Uncertainty Quantification course homework assignment lecture 11

November 23, 2023

You are given the following set of data points:

$$
\begin{aligned}
d &= \{(x_i, y_i) \mid i = 1, 2, \cdots, n = 5\} \\
&= \{(-2, -1), (-1, -0.1), (0, 0.2), (1, 0.1), (2, 0.5)\}
\end{aligned}
$$

Use the weight-space view to construct a Gaussian Process trained on this data set. Your model for the underlying function is of the form

$$
f(x) = \phi(x)^T \mathbf{w}, \tag{1}
$$

where $\phi(x)$ is an N-dimensional feature vector, and $\mathbf{w}$ are the unknown weights. Relate the model to the data via

$$
y_i = f(x_i) + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}\left(0, \sigma^2\right)
$$

where you can set $\sigma = 0.05$. Show what the likelihood function looks like, and use a zero-mean multivariate Gaussian prior for the weights, with just the identity matrix as the covariance matrix. Also show the posterior distribution for the weights, and the posterior predictive distribution for f, i.e. $p(f^*, \mathbf{w} \mid x^*, X, \mathbf{y})$.

Evaluate this last distribution at 100 equidistant points x^* in $[-2, 2]$. First, show the results of the linear model ($\phi(x) = [1, x]^T$), plotting the posterior predictive mean $\pm$ 1 standard deviation, and also show the data d. Then, enrich the feature vector with a quadratic term ($\phi(x) = [1, x, x^2]^T$), and show the same plot. Repeat this process one more time by adding x^3 to ϕ.