MA374-Financial Engineering Laboratory
Assignment 4

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February 2, 2014

Contents

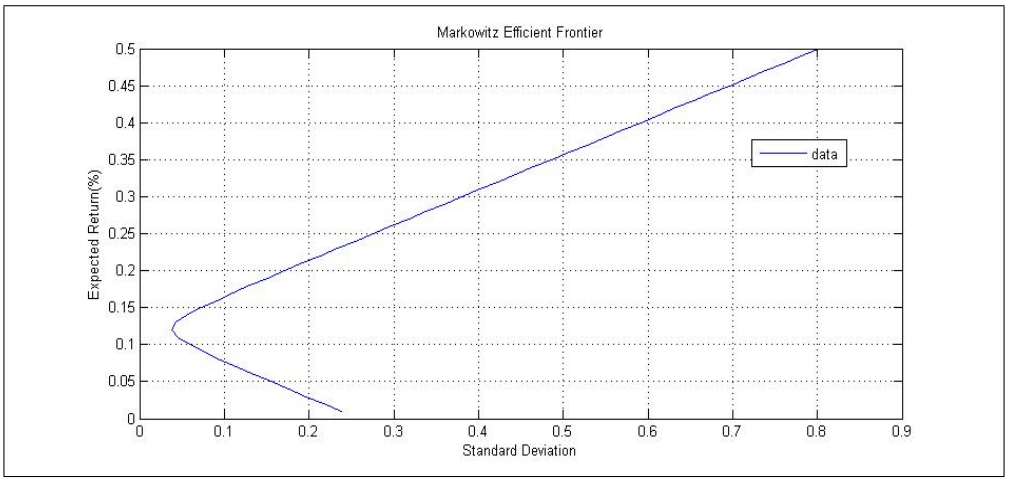
1 Question 1

We are given three assets with the following mean return vector and covariance matrix:

$$M = \begin{pmatrix} 0.1 & 0.2 & 0.15 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.005 & -0.010 & 0.004 \\ -0.010 & 0.040 & -0.002 \\ 0.004 & -0.002 & 0.023 \end{pmatrix}$$

1.1 The Markowitz Efficient Frontier



1.2 Portfolios on the MEF

Sl.No.	Return	Risk	W1	W2	W2
1	0.0100	0.0566	2.4073	-0.3927	-1.0147
2	0.0200	0.0471	2.2642	-0.3358	-0.9284
3	0.0300	0.0385	2.1211	-0.2789	-0.8422
4	0.0400	0.0308	1.9780	-0.2220	-0.7560
5	0.0500	0.0240	1.8349	-0.1651	-0.6697
6	0.0600	0.0181	1.6917	-0.1083	-0.5835
7	0.0700	0.0131	1.5486	-0.0514	-0.4972
8	0.0800	0.0090	1.4055	0.0055	-0.4110
9	0.0900	0.0058	1.2624	0.0624	-0.3248
10	0.1000	0.0034	1.1193	0.1193	-0.2385

1.3 For 15% Risk

The maximum return is 17%. For maximum return the corresponding weights are:

$$W_{max} = \begin{pmatrix} 0.1174 & 0.5174 & 0.3651 \end{pmatrix}$$

The minimum return is 5%. For minimum return the corresponding weights are:

$$W_{min} = \begin{pmatrix} 1.8349 & -0.1651 & -0.6697 \end{pmatrix}$$

1.4 For 18% return

For 18% return the minimum risk portfolio is with the weights:

