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Philosophical Challenges to Imprecise Probabilism

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Chapter 1

Introduction

1.1 Formal epistemology and Bayesianism

Formal epistemology is a branch of philosophy that applies logic and probability to a variety of epistemological issues in a spirit of analytic philosophy. Generally speaking, the property that distinguishes formal epistemology from the regular one is the wide use of mathematical methods (Bradley 2015). One of the issues that is a subject of formal epistemology is belief representation, and a very compelling theory of it is Probabilism. Beliefs are usually indirectly linked with agent's behaviour, they are a type of agent's attitude to something, they can be known by observation of behaviour or by utterances. The most common way of thinking about the objects of beliefs is taking them as propositions or sentences, and an agent's attitude towards these objects are her beliefs. Such theories are trying to give an account of what an ideal, fully rational agent's beliefs ought to look like. A very prominent theory of belief representation that adopts probabilism as one of its rational constraints is Classical Bayesianism.

One of the challenges that belief representation theory faces is uncertain reasoning, in which an agent does not possess full beliefs in propositions. In such situations, the agent is unable to answer with a binary yes or no to questions such as, "Will a tossed fair coin land on Tails?"

An agent in such a situation cannot be sure of that, her belief is partial. The proponents of the Bayesian approach will require her to have probabilistic beliefs to stay rational.

However, with that constraint, comes the requirement that all agent's beliefs ought to be probabilistic, which is sometimes problematic, especially in situations involving incomplete or non-existing evidence. One of the answers that the challenge of assessing rational credence in situations of vague evidence sparked, is the idea of using imprecise models, which assume that instead of representing agent's beliefs with precise values, an interval can be used.[1] This idea led to a fruitful discussion among the scholars. Some of them claim that imprecise probability is an intuitive way of modeling beliefs. Others were less optimistic and claim that this approach violates some other rationality principles. In order to delineate what imprecise probability is, how it can be argued for, and what challenges it faces, I shall first present the wider theoretical background of that concept, Bayesianism.

The Classical Bayesian approach to formal epistemology is the framework that emerged as an epistemological movement in the 20^{th} century sparked by first formal axiomatizations of probability (mainly the one introduced by Andrey Kolmogorov in 1933). Bayesian inference not only gained a lot of interest in philosophy (especially in epistemology, philosophy of science and philosophy of economics) but also in empirical sciences such as physics, statistics, psychology and economy. Bayesianism is a wide term that originally meant an approach that uses the formal apparatus (Talbott, 2016) in inductive logic and pragmatic self-defeat test as a method of evaluation of an account of rationality. The Classical Bayesian approach to epistemology, or as it is called *Subjective Bayesianism* is based on two pillars: probabilism and conditionalization.

[1] A set of probabilistic functions rather than one function, I will cover this more precisely later on.

1.2 Probabilism[2]

The fundamental assertion in Bayesianism is that beliefs come in degrees, in opposition to *Acceptance*, the seemingly reasonable view that an agent simply can either accept, deny or be undecided about a belief. On the Bayesian approach, to have a belief is to have a certain degree of belief in some statements, it is not a matter of having or not having a certain belief, it is a matter of being more or less confident that a statement is true. Of course, there are some claims of which you are totally certain. For example, if I ask you whether "2+2=4" is a true statement, you are tempted to answer enthusiastically "Yes, I am sure for 100%." This example shows that even in natural contexts on an everyday basis we think in terms of being more or less confident, and of course, being totally sure of something can be easily modelled by the use of degrees of beliefs. The most popular way of modelling credences[3] is by using a numerical value that lies between 0 and 1. Where 0 stands for fully believing that something is false and 1 that something is true (property of being totally confident). It can be portrayed formally as a function, where an input is a proposition and an output is a value that represents agent's attitude to that proposition, as follows: $P(A) = y$ where y is a real number between 0 and 1.

The claim that beliefs come in degrees seems intuitive, but if we want to have a theory which gives us a sufficient account of belief, we need to emphasize some constraints on rationality that will not allow for situations such as the following one:

> Michael ought to bet on a fair coin, he is confident to the degree of 0.8 ($P(H) = 0.8$) that Heads will win, when simultaneously being confident to the degree of 0.5 ($P(H) = 0.5$) that Tails will win.

A well-argued and accepted constraint that does not allow agent's to have such beliefs is

[2] "Probability" refers to probability calculus, "probabilism" has a different meaning, these two words are not synonymous. Probabilism is a doctrine that beliefs ought to be modelled with probabilistic functions.

[3] I will use "credences" and "degrees of beliefs" interchangeably throughout this work.

Probabilism. According to this principle, credences need to satisfy the axioms of probability calculus. This requirement not only does not allow for irrational beliefs such as Michael's, but also it opens new possibilities. With probability comes the possibility to apply tools such as Bayes' theorem and it is possible to derive and apply more principles. There are 3 basic probability axioms formulated by Kolmogorov (Peterson, 2009, p. 120):

Nonnegativity[4]	$1 \geq P(A) \geq 0$
Normalization	P(**T**)=1
Finite additivity	If $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$

Proposition A is the set of states which make it true, A is a subset of Ω which is the set of all possible worlds under consideration. P is a function that for any proposition returns an output which is a probabilistic degree of belief in that proposition. An agent whose belief is represented by $P(A) = 0.4$ is certain to the degree of 0.4 that A is true. If this agent does not want to violate probability axioms, she ought to be certain to the degree of 0.6 that A is false.

Nonnegativity says that probabilities cannot be larger than 1 and smaller than 0, which is intuitive, because 0 and 1 stand for "not probable" and "absolutely certain", respectively. *Normalization* says that an agent ought to be fully confident about the truth of tautologies, propositions that are true in every possible interpretation such as "It will snow tomorrow, or it will not snow tomorrow." *Finite additivity* says that if A and B are mutually exclusive (they cannot both be true), then the probability of "either A or B" is equal to the sum of the probabilities A and B. If you have a belief to the degree of 0.3 that it will rain tomorrow and you have a belief to the degree of 0.4 that it will be sunny tomorrow, then your probability that it will either be sunny or it will rain tomorrow ought to be 0.7.

There are many arguments for probabilism (Bradley, 2015; Titelbaum, 2015), I will present one of them, because it concerns the topic that will play a crucial role later on.

[4]Probability is defined here over Boole's algebra with relations of intersection and union, it can be defined differently e.g. over propositional logic, but the main idea stays the same and the differences are not essential in this context.

The accuracy argument for probabilism

This argument is motivated by accuracy-centred epistemologists, who attempt to derive all rationality principles for credences from norms related to the notion of accuracy. This kind of approach is called *veritism*.

> "According to veritism, there is a single fundamental source of value that is relevant to the epistemic evaluation of credences-it is accuracy."
> (Pettigrew, 2016, p. 10)

Accuracy of a credence function is usually a numerical value that represents credence closeness to truth, in this interpretation it is called alethic accuracy. Scoring rule is a method of measure that takes a credence function and a truth-value of a proposition in a certain world as its input and gives as an output the accuracy score of this credence according to how close it was to the truth. According to the veritism, accuracy is the only source of value for credences. The more accurate credence is, the more value it has. A proposition's truth-value is represented numerically by 1 when it is true, and 0 when it is false. If a proposition A turns out true, then credence 0.4 is less valuable than credence 0.6, because the latter one was closer, it was more accurate. But how to measure accuracy?

There are many different methods of measuring accuracy of credence. One that I shall present measures gradational accuracy (Titelbaum, 2015). The most common measure method, strongly advocated by Joyce (1998) is based on Squared Euclidean Distance. This method of evaluating credences accuracy, gained popularity because it satisfies constraints which according to many, defines reasonable scoring rule.[5] This measure is called the Brier score.[6]

$$B(c,w) := \sum_{X \in F} |v_w(X) - c(w)|^2$$

[5]These constraints on a reasonable scoring rule will be presented in the Chapter 3.

