Time Series (732A62) Lab1 Group 7

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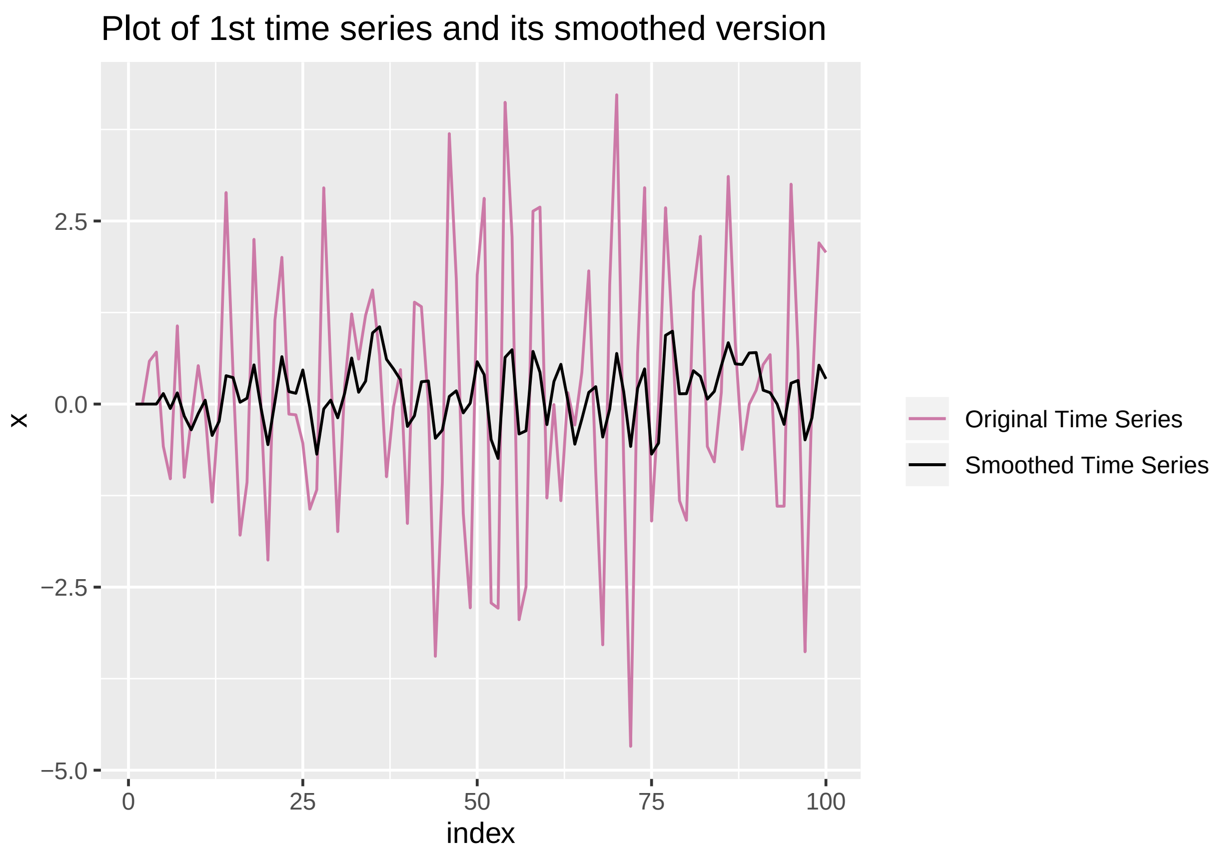
Contents

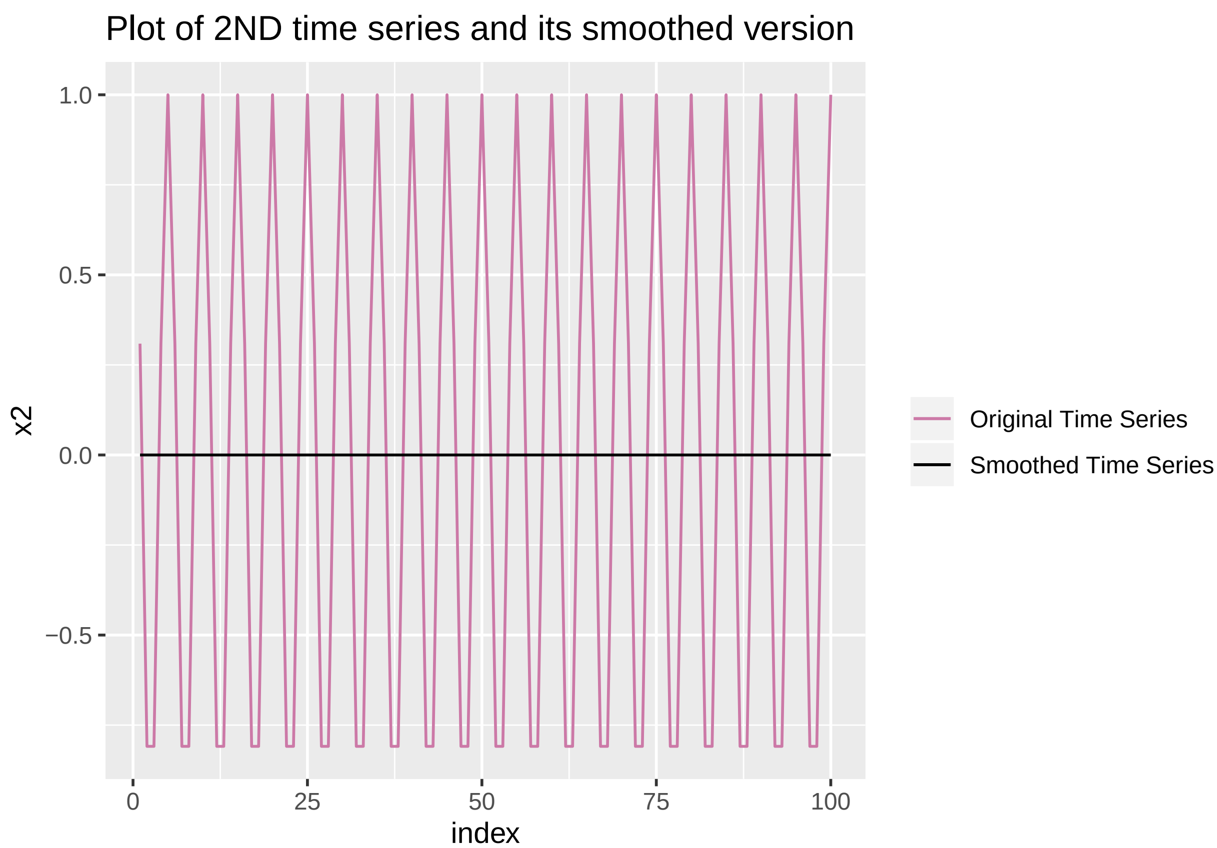
Assignment 1. Computations with simulated data

a) Generate two time series $x_t = -0.8x_{t-2} + w_t$, where $x_0 = x_1 = 0$ and $x_t = cos(\frac{2\pi t}{5})$ with 100 observations each. Apply a smoothing filter $v_t = 0.2(x_t + x_{t-1} + x_{t-2} + x_{t-3} + x_{t-4})$ to these two series and compare how the filter has affected them.

```r
set.seed(12345)

n = 100
x <- vector(length = n)
x2 <- vector(length = n)

x[1] <- 0
x[2] <- 0

#first series generation
for(i in 3:n){
  x[i] <- -0.8 * x[i-2] + rnorm(1,0,1)
}

#second series generation
for(i in 1:n){
  x2[i] <- cos(0.4*pi*i)
}

# smoothing filter function
smoothing_filter <- function(x){
v <- vector(length = length(x))
for(i in 5:length(x)){
  v[i] = 0.2 * (x[i] + x[i-1] + x[i-2] + x[i-3] + x[i-4])
}
return(v)
}

#generate smoothed series
smooth_x <- smoothing_filter(x)
smooth_x2 <- smoothing_filter(x2)

#adding everything to a dataframe
df <- cbind(x,x2,smooth_x,smooth_x2) %>% as.data.frame() %>% mutate(index=1:100)

ggplot(df, aes(x=index)) +
  geom_line(aes(y=x, color="Original Time Series")) +
  geom_line(aes(y=smooth_x, color="Smoothed Time Series")) +
  ggtitle("Plot of 1st time series and its smoothed version") +
    scale_colour_manual("", breaks = c("Original Time Series", "Smoothed Time Series"),
                        values = c("#CC79A7", "#000000"))
```




```r
ggplot(df, aes(x=index)) +
  geom_line(aes(y=x2, color="Original Time Series")) +
  geom_line(aes(y=smooth_x2, color="Smoothed Time Series")) +
  ggtitle("Plot of 2ND time series and its smoothed version") +
    scale_colour_manual("", breaks = c("Original Time Series", "Smoothed Time Series"),
                        values = c("#CC79A7", "#000000"))
```



Analysis: For the first time series the smoothing really helped the variation in the vertical axis(y-axis) has decreased, which is an expected effect of the time series smoothing. In the cosine case the smoothing results in a horizontal straight line, this is due to the fact that cosine is a periodic function and odd number of lags will always lead to a horizontal line.

b) Consider time series $x_t - 4x_{t-1} + 2x_{t-2} + x_{t-5} = w_t + 3w_{t-2} + w_{t-4} - 4w_{t-6}$. Write an appropriate R code to investigate whether this time series is casual and invertible.

Causality: ARMA(p,q) is causal iff roots $\phi(z') = 0$ are outside unit circle. eg: $x_t = 0.4x_{t-1} + 0.3x_{t-2} + w_t$, roots are -> $1 - 0.4B + 0.3B^2$

equation is: $\phi(Z) = 1 - 4B + 2B^2 + 0B^3 + 0B^4 + B^5$

```
z = c(1,-4,2,0,0,1)
polyroot(z)
```

```
## [1]  0.2936658+0.000000i -1.6793817+0.000000i  1.0000000-0.000000i
## [4]  0.1928579-1.410842i  0.1928579+1.410842i
```

```
any(Mod(polyroot(z))<=1)
```

```
## [1] TRUE
```

Invertible: ARMA(p,q) is causal iff roots $\theta(z') = 0$ are outside unit circle.

equation is: $\theta(Z) = 1 + 3B^2 + B^4 - 4B^6$

```r
z = c(1,0,3,0,1,0,-4)
polyroot(z)
```

```
## [1]  0.1375513+0.6735351i -0.1375513+0.6735351i -0.1375513-0.6735351i
## [4]  0.1375513-0.6735351i  1.0580446+0.0000000i -1.0580446+0.0000000i
```

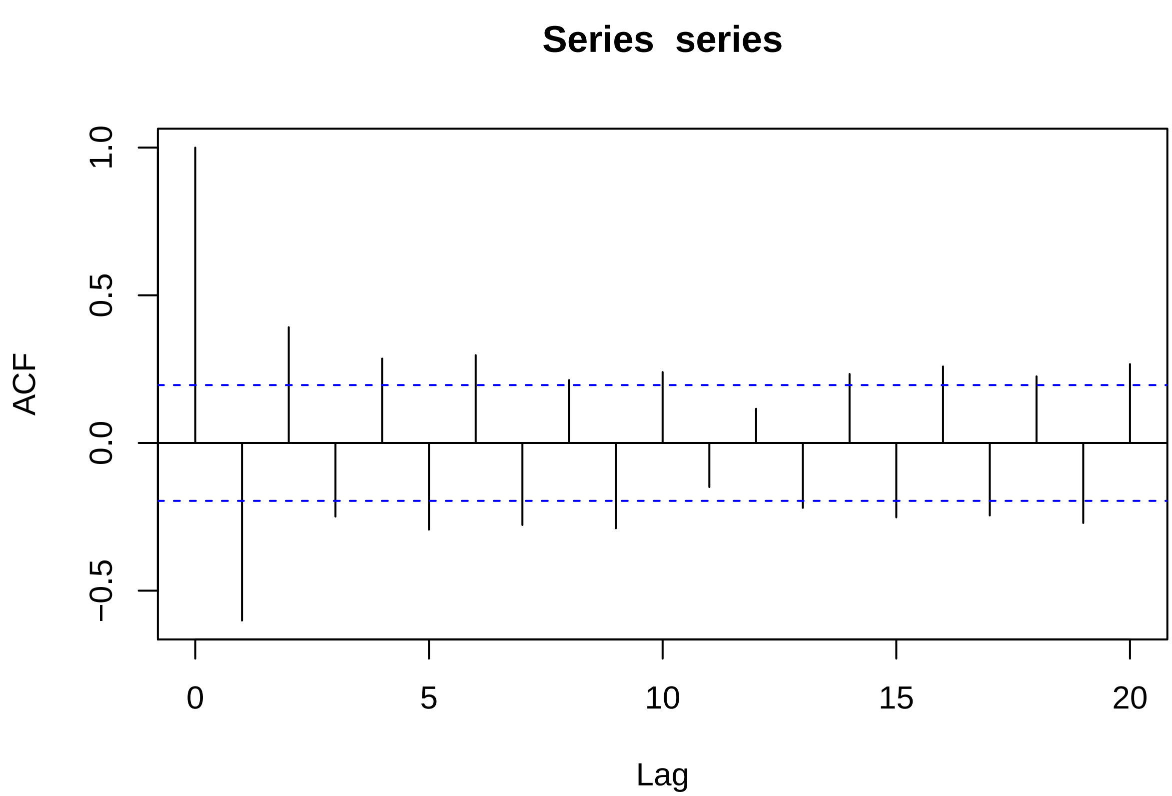
```r
any(Mod(polyroot(z))<=1)
```

```
## [1] TRUE
```

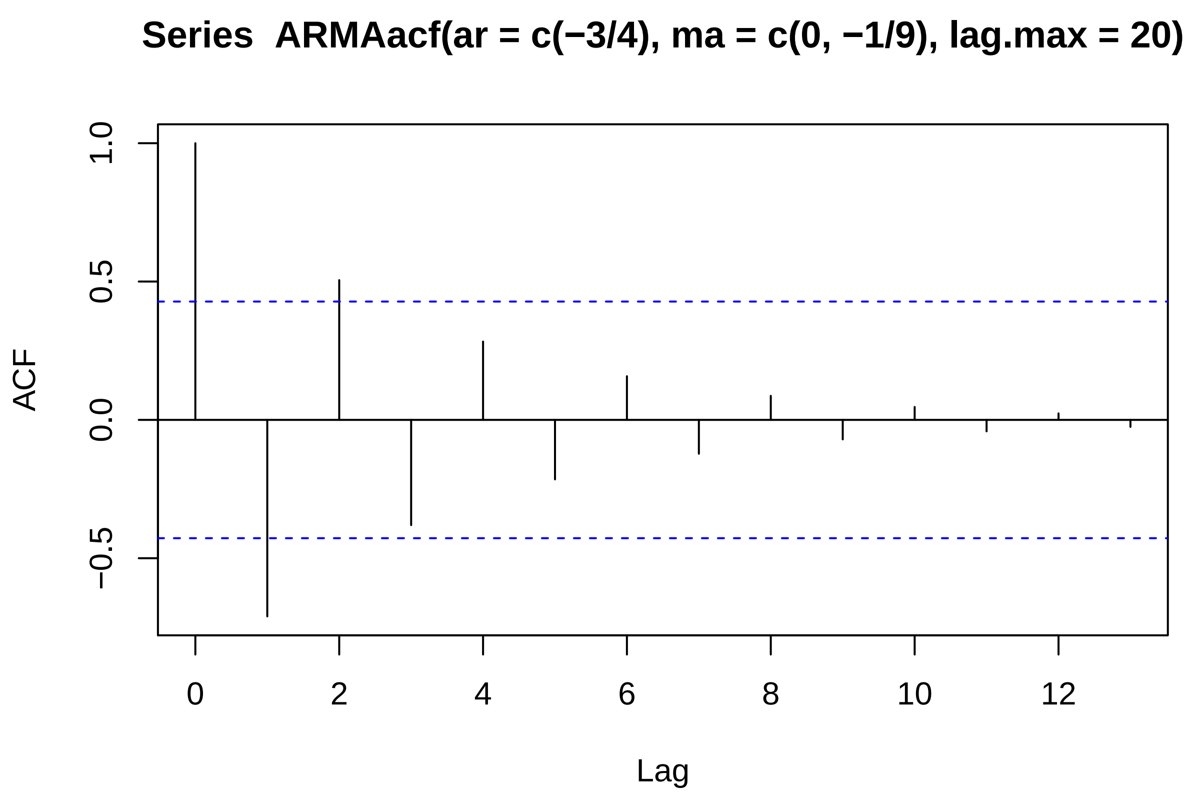
Analysis: Baring one of the roots all are inside the unit circle. Thus the time series is neither invertible or causal.

