MA374-Financial Engineering Laboratory
Assignment 7

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Contents

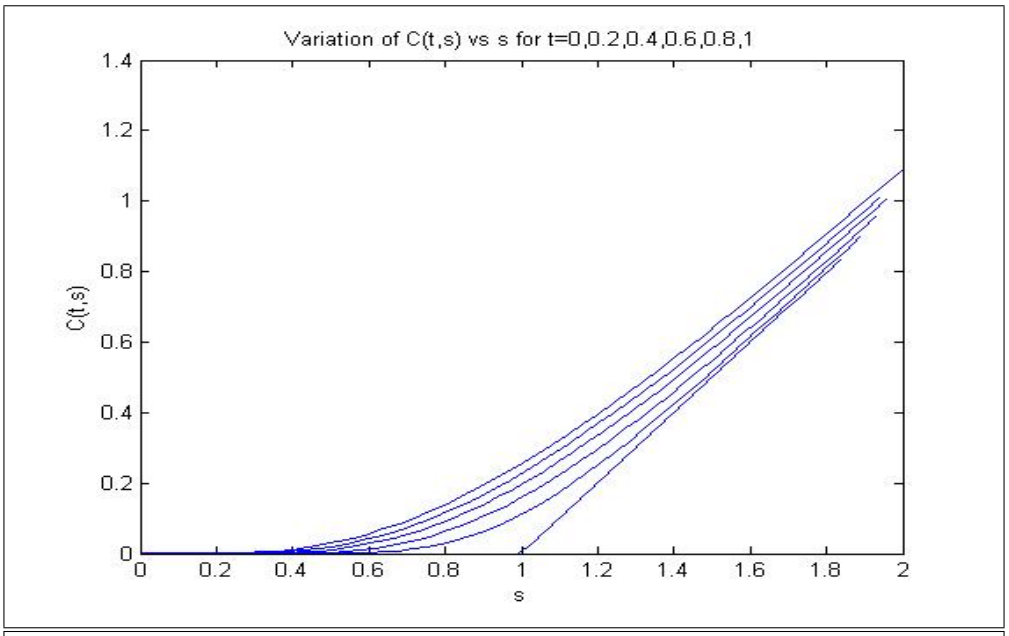
1 Question 1

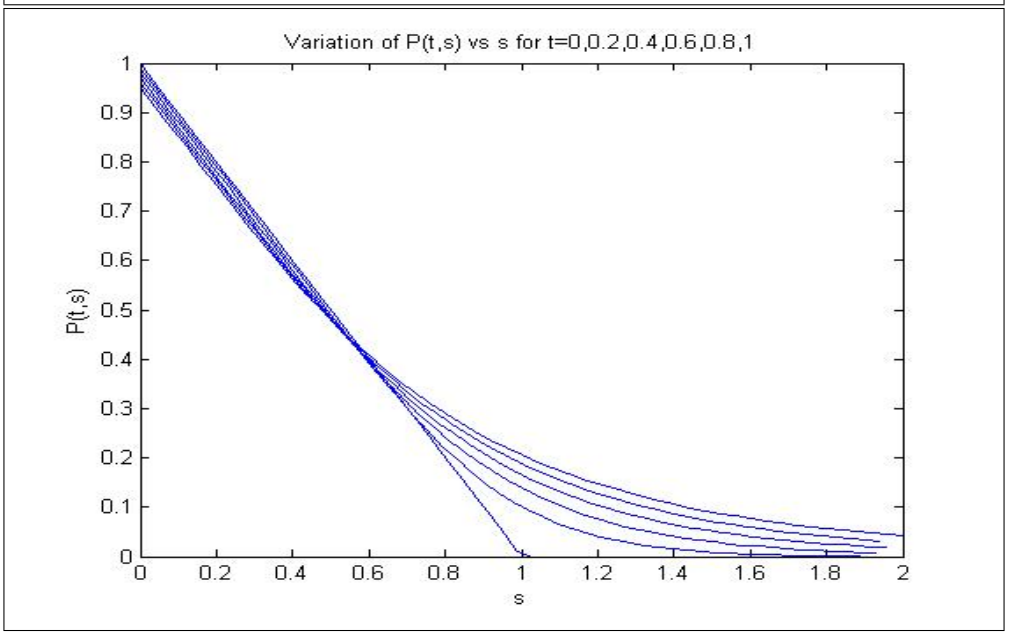
1.1 The Markowitz Efficient Frontier and the Feasible Region

We use the simple Black-Scholes Formula and its solution for C(t,s) and P(t,s).
Enter the initial stock price : 1
Enter the strike price : 1
Enter the maturity period : 1
Enter the risk free rate of return for the period : 0.05
Enter the standard deviation of the stock on which option is written : 0.6
Enter the time at which you want to find the call and put price : 0.5
Call price = 0.12878
Put price = 0.19218

