

Problemset 4

Nikolai German (12712506)

Exercise 5.2 - Local Monte Carlo Sampling

ex

(a)

Implement the above procedure in R to simulate S versions of $f(x^*)$ on the grid. What do you have to keep in mind to obtain S independent samples? Choose reasonable values for m, a, b, J , and σ .

```
gibbs.sample <- function(x_known, y_known, m, J, sigma = 1, a = 0, b = 1) {  
  checkmate::assertNumeric(x_known, any.missing = FALSE)  
  checkmate::assertNumeric(y_known, len = length(x_known), any.missing = FALSE)  
  # check a, b, sigma  
  
  x0 <- tidyr::full_seq(c(a, x_known, b), 1/(m))  
  x0_pos <- vapply(x_known, function(x) which(abs(x0 - x) < 1e-8), integer(1))  
  y0 <- rep(NA, length(x0))  
  y0[x0_pos] <- y_known  
  sims <- matrix(nrow = length(x0), ncol = J + 2)  
  sims[,1] <- x0  
  sims[,2] <- y0  
  
  for (i in seq_len(J)) {  
    if (i == 1) {  
      sims[, i+2] <- rnorm(nrow(sims), mean(y_known), sd(y_known))  
    } else {  
      for (j in seq_len(nrow(sims))) {  
        if (j %in% x0_pos) {  
          sims[j, i + 2] <- sims[j, 2]  
        }  
      }  
    }  
  }  
}
```

```

    } else if (j == 1) {
      mu <- sims[j, i + 1]
      sims[j, i + 2] <- rnorm(1, mu, sigma)
    } else if (j == nrow(sims)) {
      mu <- sims[j - 1, i + 2]
      sims[j, i + 2] <- rnorm(1, mu, sigma)
    } else {
      mu <- 0.5*(sims[j - 1, i + 2] + sims[j + 1, i + 1])
      sims[j, i + 2] <- rnorm(1, mu, sigma)
    }
  }
}
}
}
structure(sims,
          x_known = x_known,
          y_known = y_known,
          pos_x = x0_pos,
          J = J)
}

```

(b)

Use your function to generate at least $S = 250$ independent simulations $y^{*(s)}$, $s = 1, \dots, S$. Plot all simulated curves $(x^*, y^{*(s)})$ as line plots in a single figure. Highlight the known data points (x_k, y_k) using distinct markers (e.g., dots). Comment on the resulting visualization. What does it reveal about the uncertainty in the function's values across different regions of the domain? How could you formally quantify the uncertainty at each grid point x_i^* ?

```

x_known <- c(0.1, 0.12, 0.18, 0.43, 0.51, 0.68, 0.86, 0.95)
y_known <- c(0.3750, 0.3015, 0.0927, 0.2374, -0.5391, -0.0649, 0.3946, -0.4626)

sims <- gibbs.sample(x_known, y_known, 500, 250, sigma = .05)

library(ggplot2)
as.data.frame(sims) |>
  tidyr::pivot_longer(-c(V1, V2), names_to = "S", values_to = "y") |>
  ggplot(aes(V1, y, group = S)) +
  geom_line(alpha = .05) +
  geom_point(aes(y = V2), color = "red", alpha = .9, shape = 3, size = 2.5) +

```

```
labs(x = "x", y = "y") +  
theme_light()
```

