Computer Lab 1 Block 2: ENSEMBLE METHODS AND MIXTURE MODELS (732A99 Machine Learning)

Lennart Schilling (lensc874)

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1. Ensemble Methods

At first, the data from the Excel file *spambase.csv* will be imported and splitted into train and test data (50%:50%).

```
library(randomForest)
library(mboost)
library(ggplot2)

# importing data
data = read.csv2("spambase.csv")

# converting 'Spam' to 'factor' so that randomForest() does classification instead of regression
data$Spam = as.factor(data$Spam)

# dividing data into train and test set
n = dim(data)[1]
set.seed(12345)
id = sample(1:n, floor(n*(2/3)))
train = data[id,]
test = data[-id,]
```

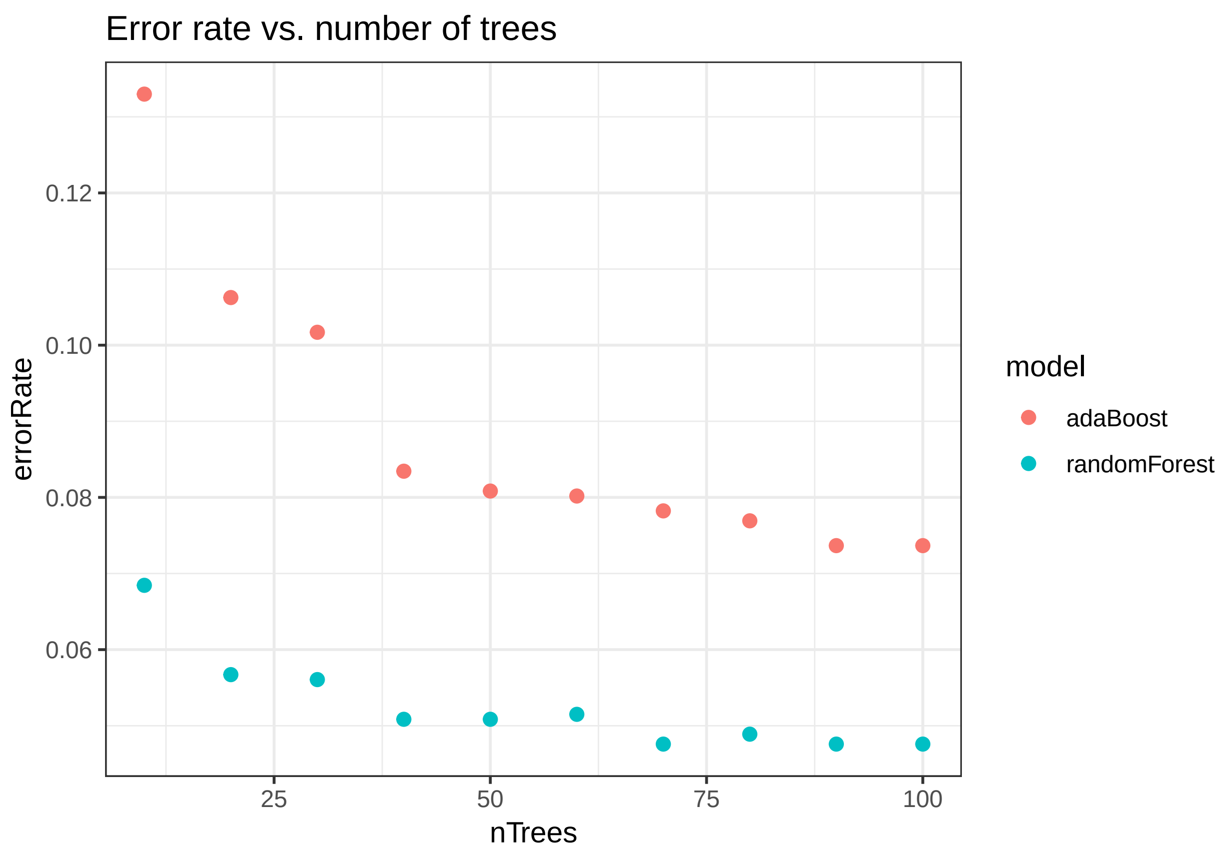
To evaluate the performance of Adaboost classification trees and random forests on the spam data, the function *modelComparison* automatically creates a plot which compares the error rates of the two algorithms for the in the input specified different number of trees (*maxNTree*). Both models will be trained with the specified train set (*trainData*). The error rates are related to the specified test set (*testX* & *testY*).

```
modelComparison = function(formula, trainData, testX, testY, maxNTree) {
  # calculating errors for random forest
  allErrors = as.data.frame(cbind(nTrees = seq(from = 10, to = maxNTree, by = 10),
                              errorRate = randomForest(formula =  formula,
                                                       data = trainData,
                                                       xtest = testX,
                                                       ytest = testY,
                                                       ntree = maxNTree)$test$err.rate[
                                                         seq(from = 10,
                                                             to = maxNTree,
                                                             by = 10),1],
                              model = "randomForest"))
  # calculating errors for adaBoost
  for (i in seq(from = 10, to = maxNTree, by = 10)) {
    adaBoostModel = blackboost(formula = formula,
                              data = trainData,
                              family = AdaExp(),
                              control = boost_control(mstop = i))
    yFit = predict(object = adaBoostModel,
```

```r
                   newdata = testX,
                   type = "class")
    error = 1 - sum(ifelse(as.numeric(as.character(testY)) ==
                             as.numeric(as.character(yFit)), 1, 0)) / length(testY)
    allErrors = rbind(allErrors,
                      cbind(nTrees = i, errorRate = error, model = "adaBoost"))
  }
  # adjusting classes
  allErrors$nTrees = as.numeric(as.character(allErrors$nTrees))
  allErrors$errorRate = as.numeric(as.character(allErrors$errorRate))
  allErrors$model = as.character(allErrors$model)
  # plotting data
  ggplot(data = allErrors,
         mapping = aes(x = nTrees, y = errorRate, color = model)) +
    geom_point(size = 2) +
    theme_bw() +
    ggtitle("Error rate vs. number of trees")
}
```

The follwing output of the function shows that the random forest algorithm generally leads to a lower test error rate for this data. In this case, the optimal choice of trees is 50, because the test error rate is minimal. Instead, the ADA boosted algorithm shows higher error rates. Especially for a small number of trees the algorithm does not deliver satisfying results compared to the random forest algorithm.

```r
modelComparison(formula = Spam ~ .,
                trainData = train,
                testX = test[,!colnames(data) %in% "Spam"],
                testY = test$Spam,
                maxNTree = 100)
```



2. Mixture Models

2.1 Code of the em()-function

To compare the results for $K = 2,3,4$, the *em*-function provides a graphical analysis for every iteration. The function includes comments which explain what I did at which step to create the EM algorithm. The function will be finally run with $K = 2,3,4$.

```
em = function(K) {
  # Initializing data
  set.seed(1234567890)
  max_it = 100 # max number of EM iterations
  min_change = 0.1 # min change in log likelihood between two consecutive EM iterations
  N = 1000 # number of training points
  D = 10 # number of dimensions
  x = matrix(nrow=N, ncol = D) # training data
  true_pi = vector(length = K) # true mixing coefficients
  true_mu = matrix(nrow = K, ncol = D) # true conditional distributions
  true_pi = c(rep(1/K, K))
  if (K == 2) {
    true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
    true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
    plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue",
         ylim = c(0,1), main = "True")
    points(true_mu[2,], type="o", xlab = "dimension", col = "red",
           main = "True")
```

```
} else if (K == 3) {
  true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
  true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
  true_mu[3,] = c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
  plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue", ylim=c(0,1),
       main = "True")
  points(true_mu[2,], type = "o", xlab = "dimension", col = "red",
         main = "True")
  points(true_mu[3,], type = "o", xlab = "dimension", col = "green",
         main = "True")
} else {
  true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
  true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
  true_mu[3,] = c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
  true_mu[4,] = c(0.3,0.5,0.5,0.7,0.5,0.5,0.5,0.5,0.4,0.5)
  plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue",
       ylim = c(0,1), main = "True")
  points(true_mu[2,], type = "o", xlab = "dimension", col = "red",
         main = "True")
  points(true_mu[3,], type = "o", xlab = "dimension", col = "green",
         main = "True")
  points(true_mu[4,], type = "o", xlab = "dimension", col = "yellow",
         main = "True")
}
z = matrix(nrow = N, ncol = K) # fractional component assignments
pi = vector(length = K) # mixing coefficients
mu = matrix(nrow = K, ncol = D) # conditional distributions
llik = vector(length = max_it) # log likelihood of the EM iterations
# Producing the training data
for(n in 1:N) {
  k = sample(1:K, 1, prob=true_pi)
  for(d in 1:D) {
    x[n,d] = rbinom(1, 1, true_mu[k,d])
  }
}
# Random initialization of the paramters
pi = runif(K, 0.49, 0.51)
pi = pi / sum(pi)
for(k in 1:K) {
  mu[k,] = runif(D, 0.49, 0.51)
}
# EM algorithm
for(it in 1:max_it) {
  # Plotting mu
    # Defining plot title
    title =  paste0("Iteration", it)
  if (K == 2) {
    plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
    points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
  } else if (K == 3) {
    plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
    points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
    points(mu[3,], type = "o", xlab = "dimension", col = "green", main = title)
```

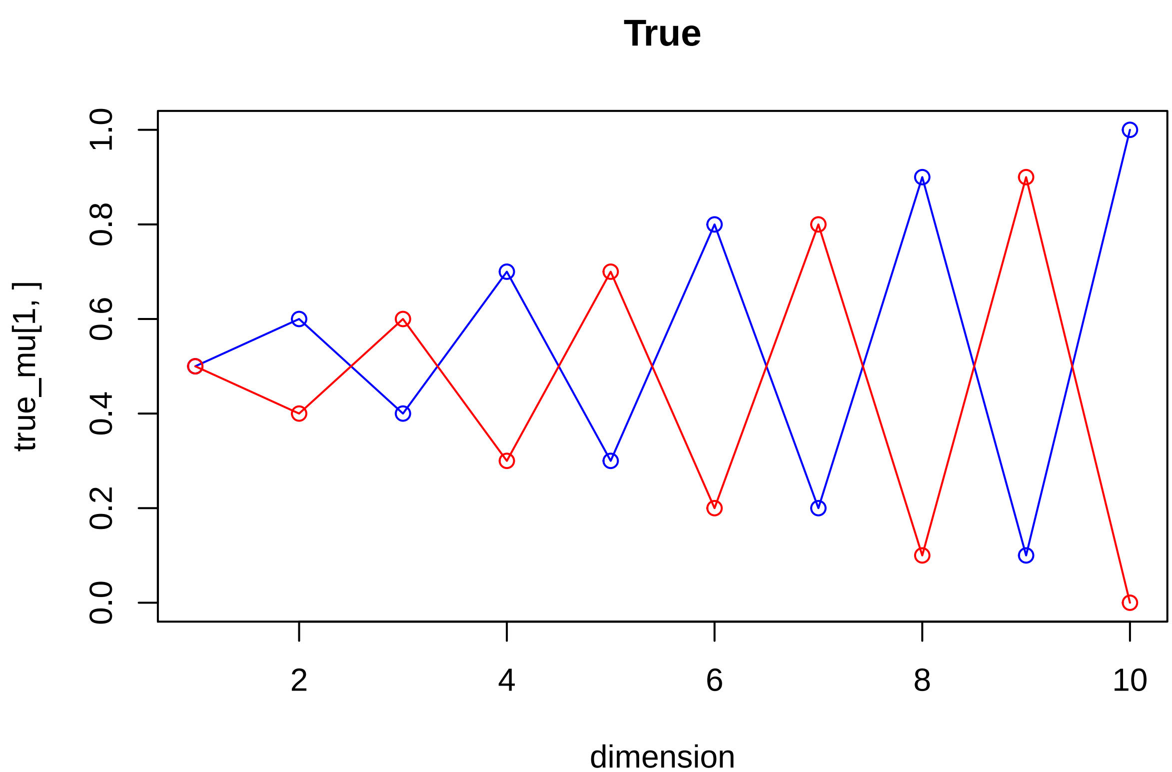
```r
  } else {
    plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
    points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
    points(mu[3,], type = "o", xlab = "dimension", col = "green", main = title)
    points(mu[4,], type = "o", xlab = "dimension", col = "yellow", main = title)
  }
  Sys.sleep(0.5)
  # E-step: Computation of the fractional component assignments
  for (n in 1:N) {
    # Creating empty matrix (column 1:K = p_x_given_k; column K+1 = p(x|all k)
    p_x = matrix(data = c(rep(1,K), 0), nrow = 1, ncol = K+1)
    # Calculating p(x|k) and p(x|all k)
    for (k in 1:K) {
      # Calculating p(x|k)
      for (d in 1:D) {
        p_x[1,k] = p_x[1,k] * (mu[k,d]^x[n,d]) * (1-mu[k,d])^(1-x[n,d])
      }
      p_x[1,k] = p_x[1,k] * pi[k] # weighting with pi[k]
      # Calculating p(x|all k) (denominator)
      p_x[1,K+1] = p_x[1,K+1] + p_x[1,k]
    }
    # Calculating z for n and all k
    for (k in 1:K) {
      z[n,k] = p_x[1,k] / p_x[1,K+1]
    }
  }
  # Log likelihood computation
  for (n in 1:N) {
    for (k in 1:K) {
      log_term = 0
      for (d in 1:D) {
        log_term = log_term + x[n,d] * log(mu[k,d]) + (1-x[n,d]) * log(1-mu[k,d])
      }
      llik[it] = llik[it] + z[n,k] * (log(pi[k]) + log_term)
    }
  }
  cat("iteration: ", it, "log likelihood: ", llik[it], "\n")
  flush.console()
  # Stop if the log likelihood has not changed significantly
  if (it != 1) {
    if (abs(llik[it] - llik[it-1]) < min_change) {
      break
    }
  }
  # M-step: ML parameter estimation from the data and fractional component assignments
  # Updating pi
  for (k in 1:K) {
    pi[k] = sum(z[,k])/N
  }
  # Updating mu
  for (k in 1:K) {
    mu[k,] = 0
    for (n in 1:N) {
```

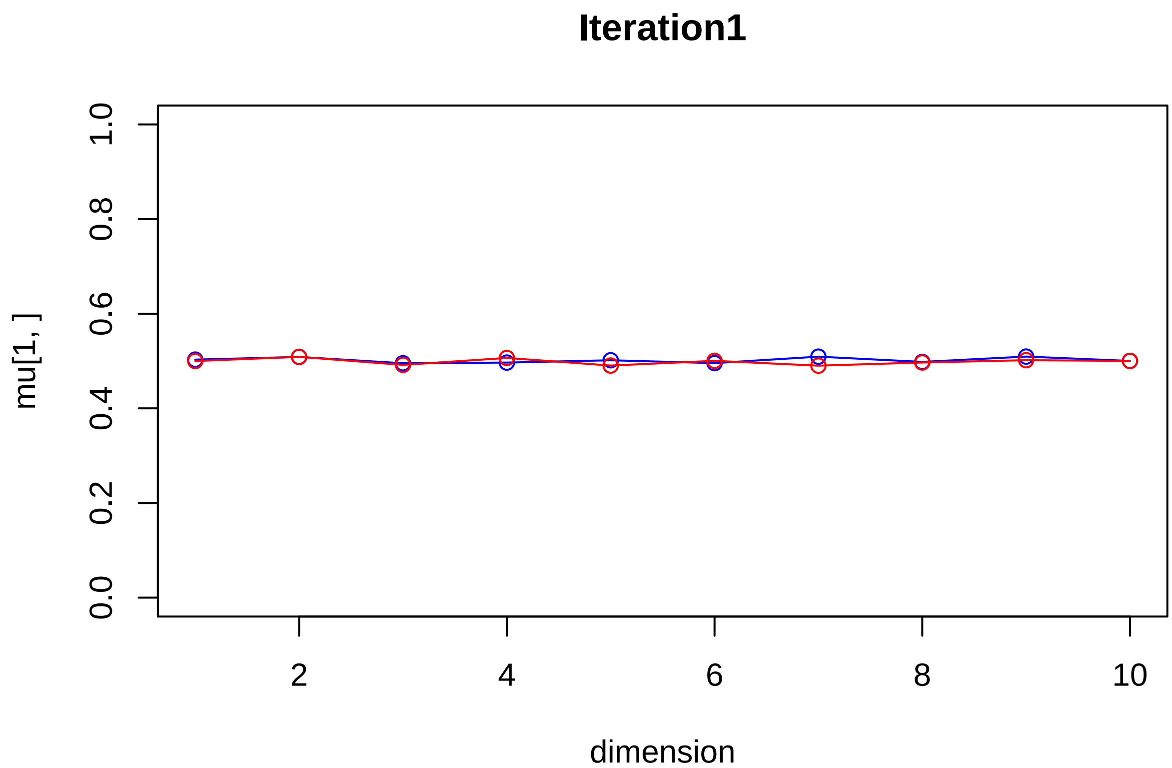
```r
        mu[k,] = mu[k,] + x[n,] * z[n,k]
      }
      mu[k,] = mu[k,] / sum(z[,k])
    }
  }
  # Printing pi, mu and development of log likelihood at the end
  return(list(
    pi = pi,
    mu = mu,
    logLikelihoodDevelopment = plot(llik[1:it],
                                    type = "o",
                                    main = "Development of the log likelihood",
                                    xlab = "iteration",
                                    ylab = "log likelihood")
  ))
}
```

2.2 K=2

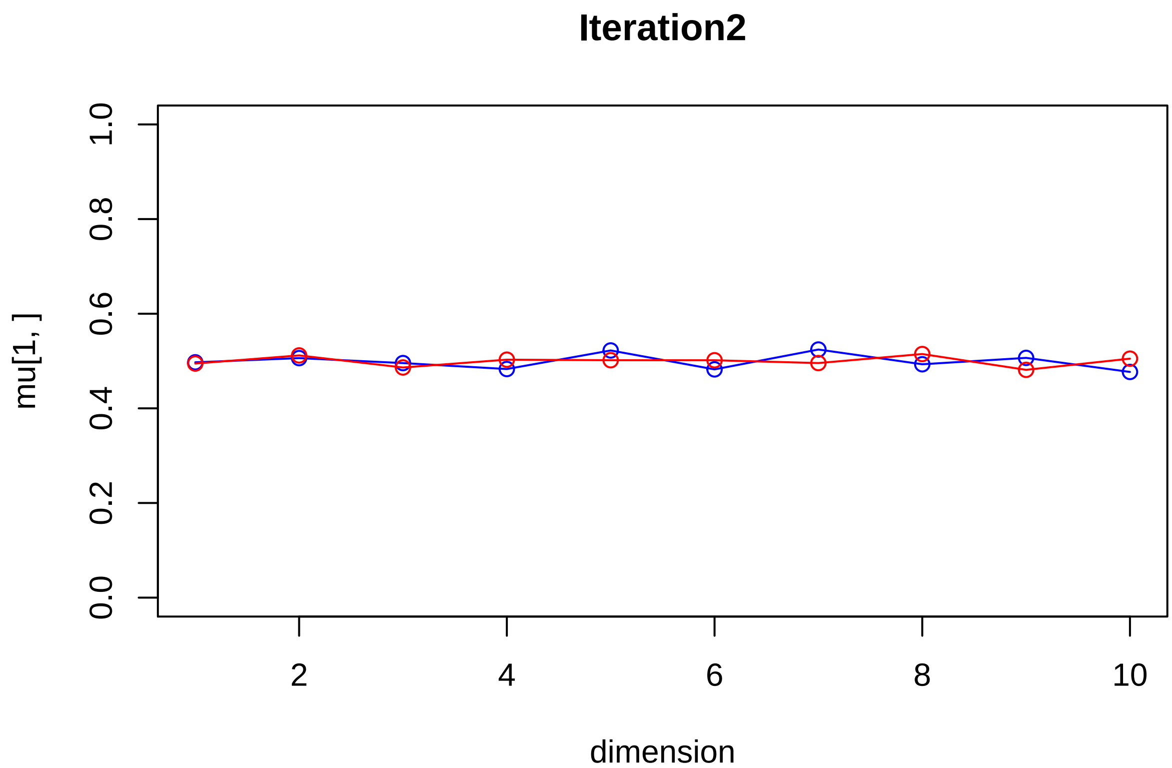
First, the function will be run for K=*2*.

```r
em(2)
```

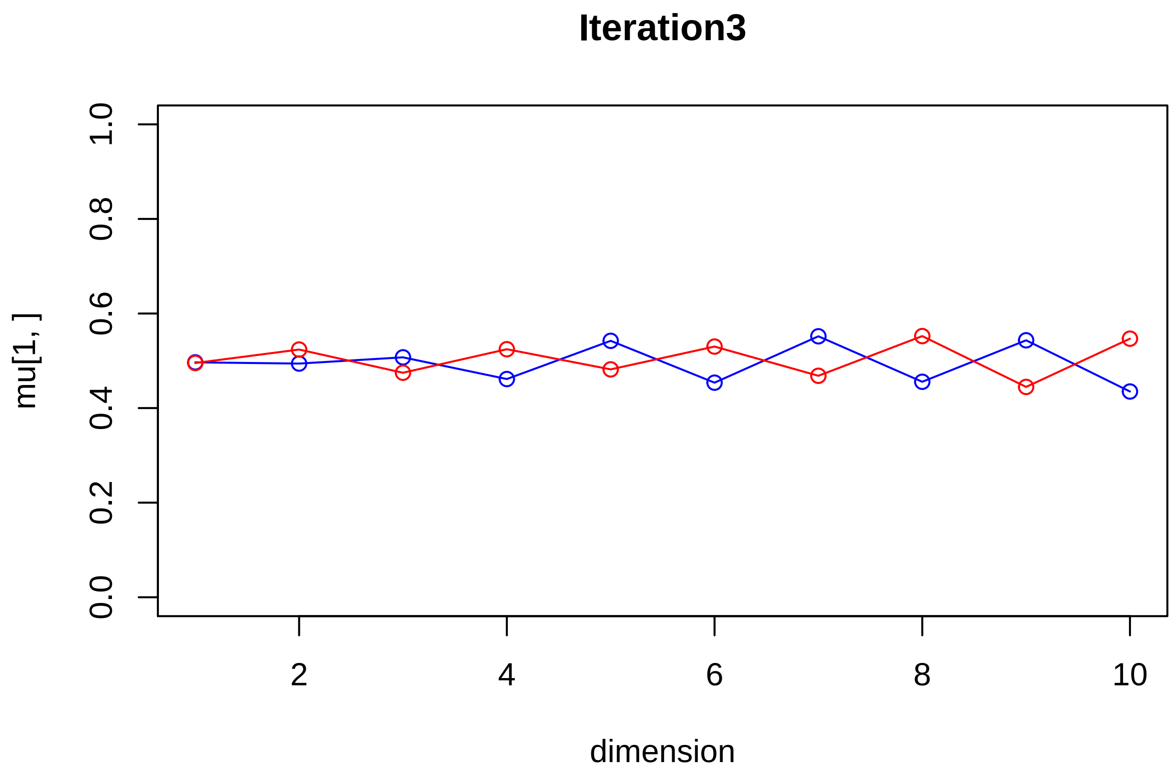




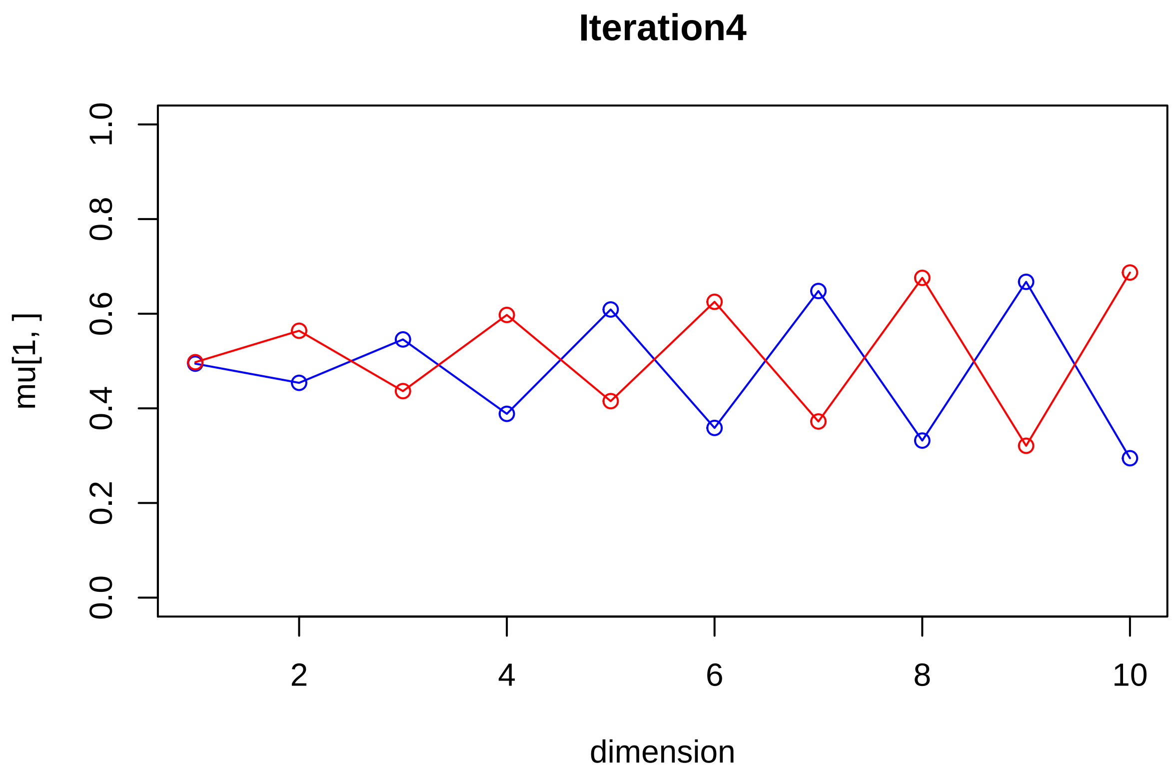

```
## iteration:  1 log likelihood:  -7623.897
```



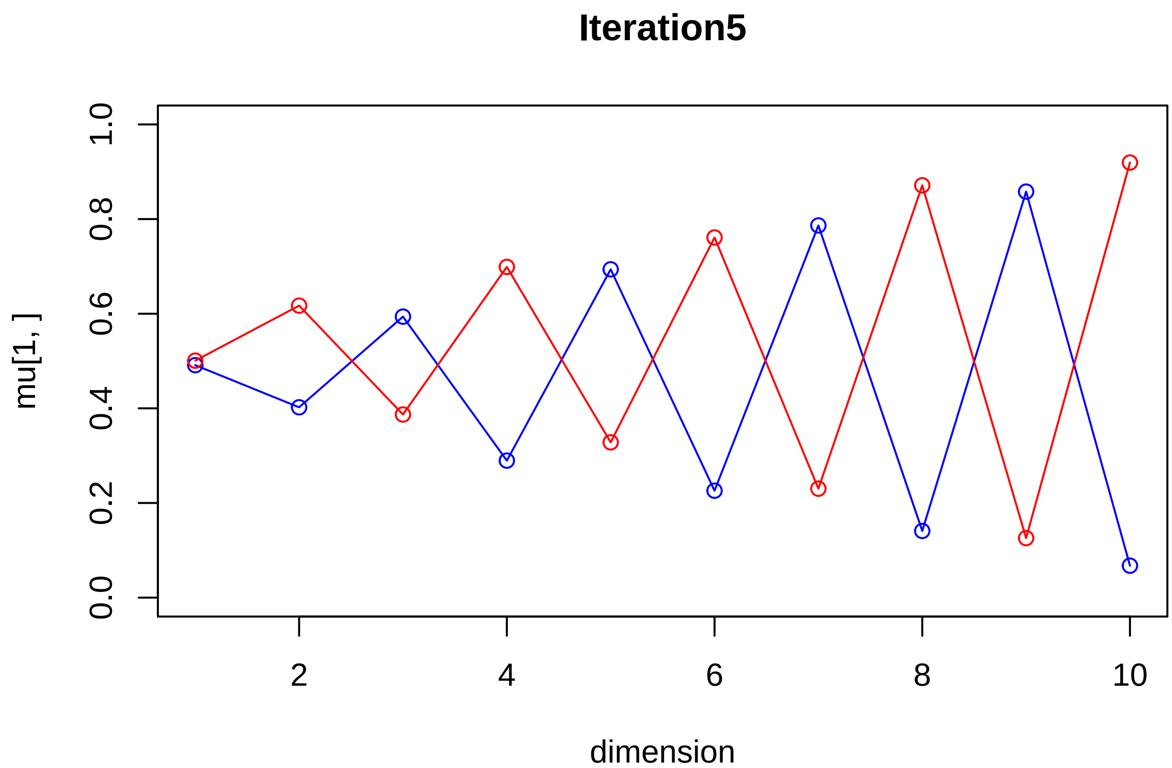

```
## iteration:  2 log likelihood:  -7610.745
```



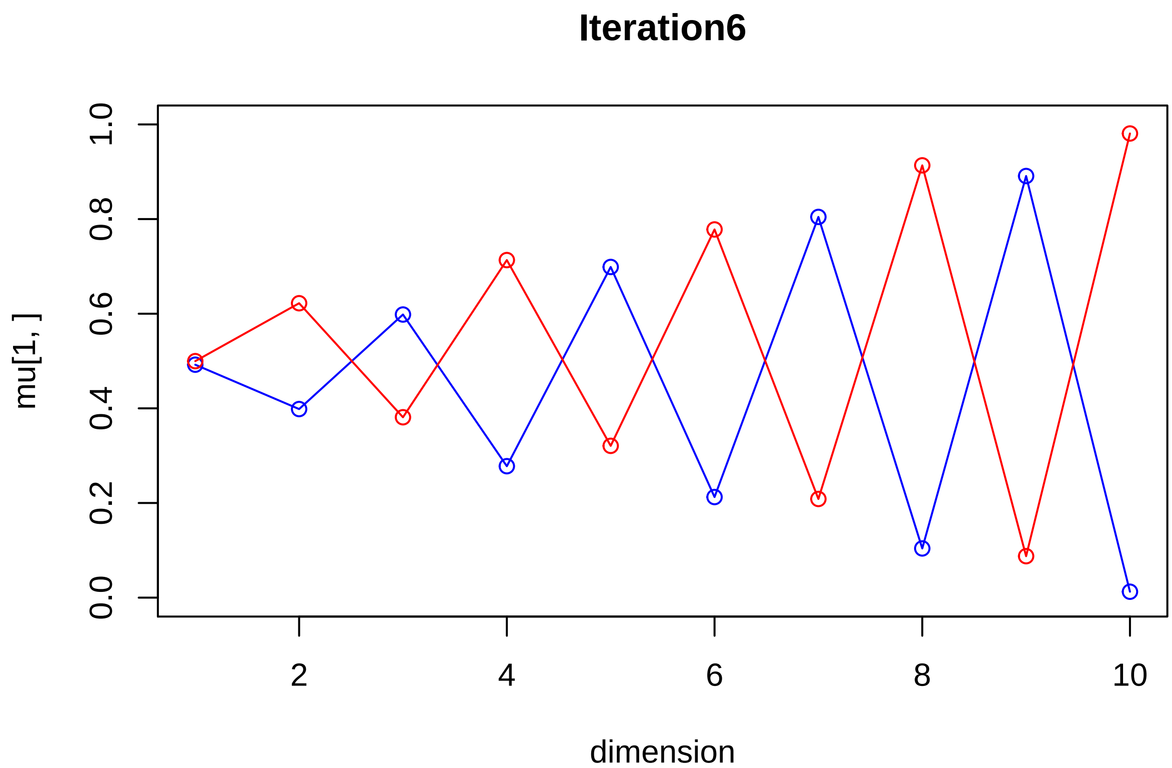

```
## iteration:  3 log likelihood:  -7463.445
```



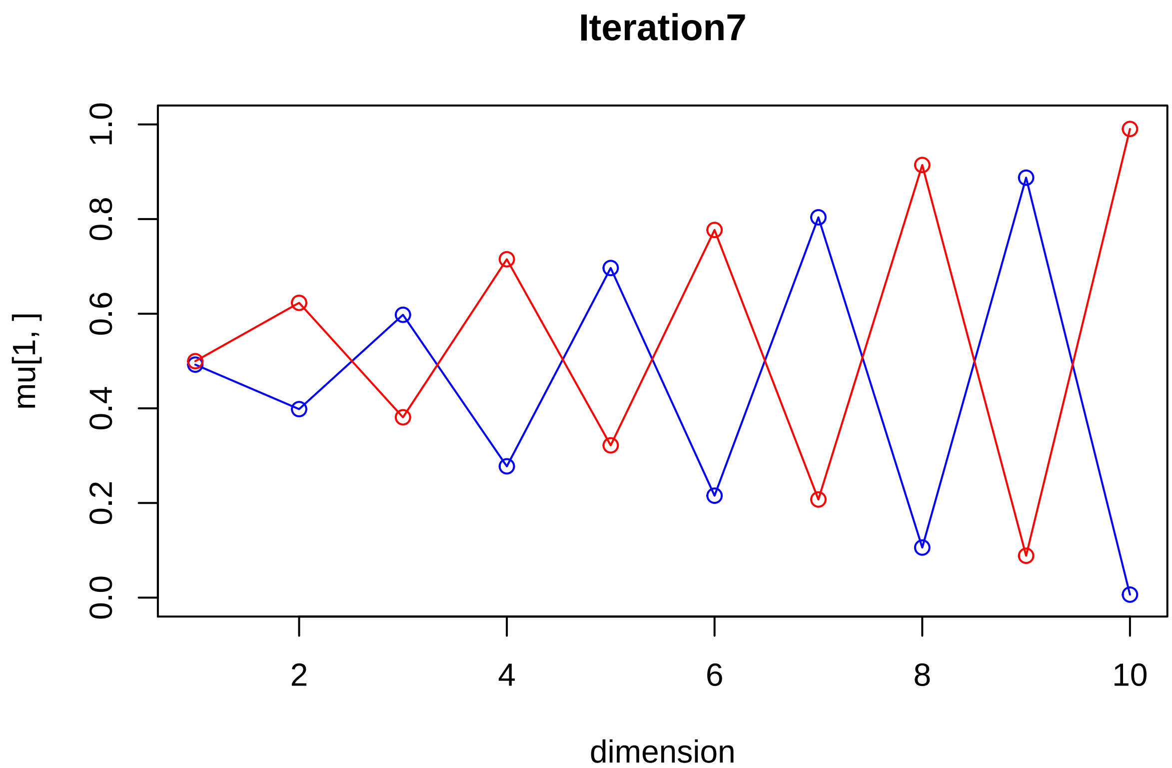

```
## iteration:  4 log likelihood:  -6575.121
```