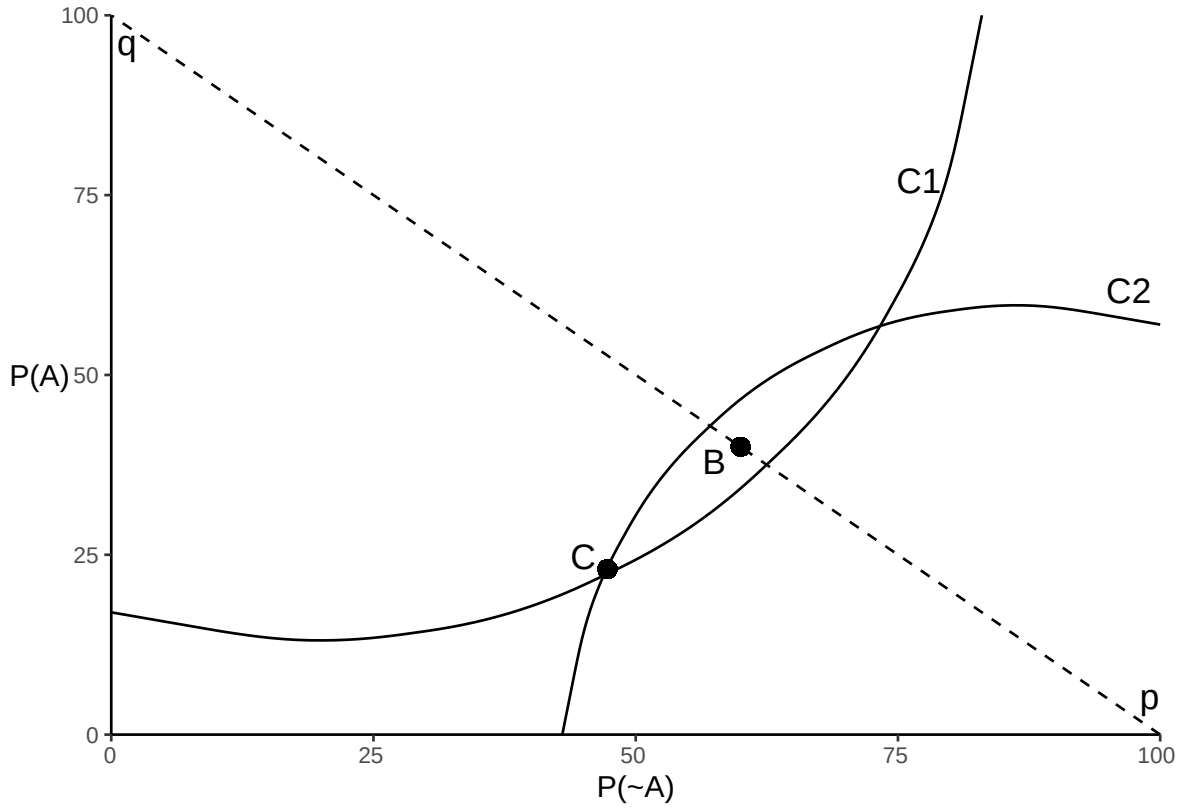
[6]This method of scoring was first formulated by Glenn W. Brier, he proposed it as a method of scoring weather forecasts (Pettigrew, 2016).

The Brier score measures a credence function's inaccuracy (or accuracy) in a certain world. The difference between accuracy and inaccuracy comes from the way as we look at the score. If we perceive the score as a representation of inaccuracy, the larger the score is, the more inaccurate a credence is, and if we look at it as a representation of accuracy, then the lower the score is, the more accurate it is. The Brier score takes the set of F of propositions to which a credence is assigned, for each proposition in F it takes the difference that comes from the subtraction of the credence from the truth-value of a single proposition, then the score is squared and summed with other proposition scores which are in the set F. Credences are represented by c, which are compared with the proposition truth-value $v_w(X)$ in a certain word w. F is the set of all propositions.

Another piece of the puzzle is the notion of *Dominance*. A set of beliefs A is dominated by a set of beliefs A' when A' is more accurate (has higher accuracy score) than belief set A in every possible world. Holding a dominated set is, of course, irrational, because there is a set of beliefs that is always better than the dominated one. The idea is to show that if a set of beliefs violates the rules of probability, then it is dominated by a set of beliefs that do not violate them. So if you want to stay rational, you ought to hold beliefs that do not violate the rules of probability.

A partial belief's accuracy is a matter of degrees, an agent needs to balance her degrees of confidence in a way that her accuracy score is as good as it can be, at least from her perspective. As it turns out, it is knowable a priori that for every nonprobabilistic belief there is a probabilistic one that dominates it in every world: it gives a higher accuracy score regardless of how the world turns out to be. The following plot is a visualization which proves that in a situation of assessing the degrees of beliefs regarding two opposite propositions, any nonprobabilistic belief is dominated by a probabilistic one.

Figure 1



Point q, which is on the one end of the dotted line, represents the belief to the degree of 1 that A is true and ~A false, p stands for belief to the degree of 1 that negation of A is true and A false. The dotted line between them represents the set of probabilistic beliefs, any point on that line will be probabilistic, so that its x and y values will sum up to 1 (e.g. point B $(0.6, 0.4)$). The axes stands for the agent's credences in ~A and A. Point C at $(0.45, 0.25)$ represents credence 0.45 in ~A and 0.25 in A. It is below the probabilistic line and its values do not sum up to 1. Curve 1 shows the exact same distance from point q that C has, Curve 2 shows the same for point p. If A turns to be false, beliefs that are inside $C2$ will get higher accuracy score, and if A will be true it would be the best for the belief to be as close as it gets to the point q. The area where the curves overlap represents all potential beliefs that dominate C, they have higher accuracy than C in every possibility. Therefore, there is a probabilistic

belief that dominates C. The claim generalizes: for any nonprobabilistic belief, there is a probabilistic belief that dominates it.

Therefore, if an agent is an accuracy-centered epistemologist and agrees that the Brier score is a salient measure of accuracy, she needs to avoid nonprobabilistic beliefs if she does not want to be considered irrational.

1.3 Conditionalization

This is the second pillar of classical Bayesianism. This principle determines how beliefs ought to be updated if an agent wants to be rational. In what situations ought an agent to update her beliefs? When she learns some new knowledge that affects her earlier beliefs. For instance: Eva is a manager in a big company. She is impatiently waiting for the report from her co-worker, because the deadline is getting closer. Her credence that her co-worker will send her report on time is represented by c. Couple days before the deadline she got a massage that her co-worker is in hospital, therefore she adopts new credence c' that the report will be ready on time. According to the Conditionalization principle, her degree of belief after obtaining new evidence ought to equal her earlier beliefs conditional on that evidence.

$$P_E(H) = P(H|E)$$

According to the Conditionalization principle, the probability of H after obtaining the evidence E ought to be equal to the conditional probability of H given E, if belief update is to be considered rational.

Bayesians that claim that there are more than these two constraints on rationality are called objective Bayesians. Subjective Bayesians hold that these two pillars are sufficient. Objective Bayesians work on further constraints, they think that rationality ought to be more strict. This view was sparked by the problem of inductive reasoning. Consider the following example:

> Howard is a crew member on the Titanic ship. At the luckless night before the ship hit the iceberg Howard's degree of belief that Titanic wil not sink on its first cruise is 1. After the hit, suprisingly Howard's belief did not change, he still thinks that Titanic is unsinkable despite the ship going down.

What is the problem here? Howard's beliefs are probabilistic as they do not violate conditionalization principle. Therefore, according to Subjective Bayesians, his beliefs are rational. Objective Bayesians thinks that it is unacceptable to claim that beliefs like that are rational. The proponents of objective bayesianism claim that there is a relation of justification between evidence (that ship is sinking) and the proposition that it will sink. This relation is objective, and therefore if an agent ignores it, she can be considered irrational. This objective relation of justification has inductive character,[7] and that is the reason why Objective Bayesians are called inductivists.

1.4 Epistemic and pragmatic role

There is an important issue that I want to emphasize. All these models ought to be understood as normative rationality constraints, that is, these rules concern the behaviour of ideal agents. The evaluation of a regular agent's rationality is a different task, which concerns psychology rather than formal epistemology.

Credences play two main roles in our lives: epistemic and pragmatic. They represent our view on certain situations, which is the first role. The second one is responsible for guiding our actions. Different theories put more pressure on one role and less on the other. Certain principles or arguments are concerned more with how well an agent's doxastic state is being grasped by the belief representation model, or more with the possible outcomes of an action.

[7] These relations are expressed by inductive probabilities. These probabilities are normative, an rational agent's credences ought to match them. They are justified by the principles that objective Bayesians propose e.g. the Principal principle.

The Dutch book theorem puts more pressure on the pragmatic role, this theorem is used to argue for probabilism by showing that using nonprobabilistic beliefs make agent susceptible to a sure loss in a betting situation. The epistemic role, however, lies more in the domain of the accuracy-centered approach. Sometimes, there is a visible tension between these two roles, and imprecise probabilities are the example of that. The imprecise model proponents are mostly concerned with the epsitemic role, they claim that in a situation in which the evidence is vague and imprecise, credences ought to be also imprecise. The precise view proponents developed their own methods of coping with severe uncertainty, which does not invoke imprecise credences. The precise model proponents do not feel the need of representing vague evidence with vague credence, and the imprecisers disagree with them.

What many perceive as a principle of the orthodox Bayesian approach is that the agent's beliefs need to be always represented by probability functions. An agent needs to have credences in any proposition she considers, even in statements such as "It will be snowing on Christmas 2027 in London." It is at least problematic for the agent to justify any sharp credence in the truth value of that statement. What credences are rational to have in such situation? And on which role of credences ought the agent focus, on an accurate representation of his epistemic state or on maximizing the possible outcome? This kind of problems motivated epistemologists to develop the notion of Imprecise Probability.

1.5 Aims of the work

The aim of this work is the analysis of the imprecise probabilities as a new rational method of belief representation. The main part, which is contained in the third chapter, concerns the validation of imprecise credal states on purely epistemic ground.

