Ex1

1)

a)

Potential Data Setup:

Variable Name	Role	Type
Price	Target	Numeric
Mileage	Feature	Numeric
days old	Feature	Numeric
service necessary	Feature	Categorical
accident-free	Feature	Categorical
...	...	...

b)

We can assume, that negative mileage or age are not possible, therefore the feature space is given as $\mathcal{X} = \mathbb{R}_0^+ \times \mathbb{R}_0^+$ and the target space as $\mathcal{Y} = \mathbb{R}$ (negative prices maybe quite uncommon, yet not impossible if we consider recycling).

c)

$\mathcal{H} = \{f | f(x) = \theta^\top x = \theta_0 + \theta_1 \cdot mileage + \theta_2 \cdot age\}$

d)

$\theta_i, \;\; i = 0, 1, 2$ need to be learned, each $\theta_i \in \mathbb{R}$, which leads us to: $\Theta = \mathbb{R}^3$.

e)

$\mathcal{L}_i(\theta) = (y_i - \theta^\top x_i)^2 = (y_i - \theta_0 - \theta_1 \cdot mileage_i + \theta_2 \cdot age_i)^2$

f)

The optimization problem is then:

$$\hat{\theta} = \underset{\theta \in \Theta}{arg\ min} \sum_{i=1}^{n} \mathcal{L}_i(\theta) = \underset{\theta \in \Theta}{arg\ min} \;\; \mathcal{L}(\theta)$$

To minimize $\mathcal{L}(\theta)$ we want to set $\frac{d\mathcal{L}(\theta)}{d\theta} = 0$.

For the Linear Model, there exists a closed form solution as long as X has full rank:

We can rewrite $\mathcal{L}(\theta)$ as $(y - X\theta)^\top (y - X\theta) = y^\top y - y^\top X\theta - \theta^\top X^\top y + \theta^\top X^\top X\theta$ and get to $\frac{d\mathcal{L}(\theta)}{d\theta} = \theta^\top X^\top X - y^\top X$. Next we obtain $\theta^\top X^\top X = y^\top X \iff X^\top X\theta = X^\top y$.

Thus obtaining the Ordinary Least Squares Estimator $\hat{\theta}_{OLS} = (X^\top X)^{-1} X^\top y$