Generalised linear model may be presented in form

$$L(\theta) = S^T\theta - A(\theta)$$

where

$$A(\theta) = \sum_i g(\psi_i^T\theta), \quad S = \sum_i Y_i\psi_i.$$

This model has following properties

$$D^2 = \sum_i g''(\psi_i^T\theta)\psi_i\psi_i^T, \quad \delta(r) = a_g\delta_\psi r, \quad a_g = \max g'''/g'', \quad \delta_\psi = \max_i \left\|D^{-1}\psi_i\right\|.$$

From condition (**??**) for likelihood function $L(\theta)$ one get

$$\left| \sqrt{2T_h(\widehat{\theta}, \widehat{\theta}_1, \widehat{\theta}_2)} - \left\| \begin{matrix} D_1(\widehat{\theta} - \widehat{\theta}_1) \\ D_2(\widehat{\theta} - \widehat{\theta}_2) \end{matrix} \right\| \right| \le c_1\delta(\sqrt{2}r).$$

Find relation between $\widehat{\theta}, \widehat{\theta}_1, \widehat{\theta}_2$

$$\left\| D(\widehat{\theta} - \theta) \right\| \le \left\| D^{-1}\{D_1D_1^{-1}\nabla L_1(\theta) + D_2D_2^{-1}\nabla L_2(\theta)\} \right\| + c_2\delta(r).$$

$$\left\| D^{-1}D_1\{D_1^{-1}\nabla L_1(\theta) - D_1(\theta - \widehat{\theta}_1)\} \right\| \le c_3\delta(r).$$

$$\left\| D^{-1}D_2\{D_2^{-1}\nabla L_2(\theta) - D_2(\theta - \widehat{\theta}_2)\} \right\| \le c_3\delta(r).$$

Define vector $\widetilde{\theta}$ close to $\widehat{\theta}$

$$\widetilde{\theta} = (D_1^2 + D_2^2)^{-1}(D_1^2\widehat{\theta}_1 + D_2^2\widehat{\theta}_2)$$

$$\left| \left\| \begin{matrix} D_1(\widehat{\theta} - \widehat{\theta}_1) \\ D_2(\widehat{\theta} - \widehat{\theta}_2) \end{matrix} \right\| - \left\| \begin{matrix} D_1(\widetilde{\theta} - \widehat{\theta}_1) \\ D_2(\widetilde{\theta} - \widehat{\theta}_2) \end{matrix} \right\| \right| \le \left\| D(\widehat{\theta} - \widetilde{\theta}) \right\| \le c_2\delta(r) + c_3\delta(\sqrt{2}r).$$

$$\left\| \begin{matrix} D_1(\widetilde{\theta} - \widehat{\theta}_1) \\ D_2(\widetilde{\theta} - \widehat{\theta}_2) \end{matrix} \right\| = \left\| D_{12}(\widehat{\theta}_2 - \widehat{\theta}_1) \right\|, \quad D_{12} = D_1D^{-1}D_2.$$

Temporary result is

$$\left| \sqrt{2T_h(\widehat{\theta}, \widehat{\theta}_1, \widehat{\theta}_2)} - \left\| D_{12}(\widehat{\theta}_2 - \widehat{\theta}_1) \right\| \right| \le c_1\delta(\sqrt{2}r) + c_2\delta(r) + c_3\delta(\sqrt{2}r).$$

Involve ξ_1 and ξ_2 by means of Fisher theorem

$$\left\| \begin{matrix} D_2D^{-1}\{D_1(\widehat{\theta}_1 - \theta_1^*) - \xi_1\} \\ D_1D^{-1}\{D_2(\widehat{\theta}_2 - \theta_2^*) - \xi_2\} \end{matrix} \right\| \le c_4\delta(\sqrt{2}r).$$

Finally,

$$\left| \sqrt{2T_h(\widehat{\theta}, \widehat{\theta}_1, \widehat{\theta}_2)} - \|D_{12}(\theta_2^* - \theta_1^*) + \xi_{12}\| \right| \le c_1\delta(\sqrt{2}r) + c_2\delta(r) + c_3\delta(\sqrt{2}r) + c_4\delta(\sqrt{2}r).$$