$$W_{18,min} = \begin{pmatrix} -0.0257 & 0.5743 & 0.4514 \end{pmatrix}$$

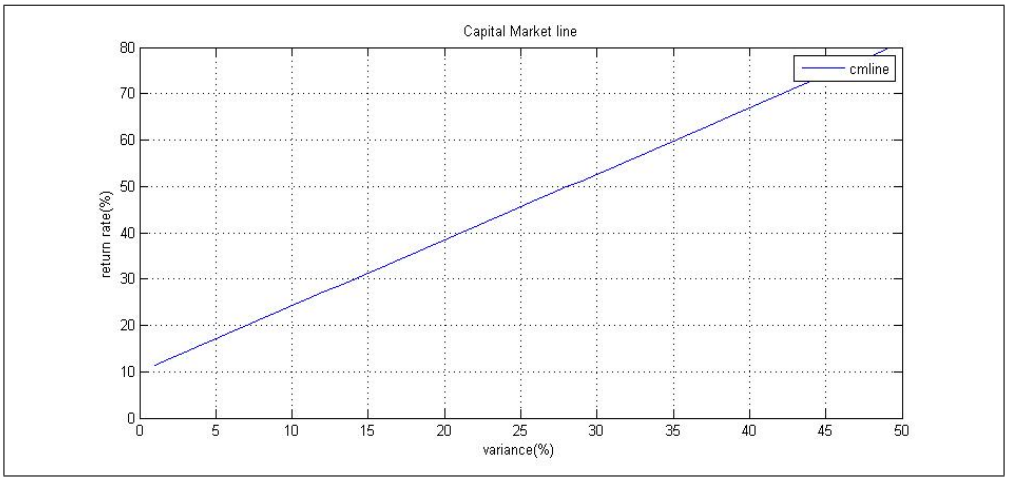
1.5 The Market Portfolio and the Capital Market Line

1.5.1 The Market Portfolio

The market portfolio with $\mu_{rf} = 10\%$ is given by the following weights:

$$W_{market} = \begin{pmatrix} 0.5937 & 0.3281 & 0.0781 \end{pmatrix}$$

1.5.2 The Capital Market Line



1.6 The Portfolios

1.6.1 With risk =10%

With risk=10% we have return=16% by Markowitz Efficient Frontier. The corresponding portfolio is:

$$W_{18,min} = \begin{pmatrix} 0.2606 & 0.4606 & 0.2789 \end{pmatrix}$$

1.6.2 With risk =25%

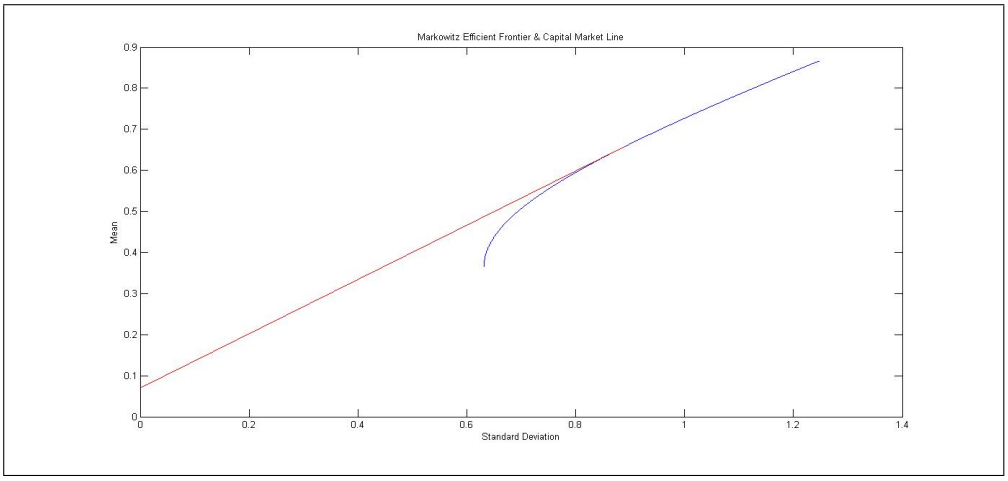
With risk=25% we have return=23% by Markowitz Efficient Frontier. The corresponding portfolio is:

$$W_{18,min} = \begin{pmatrix} -0.7413 & 0.8587 & 0.8826 \end{pmatrix}$$

2 Question 2

Here we had to calculate the following using data from online resources. The data source has been listed in the reference section.

