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# Using Partial Evaluation to Generate Compilers



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# Partial Evaluation – Introduction

- Optimization technique (Program Specialization)
  - Evaluate programs with partial input
- Paradigm connected to program generation, program analysis, …
- Tool for compiler generation

# $S_n^m$ - Theorem

- Given a function *f* with *n* + *m* arguments it is possible to construct a function *g* with n arguments such that $f(s_1, \ldots, s_m, d_1, \ldots, d_n) = g(d_1, \ldots, d_n)$
- Proven by S. Kleene (1943)
- Specialized functions could be slower than generic ones

# Program Specialization

- Given a **program** *f* with *n* + *m* arguments it is possible to construct a **program** *g* with n arguments such that $f(s_1, \ldots, s_m, d_1, \ldots, d_n) = g(d_1, \ldots, d_n)$

- Specialized programs can be optimized
  - Validation of arguments is no longer required
  - Static computations are possible
  - Parts of control flow are known
  - …

⇒ Partial Evaluators can optimize programs!

# Program Specialization

**Important:** Program Specialization (Partial Evaluation) **is not** Partial Application or Currying

```
/* Generic method */
def pow(base: Int, exp: Int): Int =
  if (exp == 0) return 1
  else return base * pow(base, exp - 1)

/* Specialized method via partial application */
def pow4(base: Int): Int =
    pow(base, 4)

/* Specialized method via partial evaluation */
def pow4(base: Int): Int =
    return base * base * base * base
```

# Execution & Programs

# Executing Programs

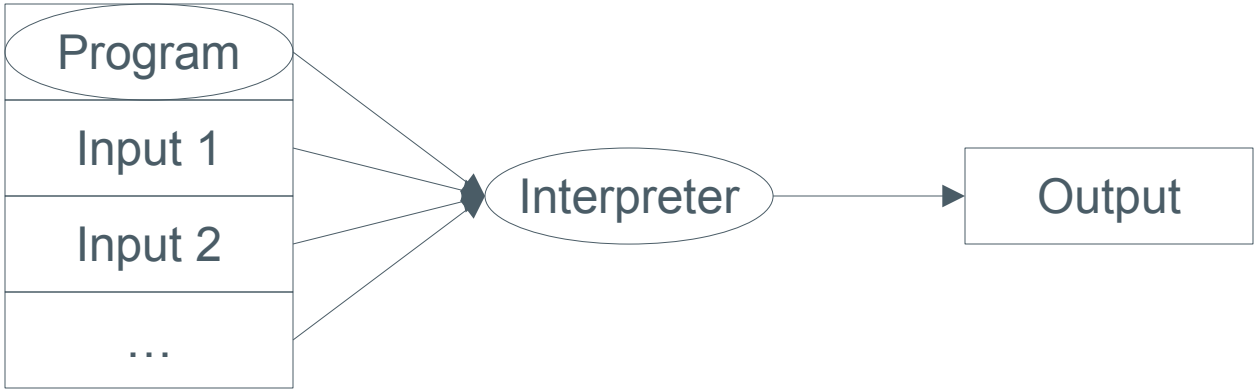
- Execution in one go
  - All inputs have to be provided
  - One output is produced

⇒ A single *computational stage*


Usually this is **not** how programs are executed!

# Interpreter

- Meaning is assigned and directly executed
  ⇒ One computational stage
- Slow: Analysis and execution share resources
- Easy to implement



```
/* abstract syntax for expressions */
sealed trait Exp
case class Number(value: Int) extends Exp
case class Variable(name: String) extends Exp
case class Comp(lhs: Exp, op: (Int, Int) => Boolean, rhs: Exp) extends Exp
case class Arith(lhs: Exp, op: (Int, Int) => Int, rhs: Exp) extends Exp

/* abstract syntax for statements */
sealed trait Stm
case class If(condition: Compare, ifTrue: Stm, ifFalse: Stm) extends Stm
case class Assign(name: String, value: Exp) extends Stm
case class Seq(first: Stm, second: Stm) extends Stm
```

```
sealed trait Exp {
    def eval(environment: Map[String, Int]): Int = this match {
      case Number(value) =>
        value

      case Variable(name) =>
        environment(name)

      case Comp(lhs, op, rhs) =>
        if (op(lhs.eval(environment), rhs.eval(environment))) 1 else 0

      case Arith(lhs, op, rhs) =>
        op(lhs.eval(environment), rhs.eval(environment))
    }
}
```

```
sealed trait Stm {
    def eval(environment: Map[String, Int]): Map[String, Int] = this match {
      case If(condition, ifTrue, ifFalse) =>
        if (condition.eval(environment) != 0) ifTrue.eval(environment)
        else ifFalse.eval(environment)

      case Assign(name, value) =>
        environment + (name -> value.eval(environment))

      case Seq(first, second) =>
        second.eval(first.eval(environment))
    }
}
```