Firstly, I present briefly the basic notions of Bayesianism: formal model of belief, credence, probabilism, conditionalization. This delineation makes further characterization of the im-

precise model possible.

In the second chapter I thoroughly describe what imprecise credences are, what the motivations of the imprecise view proponents are, how it can be argued for, and what challenges it has to overcome. I presented there two views on imprecise probabilities: that they can be rationally required or that they are only sometimes permitted. As a one way of objecting against imprecise probabilities I have presented Elga's sequential decision problem.

In the third part, I delineate an accuracy-centered view on epistemology and conduct an analysis of the imprecise standard view from that perspective. I have analyzed several recent articles that attempt to vindicate imprecise credal states, and others that present impossibility arguments. I was mainly focused on (Schoenfield, 2017), because it seems to me to pose the biggest treat for any impreciser. I also presented some possible answers to Schoenfield impossibility result.

At the end, I concluded that only a weaker version of the standard imprecise view can be defended on the ground of the accuracy-centered epistemology. This weakened version says that imprecise credal states are sometimes permitted, but never required by rationality. However, this claim is supported only by alternative, non-orthodox approaches to the accuracy-centered epistemology.

Chapter 2

Imprecise probability

2.1 Exploring the imprecise view

The idea of using imprecise credence values rather than regular, precise ones to describe agent's epistemic state was sparked by the difficulties that the standard Bayesian approach meets, mostly incomplete (non-specific) or non-existing evidence. The proponents of imprecise probability claim that rather than using a single value to describe the degree of belief of an agent, a set of probabilistic functions should be used instead.

The Classical Bayesian probabilistic model of belief is governed by the idea that credences are best represented by a single numerical value. This is fairly clear in cases in which the evidence can be easily represented formally, such as a coin toss case (when the coin is fair). In other cases, it can be problematic for an agent to describe their attitude towards certain statements that are not so specific, such as "Tomorrow there is going to be a traffic jam on my way to work." My evidence that usually there is a traffic jam at that time supports having a high credence value in that statement, let's say around 0.8, but it seems problematic to be very specific about that value. Why not 0.81? It is almost impossible for the agent to catch a

difference.

To be fair, in the example as such, the problem with justifying precise credence is not an unacceptable flaw. The main function of formal apparatus which is used to represent agent's belief is to enable work with agent's doxastic state. It is a challenge to accurately grasp the agent's psychological content that represents her beliefs (Joyce, 2010). Abductively speaking, the standard probabilistic model seems to be the best available approach to representing uncertain beliefs, but some scholars claim that there are situations where there is a better way to model credences. This new method is sparked by two difficulties that the standard, precise view meets: incomplete and non-existing evidence.

What is a consequence of Bayesianism is that any epistemic state towards a proposition ought to be probabilistic if an agent is to stay rational. With probabilism comes the claim that any agent's belief ought to be represented by a single-valued function that reflects her epistemic state[1] (Mahtani, 2019).

Sharp probabilism sounds intuitive, in many situations people act subconsciously like in a betting situation. But what about situations that look impossible to bet on, where our betting intuition seems to fail? Some people find using sharp degrees of beliefs unrealistic, because agent's everyday basis beliefs are mostly vague and not definite, and situations in which it is possible to have completely single-valued credences, are rare and happen mostly at casinos.

In my work I focus on the normative approach to rationality, which stands in opposition to the descriptive approach. This means I am interested in analyzing an ideal agent's doxastic states, or how an agent should behave to be considered rational. An agent ought to aspire to be as close to the ideal agent as possible. The Dutch book argument supports that by showing that an agent which does not have probabilistic beliefs is susceptible to a sure loss in a betting situation. Probabilism can be also argued for from an accuracy-centered position (Staffel, 2019).

[1] The issue how accurately her epistemic state ought to reflect the evidence is a different problem. Objective and subjective Bayesians argue over that.

Defining incomplete and non-existing evidence

Scholars (Elga, 2010; Ellsberg, 1961; Joyce, 2005) usually mention problems in which pieces of evidence are unspecified or there is a lack of any evidence. Both situations are pretty similar and often are used to pose the same conclusion that a precise probability model should be abandoned and replaced with the imprecise model.

Problems of this kind are characterized by vague chances, an agent is not able to detect them and represent them with a standard probability function. There may be chances that are far to being sharp, as in the following example.

You are in a betting situation where the coin will be tossed. The only piece of evidence you have is that the probability of Heads is between 0.2 and 0.8. Or there may be no chances at all, where the only piece of evidence you have is that the coin is biased to an unknown degree. Other popular examples invoke having credences in statements such "It will be raining on 27.05.2027 in London." There is a visible difference between not having evidence at all and insufficiently specific evidence, but they both invoke a doxastic state of an agent who has a hard time working with sharp probabilistic values that match their belief state. I will be relying on what scholars called incomplete evidence scenarios, but it should not be any loss because of the similarities with no evidence scenarios. I would not necessarily perceive the example with London as no evidence problem, in situations as such an agent is always trying to refer to vestigial evidence or she is trying to help herself with the knowledge from the past e.g. one might think about the fact that it is often raining in London, which can be some kind of evidence. I think that the London example is only seemingly a non-evidence problem.

Example of non-existing evidence can be as such "Imagine a game show where you are competing with other contestants over the mystery prize, which is hidden behind the doors. What makes this show original is that no one knows what the prize is." In this scenario you are not able to have any evidence about the proposition A that, a single flip-flop is the prize.

The difference between these two problems, although visible, will not be considered significant in this work.

2.2 Problems with precise probability

Imagine that on a Sunday walk in the park you notice that there is a small gathering around a wooden bench. You observe a man in the middle who pulls out one coin from a large bag full of coins and then he takes bets from the people around. All coins in the bag are biased to an unknown degree, the probability of Heads up can be any number between 0 and 1.

What beliefs are rational in this situation if you want to bet on Heads? Beliefs ought to somehow reflect the evidence, that is why imprecise probabilists claim that when the evidence is vague and not specific your credences should be too. The advocates of the precise model have a different answer, they hold on to the rule that beliefs ought to be represented by a single probabilistic value. The main focus is what credences an agent should have to act rationally, the precise view proponents intention seem to be very pragmatic in such a scenario.

The proponents of precise probability invoke a probability density function in such scenarios. This function invoke a probability distribution, which helps to estimate an objective bias of a proposition (the real bias of a proposition within the known range). It takes as an input a range of the bias and returns expectation of that objective bias which constitutes credence that an agent should adopt. Probability density function output depends on distribution that is used, standard approach emphasized by the objective Bayesians invoke uniform distribution.

In our example we have no evidence what bias b can be, so we need to assume that it can be any number between 0 and 1 ($0 \leq b \leq 1$). From a definition of probability density function we have $\int_0^1 f_c(x)dx = 1$, where c is a person credence towards H, which is a proposition (Joyce, 2010). But there is no information on how chances of Heads are distributed across all biased coins, we have no evidence about it. What should be here assumed is a controversial topic for objective Bayesians. Subjectivists says that it is completely up to the agent's doxastic state, that no evidence supports any distribution, so any distribution that an agent takes into account is rational. For the subjectivists it is rational to use uniform distribution so $f_u(x) = 1$ then $c_u(H) = 1/2$, or completely random one such as $c(H) = 1/4$, if someone thinks that b

is usually a high value. Subjectivists do not see any problem with that in a situation where no evidence could support one distribution over the others. Objectivists find it completely irrational because there is no reason to favour any possible bias, therefore probabilities should be assigned symmetrically.

The reason why objectivists claim this, comes from the *Principle of Indifference*,[2] according to which, if there is no evidence that supports favoring one possibility over the others, each possibility should be assigned equal probability. So if there are n possibilities, an agent needs to have credence $1/n$ in any possibility if there is no meaningful evidence. In our example, this reasoning supports having the degree of belief $1/2 = 0.5$ in Heads.

Objective Bayesians always need to favour one credence distribution over the others, this favoured option has status of being normative. If the agent's degree of belief would be different than this normative credence distribution, then she can be considered irrational. For that reason they need a principle that will regulate evaluating beliefs in situations of vague or non-existing evidence. Subjective Bayesians do not feel a similarly strong urge to emphasize the importance of matching chances. Subjectivists allow for a subjective perception of evidence, and for this reason they do not accept the Principle of Indifference.

Common objections to the Principle of Indifference indicate that there is sometimes more than one way of assigning credences. Those who criticize the principle try to show that the objectivists by assigning credences symmetricaly at the same time have to choose one way of dividing the possibilities.

This argument can be made clear by means of the following example. There are three balls in an urn, they have all different colors: green, red, and blue. One ball will be randomly drawn from the urn.

A common, seemingly privileged, way of dividing possibilities in this example is to assign equal probabilities, $1/3$, to every ball, because three possible events can happen: either blue, green, or red ball can be drawn. But this can be done differently. There are also two possible

[2]Principle of Indifference is justified by its proponents by maximum entropy principle, more on that in Landes and Williamson article (2013).

events: one that an agent will draw the red ball, and the other that the drawn ball would not be red. If we assign equal probabilities to these possibilities, then the probability of drawing the red ball is equal $1/2$ and not getting it also equals $1/2$.

