Statistical Inference 2 Summer Term 2025

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Exercise Sheet 1

Exercise 1.1 - Random Variable Generation

(a)) Implement the Lehmer Random Number Generator (RNG) presented in the lecture with 42 as starting value and obtain a simulated sample ($n = 1000$) of the uniform distribution.

(b) Make use of the inverse-transformation method to generate a sample $(X_1, \ldots, X_n)$ of X with distribution function

$$F(x) \ = \ 1 - e^{-2x}, \quad x \geq 0 \ .$$

Generate a histogram of your simulated sample and overlay the true density.

(c) Goodness-of-Fit Testing

(1) In `R`, perform a Kolmogorov–Smirnov test of your sample against $\text{Exp}(2)$. Have a look at `ks.test()`.

(2) Bin your sample into $k = 10$ equal-width intervals on $[0, b]$ (choose $b = \max X_i$), then perform a chi-square test comparing observed vs. expected counts under $\text{Exp}(2)$.

Interpret/Explain the result of both tests.

Exercise 1.2 - Rejection Sampling, Importance Sampling

(a) Implement a function

```r
rejection_sampler <- function(n_samples, a)
```

in R that performs **rejection sampling** to draw samples from the target density:

$$f(x) \propto \exp(-x^2), \quad x \in [-a, a]$$

with $a > 0$. Use the uniform distribution $g(x) = \mathrm{U}_{[-a,a]}(x)$ as umbrella distribution.

Hint: The density of the uniform distribution here is $g(x) = \frac{1}{2a}$.

(b) Draw a sample of size $n = 1000$ for $a = 0.1$, $a = 1$ and $a = 10$. Plot the empirical densities of the obtained samples. What do you notice?

(c) Make use of the results from (a) and (b) and perform importance sampling to estimate the following integral:

$$I_a = \int_{-a}^{a} \frac{\cos(x)}{1 + x^2}\, dx.$$

(d) Compute the standard error of your estimate.

(e) Can this method be used to evaluate I_a for large values of a, or even for $a = \infty$? ? Motivate your answer.