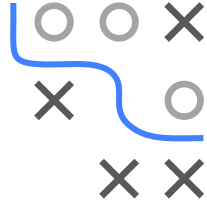
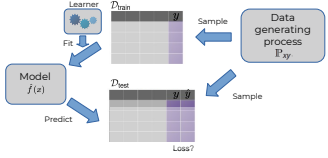
# Introduction to Machine Learning



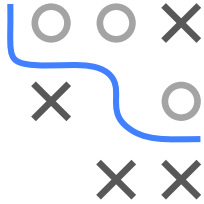
## Evaluation: In a Nutshell



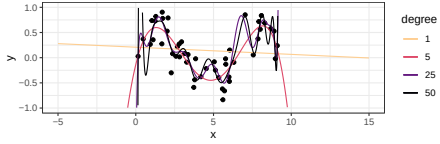

### Learning goals

- Understand what the Generalization Error is
- Get an overview on how we evaluate performance of learners
- Learn about some evaluation metrics
- Understand why we do resampling

# EVALUATING A MODEL

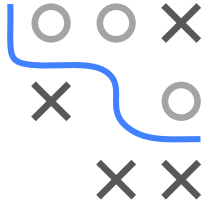


- We have seen how to train models that are optimal in some sense, relative to other possible models. However, how can we assess how good they actually are, in absolute numbers?
- Idea: Use risk $\sum_{(\mathbf{x},y)\in\mathcal{D}_{\text{train}}} \left[ L\left(y, \hat{f}(\mathbf{x})\right) \right]$ after training.
- Problem: This value can be very optimistic.
- Example: Overfitting of a polynomial regression.

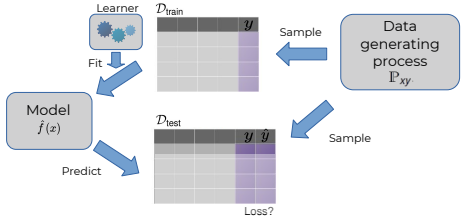



- Degree 50 will result in lowest training loss, however, degree 5 seems to be the "best" model.
- "Best" means that using new data, this model will probably produce the most meaningful predictions.

# GENERALIZATION ERROR



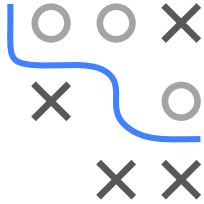
- In other words, the "best" model will generalize well and have a low **G**eneralization **E**rror.
- Formally, for a fixed model, the GE can be expressed via: $\mathrm{GE}\left(\hat{f}, L\right) := \mathbb{E}\left[L\left(y, \hat{f}(\mathbf{x})\right)\right]$,
- i.e., "what is the expected loss for a new observation?"
- Ideally, the GE should be estimated with new, unseen data.
- Usually, we have no access to new **unseen** data, though.
- Thus, we divide our data set manually into $\mathcal{D}_{\text{train}}$ and $\mathcal{D}_{\text{test}}$ and use the latter to estimate the GE via some metric $\rho()$.



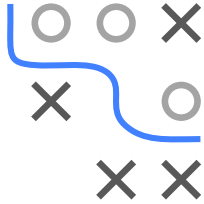
# METRICS

But what is a good metric $\rho()$?

- While we can always use the (inner) loss function that we trained the model on as outer loss to construct a metric $\rho()$, this may not always be ideal.
- For both, classification and regression there is a large variety of evaluation metrics, of which we will just cover a fraction.



# METRICS FOR CLASSIFICATION



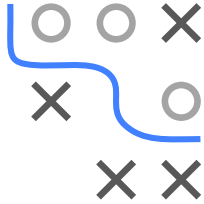
Commonly used evaluation metrics include:

- Accuracy:
  - $\rho_{ACC} = \frac{1}{m} \sum_{i=1}^{m} [y^{(i)} = \hat{y}^{(i)}] \in [0, 1]$.
  - "Proportion of correctly classified observations."
- Misclassification error (MCE):
  - $\rho_{MCE} = \frac{1}{m} \sum_{i=1}^{m} [y^{(i)} \neq \hat{y}^{(i)}] \in [0, 1]$.
  - "Proportion of incorrectly classified observations."
- Brier Score:
  - $\rho_{BS} = \frac{1}{m} \sum_{i=1}^{m} \left(\hat{\pi}^{(i)} - y^{(i)}\right)^2$
  - "Squared error btw. predicted probability and actual label."
- Log-loss:
  - $\rho_{LL} = \frac{1}{m} \sum_{i=1}^{m} \left(-y^{(i)} \log\left(\hat{\pi}^{(i)}\right) - \left(1 - y^{(i)}\right) \log\left(1 - \hat{\pi}^{(i)}\right)\right)$.
  - "Distance of predicted and actual label distribution."

The probabilistic metrics, Brier Score and Log-Loss, penalize false confidence, i.e. predicting the wrong label with high probability, heavily.

# METRICS FOR CLASSIFICATION

For hard-label classification, the confusion matrix is a useful tool:



| | | **True Class** $y$ | |
|---|---|---|---|
| | | $+$ | $-$ |
| **Pred.** $\hat{y}$ | $+$ | True Positive (TP) | False Positive (FP) |
| | $-$ | False Negative (FN) | True Negative (TN) |

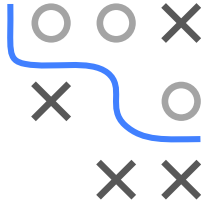
From this matrix a variety of evaluation metrics, including precision and recall, can be computed.

$$Precision = \frac{TP}{TP + FP}$$

$$Recall = \frac{TP}{TP + FN}$$

# METRICS FOR CLASSIFICATION

- Other frequently used metrics like the False Negative Rate FPR can also be derived from the confusion matrix.
- The confusion matrix below covers many of these.
- Many of these metrics may also go with different names.



| | | True condition | | | |
|---|---|---|---|---|---|
| | Total population | Condition positive | Condition negative | Prevalence = $\frac{\sum \text{Condition positive}}{\sum \text{Total population}}$ | Accuracy (ACC) = $\frac{\sum \text{True positive} + \sum \text{True negative}}{\sum \text{Total population}}$ |
| Predicted condition | Predicted condition positive | **True positive**, Power | **False positive**, Type I error | Positive predictive value (PPV), Precision = $\frac{\sum \text{True positive}}{\sum \text{Predicted condition positive}}$ | False discovery rate (FDR) = $\frac{\sum \text{False positive}}{\sum \text{Predicted condition positive}}$ |
| | Predicted condition negative | **False negative**, Type II error | **True negative** | False omission rate (FOR) = $\frac{\sum \text{False negative}}{\sum \text{Predicted condition negative}}$ | Negative predictive value (NPV) = $\frac{\sum \text{True negative}}{\sum \text{Predicted condition negative}}$ |
| | | True positive rate (TPR), Recall, Sensitivity, probability of detection = $\frac{\sum \text{True positive}}{\sum \text{Condition positive}}$ | False positive rate (FPR), Fall-out, probability of false alarm = $\frac{\sum \text{False positive}}{\sum \text{Condition negative}}$ | Positive likelihood ratio (LR+) = $\frac{\text{TPR}}{\text{FPR}}$ | Diagnostic odds ratio (DOR) = $\frac{\text{LR+}}{\text{LR−}}$ |
| | | False negative rate (FNR), Miss rate = $\frac{\sum \text{False negative}}{\sum \text{Condition positive}}$ | Specificity (SPC), Selectivity, True negative rate (TNR) = $\frac{\sum \text{True negative}}{\sum \text{Condition negative}}$ | Negative likelihood ratio (LR−) = $\frac{\text{FNR}}{\text{TNR}}$ | |

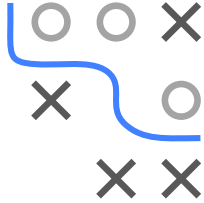
$F_1$ score = $\frac{1}{\frac{1}{2}\left(\frac{1}{\text{Recall}} + \frac{1}{\text{Precision}}\right)}$