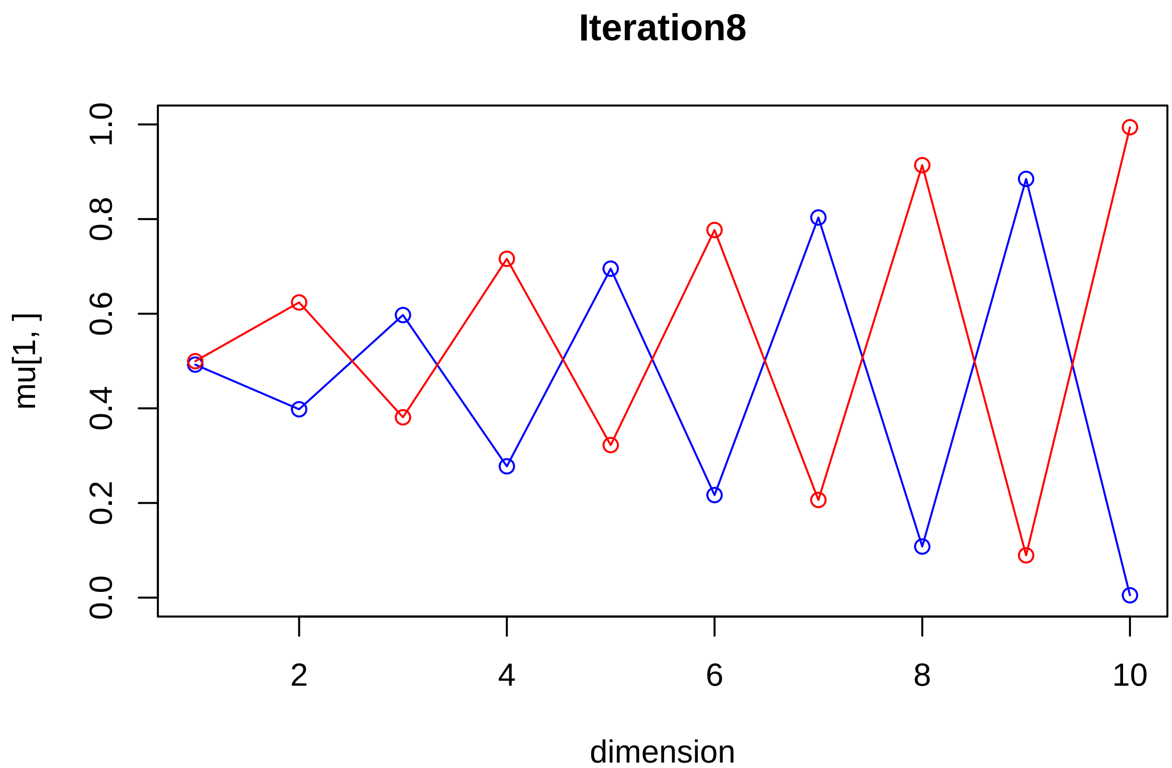

```
## iteration:  5 log likelihood:  -5731.559
```



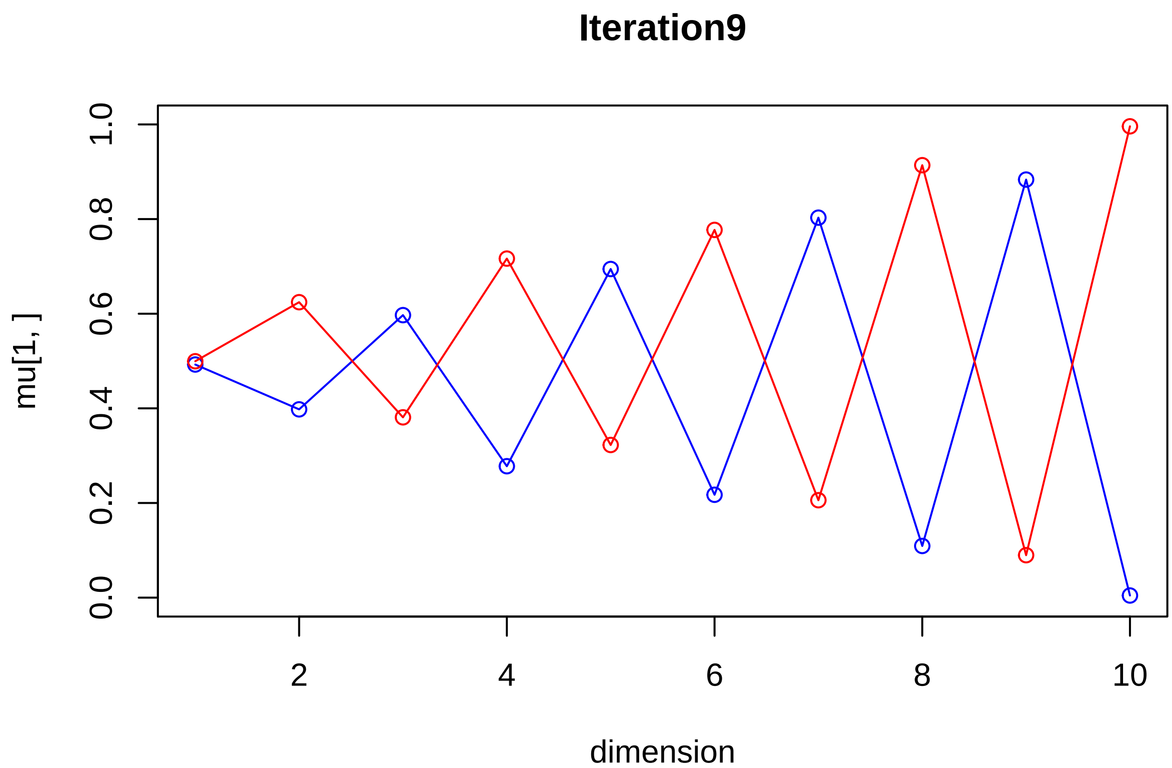

```
## iteration:  6 log likelihood:  -5656.174
```



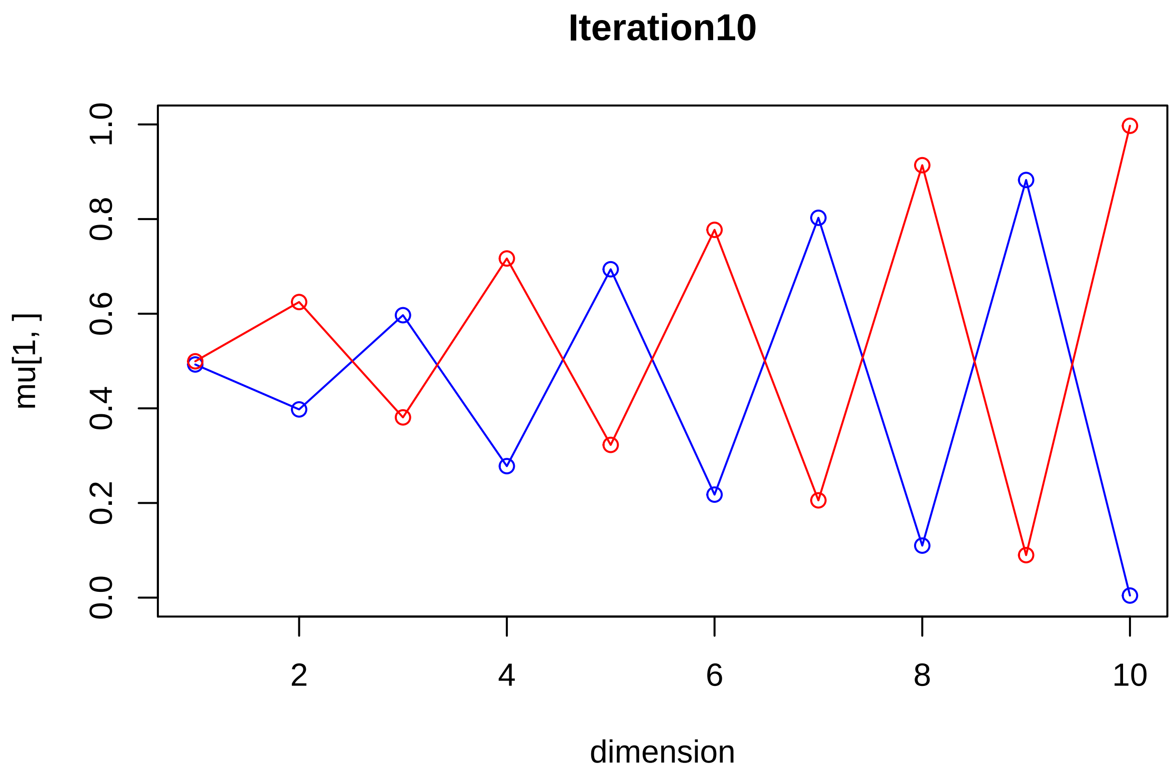

```
## iteration:  7 log likelihood:  -5648.904
```



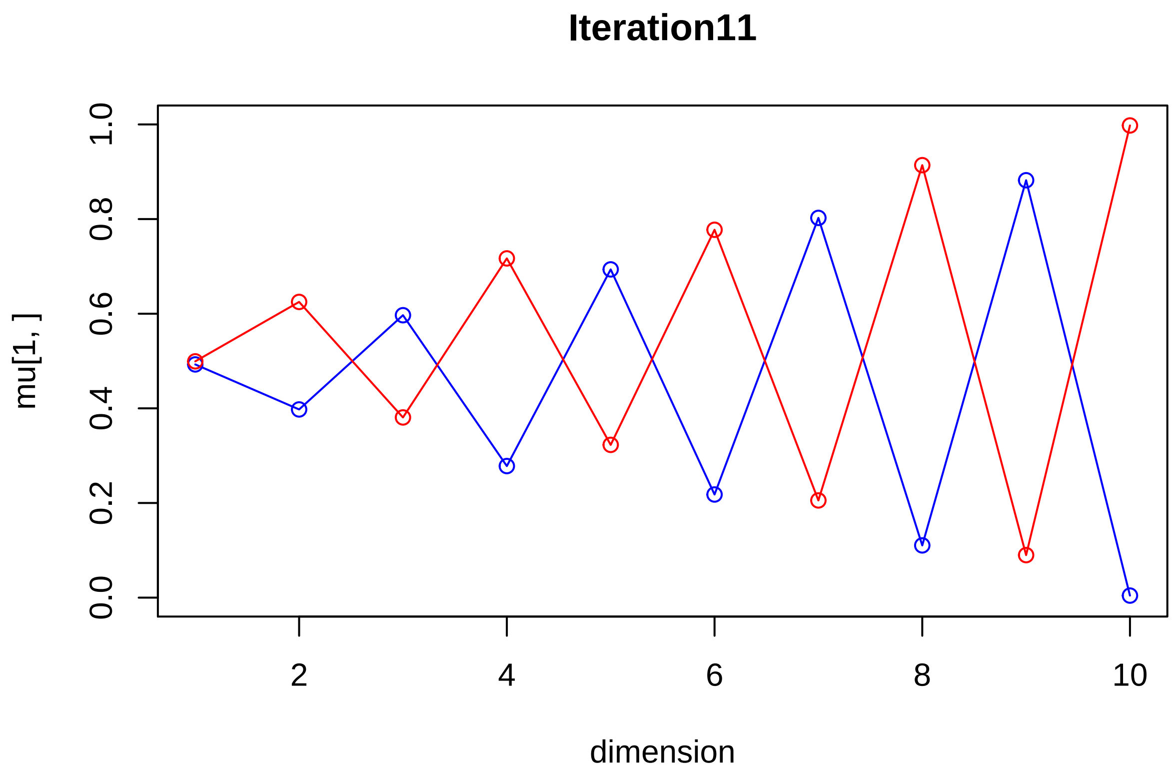

```
## iteration:  8 log likelihood:  -5646.139
```



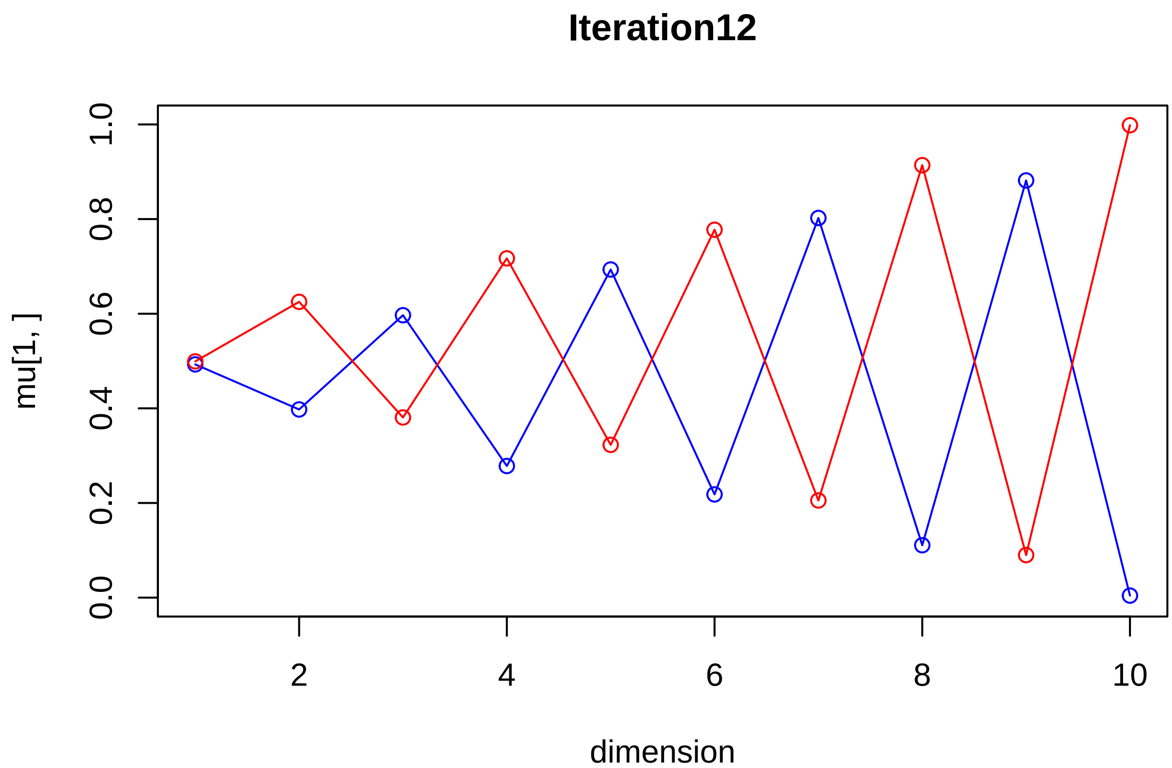

```
## iteration:  9 log likelihood:  -5644.608
```



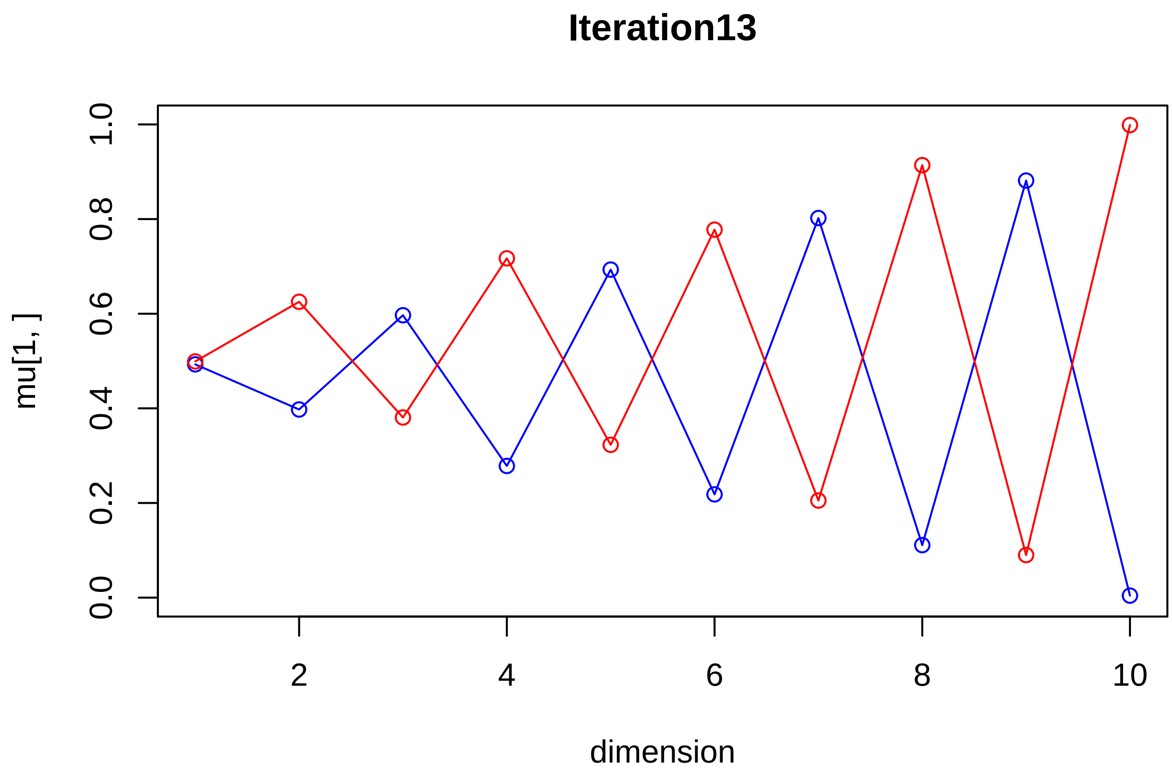

```
## iteration:  10 log likelihood:  -5643.615
```



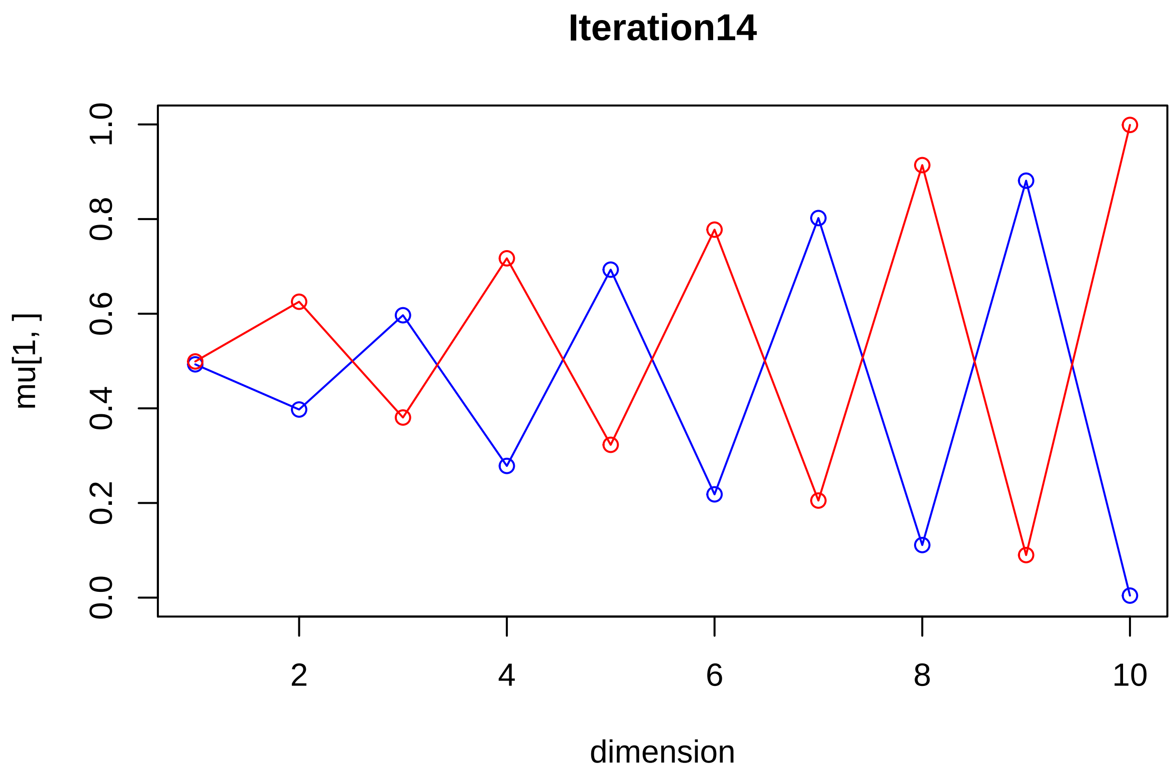

```
## iteration:  11 log likelihood:  -5642.913
```



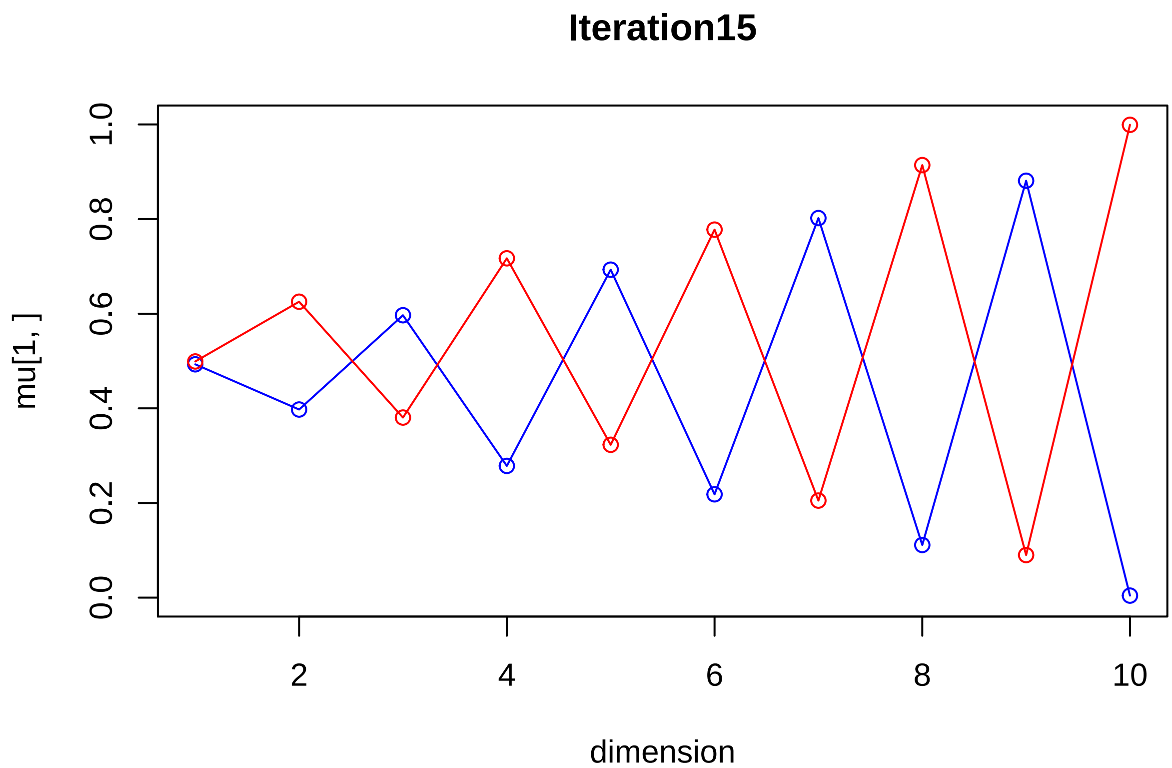

```
## iteration:  12 log likelihood:  -5642.386
```



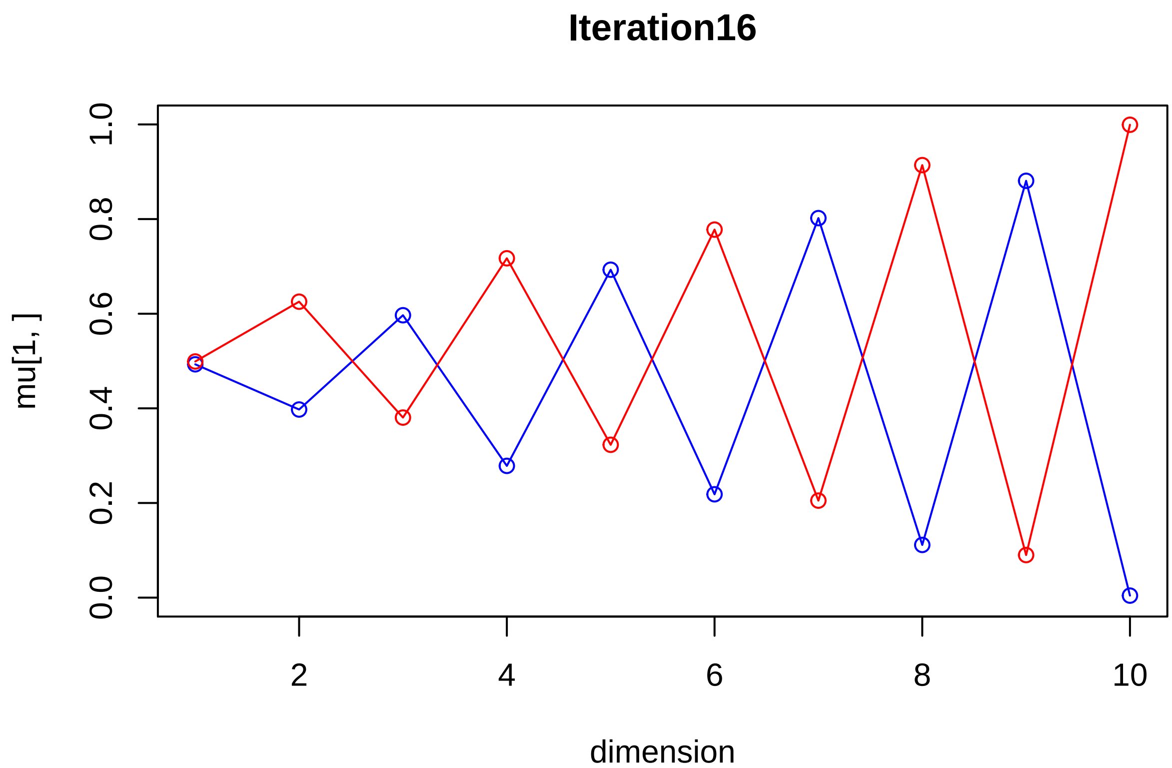

```
## iteration:  13 log likelihood:  -5641.977
```



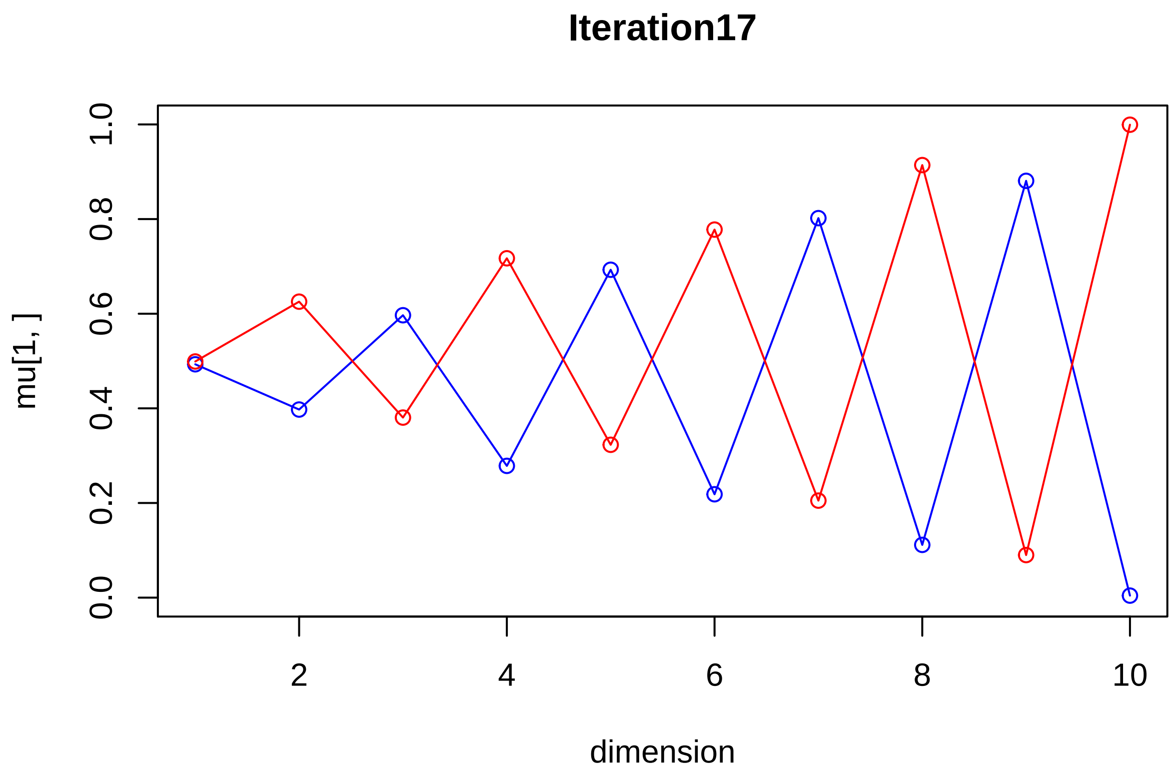

```
## iteration:  14 log likelihood:  -5641.649
```



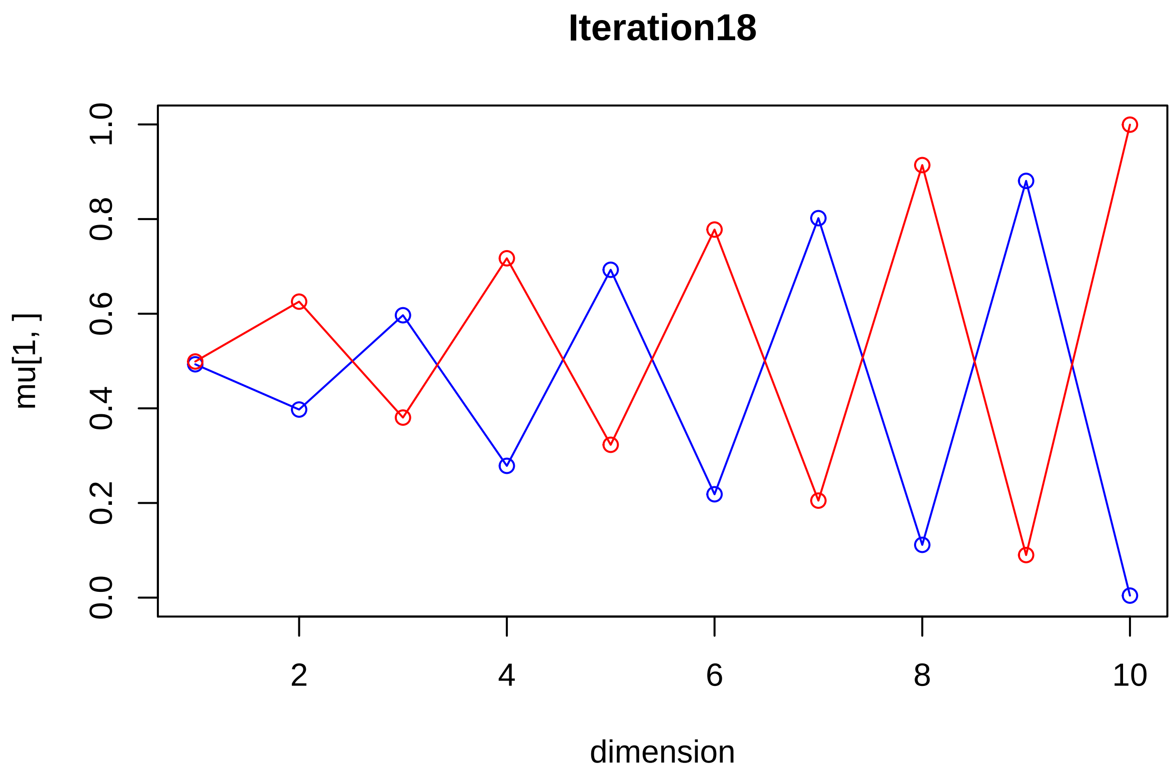

```
## iteration:  15 log likelihood:  -5641.382
```



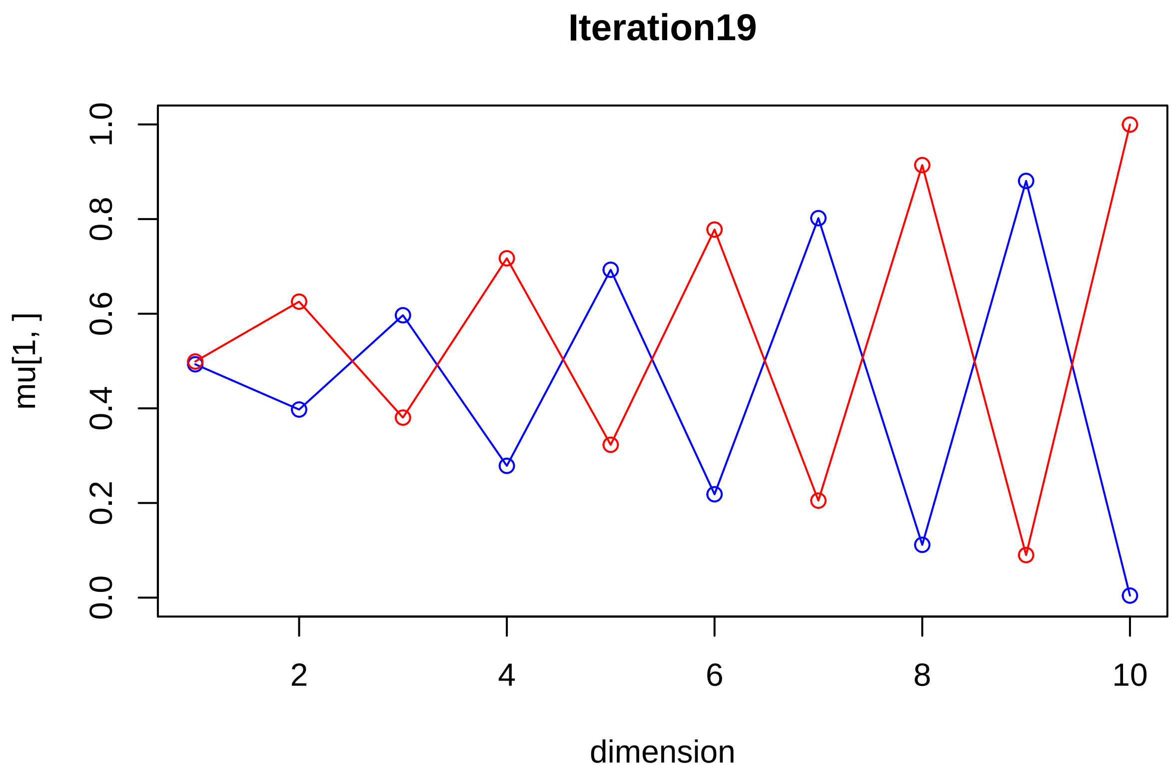

```
## iteration:  16 log likelihood:  -5641.161
```