c) Use built-in R functions to simulate 100 observations from the process $x_t + \frac{3}{4}x_{t-1} = w_t - \frac{1}{9}w_{t-2}$ compute sample ACF and theoretical ACF, use seed 54321. Compare the ACF plots.

```r
set.seed(54321)

series <- arima.sim(n = 100, list(ar = c(-3/4), ma = c(0,-1/9)))

acf(series)
```



```
acf(ARMAacf(ar = c(-3/4), ma =  c(0,-1/9), lag.max = 20))
```



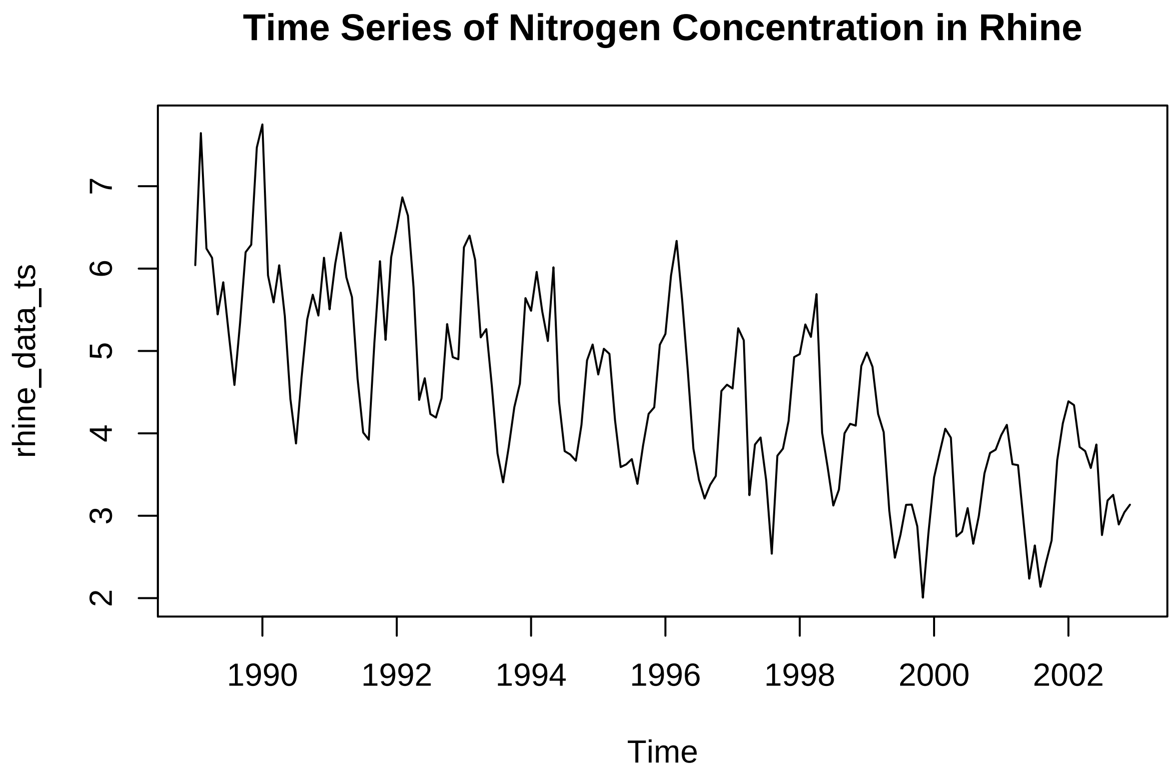
Analysis: In the theoretical ACF, only the 1 and 2ND lag components were significant, while using the sample ACF function we get many more lag components as significant.

Assignment 2. Visualization, detrending and residual analysis of Rhine data.

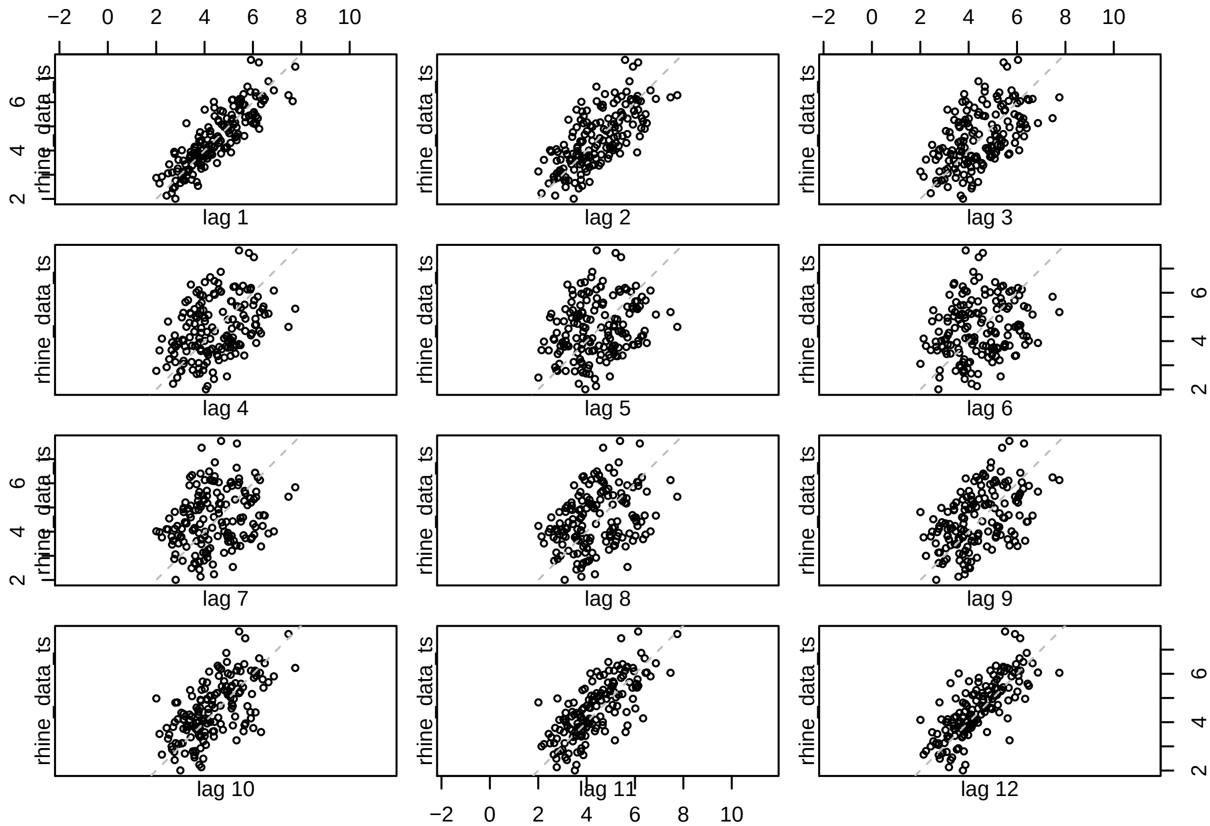
The data set Rhine.csv contains monthly concentrations of total nitrogen in the Rhine River in the period 1989-2002.

a) Import the data to R, convert it appropriately to ts object (use function ts()) and explore it by plotting the time series, creating scatter plots of x_t against $x_{t-1}, \ldots x_{t-12}$. Analyze the time series plot and the scatter plots: Are there any trends, linear or seasonal, in the time series? When during the year is the concentration highest? Are there any special patterns in the data or scatter plots? Does the variance seem to change over time? Which variables in the scatter plots seem to have a significant relation to each other?

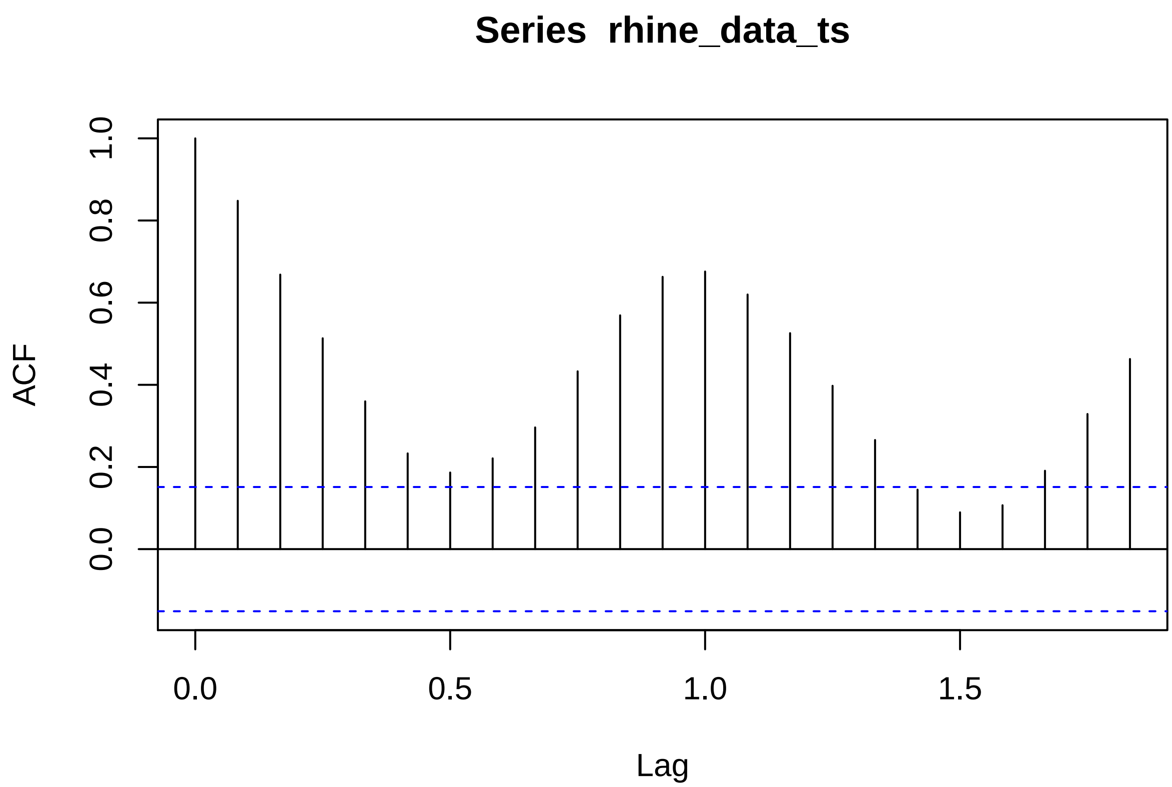
```
set.seed(12345)
rhine_data <- read.csv2("Rhine.csv")
rhine_data_ts <- ts(data = rhine_data$TotN_conc,
                    start = c(1989,1),
                    frequency = 12)

plot.ts(rhine_data_ts, main="Time Series of Nitrogen Concentration in Rhine")
```



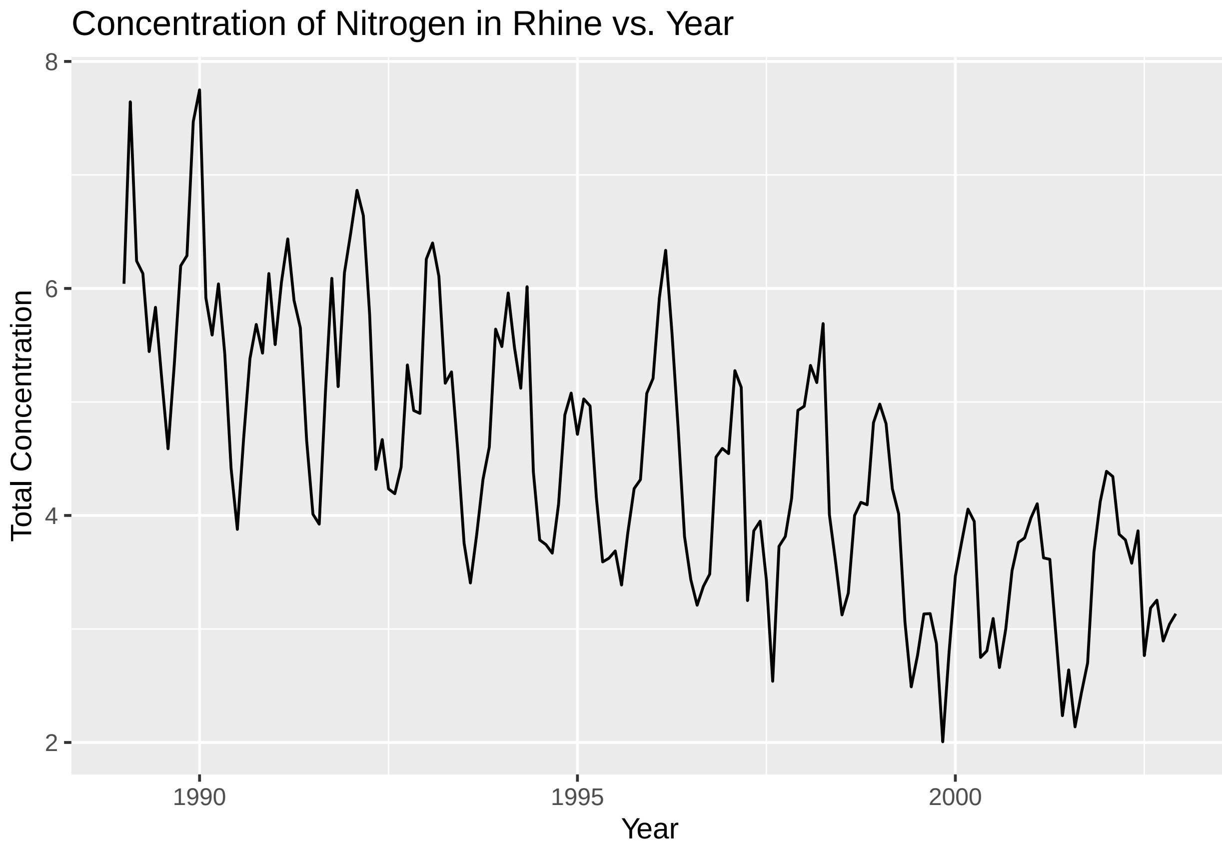

```
lag.plot(rhine_data_ts,lags = 12)
```



