Question 1

1. Equation: $x_{n+1} = (a*x_n + c)mod m$
2. Substitute Initial Numbers:
   $x_1 = (13* - 5 + 7)mod 12$
   $x_1 = (- 58)mod 12$
   $x_1 = 2$
3. Find x_2:
   $x_2 = (13*2 + 7)mod 12$
   $x_2 = (33)mod 12$
   $x_2 = 9$
4. Find x_3:
   $x_3 = (13*9 + 7)mod 12$
   $x_3 = (124)mod 12$
   $x_3 = 4$
5. Find x_4:
   $x_4 = (13*4 + 7)mod 12$
   $x_4 = (59)mod 12$
   $x_4 = 11$
6. Find x_5:
   $x_5 = (13*11 + 7)mod 12$
   $x_5 = (150)mod 12$
   $x_5 = 6$

Question 2

1. Trailing zeros are formed when a multiple of 5 is multiplied with a multiple of 2
2. Number of 5's:
   a. Initial 20: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100
   b. Extra 4: 25, 50, 75, and 100 each have two 5's so we count them twice

c. Total: 24
3. Number of 2's:
   a. Initial 50: 2, 4, 6, 8, 10, etc
   b. Extra 25: Multiples of 4's count as two each
   c. Extra 12: Multiples of 8's
   d. Extra 6: Multiples of 16's
   e. Extra 3: Multiples of 32's
   f. Extra 1: Multiples of 64's
   g. Total: 97
4. We know each pair of 2 and 5 will give a trailing zero, but we have only twenty-four 5's so we can only make 24 such pairs

Answer: 24 trailing zeros

Question 3

1. Initial:
   $n^5 - 5n^3 + 4n$
2. Take out an n:
   n$(n^4 - 5n^2 + 4)$
3. Factor using difference of squares:
   $n(n + 2)(n - 2)(n + 1)(n - 1)$
4. As shown above, $n^5 - 5n^3 + 4n$ is the product of 5 consecutive numbers so at least one of these numbers must be a multiple of 5 so the final product will be divisible by 5.

Question 4

1. Initial: $1333^{42} mod 11$
2. Reduce Exponent:
   $2^{42} mod 11$

   $(2^{21})^2 mod 11$

   $(2 * 2^{20})^2 mod 11$

   $(2 * (2^{10})^2)^2 mod 11$

   $(2 * ((2^5)^2)^2)^2 mod 11$

$(2 * ((2 * 2^4)^2)^2)^2 \ mod 11$

$(2 * ((2 * (2^2)^2)^2)^2)^2 \ mod 11$

3. Square:

   $(2 * ((2 * (4)^2)^2)^2)^2 \ mod 11$

   $(2 * ((2 * 16)^2)^2)^2 \ mod 11$
4. $mod 11$: $(2 * ((2 * 5)^2)^2)^2 \ mod 11$
5. Reduce Exponent:

   $(2 * ((10)^2)^2)^2 \ mod 11$

   $(2 * (100)^2)^2 \ mod 11$
6. $mod 11$: $(2 * (1)^2)^2 \ mod 11$
7. Reduce Exponent:

   $(2)^2 mod 11$

   $4 mod 11$

Answer: 4

Question 5

1. $309 = 112 * 2 + 85$
2. $112 = 85 * 1 + 27$
3. $85 = 27 * 3 + 4$
4. $27 = 4 * 2 + 3$
5. $4 = 3 * 1 + 1$
6. $3 = 1 * 2 + 1$
7. $1 = 1 * 1 + 0$
8. GCD(1, 0) = 1

Answer: The GCD of 309 and 112 is 1 so they are relatively prime

Question 6

Find $gcd(54, 16)$:

1. $54 * x + 16 * y = gcd(54, 16)$

2. $54 = 16 * 3 + 6$
3. $16 = 6 * 2 + 4$
4. $6 = 4 * 1 + 2$
5. $4 = 2 * 2 + 0$

Answer: $gcd(54, 16) = 2$

Rearrange Equations:
1. $6 = 54 - 16 * 3$
2. $4 = 16 - 6 * 2$
3. $2 = 6 - 4$

Diophantine:
1. $r0 = 54$ and $r1 = 16$
2. $6 = r0 - r1 * 3$
3. $4 = r1 - 6 * 2$
   $4 = r1 - (r0 - r1 * 3) * 2$
   $4 = r1 - 2r0 + 6r1$
   $4 = \ - 2r0 + 7r1$
4. $2 = 6 - 4$
   $2 = (r0 - r1 * 3) - (- 2r0 + 7r1)$
   $2 = (r0 - 3r1) + 2r0 - 7r1$
   $2 = 3r0 - 10r1$

Answer: $x = 3$ and $y =- 10$ and $gcd(54, 16) = 2$

Question 7

1. Find v:
   $33v = 1 - 112w$
   $33v = 1(mod112)$
2. Bezout's Identity:
   $33v + 112w = 1$
3. Euclid Algorithm:
   $112 = 3 * 33 + 13$
   $33 = 2 * 13 + 7$
   $13 = 1 * 7 + 6$
   $7 = 1 * 6 + 1$
4. Rewrite Algorithms:
   $1 = 7 - 1 * 6$
   $6 = 13 - 1 * 7$

$7 = 33 - 2 * 13$
$13 = 112 - 3 * 33$

5. Backwards Substitution:
    a. $1 = 7 - 1 * 6$
    $1 = 7 - (13 - 7)$
    $1 = 2 * 7 - 13$
    b. $1 = 2 * (33 - 2 * 13) - 13$
    $1 = 2 * 33 - 5 * 13$
    c. $1 = 2 * 33 - 5 * (112 - 3 * 33)$
    $1 = 2 * 33 - 5 * 112 + 15 * 33$
    $1 = \ - 5 * 112 + 17 * 33$
6. Rewrite:
    $1 = 0 + 17 * 33$
    $33^{-1}(mod 112) = 17$

Answer: 17