Uncertainty Quantification course
Homework assignment 3

September 27, 2023

We discuss this exercise in the class meeting on 4 October 2023.

Let Z be a random variable with uniform distribution, $Z \sim \mathcal{U}[-1, 1]$. We consider the function $f(Z) = \sin(2(Z-1)^2)$.

Build a gPC approximation of $f(Z)$ using orthogonal polynomials. What are suitable orthogonal polynomials here? Truncate after the 7-th degree polynomial. Plot the true function f and its approximation $f_{N=7}$. (You can use the Matlab script *legendre_projection_gaussquad.m* from the first homework assignment for this, see the Canvas page for the course, under Files - Code)

Compute the mean and variance of $f(Z)$ using Monte Carlo sampling. Compute the mean and variance of the approximation $f_{N=7}(Z)$ from the expansion coefficients. Furthermore, plot the distributions of both $f(Z)$ and $f_{N=7}(Z)$, e.g. by generating many samples and making histograms.