Of course, objective Bayesians can argue that the suggested division of alternatives is very counterintuitive and unjustified, but indicating which principle justifies the choice of a division is notorously hard. The difficulty of developing a system of inductive probabilities based on the Principle of Indifference is illustrated by the work of R. Carnap whose attempt failed due to similar challenges (Bradley, 2015).

In theory, the objectivists tried to minimize the informative cost of agent's credences, claiming that a symmetrical probability of possibilities should be less expensive than any other way. An agent's credence 0.75 in Heads in our earlier example is more informatively expensive than a symmetrical distribution, because an agent needs more evidence to justify it.

But for the proponent of imprecise probability, the symmetrical distribution of credence is still informatively expensive. Your credences ought to represent your belief state. If you adopt credence 0.5 in Heads in a situation of incomplete evidence, you seem to be more certain about that distribution than about any alternative. But this is not really true. You adopted this credence, because of the Principle of Indifference, and because other options might seem equipossible for you. So the credence distribution suggested by the proponents of precise probability does not reflect your epistemic state, you adopted it because it is an allegedly rationally practical thing to do.

If an agent adopts precise credences in the case of incomplete evidence, she has a belief that normally is built upon much more solid evidence. Precise credences does not carry information about how strong evidence they were build on. Therefore, credence that is well justified by evidence is indistinguishable from credence that does not stand on a solid ground. An observer watching her bets would not be able to guess if she has complete or incomplete evidence, e.g. in a scenario of betting on a fair dice and dice with an unknown bias. The imprecise probabilists hold that in a situation in which the evidence is vague or imprecise,

the agent's credences also ought to be vague and imprecise.

2.3 Standard imprecise model

The standard imprecise model is usually perceived by the scholars (Kaplan, 1996, 2010; Levi, 1985) as follows. This alternative model is not very different from the precise one, it still concerns probabilistic functions and real numbers. But instead of using a single probability p to represent one's belief state, a set of functions P is used instead. Functions in this set need to be probability functions if an agent wants to be considered rational. Collectively, all the functions $P = \{p_1, p_2, \ldots, p_k\}$ represent the agent's epistemic state. This probability set can be thought of as a credal committee, van Fraasen (1990) calls it one's *representor*. According to this perspective, each function represents one member of a committee, all of them represent the agent's epistemic state. Of course, like in real life, committee members can disagree, this is captured by the set of functions that are different to each other.

At first glance, using a set of probabilistic functions to represent your epistemic state might look unintuitive, especially when your degree of belief concerns a situation in which you can easily have precise credences, when the evidence is solid. But a precise credence, which normally is represented by a single function, can be easily translated into a set of functions. In such a situation, all functions in the set assign the same credence to the proposition. Using the representor rather than a regular probability function needs to have some positive features that the standard model does not have, otherwise using this apparatus would be an unnecessary complication.

Imprecise model is capable of representing comparison of the beliefs, such as: my all credence functions assign a higher value to A than to B. Standard, precise model is unable to comper degrees of belief in two propositions without a need of assigning them precise credences. But the main feature of imprecise model is the possibility of thinking of credences as intervals e.g. An agent believes to the degree of no more than 0.6 that C. In this example,

probabilistic functions in the set that represent the agent's belief assign different values to C, but they are not larger than 0.6. Therefore the interval that reflects the agent's belief is $(0, 0.6)$. Thanks to the possibility of representing beliefs with the set of functions, an agent with severely vague doxastic state does not have to rely on the precise model which would not grasp her uncertainty.

Let us go through some examples:

1. *The coin will be tossed, you collected the evidence that the coin will be biased. But all you know about the bias is that it can reach from* 0.3 *to* 0.7 *for Heads.*

If we ask the impreciser what credences in H she would have in such a situation, she would answer that her credence would be represented by the interval $(0.3, 0.7)$. Not by any single number within that interval but by the whole set of probabilistic functions that assign those values to the proposition. A Standard Bayesian however would have to assign a single value, according to the rule that a rational agent ought to have precise value credence towards any proposition. She would very likely adopt the symmetrical value 0.5 if she turns out to be an objective Bayesian, or any other value within this interval if she were a subjectivist.

2. *The coin will be tossed, you do not have any evidence about the coin, you cannot tell if it is fair or biased, and to what degree it could be biased.*

In this scenario, a proponent of imprecise probability would recommend the $(0, 1)$ interval to represent the probability of getting Heads. There is no evidence, therefore there is no reason to favour any chance. An objective Bayesian would again opt for a symmetrical arrangement of chances, and having 0.5 degree of belief in Heads. A subjective Bayesian would be fine with any precise credence in this situation.

One important issue that is being discussed among the proponents of imprecise probability are the conditions of use. Is imprecise probability another constraint on rationality, or mere possibility that an agent can use? This discussion resembles the attitudes of Objective and Subjective Bayesians: two opposite approaches, one which is very strict and another which treats imprecise probabilities more liberally. Joyce (2005), for example, requires an agent

to have credences no sharper than the evidence requires, so he treats imprecise credences in some situations as rationality constraint: an agent needs to have imprecise credences in some cases to be considered rational. On the other hand, it is possible to claim that imprecise credences are not ever required, but that they are sometimes permitted. This is a more subjective view, according to which more than one answer might be rational.

There are many doubts about the latter view. The problems appears when we try to compare two different, permitted credence distributions over one proposition. According to the view that imprecise probabilities are permitted but not required in cases of vague evidence, how is the rationality of precise and imprecise credences to be compared? The first thought would be that they are equally rational. If so, then why do we need imprecise credences if the standard model covers rationally all cases with the same effectiveness? So maybe they are not equally rational. But this only makes things worse. If there are not equal, and imprecise credences are more rational, then why should the precise ones be tolerated? If imprecise credences are not more rational than the precise ones, then why do we need them after all?

The imprecisers who think that imprecise credences are sometimes required and their opponents come up with some regulative principles. The proponents of treating imprecise credences as a requirement of rationality propose the chance grounding thesis "One's spread of credence should cover the range of chance hypotheses open by the evidence" (Mahtani, 2019, p. 110). The idea is that credence ought to match known chances precisely, even if they are vague or imprecise. So if there is a coin with a bias between $(0.4, 0.7)$, the agent is required to have credences represented by the interval $(0.4, 0.7)$. Joyce argues that this claim is too restrictive, he wants to formulate it differently. According to him (Joyce, 2010), in some situations where there is more evidence then only an information about the chances, the Chacne Grounding Thesis can lead to unintuitve results. Therefore, an agent is required by rationality to cover the range of known chances with her credence, if it is the only evidence that the agent posses.

The imprecise probabilists who claim that imprecise credences are only sometimes permit-

ted represent a subjective view on credences. They give a lot of freedom to an epistemic agent. If there is no evidence, in Example 2, the agent is considered rational even if they have precise credence 0.7, 0.5 or 0.3 in Heads. The only rule is that the agent's credence in P ought to fit inside the known chances of P. If the chances of P are in the form of an interval, the credence of the rational agent needs to fit within that interval. It does not matter if the agent's credences reflect this interval (are imprecise), or whether they took the form of a single probabilistic value. As long as they belong to the interval, an agent is considered rational. This subjective view is susceptible to criticism from the objectivist ground, that these unjustified sharp credences are very informative, which is a flaw, because vague evidence does not support specific probabilistic assignments. Agents who adopt such a doubtful sharp credence act as if they wanted to bet on a specific probabilistic outcome. By having credence equal 0.63 in Heads when the coin with unknown bias will be tossed, an agent acts as if she was almost certain that in 1000 drops the coin will land on Heads in approximately 630 times. This looks like she believes that this particular scenario is more probable than any other possibility, the agent acts as if she had the evidence that can justify it, but she does not.

The view that I find more supported is the one which concern imprecise probability as a rationality constraint. So I will focus on the more discussed approach and associate standard imprecise probabilist with the view that imprecise credences are sometimes required by rationality.

Now I will define what Schoenfield calls the Standard imprecise view (Schoenfield, 2017). This definition of the imprecise view will help in the further characterization of imprecise probability. It is a generalization of the views of the standard imprecise probabilist.

The Standard Imprecise View

In cases of incomplete or no evidence, when no evidence supports a sharp model of credences, any degree of belief represented by a precise probabilistic value is irrational. In such scenarios, agents are required to represent their doxastic state with a set of credence functions rather than a single function.

So according to that view, imprecise probabilities are sometimes required. The proponent of this standard view will also accept weak version of Lewis's Principal principle (Lewis, 1980) that an agent's credences ought to match chances when they are known.