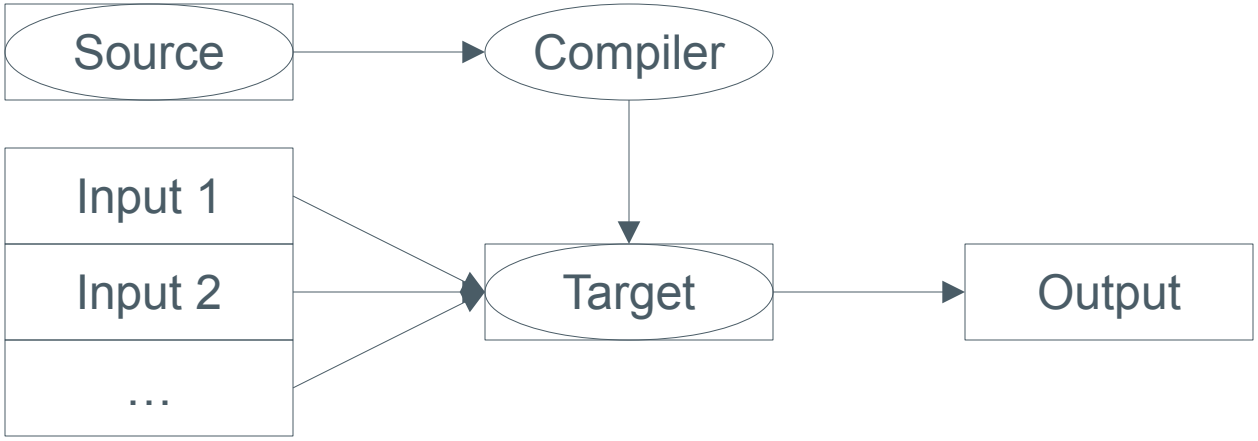
```
val program = If(Comp(                          // if (
    Arith(Variable("x"), _ * _, Number(2)),     //  (x * 2)
    _ > _, Number(12)),                         //  > 12 )
  Assign("result", Number(42)),                 // then { result := 42; }
  Assign("result", Number(12)))                 // else { result := 12; }

program.eval(Map("x" -> 6))                     // x -> 6, result -> 12

program.eval(Map("x" -> 7))                     // x -> 7, result -> 42
```

# Compiler

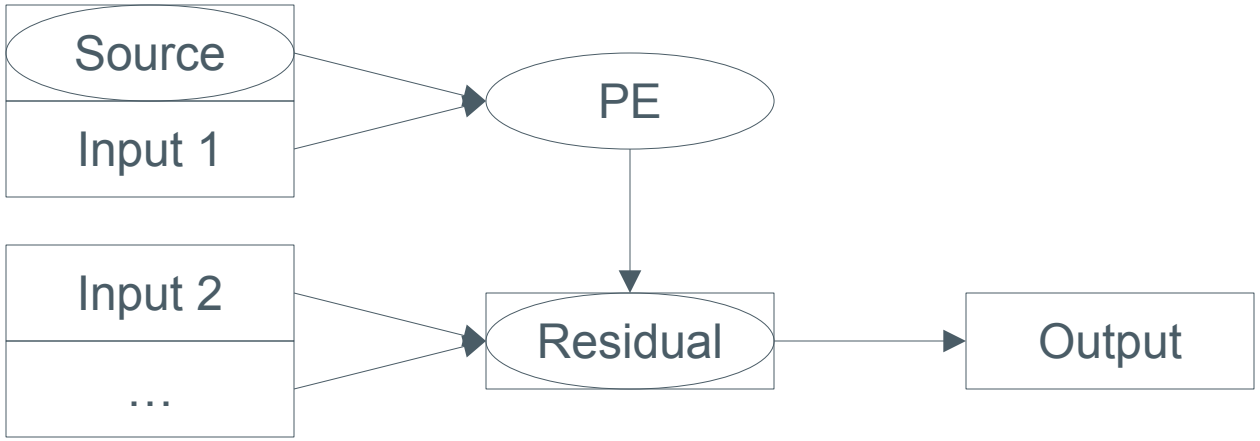
- Meaning is assigned and translated to another language
  ⇒ Two computational stages
- Faster: Analysis and execution are separated as different programs
- More complex implementation



