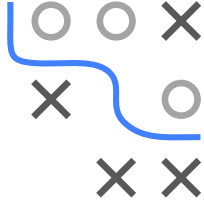
# Introduction to Machine Learning



# Evaluation: Multi-Class AUC

AUC(pos|neg) = AUC(1|3)

| | Y | $\hat{\pi}_1$ | $\hat{\pi}_2$ | $\hat{\pi}_3$ |
|---|---|---|---|---|
| pos | 1 | 0.7 | 0.2 | 0.1 |
| pos | 1 | 0.5 | 0.3 | 0.2 |
| | 2 | 0.3 | 0.5 | 0.2 |
| | 2 | 0.4 | 0.5 | 0.1 |
| neg | 3 | 0.6 | 0.1 | 0.3 |
| neg | 3 | 0.1 | 0.1 | 0.8 |

AUC(pos|neg) = AUC(3|1)

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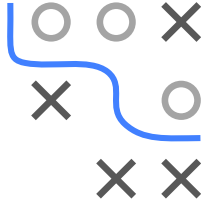
**Learning goals**

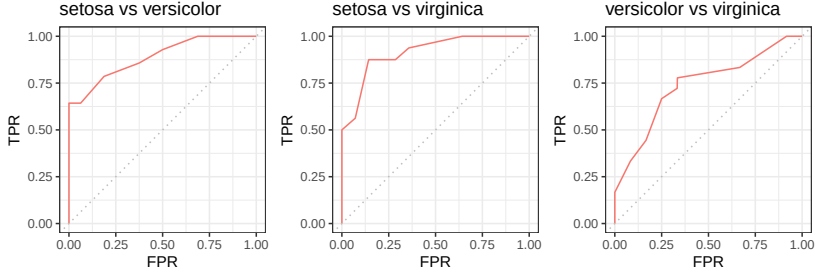
- Understand that generalizing AUC to multi-class is not trivial
- Learn how multi-class AUC can be derived

# MULTI-CLASS AUC

- AUC and other ROC metrics for binary classification
- Different ways to estimate **multi-class AUC**
- Often based on aggregated binary AUCs:
  e.g. 1-vs-1 or 1-vs-rest

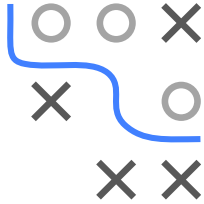


**Example:** 1-vs-1 on iris



# MULTI-CLASS AUC

- Def AUC($k \mid \ell$) for classes $k$ (pos) and $\ell$ (neg)
- Compute AUC: Subset preds to rows of true $k$ and $\ell$, use $\hat{\pi}_k$
- Interprete: Prob that random member of $\ell$ has a lower prob to belong to class $k$ than random member of class $k$.



**Example:** AUC(3|1) with $g = 3$ classes

AUC(pos|neg) = AUC(3|1)

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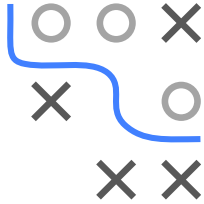
1. Subset pred rows to true classes 1 and 3
2. Use $k = 3$ as pos and $\ell = 1$ as neg class
3. Compute standard AUC with $\hat{\pi}_3$ as scores
4. AUC(3|1) = 1:
   all pos have higher $\hat{\pi}_3$ than negs

# MULTI-CLASS AUC

- For binary classes: always $\text{AUC}(1|0) = \text{AUC}(0|1)$
- For multi-class usually: $\text{AUC}(k \mid \ell) \neq \text{AUC}(\ell \mid k)$
- **Example** with $g = 3$ where $\text{AUC}(1|3) \neq \text{AUC}(3|1)$:
  - $\text{AUC}(3|1) = 1$ (RHS) as before
  - $\text{AUC}(1|3) \neq 1$ (LHS)



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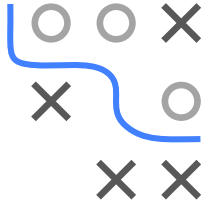
# MULTI-CLASS AUC

Hand and Till (2001) proposed to avg AUC via **1-vs-1**:

- For all class pairs, compute AUC($k \mid \ell$).

$$\text{AUC}_{MC} = \frac{1}{g(g-1)} \sum_{k \neq \ell} \text{AUC}(k|\ell) \in [0, 1].$$



**Comments**:

- Other defs use **1-vs-rest** and need to avg only $g$ AUC values
- 1-vs-rest creates imbal classes even if orig classes are balanced
- Imbalanced classes can be considered by weighting individual AUC values with class priors [Ferri et al. (2003)]