# Machine Learning:
## An Applied Econometric Approach

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based on work with Sendhil Mullainathan

in collaboration with Susan Athey and Niall Keleher

## 3. Prediction vs Estimation

# Structure of first chapter of webinar

1. Introduction

Training data $(y, x)$ $\xrightarrow{\hat{f}}$ Application data $(\hat{y} = \hat{f}(x), x)$

2. The Secret Sauce of Machine Learning

3. Prediction vs Estimation

# ML basics recap

1. Flexible functional forms
2. Limit expressiveness (regularization)
3. Learn how much to regularize (tuning)

- What do these features imply for the properties of $\hat{f}$?
- And how can we therefore use $\hat{f}$ in applied work?

# Prediction problem set-up

Given:

- Training data set $(y_1, x_1), \dots, (y_n, x_n)$ (assume iid)
  - Usually called "regression" when $y$ continuous, "classification" when $y$ discrete
- Loss function $\ell(\hat{y}, y)$

Goal:

- Prediction function $\hat{f}$ with low average loss ("risk")

$$\mathrm{L}(\hat{f}) = E_{(y,x)}\left[\ell\left(\hat{f}(x), y\right)\right]$$

where $(y, x)$ distributed same as training

# What can we learn about the world from $\hat{f}$?

- Often interested in inference on $f(x) = E[y|x]$
  → what can we learn from ML output $\hat{f}$?
- Particularly tempting when output has common form

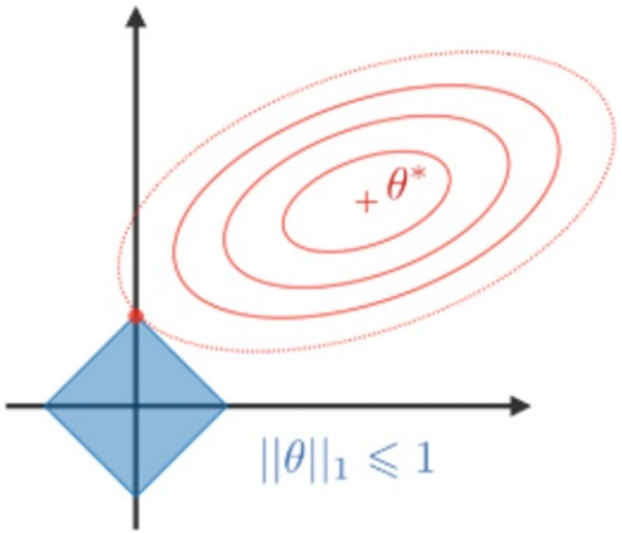
$$\hat{f}(x) = \hat{\beta}'x$$

- Unbiasedness
- Consistency
- Inference: asymptotic Normality, standard errors, tests
- Robustness

# LASSO regression

$$\min_{\hat{\beta}} \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \hat{\beta}' x_i\right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j|$$

- Selects *and* shrinks
- "Capitalist" – in doubt give all to one
- Produces sparse solutions



