 | + | 11 | · | 100 | + | 8 | · | 190 | + | 15 | · | 0 | + | 26 | · | 170 | + | 19 | · | 20 | = | 9100 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=12
* β4−α1=11
* β2−α2=8
* β3−α2=15
* β3−α3=26
* β4−α3=19

Полагая α1=0, находим β1=12 β4=11 α3=-8 β3=18 α2=3 β2=11 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α12=-4, α13=-3, α21=-5, α24=-12, α31=1, α32=3.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  | |  |  | | --- | --- | | |  | | --- | | −4 | | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −5 | | |  | |  | | --- | | 190 | |  | |  | | --- | | 0 | |  | |  |  | | --- | --- | | |  | | --- | | −12 | | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  |  | | --- | --- | | |  | | --- | | 3 | | |  | |  | | --- | | 170 | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 3 находится в пересечении строки *A*3 и столбца *B*2. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  | |  |  | | --- | --- | | |  | | --- | | −4 | | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | | − | |  | | --- | | 15 | | + | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −5 | | |  | |  | | --- | | 190 | |  | |  | | --- | | 0 | |  | |  |  | | --- | --- | | |  | | --- | | −12 | | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | | + | |  | | --- | | 26 | | − | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  |  | | --- | --- | | |  | | --- | | 3 | | |  | |  | | --- | | 170 | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Наименьшее из чисел в минусовых клетках равно 170. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 170, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  |  |  |  |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  |  |  | |  | | --- | | 20 | |  | |  | | --- | | 170 | |  |  | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  |  |  | |  | | --- | | 170 | |  |  |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 140 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 100 | | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  | | --- | | 170 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 170 | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 12 | · | 140 | + | 11 | · | 100 | + | 8 | · | 20 | + | 15 | · | 170 | + | 16 | · | 170 | + | 19 | · | 20 | = | 8590 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=12
* β4−α1=11
* β2−α2=8
* β3−α2=15
* β2−α3=16
* β4−α3=19

Полагая α1=0, находим β1=12 β4=11 α3=-8 β2=8 α2=0 β3=15 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α12=-7, α13=-6, α21=-2, α24=-9, α31=1, α33=-3.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  | |  |  | | --- | --- | | |  | | --- | | −7 | | |  | |  |  | | --- | --- | | |  | | --- | | −6 | | |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  | | --- | | 20 | |  | |  | | --- | | 170 | |  | |  |  | | --- | --- | | |  | | --- | | −9 | | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  | | --- | | 170 | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 1 находится в пересечении строки *A*3 и столбца *B*1. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | | − | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | | + | |  | | --- | | 240 | | |  | |  | | --- | | 140 | |  | |  |  | | --- | --- | | |  | | --- | | −7 | | |  | |  |  | | --- | --- | | |  | | --- | | −6 | | |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  | | --- | | 20 | |  | |  | | --- | | 170 | |  | |  |  | | --- | --- | | |  | | --- | | −9 | | | | *A*3 | |  | | --- | | 19 | | + | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | | − | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  | | --- | | 170 | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Наименьшее из чисел в минусовых клетках равно 20. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 20, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 120 | |  |  |  |  |  | |  | | --- | | 120 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  |  |  | |  | | --- | | 20 | |  | |  | | --- | | 170 | |  |  | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  | |  | | --- | | 20 | |  | |  | | --- | | 170 | |  |  |  |  | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 120 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 120 | | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  | | --- | | 170 | | |  | | --- | | 0 | | | |  | | --- | | 20 | | |  | | --- | | 170 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 12 | · | 120 | + | 11 | · | 120 | + | 8 | · | 20 | + | 15 | · | 170 | + | 19 | · | 20 | + | 16 | · | 170 | = | 8570 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=12
* β4−α1=11
* β2−α2=8
* β3−α2=15
* β1−α3=19
* β2−α3=16

Полагая α1=0, находим β1=12 β4=11 α3=-7 β2=9 α2=1 β3=16 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α12=-6, α13=-5, α21=-3, α24=-10, α33=-3, α34=-1.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 12 | |  | |  | | --- | | 15 | |  | |  | | --- | | 21 | |  | |  | | --- | | 11 | |  | |  | | --- | | 240 | | |  | |  | | --- | | 120 | |  | |  |  | | --- | --- | | |  | | --- | | −6 | | |  | |  |  | | --- | --- | | |  | | --- | | −5 | | |  | |  | | --- | | 120 | | | *A*2 | |  | | --- | | 14 | |  | |  | | --- | | 8 | |  | |  | | --- | | 15 | |  | |  | | --- | | 20 | |  | |  | | --- | | 190 | | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  | | --- | | 20 | |  | |  | | --- | | 170 | |  | |  |  | | --- | --- | | |  | | --- | | −10 | | | | *A*3 | |  | | --- | | 19 | |  | |  | | --- | | 16 | |  | |  | | --- | | 26 | |  | |  | | --- | | 19 | |  | |  | | --- | | 190 | | |  | |  | | --- | | 20 | |  | |  | | --- | | 170 | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  |  | | --- | --- | | |  | | --- | | −1 | | | | Потребности | |  | | --- | | 140 | | | |  | | --- | | 190 | | | |  | | --- | | 170 | | | |  | | --- | | 120 | | | |  | | --- | | 620 | | |

Среди чисел *α*ij нет положительных. Следовательно данный опорный план является оптимальным.

**Решение:**

Оптимальный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 120 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 120 | | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  | | --- | | 170 | | |  | | --- | | 0 | | | |  | | --- | | 20 | | |  | | --- | | 170 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 12 | · | 120 | + | 11 | · | 120 | + | 8 | · | 20 | + | 15 | · | 170 | + | 19 | · | 20 | + | 16 | · | 170 | = | 8570 | |