Exercise 4 – Classification II

Introduction to Machine Learning

Hint: Useful libraries

R

```
# you may need the following packages for this exercise sheet:

library(mlr3)
library(mlr3learners)
library(ggplot2)
library(mlbench)
library(mlr3viz)
```

Python

```
# Consider the following libraries for this exercise sheet:

# general
import numpy as np
import pandas as pd
from scipy.stats import norm
# plotting
import matplotlib.pyplot as plt
import seaborn as sns
# sklearn
from sklearn.naive_bayes import CategoricalNB # import Naive Bayes Classifier for categori
from sklearn.naive_bayes import GaussianNB # import Naive Bayes Classifier for normal dist
from sklearn.preprocessing import OrdinalEncoder
from sklearn.preprocessing import LabelEncoder
```

```python
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis as QDA
from sklearn.inspection import DecisionBoundaryDisplay
from sklearn.metrics import confusion_matrix
from sklearn.metrics import precision_recall_fscore_support
```

Exercise 1: Naive Bayes

> Learning goals
>
> Compute Naive Bayes predictions by hand

You are given the following table with the target variable `Banana`:

ID	Color	Form	Origin	Banana
1	yellow	oblong	imported	yes
2	yellow	round	domestic	no
3	yellow	oblong	imported	no
4	brown	oblong	imported	yes
5	brown	round	domestic	no
6	green	round	imported	yes
7	green	oblong	domestic	no
8	red	round	imported	no

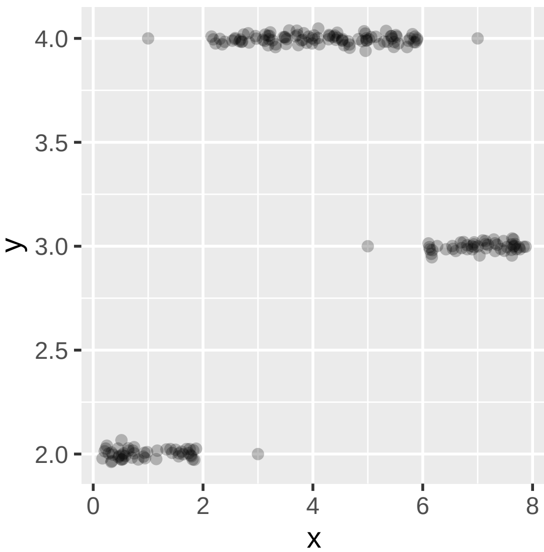
We want to use a Naive Bayes classifier to predict whether a new fruit is a `Banana` or not. Estimate the posterior probability $\hat{\pi}(\mathbf{x}_*)$ for a new observation $\mathbf{x}_* = (\text{yellow}, \text{round}, \text{imported})$. How would you classify the object?

Assume you have an additional feature `Length` that measures the length in cm. Describe in 1-2 sentences how you would handle this numeric feature with Naive Bayes.

Exercise 2: Discriminant analysis

Learning goals

1) Set up discriminant analysis by hand
2) Make predictions with discriminant analysis
3) Discuss difference between LDA and QDA



The above plot shows $\mathcal{D} = \left(\left(\mathbf{x}^{(1)}, y^{(1)}\right), \ldots, \left(\mathbf{x}^{(n)}, y^{(n)}\right)\right)$, a data set with $n = 200$ observations of a continuous target variable y and a continuous, 1-dimensional feature variable $\mathbf{x}$. In the following, we aim at predicting y with a machine learning model that takes $\mathbf{x}$ as input.

To prepare the data for classification, we categorize the target variable y in 3 classes and call the transformed target variable z, as follows:

$$z^{(i)} = \begin{cases} 1, & y^{(i)} \in (-\infty, 2.5] \\ 2, & y^{(i)} \in (2.5, 3.5] \\ 3, & y^{(i)} \in (3.5, \infty) \end{cases}$$

Now we can apply quadratic discriminant analysis (QDA):

Estimate the class means $\mu_k = \mathbb{E}(\mathbf{x}|z = k)$ for each of the three classes $k \in \{1, 2, 3\}$ visually from the plot. Do not overcomplicate this, a rough estimate is sufficient here.

Make a plot that visualizes the different estimated densities per class.

How would your plot from ii) change if we used linear discriminant analysis (LDA) instead of QDA? Explain your answer.

Why is QDA preferable over LDA for this data?

Given are two new observations $\mathbf{x}_{*1} = -10$ and $\mathbf{x}_{*2} = 7$. Assuming roughly equal class sizes, state the prediction for QDA and explain how you arrive there.

Exercise 3: Decision boundaries for classification learners

> Learning goals
>
> Get a feeling for decision boundaries produced by LDA/QDA/NB

We will now visualize how well different learners classify the three-class `mlbench::mlbench.cassini` data set.

- Generate 1000 points from `cassini` using R or import `cassini_data.csv` in Python.
- Then, perturb the `x.2` dimension with Gaussian noise (mean 0, standard deviation 0.5), and consider the classifiers already introduced in the lecture:
  - LDA (Linear Discriminant Analysis),
  - QDA (Quadratic Discriminant Analysis), and
  - Naive Bayes.

Plot the learners' decision boundaries. Can you spot differences in separation ability?

(Note that logistic regression cannot handle more than two classes and is therefore not listed here.)