Uncertainty Quantification course
Homework assignment 9

November 16, 2023

This homework assignment will be graded. Please submit one report (in pdf) per team by email **before** 22 November, in other words send it on 21 November 23:59h at the latest, to `D.T.Crommelin@uva.nl`. Reports sent in after that time will not be considered, instead the minimum grade will be given to the team. Put the names of the team members who were involved in making the assignment on the report.

Reports can have 3 pages maximal length, including figures. Please add a print-out of your Matlab or Python code (with some inline comments) as an appendix to your report, this print-out will not count towards the 3-page maximum.

You are given the model $f(\alpha, \beta) = 1.5\alpha + 0.25(\beta - 1)^2 + \cos(\pi + \alpha + \beta)$. Furthermore, you have five noisy observations y_i: 4.02, 3.97, 4.05, 3.85, 3.94. The relation between the observations and the model output is $y_i = f(\alpha, \beta) + r_i$. The r_i are all random (iid) samples from the standard normal distribution with zero mean and standard deviation 0.1, in other words $r_i \sim \mathcal{N}(0, 10^{-2})$.

Use MCMC sampling to calibrate the parameters α, β of the model using the observations. Assume a Gaussian prior $\mathcal{N}(\mu, \sigma^2 I_2)$ with mean $\mu = (0, 4)$ and covariance matrix $\sigma^2 I_2$ where I_2 is the 2×2 identity matrix and $\sigma = 0.25$. Explain what proposal distribution you use. Tune the variance of the propsal distribution to get an acceptance rate in the range 25% - 30%.

Make a figure of the MCMC samples in the (α, β) plane. How long (roughly) is the burn-in period and how long before the chain has converged? Make figures (e.g. histograms) to show the marginals of the posterior distribution for both α and β (i.e., show $p_{\text{posterior}}(\alpha|y_1, ..., y_5)$ and $p_{\text{posterior}}(\beta|y_1, ..., y_5)$). What is (your estimate of) the covariance matrix of the posterior?