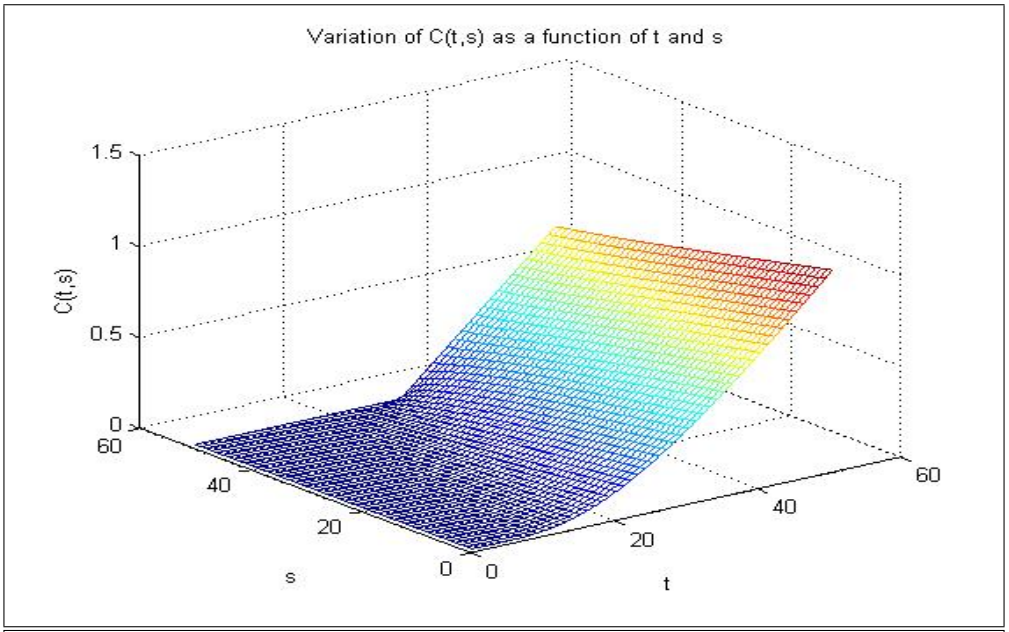
2 Question 2

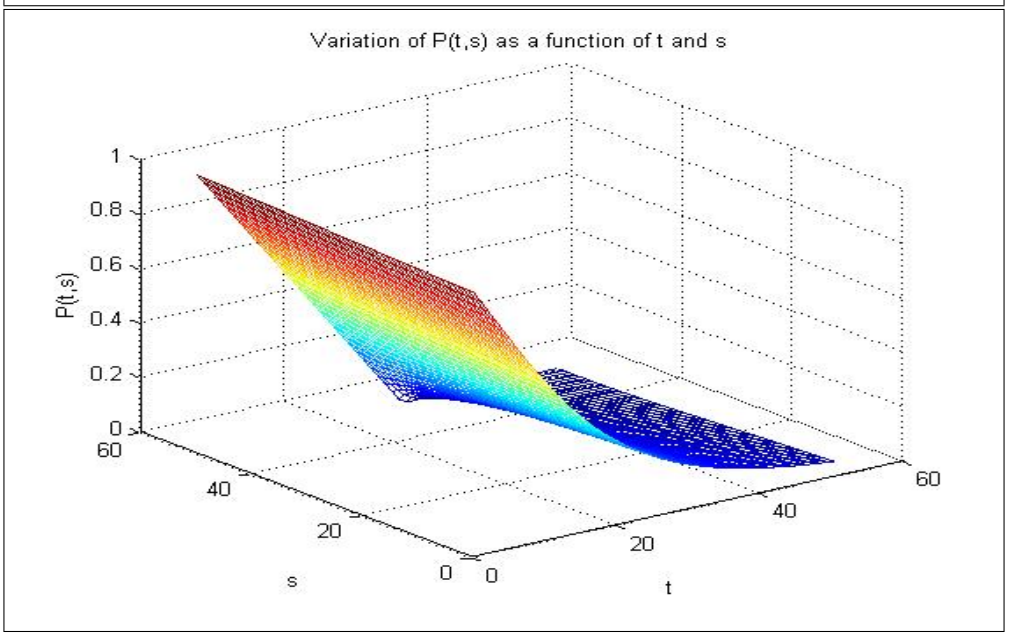
2.1 The Call and Put





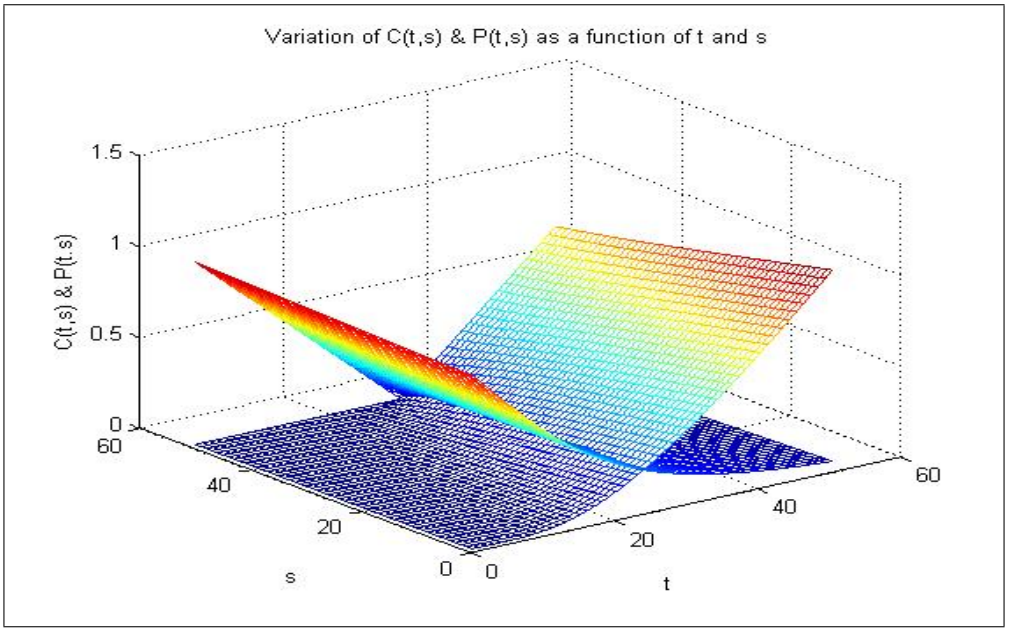
