Uncertainty Quantification course
Homework assignment 2

September 21, 2023

We discuss this exercise in the class meeting on 27 September 2023.

Consider the truncated Karhunen-Loève expansion Y_t^d, $t \in [0,1]$, with eigenfunctions $\psi_i(t) = \sin(\pi\, i\, t)$ and eigenvalues $\lambda_i = 1/(\pi\, i)^2$.

Construct and plot sample paths (i.e., realizations) of Y_t^d with $d = 10, 100, 1000$ (multiple paths for each d). Try out both normal and uniform distributions, i.e. $\hat{Y}_i \sim \mathcal{N}(0,1)$ and $\hat{Y}_i \sim U[-1,1]$.

With normal distributions ($\hat{Y}_i \sim \mathcal{N}(0,1)$ for all i), $B_t^d := \sqrt{2}\, Y_t^d$ is the KL expansion for the Brownian bridge process B_t. What is the covariance function $C(t,s)$ of the Brownian bridge? Can you prove that $(\psi_i(t), \lambda_i)$ as defined above are indeed the eigenfunctions and eigenvalues of the Brownian bridge covariance function?