Uncertainty Quantification course
Homework assignment 4

October 5, 2023

This homework assignment will be graded. Please submit one report (in pdf format) per team by email **before** 11 October, in other words send it on 10 October 23:59h at the latest, to `D.T.Crommelin@uva.nl`. Reports sent in after that time will not be considered, instead the minimum grade will be given to the team. Put the names of the team members who were involved in making the assignment on the report.

Reports can have 3 pages maximal length, including figures. Please add a print-out of your Matlab or Python code (with some inline comments) as an appendix to your report, this print-out will not count towards the 3-page maximum.

Consider the following function with two random inputs:

$$f(Z_1, Z_2) = -(Z_1 - 2)(Z_2 - 1)^3 + \exp\left[-\frac{1}{2}(Z_1 - 2)^2 - \frac{1}{10}(Z_2 - 1)^2\right]$$

The inputs Z_1, Z_2 are independent. Z_1 has a uniform distribution on $[1, 3]$, whereas Z_2 has a normal distribution with mean 1 and variance 1. Thus, $Z_1 \sim \mathcal{U}[1, 3]$ and $Z_2 \sim \mathcal{N}(1, 1)$.

The goal in this assignment is to construct a truncated gPC expansion to approximate f, and to investigate the convergence of this approximation numerically. First, what are the (2-dimensional) gPC basis functions that you use for the expansion? Compute the expansion coefficients using quadrature (you can use a numerical library routine for quadrature, e.g. from numpy). How do you truncate the expansion (i.e., how do you truncate the multi-indices)? Let n denote the total number of the 2-dim gPC basis functions used in your approximation. Compute the error $\|f - f_n\|$ in weighted L_2-norm (weighted by the probability density of Z_1, Z_2), by evaluating $f(Z_1, Z_2)$ and its approximation $f_n(Z_1, Z_2)$ on a fine numerical grid.

Repeat your computation for a range of truncations from $n = \mathcal{O}(10)$ to $n = \mathcal{O}(10^2)$. Make a plot of the error $\|f - f_n\|$ versus n. What type of convergence do you see, algebraic (power-law) or exponential? Is this what you expected to see? Motivate your answers.

Show contour plots of both f and f_n (for the latter, take largest value of n that you used) on the domain $1 \leq Z_1 \leq 3$, $-2 \leq Z_2 \leq 4$. Finally, compute (approximations of) the mean and variance of $f(Z_1, Z_2)$ from the gPC expansion coefficients.