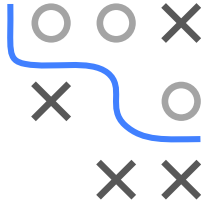
Introduction to Machine Learning

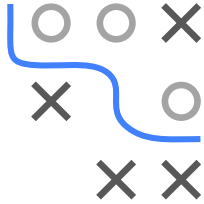


Random Forest Basics

Learning goals

- Know how random forests are defined by extending the idea of bagging
- Understand general idea to decorrelate trees
- Understand effects of hyperparameters
- RFs and overfitting

MOTIVATION



CARTs offer several appealing features:

- **Interpretability**: Easy to understand and explain
- **Invariance to rank-preserving transformations**: E.g., unaffected by scaling or shifting of features
- **Versatility**: Work on categorical and numerical data
- **Robustness to missing values**: Can work with missings

Despite these benefits, CARTs are not without drawbacks:

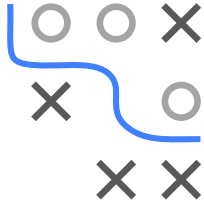
▸ Hastie, Tibshirani, and Friedman 2009

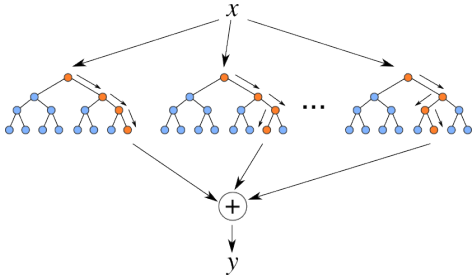
> *"Trees have one aspect that prevents them from being the ideal tool for predictive learning, namely inaccuracy."*

RANDOM FORESTS

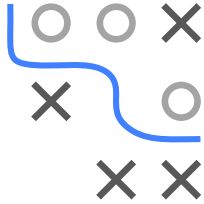
Breiman 2001

- RFs use **bagging with CARTs as BLs**
- **Random feature sampling** decorrelates the base learners
- **Fully expanded trees** further increase variability of trees





INTUITION BEHIND DECORRELATION



- Since bootstrap samples are similar, models $\hat{b}^{[m]}$ are correlated, affecting the variance of an ensemble $\hat{f}$
- We would like variance to go down linearly with ensemble size, but because of correlation we cannot really expect that
- Assuming $\text{Var}(\hat{b}^{[m]}) = \sigma^2$, $\text{Corr}(\hat{b}^{[m]}, \hat{b}^{[j]}) = \rho$, semi-formal analysis, without proper analysis of prediction error:

$$\text{Var}\left(\hat{f}\right) = \text{Var}\left(\frac{1}{M}\sum_{m=1}^{M}\hat{b}^{[m]}\right) = \frac{1}{M^2}\left(\sum_{m=1}^{M}\text{Var}(\hat{b}^{[m]}) + 2\sum_{m<j}\text{Cov}(\hat{b}^{[m]}, \hat{b}^{[j]})\right)$$

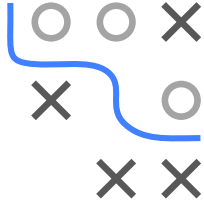
$$= \frac{1}{M^2}\left(M\sigma^2 + 2\frac{M(M-1)}{2}\rho\sigma^2\right) = (1-\rho)\frac{\sigma^2}{M} + \rho\sigma^2$$

- Ensemble variance is “convex-combo of linear-reduction and no-reduction, controlled by ρ”
- Maybe we can decorrelate trees, to reduce ensemble variance? And get less prediction error?

