

# Influence of Individuality on the Flocking Behaviour of Boids

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## Abstract

In the hereby presented work we try to build a boid model that improves on the model proposed by W. Reynolds by adding different constraints, such as obstacles and predators. After simulating the altered flocking dynamics we add another constraint to the model, namely individuality. By adding this new constraint we try to answer the question whether this individuality can be seen in the distribution of the catched boids.

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1 INTRODUCTION 2

# 1 Introduction

In 1986 W. Reynolds came up with the idea to simulate the motion of bird flocks and fish schools [1]. The general term for those flocking creatures became the boid. In his work he introduces the concept of forces acting on the different entities of the flock. Thereby every boid follows three simple rules: cohesion, alignment and separation. These simple rules result in a complex general motion, which is extremely similar to the one that can be observed in nature.

After reading through multiple papers and already presented work [2, 3] we decided to take our own approach by doing the implementation by ourselves. In the hereby presented work we try to build upon the work presented by Reynolds [1] by creating our own implementation of the flocking behaviour of boids when confronted with different environments. To the basic model we add constraints by implementing obstacles and predators.

A lot of research has already been done using the previously explained implementations, so in order to present something new we try to add individuality to the different boids by giving them an age and therefore constraints in their characteristics. To understand the effects of this individuality aspect on the flocking behaviour of the latter is the main goal of our work. We try to answer the following question: how is the boid's age and the catching rate related when a flock of birds is confronted with environmental constraints such as predators and obstacles?

# 2 Theoretical Background

In this section we give some theoretical background to the flocking behaviour of boids. We start by introducing the basic boid model and improve on it by adding constraints to the dynamics of the flock. The way these constraints are chosen is based on our own intuition and do not claim to be the best suited choice. Finally we add an individuality constraint to the single boids by attributing an age to each of them according to a predefined age distribution.

# 2.1 Basic Model

Let us introduce the basic model that describes the flocking behaviour of boids as it was thought of by W. Reynolds [1]. The simple model consists of three rules/forces that describe the steering behaviour. Here we will go over the different forces and different boid characteristics at hand.

#### 2.1.1 View

In the used model the most important characteristic of a boid is its viewing field V as it play a role in the different forces that act on the boid. This field can be described by two parameters, the viewing radius R and the viewing angle  $\alpha$ , together they form the space in which the chosen boid is able to interact with his flocking mates. An illustration of the viewing field is given in fig. 4.

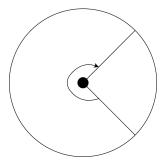


Figure 1: Illustration of the viewing field of a boid.

#### 2.1.2 Cohesion

The most intuitive force is the so called cohesion force. This rule ensures the flocking and attraction of the boids. As suggested by observing flocks and herds in nature the safest place is in the middle as potential predators are most likely to attack animals at the borders of the flock. The cohesion force therefore depends on the centre of mass (COM) of the flock and is a driving force for each boid to steer towards it. Keep in mind that every boid has only a limited field of view and that the COM he perceives is only the one of the N boids in his viewing field. Since the only chance for a separated boid not to be caught is to join a flock the driving force towards the COM linearly increases with the distance to the COM.

Mathematically this can be achieved in the following way

$$\mathbf{F_c} = c_c \left( \sum_{\mathbf{x}_n \in V} \frac{\mathbf{x}_n}{N} - \mathbf{x} \right), \tag{1}$$

where  $c_c$  is a constant,  $x_n$  are the positions of the boids in V and x is the position of the boid itself.

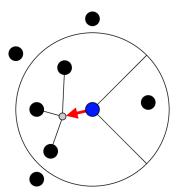


Figure 2: Illustration of cohesion force acting on a single boid.

#### 2.1.3 Alignment

For the flock to be able to perform collective steering manoeuvres the individual boids have to adjust their velocity v to the ones of their neighbours  $v_n$ . The faster the boids are able to react to their flock mates the more consistent the behaviour of the entire flock. In other words, if some of the boids are faster than others and do not adjust their velocity to the swarm the latter is due to break. Again we note that a boid is only able to align itself to the boids in his field of view V. The force is growing linearly with the difference between the boids and the flocks velocity.