2.4 Motivations for imprecise probability

Imprecise probabilities have many advantages, and a wide range of applications in empirical sciences proves their utility. Imprecise models are used in statistics, mathematical economics, engineering, physics, computer science, and many more. Imprecise probabilities are used there as a tool for dealing with uncertainty, which is an interdisciplinary issue. Many disciplines rely on different interpretations of the imprecise model. This subject is beyond my work, but the main idea stays the same. In the empirical sciences, it is used as a tool in situations where there is not enough evidence for adopting precise values. Many scientist appreciate imprecise probabilities, because they give opportunities which are beyond the capabilities of the regular approach which invokes adopting poorly justified precise values. There are many motivations for using imprecise probabilities, in my work I will focus mainly on adopting imprecise probabilities as a part of rational model of belief. What is worth noting, an imprecise probabilist does not have to necessarily agree with all the motivations that I will mention (Bradley, 2019a, 2019b).

1. Indifference and incomparability

If precise probability distribution ought to represent an agent's preferences over acts it fails to do so in cases of severe uncertainty. The precise view requires completeness of preferences, an agent needs to adopt precise credences over any proposition that she is considering, these precise values allegedly represent her epistemic state. But this requirement cannot be satisfied in situations when preferences are not complete, an agent is having a hard time to decide what she prefers. In situation of severe uncertainty, the precise credence that an agent

ends up adopting does not reflect her epistemic state, which can contain indifference or incomparability. According to the imprecise probabilists, the behavioristic premise that the final precise credence that an agent adopts reflects her epistemic state is wrong, and therefore the requirement of completeness should be abandoned. Imprecise probability does not invoke this requirement, it can model uncertainty of an agent better than the precise model does.

Behaviourists claim (Bradley, 2019a) that states of severe uncertainty: indifference and incomparability between two possibilities, are one and the same kind of preference over acts. This claim is supported by the argument that there is no visible difference in an agent behaviour, regardless of these two states. Imprecise probabilists invoke *Insensitivity to small improvements* as the argument against that claim. According to that argument, the relation of incomparability is different to the relation of indifference. This difference is not visible in the precise probability model, but it can be grasped by the imprecise one. The indifference between A and B means that an agent does not prefer A over B or B over A, they are equally preferable for an agent. When it comes to the incomparability, an agent's indecision comes from a lack of determination to prefer one good over the other. Incomparable goods are insensitive to small improvements, when indifferent goods are not.

Insensitivity to small improvements

Imagine that there are two propositions that the agent is undecided about: A and B. The agent collects new evidence about A, now it is slightly more appealing. We will mark the proposition A that is now slightly more attracitve for an agent as A^+. If the agent is still unable to decide between A^+ and B, after this small improvement, then the propositions are indifferent for the agent. If she now prefers A^+ over B, then these propositions were incomparable for her before small improvement (Bradley, 2019a).

2. Group beliefs

Imprecise model can be a powerful apparatus for modeling group beliefs. The precise model struggles with representing group opinions with a single value function. To model

group beliefs the precise view proponents use symmetrization methods. These methods are called "Opinion pooling methods", most of them does not invoke the imprecise probability framework. Standard opinion pooling approaches invoke average or weighted average of single beliefs in a group. The imprecise model of beliefs is an alternative approach. Imprecise probabilities are perceived in this case as a credal committee with members that represent single opinions, group belief is represented by a set of opinions. It is not trivial what strategy deals with group beliefs uncertainty better. For sure, an imprecise model gives some new possibilities and it is a promising body of research.

3. Suspending judgment

Precise probabilists are unable to capture the difference between the suspension of judgment and position of full decisiveness that supports precise credence. The proponents of precise model need to satisfy the rule of having precise credence about any proposition, there is no room for suspending judgment on the standard, precise view. In a situation in which suspending judgment seems reasonable, a precise probabilist would advocate using a uniform distribution. If you are unable able to bet, the best you can do, will be to have symmetrical distribution of probabilities. Therefore the standard view is not capable of modeling the suspension of judgment. Think about the following example inspired by A. Elga (2010):

> Imagine that you observe a man on a park bench, he has a large bag next to him. Suddenly he starts pulling objects from his bag. The first three objects that he pulls out are: a jar of honey, a dead pigeon, and a bar of olive soap. To what degree should you believe that the next thing that he pulls out will be a another bar of olive soap?

It is an obvious case where there is no evidence that can support the proposition whatsoever. But a precise probabilist is entailed to have precise credence in that proposition, she would probably end up with having credence 0.5 (or any other precise credence if she is a subjectivist). The advantage of the imprecise model is that it can deal with suspending judgment,

you can have a set of probabilistic functions that are inconclusive and they can be represented, for example, by the interval $(0, 1)$. Hereby, imprecise probability model is better in modeling an agent's epistemic state of absolute uncertainty, precise model is unable to represent suspension of judgment so accurately.

2.5 Problems with imprecise probability

Imprecise probability should not be perceived as a rival to the standard Bayesian view. The imprecise model is proposed to solve some difficulties that the precise view struggles with, and as I mentioned earlier, imprecise model proved its usefulness in empirical sciences (Bradley, 2015).

There are many problems which concern imprecise probability. In fact, the core element of my work will concern the challenges that imprecise probabilist need to face if she is an accuracy-centered epistemologist. But these problems will be presented in the next chapter, now I shall present briefly a critique of this model from a different point of view.

Sequential decision problem

Elga in his paper "Subjective Probabilities should be Sharp" (2010) argues that the imprecise probabilist is susceptible to taking bets that are dominated, and choosing a dominated action is irrational. Elga exposes alleged flaw of the imprecise model when it comes to taking sequence of bets. He demonstrates his objection using the following example:

> "**Bet A** If H is true, you lose \$10. Otherwise you win \$15.
> **Bet B** If H is true, you win \$15. Otherwise you lose \$10."
> (Elga, 2010, p. 4)

These bets are offered sequentially, but an agent is aware of the second bet. Firstly she is offered Bet A, which an agent can reject or accept, and then Bet B. An agent can take two bets

or not to take any. Therefore, four possible combinations are possible $\{A, B, AB, \emptyset\}$. What is important, an agent has no information about the probability of H coming out as true (or false). If an agent is an imprecise probabilist, she could represent her uncertainty of H with the interval $(0, 1)$. The precise probabilist however, would have to adopt a precise credence of H.

Apart from credences, which are guiding agent's actions there is also *expected utility*. To any action an agent assigns her expected utility, which is commonly measured numerically and linearly. The utility score depends on expected gain or loss of the action which is correlated with the state of the world. In one world an action can have high utility score and in the other much lower. In our example money play the role of utility points, expected utility of Bet A is 15 if H is false, and -10 if H turns out to be true. What is important, not choosing any bet has expected utility equal 0 in both worlds, and choosing both bets is a guaranteed gain, because in every world expected utility of that action equals 5. Therefore, choosing both bets dominates not choosing any bet. If maximizing expected utility is a rule that an rational agent ought to obey, then choosing dominated action of not choosing any bet is irrational. Elga claim that imprecise probabilist is susceptible to choosing this dominated action, therefore in situations as such the imprecise model should be abandoned.

According to Elga this is the example of a sequence of bets, and not a choice between four options that I have mentioned. By allowing an agent to have imprecise credal states, it makes her susceptible to make sequence of decisions such as, not taking Bet A and not taking Bet B. An imprecise agent, has members of her credal committee, that suggest rejecting Bet A, when it is presented lonely, and members that suggest rejecting Bet B after the first decision. So an agent can end up with irrational (dominated) sequence of decisions. Elga argues that if a sequence of actions is irrational then the decisions within the sequence are also irrational.

Elga's argument is not very strong, it only demonstrates susceptibility to choosing dominated sequence of decisions in a specific situation, it is not a type of general argument. Claim that the imprecise model should be completely abandoned because of this flaw would seem

unfounded. However, Elga's objection demands an answer from the proponents of imprecise probability.

One way to disagree with Elga is to claim that theses decisions are not separate actions, but a single decision between four mentioned possibilities (Weatherson, 2003; Williams, 2014). Elga emphasizes that an agent's epistemic state does not change within the time of two decisions, and also an agent is aware of the second bet. Therefore she can perceive it as a one decision, which she makes by answering to two offered bets.

Bradley and Steele (2014) argued that some of the principles that Elga use are in fact not true, and what follows from that is the claim that an impreciser in such situation is not susceptible to rejecting both bets. They criticize the principle that rationality of a sequence ought to be evaluated retrospectively, and not from the agent's perspective. Also, they were opposed to the view that decisions ought to be made by considering rationality at a time, by cutting out the consideration of the second action in a scenario of a sequence of decisions.

Other scholars propose even different answers, some claim that an agent in such situation would make a plan which would make her resistant to choosing dominated action (Bratman, 2012; McClenen, 1990). There is also an argument that rejecting both bets does not have to be consider severely irrational (Moss, 2015).

Chapter 3

Standard imprecise view and accuracy

3.1 Accuracy-centered perspective

In this chapter, I will try to delineate the relation between imprecise credal states and the accuracy-centered approach to epistemology. The main question is whether imprecise credal states can be justified based solely on epistemic utility, and why justyfing it is so problematic.