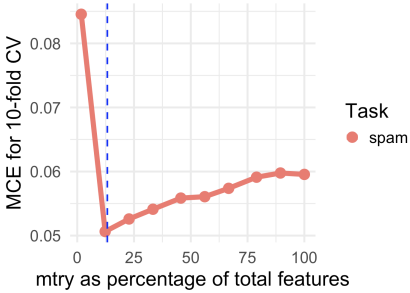
RANDOM FEATURE SAMPLING

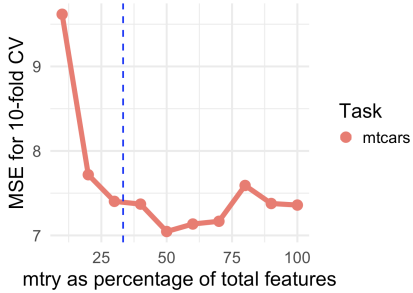
RFs decorrelate trees with a simple randomization:

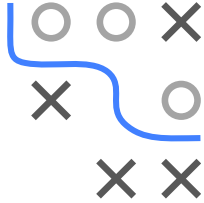
- For each node of tree, randomly draw $\texttt{mtry} \leq p$ features ($\texttt{mtry}$ = name in some implementations)
- Only consider these features for finding the best split
- Careful: Our previous analysis was simplified! The more we decorrelate by this, the more random the trees become! This also has negative effects!



EFFECT OF FEATURE SAMPLING





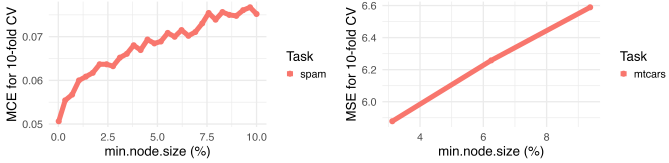


- Optimal `mtry` typically larger for regression than for classification
- Good defaults exist, but still most relevant tuning param
- Rule of thumb:
  - Classification: $\texttt{mtry} = \lfloor \sqrt{p} \rfloor$
  - Regression: $\texttt{mtry} = \lfloor p/3 \rfloor$

TREE SIZE

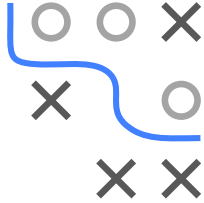
In addition to `mtry`, RFs have two other important HPs:

- Min. nr. of obs. in each decision tree node
  Default (ranger): `min.node.size = 5` ▸ Breiman 2001



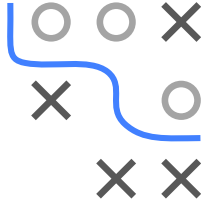
- Depth of each tree
  Default (ranger): `maxDepth` $= \infty$
- There are more alternative HPs to control depth of tree:
  minimal risk reduction, size of terminal nodes, etc.

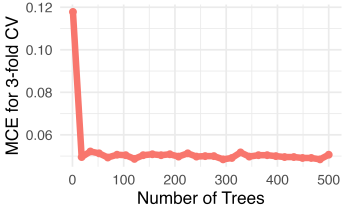
RF usually use fully expanded trees, without aggressive early stopping or pruning, to further **increase variability of each tree**. ▸ Louppe 2015