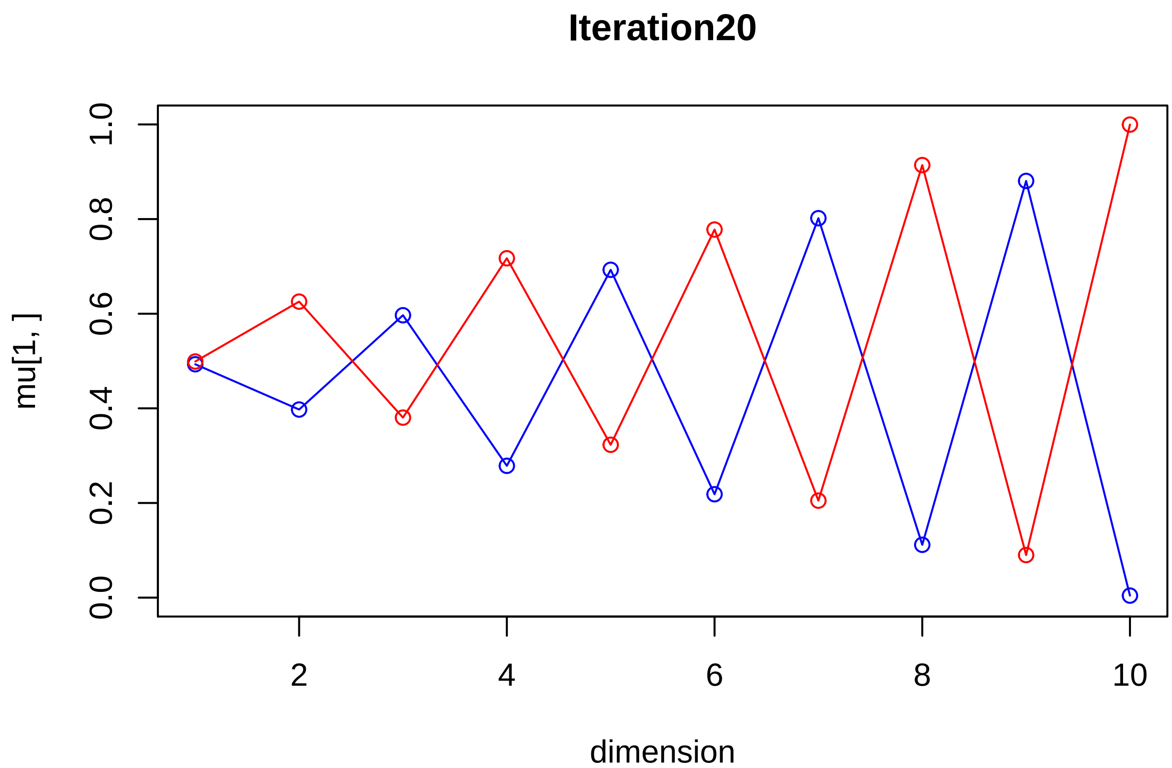

```
## iteration:  17 log likelihood:  -5640.975
```



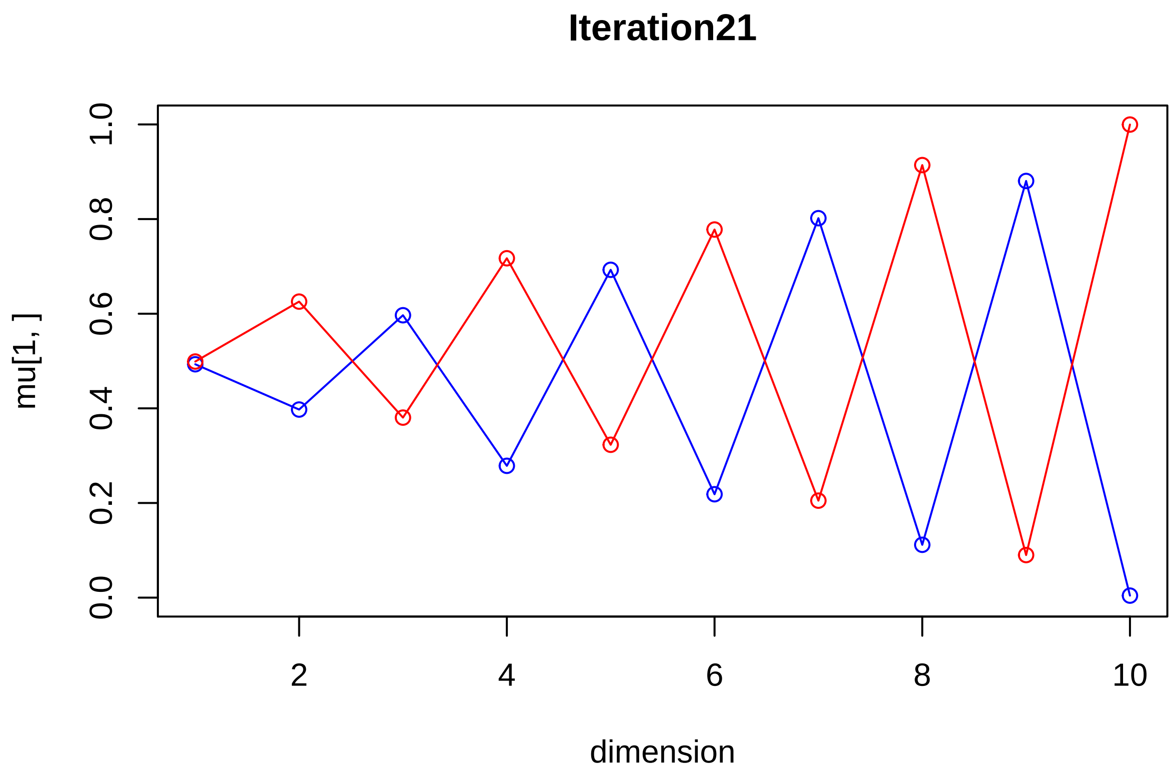

```
## iteration:  18 log likelihood:  -5640.819
```



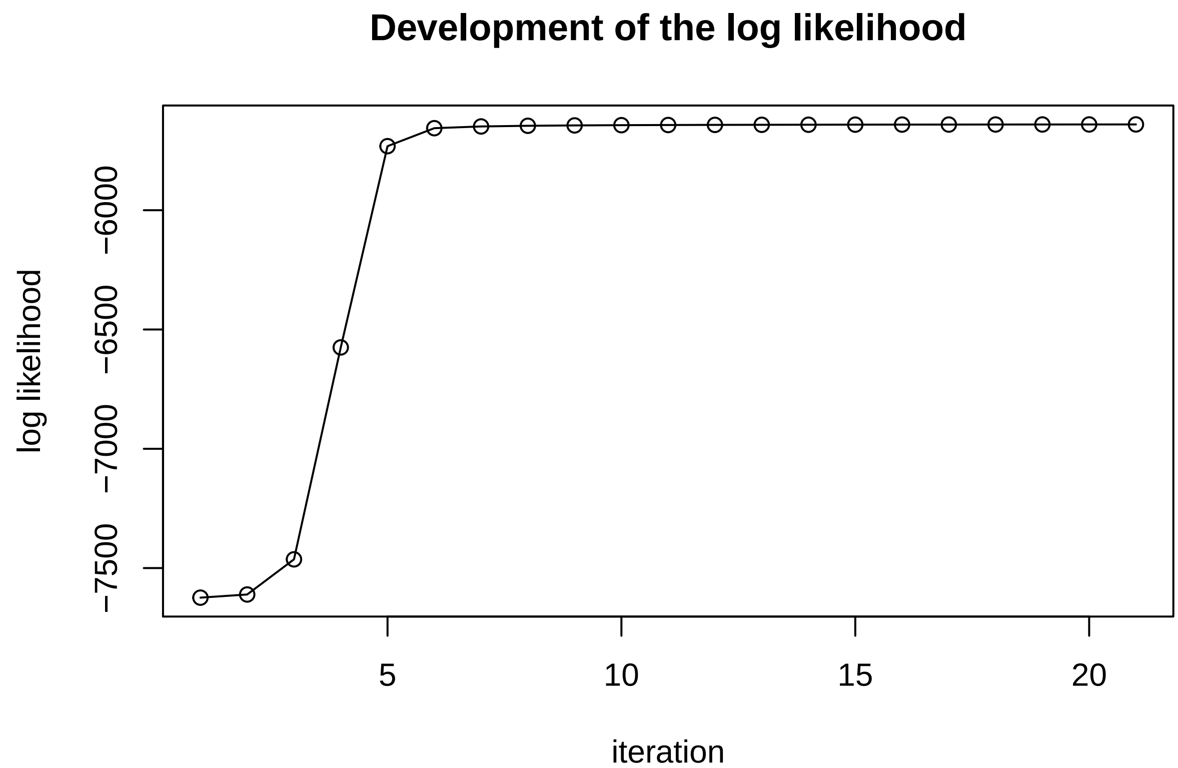

```
## iteration:  19 log likelihood:  -5640.685
```




```
## iteration:  20 log likelihood:  -5640.571
```




```
## iteration:  21 log likelihood:  -5640.473
```

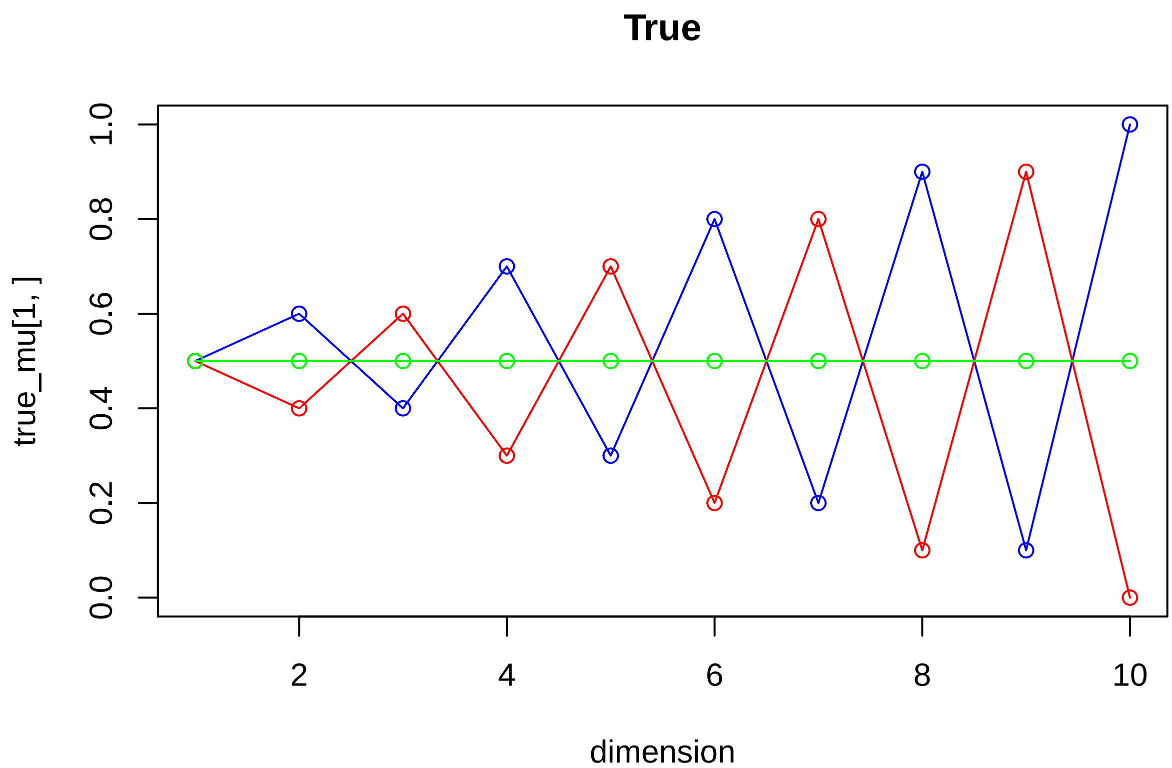


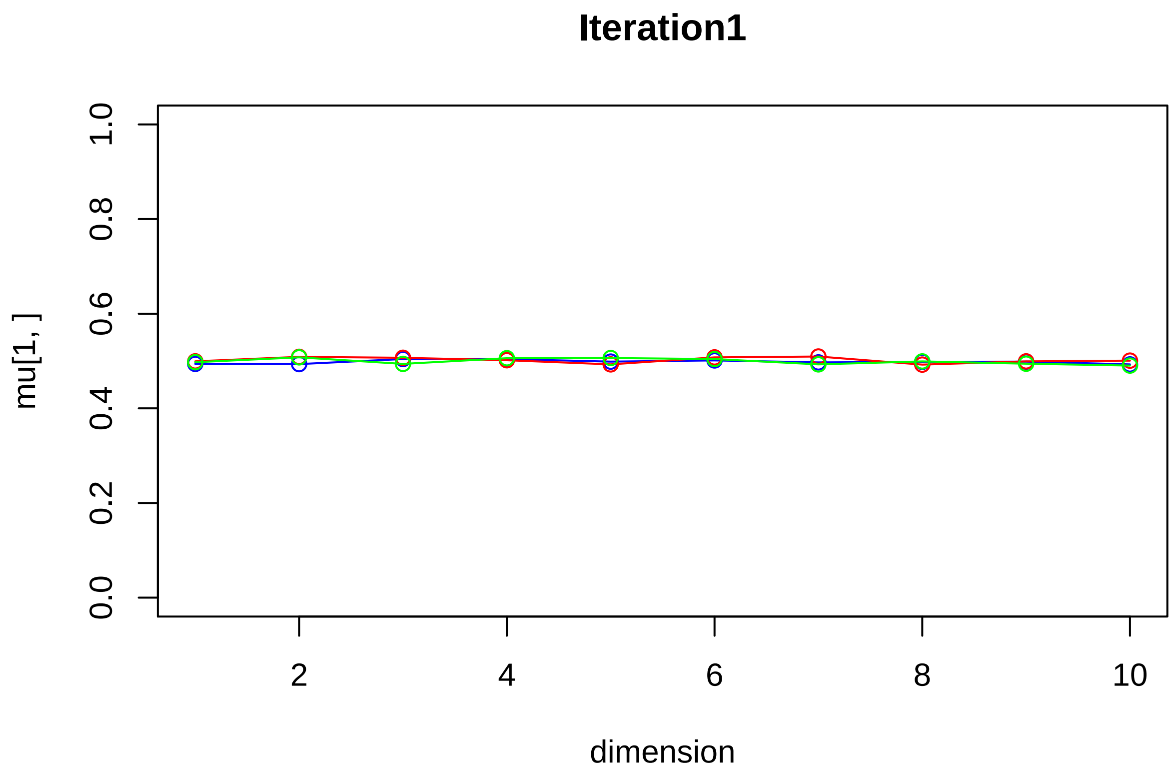

```
## $pi
## [1] 0.5110531 0.4889469
##
## $mu
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4931735 0.3974606 0.5967811 0.2785480 0.6927917 0.2184957 0.8018491
## [2,] 0.4989543 0.6255823 0.3804363 0.7171478 0.3230343 0.7778699 0.2049559
##           [,8]       [,9]       [,10]
## [1,] 0.1116477 0.88054439 0.004290353
## [2,] 0.9140913 0.08997919 0.999714736
##
## $logLikelihoodDevelopment
## NULL
```

2.3 K=3

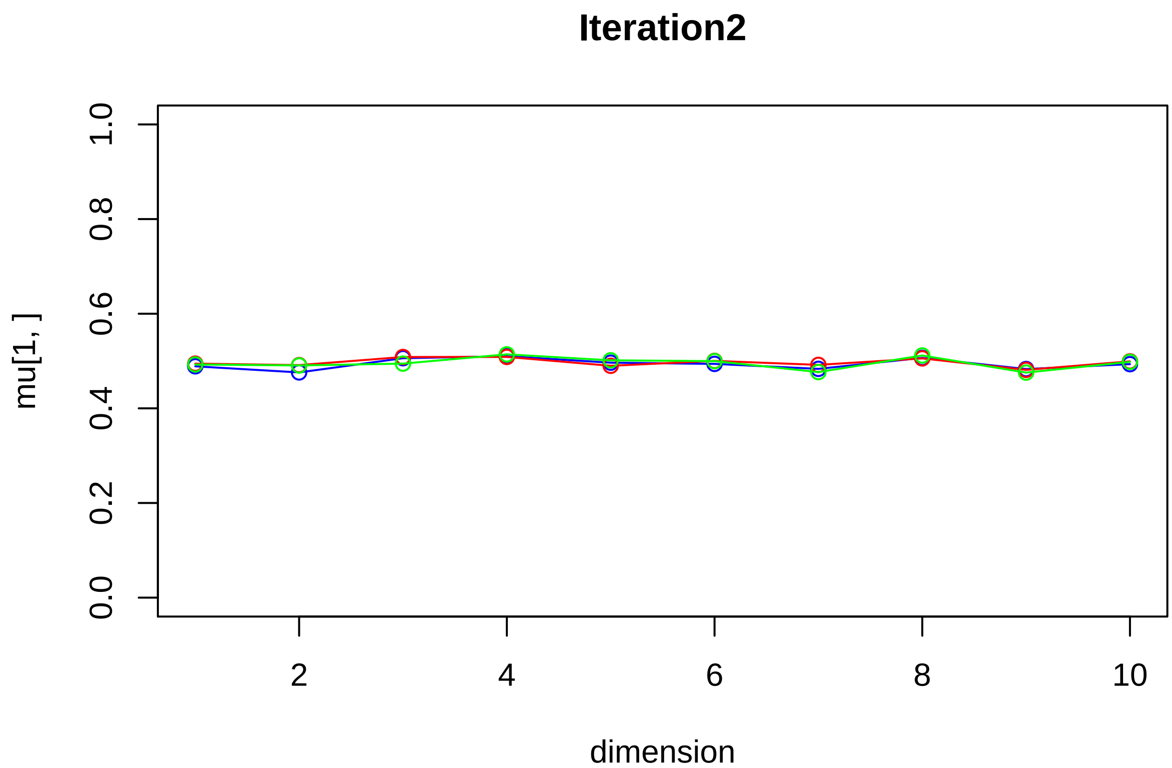
Next, the function will be run for *K=3*.

```
em(3)
```

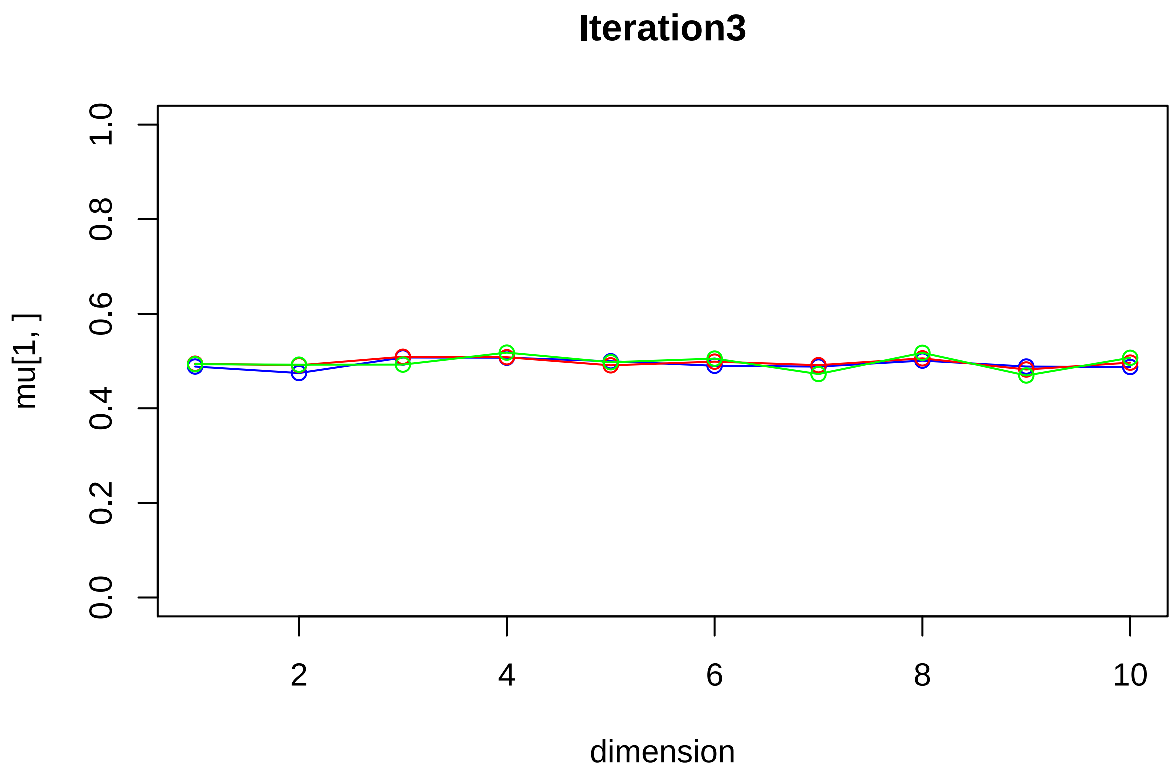

True
true_mu[1,]
0.0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
dimension



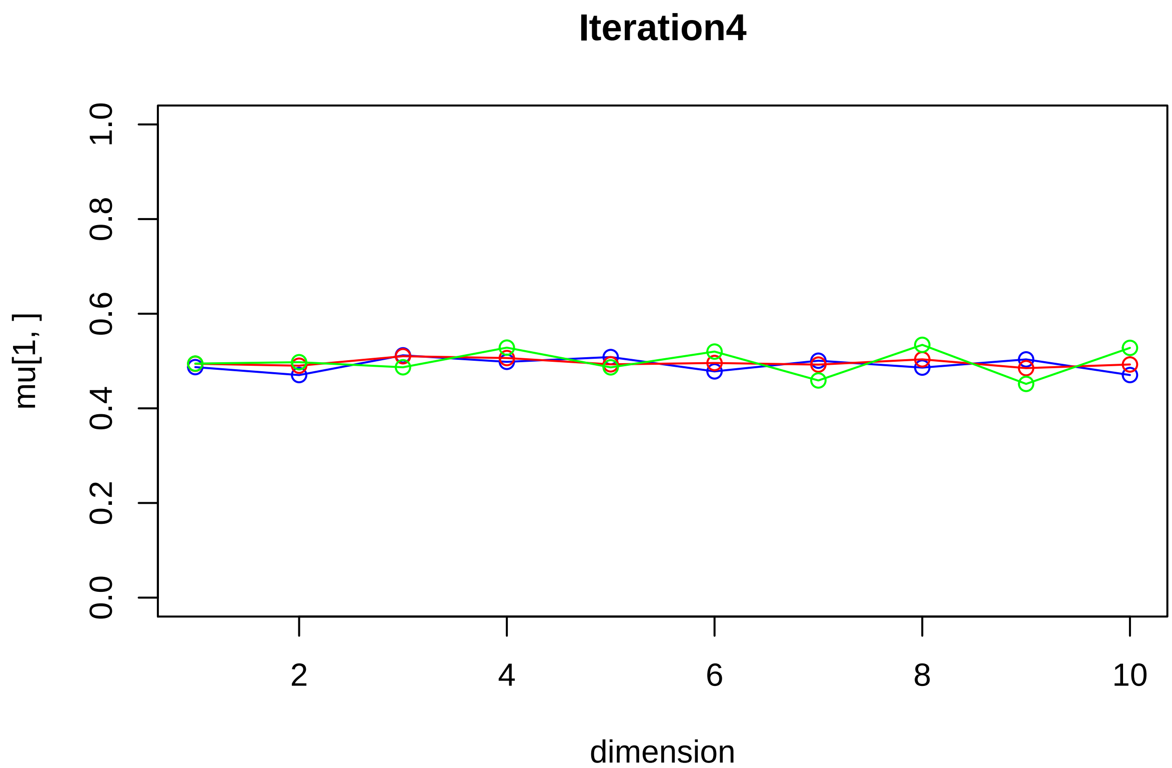

```
## iteration:  1 log likelihood:  -8029.723
```



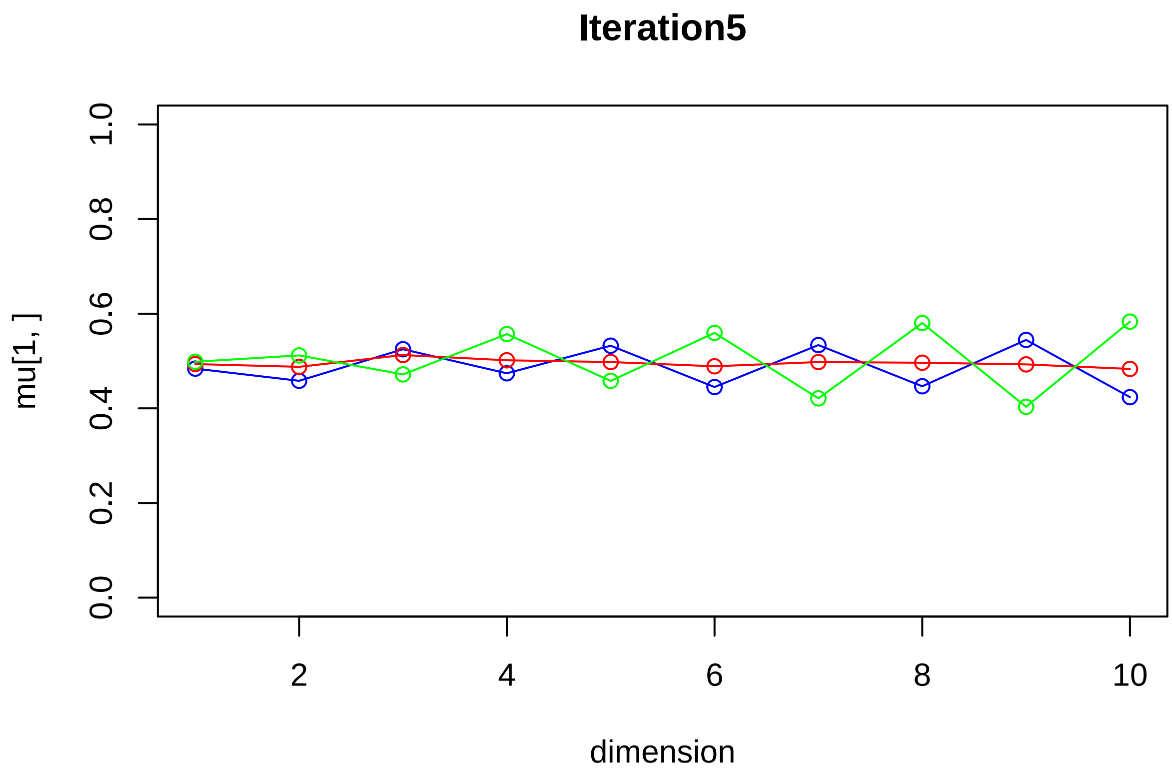

```
## iteration:  2 log likelihood:  -8027.183
```



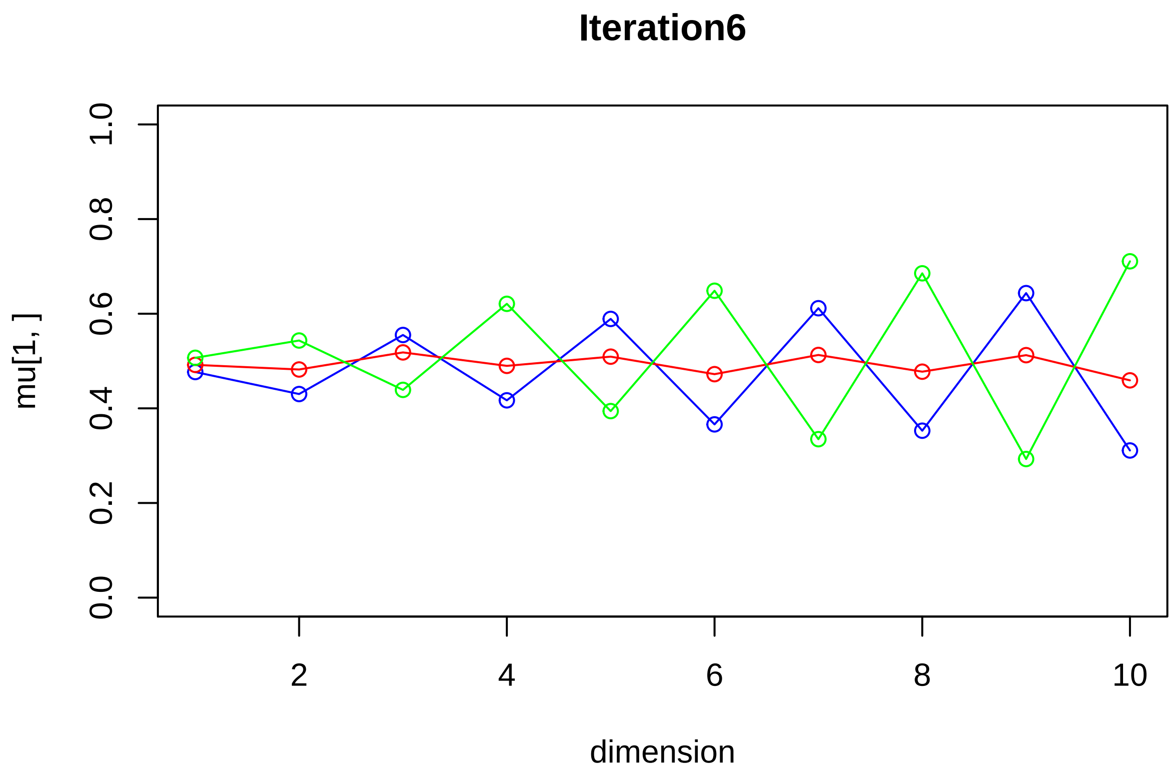

```
## iteration:  3 log likelihood:  -8024.696
```



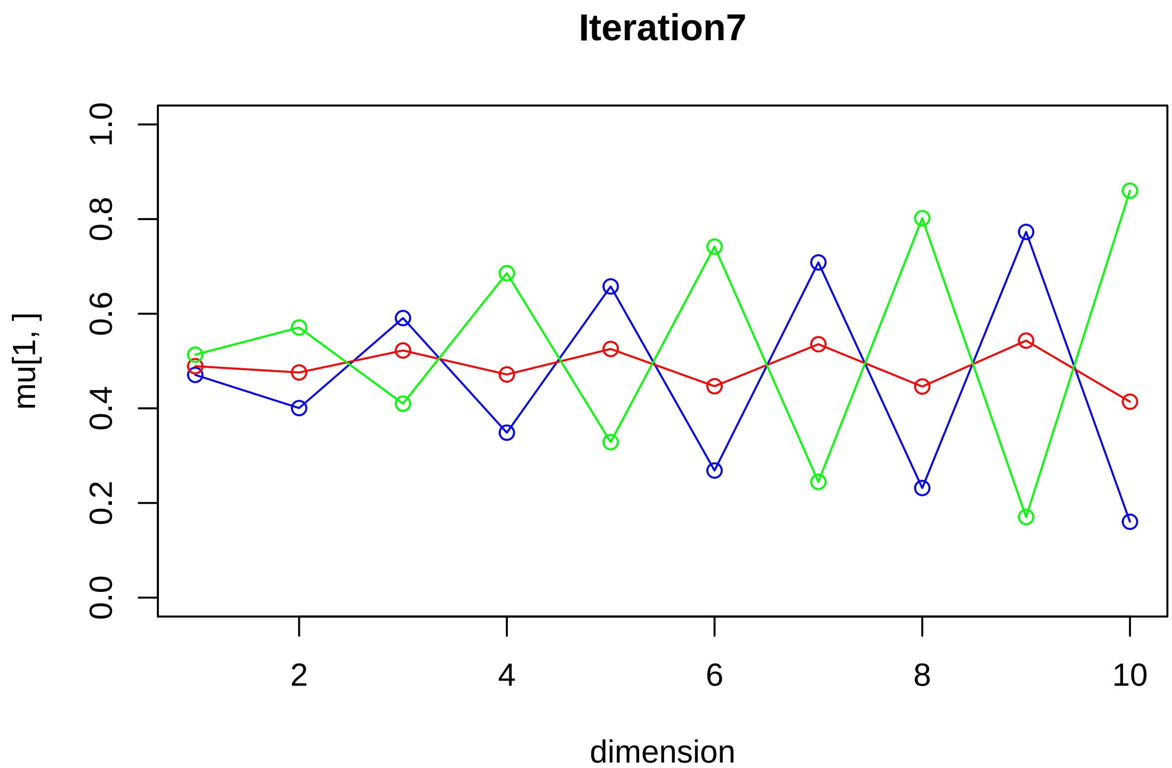

```
## iteration:  4 log likelihood:  -8005.631
```



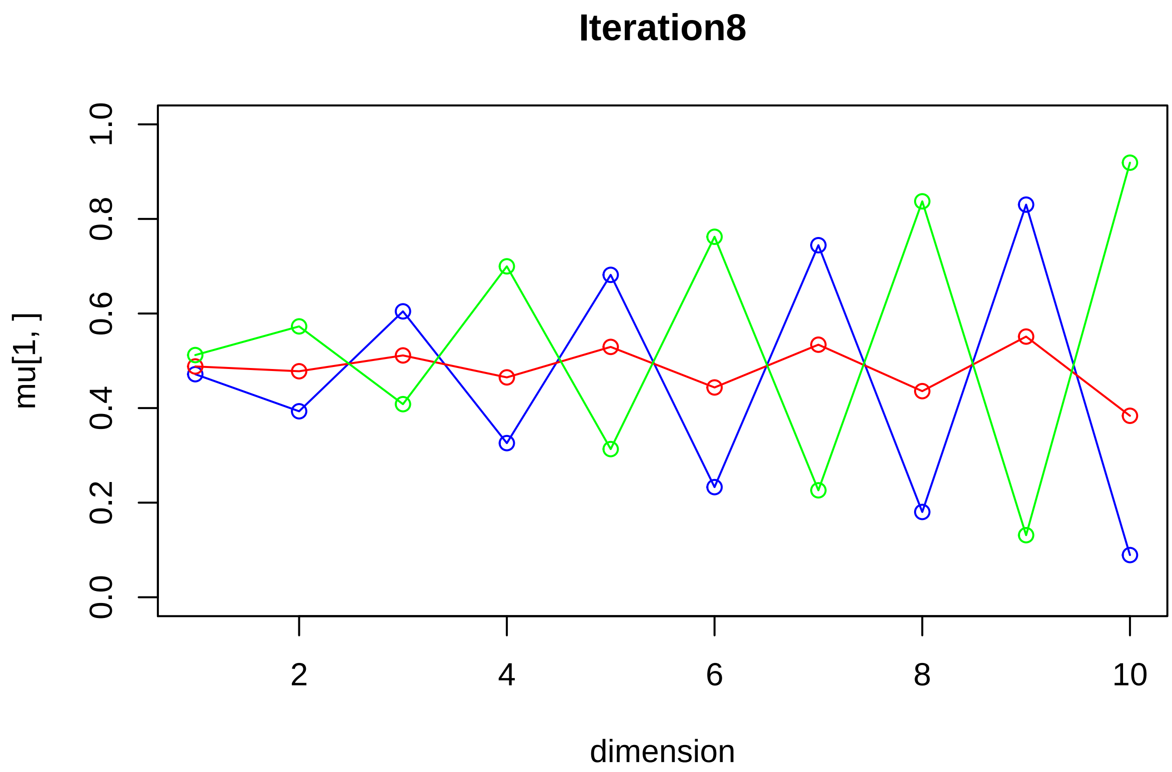

```
## iteration:  5 log likelihood:  -7877.606
```