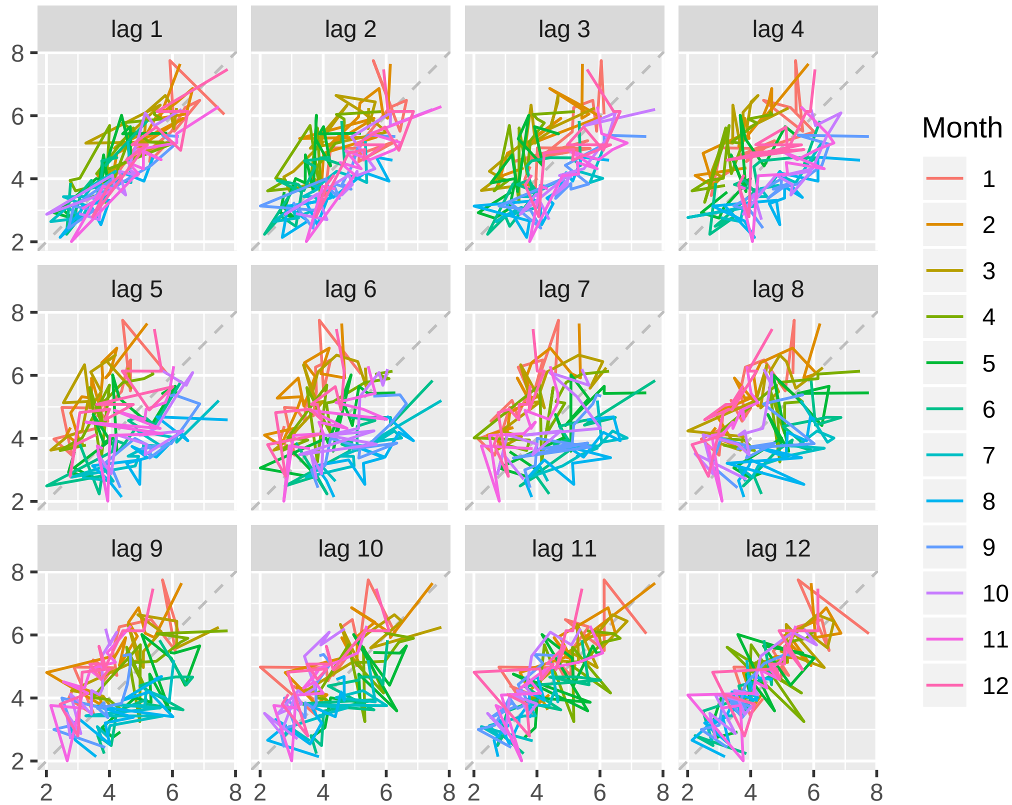

```r
acf(rhine_data_ts)
```



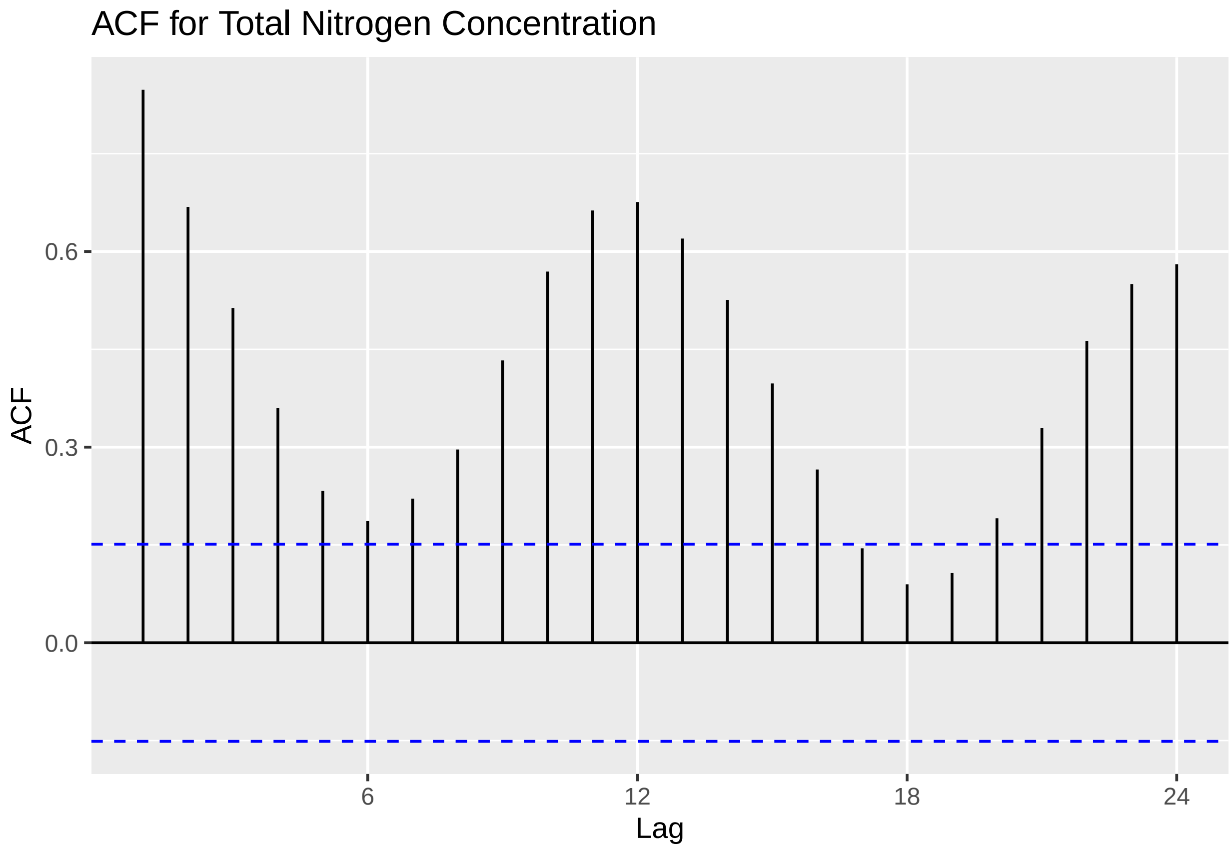

```r
#alternative
autoplot(rhine_data_ts) + ylab("Total Concentration") +xlab("Year") +
  ggtitle("Concentration of Nitrogen in Rhine vs. Year")
```




```
gglagplot(rhine_data_ts, lags = 1, set.lags = 1:12, color=FALSE)
```




```r
ggAcf(rhine_data_ts) + ggtitle("ACF for Total Nitrogen Concentration")
```



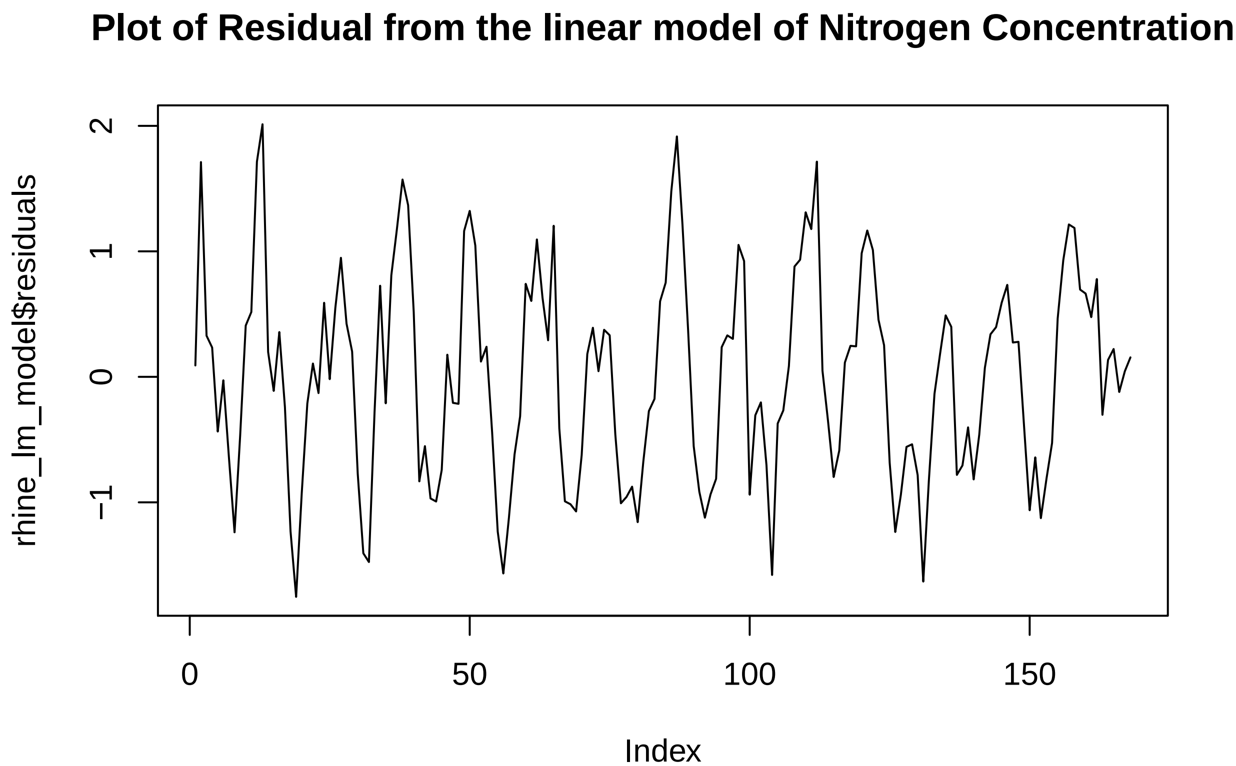
Analysis: Its evident from the time series plot that there is a decreasing trend and there is a seasonality to the concentration of nitrogen.

From the scatter plots of lag concentration vs. time we can conclude that correlation between concentration and lag is least in the middle months of the years than the beginning and end months.

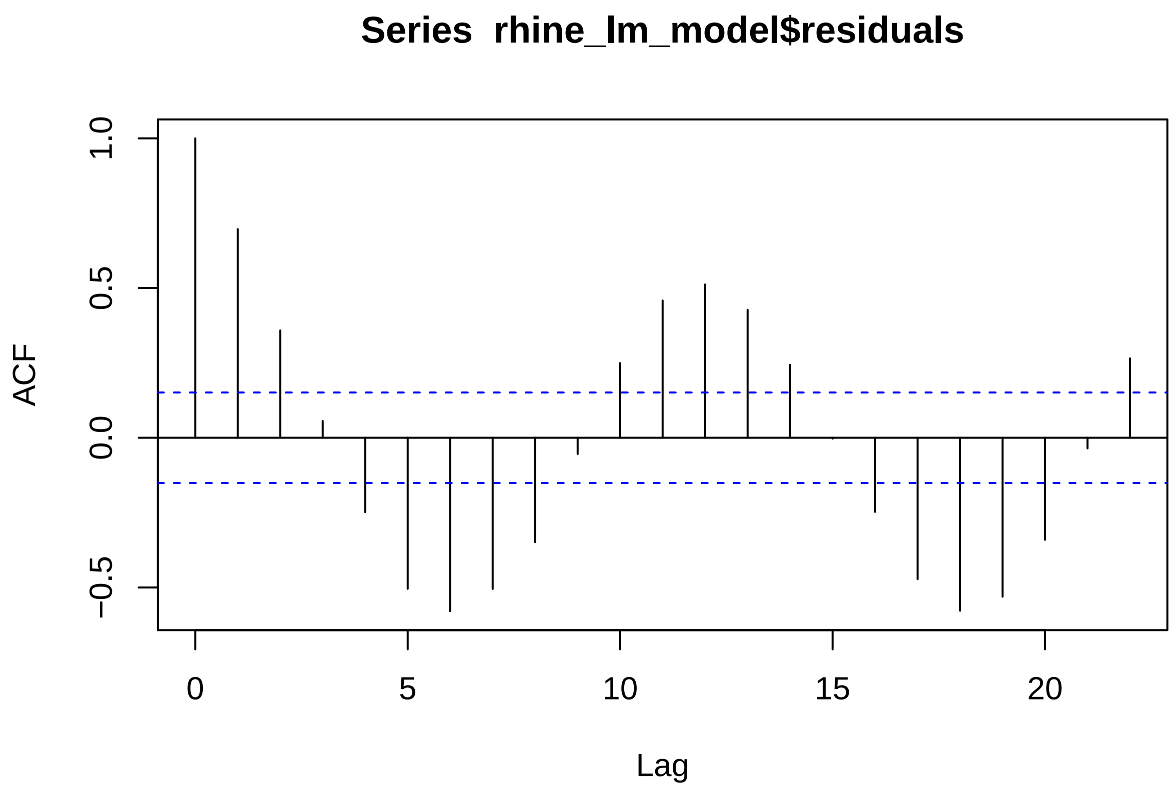
As from above we from ACF plots we can also conclude that the components of the end of the year and beginning of the year have more impact than the middle year components.

b) Eliminate the trend by fitting a linear model with respect to t to the time series. Is there a significant time trend? Look at the residual pattern and the sample ACF of the residuals and comment how this pattern might be related to seasonality of the series.

```
set.seed(12345)

rhine_lm_model <- lm(TotN_conc~Time, data=rhine_data)
plot(rhine_lm_model$residuals, type = 'l', main="Plot of Residual from the linear model of Nitrogen Conc
```




```
acf(rhine_lm_model$residuals)
```

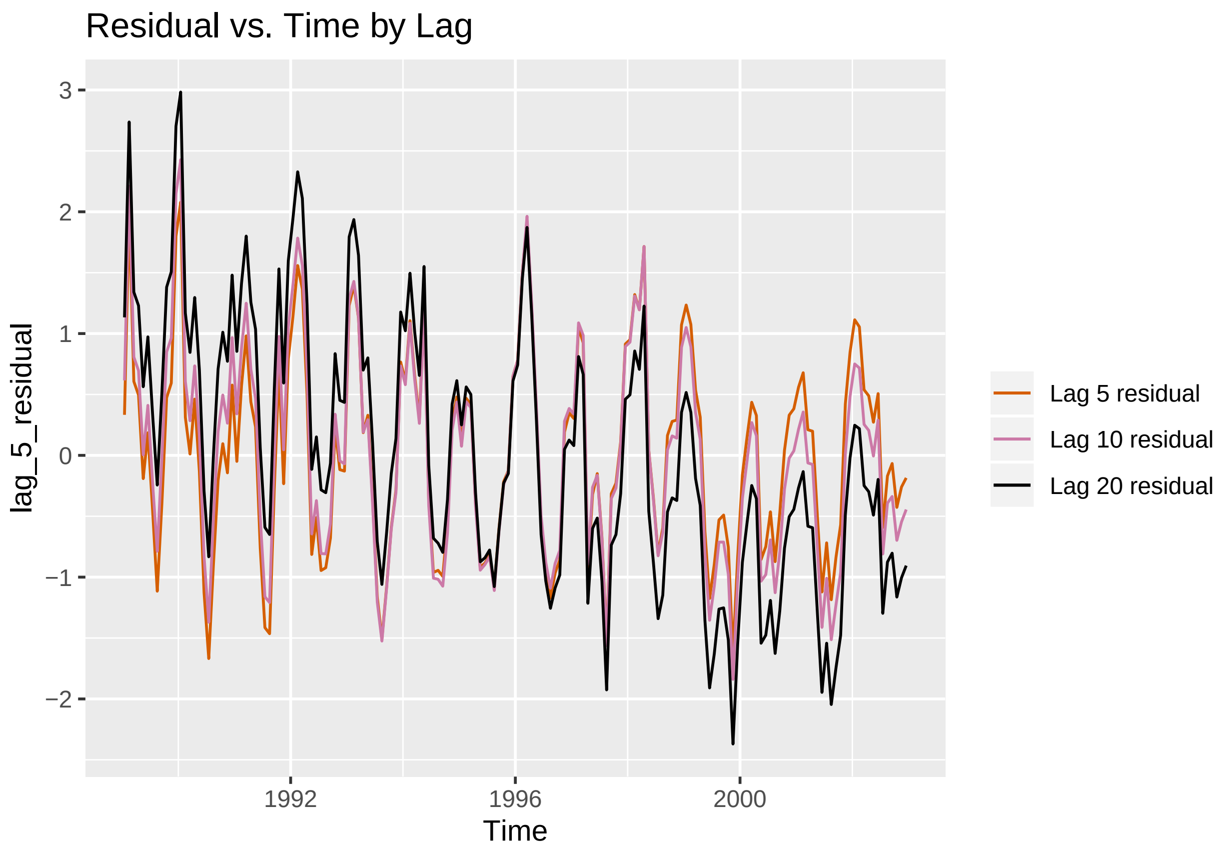


Analysis: The trend from the time series has been eliminated and the seasonality variation is limited to 1/2 units. From the ACF plot we also see that few components are needed to describe this compared to our previous approach.