# Partial Evaluator

- Mix of interpretation (execute now) and compilation (execute later)
- Generalizes creation of computational stages

**Operation:** Given program and fixed inputs, specialized program is generated



# How Do Partial Evaluators Work?

# Program Specialization as an Optimization Strategy

Three problems have to be solved:

1) Execute static computations

2) Defer dynamic computations

3) Distinguish between static and dynamic computations

# Defer Dynamic Computations

- Only parts of input are fixed/known
- Computations may depend on unknown input
- Partial Evaluator must generate a program containing these computations

  ⇒ The **residual program** performs computations at runtime

# Nothing really changes…

```
int example(int x, int y) {
  int result = 1;
  while (result < x) {
    result += result * 3 / x;
  }
  return result;
}
```

```
int example_2(int x /* int y = 2 */) {
  int result = 1;
  while (result < x) {
    result += result * 3 / x;
  }
  return result;
}
```

# … Except (Optionally) the Output Language!

```
int example(int x, int y) {
  int result = 1;
  while (result < x) {
    result += result * 3 / x;
  }
  return result;
}
```

```
def example_2(x /* y = 2 */):
  result = 1
  while (result < x):
    result += result * 3 // x

  return result
```

# Execute Static Computations

- Specialization as program optimization

- **What** can be optimized?
  - Expressions
  - Program Points

- **How** can we optimize?
  - Symbolic Computation
  - Unfolding of function calls
  - Program point specialization
  - … many more!

# Execute Static Computations – Symbolic Computation

- Computation depend on symbols and not (just) values
  - Arithmetic simplifications (simplification of expression structure)
  - Constant propagation
  - Constant folding
  - …
- Reduces / Removes computations
- Can spread static values

# Execute Static Computations – Unfolding of Calls

- Include body of (recursive) calls at call site
  - Known as *inlining* in most productive compilers today

- Allows further optimizations

**Disadvantages:**

- Increases code size

- Termination of optimization is not guaranteed

# Unfolding of Calls – Example 1

```
int pow(int base, int exponent) {
  if (exponent == 0) return 1;
  else return (base * pow(base, exponent - 1));
}
```

```c
int pow_3(int base, /* int exponent = 3 */) {
  if (3 == 0) return 1; // Branch is not executed!
  else return (base * pow(base, 3 - 1)); // Unfold!
}
```

```
int pow_3(int base, /* int exponent = 3 */) {
  return base *
       if (2 == 0) return 1;
       else return (base * pow(base, 2));
} // Unfold again! And again! And ...
```

```
int pow_3(int base, /* int exponent = 3 */) {
  return base *
         base *
           base *
             if (0 == 0) return 1; // Is 0 == 0? Yes!
             else return (base * pow(base, 0 - 1));
}
```

```
int pow_3(int base, /* int exponent = 3 */) {
  // Arithmetic simplification!
  return base * base * base * 1;
}
```

```
int pow_3(int base, /* int exponent = 3 */) {
  return base * base * base;
}
```

Partial Evaluation works and is a powerful optimization strategy!

# Unfolding of Calls – Example 2

```
def collatz: Int => Int = {
  case 1 => 0
  case n if n % 2 == 0 =>
    1 + collatz(n / 2)
  case n if n % 2 != 0 =>
    1 + collatz(3 * n + 1)
}
```

# Unfolding of Calls – Example 2

```
def collatz_1 = 0
def collatz_2 = 1
def collatz_3 = 7
def collatz_4 = 2
 ...
```

Partial Evaluation works … in these cases!