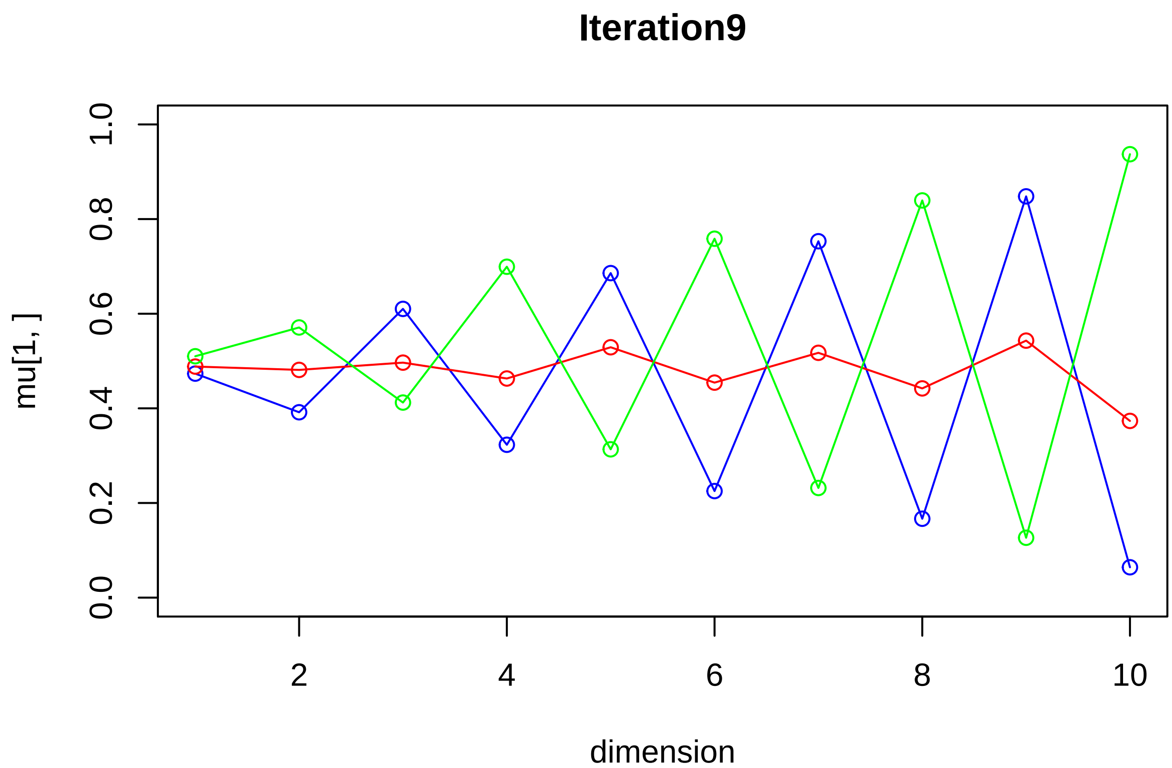

```
## iteration:  6 log likelihood:  -7403.513
```



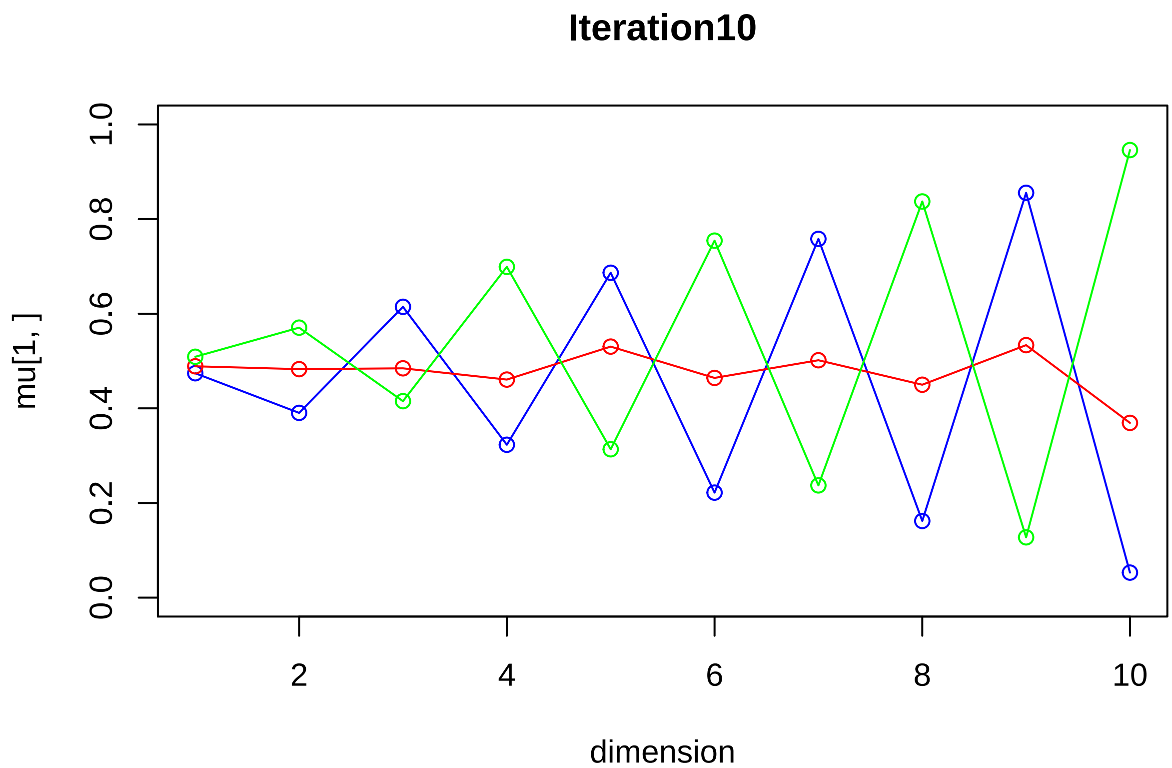

```
## iteration:  7 log likelihood:  -6936.919
```



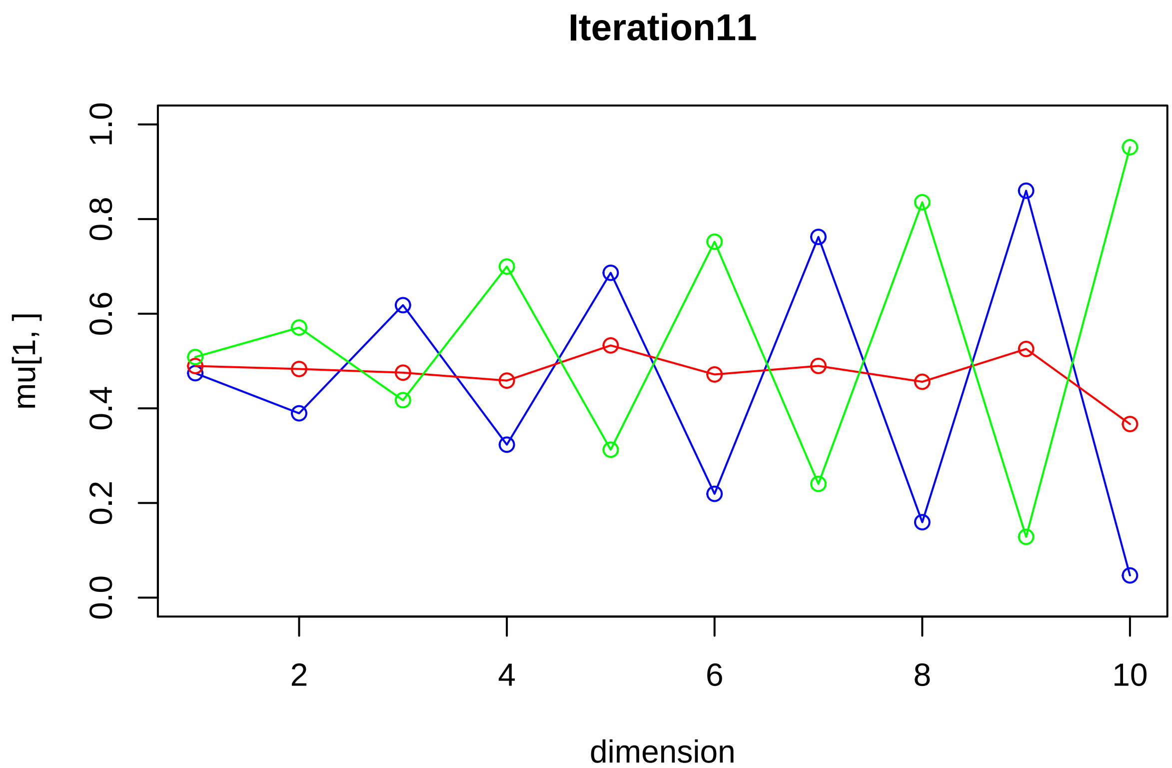

```
## iteration:  8 log likelihood:  -6818.582
```



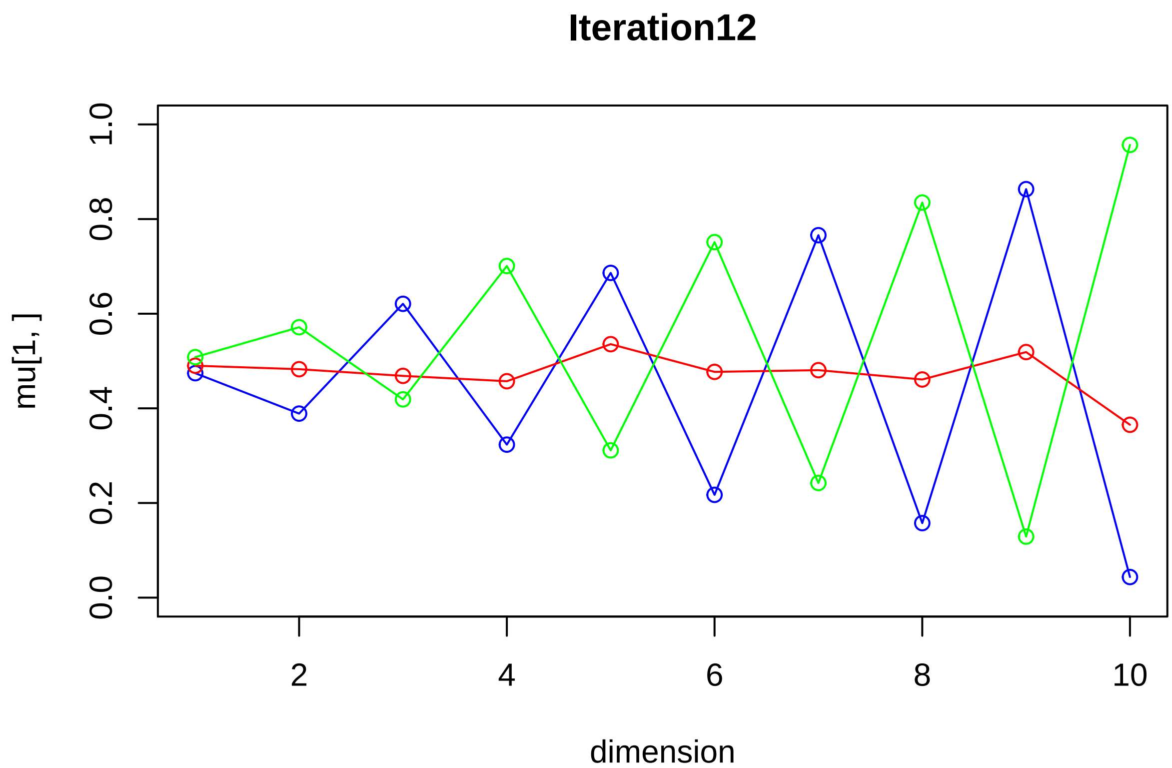

```
## iteration:  9 log likelihood:  -6791.377
```



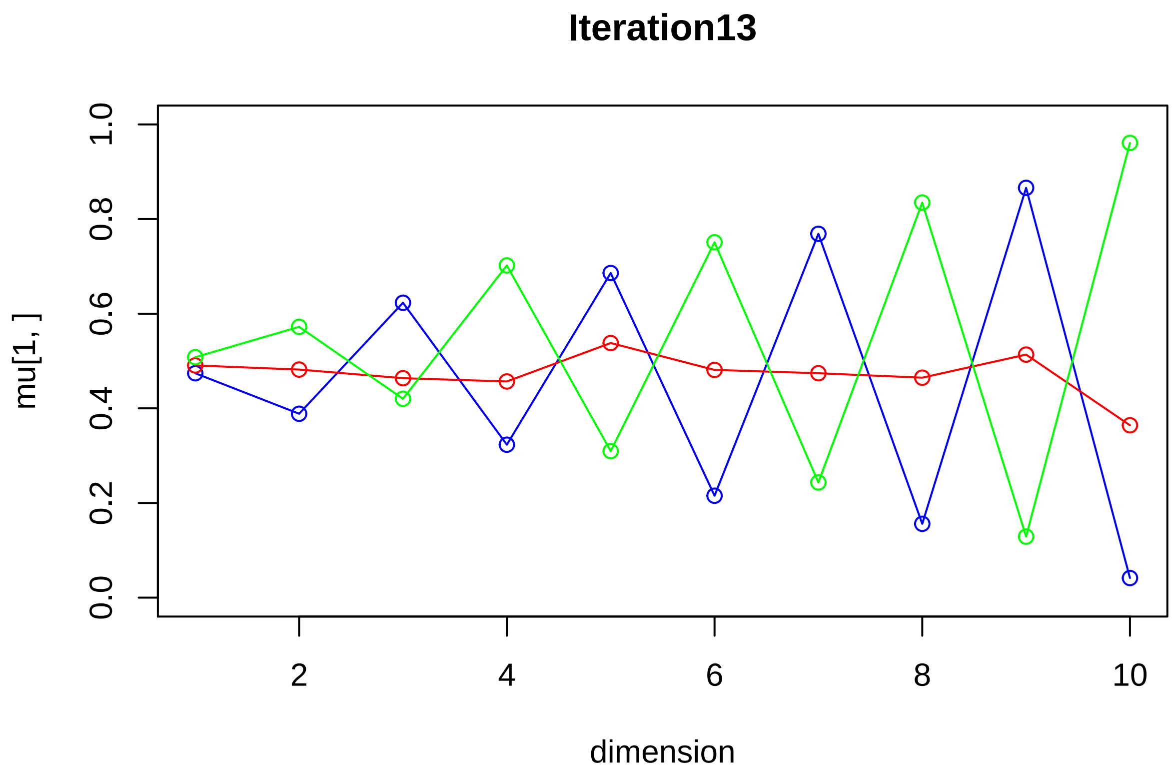

```
## iteration:  10 log likelihood:  -6780.713
```



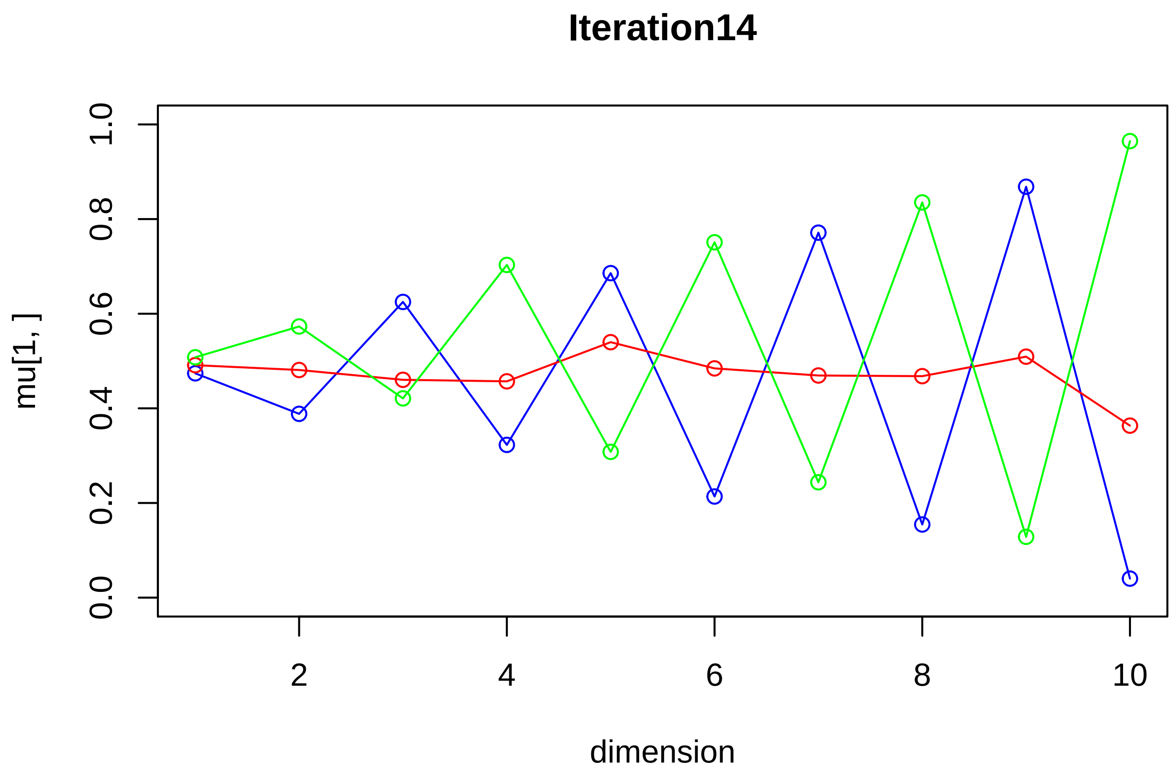

```
## iteration:  11 log likelihood:  -6774.958
```



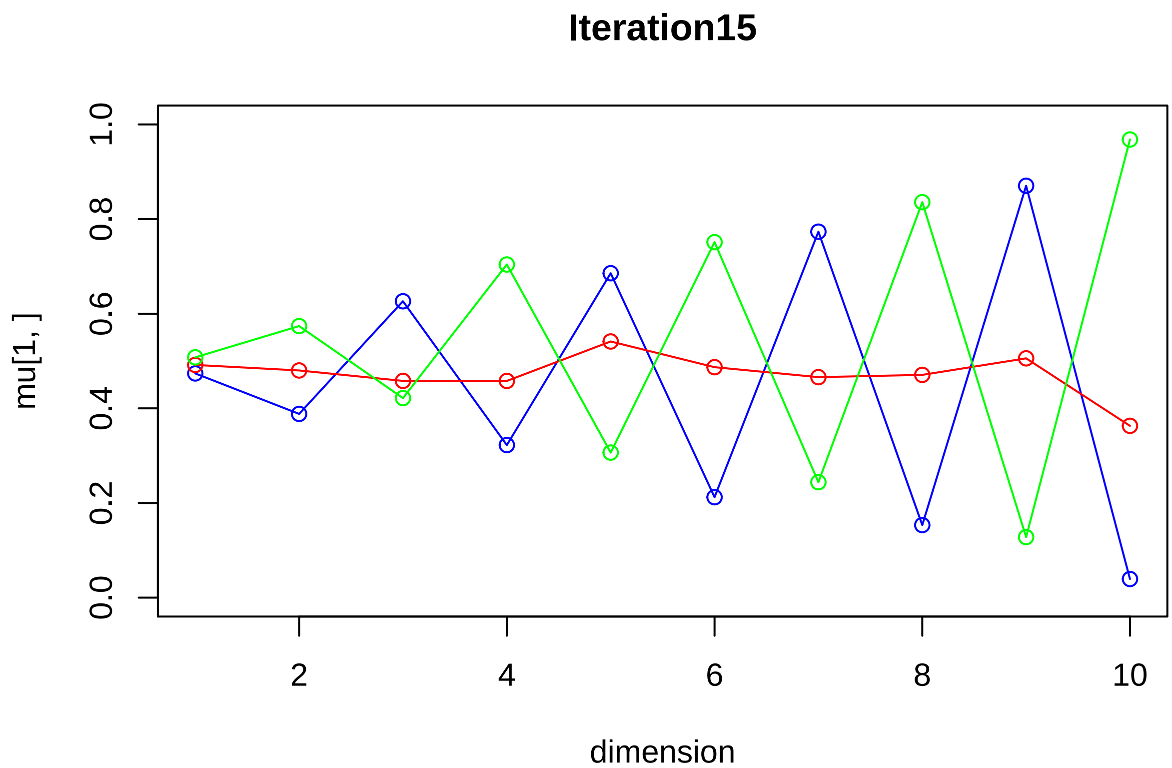

```
## iteration:  12 log likelihood:  -6771.261
```



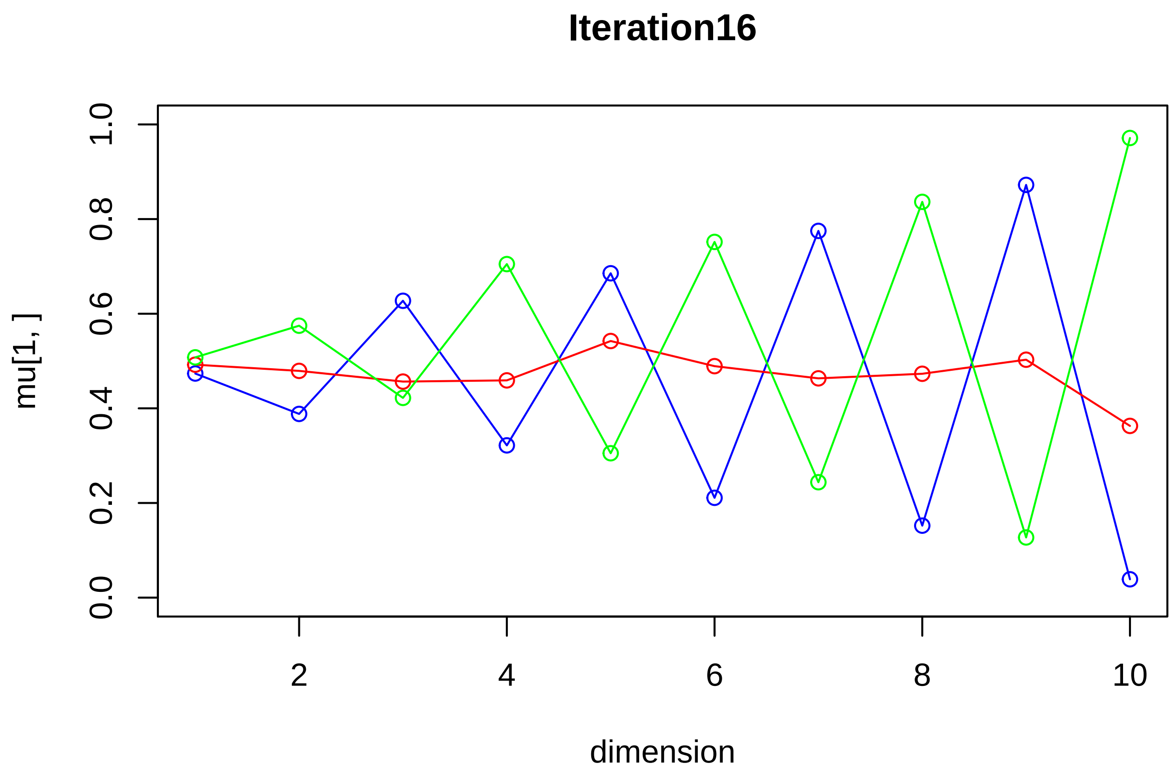

```
## iteration:  13 log likelihood:  -6768.606
```



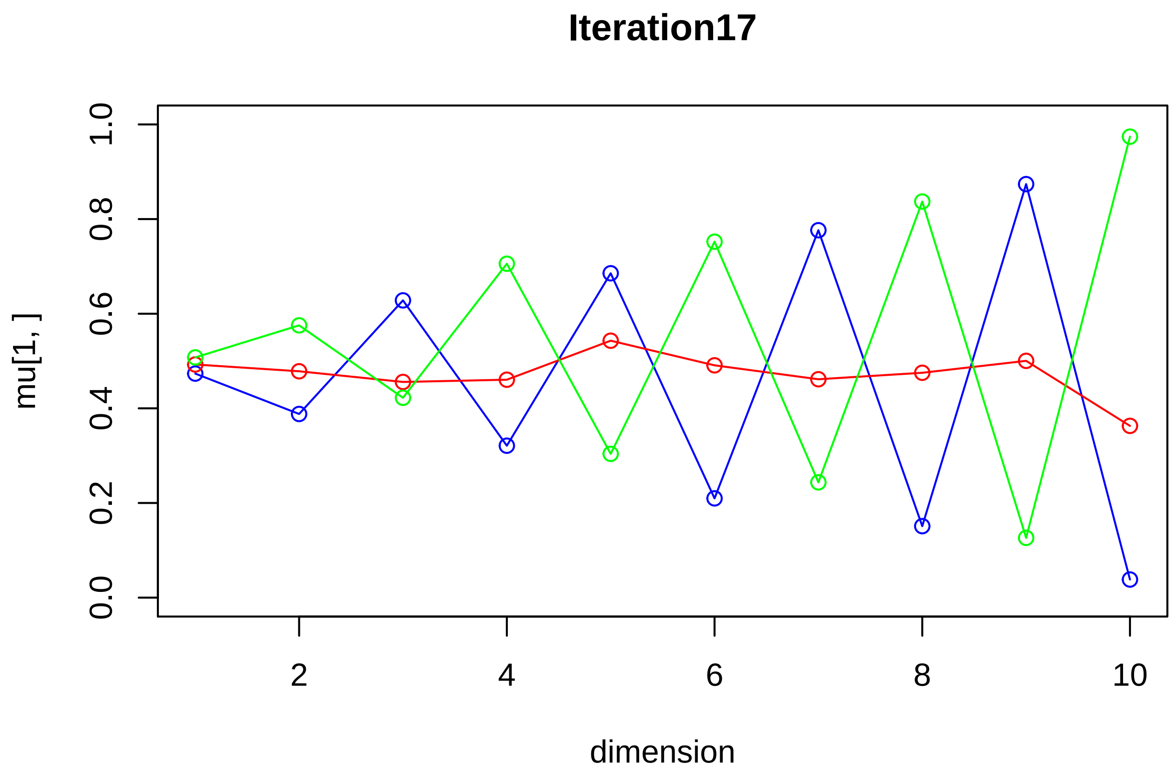

```
## iteration:  14 log likelihood:  -6766.535
```



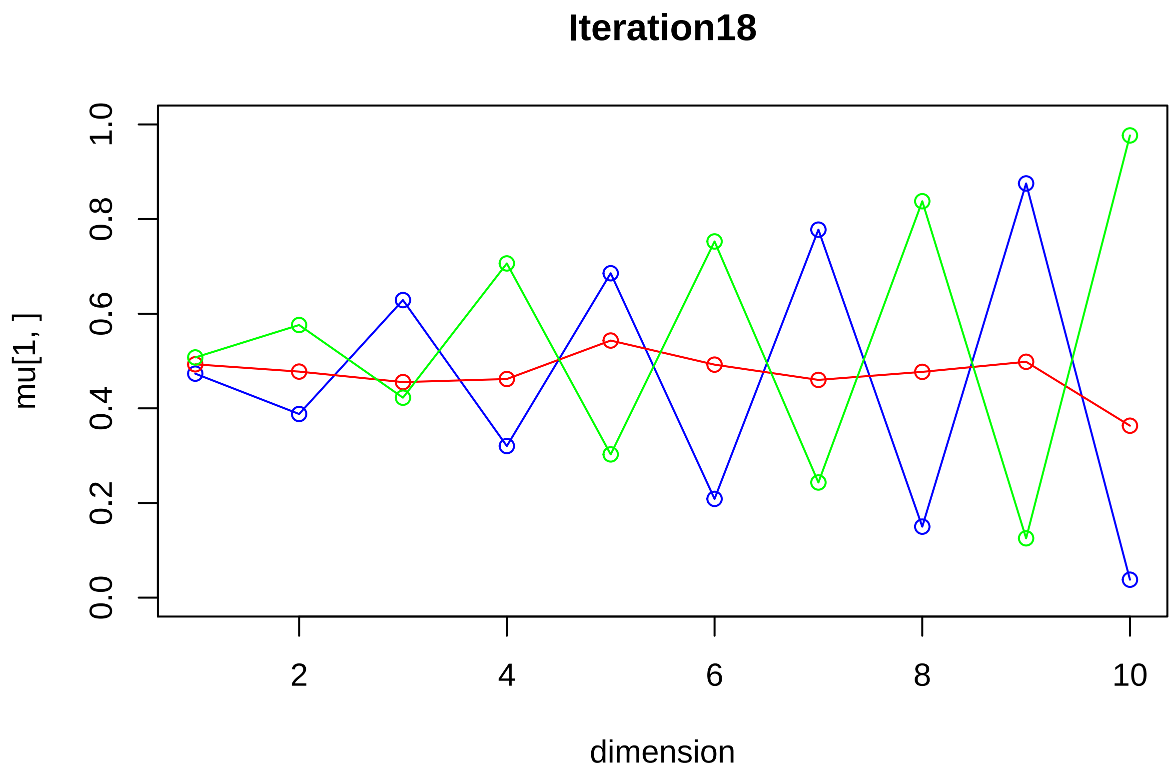

```
## iteration:  15 log likelihood:  -6764.815
```



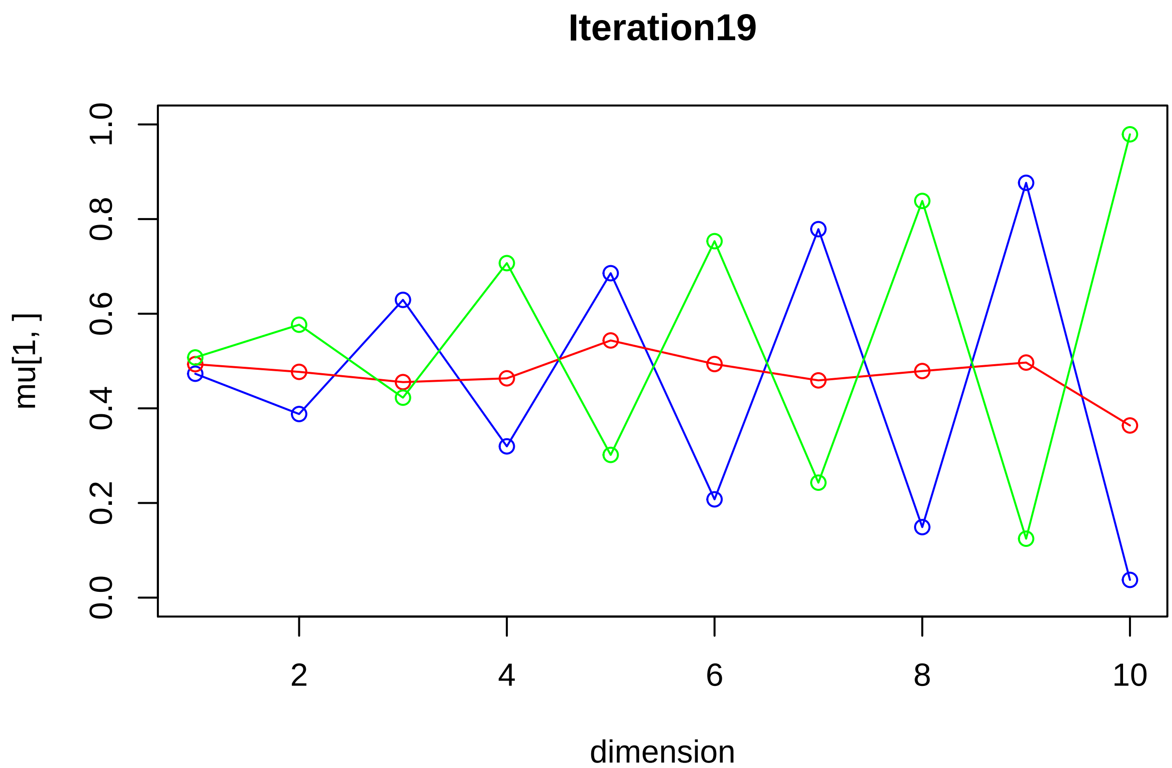

```
## iteration:  16 log likelihood:  -6763.316
```



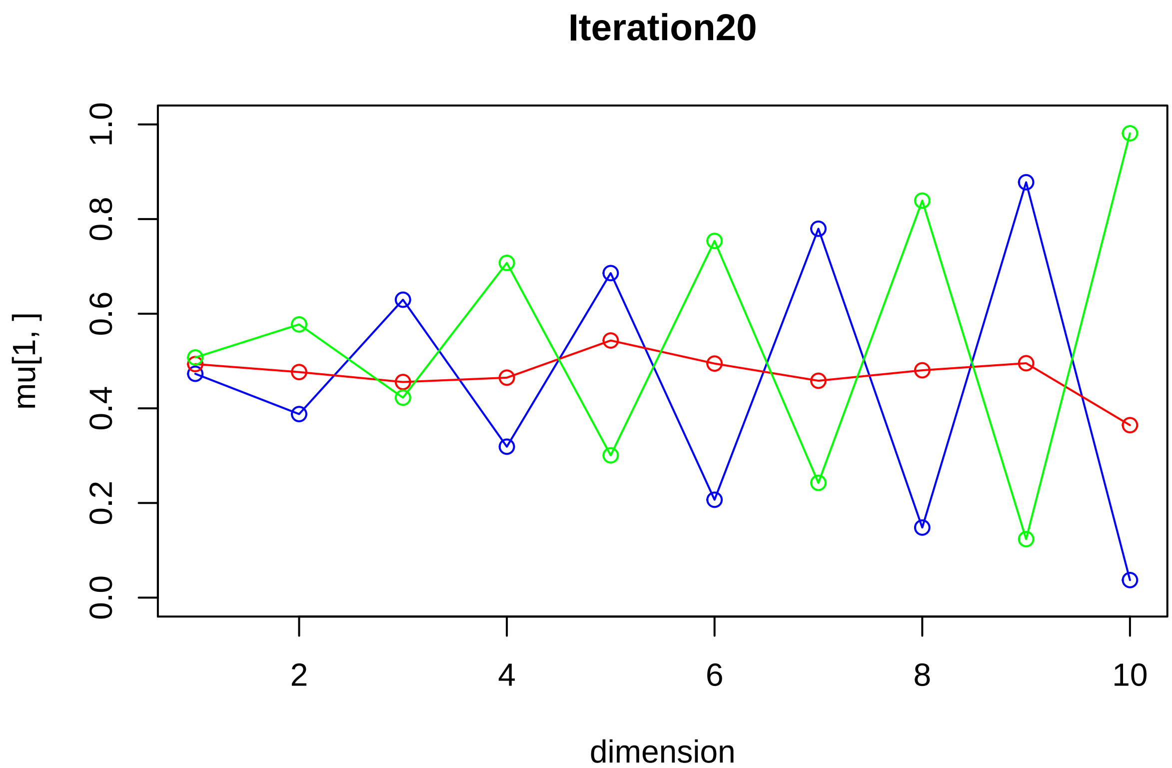

```
## iteration:  17 log likelihood:  -6761.967
```



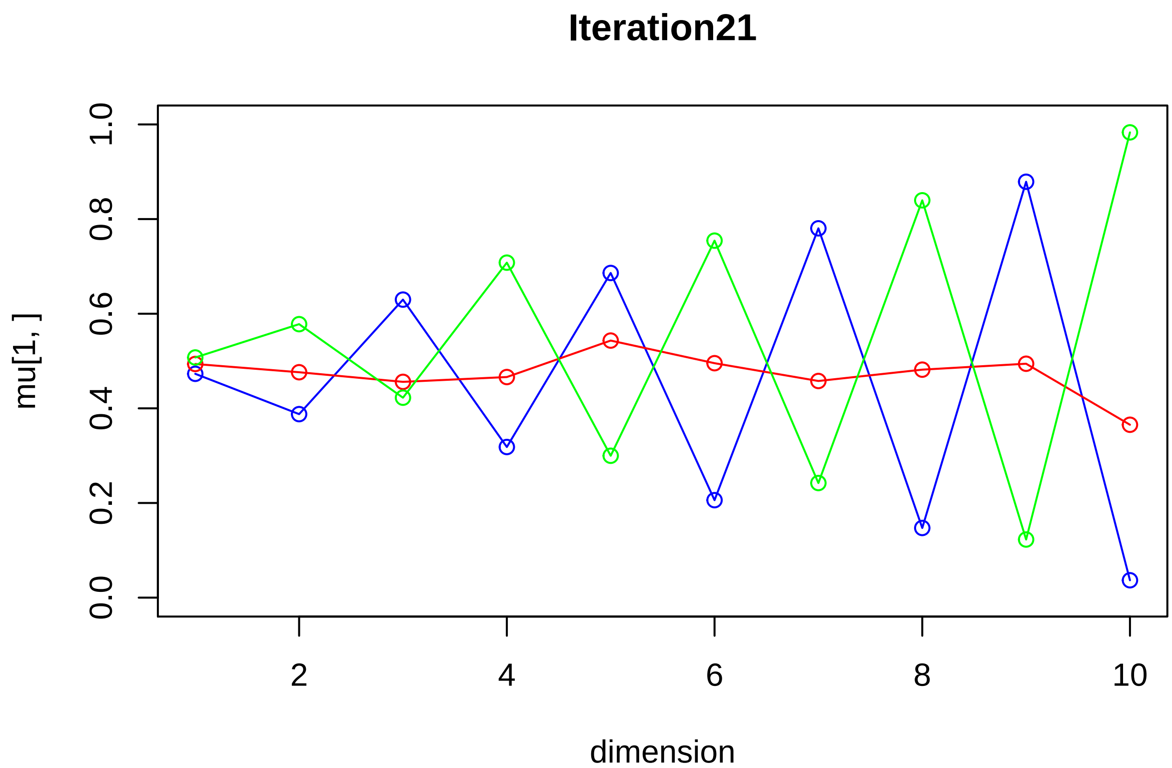

```
## iteration:  18 log likelihood:  -6760.727
```