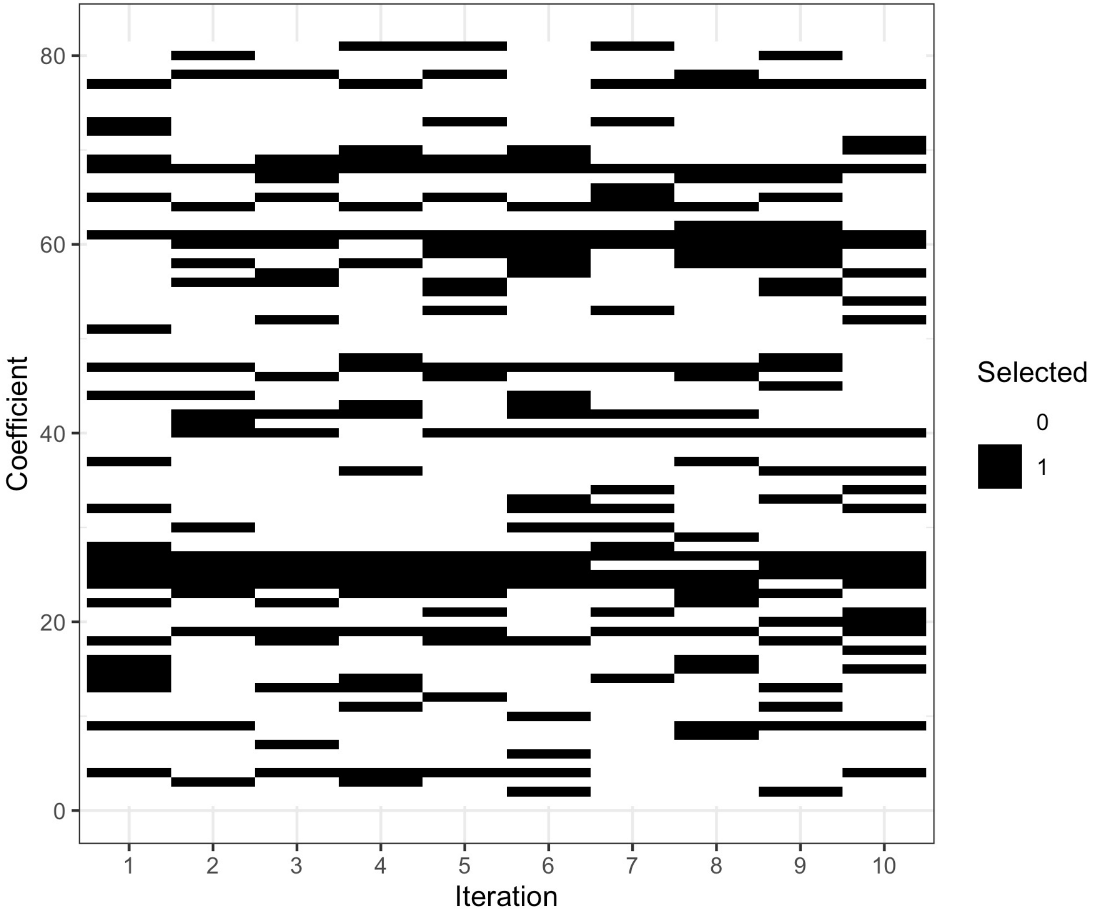
# From OLS to LASSO

$$default = \alpha + \beta_1\, income + \beta_2\, age$$
$$+\beta_3\, education + \beta_4\, creditscore$$
$$+\beta_5\, x_5 + \cdots + \beta_{27} x_{27} + \cdots \beta_{80} x_{80}$$

# LASSO selection

$$default = \alpha + \beta_1\ income + \beta_2\ age$$
$$+\beta_3\ education + \beta_4\ creditscore$$
$$+\beta_5\ x_5 + \cdots + \beta_{27}x_{27} + \cdots \beta_{80}x_{80}$$

# LASSO selection



# LASSO selection

$$default = \alpha + \beta_1\ income + \beta_2\ age$$
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# LASSO selection

$$default = \alpha + \beta_1\ income + \beta_2\ age + \beta_3\ education + \beta_4\ creditscore + \beta_5\ x_5 + \cdots + \beta_{27} x_{27} + \cdots \beta_{80} x_{80}$$

# LASSO selection

$$default = \alpha + \beta_1\ income \cancel{+ \beta_2\ age}$$
$$+\beta_3\ education + \beta_4\ creditscore$$
$$+\beta_5\ x_5 + \cdots + \beta_{27}x_{27} + \cdots \beta_{80}x_{80}$$

$$age \approx f(income, creditscore, \ldots, x_{27}, \ldots)$$

# LASSO biases

Fit LASSO with $x_1, x_2, x_3$ on

$$y = 2x_2 - x_3 + \epsilon$$

Selection biases:

- $\rho(x_2, x_3)$ large → $\hat{\beta}_3 = 0$ (compactification bias)
- $\rho(x_1, 2x_2 - x_3)$ large → $\hat{\beta}_1 \neq 0$ (expansion bias)

Size biases:

