

Homework 1

Part A

1. The outer for loop will be $\Theta(\sqrt{n})$ since it will run from 0 to \sqrt{n} . This is because n decreases by \sqrt{n} each time the loop is run and $n/\sqrt{n} = \sqrt{n}$.
2. The inner for loop will be $\Theta(n)$ as it always runs from 0 to n , even if the n decreases.
3. Since the contents of the inner for loop are $O(1)$, it does not impact the runtime of the function as a whole.
4. The Big- Θ is $\sqrt{n} * n$ or $\Theta(n\sqrt{n})$.

Part B

1. The inner most loop will increase by $2m$ since $(m + m = 2m)$ up until n . Thus this loop will be logarithmic or $\Theta(\log(n))$.
2. The second loop increases linearly up until n therefore the loop will be $\Theta(n)$.
3. The outermost loop also increases linearly like the second loop. Therefore it will also have a runtime of $\Theta(n)$.
4. The end result is $\Theta(n^2 \log(n))$ because the outermost and second loop are n^2 ($n * n = n^2$). You then must multiply this result by the innermost loop.

Part C

$$1^{st}: \theta(1) + T(n-2) + T(n-2)$$

$$= \theta(1) + 2T(n-2)$$

$$2^{nd}: \theta(1) + 2T(n-2)$$

$$\theta(1) + 2T(n-2-2)$$

$$\theta(1) + 2T(n-4)$$

$$= \theta(1) + 2(\theta(1) + T(n-4))$$

$$= \theta(3) + 4T(n-4)$$

$$3^{rd} = 4(\theta(1) + 2T(n-4-2))$$

$$= \theta(3) + 4\theta(1) + 8T(n-6)$$

$$= \theta(7) + 8T(n-6)$$

$$k^{th} = \theta(2^k - 1) + 2^k(k - 2k)$$

Part D