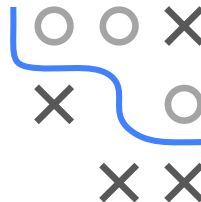
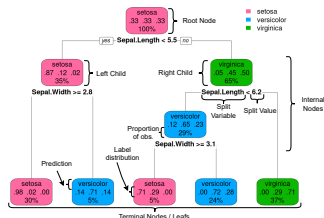
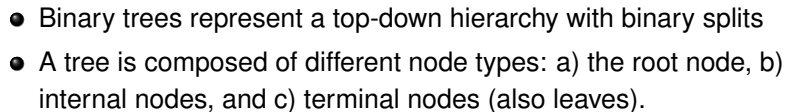


## Predictions with CART

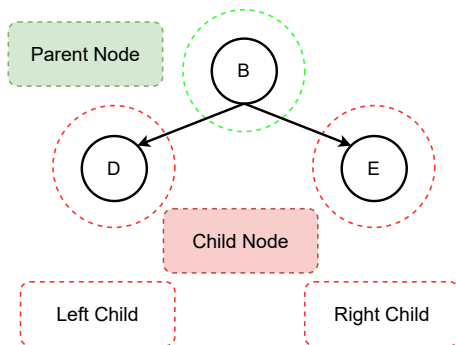


- Understand the basic structure of a tree model
- Understand that the basic idea of a tree model is the same for classification and regression
- Understand how the label of a new observation is predicted via CART
- Know hypothesis space of CART



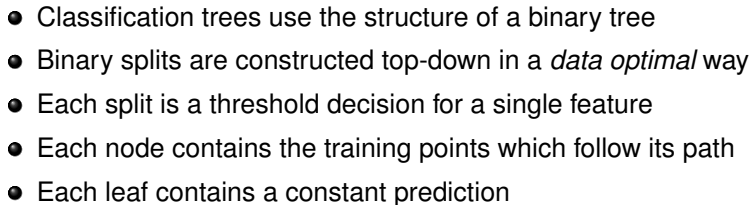


# BINARY TREES



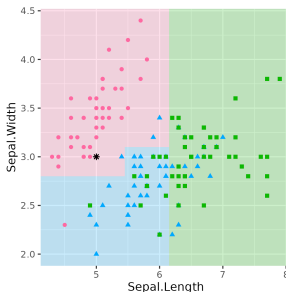
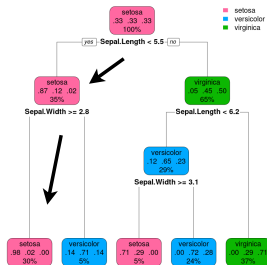
- Nodes have relative relationships, they can be:
  - Parent nodes
  - Child nodes
- Root nodes don't have parents – leaves don't have children

A 3x3 grid with a blue path starting at the top-left cell and ending at the bottom-right cell. The path moves right, then down, then right again. The cells containing the path are (1,1), (2,1), (2,2), and (3,3).



# CLASSIFICATION TREE MODEL AND PREDICTION

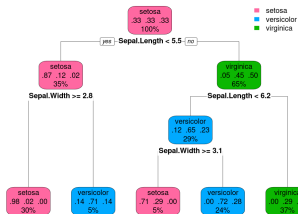
- When predicting new data (here\*: `Sepal.Length = 5`, `Sepal.Width = 3`) we use the learned split points and pass an observation through the tree
- Each observation is assigned to exactly one leaf
- Classification trees can make hard-label predictions (here: `setosa`) or predict probabilities (here: 0.98, 0.02, 0.00)



# CART AS A RULE BASED MODEL

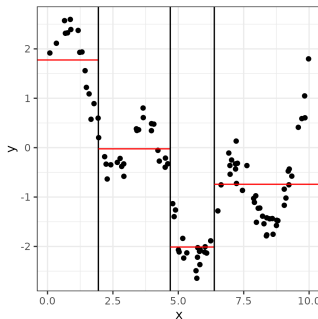
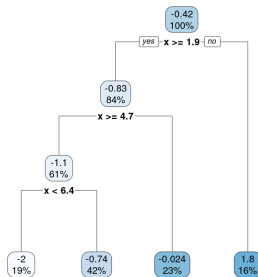
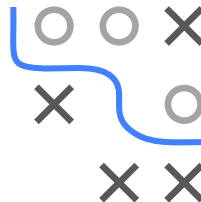
Leaf nodes can be expressed by a set of rules (left to right):

Hard label prediction	Label distribution	Sepal.Width	Sepal.Length
setosa	0.98, 0.02, 0.00	$\geq 2.8$	$< 5.5$
versicolor	0.14, 0.71, 0.14	$< 2.8$	$< 5.5$
setosa	0.71, 0.29, 0.00	$\geq 3.1$	$\geq 5.5 \text{ \& } < 6.2$
versicolor	0.00, 0.72, 0.28	$< 3.1$	$\geq 5.5 \text{ \& } < 6.2$
virginica	0.00, 0.29, 0.71	—	$\geq 6.2$



# REGRESSION TREE MODEL AND PREDICTION

- Works the same way as for classification
- But predictions in leaf nodes are a numerical scalar



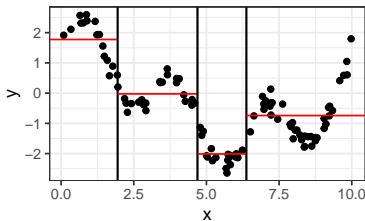
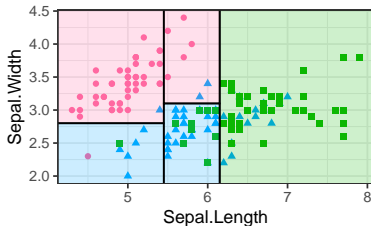
# TREE AS AN ADDITIVE MODEL

Trees divide the feature space  $\mathcal{X}$  into **rectangular regions**:

$$f(\mathbf{x}) = \sum_{m=1}^M c_m \mathbb{I}(\mathbf{x} \in Q_m),$$

where a tree with  $M$  leaf nodes defines  $M$  “rectangles”  $Q_m$ .

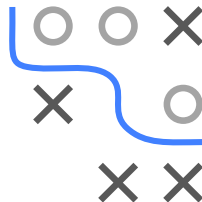
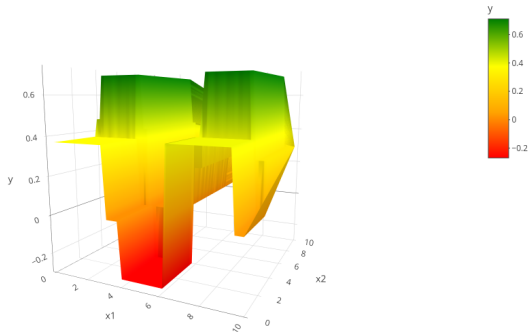
$c_m$  is the predicted numerical response, class label or class distribution in the respective leaf node.





# TREE AS AN ADDITIVE MODEL

A 2D regression example:



(For binary classification with probabilities, 2D surface looks similar.)