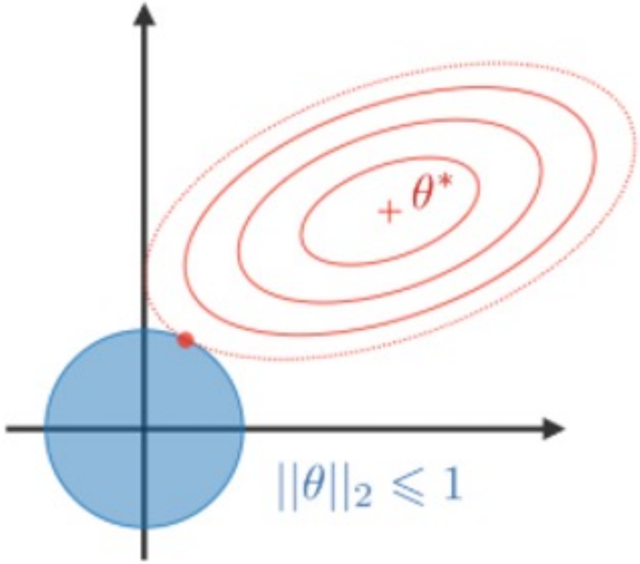
- $x_3$ not selected → $\hat{\beta}_2$ biased (omitted variable bias)
- Even if $x_2, x_3$ selected, biased toward zero (shrinkage bias)

- In high dimensions, correlations (empirically) ubiquitous

# Ridge regression

$$\min_{\hat{\beta}} \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \hat{\beta}' x_i\right)^2 + \lambda \sum_{j=1}^{k} \hat{\beta}_j^2$$

- Shrink towards zero, but never quite
- “Socialist” – in doubt distribute to multiple
- Can be interpreted as Bayesian posterior



# Ridge biases

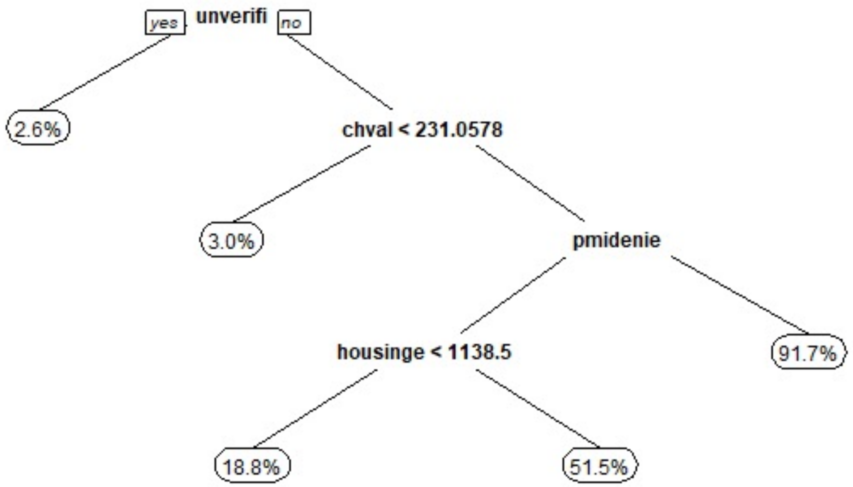
Fit ridge with $x_1, x_2$ with correlation $\rho$ on

$$y = \beta_2 x_2$$

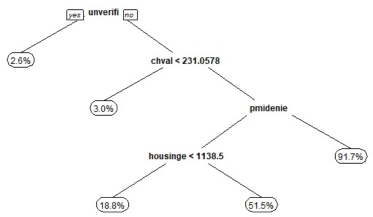
$$\rightarrow \quad \hat{\beta}_1 = \frac{\rho\lambda\beta_2}{(1+\lambda^2)-\rho^2}, \quad \hat{\beta}_2 = \frac{(1+\lambda-\rho^2)\beta_2}{(1+\lambda^2)-\rho^2}$$

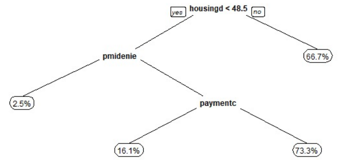
(expansion bias) (shrinkage bias)

