1. Write a program to determine the initial price of an American call and an American put option in the binomial model with the following data:

$$S(0) = 100; K = 100; T = 1; M = 100; r = 8\%; \sigma = 20\%.$$

   Use the following two sets of u and d for your program.

   (a) Set 1 : $u = e^{\sigma\sqrt{\Delta t}}$; $d = e^{-\sigma\sqrt{\Delta t}}$.

   (b) Set 2 : $u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$; $d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$.

   Here $\Delta t = \frac{T}{M}$, with M being the number of subintervals in the time interval $[0, T]$. Use the continuous compounding convention in your calculations (i.e., both in $\tilde{p}$ and in the pricing formula).
   Now, plot the initial prices of both call and put options (for both the above sets of u and d) by varying one of the parameters at a time (as given below) while keeping the other parameters fixed (as given above) :

   (a) $S(0)$.

   (b) K.

   (c) r.

   (d) σ.

   (e) M (Do this for three values of K, $K = 95, 100, 105$).

2. Write a program to determine the initial price of a *lookback* option in the binomial model, using the basic binomial algorithm (used in earlier lab assignments), with the following data:

$$S(0) = 100; T = 1; r = 8\%; \sigma = 20\%.$$

   The payoff of the *lookback* option is given by

$$V = \max_{0 \le i \le M} S(i) - S(M),$$

   where $S(i) = S(i\Delta t)$ with $\Delta t = \frac{T}{M}$ (M being the number of subintervals of the time interval $[0, T]$). Use the continuous compounding convention in your calculations (i.e., both in $\tilde{p}$ and in the pricing formula). Use the following values of u and d for your program:

$$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}; \qquad d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$$

   (a) Obtain the initial price of the option for $M = 5, 10, 25, 50$.

   (b) How do the values of options at time $t = 0$ compare for the above values of M that you have taken ?

   (c) Tabulate the values of the options at all intermediate time points for $M = 5$.

3. Repeat Problem 2 using the (Markov based) computationally efficient binomial algorithm. Make a comparative analysis of the two algorithms, like computational time, the value of M it can handle, etc.