2.2 The Surface Plots





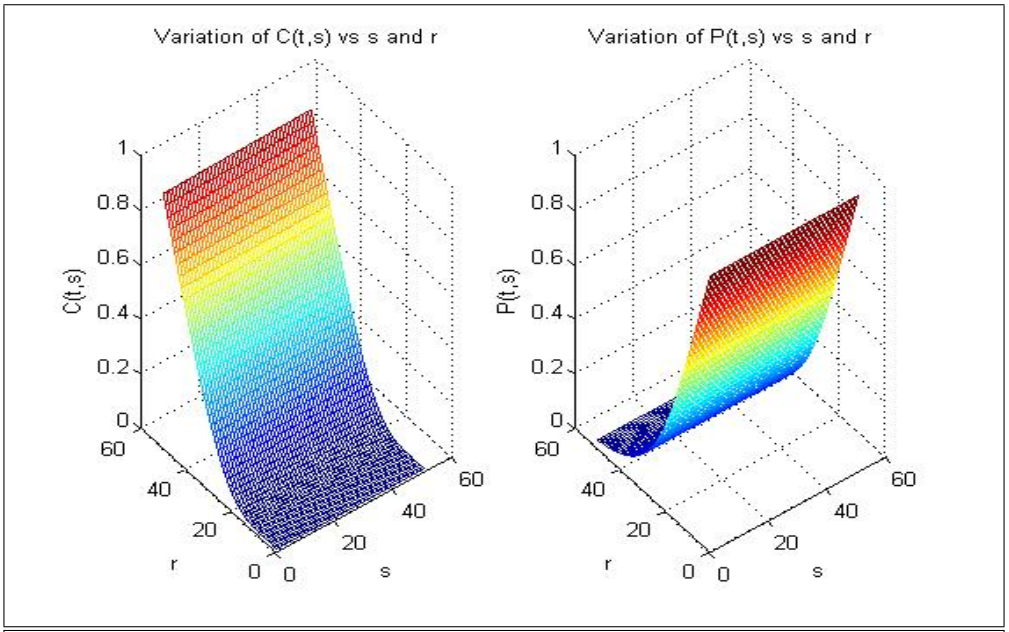
3 Question 3

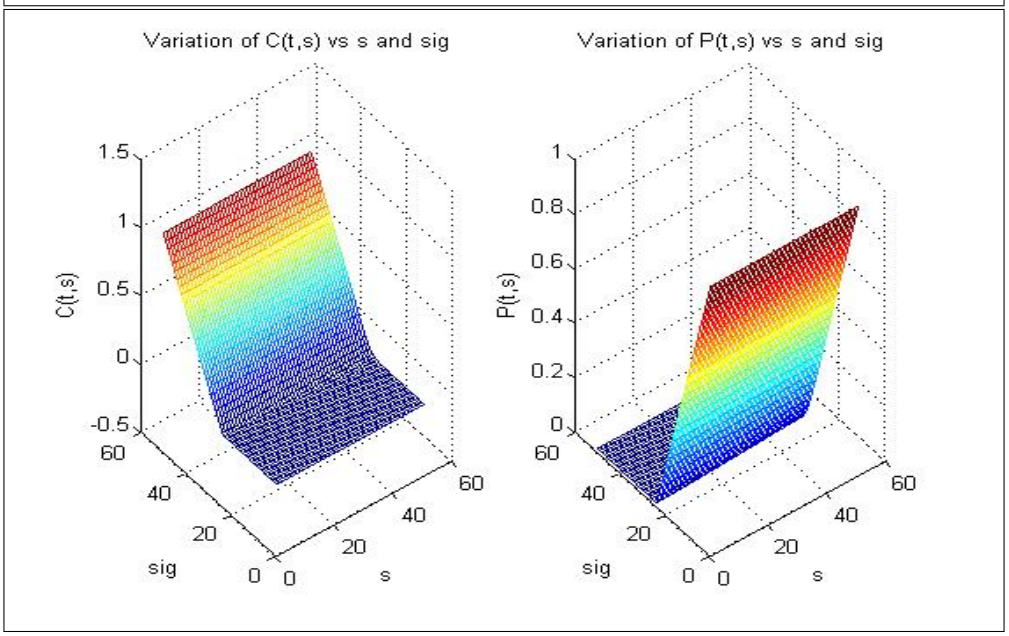
3.1 C(t,s) and P(t,s)

