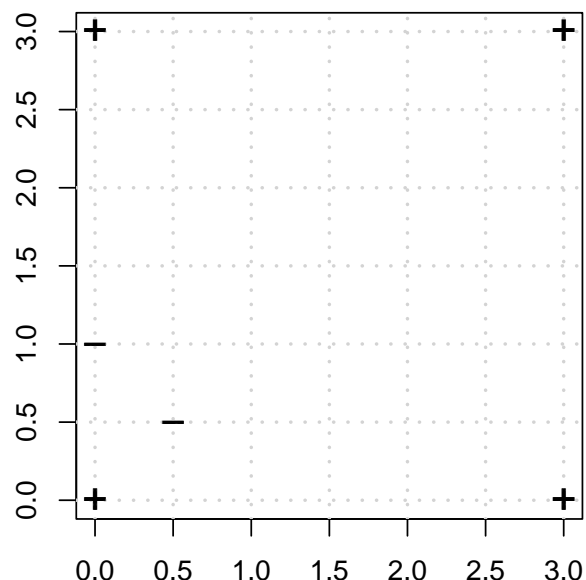


### Exercise 1: SVM – Support Vectors and Separating Hyperplane

The primal optimization problem for the two-class soft margin SVM classification is given by

$$\begin{aligned} \min_{\theta, \theta_0, \zeta^{(i)}} \quad & \frac{1}{2} \|\theta\|^2 + C \sum_{i=1}^n \zeta^{(i)} \\ \text{s.t. :} \quad & y^{(i)} (\theta^\top \mathbf{x}^{(i)} + \theta_0) \geq 1 - \zeta^{(i)}, \\ & \zeta^{(i)} \geq 0, \quad \forall i = 1, \dots, n. \end{aligned}$$



- Add the decision boundary to the figure for  $\hat{\theta} = (1, 1)^T, \hat{\theta}_0 = -2$ . (NB: This is the approximate optimum for  $C = 10$ )
- Identify the coordinates of the support vector(s) and compute the values of their slack variables  $\zeta^{(i)}$ .
- Compute the Euclidean distance of the non-margin-violating support vector(s) (i.e. support vectors that are located on the margin hyperplanes) to the decision boundary.
- What needs to be changed in the plot such that a hard margin SVM results into the same decision boundary?

### Exercise 2: SVM – Optimization

Write your own stochastic subgradient descent routine to solve the soft-margin SVM in the primal formulation.

Hints:

- Use the regularized-empirical-risk-minimization formulation, i.e., an optimization criterion without constraints.
- No kernels, just a linear SVM.
- Compare your implementation with an existing implementation (e.g., `kernlab` in R). Are your results similar? Note that you might have to switch off the automatic data scaling in the already existing implementation.