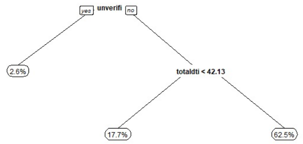
# Tree estimation

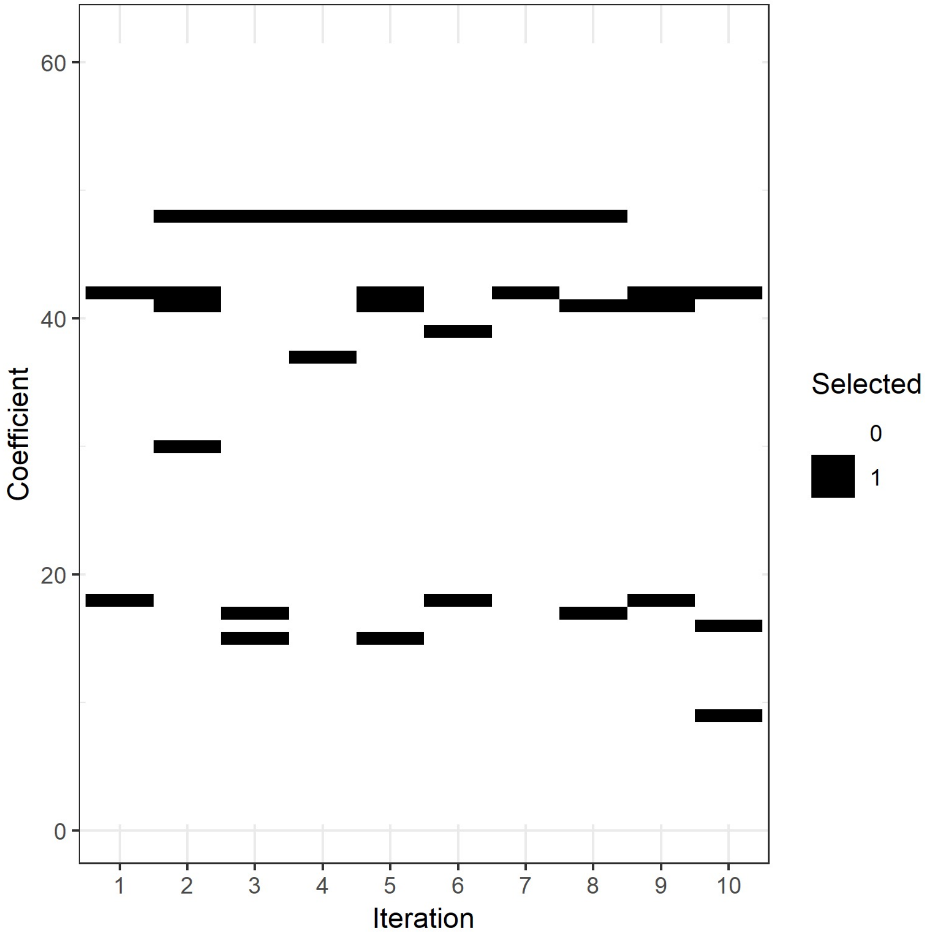


# Instability, inconsistency



# $\hat{y}$ vs $\hat{\beta}$

- Prediction ($\hat{y}$): care about out-of-sample loss min
- Estimation ($\hat{\beta}$): inference on coefficients
- In high dimensions, can have good predictions even when coefficients unstable, biased, inconsistent
- In big data, many functions that look different can have similar prediction properties, distinguishing hard
- The very features (complexity, regularization, tuning) that make prediction successful make estimation hard

# What I mean by prediction ($\hat{y}$)

- A sense in which ML does *not* deliver prediction: predict what happens under alternative policy
- Counterfactual requires structural/causal knowledge
- I mean: good fit of $\hat{y}$ to $y$ on *same* distribution

# What I mean by estimation ($\hat{\beta}$)

- A sense in which ML *does* deliver estimation: $\hat{f}(x)$ gets close to $f(x) = E[y|x]$ for minimal loss
- But only in loss norm, say $E_x\left(\hat{f}(x) - f(x)\right)^2$ – consistency not invariant to distribution of $x$
- I mean: estimation consistency $\hat{\beta} \to \beta$

# Take-aways for applied work

- ML provides quality predictions $\hat{y}$
- The prediction quality of given $\hat{f}$ comes with guarantees from hold-out
- Typically no estimation ($\hat{\beta}$) consistency
- Hence, by itself no structural interpretation or counterfactual extrapolation (causal inference)
- As a side note, inference hard (bootstrap may fail)

# Application areas for a $\hat{y}$ tool

1. Data pre-processing
2. $\hat{y}$ tasks (prediction policy problems)
3. $\hat{y}$ in the service of $\hat{\beta}$

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1. Introduction

Training data
$(y, x)$ $\xrightarrow{\hat{f}}$ Application data
$(\hat{y} = \hat{f}(x), x)$

2. The Secret Sauce of Machine Learning
(getting good $\hat{y}$)

3. Prediction vs Estimation
($\hat{y}$ vs $\hat{\beta}$)

# Thank you!

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