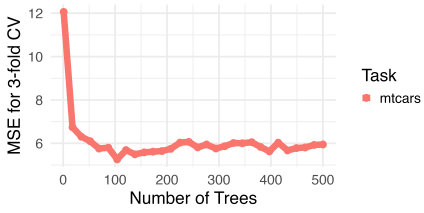


ENSEMBLE SIZE

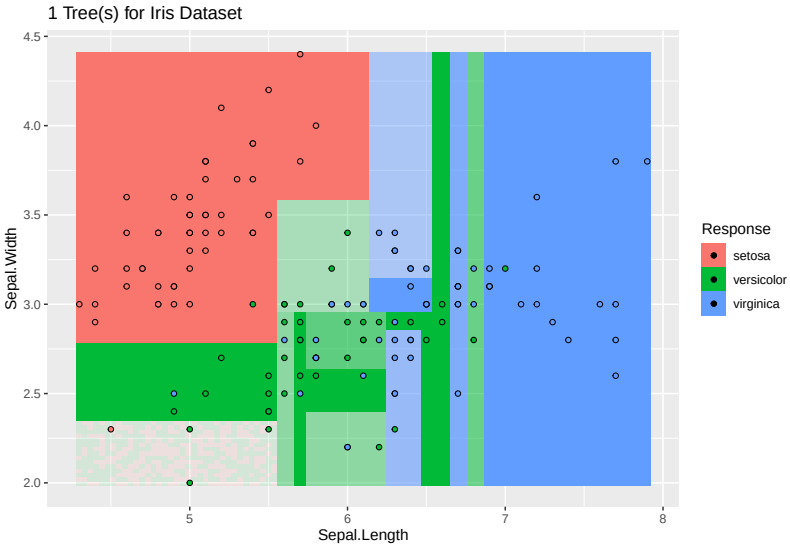
- RFs usually better if ensemble is large ▸ Breiman 2001
- But: Increases computational costs, and diminishing returns
- 100 or 500 is a sensible default
- Can also inspect the OOB error (see later)

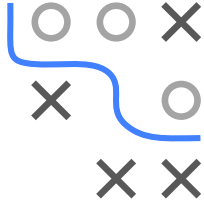






EFFECT OF ENSEMBLE SIZE





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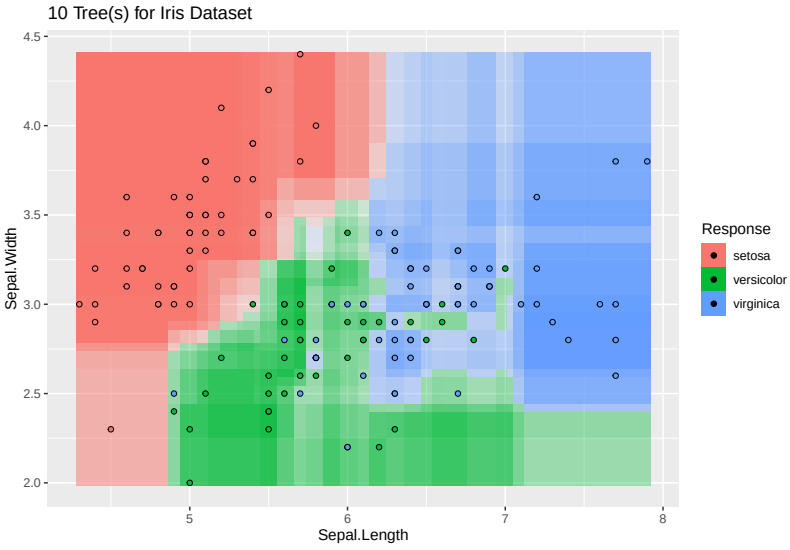

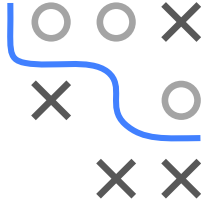

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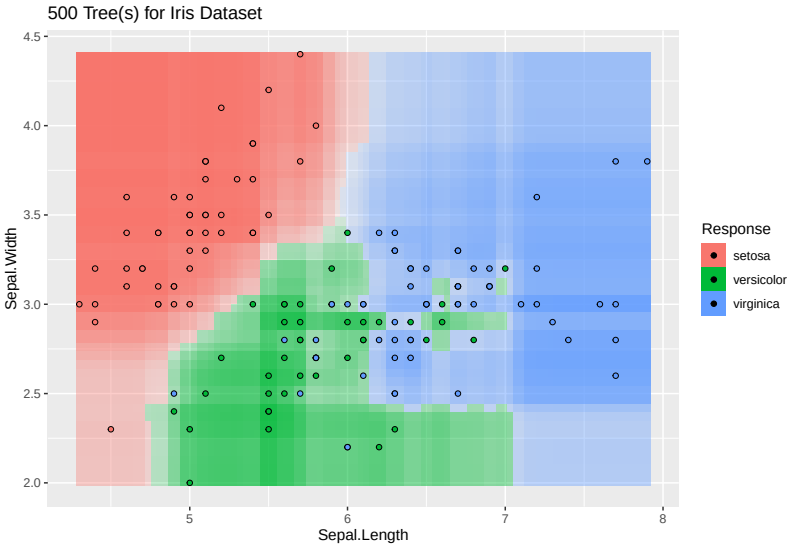
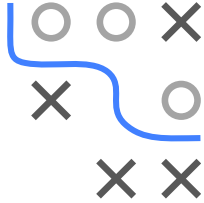
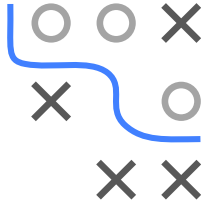
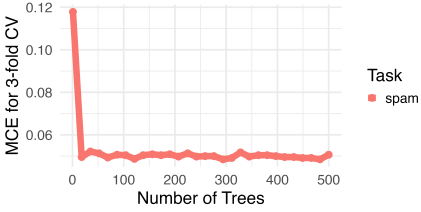


