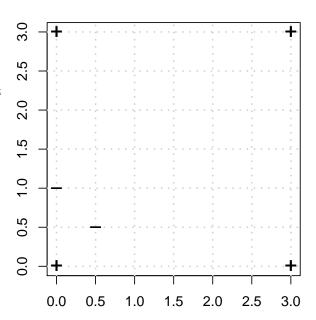
Exercise 1: SVM - Support Vectors and Separating Hyperplane

The primal optimization problem for the two-class soft margin SVM classification is given by

$$\min_{\substack{\theta, \theta_0, \mathbf{x}^{(i)}}} \quad \frac{1}{2} ||\theta||^2 + C \sum_{i=1}^n \zeta^{(i)} \\
\text{s.t.} : \quad y^{(i)} (\theta^\top \mathbf{x}^{(i)} + \theta_0) \ge 1 - \zeta^{(i)}, \\
\zeta^{(i)} \ge 0, \quad \forall i = 1, \dots, n.$$



- (a) Add the decision boundary to the figure for $\hat{\theta} = (1,1)^T$, $\hat{\theta}_0 = -2$.
- (b) Identify the coordinates of the support vector(s) and compute the values of their slack variables $\zeta^{(i)}$.
- (c) Compute the Euclidean distance of the non-margin-violating support vector(s) (i.e. support vectors that are located on the margin hyperplanes) to the decision boundary.
- (d) What needs to be changed in the plot such that a hard margin SVM results into the same decision boundary?

Exercise 2: SVM - Optimization

Write your own stochastic subgradient descent routine to solve the soft-margin SVM in the primal formulation.

Hints:

- Use the regularized-empirical-risk-minimization formulation, i.e., an optimization criterion without constraints.
- No kernels, just a linear SVM.
- Compare your implementation with an existing implementation (e.g., kernlab in R). Are your results similar? Note that you might have to switch off the automatic data scaling in the already existing implementation.