Exercise 1: HRO

Throughout the lecture, we will frequently use the `R` package `mlr3`, resp. the `Python` package `sklearn`, and its descendants, providing an integrated ecosystem for all common machine learning tasks. Let's recap the HRO principle and see how it is reflected in either `mlr3` or `sklearn`. An overview of the most important objects and their usage, illustrated with numerous examples, can be found at `https://mlr3book.mlr-org.com/chapters/chapter2/data_and_basic_modeling.html` and `https://scikit-learn.org/stable/index.html`.

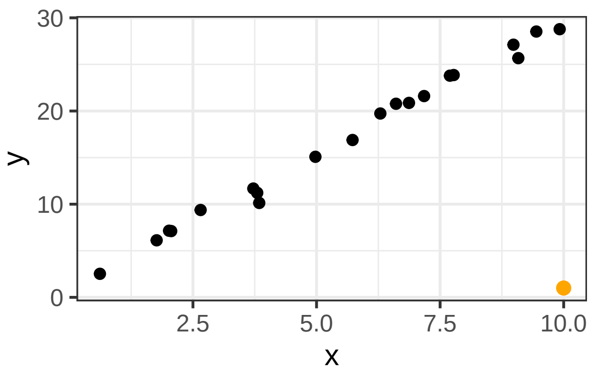
a) How are the key concepts (i.e., hypothesis space, risk and optimization) you learned about in the lecture videos implemented?

b) Have a look at `mlr3::tsk("iris")` / `from sklearn.datasets import load_iris`. What attributes does this object store?

c) Instantiate a regression tree learner (`lrn("regr.rpart")` / `DecisionTreeRegressor`). What are the different settings for this learner?
(`R` Hint: `mlr3::mlr_learners$keys()` shows all available learners.)
(`Python` Hint: Use `get_params()` to see all available settings.)

Exercise 2: Loss Functions for Regression Tasks

In this exercise, we will examine loss functions for regression tasks somewhat more in depth.

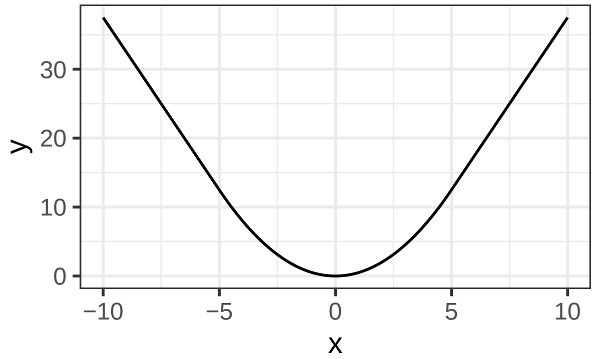


a) Consider the above linear regression task. How will the model parameters be affected by adding the new outlier point (orange) if you use

   i) $L1$ loss

   ii) $L2$ loss

   in the empirical risk? (You do not need to actually compute the parameter values.)



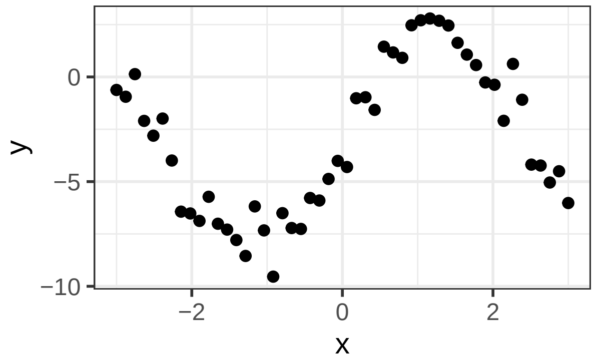
b) The second plot visualizes another loss function popular in regression tasks, the so-called *Huber loss* (depending on $\epsilon > 0$; here: $\epsilon = 5$). Describe how the Huber loss deals with residuals as compared to $L1$ and $L2$ loss. Can you guess its definition?

Exercise 3: Polynomial Regression

Assume the following (noisy) data-generating process from which we have observed 50 realizations:

$$y = -3 + 5 \cdot \sin(0.4\pi x) + \epsilon$$

with $\epsilon \sim \mathcal{N}(0, 1)$.



a) We decide to model the data with a cubic polynomial (including intercept term). State the corresponding hypothesis space.

b) State the empirical risk w.r.t. $\boldsymbol{\theta}$ for a member of the hypothesis space. Use $L2$ loss and be as explicit as possible.

c) We can minimize this risk using gradient descent. Derive the gradient of the empirical risk w.r.t $\boldsymbol{\theta}$. [Only for lecture group A]

d) Using the result for the gradient, explain how to update the current parameter $\boldsymbol{\theta}^{[t]}$ in a step of gradient descent. [Only for lecture group A]

e) You will not be able to fit the data perfectly with a cubic polynomial. Describe the advantages and disadvantages that a more flexible model class would have. Would you opt for a more flexible learner?