# Uncertainty Quantification course
# Homework assignment 1

September 13, 2023

We discuss the exercises in the class meeting on 20 September 2023.

Exercise 1: Let $Z$ is a random variable with beta distribution on the interval [-1,1]. The probability density function (pdf) of $Z$ is $\rho(z) \propto (1-z)^6(1+z)^2$. What is the set of polynomials that are orthogonal with respect to the pdf of $Z$? List the first 3 polynomials of this set. What are their normalization constants (denoted $\gamma_n$ in the book by Xiu)?

Exercise 2: Let $f$ be the function $f(x) = \exp(2x + \sin(4x))$ with $x \in I := [-1, 1]$. Approximate $f$ by (i) orthogonal projection on Legendre polynomials, and (ii) interpolation. What is the approximation error $\|f - \tilde{f}\|_{L^2_w} = \left( \int_I |f(x) - \tilde{f}(x)|^2 \, dx \right)^{1/2}$ in both cases (where $\tilde{f}$ denotes the approximating function)? How does the error change when increasing the order of the approximation?