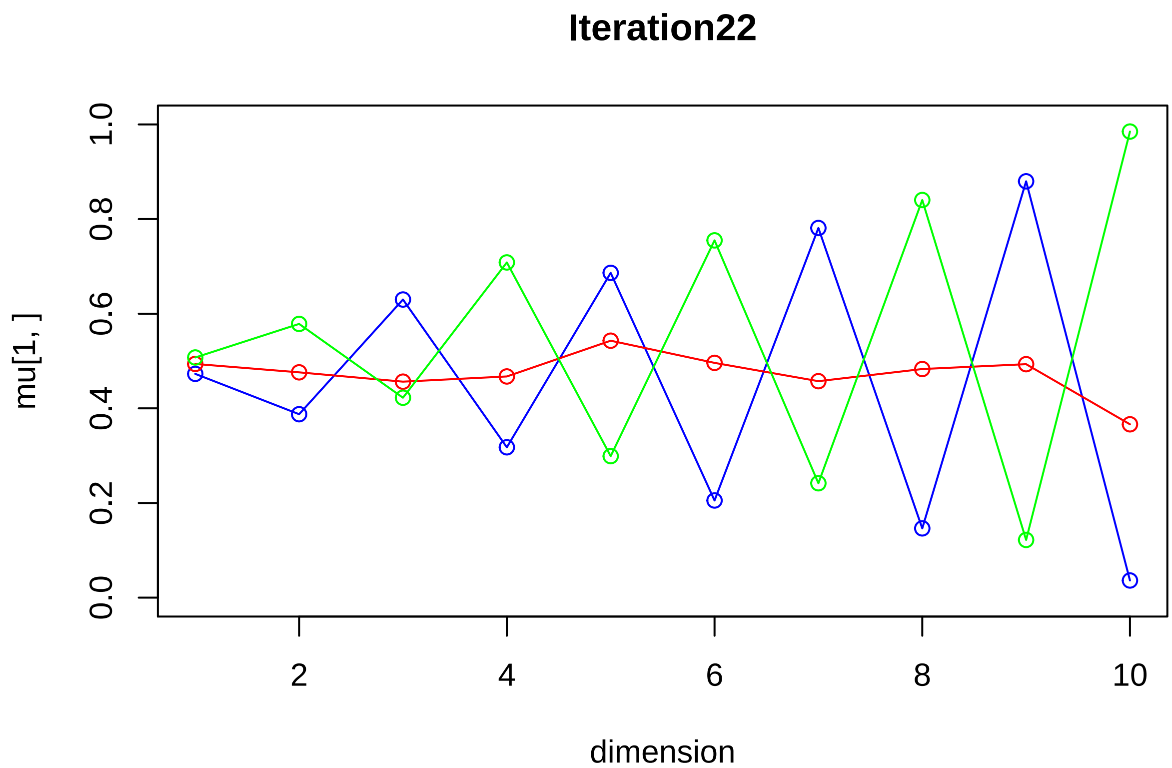

```
## iteration:  19 log likelihood:  -6759.572
```



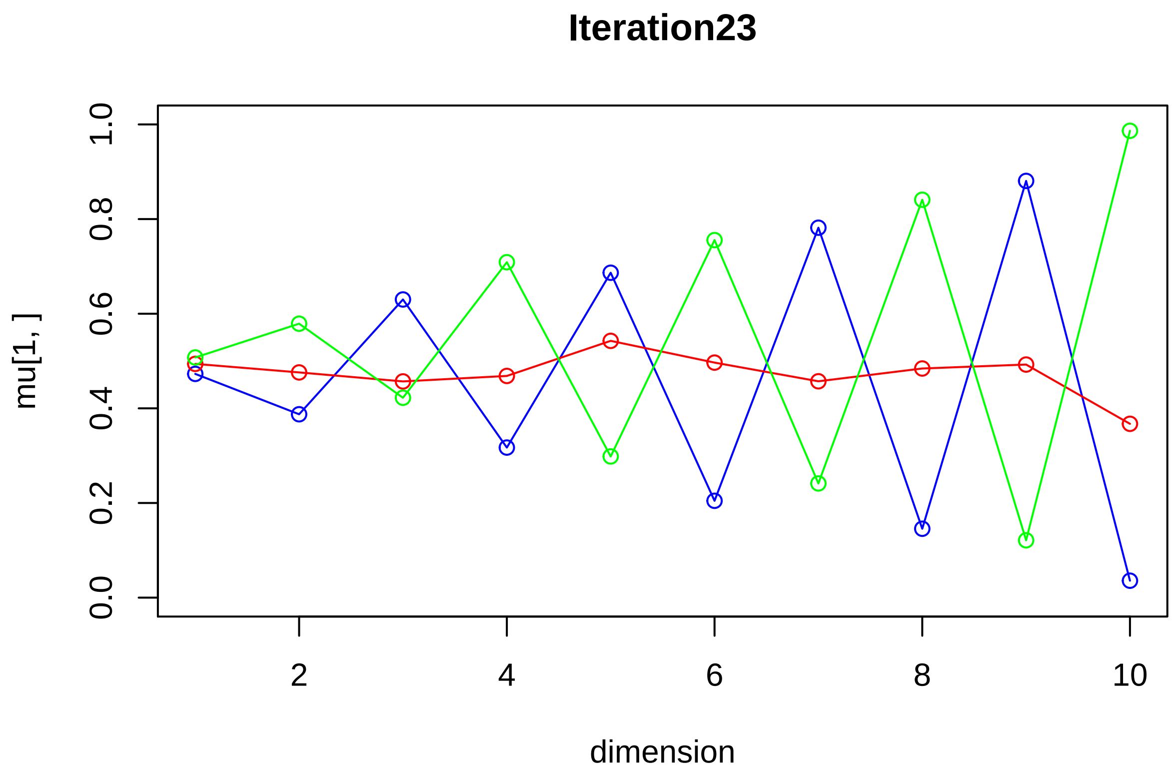

```
## iteration:  20 log likelihood:  -6758.491
```



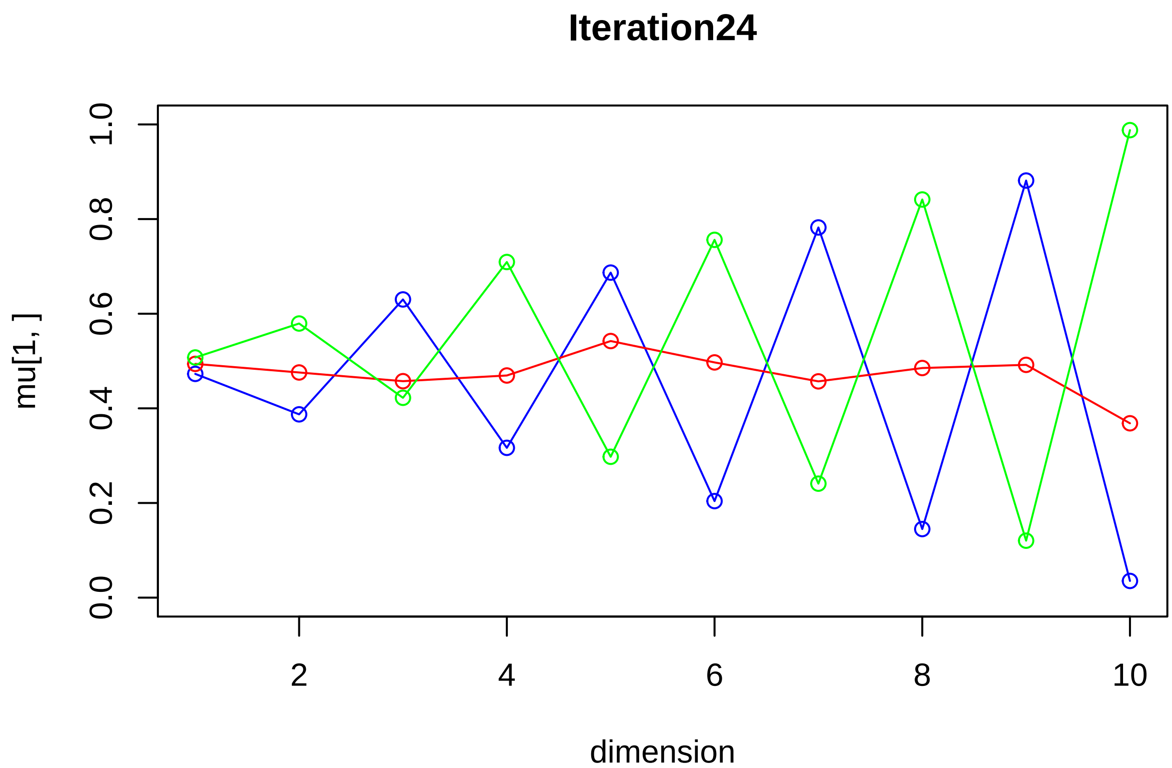

```
## iteration:  21 log likelihood:  -6757.475
```



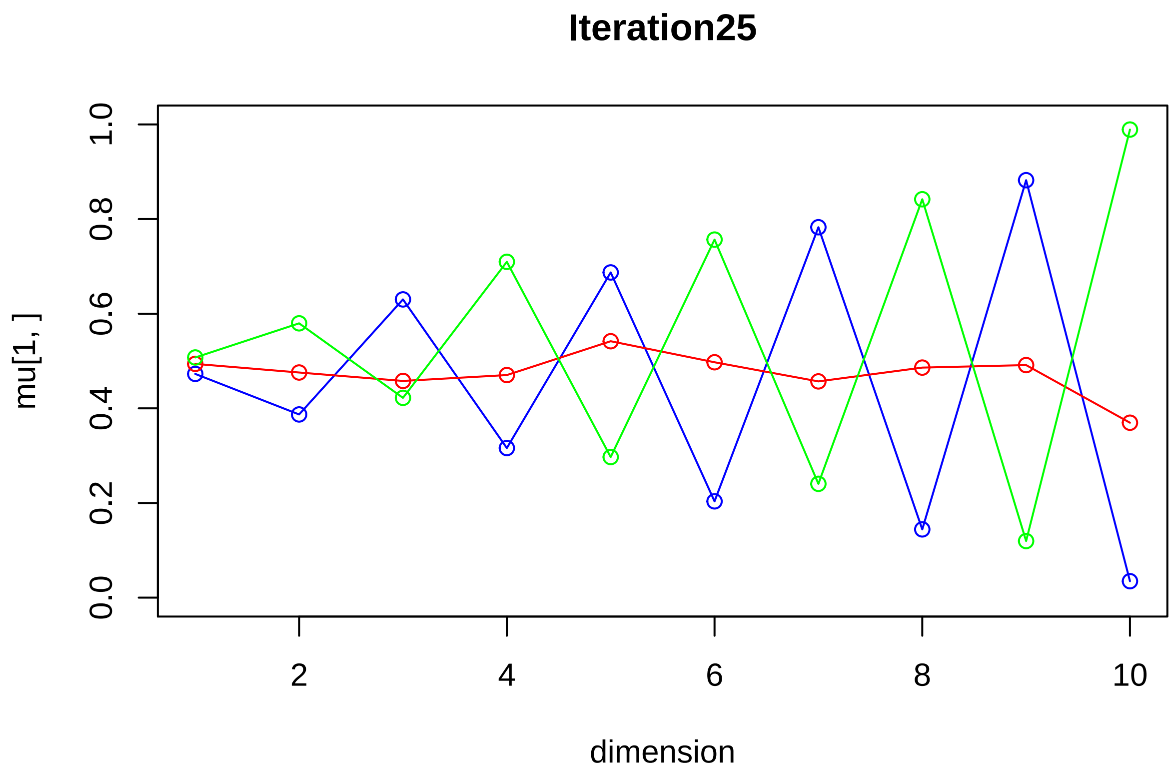

```
## iteration:  22 log likelihood:  -6756.521
```



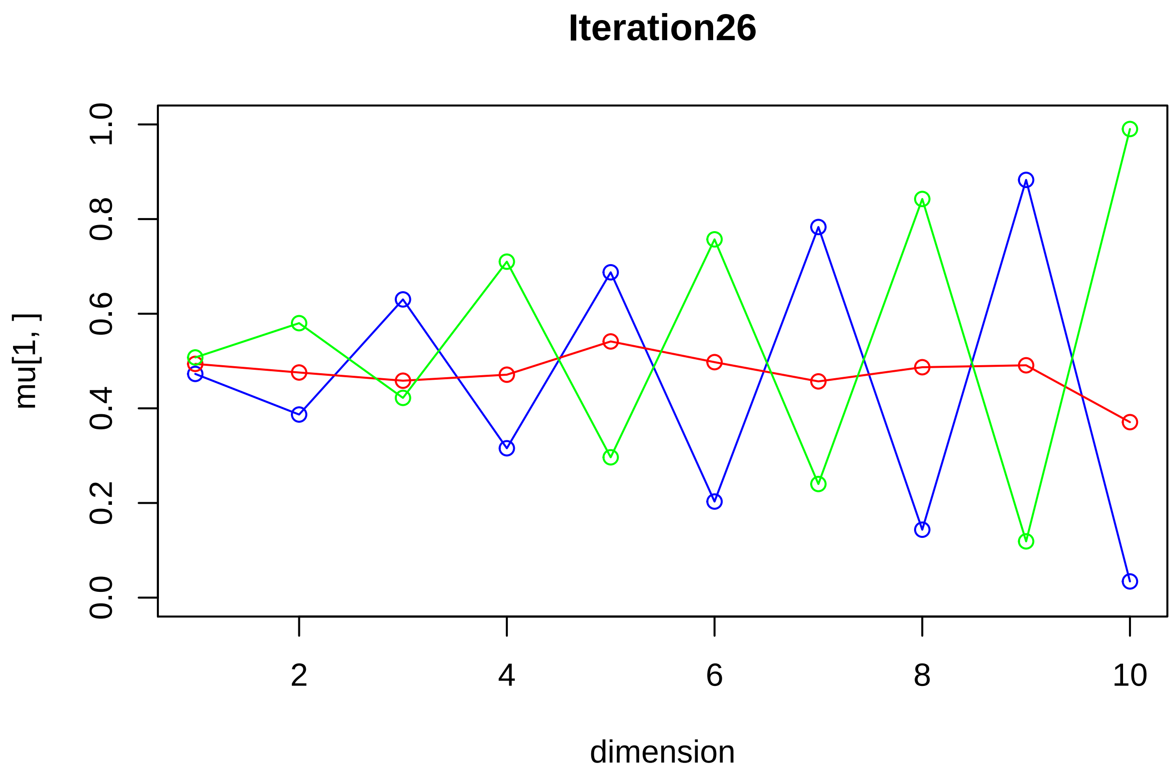

```
## iteration:  23 log likelihood:  -6755.625
```



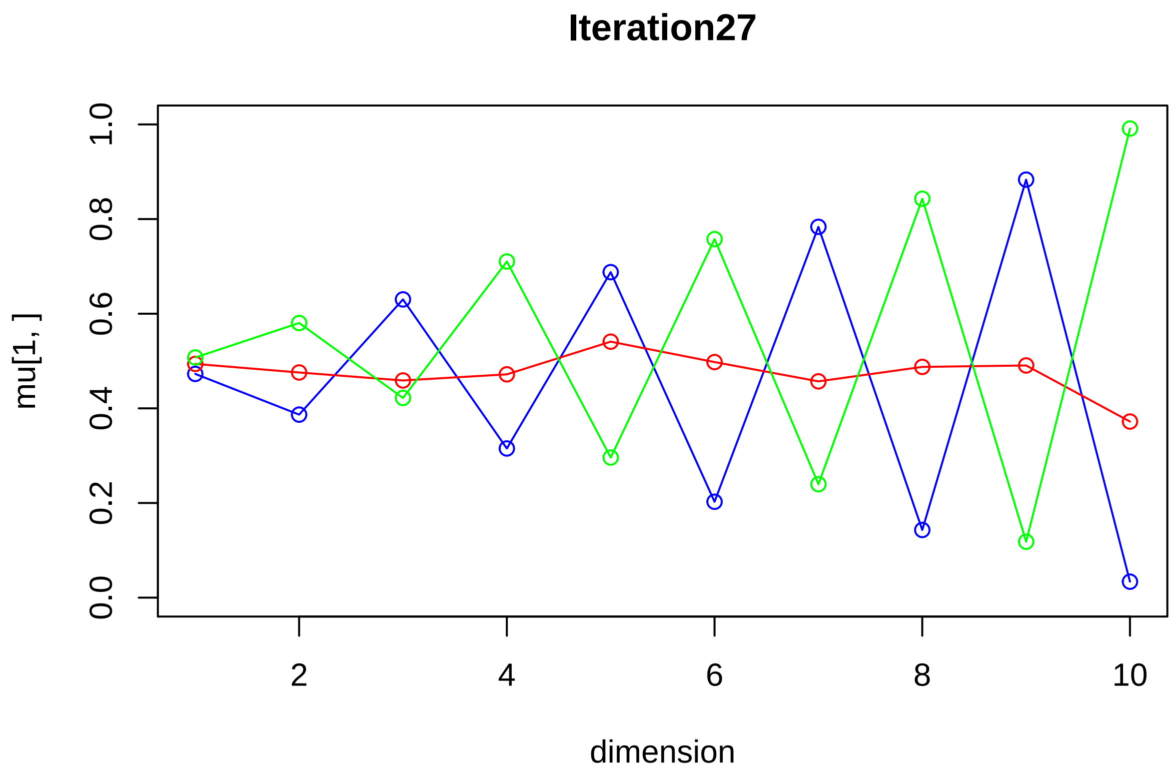

```
## iteration:  24 log likelihood:  -6754.784
```



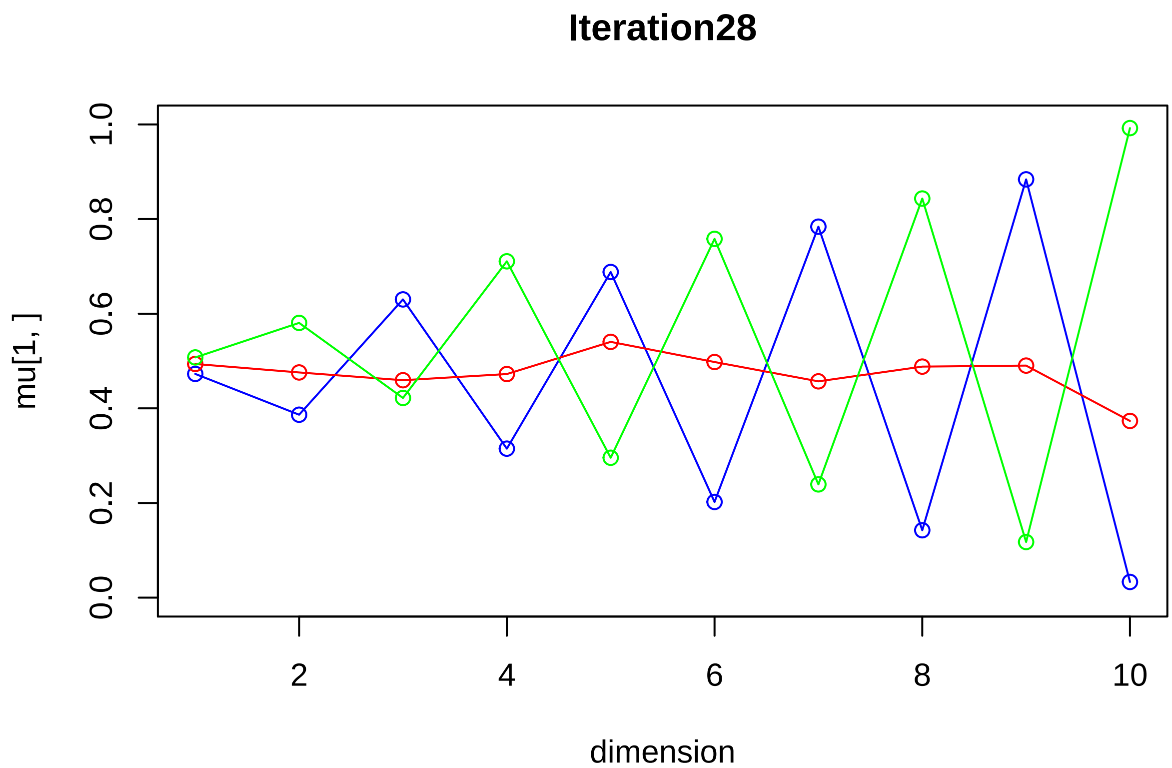

```
## iteration:  25 log likelihood:  -6753.996
```



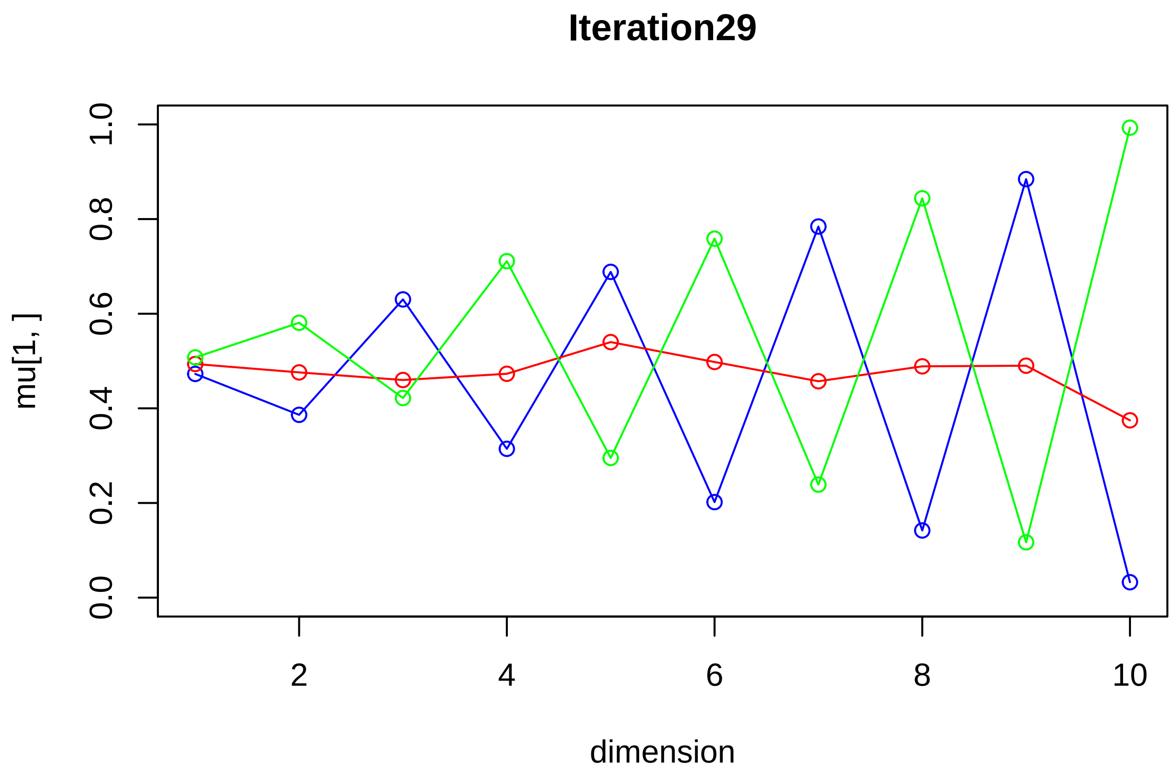

```
## iteration:  26 log likelihood:  -6753.26
```



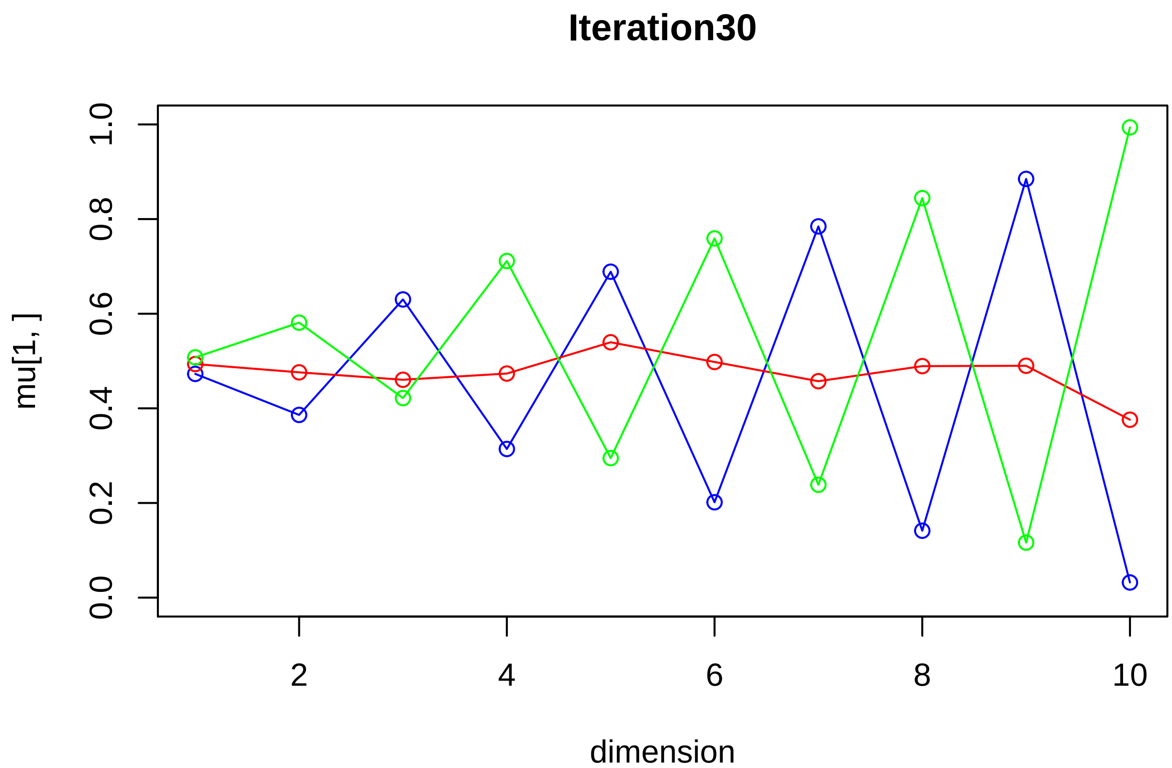

```
## iteration:  27 log likelihood:  -6752.571
```



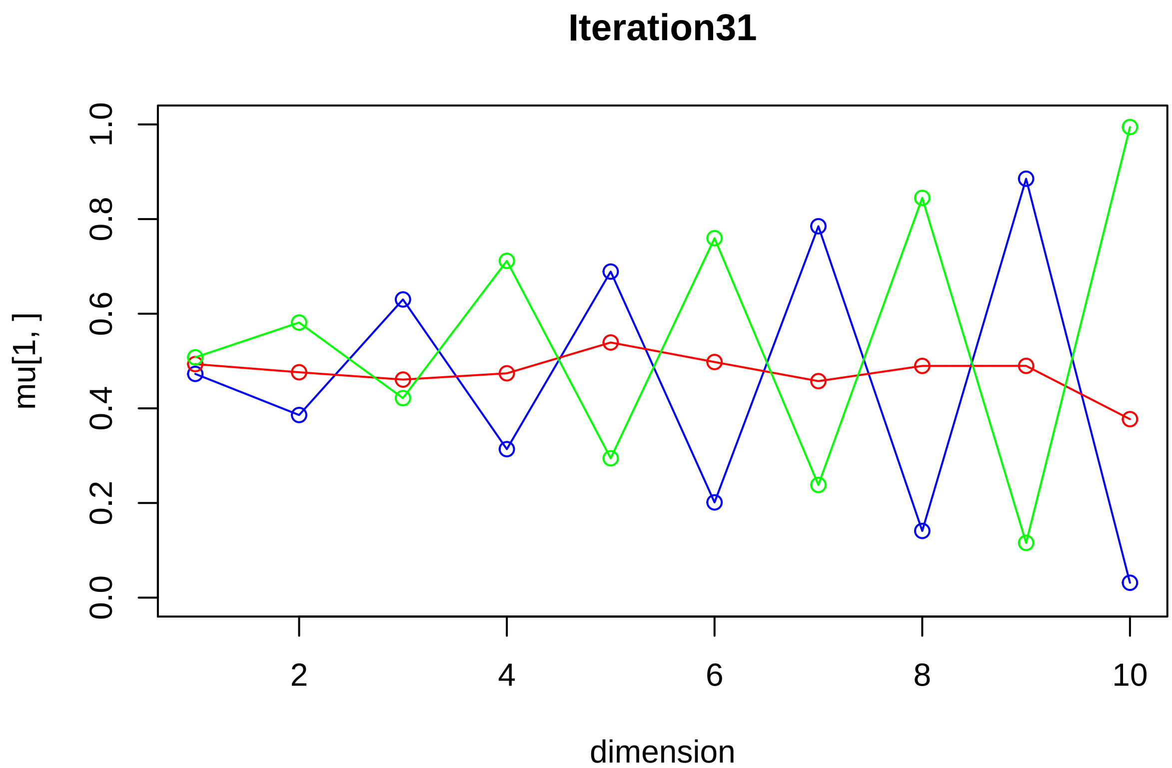

```
## iteration:  28 log likelihood:  -6751.928
```



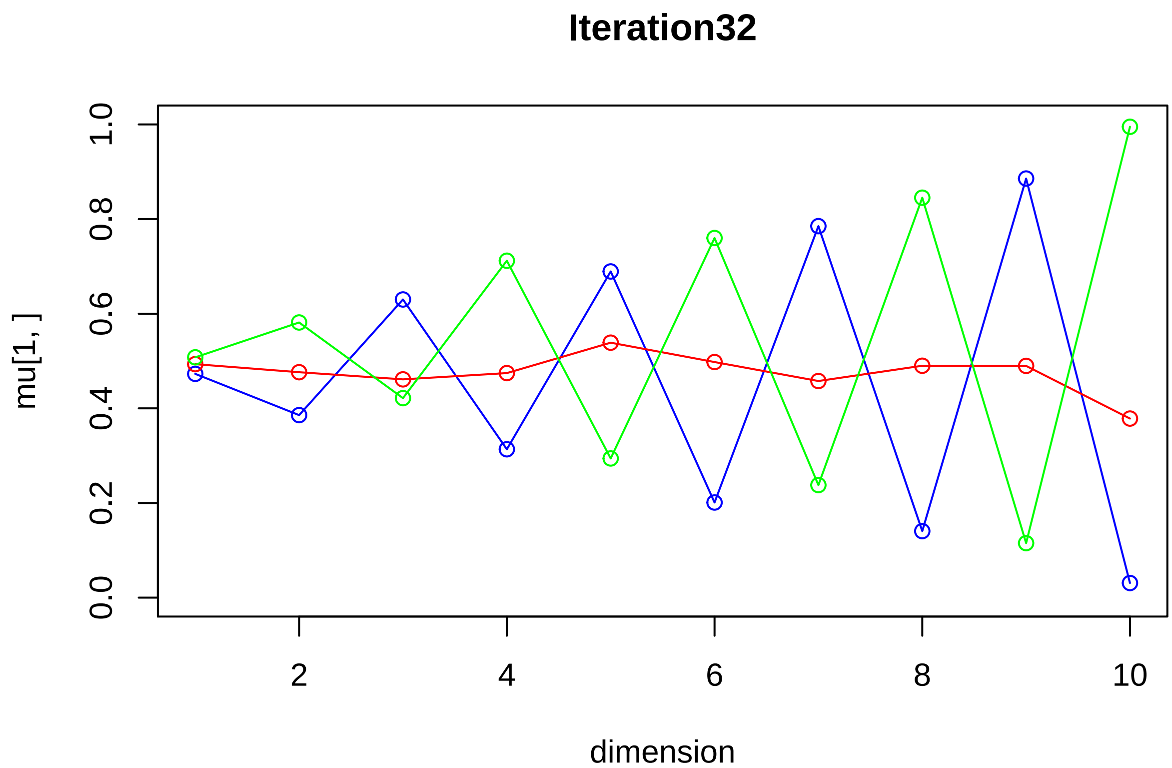

```
## iteration:  29 log likelihood:  -6751.328
```



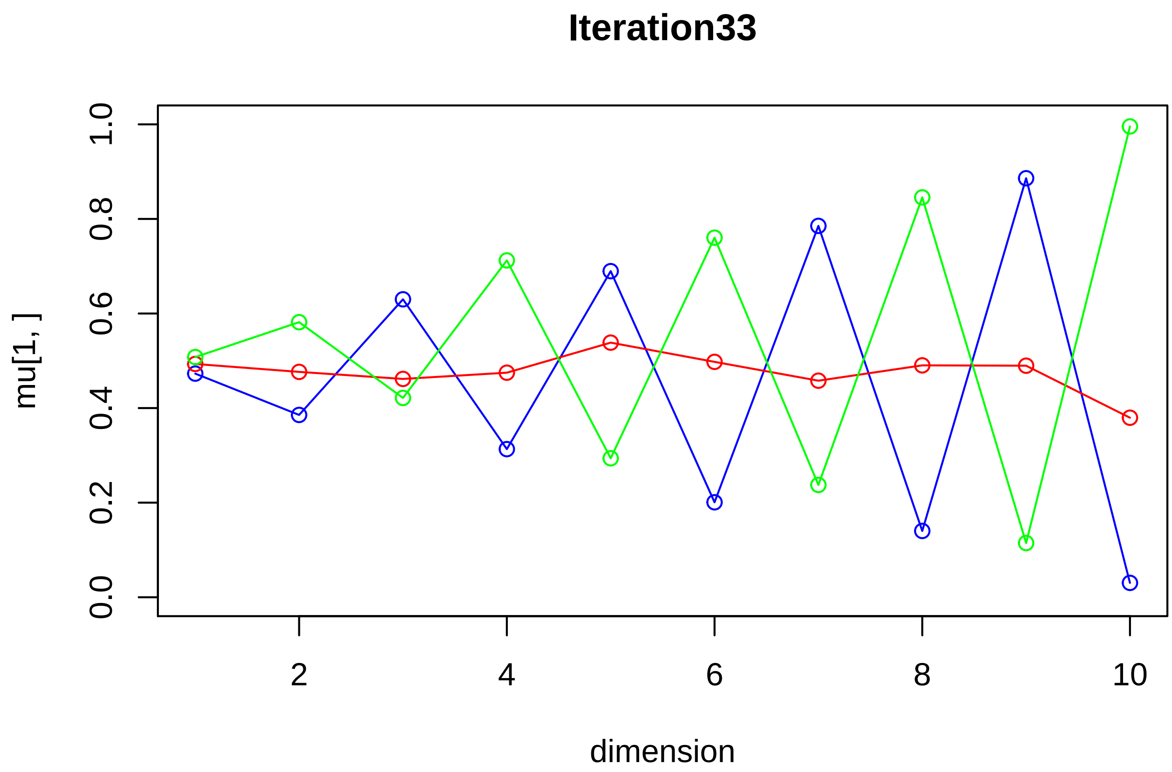

```
## iteration:  30 log likelihood:  -6750.768
```