The accuracy firsters claim that accuracy is the main source of epistemic value, in the standard approach they consider alethic accuracy. The pursuit of truth is, for the proponents of that view, the pursuit of accuracy, being as close to the truth as possible. Many norms that are derived from the accuracy-centered approach do not directly guide an agent to be more accurate, they work negatively. Violating them leads to unintuitive or strictly irrational results.

However, imprecise credences are originally based on the evidential premise, that they reflect vagueness of evidence better than precise credences. The accuracy-centered approach is concerned with evidence only secondarily because following the known chances leads to adopting more accurate credal states (Principal Principle). If a valid argument for the standard imprecise view could be constructed on the accuracy-centered ground, then it would mean

that having imprecise credences in some situations leads to a higher accuracy score (alethic accuracy). There is no measure of being accurate to chances, at least not in a standard interpretation. Vindicating imprecise credal states requires proving that they lead to more accurate beliefs.

Accuracy firsters (Joyce, 1998; Pettigrew, 2016) provided some respected constraints on rationality like: Probabilism, Conditionalization, Principal Principle. The proponents of imprecise credences would like to fit the imprecise standard view among them. Before delineating arguments and objections I will try to present more thoroughly what accuracy-centered epistemology is.

The accuracy-centered doctrine contains many principles and constraints on rationality. Of course, there is a constant discussion between proponents about what principles are valid. All theories need to contain a scoring rule which measures the accuracy of credal states. There are many constraints on these rules, the most popular rule is the Brier score, because it meets the most advocated constraints. But it should be remembered that there is a set of such scoring rules, which meet certain constraints. I will present shortly some principles advocated by Joyce (1998, 2009), which is one of the most recognizable proponents of this account. A scoring rule that satisfies them can be considered reasonable.

Extensionality – "For every truth-value x and every belief state b, there is a single degree of inaccuracy $I(b,x)$ representing how inaccurate belief b is. Moreover, this degree depends only upon b and the truth-value x of the proposition Φ of interest." (Mayo-Wilson & Wheeler, 2016, p. 61)

Continuity – Inaccuracy function is a continuous function. If there are two similar beliefs states c and c', then their inaccuracy scores are also similar.

Quantifiability – Inacuraccies can be numerically quantified by a real number.

Admissibility – There can be no belief distribution that is more accurate in every world, than the one that an agent adopts if the agent's credence distribution is measured by a rational scoring rule.

Strict Propriety – A rational agent cannot expect herself to have a worse accuracy score of her belief than a different credence that she considers. An agent should always adopt credence that seems to her to be the most rational.

The mentioned set of reasonable scoring rules contains rules which satisfy the above constraints. The most challenging part of arguing for the imprecise view from the accuracy-centered perspective is scoring them. It is problematic to grasp the accuracy of an interval, it can be measured by summing an accuracy of all functions within the set (after agreeing that this set is finite), or accuracy of a lower and upper envelope, or by reducing interval to its middle value. Seidenfield, Schervish and Kadane (2012) however argued that there can be no strictly proper scoring rule which supports the standard imprecise view. Also, Mayo-Wilson and Wheeler (2016) provide a similar argument that the above constraints conflict with the imprecise standard view, that either Continuity, Quantifiability or Immodesty needs to be abandoned. These kinds of objections to imprecise credences are called impossibility theorems. They argue that it is not possible to construct an argument for the imprecise standard view which is grounded on the accuracy-centered view and which does not violate some of the core principles.

I shall present more thoroughly a different impossibility argument and two interesting answers to that objection. I think that they provide a good representation of the condition of the imprecise standard view. Imprecise probabilities are in the defensive position and the majority of accuracy firsters reject vindication of them.

3.2 Schoenfield's imposibility argument

Schoenfield argues that the veritist claim that the rationality of beliefs is evaluated solely by their accuracy to the truth, is incompatible with the standard imprecise view, mainly with the claim that imprecise credences are sometimes rationally required. She also gives another, stronger argument that they can not be ever permitted. Her main argument is based on the

accuracy firster's admired principle of Admissibility. This principle is well-argued and defended by Joyce (2009)[1] who claim that finding probabilistic beliefs weakly dominated is an unbearable flaw of any reasonable scoring rule. To avoid this flaw, the Admissibility[2] principle is needed.

> *Probabilistic Admissibility* - "Take any partition X and any probabilistic belief state b defined over X. (A belief state is probabilistic if it is a set that contains only probability functions.) There can be no belief state b' defined over X such that b' is more accurate than b for some $X_i \in X$, and no less accurate than b for any $X_i \in X$. (In other words: probabilistic belief states cannot be dominated.)" (Schoenfield, 2017)

Probabilistic beliefs cannot be dominated, it follows that if there are two probabilistic beliefs, precise and imprecise, one cannot have larger accuracy score[3] in some $X_i \in X$ and be no less accurate in any proposition in X then the other. If this happen one of them dominates the other, which is forbidden.

If epistemic utility is the only property of a belief that ought to be considered then it cannot be rationally required to adopt an imprecise belief state, because it is not more accurate than the precise one. If it would be more accurate, it would violate the Probabilistic Admissibility. According to Schoenfield if two belief states are indifferent in means of accuracy, an agent which is an accuracy firster should not prefer one over the other. It cannot be rationally required to adopt belief state b, which is no less accurate than belief state b'.

[1] Joyce outlines his accuracy first approach to epistemology in his article "A Nonpragmatic Vindication of Probabilism" (1998), where the doctrine of veritism is delineated. In his next article from 2009, he expands his earlier principles.

[2] The Admissibility principle is variously defined. Its different forms relate to the context in which it is defined. Joyce (2009) mentions "coherent admissibility" and Schoenfield "probabilistic admissibility". The main idea stays the same, forbidding rational beliefs to be even weakly dominated.

[3] Schoenfield made two arguments against the requirement of adopting imprecise credences, one which concerns scoring rules which use a single value, and one which scores them imprecisely. The result stays the same.

> For every imprecise belief state, there is a precise belief state that is no less accurate than the imprecise one. Therefore adopting an imprecise belief state cannot be rationally required.

Schoenfield's argument is epistemically undemanding, there is no need to adopt controversial principles. It is also remarkably general, it does not refer to a class of scenarios in which it is irrational to require imprecise credences, it has a much wider range. Her argument undermines the proponents of imprecise probabilities belief that for any imprecise belief state b there is some body of evidence that requires b. This argument shows that it is not possible to argue for the imprecise standard view from the accuracy-centered approach to epistemology. Because epistemic utility scores cannot mark imprecise credences as being more accurate than precise ones.

Schoenfield's objection to imprecise credences gets even stronger. She claims that imprecise credences cannot be even rationally permitted because this leads to tension between imprecise beliefs and the Principal Principle.

Recall the first argument which was based on two credences, precise and imprecise, over some partition X. None of them can be rationally required because they have the same accuracy score, they are epistemically indifferent. But what if one of these beliefs is 0.5 and the second $(0.2, 0.8)$ in Tails in a scenario of betting on a fair coin. It is completely unintuitive result that the 0.5 credence cannot be rationally required, moreover, not adopting this precise belief seems irrational. According to the Principal Principle, a rational agent needs to adopt 0.5, because it matches known chances. It might sound like an evidential claim, but this principle is actually well-argued from the accuracy-centered view as a constraint of rational beliefs (Pettigrew, 2016).

With this in mind, an accuracy firster can either reject the Principal Principle or imprecise view, that unsharp credences are sometimes permitted. The principle is much more valuable and well-argued from the epistemic ground than the imprecise view, therefore rejecting the possibility of adopting imprecise credences is more rational.

Unfortunately, Schoenfield's arguments showed that imprecise credal states, whose use is supported by evidence, cannot be rationally required or even permitted in a framework that is solely based on the pursuit of alethic accuracy. Her objection applies to scoring rules which were presented by Seidenfield, Schervish and Kadane (2012) and Konek (forthcoming), according to Schoenfiled's impossibility result, these two theories which tried to vindicate imprecise scoring rules violate the Probabilistic Admissibility principle.

On top of mentioned earlier advantages of her argumentation, she does not specify a scoring rule except for it being reasonable. She considers separate arguments with scoring rules which is a regular single value score and the second that uses interval to represent beliefs accuracy. In the latter argument, she bases her argument only on a single example, without a generalization, which can be considered as a flaw.

Does constraint of matching known chances support imprecise beliefs? From the evidential ground, it might in a scenario of vague evidence, but her argument concerns the accuracy-centered view. It seems that there is more about credences than their accuracy, an additional property of representing an agent's unspecific doxastic state and representing known chances. This property however is not recognized by the accuracy-first methods of vindicating rationality. Dominance and evidential considerations can conflict, and it can be seen in our case.

Consider two agent's Robert and Mark, they both hold the same belief, accuracy score of their beliefs is therefore also the same. Robert's belief is well justified by his evidence, but Mark's belief is not, actually it is the opposite of what his evidence support. On the alethic accuracy ground their beliefs have the same rationality score, but it is not that obvious if someone values more matching known chances.