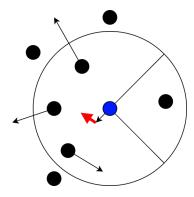


Figure 3: Illustration of the alignment force acting on a boid.

Formally the alignment force is given by

$$\boldsymbol{F}_a = c_a \left( \sum_{\boldsymbol{x}_n \in V} \frac{\boldsymbol{v}_n}{N} - \boldsymbol{v} \right), \tag{2}$$

where  $c_a$  is a constant.

#### 2.1.4 Separation

The third rule defined by W. Reynolds is the separation force. This force counteracts the two previously described ones and prevents the boids from colliding with each other during the flocking process. This allows for a minimal distance between the boids. Keep in mind that this minimal distance can not be too big as this would allow potential predators to easily isolate single boids.

Mathematically separation can be achieved in the following way

$$\boldsymbol{F}_{s} = -c_{s} \sum_{\substack{\boldsymbol{x}_{n} \in B \\ |\boldsymbol{x}_{n} - \boldsymbol{x}| \leq R_{s}}} \frac{\boldsymbol{x}_{n} - \boldsymbol{x}}{|\boldsymbol{x}_{n} - \boldsymbol{x}|},$$
(3)

where  $c_s$  is a constant and B is the set of all boids. Note that the selection of considered boids and their positions  $x_n \in B$  is not based on the boids in V but on all boids which are in a certain range  $R_s$ .

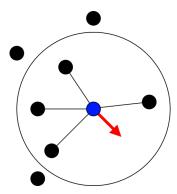


Figure 4: Illustration of the separation force acting on a single boid. Circle surrounding the blue boid has radius  $R_s$ .

## 2.2 Obstacles

As a first improvement to the basic boid model we introduce obstacles to the environment. This first environmental constraint on the dynamics of the flock can be added to the model by using the same force as the separation force between two boids.

This results in the following rule

$$\boldsymbol{F}_{oa} = -c_{oa} \sum_{\substack{\boldsymbol{o}_n \in O \\ |\boldsymbol{o}_n - \boldsymbol{x}| \le R_O}} \frac{\boldsymbol{o}_n - \boldsymbol{x}}{|\boldsymbol{o}_n - \boldsymbol{x}|}, \tag{4}$$

where  $c_{oa}$  is a constant,  $o_n \in O$  are the positions of the obstacles contained in the set of all obstacles O. Note that only obstacles in the range  $R_O$  are considered during the computation of the force.

#### 2.3 Predators

As a second improvement we add predators to the environment. The goal of these predators is to catch the boids while they try to escape. In the following we take a look at the rules which apply to the predators and subsequently to the boids.

#### 2.3.1 Predator Behaviour

Like in the boid model predators should not collide into each other nor into objects. Therefore the same separation and obstacle avoidance forces (see eq. (3) and eq. (4)) with different scaling constants

are acting on them.

In addition to those two forces a third force is added which drives the predators towards the boids. We call this force the attacking force  $\mathbf{F}_{at}$ , which accelerates the predator towards the nearest boid in his viewing field v. This nearest boid is determined using a simple distance measure.

Formally the attacking force is given by

$$\boldsymbol{F}_{at} = \begin{cases} c_{at} \frac{\boldsymbol{x}_{\text{boid}} - \boldsymbol{x}}{|\boldsymbol{x}_{\text{boid}} - \boldsymbol{x}|}, & \text{with } \boldsymbol{x}_{\text{boid}} = \arg\min_{n} |\boldsymbol{x}_{n} - \boldsymbol{x}| \\ 0, & \text{if } V = \emptyset, \end{cases}$$
 (5)

where  $c_{at}$  is a constant and  $x_{\text{boid}}$  is the position of the boid. This means that the predator always attacks the nearest boid with full force.