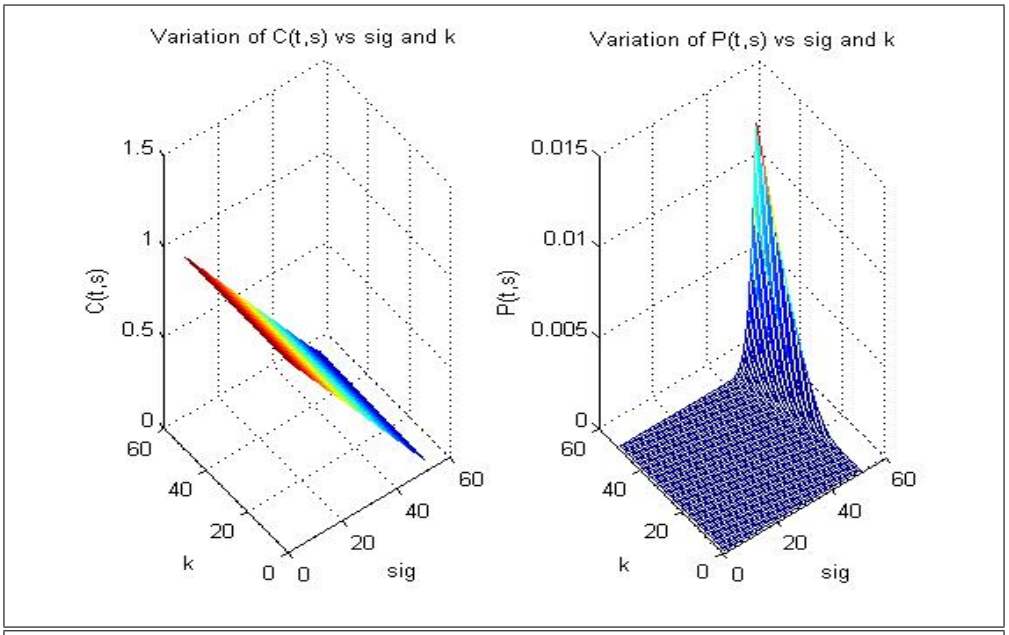


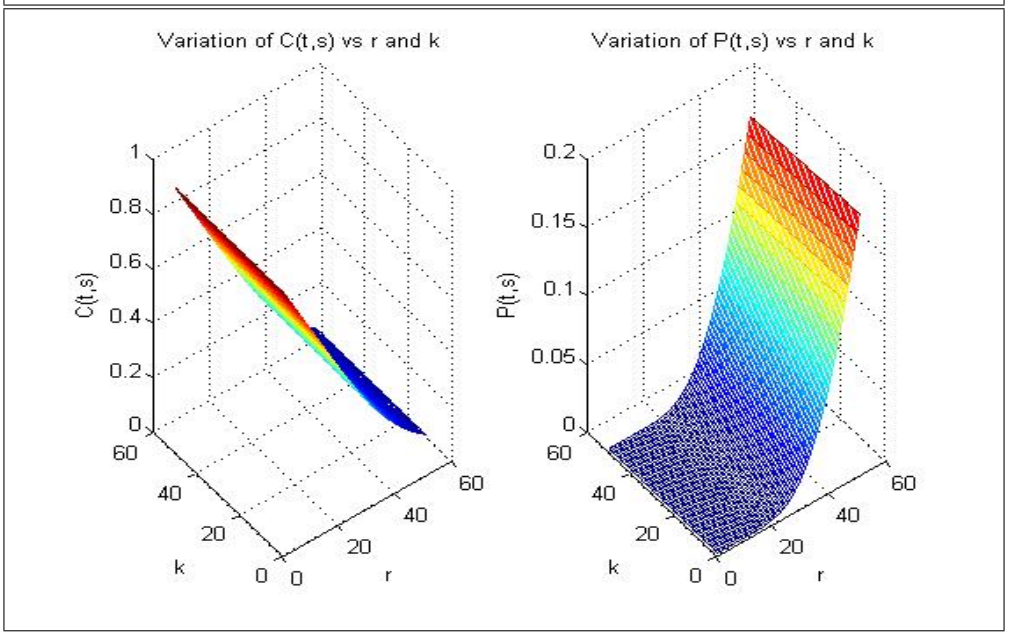
4 Question 4

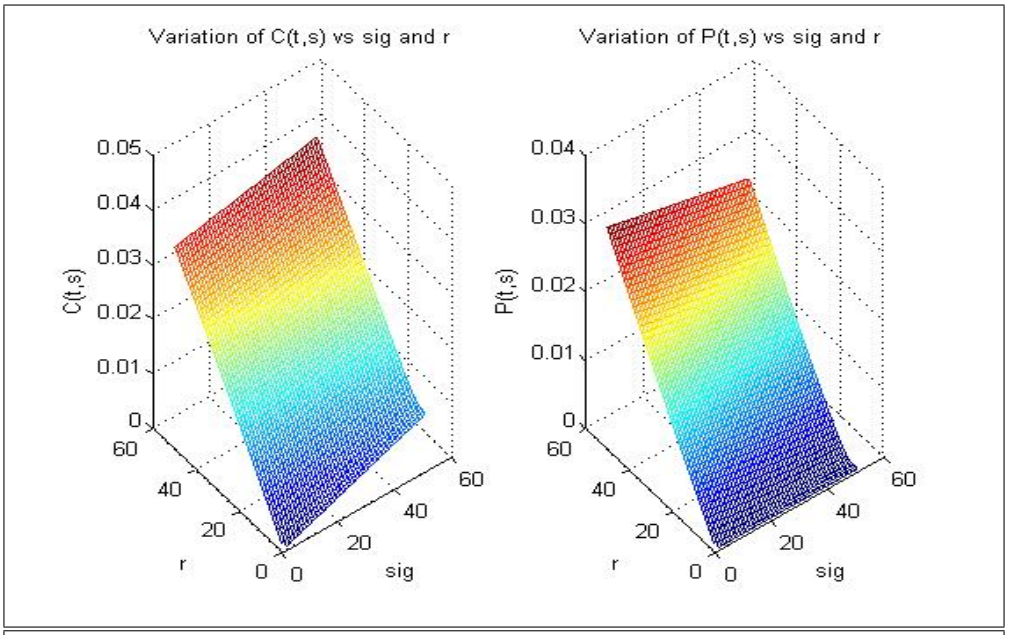
4.1 Sensitivity Analysis of C(t,s) and P(t,s)

