Uncertainty Quantification course
Homework assignment 12

December 7, 2023

We discuss this exercise in the class meeting on 13 December 2023.

Let Z_1, Z_2 and Z_3 be independent, continuous random variables all with uniform distribution on $[0, 1]$. Let $f : [0, 1]^3 \to \mathbb{R}$ be the function

$$f(Z) = Z_1 + Z_2^2 + Z_1\, Z_2 + Z_2\, Z_3^2$$

What is the HDMR or Sobol representation of this function? Compute all expansion terms ($f_0, f_1(z_1)$, etc).

Let $Y = f(Z)$. Use Monte Carlo sampling to estimate the variance $D = \mathrm{Var}(Y)$. Furthermore, use Monte Carlo sampling to estimate the variances of all the expansion terms. Make sure that you use enough samples to get reliable estimates. Finally, compute the Sobol indices from these estimates.