#### 2.3.2 Predator Avoidance

The boids need to avoid the predators in order not to be caught. This can be done by applying a repellent force away from approaching predators. We again use the same force as the collision avoidance force (see eq. (3) and eq. (4)) with a different scaling factor

$$\boldsymbol{F}_{pa} = -c_{pa} \sum_{\substack{\boldsymbol{p}_n \in P \\ |\boldsymbol{p}_n - \boldsymbol{x}| \le R_p}} \frac{\boldsymbol{p}_n - \boldsymbol{x}}{|\boldsymbol{p}_n - \boldsymbol{x}|},\tag{6}$$

where  $c_{pa}$  is a constant,  $p_n \in P$  are the positions of the predators contained in the set of all predators P. Note that the boids will only consider the predators in the range  $R_p$ . This design choice is based on observations.

# 2.4 Individuality

As a third improvement to the basic model we add individuality to each of the boids of the flock. In our case the individuality is determined by the age  $\in [0,1]$  of each boid. Poorer properties in movement and sight for young (age  $\simeq 0$ ) and old (age  $\simeq 1$ ) boids are achieved by varying the size of V and the maximum speed at which they can move. The ages are assigned at random based on an equal distribution.

The constrains are formulated as follows

$$r_{\text{view}} = r_{\text{view,global}} - c_{\text{r,view}} \cdot (\text{age} - 0.5)^2,$$
 (7)

$$\alpha_{\text{view}} = \alpha_{\text{view,global}} - c_{\alpha,\text{view}} \cdot (\text{age} - 0.5)^2,$$
 (8)

$$v_{\text{max}} = v_{\text{max,global}} - c_{\text{maxspeed}} \cdot (\text{age} - 0.5)^2,$$
 (9)

where r is a radius,  $\alpha$  an angle and v a velocity. The index 'global' refers to a globally set value which is the same for all boids, while the constants c are used to adjust the strength of the constrain. Figure 5 shows that boids with an age of 0.5 are the fastest and best seeing ones while those with an age of 0 or 1 are the worst ones.

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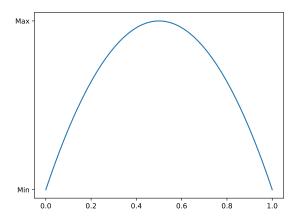


Figure 5: Parabolic distribution of the features of the boids given their age.

# 2.5 Full Model

In summary we build a model that is composed of three different modules:

- boids with different characteristics
- predators
- obstacles.

The velocities and positions of predators and boids are updated in similar fashion for each time step dt. First we add up all forces described above and get a total force  $F_{\text{tot}}$ . Then the velocity is updated

$$\mathbf{v}_{\text{new.uncut}} = \mathbf{v}_{\text{old}} + \mathbf{F}_{\text{tot}}.$$
 (10)

Notice that without units the mass is chosen to be the same value as the time step, this can be done without loss of generality. Next the velocity  $v_{\text{new,uncut}}$  is cut to a minimal magnitude  $v_{\text{min}}$  and maximal magnitude  $v_{\text{max}}$  in the following fashion

$$\boldsymbol{v}_{\text{new}} = \begin{cases} \frac{\boldsymbol{v}_{\text{new,uncut}}}{|\boldsymbol{v}_{\text{new,uncut}}|} \cdot v_{\text{max}} & \text{if } v_{\text{max}} < |\boldsymbol{v}_{\text{new,uncut}}|, \\ \boldsymbol{v}_{\text{new,uncut}} & \text{if } v_{\text{min}} < |\boldsymbol{v}_{\text{new,uncut}}| < v_{\text{max}}, \\ \frac{\boldsymbol{v}_{\text{new,uncut}}}{|\boldsymbol{v}_{\text{new,uncut}}|} \cdot v_{\text{min}} & \text{if } |\boldsymbol{v}_{\text{new,uncut}}| < v_{\text{min}}. \end{cases}$$

$$(11)$$

This velocity  $v_{\text{new}}$  is the final speed of the boid or predator. Making these cuts hinders the animals from getting to fast and obligates them to move with a minimal speed. Having a minimal speed even if no forces apply to the animal should help a single boid to find a flock and a predator to find its prey. The updated position is given by

$$\boldsymbol{x}_{\text{new}} = \boldsymbol{x}_{\text{old}} + \text{dt} \cdot \boldsymbol{v}_{\text{new}}.$$
 (12)

In the next time step the position and velocity of the prior step will be marked with the index 'old'.