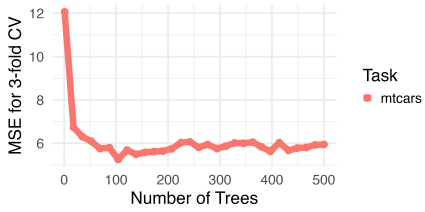


CAN RF OVERFIT? ▸ Probst and Boulesteix 2018



- Just like any other learner, RFs **can** overfit!
- However, RFs generally **less** prone to overfitting than individual CARTs.
- Overly complex trees can *still* lead to overfitting!
  If most trees capture noise, so does the RF.
- But randomization and averaging helps.

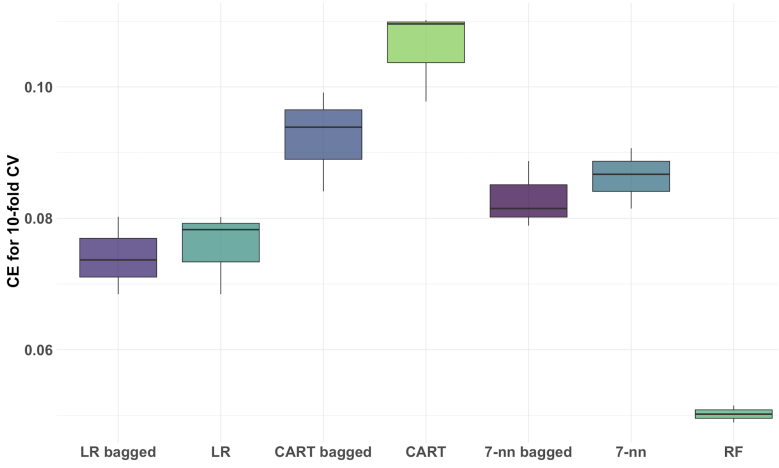




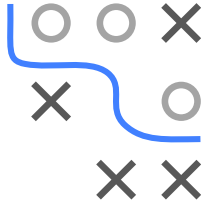
Since each tree is trained *individually and without knowledge of previously trained trees*, increasing `ntrees` generally reduces variance **without increasing the chance of overfitting!**

RF IN PRACTICE

Benchmarking bagged ensembles with 100 BLs each on `spam` versus RF ($\texttt{ntrees} = 100$, $\texttt{mtry} = \sqrt{p}$, $\texttt{minnode} = 1$), we see how well RF performs!



DISCUSSION



Advantages:

- Most advantages of trees also apply to RF: not much preprocessing required, missing value handling, etc.
- Easy to parallelize
- Often work well (enough)
- Works well on high-dimensional data
- Works well on data with irrelevant “noise” variables

Disadvantages:

- Same extrapolation problem as for trees
- Harder to interpret than trees
  (but many extra tools are nowadays available for interpreting RFs)
- Implementation can be memory-hungry
- Prediction can be computationally demanding for large ensembles