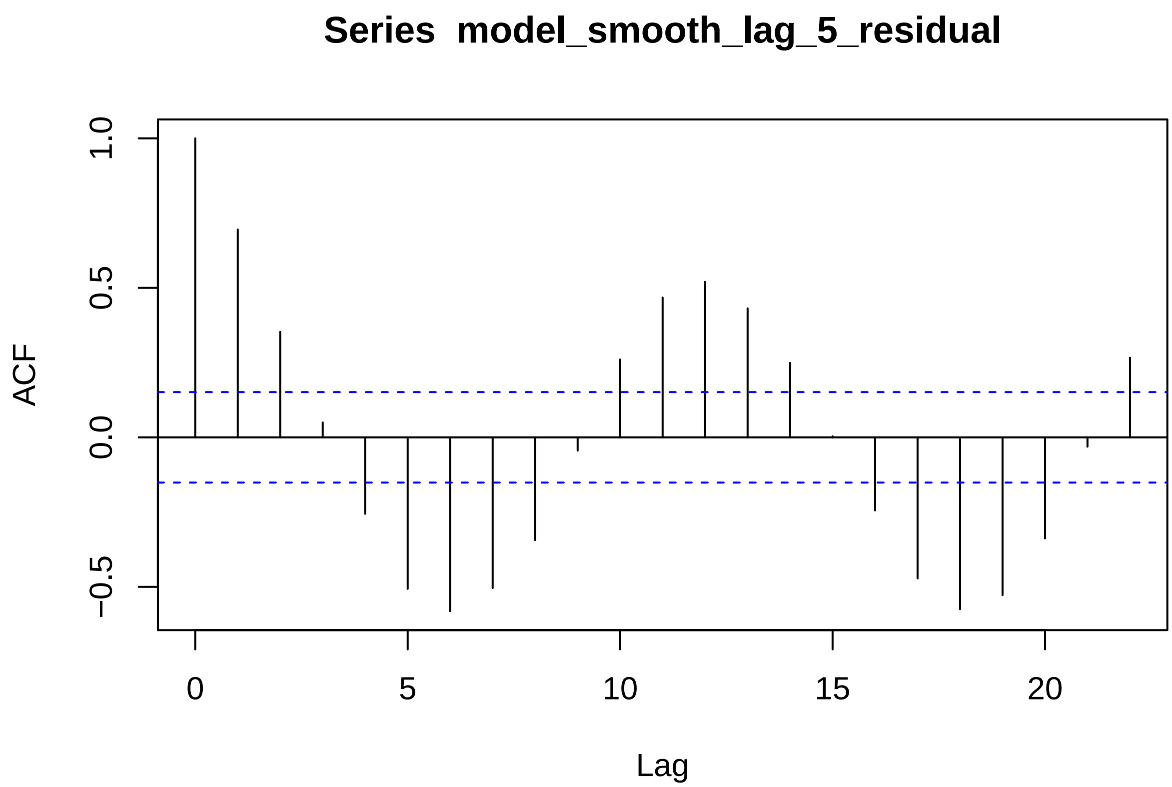
c) Eliminate the trend by fitting a kernel smoother with respect to t to the time series (choose a reasonable bandwidth yourself so the fit looks reasonable). Analyze the residual pattern and the sample ACF of the residuals and compare it to the ACF from step b). Conclusions? Do residuals seem to represent a stationary series?

```
set.seed(12345)

model_smooth_lag_5 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=5)
model_smooth_lag_10 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=10)
model_smooth_lag_20 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=20)

model_smooth_lag_5_residual <- rhine_data$TotN_conc - model_smooth_lag_5$y
model_smooth_lag_10_residual <- rhine_data$TotN_conc - model_smooth_lag_10$y
model_smooth_lag_20_residual <- rhine_data$TotN_conc - model_smooth_lag_20$y

residual_df <- cbind(model_smooth_lag_5_residual, model_smooth_lag_10_residual,
                     model_smooth_lag_20_residual, rhine_data$Time) %>% as.data.frame()

colnames(residual_df) <- c("lag_5_residual", "lag_10_residual", "lag_20_residual", "Time")

ggplot(residual_df, aes(x=Time)) +
  geom_line(aes(y=lag_5_residual, color="Lag 5 residual")) +
  geom_line(aes(y=lag_10_residual, color="Lag 10 residual")) +
```

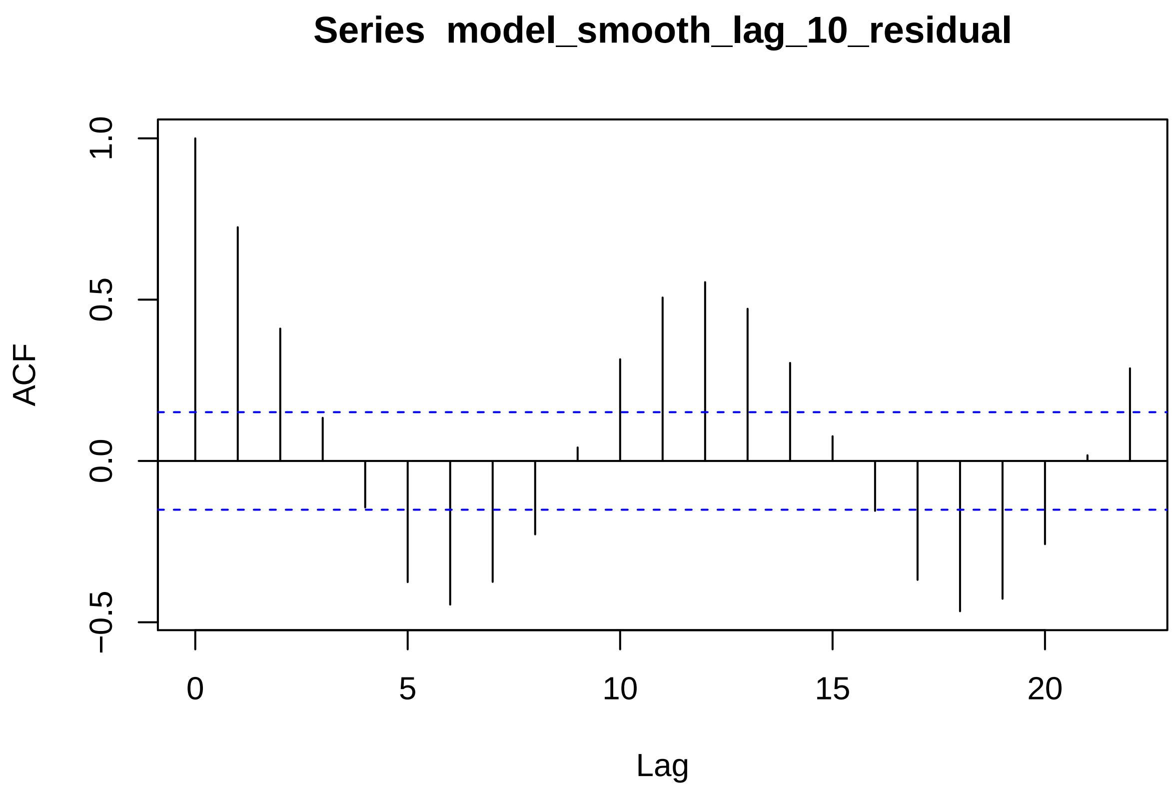
```r
  geom_line(aes(y=lag_20_residual, color="Lag 20 residual")) +
  ggtitle("Residual vs. Time by Lag") +
    scale_colour_manual("", breaks = c("Lag 5 residual", "Lag 10 residual", "Lag 20 residual"),
                        values = c("#CC79A7", "#000000", "#D55E00"))
```



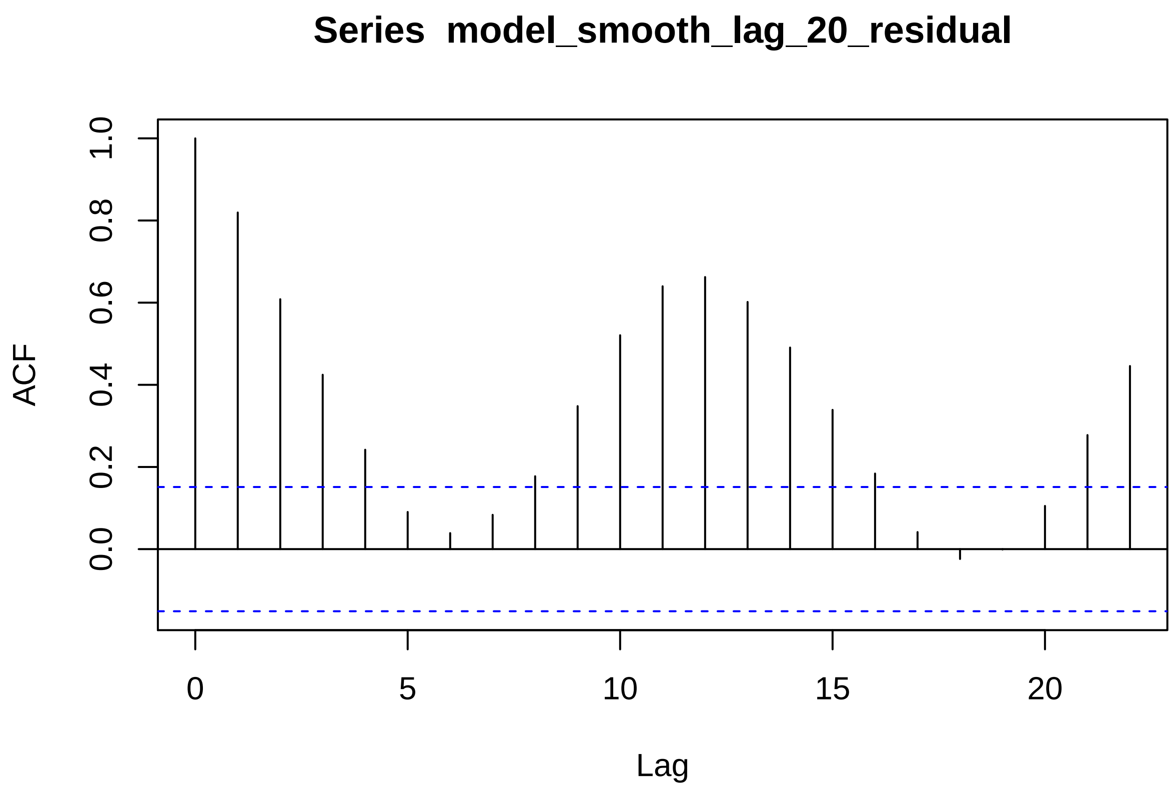

```r
acf(model_smooth_lag_5_residual)
```




```
acf(model_smooth_lag_10_residual)
```



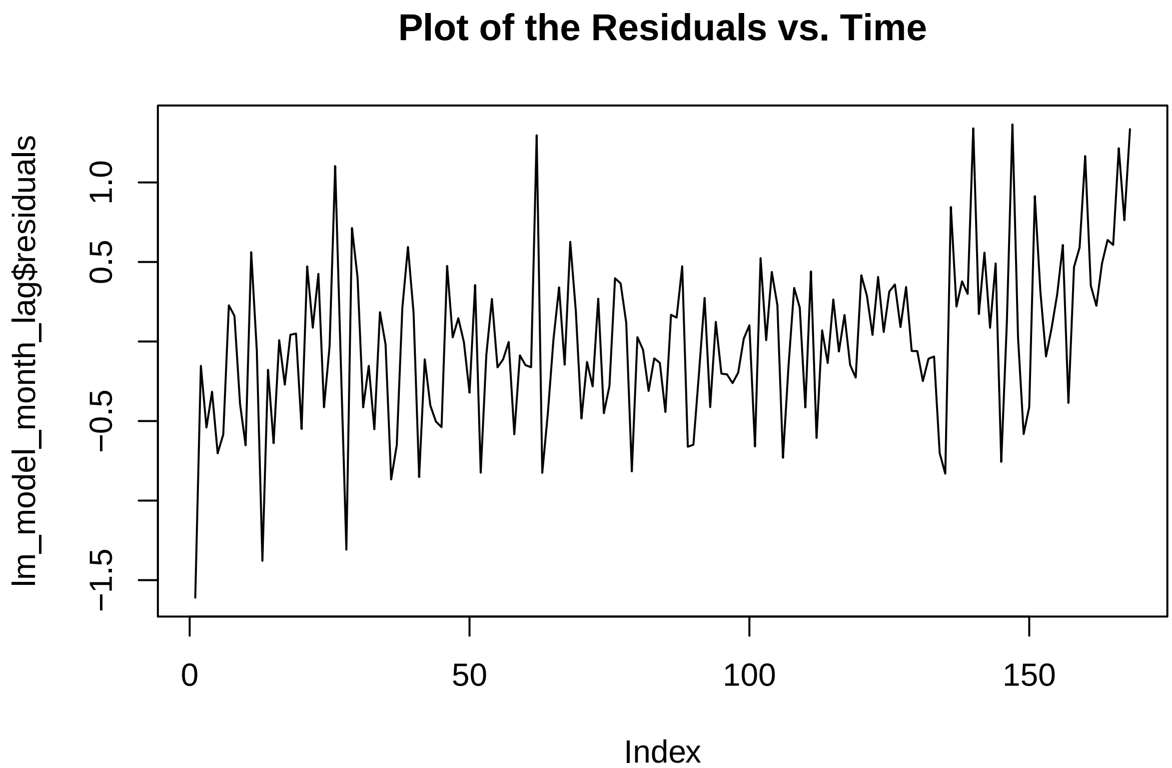

```
acf(model_smooth_lag_20_residual)
```