2.1 The Markov Efficient Frontier and The CML

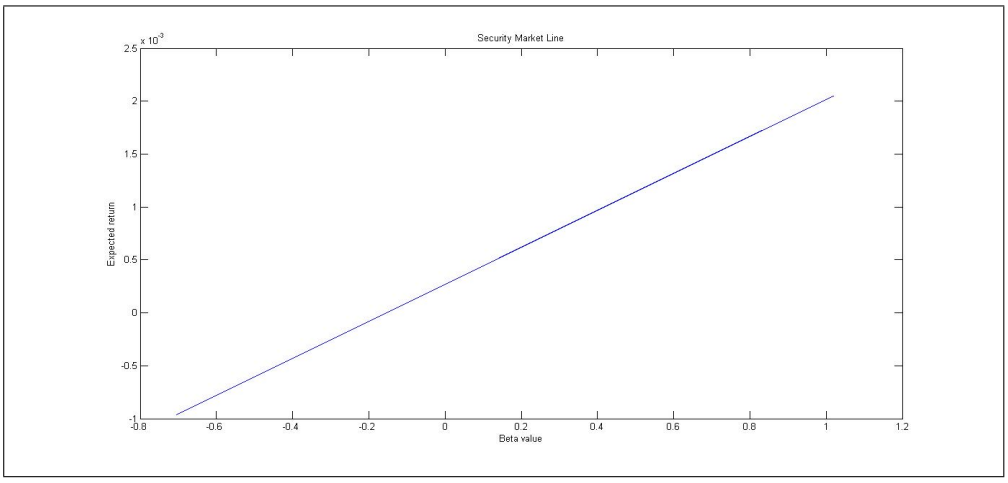


2.2 The Market Portfolio

The Market Portfolio is the Portfolio with the following mean and standard deviation:

Mean Return:0.6576
Standard Deviation:0.8900

2.3 The Security Market Line



3 Matlab Codes

3.1 Question 1

```
 clear;
% Part A
%-------------------------------------------------------------------------------
%script to calculate the markov efficient frontier
mu=(1:50)/100;%for fifty points
z=[0.1;0.2;0.15];%the mean return vector
C=[0.005,-0.010,0.004;-0.010,0.040,-0.002;0.004,-0.002,0.023];% the covariance
z1=ones(3,1);
A=z1'*((C)^-1)*z1;
if(A<0)
    display('error');
end
B=z1'*((C)^-1)*z;
C1=z'*((C)^-1)*z;
delta=(A*C1)-(B*B);
if(delta<0)
    display('error');
end
variance=zeros(1,50);
i=1;
while(i<=50)
    variance(i)=sqrt(((A*mu(i)*mu(i))-(2*B*mu(i))+C1)/delta);% applying the fo
    i=i+1;
```

```
end
%plot(variance,mu);
%-------------------------------------------------------------------------
%Part B
%-------------------------------------------------------------------------
wg=(C\z1)/A;%this is the weights for the global minimum variance portfolio
% we need another point on the efficient frontier so we calculate the
% tangency portfolio with R=0%
wt=(C\z)/B;
i=1;
weight=zeros(3,50);
ret=zeros(1,50);
sd=zeros(1,50);
while(i<=50)
    % Calculate Lambda and Gamma
    lambda= (C1 - (mu(i)*B))/delta;
    gamma=  ((mu(i)*A)-B)/delta;
    w_s=((lambda*A)*wg)+((gamma*B)*wt);
    ret(i)=w_s'*z;
    sd(i)=w_s'*C*w_s;
    weight(:,i)=w_s;
    i=i+1;
end
%tabulate weights ,return, and risk
ret
sd
weight(1,:)
weight(2,:)
weight(3,:)
%-------------------------------------------------------------------------
%Part C
%-------------------------------------------------------------------------
%15 percent risk from the markowitz graph max_return=17%, min_return=5%
% the protfoli 1
lambda= (C1 - (.17*B))/delta;
gamma=  ((.17*A)-B)/delta;
display('for maximum return');
w_s=((lambda*A)*wg)+((gamma*B)*wt)
% the protfoli 1
lambda= (C1 - (.05*B))/delta;
gamma=  ((.05*A)-B)/delta;
display('for minimum return');
w_s=((lambda*A)*wg)+((gamma*B)*wt)
```

```
%-------------------------------------------------------------------------
%Part D
%-------------------------------------------------------------------------
display('Weights of the Portfolio(at 18% return)');
weight(:,18)
%-------------------------------------------------------------------------
%Part E
%-------------------------------------------------------------------------
display('the market portfolio');
display('the market portfolio weights');
w_market=(C\(z-0.1*z1))/(B-(A*.1))
display('the market portfolio return rate');
ereturn_market=(C1-B*0.1)/(B-(A*.1))
display('the market portfolio variance')
sd_market=(C1-2*0.1*B+(0.01*A))/((B-(A*.1))^2)
variance1=(1:50)/10;
cmline=((variance1*(ereturn_market-0.1))/(sd_market));
cmline=cmline+(10*ones(1,50));
%plot the cmline from the workspace to observe the CML
%-------------------------------------------------------------------------
%Part F
%-------------------------------------------------------------------------
%risk =10% expected return from markowitz efficient frontier is 16%
display('risky asset weight');
y=(.16-0.1)/(ereturn_market-.1)
display('risk-free asset weight');
1-y
%risk =25% expected return from markowitz efficient frontier is 23%
display('risky asset weight');
y=(.23-0.1)/(ereturn_market-.1)
display('risk-free asset weight');
1-y
% the protfoli 1
lambda= (C1 - (.16*B))/delta;
gamma=  ((.16*A)-B)/delta;
display('for 10% risk');
w_s=((lambda*A)*wg)+((gamma*B)*wt)
% the protfoli 2
lambda= (C1 - (.23*B))/delta;
gamma=  ((.23*A)-B)/delta;
display('for 25% risk');
w_s=((lambda*A)*wg)+((gamma*B)*wt)
```