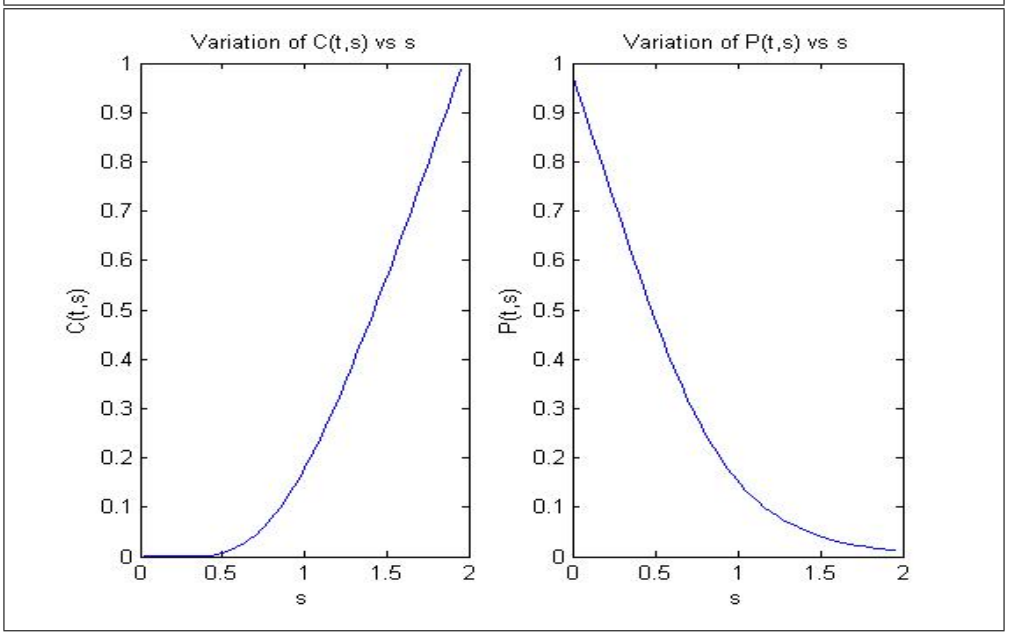


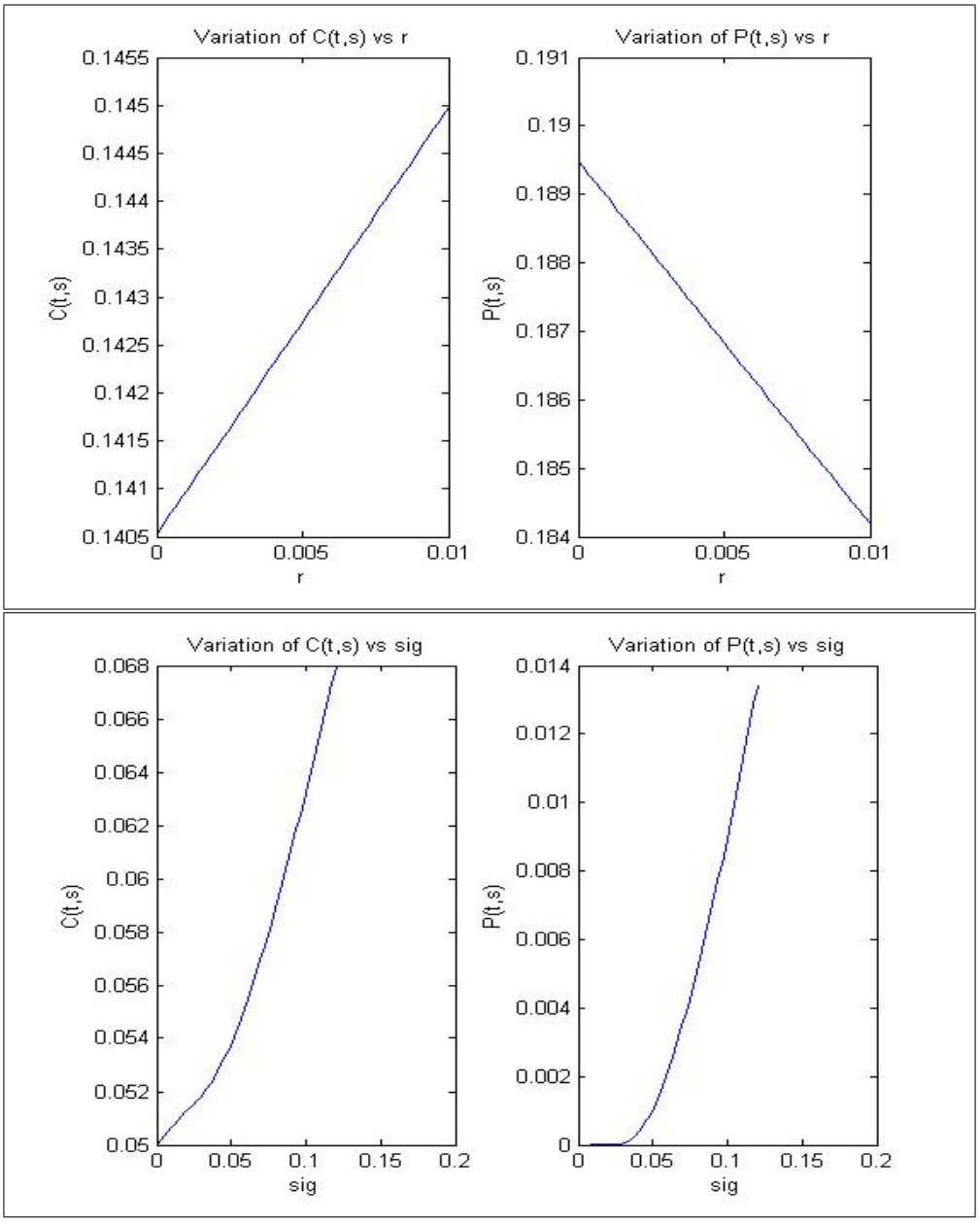


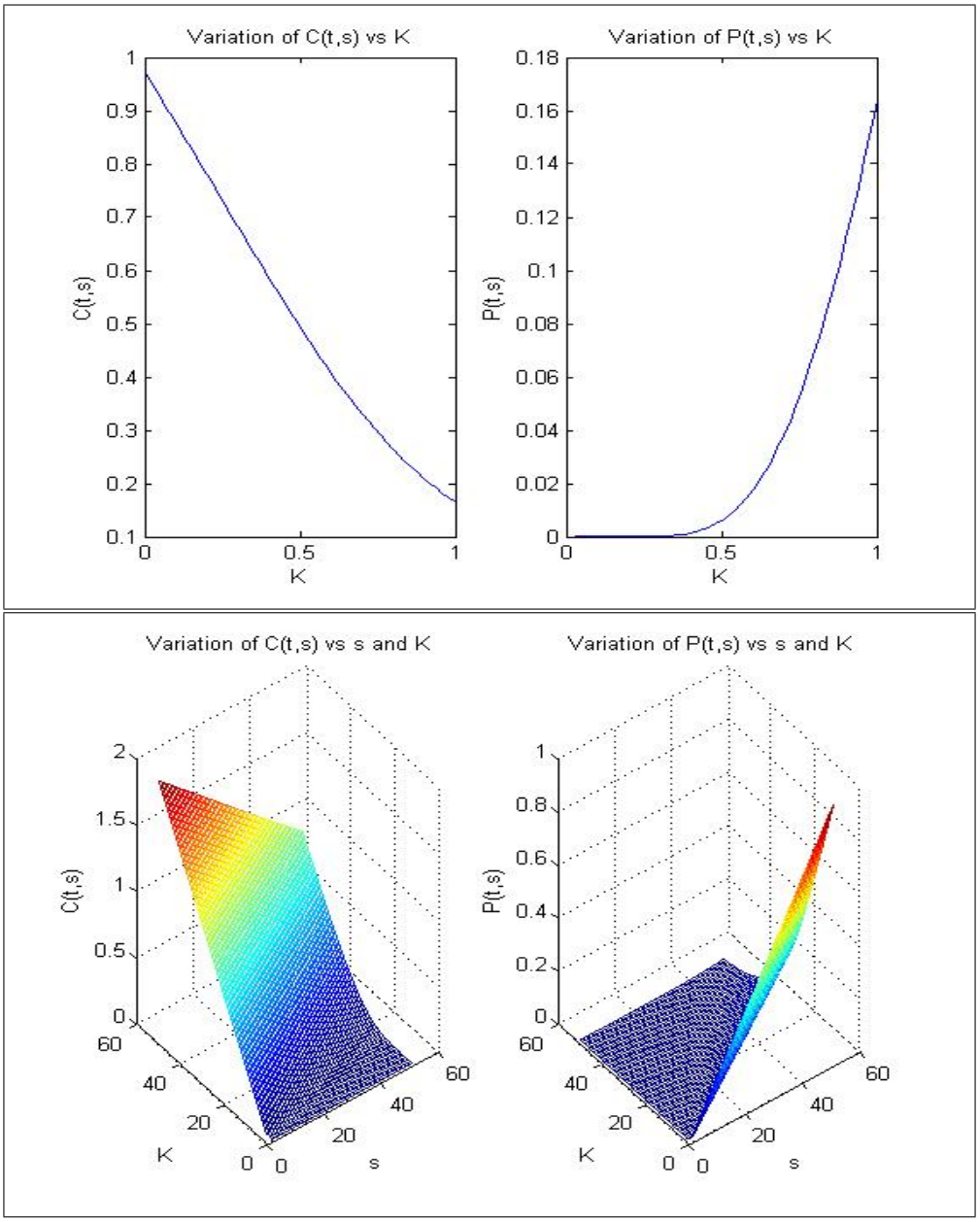

Variation of C(t,s) vs sig and k
C(t,s)
1.5
1
0.5
0
60
40
20
0
k
0
20
40
60
sig
Variation of P(t,s) vs sig and k
P(t,s)
0.015
0.01
0.005
0
60
40
20
0
k
0
20
40
60
sig


Variation of C(t,s) vs r and k
C(t,s)
1
0.8
0.6
0.4
0.2
0
60
40
20
0
k
0
20
40
60
r
Variation of P(t,s) vs r and k
P(t,s)
0.2
0.15
0.1
0.05
0
60
40
20
0
k
0
20
40
60
r


Variation of C(t,s) vs sig and r
C(t,s)
0.05
0.04
0.03
0.02
0.01
0
60
40
20
0
r
0
20
40
60
sig
Variation of P(t,s) vs sig and r
P(t,s)
0.04
0.03
0.02
0.01
0
60
40
20
0
r
0
20
40
60
sig


Variation of C(t,s) vs s
C(t,s)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
s
Variation of P(t,s) vs s
P(t,s)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
s


Variation of C(t,s) vs r
C(t,s)
r
0.1455
0.145
0.1445
0.144
0.1435
0.143
0.1425
0.142
0.1415
0.141
0.1405
0
0.005
0.01
Variation of P(t,s) vs r
P(t,s)
r
0.191
0.19
0.189
0.188
0.187
0.186
0.185
0.184
0
0.005
0.01
Variation of C(t,s) vs sig
C(t,s)
sig
0.068
0.066
0.064
0.062
0.06
0.058
0.056
0.054
0.052
0.05
0
0.05
0.1
0.15
0.2
Variation of P(t,s) vs sig
P(t,s)
sig
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.05
0.1
0.15
0.2



5 Matlab Codes

5.1 Question 1

```
clear;
clc;
s0=input('Enter the initial stock price : ');
```

```
k=input('Enter the strike price : ');
T=input('Enter the maturity period : ');
r=input('Enter the risk free rate of return for the period : ')
    ;
sig=input('Enter the standard deviation of the stock on which
    option is written : ');
t=input('Enter the time at which you want to find the call and
    put price : ');
w=[0; cumsum(randn(100000,1))]/sqrt(100000);
s=s0*exp(sig*w(t*1000)+(r-0.5*sig*sig)*t);
d1=(log(s/k)+(r+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
d2=(log(s/k)+(r-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
c=normcdf(d1)*s-normcdf(d2)*k*exp(-r*(T-t));
disp(['Call price = ' num2str(c)]);
p=normcdf(-d2)*k*exp(-r*(T-t))-normcdf(-d1)*s;
disp(['Put price = ' num2str(p)]);
```

5.2 Question 2

```
  clear;
clc;

k=1;
T=1;
r=0.05;
sig=0.6;

w=[0; cumsum(randn(100000,1))]/sqrt(100000);