# 3 Implementation

In this third section we shine some light on the implementation of the previously described boid model and on the improvements we decided to make. These explanations are split into five main parts: the environment, the boids, the predators, the individuality component and the simulation. We used **Python** and uploaded our code to **GitHub**<sup>1</sup>. A summary of all of the used parameters can be found in the appendix. Note the most of the parameters were determined by intuition or observations of the visual rendering of the simulation.

<sup>1</sup>https://github.com/Hexabug/boids

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#### 3.1 Environment

The environment is the place in which our simulation runs. Since we are only able to create a space of finite extension it is crucial to decide how the animals should behave when reaching the edge. Setting the edge to be handled as an object would slow down approaching boids suddenly and make them an easy prey. Thus we decide to connect the edges: left to right and top to bottom. The newly created map can be visualised as a sphere. This creates some implementation challenges as the animals should be able to get the correct distance to the other ones considering the connected edges. We resolve this issue by defining four virtual domains appended to the edges of the real domain. Each of these virtual domains contains a virtual position of the animal. This ensures that all animals are able to estimate proper distances and therefore compute the correct force vectors. The described virtual domains can be seen in fig. 6.

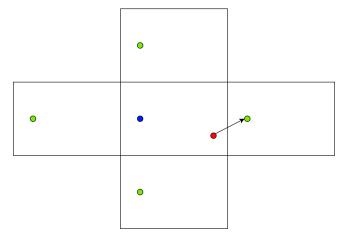


Figure 6: Domain with four virtual domains. The predator (red) is attacking a virtual copy (green) of the real boid (blue) which leads him over the right edge. Thus the predator will reappear on the left edge close to the boid, proceeding his attack towards the real boid.

#### 3.2 Boids

We define helper functions that are used to construct the different forces acting on a single boid. Those functions consider

- position (horizontal position, vertical position)
- velocity (horizontal velocity, vertical velocity)
- view (viewing radius, viewing angle)
- size
- age

of the boids. A first function determines the boids inside of the viewing field V of the boid in question, while two other functions compute the average position and the average velocity of the boids in V. A fourth function is used to determine boids, predators and objects within a certain range (i.e. the viewing field but without the restriction of the viewing angle). This information is then used to compute all repulsive forces. The resulting time update for each boid is based on the addition of the previously described forces, while restricting the speed as described in section 2.5.

#### 3.3 Predators

The idea behind the implementation of the predators is similar to the one of the boids. Here we also use helper functions which consider

• position (horizontal position, vertical position)

- velocity (horizontal velocity, vertical velocity)
- view (viewing radius, viewing angle)
- size.

of the predators. In contrast to the boids only three forces are acting on the predators as they do not follow a flocking pattern. A first function determines the nearest boid in sight and a second function determines the position of all the predators and obstacles within a certain range. Based on this information the forces acting on the predator are computed. The resulting time update for each predator is the addition of the previously described forces, while restricting the speed as described in section 2.5.

# 3.4 Individuality

To implement individuality we used a function which introduced a small constrain to the globally set parameters of the boids (see Appendix). The constrain simply follows the rules as explained in section 2.4.

### 3.5 Simulation

Finally we take a closer look at the internal processes when a simulation is run. Since several simulation runs are necessary to obtain a meaningful result we applied multiprocessing to our model which enables the parallel processing of n simulation runs (where n is equal to the number of CPU kernels). Every simulation run starts with the initialisation of the domain on which a predefined number of obstacles is set at random. Then the boids are initialised at random in a centred circle of the domain keeping away from the obstacles. To ensure an initial distance between predators and boids the predators are initialised outside the inner circle.