⇒ But we have to ***actively prevent*** infinite loops

# Execute Static Computations – Program Point Specialization

- Creation of specialized definitions

- Parts of a program occur multiple times with identical specialized inputs
  - Only one specialized definition is required
  - Specialized definition can be shared between instances

**Disadvantages:**

- Potential increase in code size (not as much as unfolding)

- Termination of optimization is not guaranteed

# Program Point Specialization – Example 1

```
def ack(m: Int, n: Int): Int =
  if (m == 0) n + 1
  else if (n == 0) ack(m - 1, 1)
  else ack(m - 1, ack(m, n - 1))
```

```
def ack_2(/* m: Int = 2 */ n: Int): Int =
 if (2 == 0) n + 1 // Is 2 == 0? No!
 else if (n == 0) ack(1, 1)
 else ack(1, ack(2, n - 1))

// Multiple recursive calls with m = 2 and m = 1
```

```
def ack_2(/* m: Int = 2 */ n: Int): Int =
  else if (n == 0) ack_1(1)
  else ack_1(ack_2(n - 1))

def ack_1(/* m: Int = 1 */ n: Int): Int =
  if (1 == 0) n + 1
  else if (n == 0) ack(0, 1)
  else ack(0, ack(1, n - 1))
```

```
def ack_2(/* m: Int = 2 */ n: Int): Int =
  else if (n == 0) ack_1(1)
  else ack_1(ack_2(n - 1))

def ack_1(/* m: Int = 1 */ n: Int): Int =
  if (1 == 0) n + 1
  else if (n == 0) ack_0(1)
  else ack_0(ack_1(n - 1))

def ack_0(/* m: Int = 0 */ n: Int): Int =
  if (0 == 0) n + 1 // Is 0 == 0? Yes!
  else if (n == 0) ack(-1, 1)
  else ack(-1, ack(0, n - 1))
```

```
def ack_2(/* m: Int = 2 */ n: Int): Int =
  else if (n == 0) ack_1(1)
  else ack_1(ack_2(n - 1))

def ack_1(/* m: Int = 1 */ n: Int): Int =
  if (1 == 0) n + 1
  else if (n == 0) ack_0(1)
  else ack_0(ack_1(n - 1))

def ack_0(/* m: Int = 0 */ n: Int): Int =
  if (0 == 0) n + 1
// ack_0 is not recursive! Unfold it!
```

```
def ack_2(/* m: Int = 2 */ n: Int): Int =
  else if (n == 0) ack_1(1)
  else ack_1(ack_2(n - 1))

def ack_1(/* m: Int = 1 */ n: Int): Int =
  if (1 == 0) n + 1
  else if (n == 0) 2
  else ack_1(n - 1) + 1
```

# Distinguish between static and dynamic computations

- Static computations can be performed by the Partial Evaluator
- Dynamic computations have to be included in the residual program
- **Division** between static and dynamic depends on input
- Two approaches for division:
  - Offline Partial evaluation
  - Online Partial evaluation

# Online Partial Evaluation

- Division is created “on the fly“ during specialization
- Fixed input is given (similar to environment in interpreter)
- Program is traversed, performing calculations and optimizations
  - Values are used to decide control flow, etc.
  - Usually yields better optimization

⇒ We performed *online* partial evaluation!

# Offline Partial Evaluation

- Division is created during separate analysis
  - Binding Time Analysis

- Annotations in source code decide which parts are static and dynamic
  - Inserted by programmer
  - Propagated, analyzed automatically

- Conservative approach
  - Depends on knowledge of what inputs are static/dynamic annotated
  - Does not utilize value of inputs

# Offline Partial Evaluation – Example

```
def ack(m: Int, n: Int): Int =
  if (m == 0) n + 1
  else if (n == 0) ack(m - 1, 1)
  else ack(m - 1, ack(m, n - 1))
```

Annotations mark dynamic computations

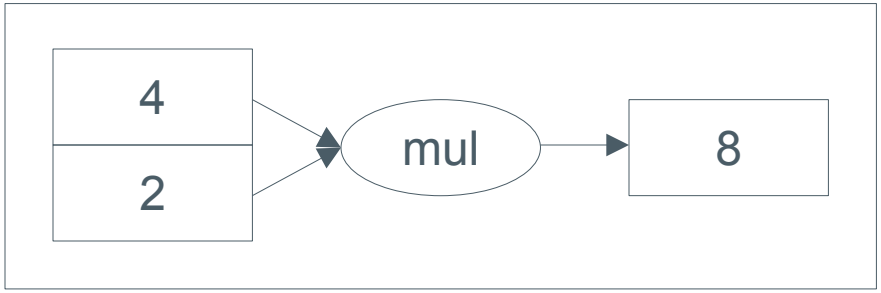
```
def ack_2(/* m = 2 */ n: Int): Int =
  if (n == 0) ack(1, 1)
  else ack(1, ack(2, n - 1))
```