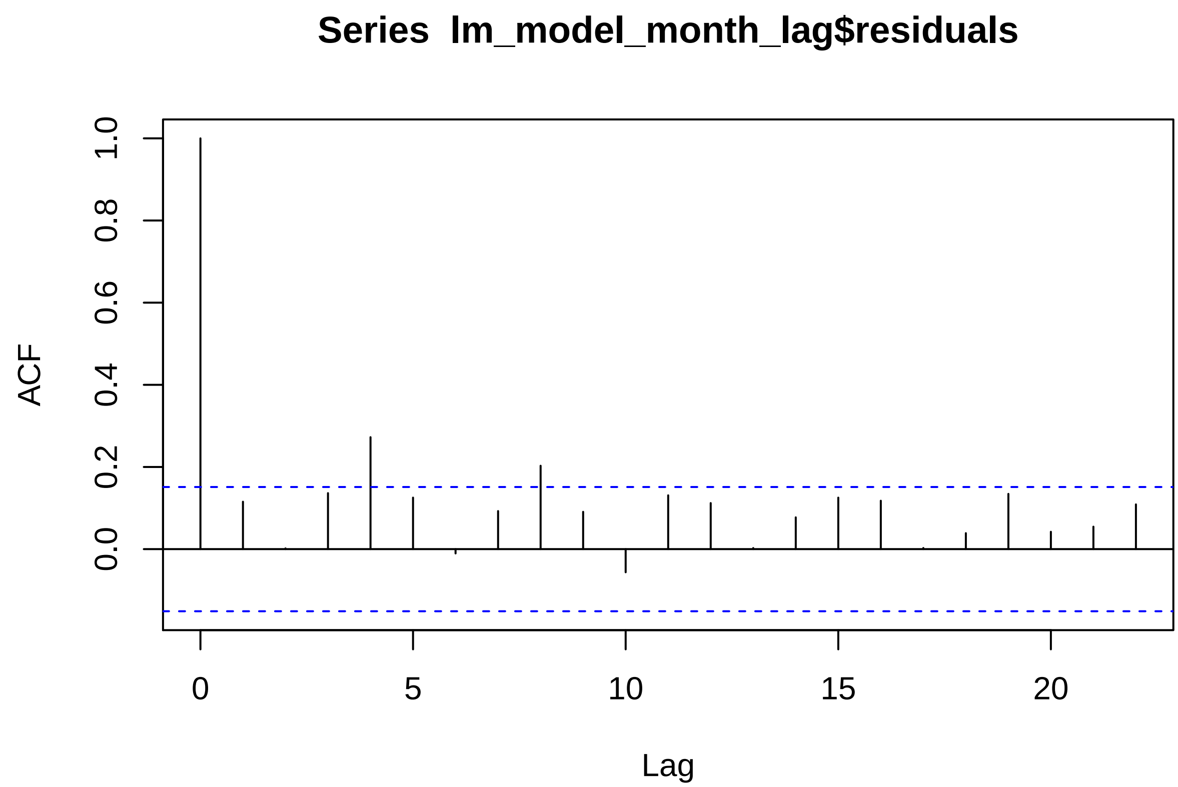
Analysis: In the above steps we have compared the residual pattern from the linear model, kernel smoother with bandwidth 4, 10 and bandwidth 20. Increasing the bandwidth does not seems to have much impact on the residual pattern and it is evident that it is not stationary, there is a large seasonality effect and a downwards trend still present in the plot.

d) Eliminate the trend by fitting the following so-called seasonal means model: $x_t = \alpha_0 + \alpha_1 t + \beta_1 I(month = 2) + ... + \beta_{12} I(month = 12) + w_t$, where I(x)=1 is an identity function. Fitting of this model will require you to augment data with a categorical variable showing the current month, and then fitting a usual linear regression. Analyze the residual pattern and the ACF of residuals.

```
set.seed(12345)

rhine_data_wide <- rhine_data
rhine_data_wide$dummy <- "1"
rhine_data_wide$Month <- paste0("Month_",rhine_data_wide$Month)
rhine_data_wide <- dcast(rhine_data_wide,
                         formula = TotN_conc+Year+Time~Month, value.var = "dummy", fill = "0")

lm_model_month_lag <- lm(data=rhine_data_wide,
                        TotN_conc~Time+Month_1+Month_2+Month_3+Month_4+Month_5+Month_6+Month_7+
                          Month_8+Month_9+Month_10+Month_11+Month_12)


plot(lm_model_month_lag$residuals, type = 'l', main="Plot of the Residuals vs. Time")
```




```r
acf(lm_model_month_lag$residuals)
```



Analysis: From the ACF plot we can see that the time series is not dependents on the components (barring maybe 1) and thus is stationary. From the residual plot we can see that there is a small trend, we can use some variable elimination to refine the model.

e) Perform step-wise variable selection in model from step d). Which model gives you the lowest AIC value? Which variables are left in the model?

```
set.seed(12345)

lm_model_month_lag_step <- stepAIC(lm_model_month_lag,
                                   scope = list(upper = ~Time+Month_1+Month_2+
                                                    Month_3+Month_4+Month_5+Month_6+Month_7+
                                                    Month_8+Month_9+Month_10+Month_11+Month_12,
                                                              lower = ~1), trace = TRUE,
                                   direction="backward")
```

```
## Start:  AIC=-202.02
## TotN_conc ~ Time + Month_1 + Month_2 + Month_3 + Month_4 + Month_5 +
##     Month_6 + Month_7 + Month_8 + Month_9 + Month_10 + Month_11 +
##     Month_12
##
##
## Step:  AIC=-202.02
## TotN_conc ~ Time + Month_1 + Month_2 + Month_3 + Month_4 + Month_5 +
##     Month_6 + Month_7 + Month_8 + Month_9 + Month_10 + Month_11
##
##            Df Sum of Sq     RSS      AIC
```

```
## - Month_4   1     0.200  43.436 -203.249
## - Month_1   1     0.220  43.456 -203.170
## - Month_3   1     0.331  43.567 -202.743
## <none>                   43.237 -202.023
## - Month_2   1     1.440  44.677 -198.517
## - Month_11  1     2.305  45.541 -195.297
## - Month_5   1     3.274  46.511 -191.760
## - Month_10  1     3.401  46.637 -191.303
## - Month_9   1     7.853  51.089 -175.986
## - Month_6   1     8.215  51.452 -174.797
## - Month_7   1    14.321  57.557 -155.959
## - Month_8   1    16.488  59.725 -149.749
## - Time      1   118.387 161.624   17.499
##
## Step:  AIC=-203.25
## TotN_conc ~ Time + Month_1 + Month_2 + Month_3 + Month_5 + Month_6 +
##     Month_7 + Month_8 + Month_9 + Month_10 + Month_11
##
##            Df Sum of Sq     RSS      AIC
## <none>                   43.436 -203.249
## - Month_1   1     0.640  44.077 -202.790
## - Month_3   1     0.851  44.288 -201.988
## - Month_11  1     2.235  45.671 -196.819
## - Month_2   1     2.706  46.142 -195.096
## - Month_5   1     3.355  46.791 -192.748
## - Month_10  1     3.502  46.938 -192.223
## - Month_9   1     8.868  52.304 -174.036
## - Month_6   1     9.317  52.753 -172.602
## - Month_7   1    16.912  60.348 -150.004
## - Month_8   1    19.636  63.072 -142.586
## - Time      1   118.194 161.630   15.506
```

```r
colnames(lm_model_month_lag_step$model)
```

```
##  [1] "TotN_conc" "Time"      "Month_1"   "Month_2"   "Month_3"
##  [6] "Month_5"   "Month_6"   "Month_7"   "Month_8"   "Month_9"
## [11] "Month_10"  "Month_11"
```

Analysis: The final terms in the model are given above, this model had the least AIC.

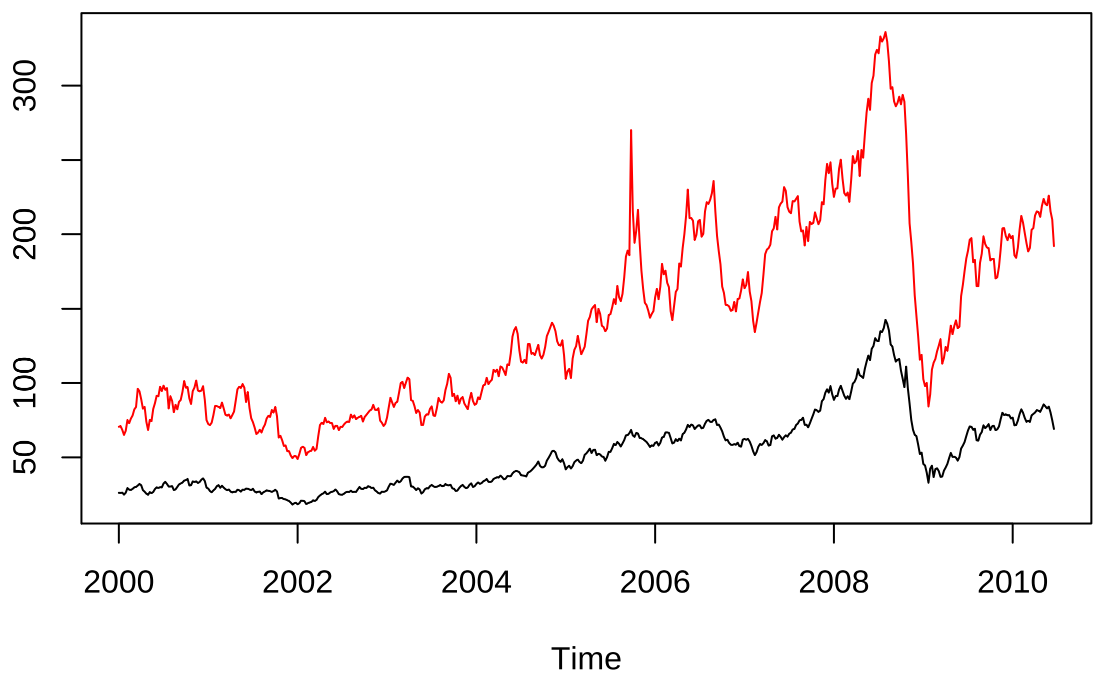
Assignment 3. Analysis of oil and gas time series.

Weekly time series oil and gas present in the package astsa show the oil prices in dollars per barrel and gas prices in cents per dollar.

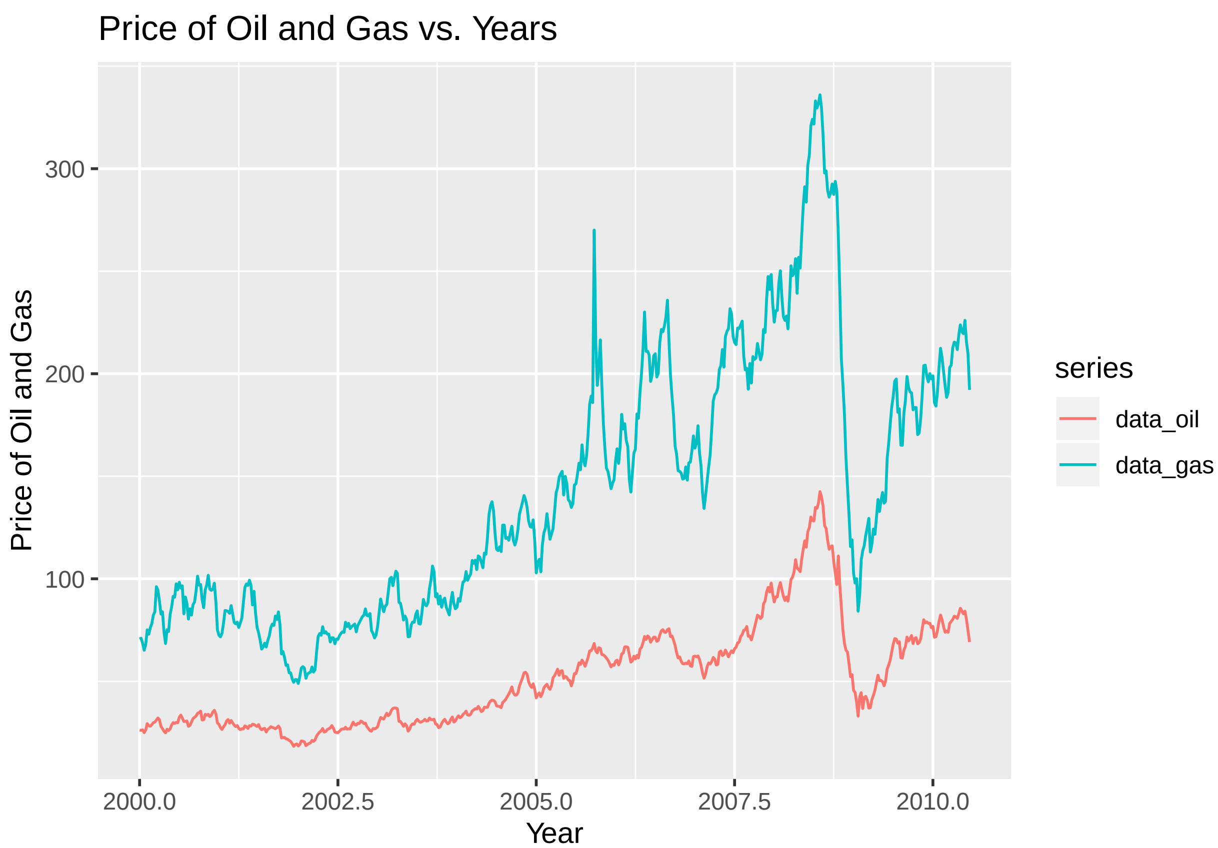
a) Plot the given time series in the same graph. Do they look like stationary series? Do the processes seem to be related to each other? Motivate your answer.

```r
set.seed(12345)

data_oil <- astsa::oil
data_gas <- astsa::gas
```

```r
ts.plot(data_oil, data_gas, gpars = list(col = c("black", "red")))
```



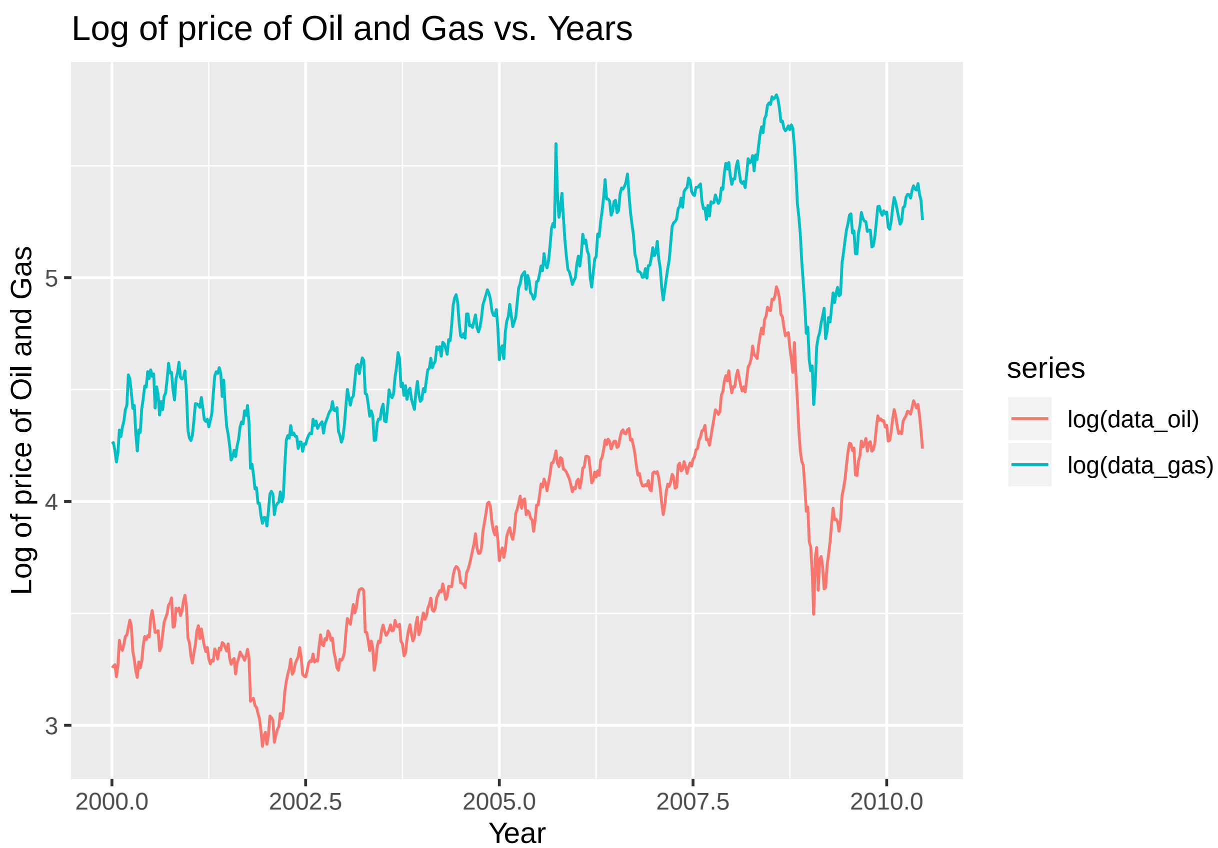

```r
#alternative

autoplot(ts(cbind(data_oil, data_gas), start = 2000, frequency = 52)) +
           ylab("Price of Oil and Gas") +xlab("Year") +
           ggtitle("Price of Oil and Gas vs. Years")
```



Analysis: No the series is not stationary since we see that it is time dependent. Both the series seemed to be correlated to each other.

b) Apply log-transform to the time series and plot the transformed data. In what respect did this transformation made the data easier for the analysis?