s0=0:0.04:2;
c=zeros(1,length(s0));
p=zeros(1,length(s0));
s=zeros(1,length(s0));
tt=[0  0.2  0.4  0.6  0.8  1];

for i=1:length(tt)
    t=tt(i);
    for j=1:51
        s(j)=s0(j)*exp(sig*w(t*1000+1)+(r-0.5*sig*sig)*t);
        d1=(log(s(j)/k)+(r+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
        d2=(log(s(j)/k)+(r-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
        c(j)=normcdf(d1)*s(j)-normcdf(d2)*k*exp(-r*(T-t));
    end
    plot(s(:),c(:));
    hold on;
```

```
end
title('Variation of C(t,s) vs s for t=0,0.2,0.4,0.6,0.8,1');
xlabel('s');
ylabel('C(t,s)');
hold off;

figure();
for i=1:length(tt)
    t=tt(i);
    for j=1:51
        s(j)=s0(j)*exp(sig*w(t*1000+1)+(r-0.5*sig*sig)*t);
        d1=(log(s(j)/k)+(r+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
        d2=(log(s(j)/k)+(r-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
        p(j)=normcdf(-d2)*k*exp(-r*(T-t))-normcdf(-d1)*s(j);
    end
    plot(s(:),p(:));
    hold on;
end
title('Variation of P(t,s) vs s for t=0,0.2,0.4,0.6,0.8,1');
xlabel('s');
ylabel('P(t,s)');
hold off;

clear all;

s0=0:0.04:2;
t=0:0.02:1;
k=1;
T=1;
r=0.05;
sig=0.6;
w=[0; cumsum(randn(100000,1))]/sqrt(100000);

figure();
for j=1:51
    for i=1:51
        s(j)=s0(j)*exp(sig*w(floor(t(i)+1))+(r-0.5*sig*sig)*t(i
            ));
        d1=(log(s(j)/k)+(r+0.5*sig*sig)*(T-t(i)))/(sig*sqrt(T-t
            (i)));
        d2=(log(s(j)/k)+(r-0.5*sig*sig)*(T-t(i)))/(sig*sqrt(T-t
            (i)));
        c(i,j)=normcdf(d1)*s(j)-normcdf(d2)*k*exp(-r*(T-t(i)));
```

```
    end
end
mesh(c);
title('Variation of C(t,s) as a function of t and s');
xlabel('t');
ylabel('s');
zlabel('C(t,s)');

figure();
for j=1:51
    for i=1:51
        s(j)=s0(j)*exp(sig*w(floor(t(i)+1))+(r-0.5*sig*sig)*t(i
            ));
        d1=(log(s(j)/k)+(r+0.5*sig*sig)*(T-t(i)))/(sig*sqrt(T-t
            (i)));
        d2=(log(s(j)/k)+(r-0.5*sig*sig)*(T-t(i)))/(sig*sqrt(T-t
            (i)));
        p(i,j)=normcdf(-d2)*k*exp(-r*(T-t(i)))-normcdf(-d1)*s(j
            );
    end
end
mesh(p);
title('Variation of P(t,s) as a function of t and s');
xlabel('t');
ylabel('s');
zlabel('P(t,s)');
```

