Exercise 1 – ML Basics

Introduction to Machine Learning

Exercise 1: Car price prediction

Learning goals

1) Translate real-world problem into ML concepts
2) Use proper mathematical notation for those concepts

Imagine you work at a second-hand car dealer and are tasked with finding for-sale vehicles your company can acquire at a reasonable price. You decide to address this challenge in a data-driven manner and develop a model that predicts adequate market prices (in EUR) from vehicles' properties.

Only for lecture group B

Characterize the task at hand: supervised or unsupervised? Regression or classification? Learning to explain or learning to predict? Justify your answers.

How would you set up your data? Name potential features along with their respective data type and state the target variable.

Assume now that you have data on vehicles' age (days), mileage (km), and price (EUR). Explicitly define the feature space $\mathcal{X}$ and target space $\mathcal{Y}$.

You choose to use a linear model (LM) for this task. The LM models the target as a linear function of the features with Gaussian error term.

State the hypothesis space for the corresponding model class. For this, assume the parameter vector θ to include the intercept coefficient.

Which parameters need to be learned? Define the corresponding parameter space Θ.

State the loss function for the i-th observation using $L2$ loss.

Now you need to optimize this risk to find the best parameters, and hence the best model, via empirical risk minimization. State the optimization problem formally and list the necessary steps to solve it.

Congratulations, you just designed your first machine learning project!

Exercise 2: Vector calculus

The whole exercise is only for lecture group A!

Learning goals

1. Understand how vector-valued functions work
2. Perform calculus on vectors and matrices

Consider the following function performing matrix-vector multiplication: $f(\mathbf{x}) = \mathbf{A}\mathbf{x}$, where $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{x} \in \mathbb{R}^{n \times 1}$.

What is the dimension of $f(\mathbf{x})$? Explicitly state the calculation for the i-th component of $f(\mathbf{x})$.

Now, consider the gradient (derivative generalized to multivariate functions) $\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}}$ (a.k.a. $\nabla_{\mathbf{x}} f(\mathbf{x})$).

What is the dimension of $\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}}$?

Compute $\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}}$.

Had trouble with this exercise?

- For the upcoming contents, you need to be able to handle **matrix-valued computations** (multiplication, transposition etc.) and also matrix-valued **calculus**.
- For more explanations and exercises, including a useful collection of rules for calculus, we recommend the book "Mathematics for Machine Learning" (https://mml-book.github.io/book/mml-book.pdf).