





$${}^0_1 T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2 T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3 T = \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_4 T = \begin{bmatrix} c_4 & -s_4 & 0 & L_3 \\ 0 & 0 & 1 & 0 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4_E T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_E \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$i-1$	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1	0	0	d_1	θ_1
1	2	90°	0	0	θ_2
2	3	0°	L_2	0	θ_3
3	4	-90°	L_3	0	θ_4
4	E	0°	0	d_E	0°

$${}^0_E T = \begin{bmatrix} c_1 c_4 c_{23} - s_1 s_4 & -s_1 c_4 - c_1 s_4 c_{23} & -c_1 s_{23} & (L_2 s_2 + L_3 c_{23} - d_E s_{23}) c_1 \\ c_1 s_4 + s_1 c_4 c_{23} & c_1 c_4 - s_1 s_4 c_{23} & -s_1 s_{23} & (L_2 c_2 + L_3 c_{23} - d_E s_{23}) s_1 \\ c_4 s_{23} & -s_4 s_{23} & c_{23} & d_1 + L_2 s_2 + L_3 s_{23} + d_E c_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_1 : If $p_x, p_y \neq 0$: $\theta_1 = \text{atan2}(p_y, p_x)$
 Else if $r_{13}, r_{23} \neq 0$: $\theta_1 = \text{atan2}(-r_{23}, -r_{13})$
 Else (singularity, $c_{23} = \pm 1$)

$c_{23} = +1$: $\theta_1 + \theta_4 = \text{atan2}(r_{21}, r_{11})$
 $c_{23} = -1$: $\theta_4 - \theta_1 = \text{atan2}(r_{21}, -r_{11})$

θ_4 : If $r_{31}, r_{32} \neq 0$: $\theta_4 = \text{atan2}(-r_{32}, r_{31})$
 Else (singularity, $c_{23} = \pm 1$)

$\theta_2 + \theta_3$: $s_{23} = -\text{sign}(r_{13}) \cdot \text{sign}(c_1) \cdot \sqrt{r_{13}^2 + r_{23}^2} \rightarrow \theta_2 + \theta_3 = \text{atan2}(-\text{sign}(r_{13}) \cdot \text{sign}(c_1) \cdot \sqrt{r_{13}^2 + r_{23}^2}, r_{33})$
 $= -\text{sign}(r_{23}) \cdot \text{sign}(s_1) \cdot \sqrt{r_{13}^2 + r_{23}^2} \rightarrow \theta_2 + \theta_3 = \text{atan2}(-\text{sign}(r_{23}) \cdot \text{sign}(s_1) \cdot \sqrt{r_{13}^2 + r_{23}^2}, r_{33})$

\uparrow If c_1 or $s_1 = 0$ use the other.
 Alternatively: $s_{23} = -r_{13}/c_1$ or $-r_{23}/s_1 \rightarrow \theta_2 + \theta_3 = \begin{cases} \text{atan2}(-\frac{r_{13}}{c_1}, r_{33}) \\ \text{atan2}(-\frac{r_{23}}{s_1}, r_{33}) \end{cases} \leftarrow \text{easier to code.}$

θ_2 : $\theta_2 = \text{atan2}(p_z - d_E c_{23} - L_3 s_{23} - d_1, p_x/c_1 + d_E s_{23} - L_3 c_{23})$
 $\text{atan2}(p_z - d_E c_{23} - L_3 s_{23} - d_1, p_y/s_1 + d_E s_{23} - L_3 c_{23})$

θ_3 : $\theta_3 = (\theta_2 + \theta_3) - \theta_2$