Introduction

This live script contains examples of reachable workspaces for 3D robots. For each example this includes:
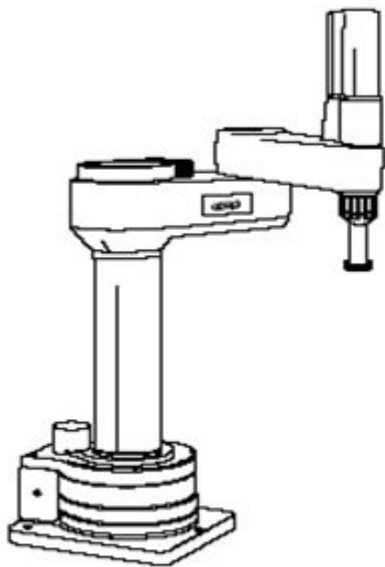
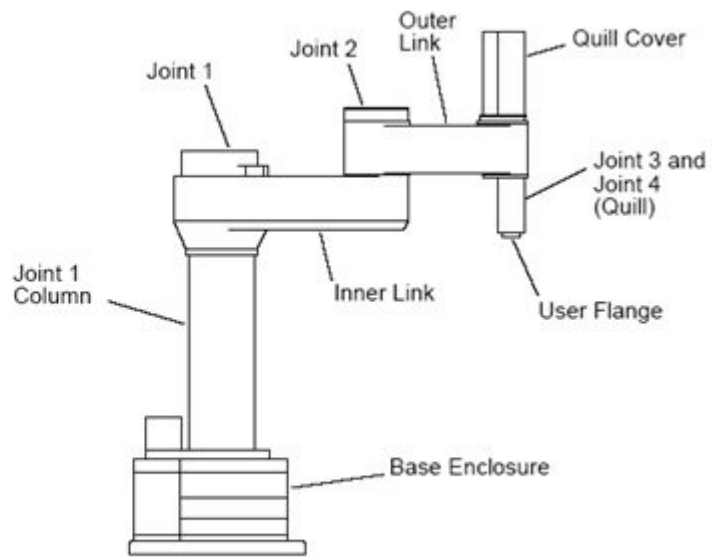- DH Parameters
- Generated Workspace

```
% Symbolic vars

syms theta1 theta2 theta3 theta4 theta5 theta6 real
syms d1 d2 d3 d4 real
```

Example 1 - AdeptThree Robot Arm

This is an example workspace search for the 4 DOF AdeptThree Robot Arm.

(a)   (b)

```
% DH parameters
test_dh = [18.5 0 42 theta1; ...
           16 pi 0 theta2; ...
           0 0 d3 0; ...
           0 0 7 theta4]
```

```
test_dh =
```

$$\begin{pmatrix} \frac{37}{2} & 0 & 42 & \theta_1 \\ 16 & \pi & 0 & \theta_2 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 7 & \theta_4 \end{pmatrix}$$

```
% Parameter ranges
theta1_range = arr2Rad(linspace(0,300, 50));
theta2_range = arr2Rad(linspace(0,300, 50));
d3_range = linspace(0,12, 50);
% Note the specification states 540°, but anything past 360° is redundant
theta4_range = arr2Rad(linspace(0,360, 50));
test_map = containers.Map({'theta1', 'theta2', 'd3','theta4'}, ...
    {theta1_range, theta2_range, d3_range, theta4_range});
% Workspace plotting function
plot3dworkspace(test_dh, test_map, @get_alternative_dh_transform)
```


Reachable Workspace
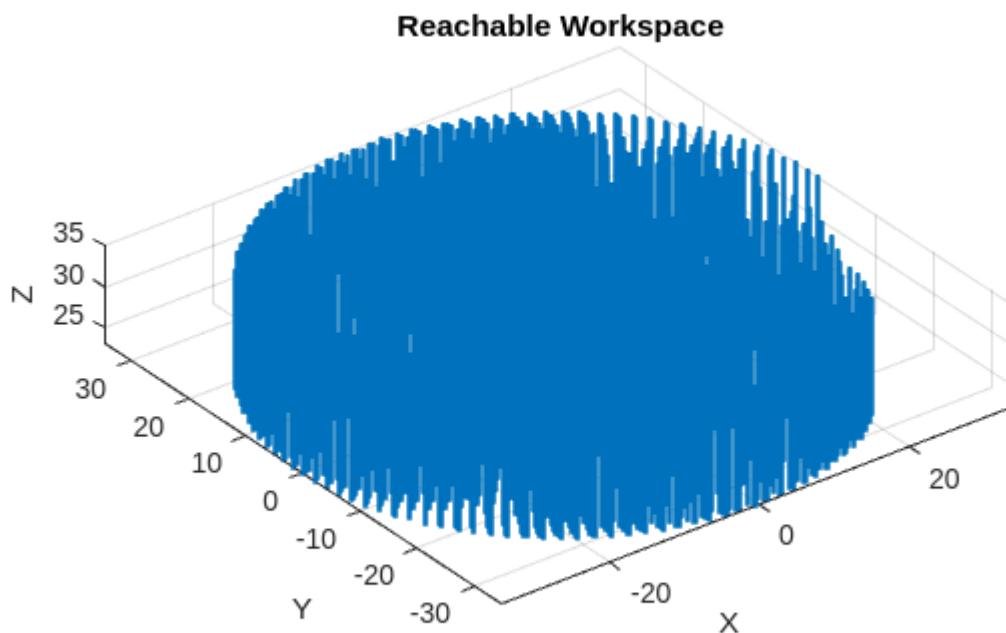
Example 2 - PUMA 560

