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Counting

Question 1

Answer: There are 312 unique strings

This is because there are 3 ways you can place the first letter. Afterwards there are 2 ways for you to place the second letter. Now there is one empty spot that can be filled with any letter of the alphabet, thus there are 26 options. Multiplying this (26 * 2 * 3) together we get 156 options. Since there are 2 different options for the first letter, we multiply this by 2 to get 312 ways.

Question 2

Answer: 121 different events

We have 3 distinguishable passengers getting off on 5 distinguishable floors. Thus we have 5^3 or 125 ways of distributing the passengers. We must also remove combinations where Alice gets off at the third floor, this is when she gets off the third floor by herself, with Bob, with Carlos, and with both Bob and Carlos. This is four different combinations, thus we only have 121 ways.

Question 3

Part 1

Answer: 381

Select 3 cards such that 1 or 0 are fire and either 0 or 1 are water.

All Other: 9C3

Two are Other and 1 Fire: 9C2*5C1

Two are Other and 1 is Water: 9C2 * 7C1

One of Each Type: 7C1 * 5C1 * 9C1

Adding each type together we get 381

Part 2

Three types and 10 cards: 3^10

Question 4

Answer: 122 ways
If we use the formula for stars and bars where we have 4 bars and 6 stars, we get 9 choose 4 or 126 ways. Now we must account for combinations where all the stars are in one "box" as one copier can not make 6 copies. There are 4 ways to place all the copies onto one copier. Thus there are 122 ways.

Question 5

Answer: 1, 176 ways

2 Node Trees - 2 Ways:
1. Root with a Right Child
2. Root with a Left Child

3 Node Trees - 5 Ways:
1. Root with L and R childs
2. Root with a 2 Node Tree on the Left (2)
3. Root with a 2 Node Tree on the Right (2)

4 Node Trees - 14 Ways:
1. Root with a 3 Node Tree on the Left (5)
2. Root with a 3 Node Tree on the Right (5)
3. Root with a 2 Node Tree on the Left and a right child (2)
4. Root with a 2 Node Tree on the Right and a left child (2)

5 Node Trees - 42 Ways:
1. Root with a 4 Node Tree on the Left (14)
2. Root with a 4 Node Tree on the Right (14)
3. Root with a 3 Node Tree on the Left and a right child (5)
4. Root with a 3 Node Tree on the Right and a left child 5)
5. Root with a 2 Node Tree on the left and right (4)

Since the BST tree's root has value 8, and its left child has value 5 then the right side can have at most 4 nodes (9, 10, 11, 12). The left side's left child will have a left node of size 5 (0, 1, 2, 3, 4) and a right child of size 2 (6, 7). When you multiply the different ways to arrange a 4 nodes, 5 nodes, and 2 nodes you get (14 * 42 * 2) 1, 176 ways.