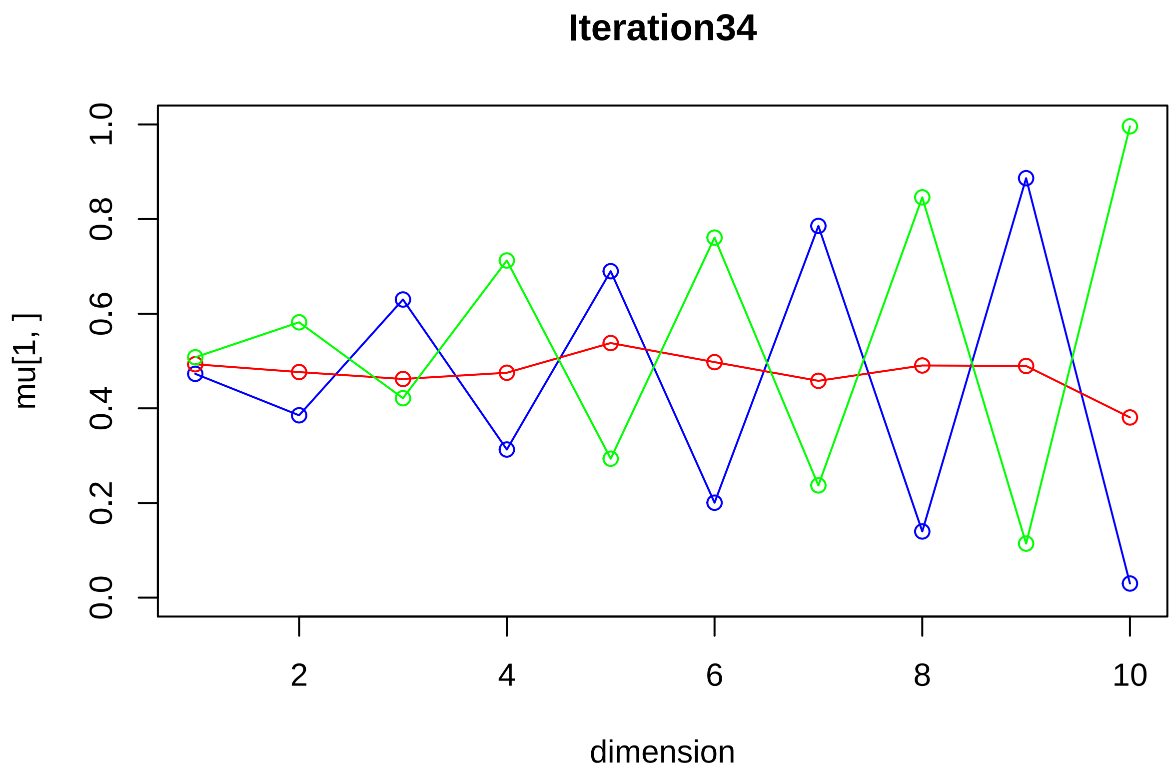

```
## iteration:  31 log likelihood:  -6750.246
```



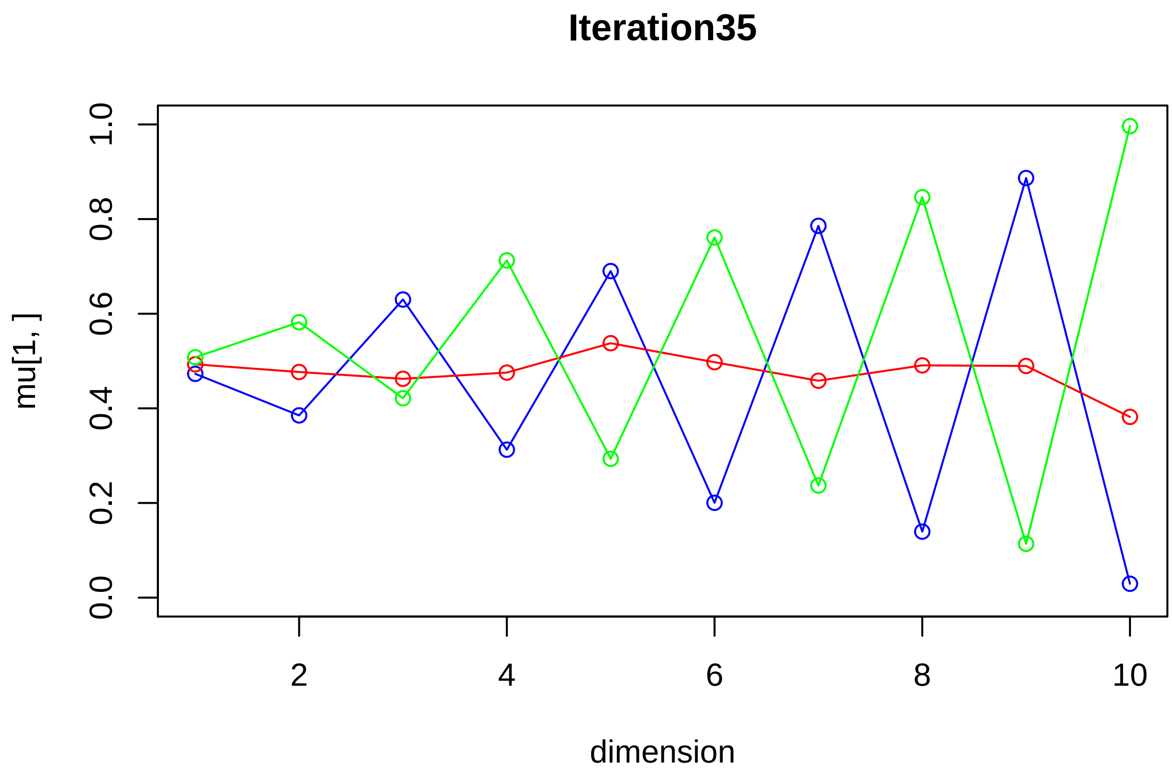

```
## iteration:  32 log likelihood:  -6749.758
```



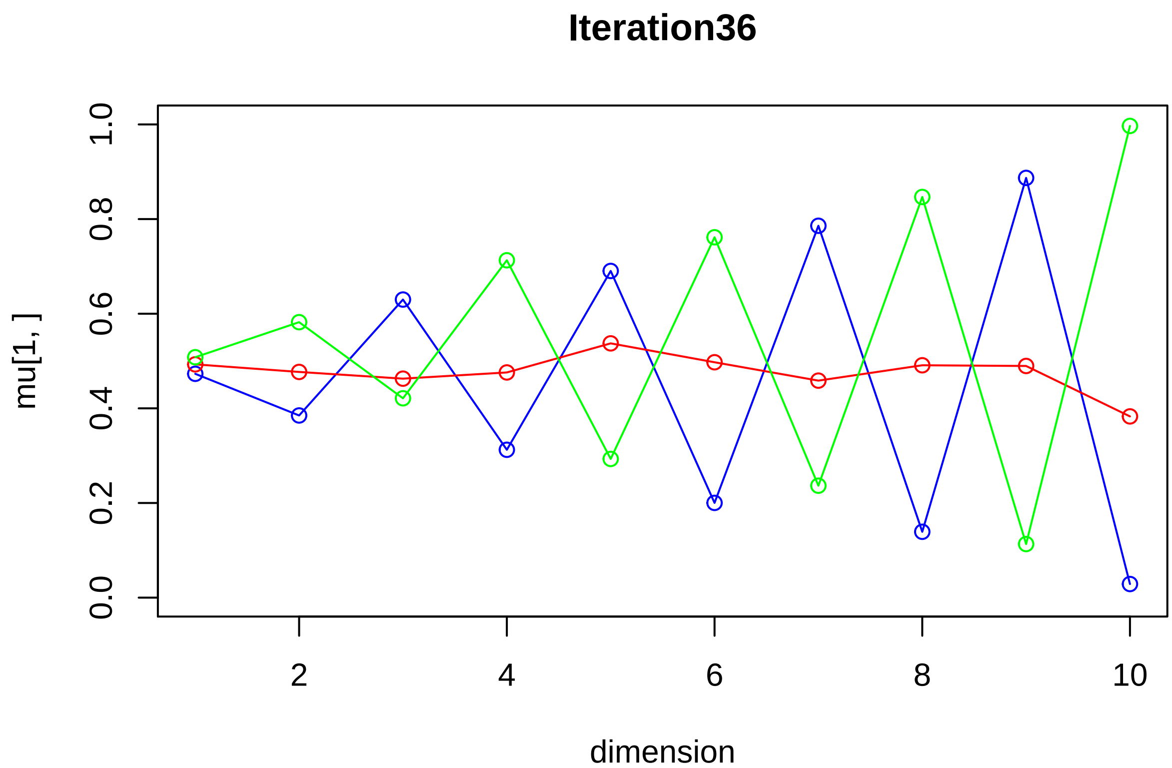

```
## iteration:  33 log likelihood:  -6749.304
```



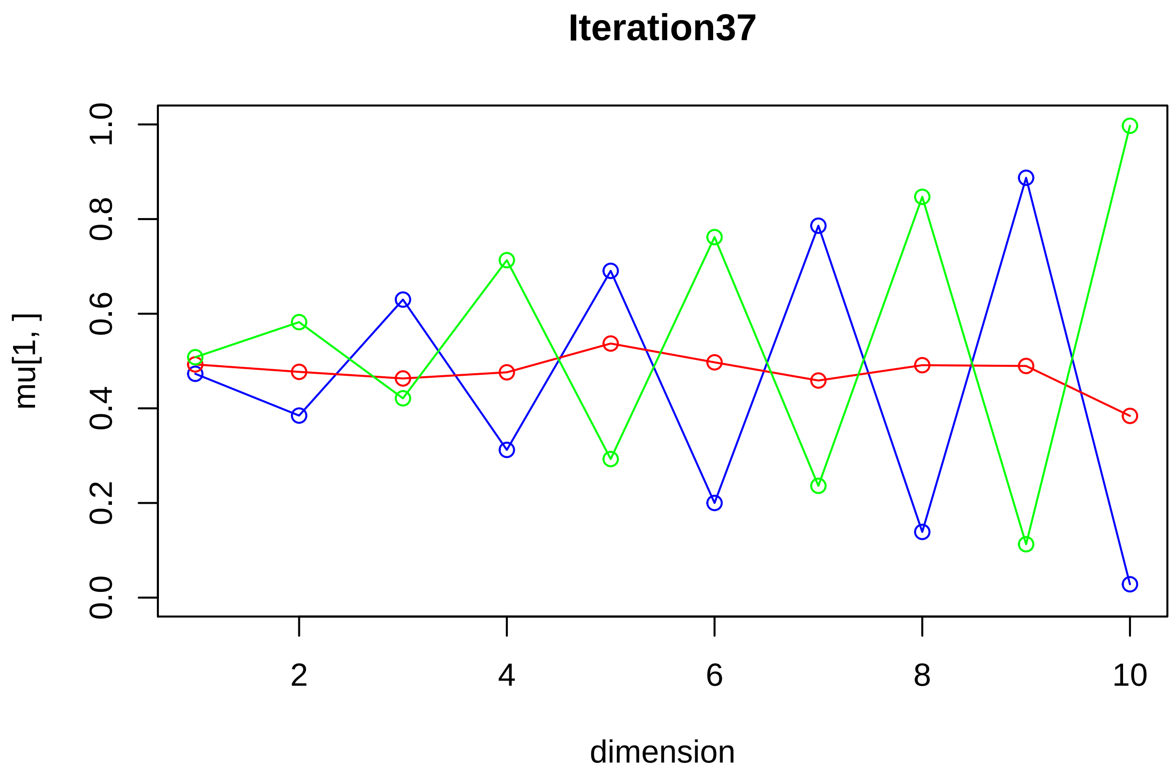

```
## iteration:  34 log likelihood:  -6748.88
```



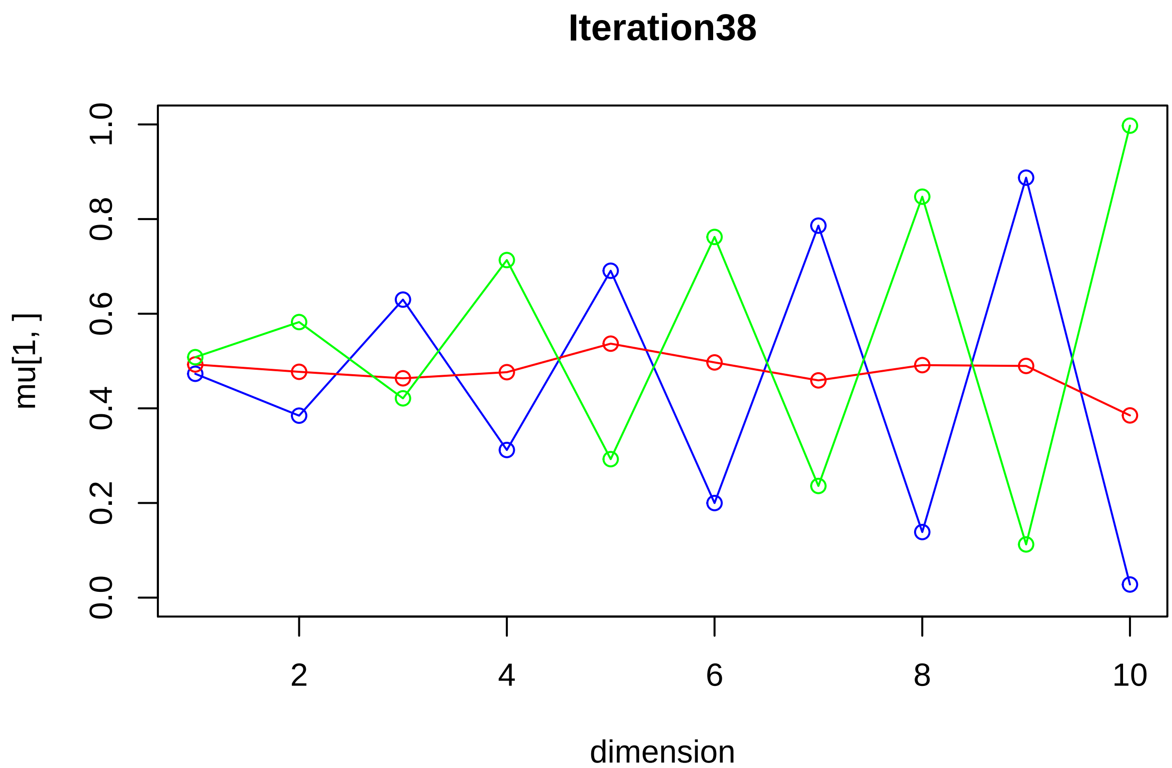

```
## iteration:  35 log likelihood:  -6748.484
```



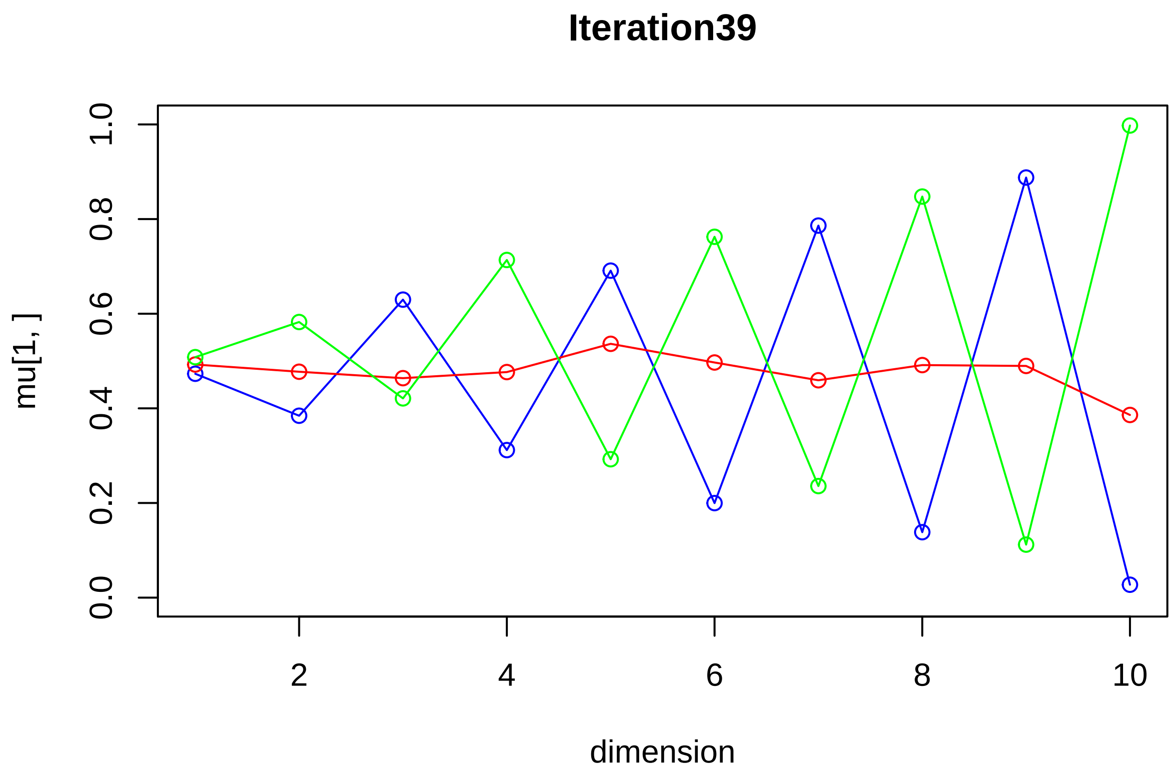

```
## iteration:  36 log likelihood:  -6748.114
```



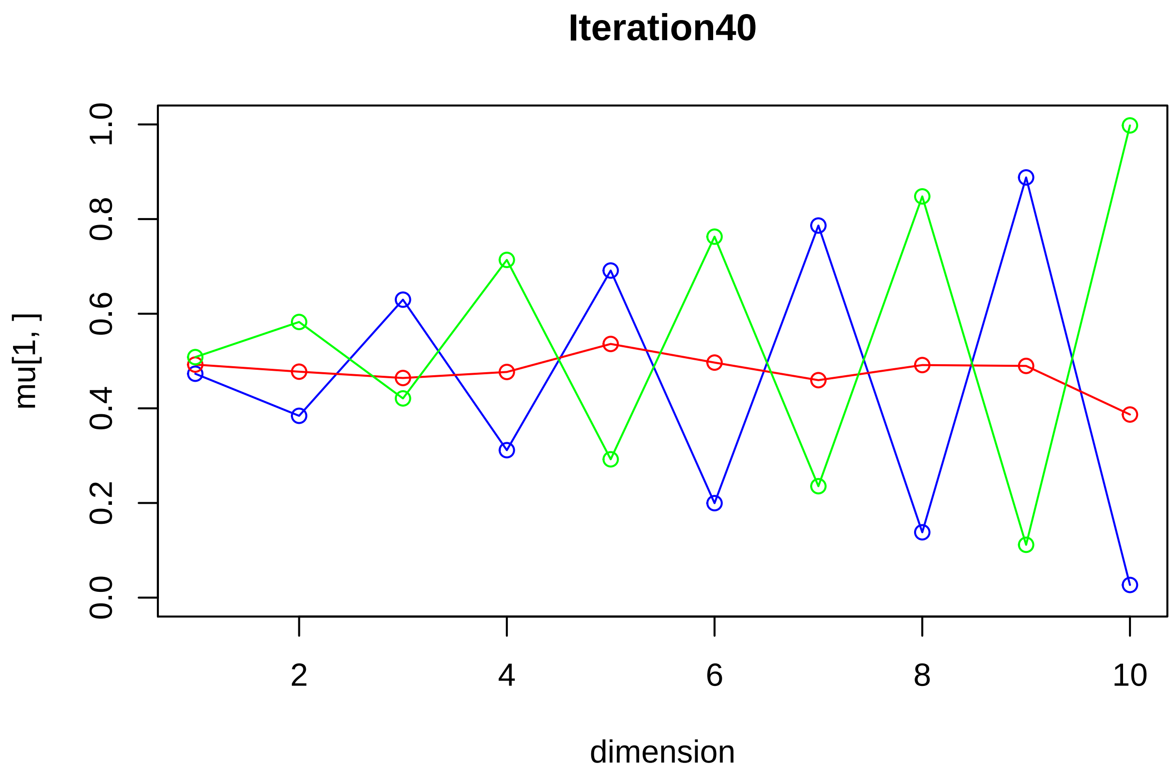

```
## iteration:  37 log likelihood:  -6747.767
```



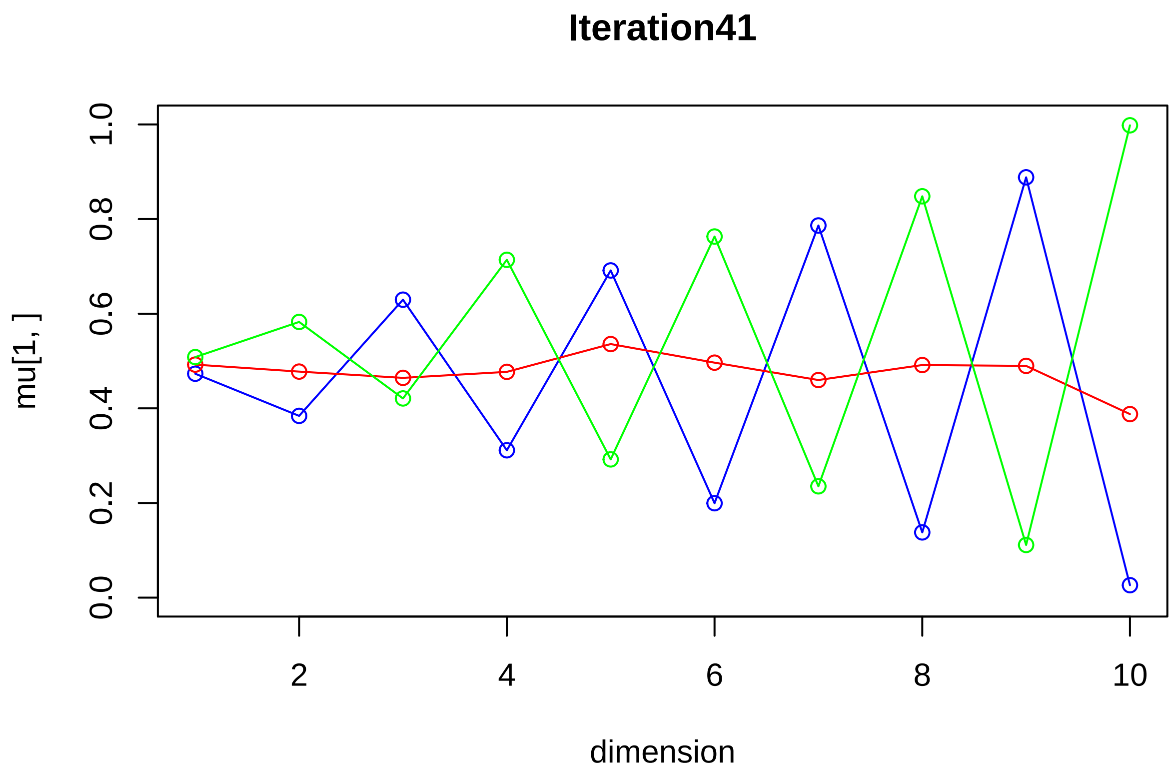

```
## iteration:  38 log likelihood:  -6747.444
```



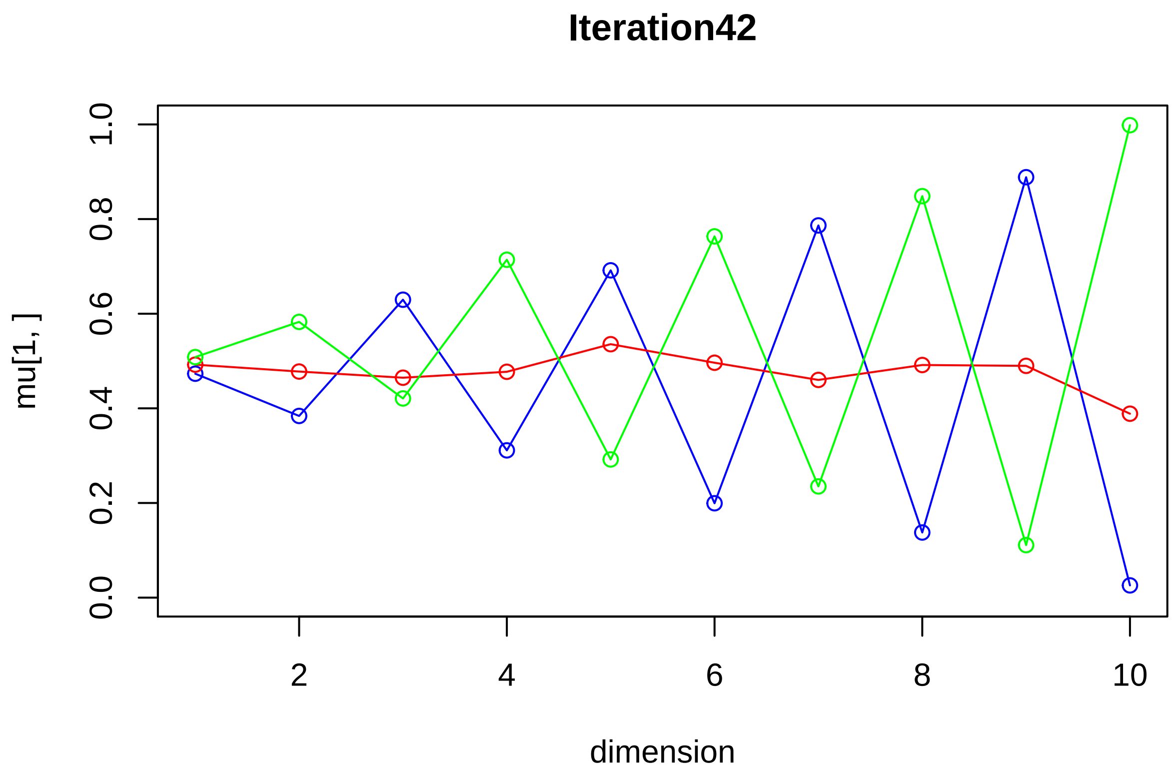

```
## iteration:  39 log likelihood:  -6747.14
```



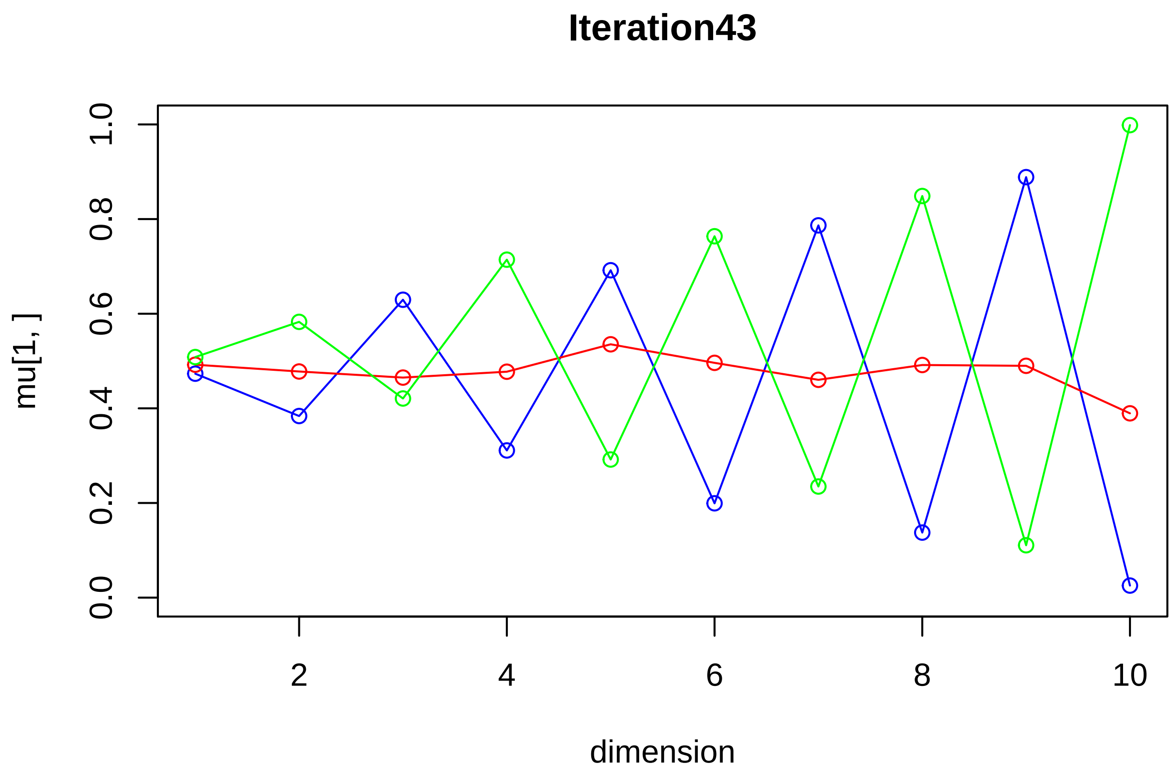

```
## iteration:  40 log likelihood:  -6746.856
```



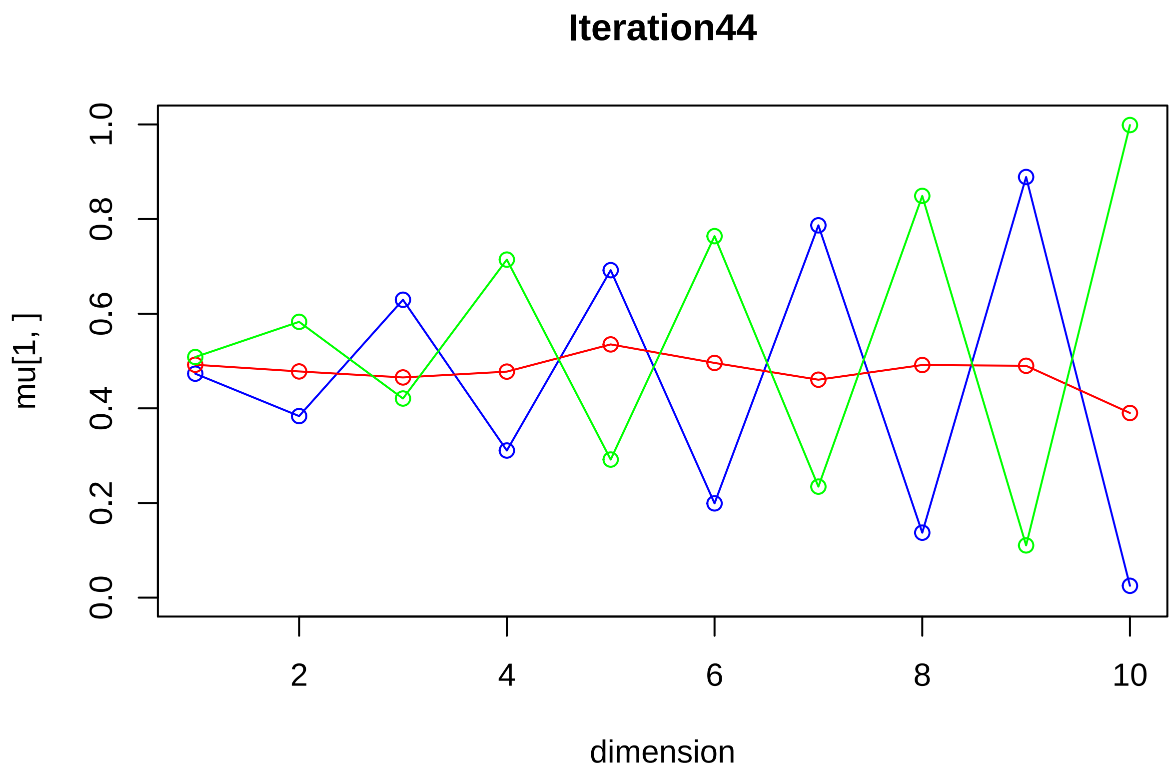

```
## iteration:  41 log likelihood:  -6746.589
```



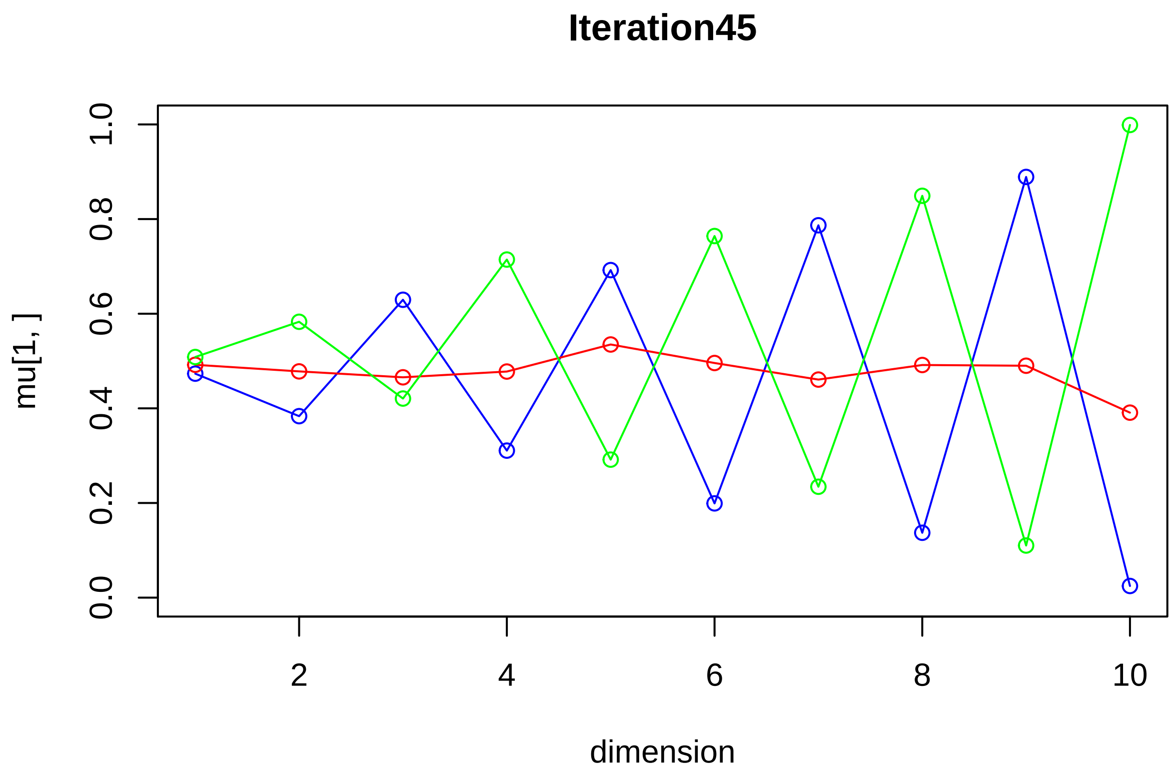

```
## iteration:  42 log likelihood:  -6746.338
```