After the initialisation step the model runs as described in section 2.5, thus the predators start hunting the boids while the latter try to avoid the predators. Since the predators are slightly faster than the boids sooner or later a boid will be caught. We consider a boid to be caught if the distance between boid and predator falls short the added size of predator and boid. A caught boid is removed form the map while his age is saved for further review.

A single simulation run ends when a predefined number n of boids is caught. Since we are interested in the age distribution of the first caught boids, we need to ensure that the number n is much lower than the total number of boids |B|. Lastly the ages of the caught boid from all simulation runs are merged and saved.

# 4 Results and Discussion

In this fourth section we go over the results we were able to attain and discuss them. To finish we round up our work with ideas on how to improve onto our model.

### 4.1 Basic Behaviour

The implementation of the basic flocking behaviour as described in section 2.1 provides the expected results. After initialising the boids at random, small flocks start to form in which the boids try to align their velocities while keeping their distance. After a small amount of time the previous flocks form a larger one in which all boids align. The described behaviour can be seen in fig. 9.

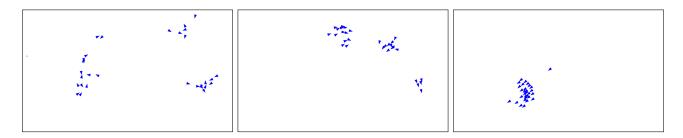


Figure 7: Basic flocking behaviour of boids. Left: the boids are initialised at random positions with random velocities. Middle: small flocks start to form where the boids are aligned. Right: one big flock is formed and all the boids velocities are aligned.

#### 4.2 Behaviour with Environmental Constraints

When adding different obstacles to create environmental constraints as discussed in section 2.2 the flocking behaviour is still visible but the reaction of the boids to the obstacles changes the previously observed dynamics. Flocks are still forming in which the boids align but they stay rather small. Due to the objects one big flock nearly never forms as it is always broken apart.

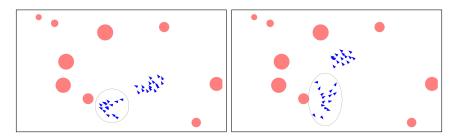


Figure 8: Flocking behaviour with obstacles as environmental constraints. Left: the left flock approaches an obstacle. Right: the same flock is reflected at the obstacle.

#### 4.3 Interaction with Predators

The addition of predators again results in a change in the behaviour of the boids. The small flocks move in between the objects and try to avoid the predators while the predators try to isolate a single boid from its flock. As in our model the predators maximum velocity is chosen to be higher than the one of the boids a predator is nearly always able to catch the boid it has targeted.

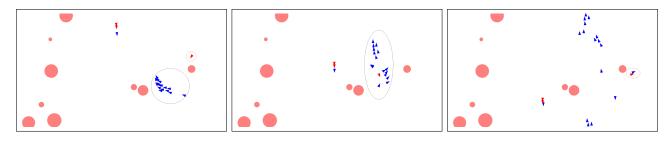


Figure 9: Basic flocking behaviour of boids. Left: the boids are initialised at random positions with random velocities. Middle: small flocks start to form where the boids are aligned. Right: one big flock is formed and all the boids velocities are aligned.

#### 4.4 Age Distribution of Caught Boids

To answer the main question of this report, namely how do age and catching rate correlate, we decided to vary a specific parameter while keeping the others fixed according to the tables in the appendix.

### 4.4.1 Varying Parameters of the Age Distribution

While varying the parameters of the age distribution, i.e.  $c_{r,\text{view}}$ ,  $c_{\alpha,\text{view}}$  and  $c_{\text{maxspeed}}$  we were expecting to see the same parabolic shape in the age distribution of the caught boids. After running multiple tests we were able to determine that a restriction onto the maximum speed of the boids had the biggest impact on the age distribution.

