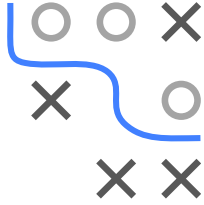
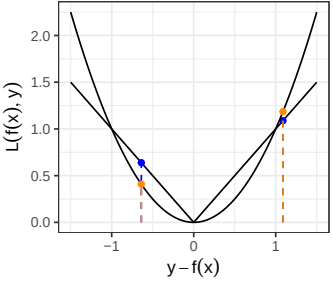
Introduction to Machine Learning



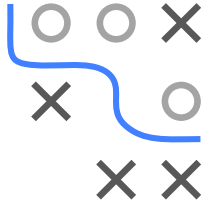
Supervised Regression: Linear Models with $L1$ Loss



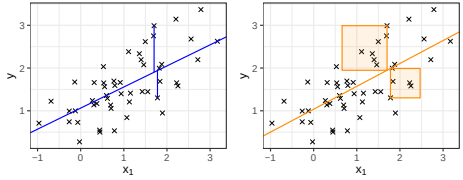
Learning goals

- Understand difference between $L1$ and $L2$ regression
- See how choice of loss affects optimization & robustness

ABSOLUTE LOSS

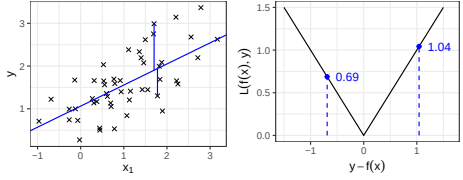


- *L*2 regression minimizes quadratic residuals – wouldn't **absolute** residuals seem more natural?

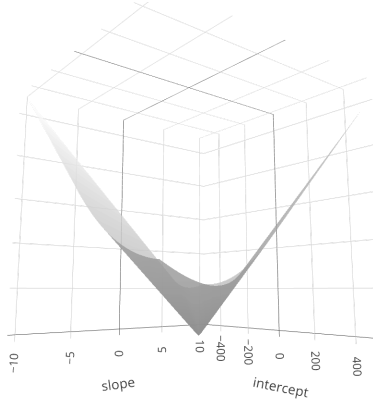


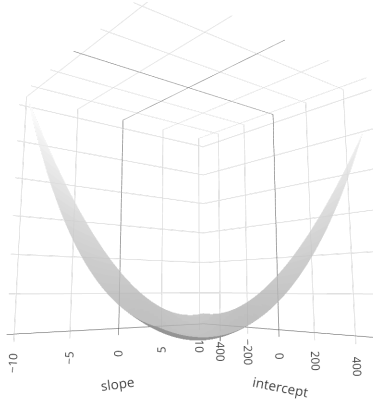
- *L*1 **loss / absolute error / least absolute deviation (LAD)**

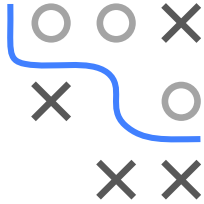
$$L(y, f(\mathbf{x})) = |y - f(\mathbf{x})|$$



$L1$ **VS** $L2$ **– LOSS SURFACE**





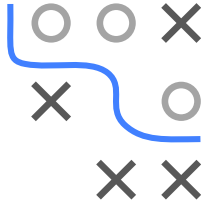


$L1$ loss (left) harder to optimize than $L2$ loss (right)

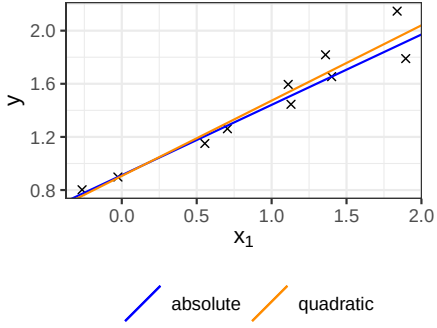
- Convex but **not differentiable** in $y - f(\mathbf{x}) = 0$
- No analytical solution

