Exercise 1: Naive Bayes

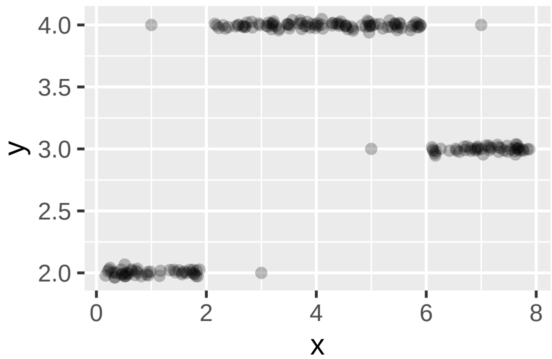
You are given the following table with the target variable `Banana`:

ID	Color	Form	Origin	Banana
1	yellow	oblong	imported	yes
2	yellow	round	domestic	no
3	yellow	oblong	imported	no
4	brown	oblong	imported	yes
5	brown	round	domestic	no
6	green	round	imported	yes
7	green	oblong	domestic	no
8	red	round	imported	no

a) We want to use a Naive Bayes classifier to predict whether a new fruit is a `Banana` or not. Estimate the posterior probability $\hat{\pi}(\mathbf{x}_*)$ for a new observation $\mathbf{x}_* = (\text{yellow}, \text{round}, \text{imported})$. How would you classify the object?

b) Assume you have an additional feature `Length` that measures the length in cm. Describe in 1-2 sentences how you would handle this numeric feature with Naive Bayes.

Exercise 2: Discriminant Analysis



The above plot shows $\mathcal{D} = \left(\left(\mathbf{x}^{(1)}, y^{(1)}\right), \dots, \left(\mathbf{x}^{(n)}, y^{(n)}\right)\right)$, a data set with $n = 200$ observations of a continuous target variable y and a continuous, 1-dimensional feature variable $\mathbf{x}$. In the following, we aim at predicting y with a machine learning model that takes $\mathbf{x}$ as input.

a) To prepare the data for classification, we categorize the target variable y in 3 classes and call the transformed target variable z, as follows:

$$z^{(i)} = \begin{cases} 1, & y^{(i)} \in (-\infty, 2.5] \\ 2, & y^{(i)} \in (2.5, 3.5] \\ 3, & y^{(i)} \in (3.5, \infty) \end{cases}$$

Now we can apply quadratic discriminant analysis (QDA):

i) Estimate the class means $\mu_k = \mathbb{E}(\mathbf{x}|z = k)$ for each of the three classes $k \in \{1, 2, 3\}$ visually from the plot. Do not overcomplicate this, a rough estimate is sufficient here.

ii) Make a plot that visualizes the different estimated densities per class.

iii) How would your plot from ii) change if we used linear discriminant analysis (LDA) instead of QDA? Explain your answer.

iv) Why is QDA preferable over LDA for this data?

b) Given are two new observations $\mathbf{x}_{*1} = -10$ and $\mathbf{x}_{*2} = 7$. State the prediction for QDA and explain how you arrive there.

Exercise 3: Decision Boundaries for `sklearn` Learners

We will now visualize how well different learners classify the three-class `cassini` data set. Import `cassini_data.csv`, perturb the `x.2` dimension with Gaussian noise (mean 0, standard deviation 0.5), and consider the classifiers already introduced in the lecture:

- LDA,
- QDA, and
- Naive Bayes.

Plot the learners' decision boundaries. Can you spot differences in separation ability?

(Note that logistic regression cannot handle more than two classes and is therefore not listed here.)