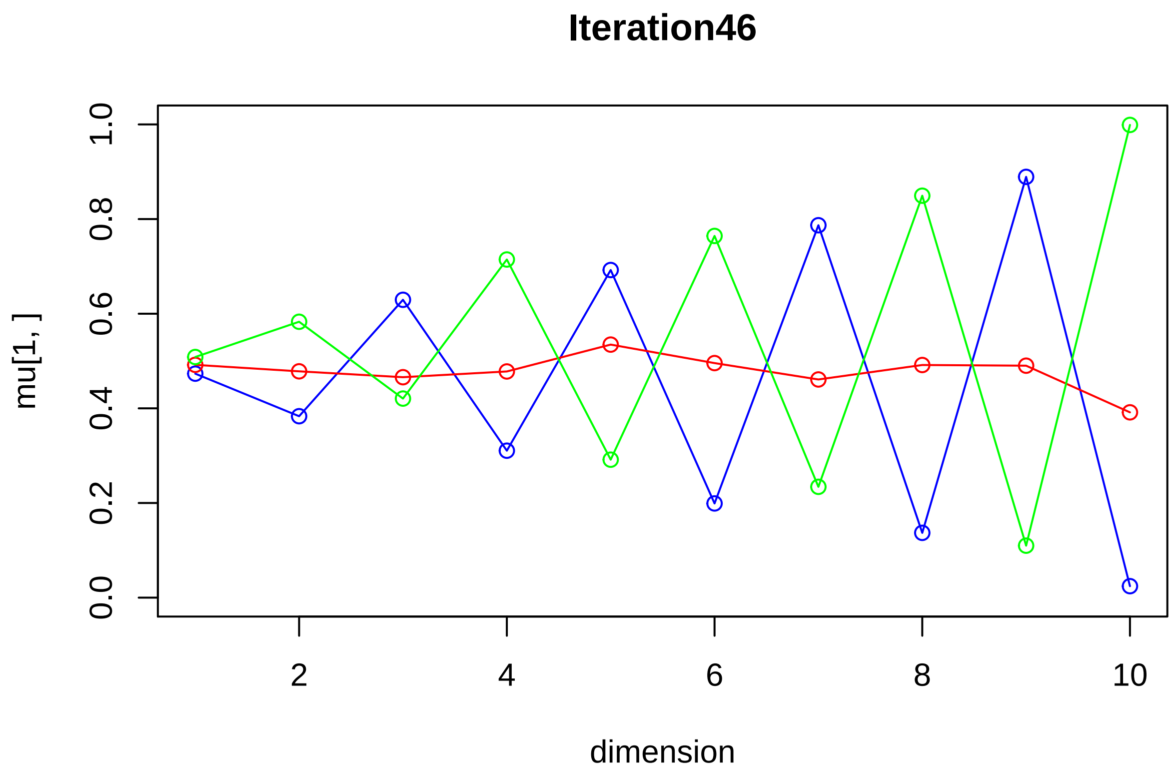

```
## iteration:  43 log likelihood:  -6746.102
```



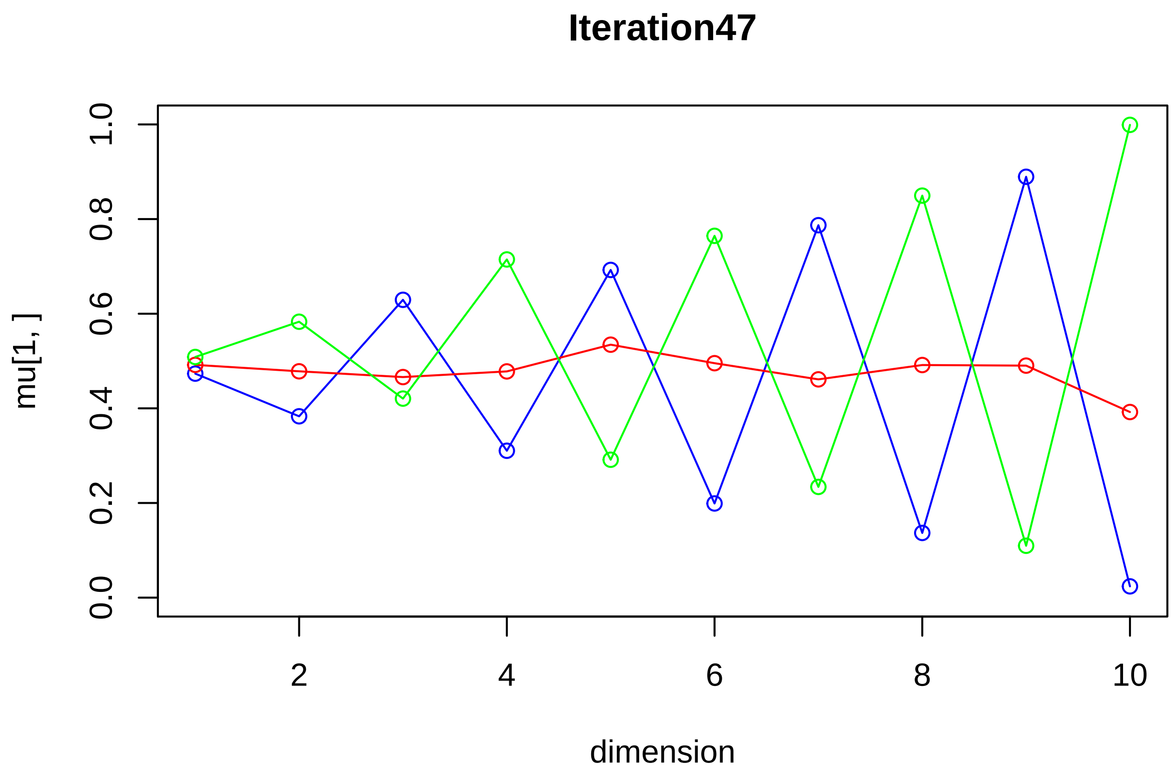

```
## iteration:  44 log likelihood:  -6745.88
```



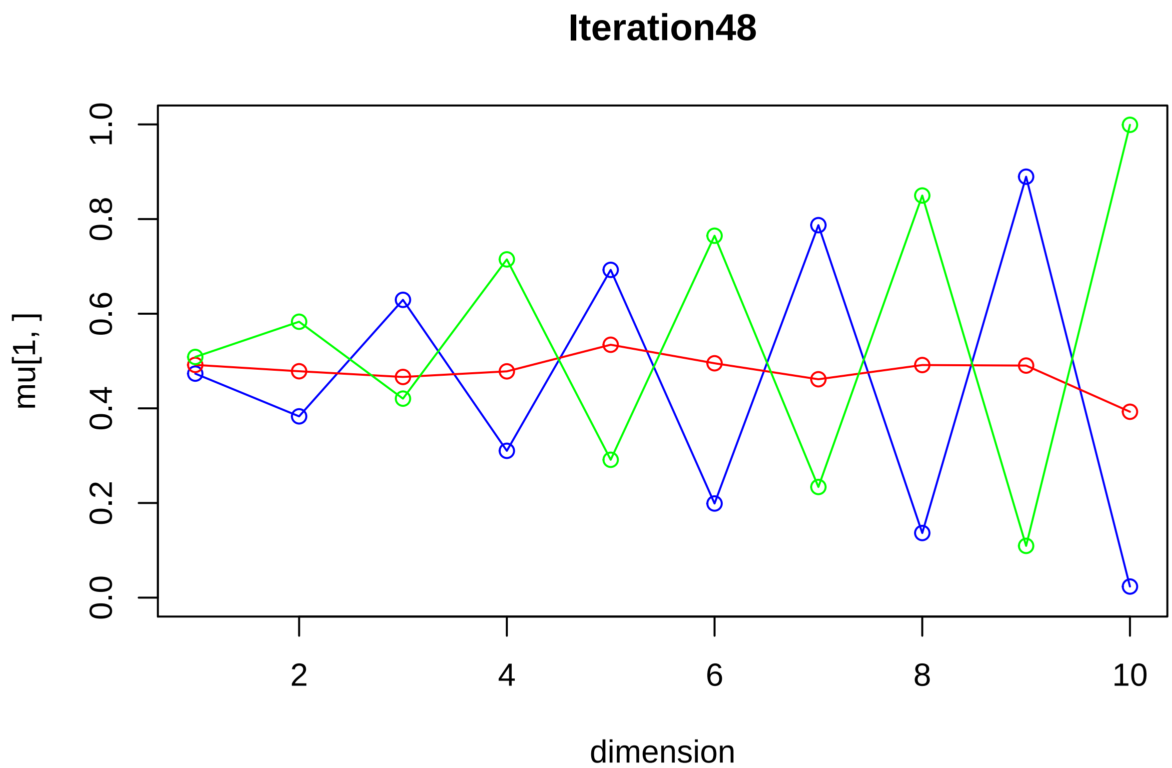

```
## iteration:  45 log likelihood:  -6745.67
```



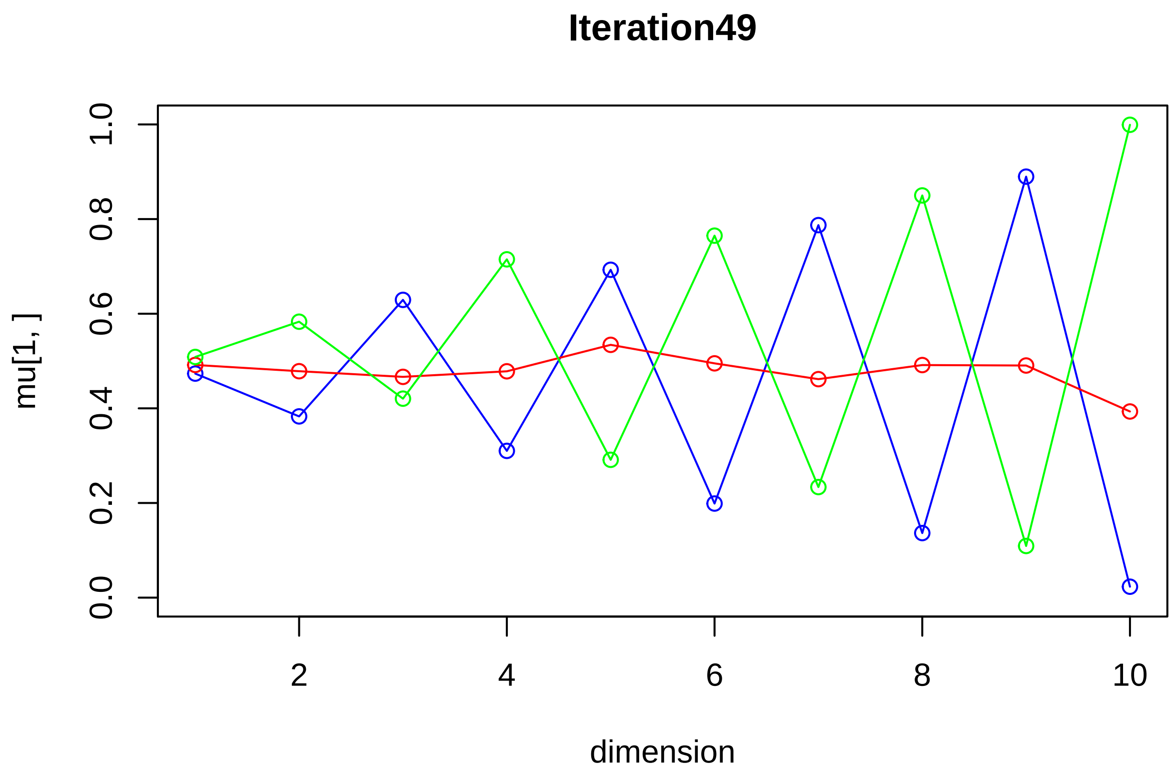

```
## iteration:  46 log likelihood:  -6745.472
```



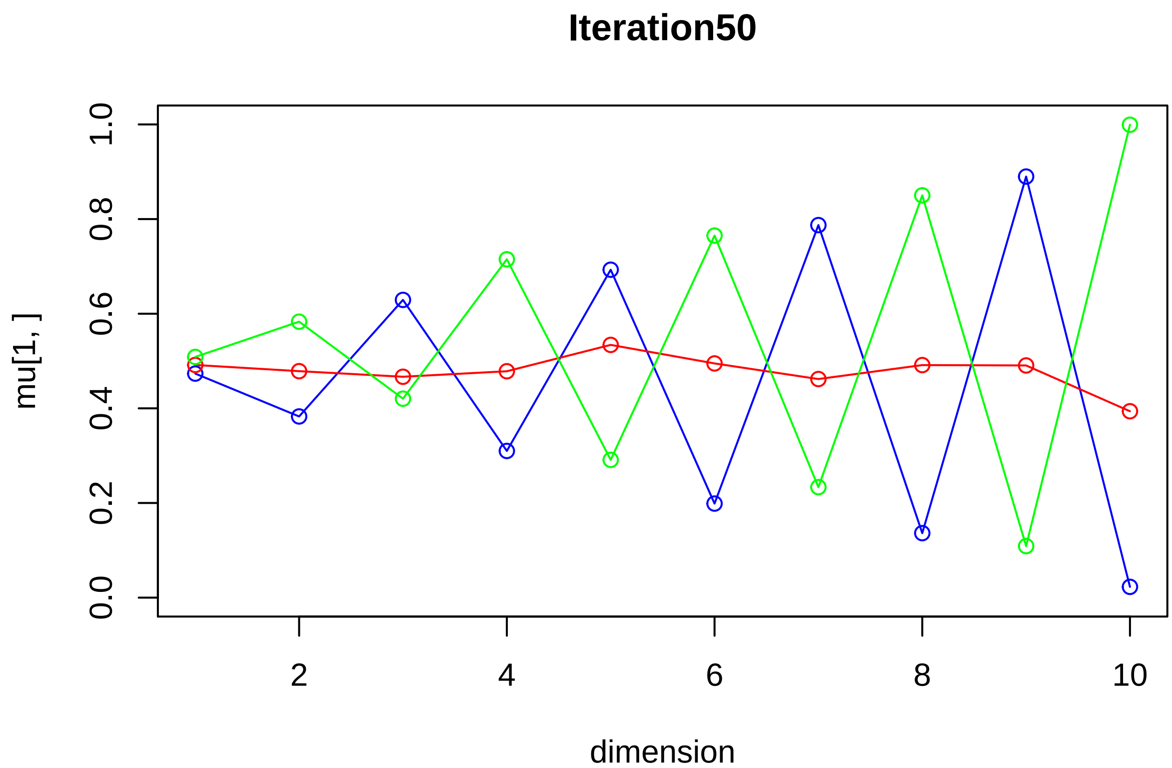

```
## iteration:  47 log likelihood:  -6745.285
```



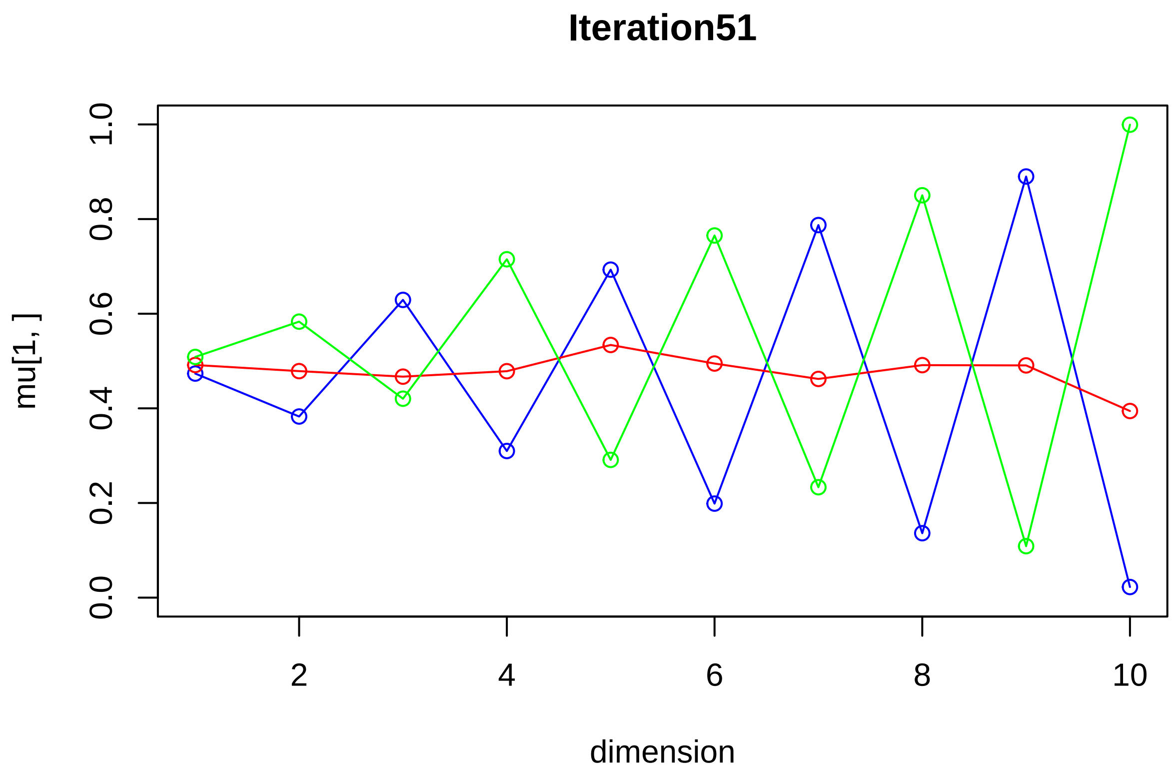

```
## iteration:  48 log likelihood:  -6745.108
```



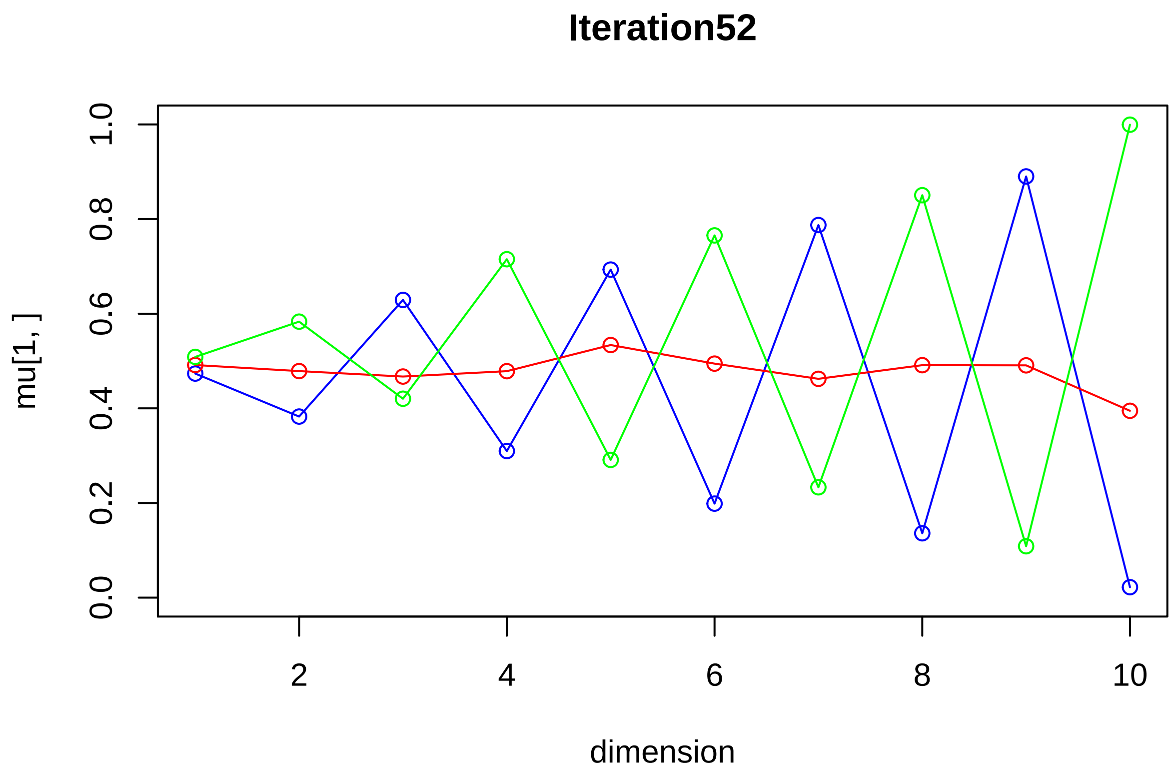

```
## iteration:  49 log likelihood:  -6744.939
```



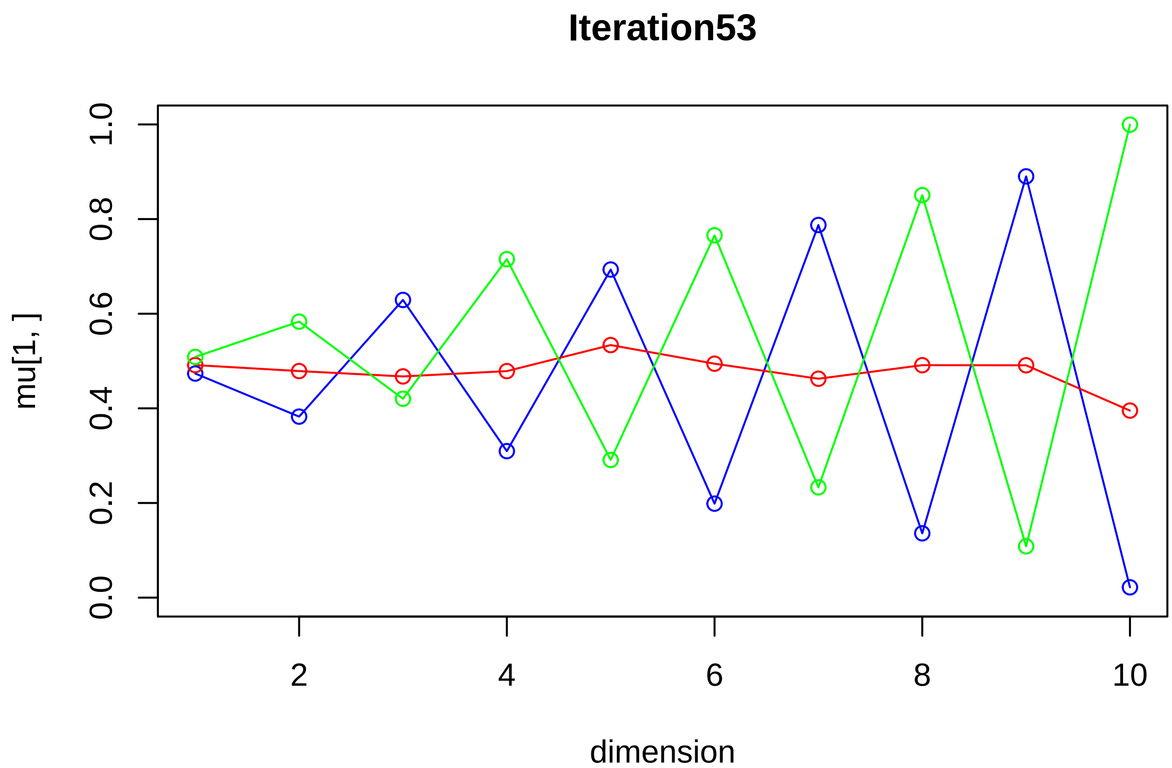

```
## iteration:  50 log likelihood:  -6744.78
```



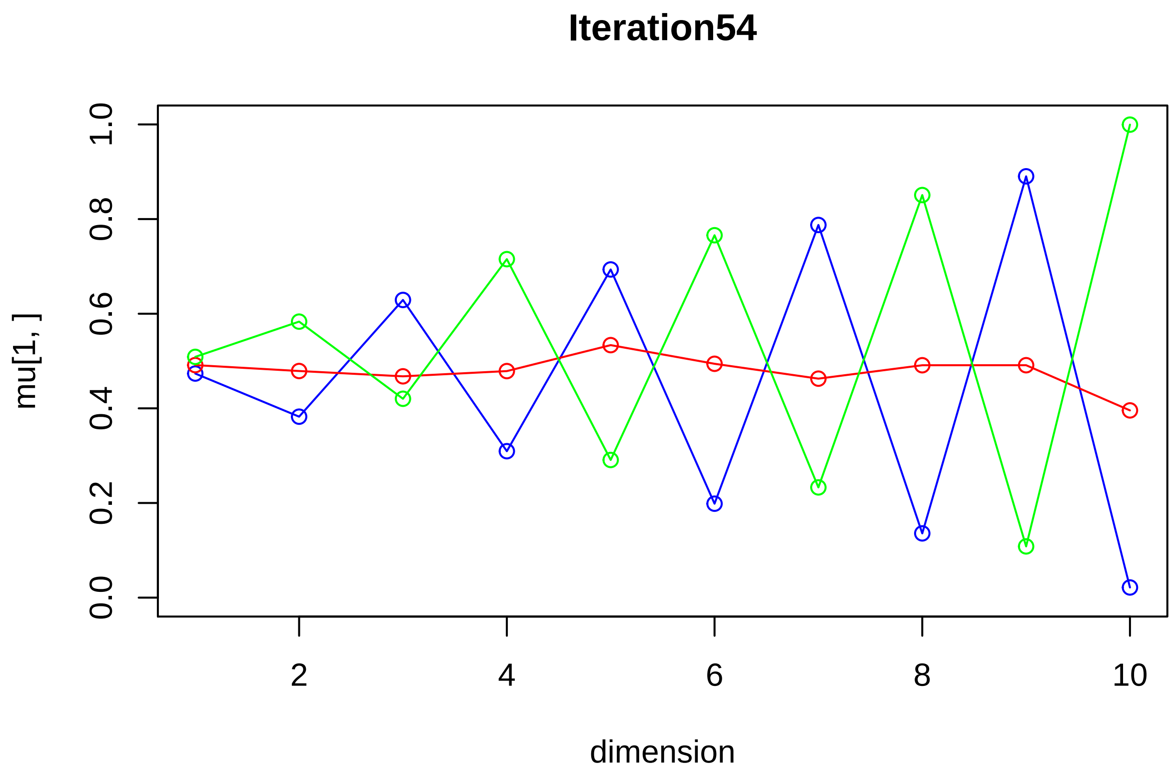

```
## iteration:  51 log likelihood:  -6744.627
```



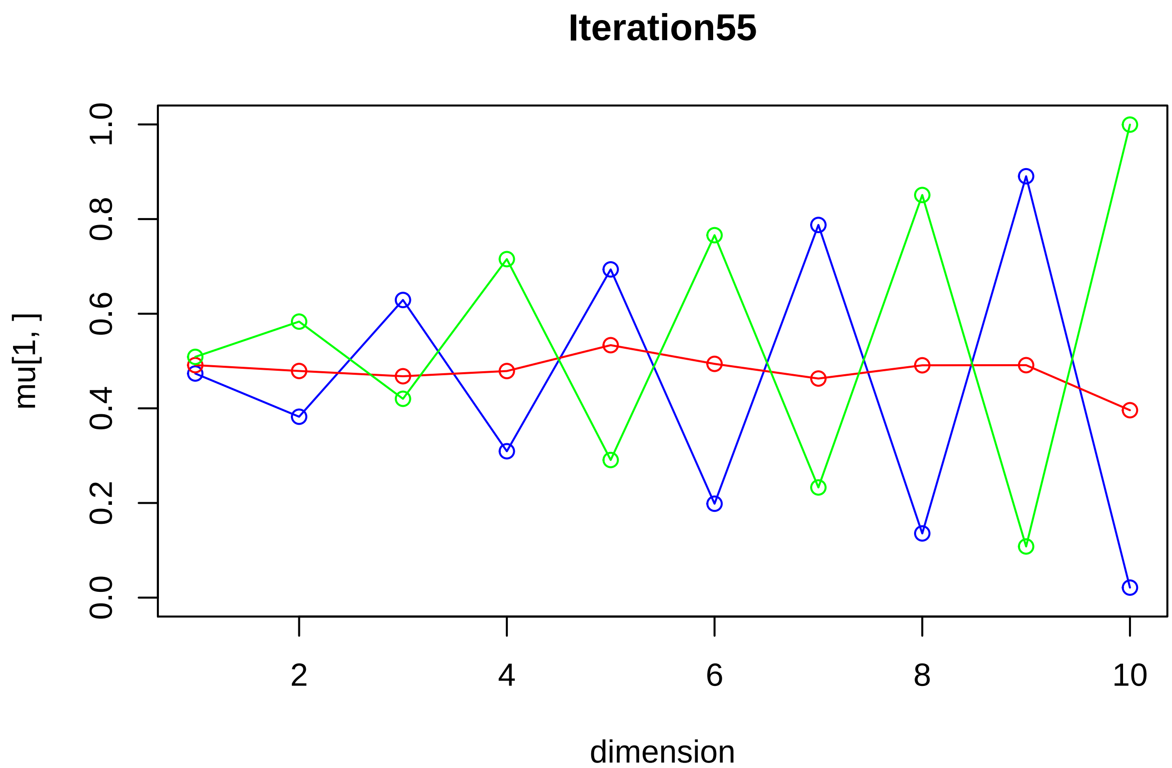

```
## iteration:  52 log likelihood:  -6744.483
```



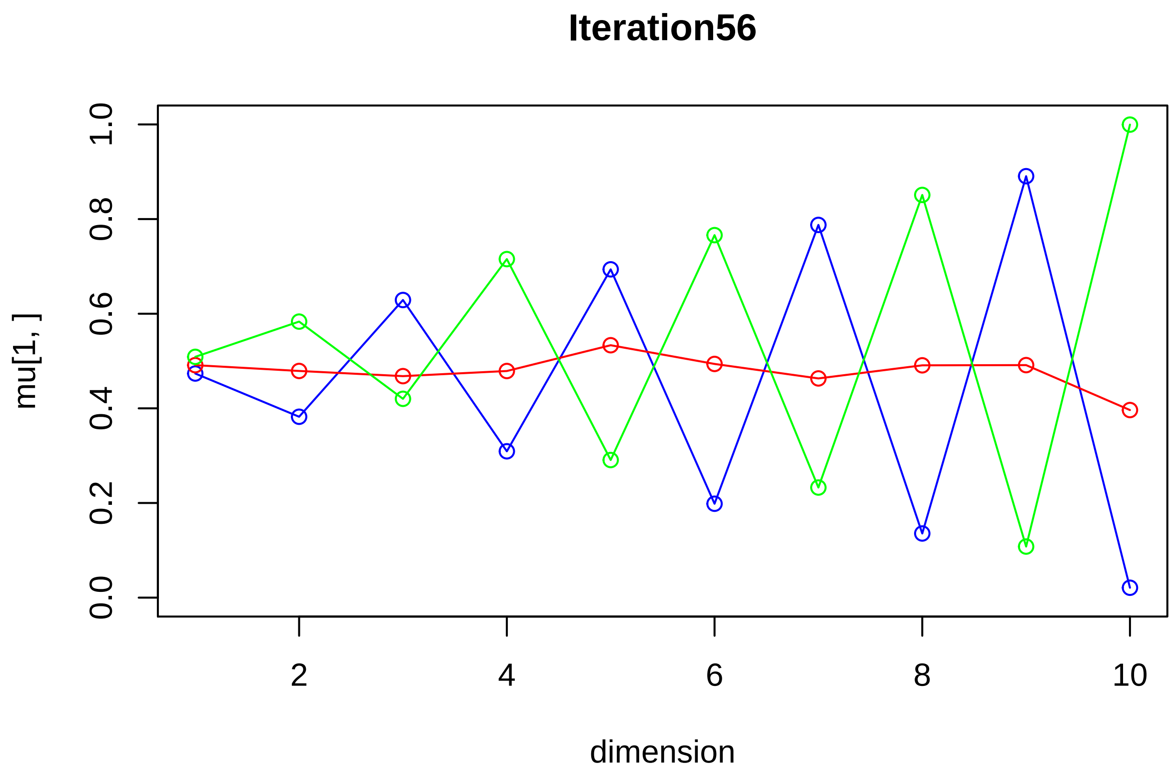

```
## iteration:  53 log likelihood:  -6744.344
```



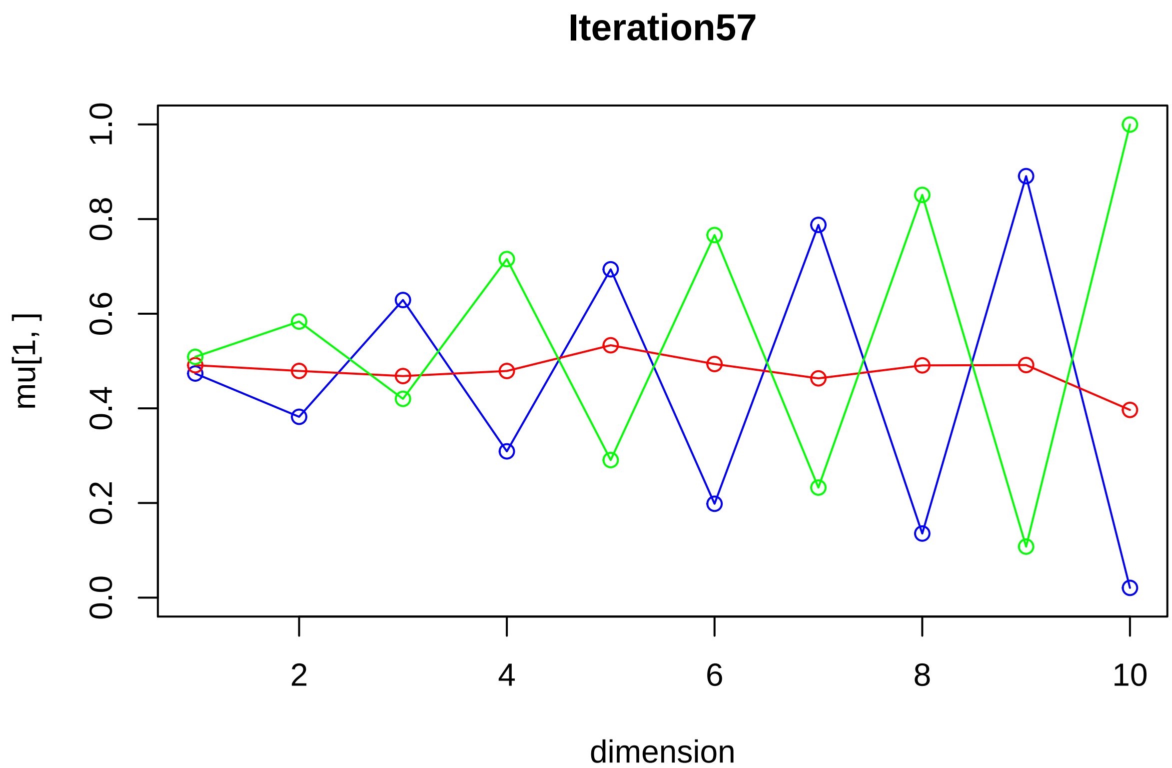

```
## iteration:  54 log likelihood:  -6744.212
```