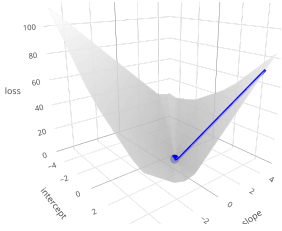
*L*1 **VS** *L*2 **– ESTIMATED PARAMETERS**

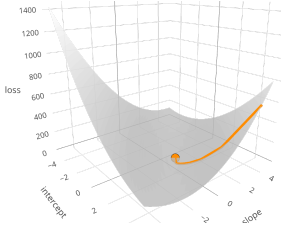
- Results of *L*1 and *L*2 regression often not that different
- Simulated data: $y^{(i)} = 1 + 0.5x_1^{(i)} + \epsilon^{(i)}, \quad \epsilon^{(i)} \overset{i.i.d}{\sim} \mathcal{N}(0, 0.01)$



	intercept	slope
*L*1	0.91	0.53
*L*2	0.91	0.57

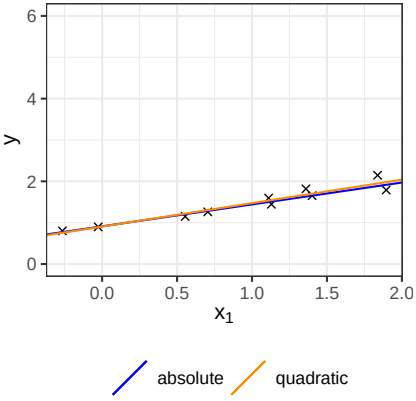


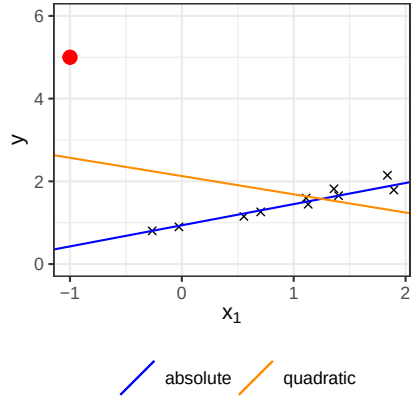


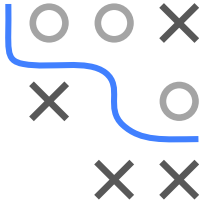


*L*1 **VS** *L*2 **– ROBUSTNESS**

- *L*2 quadratic in residuals $\rightsquigarrow$ outlying points carry lots of weight
- E.g., $3\times$ residual $\Rightarrow$ $9\times$ loss contribution
- *L*1 more **robust** in presence of outliers (example ctd.):

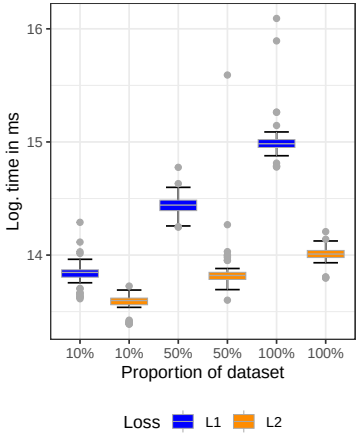






*L*1 VS *L*2 – OPTIMIZATION COST

- Real-world `weather` problem $\rightsquigarrow$ predict mean temperature
- Compare **time** to fit *L*1 (`quantreg::rq()`) vs *L*2 (`lm::lm()`) for different dataset proportions (repeat 50$\times$)



Loss

	Fitted: *L*1	Fitted: *L*2
Total *L*1 loss	8.98×10^4	8.99×10^4
Total *L*2 loss	5.83×10^6	5.81×10^6

Estimated coefficients

x_j	*L*1: $\hat{\theta}_j$	*L*2: $\hat{\theta}_j$
`Max_temperature`	0.553	0.563
`Min_temperature`	0.441	0.427
`Visibility`	0.026	0.041
`Wind_speed`	0.002	0.010
`Max_wind_speed`	−0.026	−0.039
`(Intercept)`	−0.380	−0.102

*L*1 **slower** to optimize!