Essential Mathematics for Data Scientists

Assessment 2D: MATLAB logic exercise

This exercise forms a part of your code workbook. Include answers to the tasks below in your workbook as appropriately commented MATLAB code.

Like many programming languages, MATLAB has support for logical (Boolean) variables. In MATLAB, the truth values T and F are specified using `true` and `false`. However, it should be noted that when MATLAB displays logical variables, it shows true values as "`1`" and false values as "`0`".

Of the logical connectives we have covered, MATLAB supports the small selection summarised in the following table:

Connective	MATLAB function	MATLAB operator
AND	`and`	`&`
OR	`or`	`|`
NOT	`not`	`~`

This means, given logical variables `p` and `q`, the proposition $p \wedge q$ can be evaluated in MATLAB as either `and(p,q)` or `p&q`.

In this exercise, we are interested in using MATLAB to determine if the proposition

$$(p \rightarrow q) \wedge \neg q \leftrightarrow p \vee q$$

is a tautology or a contradiction (if either). This can of course be achieved after enough simplification with the laws of logic, but it should be much quicker to instead use MATLAB to evaluate the above proposition for every combination of truth values for p and q (in effect, computing the truth table for the proposition).

For this exercise, we encourage extensive use of MATLAB's anonymous function notation which allows functions to be defined succinctly in the same way you would define a variable. e.g.

```
add = @(x,y) x+y
```

defines a function that adds its inputs. e.g. `add(1,2)` would return `3`.

Tasks

1.   To begin with, we need to implement some of the logical connectives that MATLAB is missing. Consider the IF-THEN connective. Using the implication law:

$$p \rightarrow q \equiv \neg p \vee q$$

Write an `ifthen` function that defines IF-THEN in terms of NOT and OR, both of which MATLAB knows by default. **(2 marks)**

2.   Next, we need an IF-AND-ONLY-IF connective function `iff`. Use the relevant law of logic to write this function in terms of `ifthen`. **(3 marks)**

3.   To make it easier to evaluate the proposition
$$(p \rightarrow q) \land \neg q \leftrightarrow p \lor q,$$
use the `ifthen` and `iff` functions to define a `proposition` function that does this for a given `p` and `q`. **(3 marks)**

4.   Use this `proposition` function to determine whether the above proposition is a tautology, contradiction or neither. **(2 marks)**