This is an example using 6 DOF PUMA 560.

```
% DH parameters
test_dh = [0 -pi/4 671 theta1; ...
```

```
            431 0 139 theta2; ...
            -20 pi/4 0 theta3; ...
            0 -pi/4 431 theta4; ...
            0 pi/4 0 theta5; ...
            0 0 56 theta6;]
```

test_dh =

$$\begin{pmatrix} 0 & -\dfrac{\pi}{4} & 671 & \theta_1 \\ 431 & 0 & 139 & \theta_2 \\ -20 & \dfrac{\pi}{4} & 0 & \theta_3 \\ 0 & -\dfrac{\pi}{4} & 431 & \theta_4 \\ 0 & \dfrac{\pi}{4} & 0 & \theta_5 \\ 0 & 0 & 56 & \theta_6 \end{pmatrix}$$

```
% Parameter ranges
theta1_range = arr2Rad(linspace(-160,160,50));
theta2_range = arr2Rad(linspace(-225,45,50));
theta3_range = arr2Rad(linspace(-45,225,50));
theta4_range = arr2Rad(linspace(-110,170,5));
theta5_range = arr2Rad(linspace(-100,100,5));
theta6_range = arr2Rad(linspace(-180,180,5));

test_map = containers.Map({'theta1', 'theta2', 'theta3','theta4', 'theta5',
'theta6'}, ...
    {theta1_range, theta2_range, theta3_range, theta4_range, theta5_range,
theta6_range})
```

test_map =
  Map with properties:

        Count: 6
      KeyType: char
    ValueType: any

```
% Workspace plotting function + timing
plot3dworkspace(test_dh, test_map, @get_alternative_dh_transform, true)
```

final transformation matrix from base to end-effector:

$$
\begin{pmatrix}
-\sin(\theta_6)\,\sigma_3 - \cos(\theta_6)\,\sigma_8 & \sin(\theta_6)\,\sigma_8 - \cos(\theta_6)\,\sigma_3 & \sigma_{10} - \sigma_9 + \dfrac{\sqrt{2}\,\sin(\theta_5)\,\sigma_{17}}{2} & 431\cos(\theta_1)\cos(\theta_2) - \sigma_2 - 2 \\[2em]
-\sin(\theta_6)\,\sigma_4 - \cos(\theta_6)\,\sigma_7 & \sin(\theta_6)\,\sigma_7 - \cos(\theta_6)\,\sigma_4 & \sigma_{12} - \sigma_{11} + \dfrac{\sqrt{2}\,\sin(\theta_5)\,\sigma_{16}}{2} & \sigma_2 + 431\cos(\theta_2)\sin(\theta_1) - 2 \\[2em]
\sin(\theta_6)\,\sigma_5 - \cos(\theta_6)\,\sigma_6 & \sin(\theta_6)\,\sigma_6 + \cos(\theta_6)\,\sigma_5 & \sigma_{13} - \sigma_{14} - \dfrac{\sqrt{2}\,\sin(\theta_5)\,\sigma_{15}}{2} & \dfrac{431\cos(\theta_2)\sin(\theta_3)}{2} - \dfrac{431}{} \\[2em]
0 & 0 & 0 &
\end{pmatrix}
$$

where

$$\sigma_1 = \frac{139\,\sqrt{2}\,\sin(\theta_1)}{2}$$

$$\sigma_2 = \frac{431\,\sin(\theta_1)}{2}$$

$$\sigma_3 = \sigma_9 + \sigma_{10} + \frac{\sqrt{2}\,\cos(\theta_5)\,\sigma_{17}}{2}$$

$$\sigma_4 = \sigma_{11} + \sigma_{12} + \frac{\sqrt{2}\,\cos(\theta_5)\,\sigma_{16}}{2}$$

$$\sigma_5 = \sigma_{13} + \sigma_{14} + \frac{\sqrt{2}\,\cos(\theta_5)\,\sigma_{15}}{2}$$