However, accuracy firsters do not completely ignore the evidential property of a belief. The Principal Principle is structured from a completely epistemic ground, even though it concerns evidence. The argument for that principle though does not favourevidence, it is guided onto the pursuit of truth, like the rest of accuracy-first principles. Matching known chances is simply, epistemically beneficial.

Schoenfield's argument seems very strong, but there are some alternative approaches to the vindication of imprecise probabilities in an accuracy-centered epistemology program. The word "alternative" should be emphasized because these proposals do not simply reject one principle and favourthe other, these projects are very unorthodox.

3.3 From alethic to chancy accuracy

One way to avoid impossibility results, when being an advocate of the imprecise view, is to shift from alethic to chancy accuracy. This vindication of imprecise credences is proposed by Carr (2015). She proposed an argument based on the accuracy-centered approach for chance-grounded, imprecise credences. The core element of that idea is the different interpretation of accuracy, instead of alethic accuracy, where a scoring rule measures inaccuracy of a credence to a true proposition, she proposed measuring how accurately credences represent chances.

Carr is a proponent of the Chance Grounding Thesis, she accommodates it because of the belief that imprecise credences are a better option in cases of severe uncertainty. Her interpretation of that thesis: "... for each proposition A, evidence determines some set of possible chances of A at an appropriate time t. A rational agent's upper credence in A will be equal to the upper evidentially possible chance of A at t, and her lower credence in A will be equal to the lower evidentially possible chance of A at t" (Carr, 2015, p. 71). Imprecise credences can cover the range of fuzzy chances. With the belief that imprecise credences are useful, she tries to delineate epistemic utility theory that can justify them.

Credences should be vindicated by chances, just like full beliefs are vindicated by a truth value. Vindication is understood by her, as objective correctness. Carr presented a general method of measuring credal accuracy to chances, of how far they are to be objectively correct. The measurement focus on how well-tuned the lower and upper envelope of an imprecise credence function (which is a set of probabilistic functions) is, with chances. Alethic value is not important in that measurement, the only property that is rewarded is accuracy to the chances.

The constraints she proposed on a scoring rule are Truth-Directedness and Non-triviality.

The pursuit of a chancy accuracy in that account is motivated by putting the standard decision rule constraint in a slightly modified version, dominance. It is similar to the Probabilistic Admissibility principle, because it concerns weak dominance[4]. She claims that a rational agent cannot adopt imprecise credence C, when C is lower, weakly dominated, or when C is upper, weakly dominated by any belief state that is not lower-dominated. Weak dominance in that context is defined as follows, C weakly dominates D if at all worlds C is at least as accurate as D, and in some worlds, it is more accurate than D.

The main line in Carr's vindication of imprecise standard view from the accuracy-centered perspective focuses on the belief that credences have more properties, and as follows more than one accuracy. They can be evidence accurate, or alethic accurate, or maybe there are even some other accuracies. Focusing on a lower and upper accuracy leads to a possible argument for imprecise credences when measured to chances, and not truth-value.

What is interesting, Carr does not disagree with Schoenfield, that the requirement to adopt imprecise beliefs cannot be justified in terms of inaccuracy when accuracy is understood in a standard way. The main thing that Carr wants to show is that there are more accuracy properties. And from a different perspective (evidential) imprecise credal states can be vindicated.

Different accuracies can be in tension, it is not obvious then what measure of accuracy (alethic or chancy) should be taken into account. There is a lack of a decision rule that can weigh the measures, so the choice seems very subjective. And still of course if someone is a proponent of a classical accuracy measure, then Schoenfield's argument should be in her mind, that imprecise credences are forbidden.

Objection to Carr's argument comes from the complexity of her theory, when Joyce (1998, 2009) presented his probability argument, his constraints were fairly simple and intuitive. If his principles (Truth-Directedness Extensionality, Convexity) ought to be used instead of theses proposed by Carr and we allow using the mean of the upper and lower envelope to

[4]Actually Carr's dominance principle is more sophisticated when it comes to time and evidence, but in this more general context, this simplified description should be enough.

represent them, then it can be argued that every imprecise credence will be dominated by a precise one. Carr's result unfortunately still does not create a safe spot for imprecise credences. She herself stays very skeptical about creating a purely epistemic rational constraint that will support the claim that imprecise credences are sometimes required by the evidence.

Her argument works as long as someone will agree on the Chance Grounding Thesis and other principles that Carr proposed. In this context, matching chances is perceived as a purely accuracy concerned, epistemic premise. Still, though it concerns only non-standard, chancy accuracy. The most significant thing that she presented is that there is more than one accuracy and it is not trivial what credence property should be taken into account.

3.4 Indeterminate credal states

Levinstein's (Levinstein, 2019) goal is to once again try to motivate imprecise credal states from the accuracy-centered, alethic perspective. He tries to defend the claim that a rational agent can sometimes end up with imprecise credences in his pursuit of accuracy, therefore he argues that imprecise beliefs are sometimes rationally permitted, from a purely epistemic point of view. His vindication is based on the belief that imprecise credences can be understood as indeterminate credences. He also shows that a scoring rule can be also indeterminate as well as a decision rule, it results in indeterminate agent's behavior. If his account is acceptable, then impossibility arguments lose their strength.

What does it mean for an imprecise credal state to be indeterminate? In the classical view of the imprecise belief, an agent identifies his doxastic state with a *set* of probability functions. This is determinate view on credences, according to Levinstein. In a situation where an agent represents his doxastic state with a single *member* of the set, and it is indeterminate with which member of that set an agent identifies with, we are dealing with an indeterminate, imprecise credal state. This division is argued on a ground of supervaluationist semantics (Rinard, 2015).

We cannot score indeterminate credal state represented by the interval, because we do not know which probabilistic function within the interval matches the agent's belief. Assigning one score to the interval is a category mistake, the agent's inaccuracy score is indeterminate. With that view, there is no way of scoring an agent's representor as a whole, so an impossibility argument cannot be executed, because this kind of argument assumes that imprecise credal state inaccuracy can be measured (precisely or imprecisely). But do these indeterminate doxastic states make sense at all?

All that an agent cares about in the accuracy-centered view is accuracy, but without specifying what accuracy. Levinstein argues that if an agent can have indeterminate alethic values, then she is permitted to have indeterminate credences. There is more than one method of scoring credences and different scoring rules grasp closeness to the truth differently.

Levinstein presents two commandments that were strongly promoted in formal epistemology by Isaac Levi (1967, 1984, 2004) and originally formulated by William James (James, 1896).

Shun error! Believe truth!

These two commandments represent two different epistemic approaches. Believing truth is a commandment that would be satisfied by being open towards all belief credal states, to not to miss the truth. Avoiding error is the opposite approach, an agent can even suspend her judgment to maximize it, if she does not have enough trust in any belief or adopt credal state close to the 1 or 0.

In the context of measuring accuracy, these two rules reflect two different attitudes to judging credal doxastic states. Credence close to 0 or 1 has the chance to be extremely accurate, by the price of huge risk of being absolutely inaccurate. Credences that are close to the middle represent more careful attitude.

These attitudes are grasped differently by different scoring and decision rules. If an agent's attitude to scoring and decision rules is general, with just a few constraints on them, there is

a class of these rules that represents different attitudes to which an agent is indeterminate. If this set of rules is indeterminate, then there is room for indeterminate imprecise credal states.

His argument is supported by the claim that no single scoring rule seems to be the best. Accuracy-firsters are the proponents of proper scoring rules (Joyce, 2009; Pettigrew 2016). But there is a class of rules that satisfy this constraint (and a few others that are claimed by accuracy-firsters). Brier score seems to dominate among scholars but there is also *Log Score* or *Spherical Score* (Levintein, 2019, p. 14), and infinitely many different, undefined, proper scoring rules. These three mentioned by name rules promote different proportions of epistemic values (avoiding error, believing truth).

On top of that, there are also different decision rules e.g. *Minimax* or *Minimin*ˆ,[Minimax rule says that the credence that has the best worst-case outcome ought to be selected. Minimax rule suggest selecting credence that has the best best-case outcome.] which again promote different epistemic values. There is also an interesting selection rule which can reflect any proportion of these two epistemic values, *Hurwicz rule*[5].

With that observation in mind, Levinstein claims that imprecise credal states are permitted because of their indeterminacy in cases of severe uncertainty. Furthermore, his view does not violate the Principal Principle and the proposed set of scoring rules contains only strictly proper ones. He claims that every mix of proportions of epistemic values obeys the Principal Principle.

His argument does not sound very strong, in fact, he did not support the standard imprecise view, because on his account imprecise credences are only sometimes permitted and are not a constraint of rationality. His argument is not very positive, he showed that indeterminate credences do not violate rationality constraints, which is valid only in his exotic interpretation of imprecise credences. Indeterminate interpretation is not only different from the standard view on imprecise credences, it is the complete opposite. In the standard view agent's credence **is** the imprecise interval, and not, specific, undecided, probabilistic function within it.