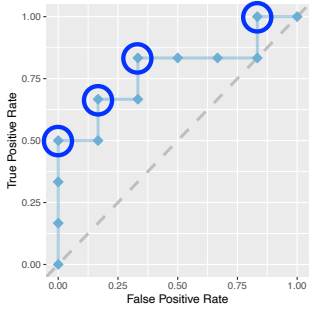
▸ Clickable version/picture source

▸ Interactive diagram

# ROC-CURVE

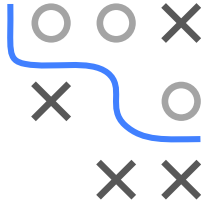
- The ROC-Curve allows to evaluate binary classifiers beyond single metrics. It compares classifiers using their TPR and FPR, for different thresholds.
- We aim to identify good thresholds that dominate others.
- The area under this curve (AUC) can also be used as metric.





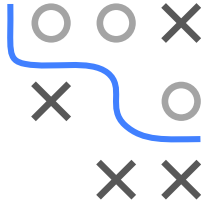
# METRICS FOR REGRESSION

Commonly used evaluation metrics include:



- Sum of Squared Errors (SSE): $\rho_{SSE} = \sum_{i=1}^{m} (y^{(i)} - \hat{y}^{(i)})^2$
- Mean Squared Error (MSE): $\rho_{MSE} = \frac{1}{m} \sum_{i=1}^{m} SSE$
- Root Mean Squared Error (RMSE): $\rho_{RMSE} = \sqrt{MSE}$
- R-Squared: $\rho_{R^2} = 1 - \frac{\sum_{i=1}^{m} (y^{(i)} - \hat{y}^{(i)})^2}{\sum_{i=1}^{m} (y^{(i)} - \bar{y})^2}$
- Mean Absolute Error (MAE):
  $\rho_{MAE} = \frac{1}{m} \sum_{i=1}^{m} |y^{(i)} - \hat{y}^{(i)}| \in [0; \infty)$

# IMPROVING ESTIMATION OF GE



We can estimate the GE with one test data set via:

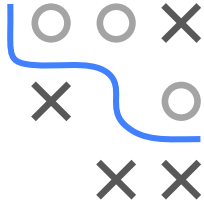
$$\widehat{\mathrm{GE}}(\hat{f}, L) := \frac{1}{m} \sum_{(\mathbf{x},y)\in\mathcal{D}_{\text{test}}} \left[ L\left(y, \hat{f}(\mathbf{x})\right) \right],$$

i.e. we compute the selected metric $L\left(y, \hat{f}(\mathbf{x})\right)$ for each observation in the test set and compute the mean.

This will give an appropriate estimate for the GE. However, with only a few test observations (small $m$), this estimate will be unstable or, in other words, have high variance. We have two options to decrease it:

- Increase $m$.
- Compute $\widehat{\mathrm{GE}}(\hat{f}, L)$ for multiple test sets and aggregate them.

# RESAMPLING



As we do not have access to infinite data and increasing $m$ will mean a reduction of the number of training observations, aggregation over $B$ sets is the preferred option:

$$\mathcal{J} = ((J_{\text{train},1}, J_{\text{test},1}), \ldots, (J_{\text{train},B}, J_{\text{test},B})) \,.$$

We compute $\widehat{\text{GE}}(\hat{f}, L)$ for each set and aggregate the estimates.
These $B$ distinct sets are generated through **resampling**.

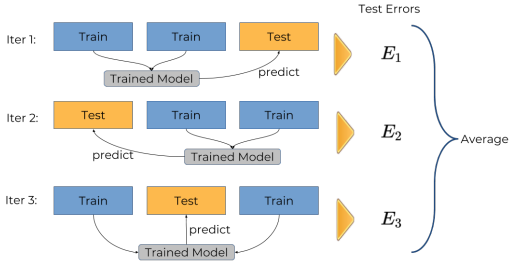
There exist a number of well-established resampling strategies:

- (Repeated) Hold-out / Subsampling
- Cross validation
- Bootstrap

# RESAMPLING

All methods aim to generate the train-test splits $\mathcal{J}$ by splitting the full data set repeatedly. The model is trained on the respective train set and evaluated on the test set.

**Example:** 3-fold cross validation




In order to robustify performance estimates, we can repeat resampling.