Figure 10 shows that our assumption was right and that a parabolic shape in the age distribution of the caught boids is clearly visible. It also shows that the age distribution is explicitly dependent on the parameter  $c_{\rm maxspeed}$ , as a lower value results in a distribution closer to an equal distribution but a higher value in a parabolic distribution.

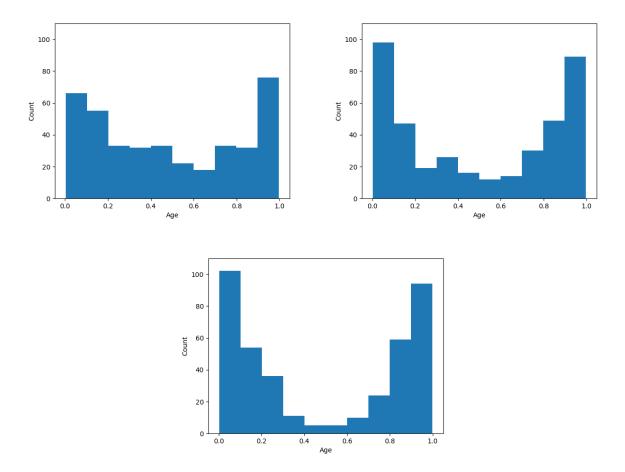


Figure 10: Age distribution of caught boids, with the parameters  $c_{\text{maxspeed}} = 20$  (left),  $c_{\text{maxspeed}} = 50$  (right) and  $c_{\text{maxspeed}} = 200$  (bottom). Note that the parabolic shape is clearly visible.

#### 4.4.2 Varying Number of Obstacles

When varying the number of obstacles our assumption was that the correlation between the age distribution of the caught boids and their age would digress. This would be the result of a more chaotic flocking behaviour, as the forces repelling the boids from the obstacles are strong.

After running multiple tests with different numbers of obstacles we were not able to verify our hypothesis. Instead the number of obstacles had no visible effect on the age distribution of the caught boids. Figure 11 shows the non existent effect of the number of boids. We want to note that simulations with over 40 objects were not possible, since we often faced scenarios where the predators and boids were cut off from each other.

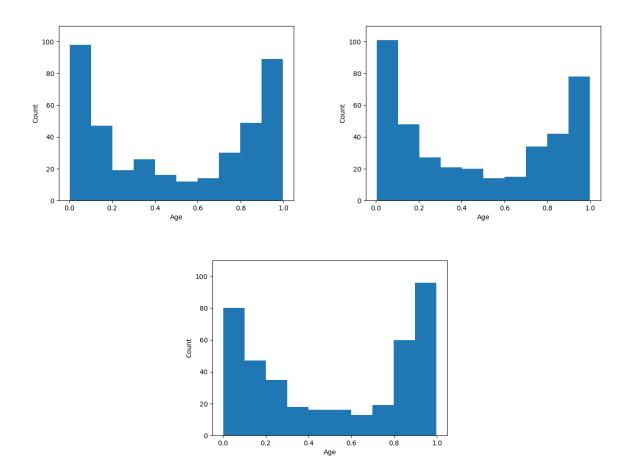


Figure 11: Age distribution of caught boids according to the number of obstacles: 4 (left), 20 (right), 40 (bottom).

# 4.5 Improvements to the Model

After running a lot of simulations to determine the different parameters of our model we noticed that some aspects of the latter could be improved on. In the following we will go into more detail on how these improvements could look like.

## 4.5.1 Boid Characteristics

Our model does not take into account that the boids are animals and that they have limited energy supplies. To improve on our model it would be interesting to give the boids a stamina component to thereby ensure that they can be exhausted. Thus their motion over time could become slower. Furthermore one could imagine adding 'food' into the environment to thereby increase or replenish the stamina of an exhausted boid.

#### 4.5.2 Boid-Obstacle Interaction

The force by which boids (and predators) interact with the obstacles that can be found in their environment is lacking complexity. In our model it is a radial force pushing the boids away from the obstacle's centre, this leads to an unnatural behaviour when the boids face an object head on. In that case the boids are mirrored away instead of, as one would expect when observing real animals, circumventing the object at hand. To ensure a better behaviour the force should take into consideration that the boid's direction should be changed as little as possible. In fig. 12 the previously described interaction can be seen.