Offline partial evaluation often yields less optimized results

# Multi-Stage Computation & Futamura Projections

# Computations as Equations

- The execution of programs can be described with equations
  - Programs are functions
  - Inputs are arguments to this function
  - Outputs are result of function




```
8 = ⟦mul⟧ (4, 2)
```

**Note:** Equality describes the output of both sides, not the way it is produced!

```
8 = ⟦mul⟧ (4, 2)
  = ⟦add⟧ (5, 3)
```

```
9 = ⟦collatz⟧ (12)
  = ⟦collatz⟧ (13)
  = ⟦collatz_12⟧ ()
```

```
output = ⟦example⟧ (x, y)
       = ⟦example⟧ (x, 2)
       = ⟦example_2⟧ (x)
```

$$
\begin{aligned}
\text{output} &= [\![\text{program}]\!]\ (\text{input}_1, \text{input}_2) \\
&= [\![\text{interpreter}]\!]\ (\text{program}, \text{input}_1, \text{input}_2) \\
&= [\![\ [\![\text{compiler}]\!]\ (\text{program})\ ]\!]\ (\text{input}_1, \text{input}_2)
\end{aligned}
$$

$$
\begin{aligned}
\text{output} &= [\![\text{program}]\!]\ (\text{input}_1, \text{input}_2) \\
&= [\![\ [\![\text{pe}]\!]\ (\text{program}, \text{input}_1)\ ]\!]\ (\text{input}_2) \\
&= [\![\ \text{program}_{\text{input1}}\ ]\!]\ (\text{input}_2)
\end{aligned}
$$

# Computations as Equations – Partial Evaluators

- Partial Evaluators can separate „function arguments“
  - Programs with two inputs become two programs with one input
  - Additional computational stages are introduced

- Equations show computational stages as nested ⟦ ⟧ brackets

- Semantic equality of programs can be shown independent of comp. stages

# Futamura Projections

- Yoshihiko Futamura presented projections in 1970s

- Partial Evaluation can not only optimize programs
  - Some combinations seemingly create new functionality

- In total: **3** Projections

- Links between *Partial Evaluation* and *Compiler Construction*

# First Futamura Projection

```
output = ⟦interpreter⟧ (program, input)
```

- An interpreter accepts multiple inputs

⇒ Partial Evaluator can specialize an Interpreter with respect to a source program!

# First Futamura Projection

```
output = ⟦interpreter⟧ (program, input)
       = ⟦⟦pe⟧ (interpreter, program)⟧ (input)
       =          ⟦target⟧             (input)
```

- Specialization of an Interpreter with respect to a program yields:

  **Compiled Program**

- Partial Evaluator can act as compiler!

```
val program = If(Comp(                              // if (
    Arith(Variable("x"), _ * _, Number(2)),         //  (x * 2)
    _ > _, Number(12)),                             //  > 12 )
  Assign("result", Number(42)),                     // then { result := 42; }
  Assign("result", Number(12)))                     // else { result := 12; }


program.eval (input) // Specialize interpreter to fixed program
```

```
if (Comp(
      Arith(Variable("x"), _ * _, Number(2)),
      _ > _, Number(12)).evaluate(input) != 0)  // Unfold .evaluate
    Assign("result", Number(42)).evaluate(input)
else
    Assign("result", Number(12)).evaluate(input)
```

```
if (Arith(Variable("x"), _ * _, Number(2))
      .evaluate(input) >
      Number(12).evaluate(input))  // Unfold .evaluate again
    Assign("result", Number(42)).evaluate(input)
else
    Assign("result", Number(12)).evaluate(input)
```

```
if (Variable("x").evaluate(input) * Number(2).evaluate(input))
        > 12) // Unfold .evaluate again ...
    Assign("result", Number(42)).evaluate(input)
else
    Assign("result", Number(12)).evaluate(input)
```

```
if (input(“x”) * 2 > 12)
    Assign("result", Number(42)).evaluate(input) // Unfold .evalute
else
    Assign("result", Number(12)).evaluate(input) // of assignments
```

```
if (input(“x”) * 2 > 12)                    // if (x * 2 > 12 )
    input + (“result” -> 42)                // then { result := 42; }
else
    input + (“result” -> 12)                // else { result := 12; }
```

Source program was translated to another language!