[5]Konek (forthcoming) basing on that rule created an argument for imprecise credal states from the accuracy-centered perspective, unfortunately his argument is susceptible to Schoenfields impossibility argument

That is why his vindication should have the status of an alternative, exotic one.

Chapter 4

Conclusions

In my work I have analyzed the issue of imprecise probabilities and its relation with accuracy-centered view one epistemology, starting from general delineation of Bayesianism. First, I have presented thorough description of imprecise credences. The second part of my work concerned the possibility of arguing for imprecise credences from the accuracy-centered perspective, which contains a description and analysis of most representative arguments and objections on that issue.

The intuition that vague evidence ought to be represented by vague credences seems to be indefensible as a rationality constraint from veritist view. A purely alethic argument for the standard imprecise view will lead to serious consequences, what Schoenfield accurately showed. These consequences are untenable for a rational agent, that is a standard proponent of accuracy-centered epistemology. Also, Seidenfield, Schervish and Kadane (2012) showed that the scoring rule for imprecise credal states cannot be strictly proper, and Mayo-Wilson and Wheeler (2016) showed that, constraint of Strict Propriety conflicts with unsharp credences.

Vindication of imprecise credal states requires an alternative approach to accuracy-centered epistemology. Both approaches that I have presented require adopting the Chance Grounding

Thesis. Carr argues that this vindication is possible in a different interpretation of accuracy, she proposed chancy accuracy. Levinstein adopted a new interpretation of fuzzy credal states, he claims that they are indeterminate. These two arguments give some room for imprecise credences, but they fail to provide a proper vindication of imprecise standard view (that imprecise credal states are sometimes required by rationality) that is not susceptible to Schoenfield's impossibility argument.

All these arguments suggest that the core issue of vindication of imprecise credal states from the accuracy-centered perspective, is the problematic relationship between alethic and evidential norms. In the accuracy-first approach, purely epistemic norms aimed at truth values dominate. But they also do lead to some constraints about evidence e.g. Principal Principle. Evidential constraints, therefore, are a secondary worry for an accuracy-centered epistemologist. The standard imprecise view, however, seems to always flow from evidential worries about rationality.

Veritist would say that if imprecise credal states are not compatible with the accuracy-centered point of view then the imprecise standard view is not worth her attention. Veritism has an undeniably strong position in Bayeasianism. The Accuracy-first approach to epistemology is a program whose main focus is theoretical rationality. But as we have seen there is a tension within this concept of rationality, pursuit of truth and representation of agent's belief state can lead to the equivocation of the word "Rational". This discussion led to creating more awareness about different features of credal states which in the standard view on accuracy-centered epistemology are reduced to alethic inaccuracy score.

Still, it is worrying that there is no imprecise credal states validation from this particular view on epistemology. They seem to be the more intuitive answer to severe uncertainty than precise credences, and also, there are some positive arguments for it. But as we have argued it can be only validated by alternative approaches, that have more respect towards evidence, and still, they struggle to provide a solid requirement for the standard imprecise view.

What does it mean for imprecise probabilities? Well, they will still be in use, as I have

stated earlier they prove their usefulness in some of the empirical disciplines. Our result concerns only the alethic side of the theoretical rationality, which is unable to accommodate standard imprecise view in the most popular interpretation. Imprecise probabilities still will be an interesting, and unobvious body of research.

Bibliography

- Bradley D., (2015), *A Critical Introduction to Formal Epistemology*, Bloomsbury Academic, Chennai.
- Bradley S., (2019a), *Imprecise Probabilities*, "The Stanford Encyclopedia of Philosophy", https://plato.stanford.edu/archives/spr2019/entries/imprecise-probabilities/, (entry: 18.04.2021).
- Bradley S., (2019b), *Imprecise Probabilities*, in "Computer Simulation Validation", C. Beisbart & N. J. Saam (eds.), 525-540, Springer, Cham.
- Bradley S. & Steele K., (2014), *Should Subjective Probabilities be Sharp?*, "Episteme", 11: 277–289.
- Bratman M., (2012), *Time, rationality and self-governance*, "Philosophical Issues" 22(1): 73–88.
- Elga A., (2010), *Subjective Probabilities should be Sharp*, "Philosophers' Imprint", 10.
- Ellsberg D., (1961), *Risk, ambiguity and the Savage axioms*, "Quarterly Journal of Economics", 75: 643–696.
- van Fraasen B., (1990), *Figures in a Probability Landscape*, in "Truth or Consequences", M. Dunn & A. Gupta (eds.), 345–356, Springer, Dordrecht.
- Genin K. & Huber F., (2021), *Formal Representations of Belief*, "The Stanford Encyclopedia of Philosophy", https://plato.stanford.edu/archives/spr2021/entries/formal-belief/, (entry: 13.03.2021).

- James W., (1896), *The Will to Believe and Other Essays in Popular Philosophy*, Longmans, Green & Company, New York.
- Joyce J., (1998), *A Nonpragmatic Vindication of Probabilism*, "Philosophy of Science", 65(4): 575–603.
- Joyce J., (2005), *How Probabilities Reflect Evidence*, "Philosophical Perspectives", 19: 153–178.
- Joyce J., (2009), *Accuracy and Coherence: Prospects for an Alethic Epistemology of Partial Belief*, in "Degrees of Belief", F. Huber & C. Schmidt-Petri (eds.), 263–97, Springer, New York.
- Joyce J., (2010), *A Defense of Imprecise Credences in Inference and Decision Making*, "Philosophical Perspectives", 24(1): 281–323.
- Kaplan M., (1996), *Decision theory as philosophy*, Cambridge University Press.
- Kaplan M., (2010), *In Defense of Modest Probabilism*, "Synthese", 176(1): 41–55.
- Konek J., (forthcoming), *Epistemic Conservativity and Imprecise Credence*, "Philosophy and Phenomenological Research".
- Landes J. & Williamson J., (2013), *Objective Bayesianism and the Maximum Entropy Principle*, "Entropy", 15(9): 3528-3591.
- Levi I., (1967), *Gambling with Truth*, Knopf, New York.
- Levi I., (1984), *Decisions and Revisions*, Cambridge University Press, Cambridge.
- Levi I., (1985), *Imprecision and Indeterminacy in Probability Judgment*, "Philosophy of Science", 52: 390–409.
- Levi I., (1991), *The Fixation of Belief and its Undoing: Changing Beliefs through Inquiry*, Cambridge University Press, Cambridge.
- Levinstein B.A., (2019), *Imprecise Epistemic Values and Imprecise Credences*, "Australasian Journal of Philosophy", 97(4): 741-760.
- Lewis D., (1980), *A subjectivist's guide to objective chance*, in "Studies in inductive logic and probability", R. C. Jeffrey (eds.), 83–132, University of California Press.

- Mahtani A., (2019), *Imprecise Probabilities*, in "The Open Handbook of Formal Epistemology", R. Pettigrew & J. Weisberg (eds.), PhilPapers Foundation, 107-130.
- Mayo-Wilson C. & Wheeler G., (2016), *Scoring Imprecise Credences: A Mildly Immodest Proposal*, "Philosophy and Phenomenological Research", 93(1): 55–78.
- McClenen F., (1990), *Rationality and dynamic choice: Foundational explorations*, Cambridge University Press, New York.
- Moss S., (2015), *Credal dilemmas*, "Noûs", 49(4): 665–683.
- Peterson M., (2009), *An Introduction to Decision Theory*, Cambridge University Press, Cambridge.
- Pettigrew R., (2016), *Accuracy and the Laws of Credence*, Oxford University Press, Oxford.
- Pettigrew R., (2019), *Epistemic Utility Arguments for Probabilism*, "The Stanford Encyclopedia of Philosophy", https://plato.stanford.edu/archives/win2019/entries/epistemic-utility/, (entry: 18.03.2021).
- Rinard S., (2015), *A Decision Theory for Imprecise Probabilities*, "Philosophers' Imprint", 15(7): 1–16.
- Talbott W., (2016), *Bayesian Epistemology*, "The Stanford Encyclopedia of Philosophy", https://plato.stanford.edu/archives/win2016/entries/epistemology-bayesian/, (entry: 01.04.2021).
- Titelbaum M., (2015), *Fundamentals of Bayesian Epistemology*, Unpublished manuscript.
- Weatherson B., (2003), *From classical to intuitionistic probability*, "Notre Dame Journal of Formal Logic", 44(2): 111–123.
- Williams J. R. G., (2014), *Decision-making under indeterminacy*, "Philosophers' Imprint", 14(4): 1–34.
- Schoenfield M., (2017), *The accuracy and rationality of imprecise credences*, "Noûs", 51(4): 667–685.

- Seidenfeld T., Schervish M.J. & Kadane J.B., (2012), *Forecasting with Imprecise Probabilities*, "International Journal of Approximate Reasoning", 53(8): 1248–61.
- Staffel J., (2019), *Unsettled Thoughts*, Oxford University Press, New York.