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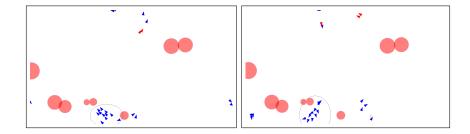


Figure 12: Unnatural obstacle interaction. Left: a flock of boids is approaching some obstacles. Right: the flock is mirrored away from the obstacles instead of circumventing them.

#### 4.5.3 Boid-Predator Interaction

Like the force by which boids are repelled from obstacles the force by which they are repelled from predators is formulated in a simple manner. If one now assumes that the boids possess some kind of intelligence they could start to use their environment to their advantage by using obstacles to escape from the predators. In other work [2] this effect is achieved by adding a force that is acting on the boids which forces them to bring an object between them and the predator that is chasing them.

# 5 Conclusion

In the presented work we built upon the work of Reynolds and expanded the constraints acting onto a flock of boids. We started by introducing the theory behind the flocking behaviour of boids and then added different constraints to the model by adding obstacles, predators and individuality. We then explained how the theory can be implemented into a simulation which was used to run several tests. These tests consisted of determining the effects of different parameters onto the age distribution of the caught boids. We were able to show that the individuality is correlated to the probability of being caught. Furthermore we could show that the parabolic strength of the constraints is reflected in the age distribution of the caught boids. Other than expected we could not determine any correlation between the number of initialised obstacles and the distribution of the caught boids.

We think that our proposed model provides a good starting point for anyone interested in this area as it is simple to understand while providing a lot of features. In the last section we proposed improvements that could be implemented without changing the structure of our model or implementation.

# A Model Parameters

Table 1: Environmental parameters.

Parameter	Value	
Number of boids	35	
Boid size	10	
Number of predators	3	
Predator size	10	
Number of obstacles	4	
Max obstacle size	35	
Domain size	[4000:4000]	
Time step dt	0.08	
Max caught boids	4	
Number of simulation runs	100	

Table 2: Parameters of the boid class.

Parameter	Acronym	Value
BOID RANGE	R	950.0
BOID VIEW ANGLE	$\alpha$	110
BOID COLLISION DISTANCE	$R_S$	45.0
OBSTACLE COLLISION DISTANCE	$R_O$	250.0
PREDATOR COLLISION DISTANCE	$R_P$	300.0
MAX SPEED	$v_{\rm max}$	150.0
MIN SPEED	$v_{ m min}$	25.0
COHESION FACTOR	$c_c$	0.5
ALIGNMENT FACTOR	$c_a$	0.045
BOID AVOIDANCE FACTOR	$c_s$	10
OBSTACLE AVOIDANCE FACTOR	$c_{oa}$	200.0
PREDATOR AVOIDANCE FACTOR	$c_{pa}$	500.0
CONSTRAIN ON VIEWING RADIUS	$c_{ m r,view}$	20
CONSTRAIN ON INDIVIDUAL VIEWING ANGLE	$c_{\alpha, \mathrm{view}}$	20
CONSTRAIN ON MAX SPEED	$c_{\rm maxspeed}$	50

Table 3: Parameters of the predator class.

Parameter	Acronym	Value
PREDATOR RANGE	R	1000.0
PREDATOR VIEW ANGLE	$\alpha$	110
PREDATOR COLLISION DISTANCE	$R_P$	45.0
OBSTACLE COLLISION DISTANCE	$R_O$	250.0
MAX SPEED	$v_{ m max}$	158.0
MIN SPEED	$v_{ m min}$	55.0
PREDATOR AVOIDANCE FACTOR	$c_{pa}$	30
OBSTACLE AVOIDANCE FACTOR	$c_{oa}$	200.0
ATTACK FACTOR	$c_{at}$	100.00

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