Uncertainty Quantification course
Homework assignment 5

October 12, 2023

We discuss this exercise in the class meeting on 18 October 2023.

Consider the 1-dimensional ordinary differential equation (ODE):

$$\frac{du}{dt} = -\frac{1}{2}u^2 + \alpha u \tag{1}$$

with initial condition $u(t = 0, Z) = 1/10$. The only random parameter is α, given by $\alpha(Z) = 1 + Z$ where Z has standard normal distribution, $Z \sim \mathcal{N}(0, 1)$.

Apply the stochastic Galerkin method to this system. What are appropriate orthogonal polynomials? What is the system of coupled ODEs that results from the stochastic Galerkin method? (for expectations of triple products of the polynomials, you can use the expression in Xiu's book, see section 6.2)

Write a program for numerical integration of the ODE (1) from $t = 0$ to $t = 2$. Use Monte Carlo sampling to estimate the mean and the variance of $u(t = 2, Z)$. Experiment with the number of samples to assess how many samples you need to get two digits of accuracy in the estimates.

Write a program for numerical integration of the coupled system resulting from the stochastic Galerkin method, from $t = 0$ to $t = 2$. Use it to compute the approximate gPC solution $v_N(t, Z)$ with a small number of terms (e.g. $N = 5$) in the gPC expansion. Plot the mean and variance of $v_N(t, Z)$ (computed from the expansion coefficients) in the time interval $t \in [0, 2]$. Compare the mean and variance at $t = 2$ with the Monte Carlo estimates obtained before.