# Second Futamura Projection

```
target = ⟦pe⟧ (interpreter, program)
```

- A Partial Evaluator accepts multiple inputs

⇒ Partial Evaluator can specialize a Partial Evaluator with respect to an Interpreter!

# Second Futamura Projection

```
target = ⟦pe⟧ (interpreter, program)
       = ⟦⟦pe⟧ (pe, interpreter)⟧ (program)
       =      ⟦compiler⟧          (program)
```

- Specialization of a Partial Evaluator with respect to an Interpreter yields:

  **Compiler**

- Partial Evaluator can act as compiler generator!

# Third Futamura Projection

```
compiler = ⟦pe⟧ (pe, interpreter)
```

- A Partial Evaluator accepts multiple inputs *again*

⇒ Partial Evaluator can specialize a Partial Evaluator
with respect to a Partial Evaluator!

# Third Futamura Projection

```
compiler = ⟦pe⟧ (pe, interpreter)
         = ⟦⟦pe⟧ (pe, pe)⟧ (interpreter)
         =  ⟦compiler-gen⟧ (interpreter)
```

- Specialization of a Partial Evaluator with respect to a Partial Evaluator yields:

  **Compiler Compiler** or **Compiler Generator**

- Partial Evaluator can act as compiler generator *generator*!

# Fourth Futamura Projection?

```
compiler-gen = ⟦pe⟧ (pe, pe)
```

- A Partial Evaluator accepts multiple inputs *again*²

⇒ Partial Evaluator can specialize a Partial Evaluator
with respect to a Partial Evaluator!

# Fourth Futamura Projection?

```
compiler-gen = ⟦pe⟧ (pe, pe)
             = ⟦⟦pe⟧ (pe, pe)⟧ (pe)
             = ⟦⟦⟦pe⟧ (pe, pe)⟧ (pe)⟧ (pe)
            ...
```

- No new functionality arises from further specialization
  - Nothing can change, since all parts of the equation are `pe`
- Speed-Ups are possible!

# Applications & Limits

# Applications of Partial Evaluation

- Programs with a fixed (or rarely changing) configuration
  - Web Servers with configuration file
  - Rendering of a fixed scene
- Domain Specific Languages
  - Specialization to programs removes interpretation overhead
- General Software Development

# Limits of Partial Evaluation – Termination

- Limited depth of optimizations (Unfolding, Symbolic Computations)
  - Unfolding has to terminate with call to (generic) definition

- Limited amount of created definitions (Program Point Specialization)
  - Amount of definitions has to be limited

Optimization  Termination

# Limits of Partial Evaluation

- Speed-Up is hard to predict
  - Binding Time Analysis makes predictions easier (at cost of performance)
  - More analysis requires more time ⇒ Less overall gain
- Difficult to use without knowledge or experience
- Cannot invent new optimizations

# Conclusion

# Conclusion

- Partial Evaluation is **not** a replacement for compilers / compiler writers
  - Humans are required to implement optimization techniques

- Applications are limited
  - No “one size fits all“-solution
  - Varying benefits in different areas

- More development required (theoretical and practical)

# Further Reading


- S. Kleene. “General Recursive Functions of Natural Numbers”. In: *Mathematische Annalen* 112.1 (Dec. 1936), pp. 727–742.
- N. Jones, C. Gomard, and P. Sestoft. *Partial Evaluation and Automatic Program Generation*. Prentice Hall, 1993. ISBN: 0130202495.
- N. Jones. “An Introduction to Partial Evaluation”. In: *ACM Computing Surveys* 28.3 (1996), pp. 480–503.
- Y. Futamura. “Partial Evaluation of Computation Process–An Approach to a CompilerCompiler”. In: *Higher-Order and Symbolic Computation* 12.4 (1999), pp. 381–391.

# Discussion