3.2 Question 2

```
  close all;
clear all;

[num] = xlsread('data.xlsx');
[n1 n2] = size(num);

for i = 1:(n1-1)
    m_temp(i,:) = (num(i+1,:)-num(i,:))./num(i,:);
end
for i = 1:n2
    m(i) = geomean(1+m_temp(:,i))-1;
end
m = ((1+m).^252)-1;
c = cov(num);
u = ones(1,n2);

w_opt = (u*inv(c))/(u*inv(c)*u');
m_opt = w_opt*m';
sig_opt = sqrt(w_opt*c*w_opt');

p1 = zeros(5000,2);
p1(1,1) = sig_opt;
p1(1,2) = m_opt;

for i = 2:5000
    p1(i,2) = m_opt+(i-1)*0.0001;
    w_temp = (det([1 u*inv(c)*m';p1(i,2) m*inv(c)*m'])*u*inv(c) + det([u*inv(c
    p1(i,1) = sqrt(w_temp*c*w_temp');
end

plot(p1(:,1),p1(:,2))
title('Markowitz Efficient Frontier & Capital Market Line');
xlabel('Standard Deviation');
ylabel('Mean');
hold on;

disp('***********************************************');
s1 = (p1(1,2)-0.07)/p1(1,1);
j = 1;
for i = 2:5000
    s2 = (p1(i,2)-0.07)/p1(i,1);
```

```
    if(s2 > s1)
        j = i;
        s1 = s2;
    end
end
m_m=p1(j,2);
sig_m=p1(j,1);
fprintf('Market mean = ');
disp(m_m);
fprintf('Market Standard Deviation = ');
disp(sig_m);
x=[0,sig_m];
y=[0.07,m_m];
plot(x,y,'red')
hold off;
disp('************************************************');

rf = (1.07^(1/252))-1;
[num]=xlsread('market.xlsx');
for i = 1:(n1-1)
    mr(i)=(num(i+1)-num(i))/num(i);
end
for i = 1:n2
    y = m_temp(:,i)-rf;
    x = [ones(n1-1,1) mr'-rf];
    temp = inv(x'*x)*x'*y;
    beta(i) = temp(2);
end
temp = geomean(1+mr)-1;
er = rf+beta*(temp-rf);
figure();
plot(beta,er)
title('Security Market Line');
xlabel('Beta value');
ylabel('Expected return');
```

4 References

- https://www.aaii.com/computerizedinvesting/article/mean-variance-optimization-multi-asset-portfolio.mobile
- http://www.calculatinginvestor.com/2011/06/07/efficient-frontier-1/...