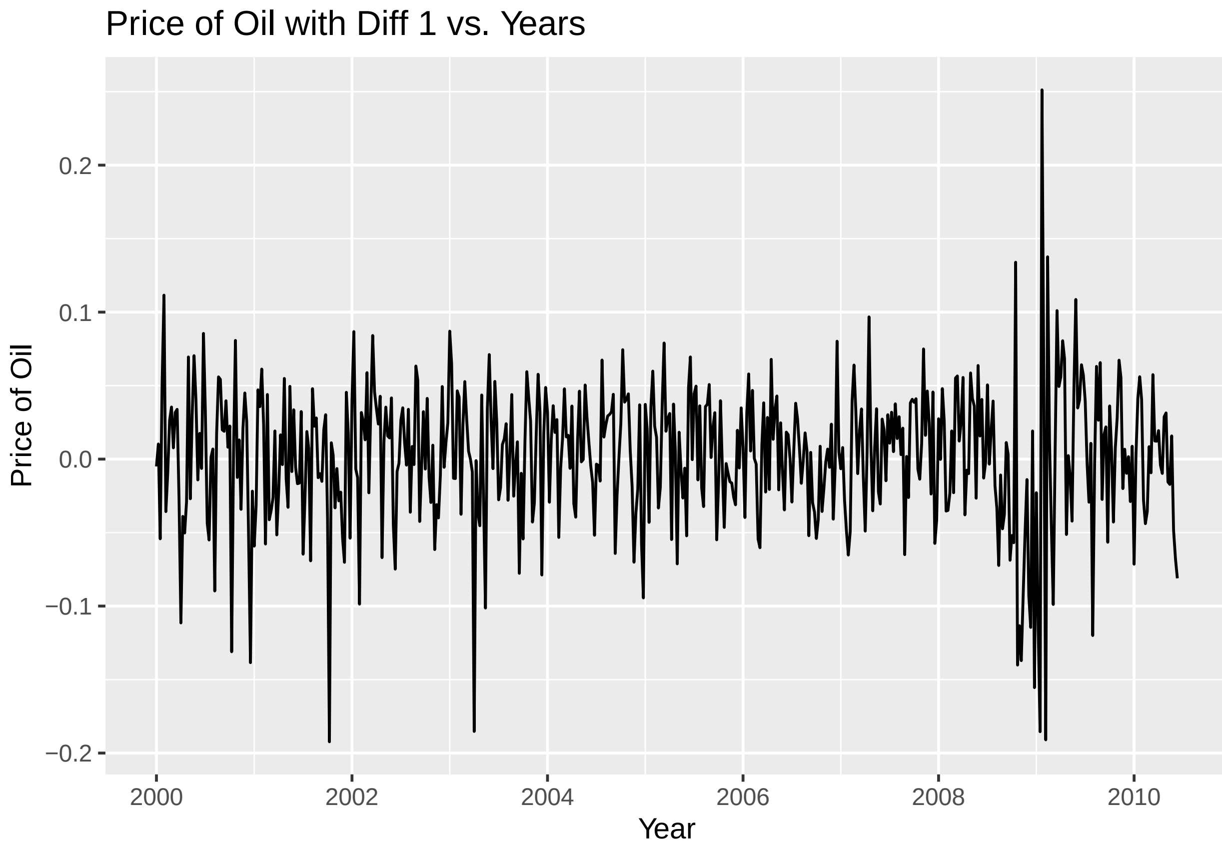
```
set.seed(12345)

autoplot(ts(cbind(log(data_oil), log(data_gas)), start = 2000, frequency = 52)) +
           ylab("Log of price of Oil and Gas") +xlab("Year") +
           ggtitle("Log of price of Oil and Gas vs. Years")
```



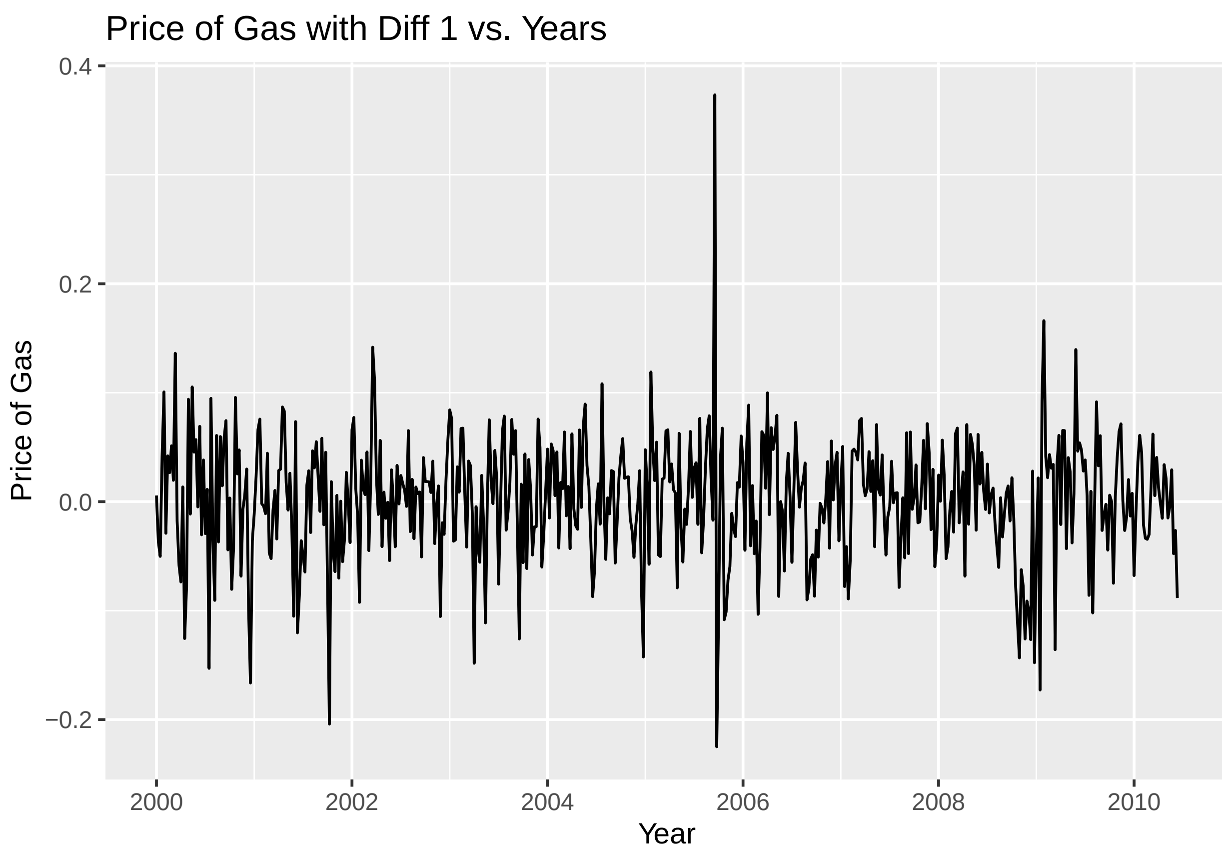
Analysis: Taking log reduces the variation(along the y-axis) so its easier to compare.

c) To eliminate trend, compute the first difference of the transformed data, plot the detrended series, check their ACFs and analyze the obtained plots. Denote the data obtained here as x_t(oil) and y_t(gas).

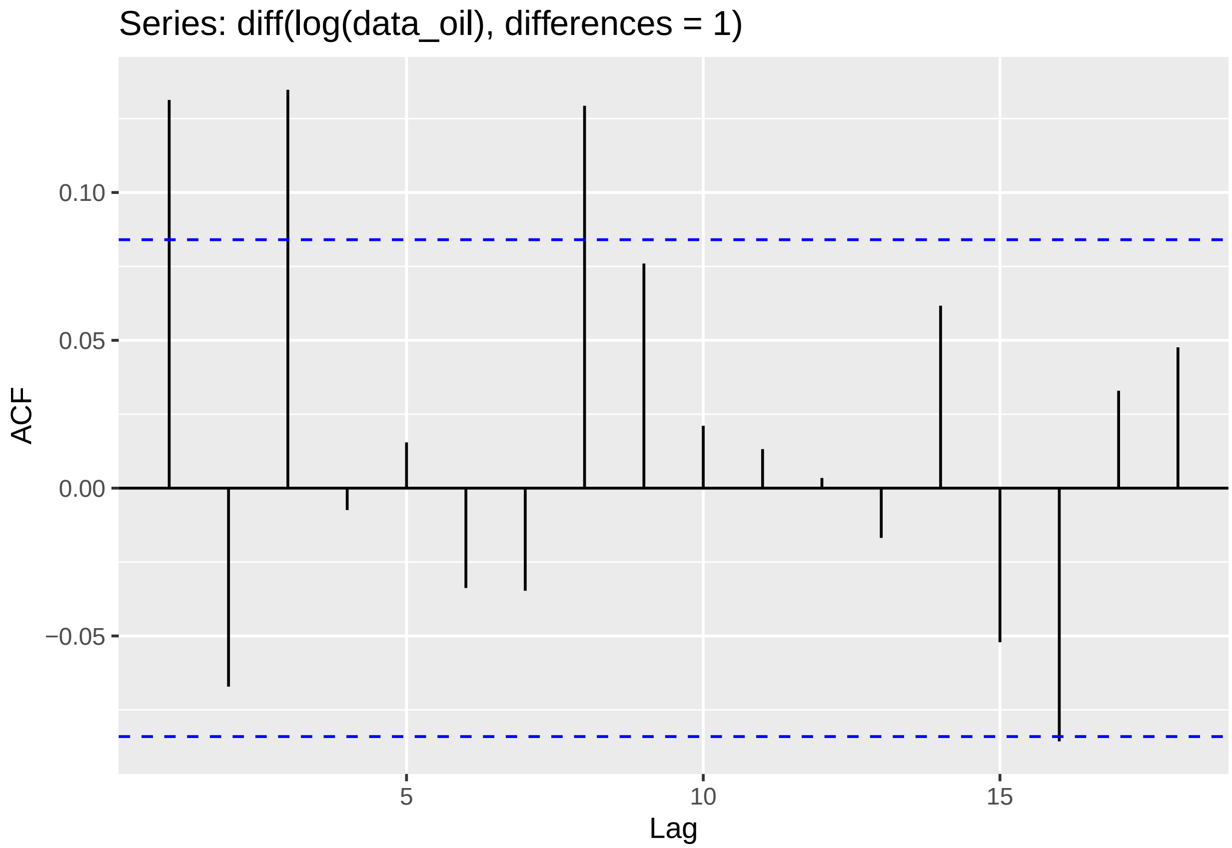
```
set.seed(12345)

autoplot(ts(diff(log(data_oil), differences = 1), start = 2000, frequency = 52)) +
          ylab("Price of Oil") +xlab("Year") +
          ggtitle("Price of Oil with Diff 1 vs. Years")
```



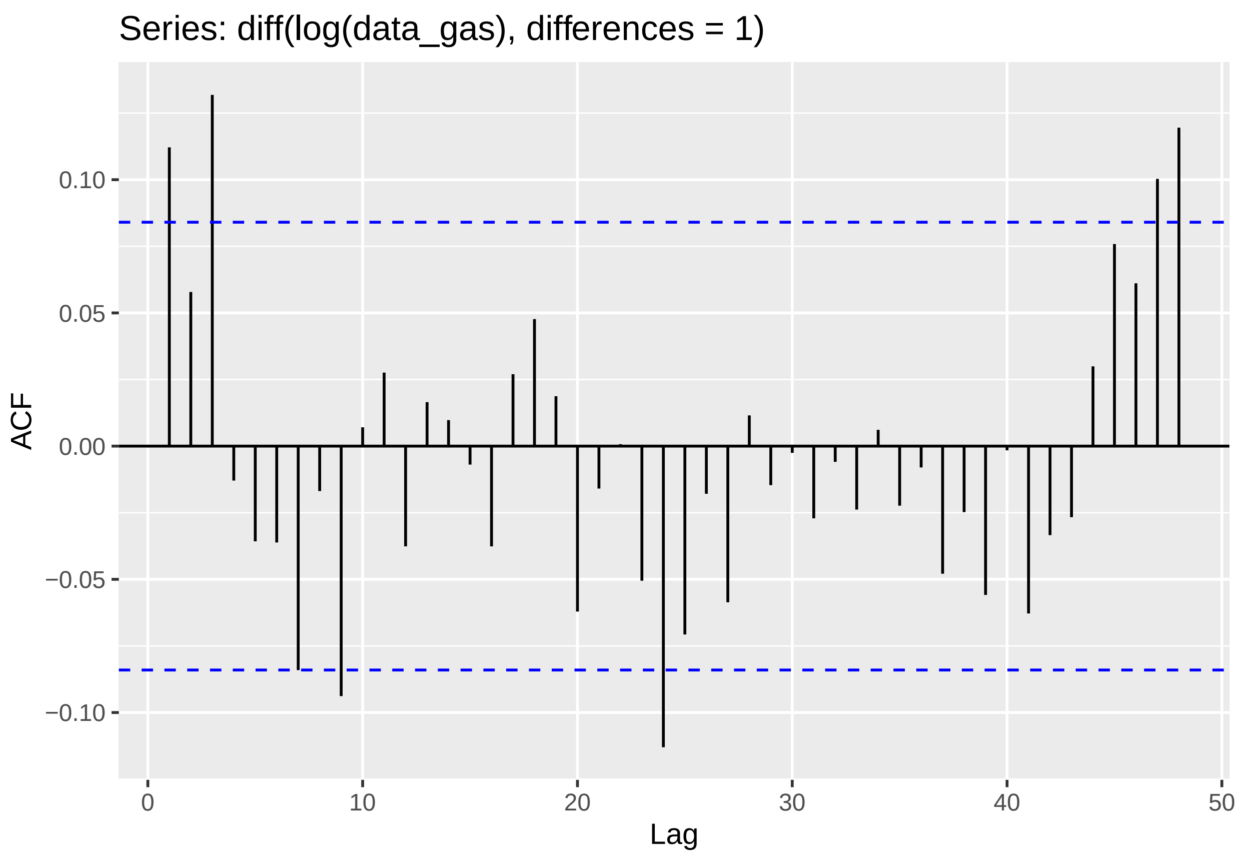

```r
autoplot(ts(diff(log(data_gas), differences = 1), start = 2000, frequency = 52)) +
          ylab("Price of Gas") +xlab("Year") +
          ggtitle("Price of Gas with Diff 1 vs. Years")
```




```
ggAcf(diff(log(data_oil), differences = 1), data_oil)
```




```
ggAcf(diff(log(data_gas), differences = 1), data_gas)
```

