

**Solution 1:**

- 1) The generalization error is the expected (future) performance of a learner  $\mathcal{I}$  trained on  $n_{\text{train}}$  observations with performance measure  $\rho$ . As any such performance estimate depends on the concrete sampling of  $\mathcal{D}_{\text{test}}$  from  $\mathbb{P}_{xy}$ , we are interested in the limit of this expectation value, as  $n_{\text{test}} \rightarrow \infty$ .
- 2) One samples  $\mathcal{D}_{\text{train}}$  of size  $n_{\text{train}} = 100$  and  $\mathcal{D}_{\text{test}}$  of size  $n_{\text{test}}$   $K$  times from  $\mathbb{P}_{xy}$  (independently). Each time, the learner  $\mathcal{I}$  is trained on  $\mathcal{D}_{\text{train},k}$ , and the respective performance  $\rho$  is evaluated on  $\mathcal{D}_{\text{test},k}$ . For  $K, n_{\text{test}} \rightarrow \infty$ , the average performance  $\frac{1}{K} \sum_{k=1}^K \rho(\mathbf{y}_{J_{\text{test},k}}, \mathbf{F}_{J_{\text{test},k}, \mathcal{I}(\mathcal{D}_{\text{train},k})})$  converges to  $\text{GE}(\mathcal{I}, n_{\text{train}} = 100, \rho)$ .
- 3) As  $n_{\text{train}}$  must be smaller than  $n$ , the estimator is a pessimistically biased estimator of  $\text{GE}(\mathcal{I}, n, \rho)$ , as we are not using all available data for training. In the context of regression tasks and performance measures MSE or MAE, pessimistic bias means:

$$\mathbb{E} \left[ \widehat{\text{GE}}_{J_{\text{train}}, J_{\text{test}}}(\mathcal{I}, |J_{\text{train}}|, \rho) \right] > \text{GE}(\mathcal{I}, n, \rho) \quad (1)$$

- 4) If one chooses a large  $n_{\text{train}}$ ,  $n_{\text{test}}$  is small, and the estimator has a large variance.