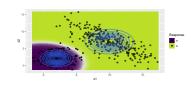
Introduction to Machine Learning

Classification: Naive Bayes



Learning goals

- Understand the idea of Naive Bayes
- Understand in which sense Naive Bayes is a special QDA model



NAIVE BAYES CLASSIFIER

NB is a generative multiclass technique. Remember: We use Bayes' theorem and only need $p(\mathbf{x}|y=k)$ to compute the posterior as:

$$\pi_k(\mathbf{x}) = \mathbb{P}(y = k \mid \mathbf{x}) = \frac{\mathbb{P}(\mathbf{x}|y = k)\mathbb{P}(y = k)}{\mathbb{P}(\mathbf{x})} = \frac{\rho(\mathbf{x}|y = k)\pi_k}{\sum\limits_{j=1}^g \rho(\mathbf{x}|y = j)\pi_j}$$



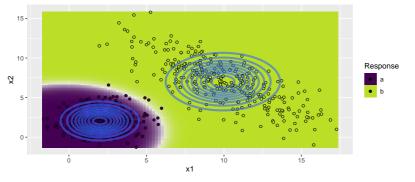
NB is based on a simple **conditional independence assumption**: the features are conditionally independent given class *y*.

$$p(\mathbf{x}|y=k) = p((x_1, x_2, ..., x_p)|y=k) = \prod_{j=1}^p p(x_j|y=k).$$

So we only need to specify and estimate the distribution $p(x_j|y=k)$, which is considerably simpler as this is univariate.

NB: NUMERICAL FEATURES

We use a univariate Gaussian for $p(x_j|y=k)$, and estimate (μ_j, σ_j^2) in the standard manner. Because of $p(\mathbf{x}|y=k) = \prod_{j=1}^p p(x_j|y=k)$, the joint conditional density is Gaussian with diagonal but non-isotropic covariance structure, and potentially different across classes. Hence, NB is a (specific) QDA model, with quadratic decision boundary.





NB: CATEGORICAL FEATURES

We use a categorical distribution for $p(x_j|y=k)$ and estimate the probabilities p_{kjm} that, in class k, our j-th feature has value m, $x_j=m$, simply by counting the frequencies.

$$p(x_j|y=k)=\prod_{m}p_{kjm}^{[x_j=m]}$$

Because of the simple conditional independence structure it is also very easy to deal with mixed numerical / categorical feature spaces.



LAPLACE SMOOTHING

If a given class and feature value never occur together in the training data, then the frequency-based probability estimate will be zero.

This is problematic because it will wipe out all information in the other probabilities when they are multiplied.

A simple numerical correction is to set these zero probabilities to a small value to regularize against this case.



NAIVE BAYES: APPLICATION AS SPAM FILTER

- In the late 90s, Naive Bayes became popular for e-mail spam filter programs
- Word counts were used as features to detect spam mails (e.g., "Viagra" often occurs in spam mail)
- Independence assumption implies: occurrence of two words in mail is not correlated
- Seems naive ("Viagra" more likely to occur in context with "Buy now" than "flower"), but leads to less required parameters and therefore better generalization, and often works well in practice.