5.3 Question 3

```
  close all;
clear;
clc;

s0=0:0.04:2;
t=0:0.02:1;
k=1;
T=1;
r=0.05;
sig=0.6;
w=[0; cumsum(randn(100000,1))]/sqrt(100000);

for j=1:51
    for i=1:51
```

```
        s(j)=s0(j)*exp(sig*w(floor(t(i)+1))+(r-0.5*sig*sig)*t(i
            ));
        d1=(log(s(j)/k)+(r+0.5*sig*sig)*(T-t(i)))/(sig*sqrt(T-t
            (i)));
        d2=(log(s(j)/k)+(r-0.5*sig*sig)*(T-t(i)))/(sig*sqrt(T-t
            (i)));
        c(i,j)=normcdf(d1)*s(j)-normcdf(d2)*k*exp(-r*(T-t(i)));
        p(i,j)=normcdf(-d2)*k*exp(-r*(T-t(i)))-normcdf(-d1)*s(j
            );
    end
end
mesh(c);
hold on;
mesh(p);
title('Variation of C(t,s) & P(t,s) as a function of t and s');
xlabel('t');
ylabel('s');
zlabel('C(t,s) & P(t.s)');
hold off;
```

5.4 Question 4

```
  clear;
clc;

k=1;
T=1;
r=0.05;
sig=0.6;
s0=1;
w=[0; cumsum(randn(10000,1))]/sqrt(10000);
t=0.5;

s0=0:0.04:2;
for j=1:51
    s(j)=s0(j)*exp(sig*w(t*1000+1)+(r-0.5*sig*sig)*t);
    d1=(log(s(j)/k)+(r+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
    d2=(log(s(j)/k)+(r-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
    c(j)=normcdf(d1)*s(j)-normcdf(d2)*k*exp(-r*(T-t));
    p(j)=normcdf(-d2)*k*exp(-r*(T-t))-normcdf(-d1)*s(j);
end
subplot(1,2,1);
plot(s(:),c(:));
title('Variation of C(t,s) vs s');
```

```
xlabel('s');
ylabel('C(t,s)');
subplot(1,2,2);
plot(s(:),p(:));
title('Variation of P(t,s) vs s');
xlabel('s');
ylabel('P(t,s)');
s0=1;

r=0:0.0002:0.01;
for j=1:51
    s(j)=s0*exp(sig*w(t*1000+1)+(r(j)-0.5*sig*sig)*t);
    d1=(log(s(j)/k)+(r(j)+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
    d2=(log(s(j)/k)+(r(j)-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
    c(j)=normcdf(d1)*s(j)-normcdf(d2)*k*exp(-r(j)*(T-t));
    p(j)=normcdf(-d2)*k*exp(-r(j)*(T-t))-normcdf(-d1)*s(j);
end
figure();
subplot(1,2,1);
plot(r(:),c(:));
title('Variation of C(t,s) vs r');
xlabel('r');
ylabel('C(t,s)');
subplot(1,2,2);
plot(r(:),p(:));
title('Variation of P(t,s) vs r');
xlabel('r');
ylabel('P(t,s)');
r=0.05;

sig=0:0.0024:0.12;
for j=1:51
    s(j)=s0*exp(sig(j)*w(t*1000+1)+(r-0.5*sig(j)*sig(j))*t);
    d1=(log(s(j)/k)+(r+0.5*sig(j)*sig(j))*(T-t))/(sig(j)*sqrt(T
        -t));
    d2=(log(s(j)/k)+(r-0.5*sig(j)*sig(j))*(T-t))/(sig(j)*sqrt(T
        -t));
    c(j)=normcdf(d1)*s(j)-normcdf(d2)*k*exp(-r*(T-t));
    p(j)=normcdf(-d2)*k*exp(-r*(T-t))-normcdf(-d1)*s(j);
end
figure();
subplot(1,2,1);
plot(sig(:),c(:));
```

```
title('Variation of C(t,s) vs sig');
xlabel('sig');
ylabel('C(t,s)');
subplot(1,2,2);
plot(sig(:),p(:));
title('Variation of P(t,s) vs sig');
xlabel('sig');
ylabel('P(t,s)');
sig=0.6;

k=0:0.02:1;
for j=1:51
    s=s0*exp(sig*w(t*1000+1)+(r-0.5*sig*sig)*t);
    d1=(log(s/k(j))+(r+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
    d2=(log(s/k(j))+(r-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t));
    c(j)=normcdf(d1)*s-normcdf(d2)*k(j)*exp(-r*(T-t));
    p(j)=normcdf(-d2)*k(j)*exp(-r*(T-t))-normcdf(-d1)*s;
end
figure();
subplot(1,2,1);
plot(k(:),c(:));
title('Variation of C(t,s) vs K');
xlabel('K');
ylabel('C(t,s)');
subplot(1,2,2);
plot(k(:),p(:));
title('Variation of P(t,s) vs K');
xlabel('K');
ylabel('P(t,s)');
k=1;

k=0:0.02:1;
s0=0:0.04:2;
for i=1:51
    for j=1:51
        s(i)=s0(i)*exp(sig*w(t*1000+1)+(r-0.5*sig*sig)*t);
        d1=(log(s(i)/k(j))+(r+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t
            ));
        d2=(log(s(i)/k(j))+(r-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t
            ));
        c(i,j)=normcdf(d1)*s(i)-normcdf(d2)*k(j)*exp(-r*(T-t));
        p(i,j)=normcdf(-d2)*k(j)*exp(-r*(T-t))-normcdf(-d1)*s(i
            );
```

```
    end
end
figure();
subplot(1,2,1);
mesh(c);
title('Variation of C(t,s) vs s and K');
xlabel('s');