$$\sigma_6 = \cos(\theta_5)\,\sigma_{23} - \sin(\theta_5)\,\sigma_{15}$$

$$\sigma_7 = \sin(\theta_5)\,\sigma_{16} - \cos(\theta_5)\,\sigma_{21}$$

$$\sigma_8 = \sin(\theta_5)\,\sigma_{17} - \cos(\theta_5)\,\sigma_{19}$$

$$\sigma_9 = \frac{\sqrt{2}\,\sigma_{18}}{2}$$

$$\sigma_{10} = \frac{\sqrt{2}\,\sin(\theta_5)\,\sigma_{19}}{2}$$

$$\sigma_{11} = \frac{\sqrt{2}\,\sigma_{20}}{2}$$

$$\sigma_{12} = \frac{\sqrt{2}\,\sin(\theta_5)\,\sigma_{21}}{2}$$

$$\sigma_{13} = \frac{\sqrt{2}\,\sigma_{22}}{2}$$

4

```
expressions for position:
x = 431*cos(theta1)*cos(theta2) - (431*sin(theta1))/2 - 20*cos(theta3)*(cos(theta1)*cos(theta2) - (2^(1/2)
y = (431*sin(theta1))/2 + 431*cos(theta2)*sin(theta1) - 20*cos(theta3)*(cos(theta2)*sin(theta1) + (2^(1/2)
z = (431*cos(theta2)*sin(theta3))/2 - (431*sin(theta2)*sin(theta3))/2 + (139*2^(1/2))/2 - (431*2^(1/2)*sin
allocating 15625000 possible DH parameter combinations...
calculating positions from expressions...
```


Reachable Workspace
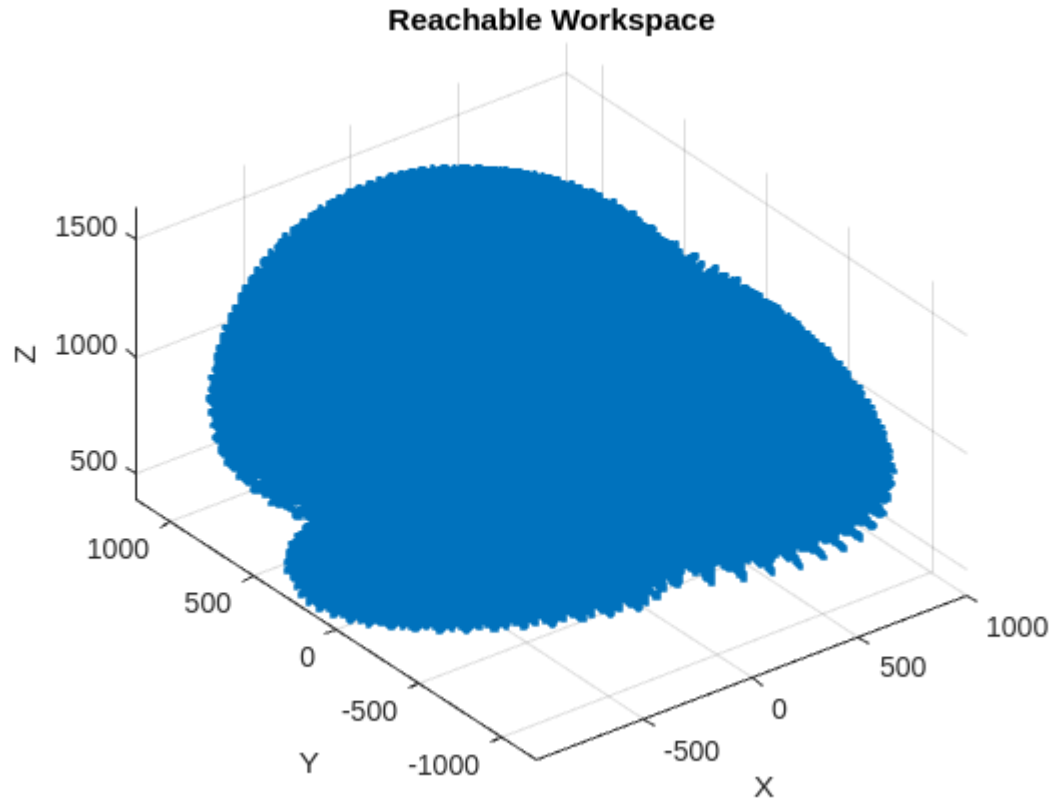
```matlab
% A somewhat convoluted PDF export procedure to save a PDF of the .mlx file
% for github. You can ignore this.
file_name = 'plot3dworkspace_examples';
current_mlx = which(file_name);
[path_to_file, name, ext] = fileparts(current_mlx);
mlx_path = fullfile(path_to_file, (file_name + ".mlx"));
pdf_path = fullfile(path_to_file, (file_name + ".pdf"));
export(mlx_path, pdf_path);
```

```matlab
function out = arr2Rad(A)
    out = arrayfun(@(angle) deg2rad(angle), A);
end

function T = get_alternative_dh_transform(a,alpha,d,theta)
T = [cos(theta) -cos(alpha)*sin(theta) sin(alpha)*sin(theta) a*cos(theta)
     sin(theta) cos(alpha)*cos(theta) -sin(alpha)*sin(theta) a*sin(theta)
     0 sin(alpha) cos(alpha) d
     0 0 0 1];
```

```
end
```