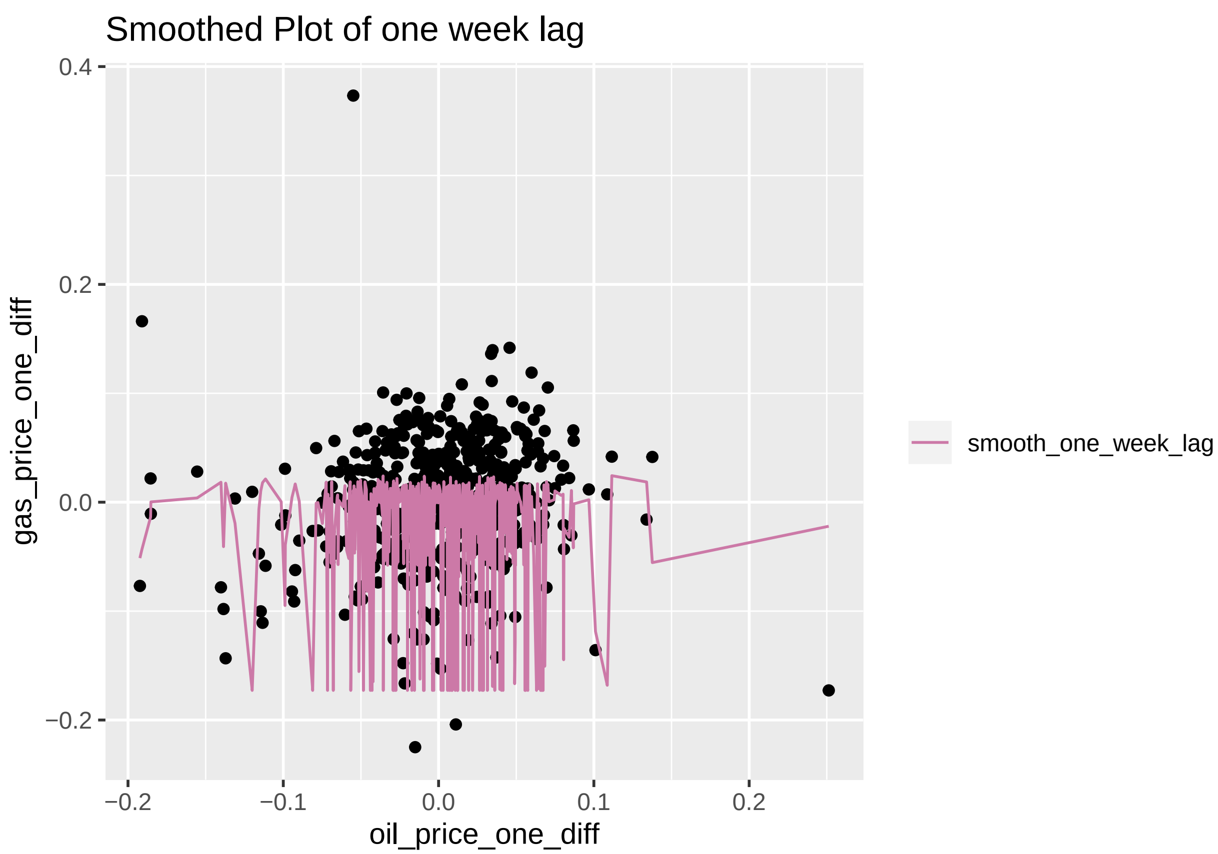


Analysis: We can say that from the ACF plot that the components that are needed to describe the series have reduced and both the series appear to be stationary barring a few points/peaks.

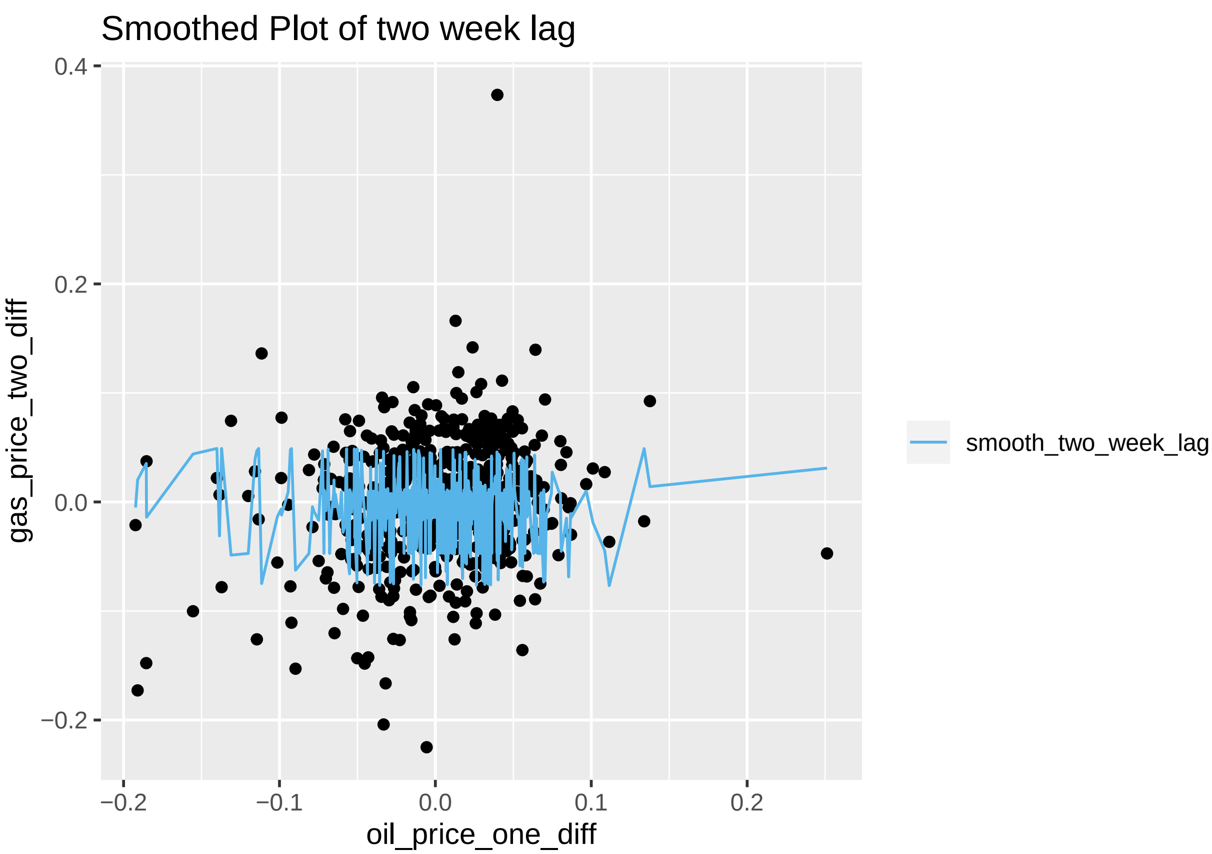
d) Exhibit scatter plots of x_t and y_t for up to three weeks of lead time of x_t include a non-parametric smoother in each plot and comment the results: are there outliers? Are the relationships linear? Are there changes in the trend?

```
set.seed(12345)

oil_price_one_diff <- diff(log(data_oil), differences = 1)
gas_price_one_diff <- diff(log(data_gas), differences = 1)

df <- data.frame(oil_price_one_diff=as.matrix(oil_price_one_diff),
           gas_price_one_diff = as.matrix(gas_price_one_diff),
                      time=time(oil_price_one_diff))

df <- na.omit(df)

df$gas_price_one_diff = lag(df$gas_price_one_diff,1)
df$gas_price_two_diff = lag(df$gas_price_one_diff,2)
df$gas_price_three_diff = lag(df$gas_price_one_diff,3)


df <- na.omit(df)

df$smooth_one_week_lag <- ksmooth(x = df$oil_price_one_diff, y = df$gas_price_one_diff, bandwidth = 0.05
df$smooth_two_week_lag <- ksmooth(x = df$oil_price_one_diff, y = df$gas_price_two_diff, bandwidth = 0.05
```

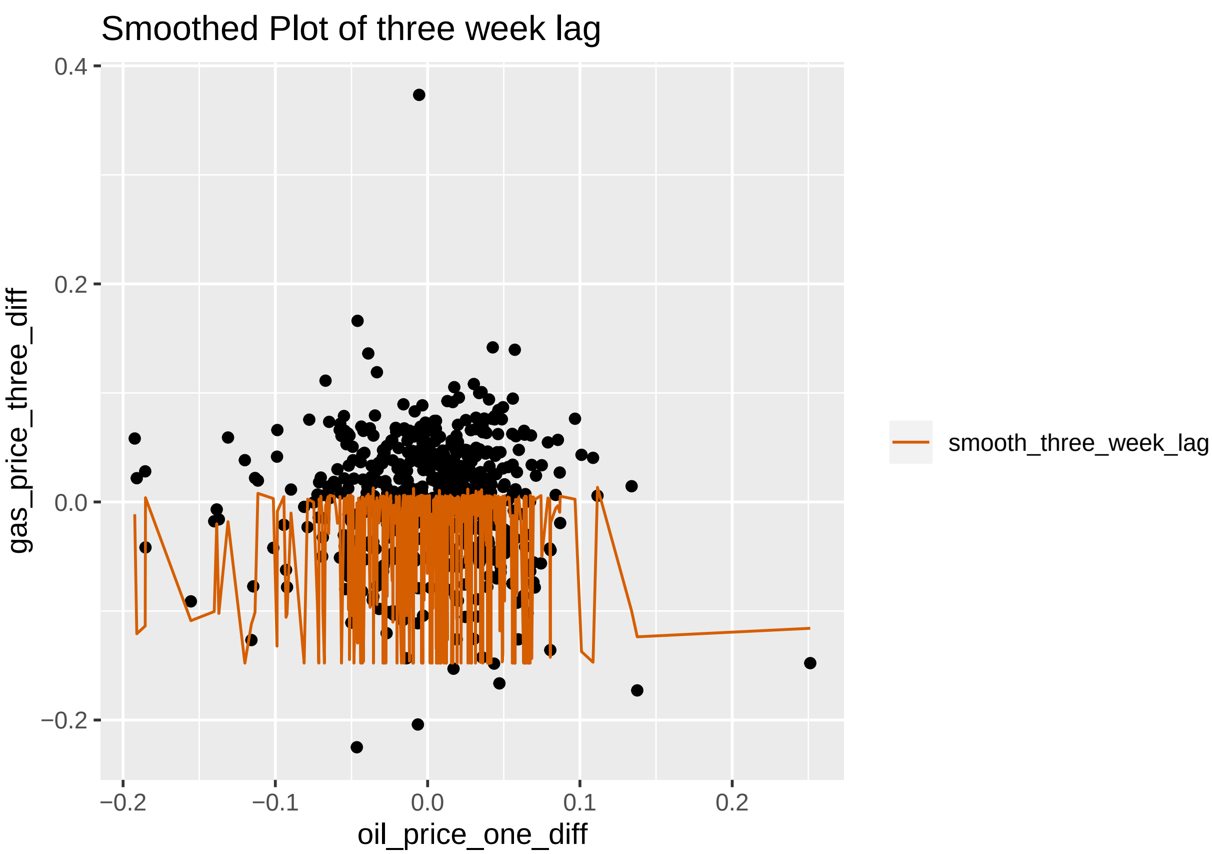
```r
df$smooth_three_week_lag <- ksmooth(x = df$oil_price_one_diff, y = df$gas_price_three_diff, bandwidth =

df <- na.omit(df)

ggplot(data=df, aes(x=oil_price_one_diff, y = gas_price_one_diff)) + geom_point() +
  geom_line(aes(y= smooth_one_week_lag, color= "smooth_one_week_lag")) +
      scale_colour_manual("", breaks = c("smooth_one_week_lag"),
                          values = c("#CC79A7")) +
  ggtitle("Smoothed Plot of one week lag")
```




```r
ggplot(data=df, aes(x=oil_price_one_diff, y = gas_price_two_diff)) + geom_point() +
    geom_line(aes(y= smooth_two_week_lag, color= "smooth_two_week_lag")) +
      scale_colour_manual("", breaks = c("smooth_two_week_lag"),
                          values = c("#56B4E9")) +
  ggtitle("Smoothed Plot of two week lag")
```




```
ggplot(data=df, aes(x=oil_price_one_diff, y = gas_price_three_diff)) + geom_point() +
    geom_line(aes(y= smooth_three_week_lag, color= "smooth_three_week_lag")) +
      scale_colour_manual("", breaks = c("smooth_three_week_lag"),
                          values = c("#D55E00")) +
  ggtitle("Smoothed Plot of three week lag")
```



Analysis: The correlation is a weakly increasing one. There are outliers present in all three plots, the lag of weeks has little impact on the plots.

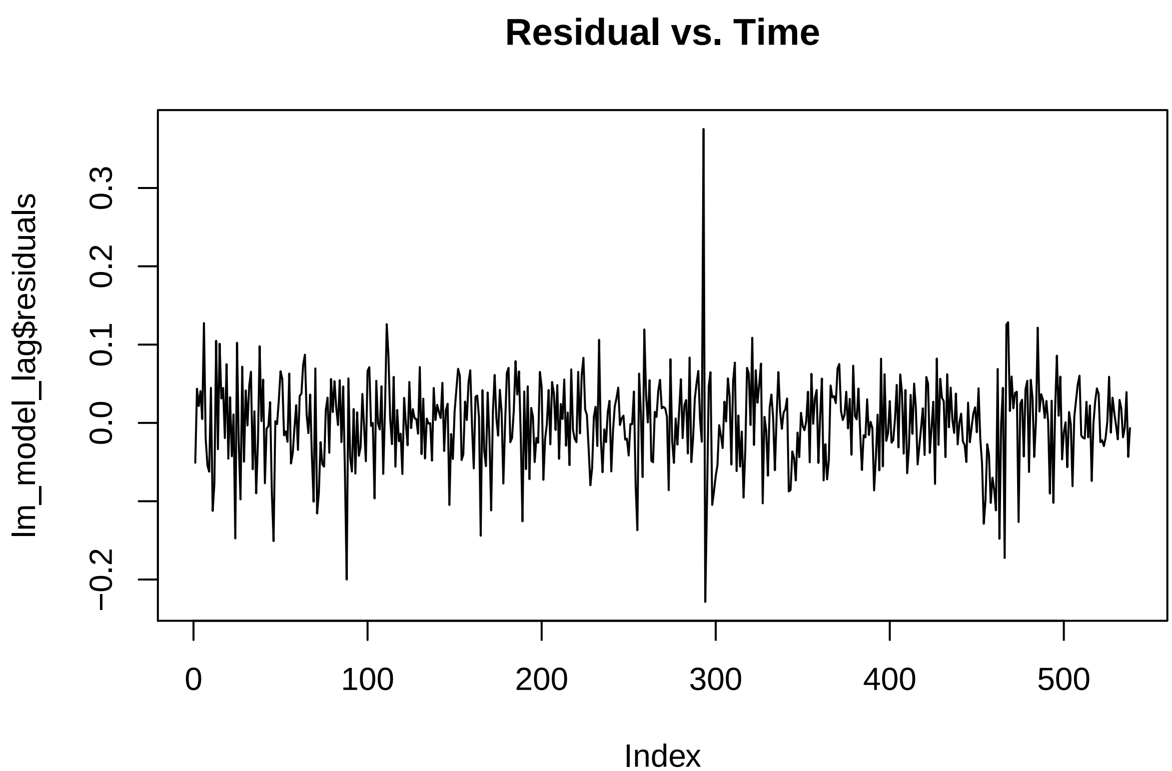
e) Fit the following model: $y_t = \alpha_0 + \alpha_1 I(x_t > 0) + \beta_1 x_t + \beta_2 x_{t-1} + w_t$ and check which coefficients seem to be significant. How can this be interpreted? Analyze the residual pattern and the ACF of the residuals.

```r
set.seed(12345)

df$oil_price_two_diff = lag(df$oil_price_one_diff,2)
df$x_t_more_zero <- ifelse(df$oil_price_one_diff>0,"1","0")
lm_model_lag <- lm(data=df, formula = gas_price_one_diff~x_t_more_zero+
                     oil_price_one_diff+oil_price_two_diff)
summary(lm_model_lag)
```

