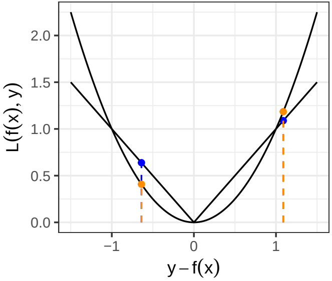
Introduction to Machine Learning

Supervised Regression: Linear Models with $L1$ Loss

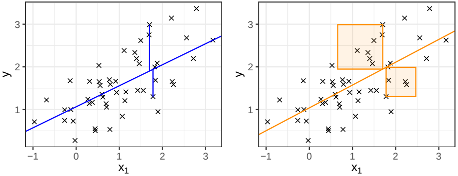


Learning goals

- Understand difference between $L1$ and $L2$ regression
- See how choice of loss affects optimization & robustness

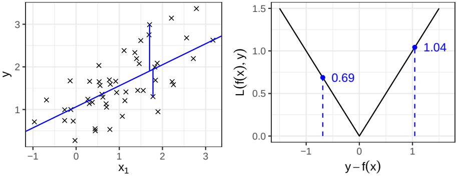
ABSOLUTE LOSS

- $L2$ regression minimizes quadratic residuals – wouldn't **absolute** residuals seem more natural?

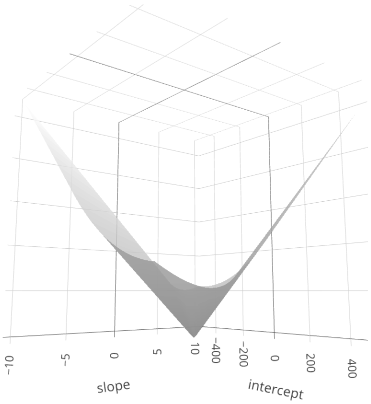


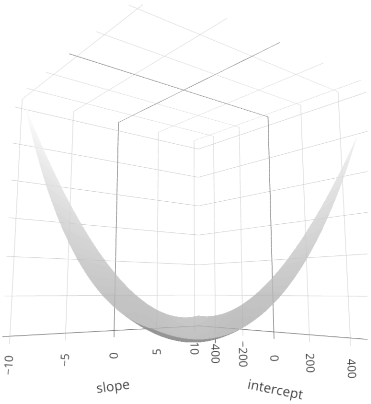
- $L1$ **loss / absolute error / least absolute deviation (LAD)**

$$L\left(y, f(\mathbf{x})\right) = \left|y - f(\mathbf{x})\right|$$



$L1$ VS $L2$ – LOSS SURFACE

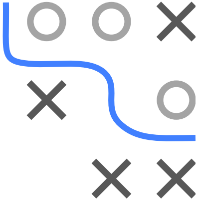




$L1$ loss (left) harder to optimize than $L2$ loss (right)

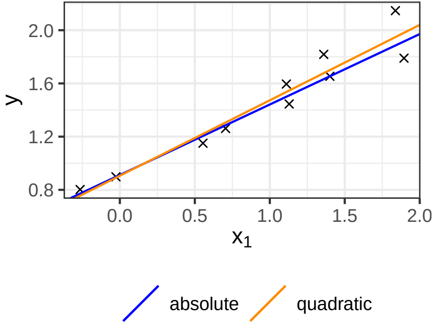
- Convex but **not differentiable** in $y - f(\mathbf{x}) = 0$
- No analytical solution

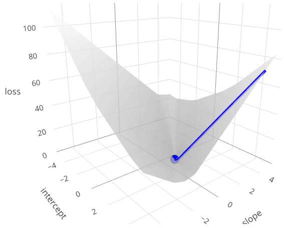
$L1$ VS $L2$ – ESTIMATED PARAMETERS

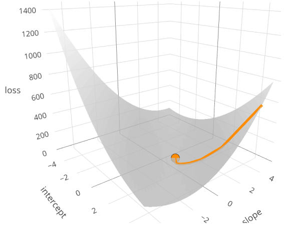


- Results of $L1$ and $L2$ regression often not that different
- Simulated data: $y^{(i)} = 1 + 0.5x_1^{(i)} + \epsilon^{(i)}, \quad \epsilon^{(i)} \overset{i.i.d}{\sim} \mathcal{N}(0, 0.01)$

	intercept	slope
$L1$	0.91	0.53
$L2$	0.91	0.57

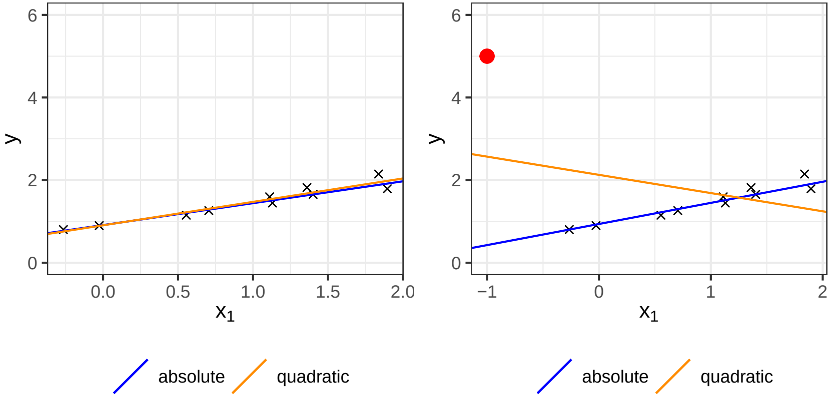






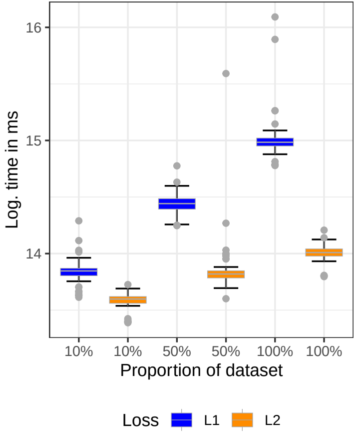
$L1$ VS $L2$ – ROBUSTNESS

- $L2$ quadratic in residuals $\leadsto$ outlying points carry lots of weight
- E.g., $3\times$ residual $\Rightarrow 9\times$ loss contribution
- $L1$ more **robust** in presence of outliers (example ctd.):



$L1$ VS $L2$ – OPTIMIZATION COST

- Real-world $\texttt{weather}$ problem $\rightsquigarrow$ predict mean temperature
- Compare **time** to fit $L1$ ($\texttt{quantreg::rq()}$) vs $L2$ ($\texttt{lm::lm()}$) for different dataset proportions (repeat 50$\times$)



Loss

	Fitted: $L1$	Fitted: $L2$
Total $L1$ loss	8.98×10^4	8.99×10^4
Total $L2$ loss	5.83×10^6	5.81×10^6

Estimated coefficients

x_j	$L1$: $\hat{\theta}_j$	$L2$: $\hat{\theta}_j$
Max_temperature	0.553	0.563
Min_temperature	0.441	0.427
Visibility	0.026	0.041
Wind_speed	0.002	0.010
Max_wind_speed	−0.026	−0.039
(Intercept)	−0.380	−0.102

$L1$ **slower** to optimize!