ylabel('K');
zlabel('C(t,s)');
subplot(1,2,2);
mesh(p);
title('Variation of P(t,s) vs s and K');
xlabel('s');
ylabel('K');
zlabel('P(t,s)');
k=1;
s0=1;

s0=0:0.04:2;
r=0:0.0002:0.01;
for i=1:51
    for j=1:51
        s(i)=s0(i)*exp(sig*w(t*1000+1)+(r(j)-0.5*sig*sig)*t);
        d1=(log(s(i)/k)+(r(j)+0.5*sig*sig)*(T-t))/(sig*sqrt(T-t
            ));
        d2=(log(s(i)/k)+(r(j)-0.5*sig*sig)*(T-t))/(sig*sqrt(T-t
            ));
        c(i,j)=normcdf(d1)*s(i)-normcdf(d2)*k*exp(-r(j)*(T-t));
        p(i,j)=normcdf(-d2)*k*exp(-r(k)*(T-t))-normcdf(-d1)*s(i
            );
    end
end
figure();
subplot(1,2,1);
mesh(c);
title('Variation of C(t,s) vs s and r');
xlabel('s');
ylabel('r');
zlabel('C(t,s)');
subplot(1,2,2);
mesh(p);
title('Variation of P(t,s) vs s and r');
xlabel('s');
```

```
ylabel('r');
zlabel('P(t,s)');
r=0.05;
s0=1;

s0=0:0.04:2;
sig=0:0.0024:0.12;
for i=1:51
    for j=1:51
        s(i)=s0(i)*exp(sig(j)*w(t*1000+1)+(r-0.5*sig(j)*sig(j))
            *t);
        d1=(log(s(i)/k)+(r+0.5*sig(j)*sig(j))*(T-t))/(sig(j)*
            sqrt(T-t));
        d2=(log(s(i)/k)+(r-0.5*sig(j)*sig(j))*(T-t))/(sig(j)*
            sqrt(T-t));
        c(i,j)=normcdf(d1)*s(i)-normcdf(d2)*k*exp(-r*(T-t));
        p(i,j)=normcdf(-d2)*k*exp(-r*(T-t))-normcdf(-d1)*s(i);
    end
end
figure();
subplot(1,2,1);
mesh(c);
title('Variation of C(t,s) vs s and sig');
xlabel('s');
ylabel('sig');
zlabel('C(t,s)');
subplot(1,2,2);
mesh(p);
title('Variation of P(t,s) vs s and sig');
xlabel('s');
ylabel('sig');
zlabel('P(t,s)');
sig=0.6;
s0=1;

sig=0:0.0024:0.12;
k=0:0.02:1;
for i=1:51
    for j=1:51
        s(i)=s0*exp(sig(i)*w(t*1000+1)+(r-0.5*sig(i)*sig(i))*t)
            ;
        d1=(log(s(i)/k(j))+(r+0.5*sig(i)*sig(i))*(T-t))/(sig(i)
            *sqrt(T-t));
```

```
        d2=(log(s(i)/k(j))+(r-0.5*sig(i)*sig(i))*(T-t))/(sig(i)
            *sqrt(T-t));
        c(i,j)=normcdf(d1)*s(i)-normcdf(d2)*k(j)*exp(-r*(T-t));
        p(i,j)=normcdf(-d2)*k(j)*exp(-r*(T-t))-normcdf(-d1)*s(i
            );
    end
end
figure();
subplot(1,2,1);
mesh(c);
title('Variation of C(t,s) vs sig and k');
xlabel('sig');
ylabel('k');
zlabel('C(t,s)');
subplot(1,2,2);
mesh(p);
title('Variation of P(t,s) vs sig and k');
xlabel('sig');
ylabel('k');
zlabel('P(t,s)');
sig=0.6;
k=1;

r=0:0.0002:0.01;
k=0:0.02:1;
for i=1:51
    for j=1:51
        s(i)=s0*exp(sig*w(t*1000+1)+(r(i)-0.5*sig*sig)*t);
        d1=(log(s(i)/k(j))+(r(i)+0.5*sig*sig)*(T-t))/(sig*sqrt(
            T-t));
        d2=(log(s(i)/k(j))+(r(i)-0.5*sig*sig)*(T-t))/(sig*sqrt(
            T-t));
        c(i,j)=normcdf(d1)*s(i)-normcdf(d2)*k(j)*exp(-r(i)*(T-t
            ));
        p(i,j)=normcdf(-d2)*k(j)*exp(-r(i)*(T-t))-normcdf(-d1)*
            s(i);
    end
end
figure();
subplot(1,2,1);
mesh(c);
title('Variation of C(t,s) vs r and k');
xlabel('r');
```

```
ylabel('k');
zlabel('C(t,s)');
subplot(1,2,2);
mesh(p);
title('Variation of P(t,s) vs r and k');
xlabel('r');
ylabel('k');
zlabel('P(t,s)');
r=0.05;
k=1;

r=0:0.0002:0.01;
sig=0:0.0024:0.12;
for i=1:51
    for j=1:51
        s(i)=s0*exp(sig(i)*w(t*1000+1)+(r(j)-0.5*sig(i)*sig(i))
            *t);
        d1=(log(s(i)/k)+(r(j)+0.5*sig(i)*sig(i))*(T-t))/(sig(i)
            *sqrt(T-t));
        d2=(log(s(i)/k)+(r(j)-0.5*sig(i)*sig(i))*(T-t))/(sig(i)
            *sqrt(T-t));
        c(i,j)=normcdf(d1)*s(i)-normcdf(d2)*k*exp(-r(j)*(T-t));
        p(i,j)=normcdf(-d2)*k*exp(-r(j)*(T-t))-normcdf(-d1)*s(i
            );
    end
end
figure();
subplot(1,2,1);
mesh(c);
title('Variation of C(t,s) vs sig and r');
xlabel('sig');
ylabel('r');
zlabel('C(t,s)');
subplot(1,2,2);
mesh(p);
title('Variation of P(t,s) vs sig and r');
xlabel('sig');
ylabel('r');
zlabel('P(t,s)');
```

6 References

- wikipedia...