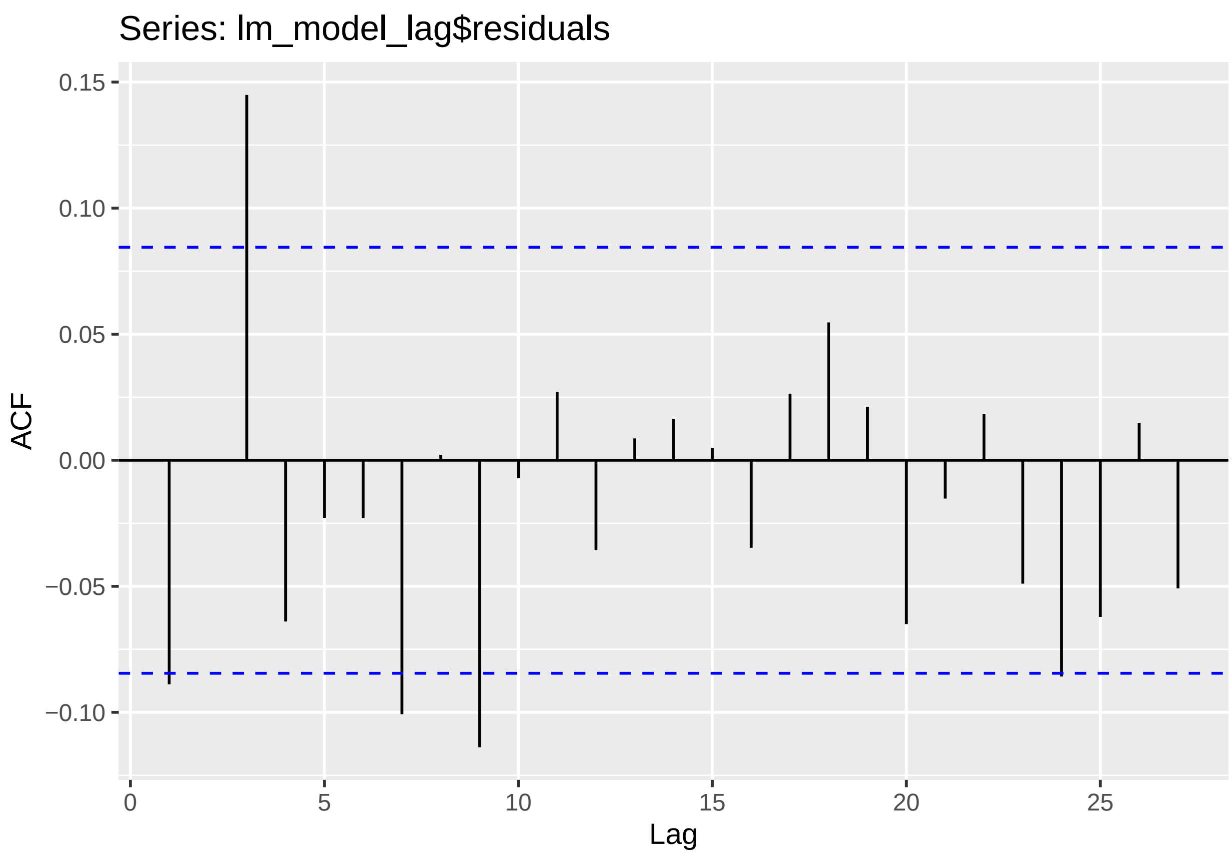
```
##
## Call:
## lm(formula = gas_price_one_diff ~ x_t_more_zero + oil_price_one_diff +
##     oil_price_two_diff, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.22855 -0.03191  0.00366  0.03692  0.37535
##
## Coefficients:
##                      Estimate Std. Error t value  Pr(>|t|)
## (Intercept)         -0.002139   0.004588  -0.466     0.641
```

```
## x_t_more_zero1      0.006265   0.007304   0.858     0.391
## oil_price_one_diff  0.084851   0.077622   1.093     0.275
## oil_price_two_diff  0.222373   0.050788   4.378 0.0000144 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05511 on 534 degrees of freedom
##   (2 observations deleted due to missingness)
## Multiple R-squared:  0.04571,    Adjusted R-squared:  0.04035
## F-statistic: 8.526 on 3 and 534 DF,  p-value: 0.00001538
```

```
plot(lm_model_lag$residuals, type = 'l', main="Residual vs. Time")
```




```
ggAcf(lm_model_lag$residuals)
```



Analysis: According to the summary none of the variable seems to be significant. The dummy variable has a positive coefficient which means that an increase in oil will also increase the gas.

We can also see from the plot of residuals that the residual seems to be constant over time with the exception of some outliers, thus the series is reaching the stationary conditions barring some regions.

The ACF plots confirms our findings from the residual plot that there is one single components which shows some auto-correlation.

Appendix

```
knitr::opts_chunk$set(echo = TRUE)
options(scipen=999)

library("tidyverse") #ggplot and dplyr
library("gridExtra") # combine plots
library("knitr") # for pdf
library("fpp2") #timeseries with autoplot and stuff
library("reshape2") #reshape the data
library("MASS") #StepAIC
library("astsa") #dataset oil and gas is present here
library("zoo") #dataset oil and gas is present here


# The palette with black:
```

```r
cbbPalette <- c("#000000", "#E69F00", "#56B4E9", "#009E73",
                "#F0E442", "#0072B2", "#D55E00", "#CC79A7")
set.seed(12345)
set.seed(12345)

n = 100
x <- vector(length = n)
x2 <- vector(length = n)

x[1] <- 0
x[2] <- 0

#first series generation
for(i in 3:n){
  x[i] <- -0.8 * x[i-2] + rnorm(1,0,1)
}

#second series generation
for(i in 1:n){
  x2[i] <- cos(0.4*pi*i)
}

# smoothing filter function
smoothing_filter <- function(x){
v <- vector(length = length(x))
for(i in 5:length(x)){
  v[i] = 0.2 * (x[i] + x[i-1] + x[i-2] + x[i-3] + x[i-4])
}
return(v)
}

#generate smoothed series
smooth_x <- smoothing_filter(x)
smooth_x2 <- smoothing_filter(x2)

#adding everything to a dataframe
df <- cbind(x,x2,smooth_x,smooth_x2) %>% as.data.frame() %>% mutate(index=1:100)

ggplot(df, aes(x=index)) +
  geom_line(aes(y=x, color="Original Time Series")) +
  geom_line(aes(y=smooth_x, color="Smoothed Time Series")) +
  ggtitle("Plot of 1st time series and its smoothed version") +
    scale_colour_manual("", breaks = c("Original Time Series", "Smoothed Time Series"),
                        values = c("#CC79A7", "#000000"))

ggplot(df, aes(x=index)) +
  geom_line(aes(y=x2, color="Original Time Series")) +
  geom_line(aes(y=smooth_x2, color="Smoothed Time Series")) +
  ggtitle("Plot of 2ND time series and its smoothed version") +
    scale_colour_manual("", breaks = c("Original Time Series", "Smoothed Time Series"),
                        values = c("#CC79A7", "#000000"))

z = c(1,-4,2,0,0,1)
```

```
polyroot(z)
any(Mod(polyroot(z))<=1)
z = c(1,0,3,0,1,0,-4)
polyroot(z)
any(Mod(polyroot(z))<=1)
set.seed(54321)

series <- arima.sim(n = 100, list(ar = c(-3/4), ma = c(0,-1/9)))

acf(series)
acf(ARMAacf(ar = c(-3/4), ma =  c(0,-1/9), lag.max = 20))
set.seed(12345)
rhine_data <- read.csv2("Rhine.csv")
rhine_data_ts <- ts(data = rhine_data$TotN_conc,
                    start = c(1989,1),
                    frequency = 12)

plot.ts(rhine_data_ts, main="Time Series of Nitrogen Concentration in Rhine")
lag.plot(rhine_data_ts,lags = 12)
acf(rhine_data_ts)

#alternative
autoplot(rhine_data_ts) + ylab("Total Concentration") +xlab("Year") +
  ggtitle("Concentration of Nitrogen in Rhine vs. Year")

gglagplot(rhine_data_ts, lags = 1, set.lags = 1:12, color=FALSE)
ggAcf(rhine_data_ts) + ggtitle("ACF for Total Nitrogen Concentration")

set.seed(12345)

rhine_lm_model <- lm(TotN_conc~Time, data=rhine_data)
plot(rhine_lm_model$residuals, type = 'l', main="Plot of Residual from the linear model of Nitrogen Conc
acf(rhine_lm_model$residuals)

set.seed(12345)

model_smooth_lag_5 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=5)
model_smooth_lag_10 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=10)
model_smooth_lag_20 <- ksmooth(x = rhine_data$Time, y = rhine_data$TotN_conc, bandwidth=20)

model_smooth_lag_5_residual <- rhine_data$TotN_conc - model_smooth_lag_5$y
model_smooth_lag_10_residual <- rhine_data$TotN_conc - model_smooth_lag_10$y
model_smooth_lag_20_residual <- rhine_data$TotN_conc - model_smooth_lag_20$y

residual_df <- cbind(model_smooth_lag_5_residual, model_smooth_lag_10_residual,
                     model_smooth_lag_20_residual, rhine_data$Time) %>% as.data.frame()

colnames(residual_df) <- c("lag_5_residual", "lag_10_residual", "lag_20_residual", "Time")

ggplot(residual_df, aes(x=Time)) +
  geom_line(aes(y=lag_5_residual, color="Lag 5 residual")) +
  geom_line(aes(y=lag_10_residual, color="Lag 10 residual")) +
  geom_line(aes(y=lag_20_residual, color="Lag 20 residual")) +
```

```
  ggtitle("Residual vs. Time by Lag") +
    scale_colour_manual("", breaks = c("Lag 5 residual", "Lag 10 residual", "Lag 20 residual"),
                        values = c("#CC79A7", "#000000", "#D55E00"))


acf(model_smooth_lag_5_residual)
acf(model_smooth_lag_10_residual)
acf(model_smooth_lag_20_residual)
set.seed(12345)

rhine_data_wide <- rhine_data
rhine_data_wide$dummy <- "1"
rhine_data_wide$Month <- paste0("Month_",rhine_data_wide$Month)
rhine_data_wide <- dcast(rhine_data_wide,
                         formula = TotN_conc+Year+Time~Month, value.var = "dummy", fill = "0")

lm_model_month_lag <- lm(data=rhine_data_wide,
                    TotN_conc~Time+Month_1+Month_2+Month_3+Month_4+Month_5+Month_6+Month_7+
                      Month_8+Month_9+Month_10+Month_11+Month_12)


plot(lm_model_month_lag$residuals, type = 'l', main="Plot of the Residuals vs. Time")
acf(lm_model_month_lag$residuals)

set.seed(12345)

lm_model_month_lag_step <- stepAIC(lm_model_month_lag,
                                   scope = list(upper = ~Time+Month_1+Month_2+
                                                    Month_3+Month_4+Month_5+Month_6+Month_7+
                                                    Month_8+Month_9+Month_10+Month_11+Month_12,
                                                          lower = ~1), trace = TRUE,
                                   direction="backward")

colnames(lm_model_month_lag_step$model)
set.seed(12345)

data_oil <- astsa::oil
data_gas <- astsa::gas

ts.plot(data_oil, data_gas, gpars = list(col = c("black", "red")))

#alternative

autoplot(ts(cbind(data_oil, data_gas), start = 2000, frequency = 52)) +
           ylab("Price of Oil and Gas") +xlab("Year") +
           ggtitle("Price of Oil and Gas vs. Years")
set.seed(12345)

autoplot(ts(cbind(log(data_oil), log(data_gas)), start = 2000, frequency = 52)) +
           ylab("Log of price of Oil and Gas") +xlab("Year") +
           ggtitle("Log of price of Oil and Gas vs. Years")
set.seed(12345)
```

```
autoplot(ts(diff(log(data_oil), differences = 1), start = 2000, frequency = 52)) +
          ylab("Price of Oil") +xlab("Year") +
          ggtitle("Price of Oil with Diff 1 vs. Years")

autoplot(ts(diff(log(data_gas), differences = 1), start = 2000, frequency = 52)) +
          ylab("Price of Gas") +xlab("Year") +
          ggtitle("Price of Gas with Diff 1 vs. Years")

ggAcf(diff(log(data_oil), differences = 1), data_oil)
ggAcf(diff(log(data_gas), differences = 1), data_gas)

set.seed(12345)

oil_price_one_diff <- diff(log(data_oil), differences = 1)
gas_price_one_diff <- diff(log(data_gas), differences = 1)

df <- data.frame(oil_price_one_diff=as.matrix(oil_price_one_diff),
          gas_price_one_diff = as.matrix(gas_price_one_diff),
                    time=time(oil_price_one_diff))

df <- na.omit(df)

df$gas_price_one_diff = lag(df$gas_price_one_diff,1)
df$gas_price_two_diff = lag(df$gas_price_one_diff,2)
df$gas_price_three_diff = lag(df$gas_price_one_diff,3)


df <- na.omit(df)

df$smooth_one_week_lag <- ksmooth(x = df$oil_price_one_diff, y = df$gas_price_one_diff, bandwidth = 0.05
df$smooth_two_week_lag <- ksmooth(x = df$oil_price_one_diff, y = df$gas_price_two_diff, bandwidth = 0.05
df$smooth_three_week_lag <- ksmooth(x = df$oil_price_one_diff, y = df$gas_price_three_diff, bandwidth =

df <- na.omit(df)

ggplot(data=df, aes(x=oil_price_one_diff, y = gas_price_one_diff)) + geom_point() +
  geom_line(aes(y= smooth_one_week_lag, color= "smooth_one_week_lag")) +
      scale_colour_manual("", breaks = c("smooth_one_week_lag"),
                        values = c("#CC79A7")) +
  ggtitle("Smoothed Plot of one week lag")

ggplot(data=df, aes(x=oil_price_one_diff, y = gas_price_two_diff)) + geom_point() +
    geom_line(aes(y= smooth_two_week_lag, color= "smooth_two_week_lag")) +
      scale_colour_manual("", breaks = c("smooth_two_week_lag"),
                        values = c("#56B4E9")) +
  ggtitle("Smoothed Plot of two week lag")


ggplot(data=df, aes(x=oil_price_one_diff, y = gas_price_three_diff)) + geom_point() +
    geom_line(aes(y= smooth_three_week_lag, color= "smooth_three_week_lag")) +
      scale_colour_manual("", breaks = c("smooth_three_week_lag"),
                        values = c("#D55E00")) +
  ggtitle("Smoothed Plot of three week lag")
```

```r
set.seed(12345)

df$oil_price_two_diff = lag(df$oil_price_one_diff,2)
df$x_t_more_zero <- ifelse(df$oil_price_one_diff>0,"1","0")
lm_model_lag <- lm(data=df, formula = gas_price_one_diff~x_t_more_zero+
                     oil_price_one_diff+oil_price_two_diff)
summary(lm_model_lag)

plot(lm_model_lag$residuals, type = 'l', main="Residual vs. Time")
ggAcf(lm_model_lag$residuals)
```