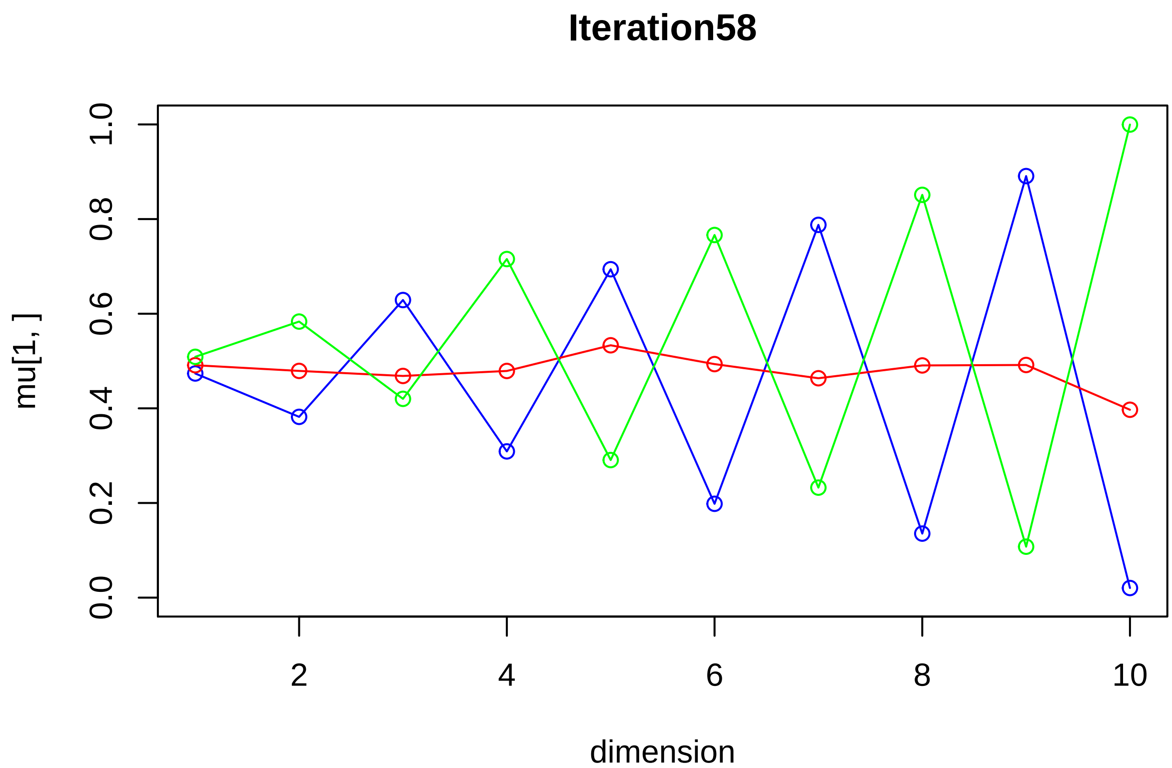

```
## iteration:  55 log likelihood:  -6744.086
```



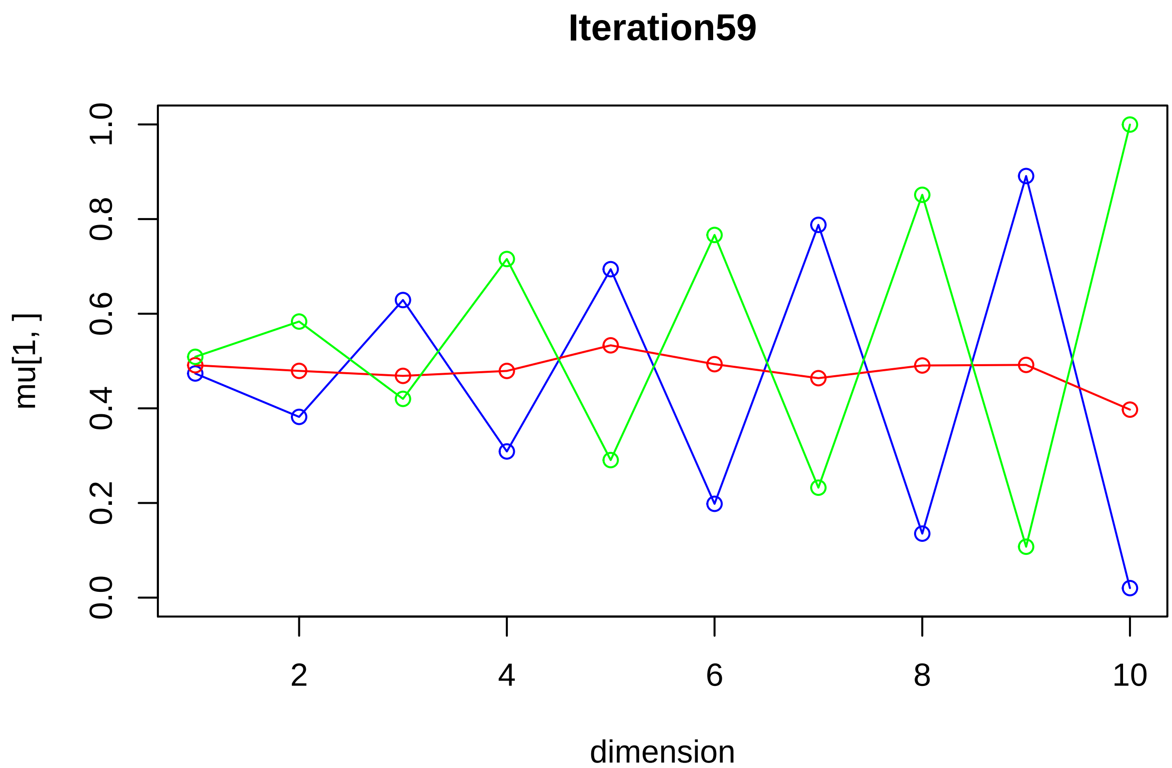

```
## iteration:  56 log likelihood:  -6743.964
```



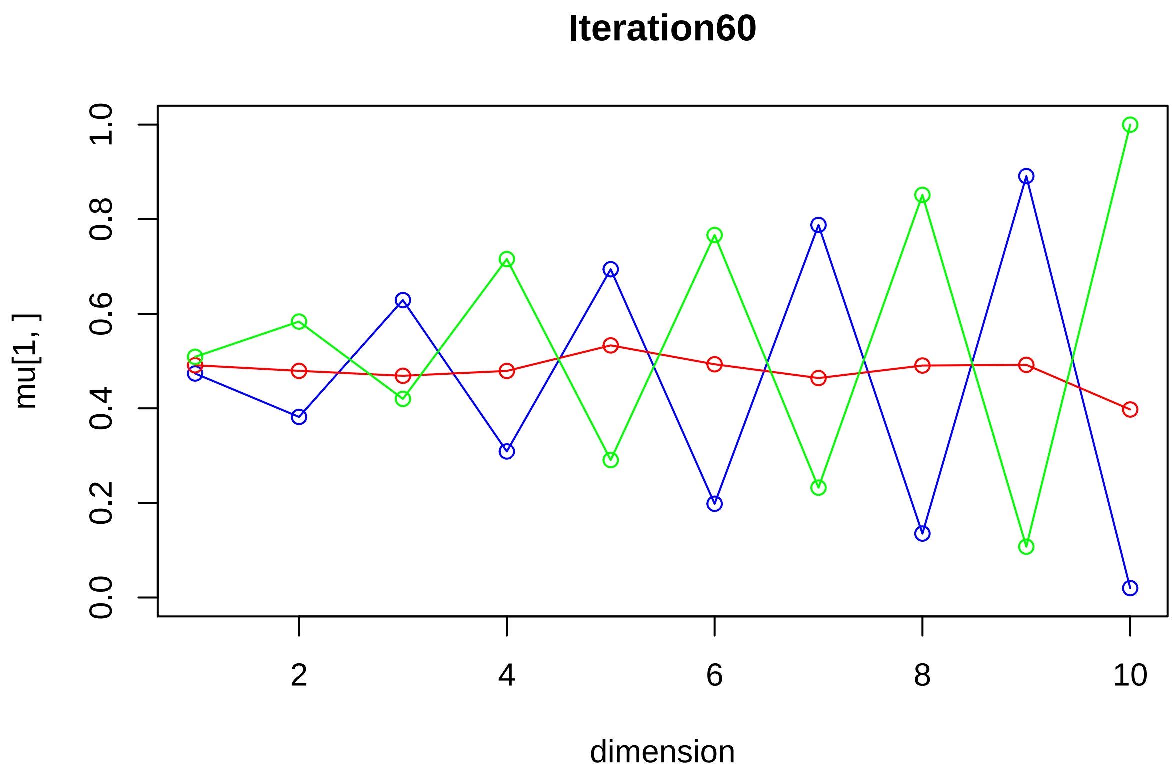

```
## iteration:  57 log likelihood:  -6743.848
```



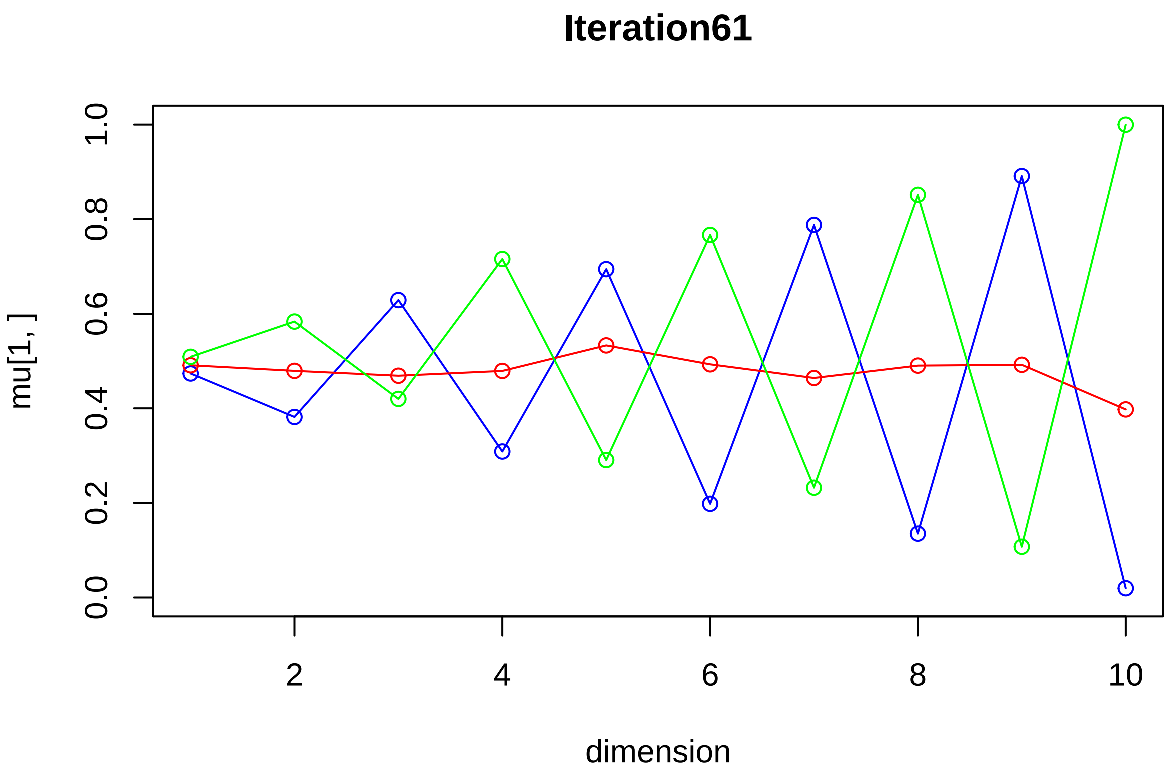

```
## iteration:  58 log likelihood:  -6743.736
```



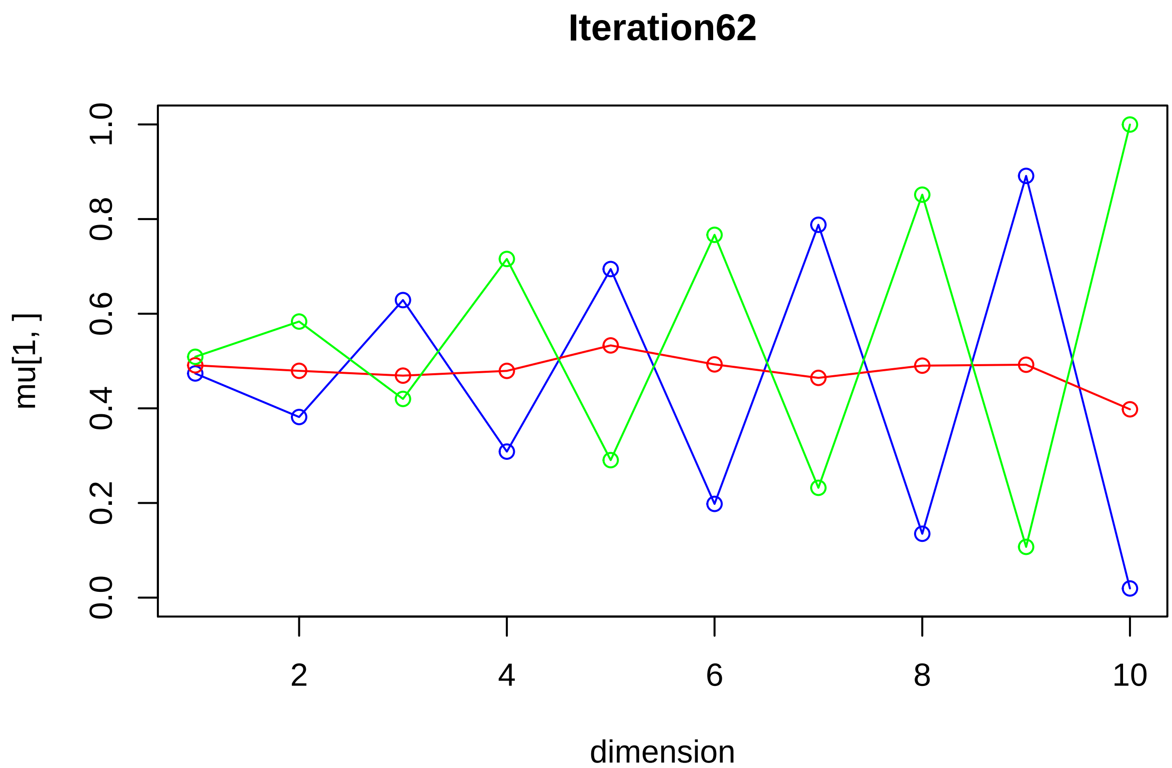

```
## iteration:  59 log likelihood:  -6743.628
```



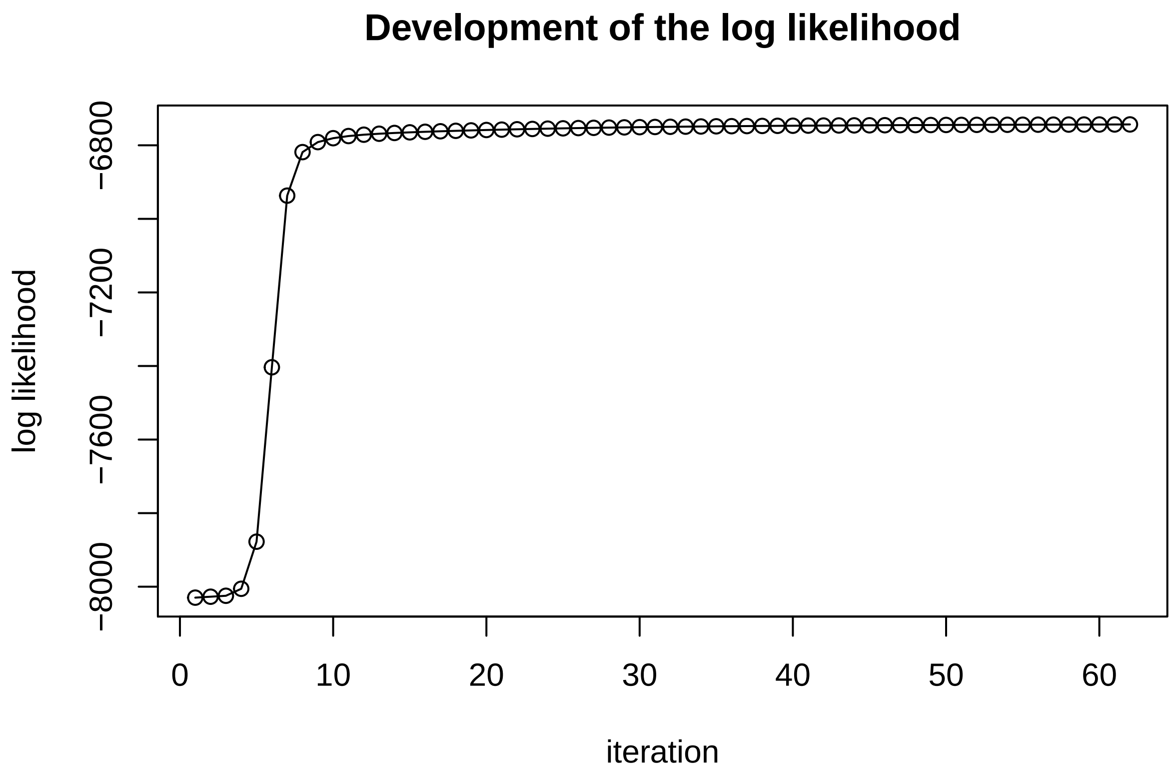

```
## iteration:  60 log likelihood:  -6743.524
```




```
## iteration:  61 log likelihood:  -6743.423
```




```
## iteration:  62 log likelihood:  -6743.326
```

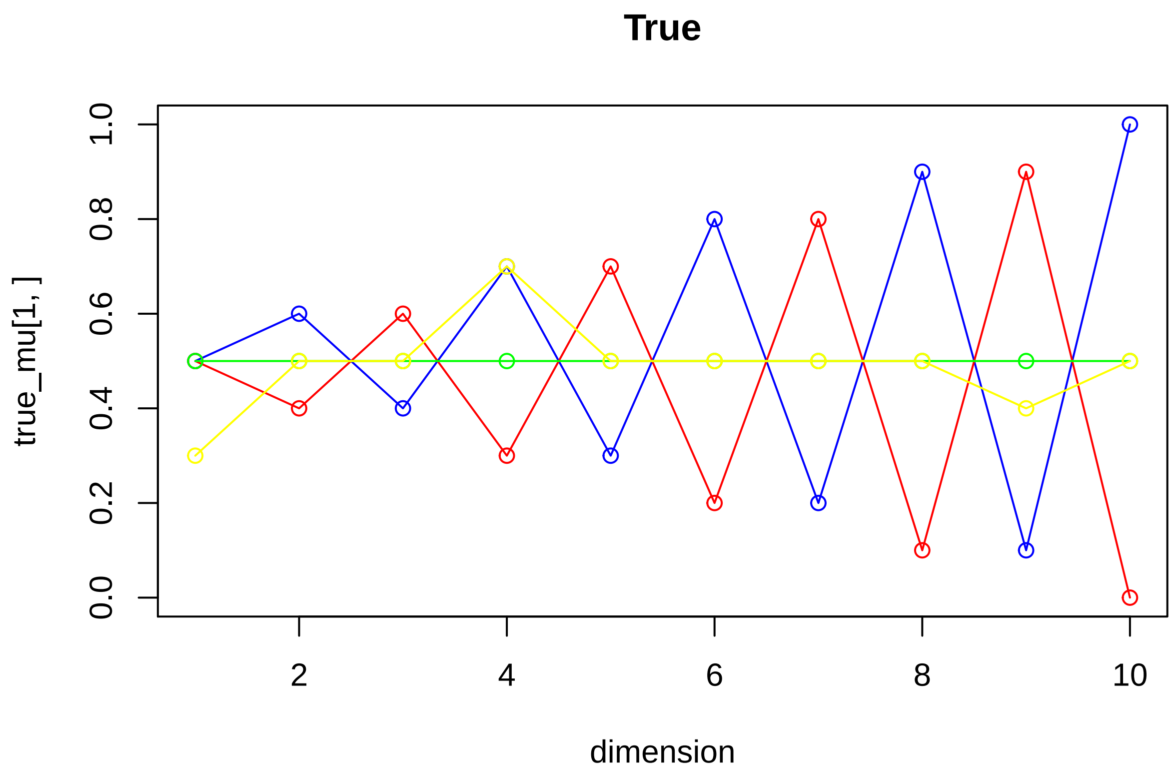


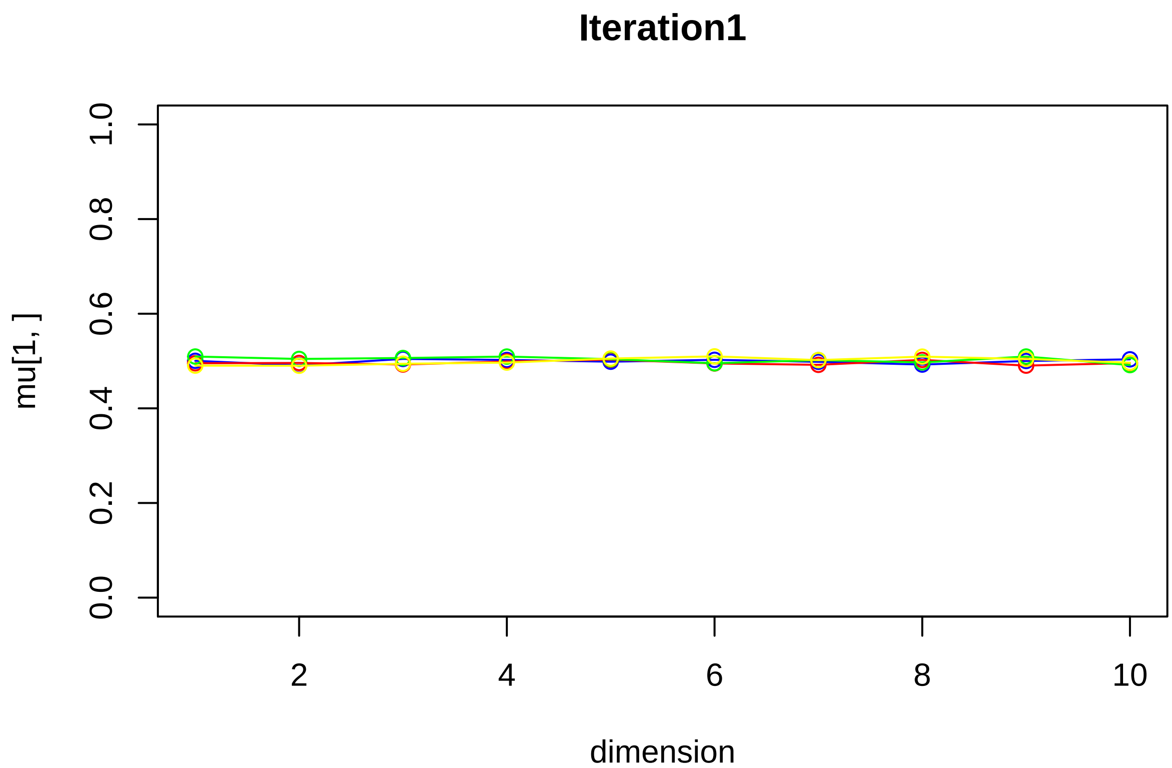

```
## $pi
## [1] 0.3259592 0.3044579 0.3695828
##
## $mu
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4737193 0.3817120 0.6288021 0.3086143 0.6943731 0.1980896 0.7879447
## [2,] 0.4909874 0.4793213 0.4691560 0.4791793 0.5329895 0.4928830 0.4643990
## [3,] 0.5089571 0.5834802 0.4199272 0.7157107 0.2905703 0.7667258 0.2320784
##           [,8]      [,9]      [,10]
## [1,] 0.1349651 0.8912534 0.01937869
## [2,] 0.4902682 0.4922194 0.39798407
## [3,] 0.8516111 0.1072226 0.99981353
##
## $logLikelihoodDevelopment
## NULL
```

2.4 K=4

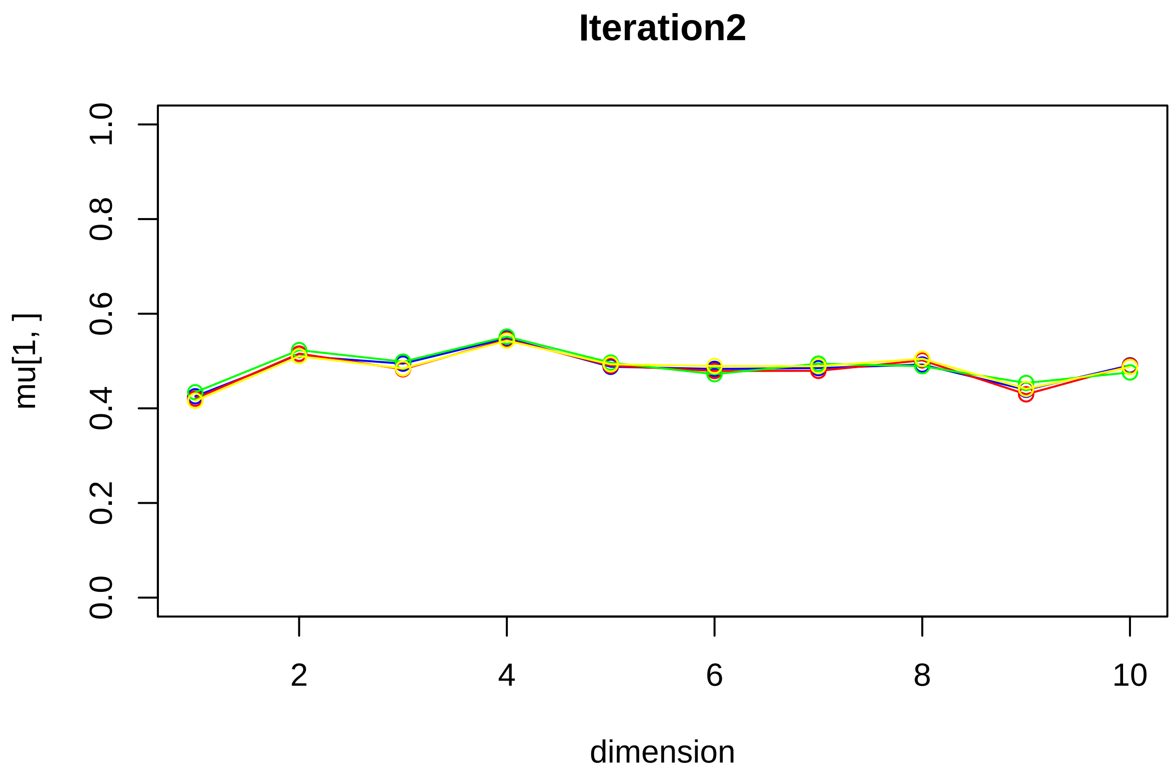
Finally, the function will be run for $K=4$.

```r
em(4)
```

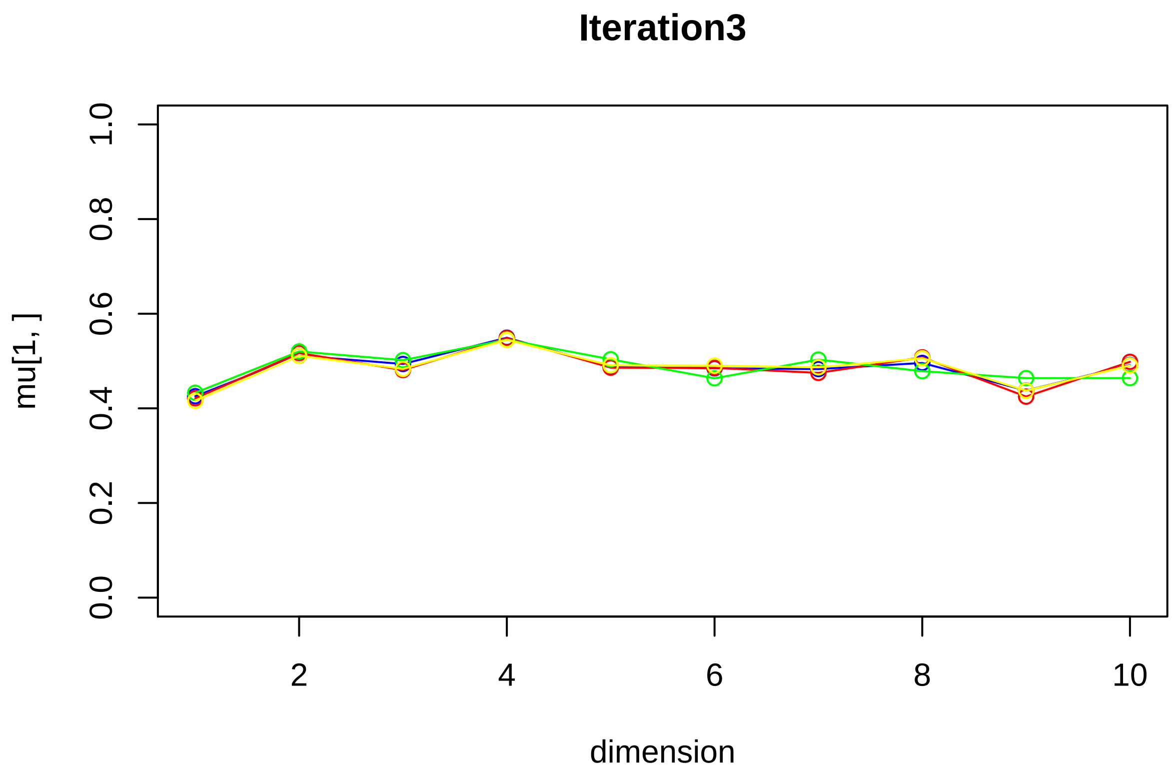

True
true_mu[1,]
0.0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
dimension



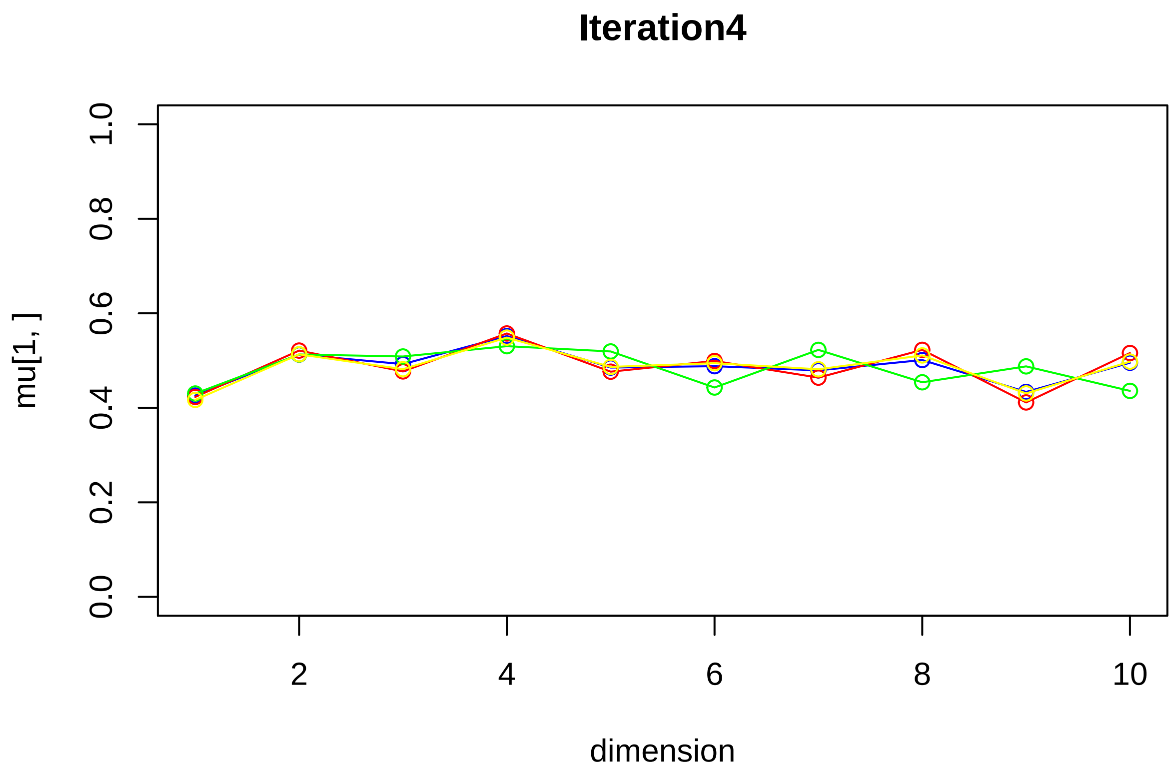

```
## iteration:  1 log likelihood:  -8316.904
```



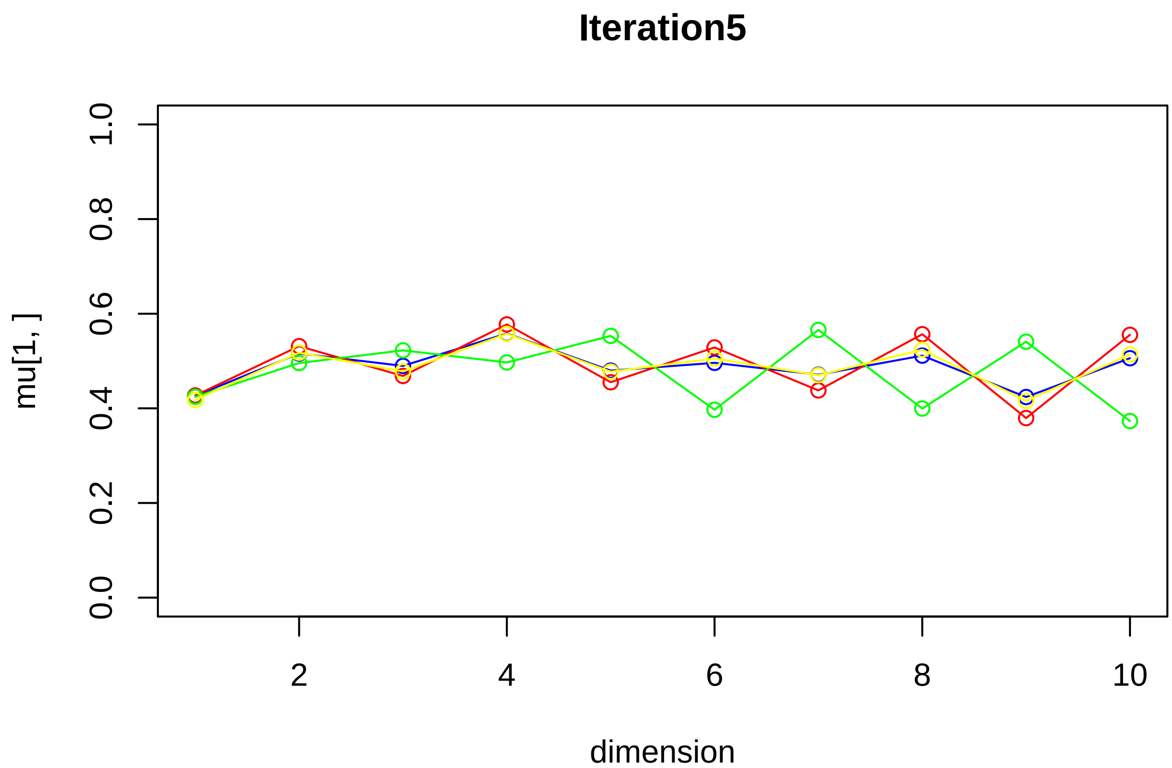

```
## iteration:  2 log likelihood:  -8291.114
```



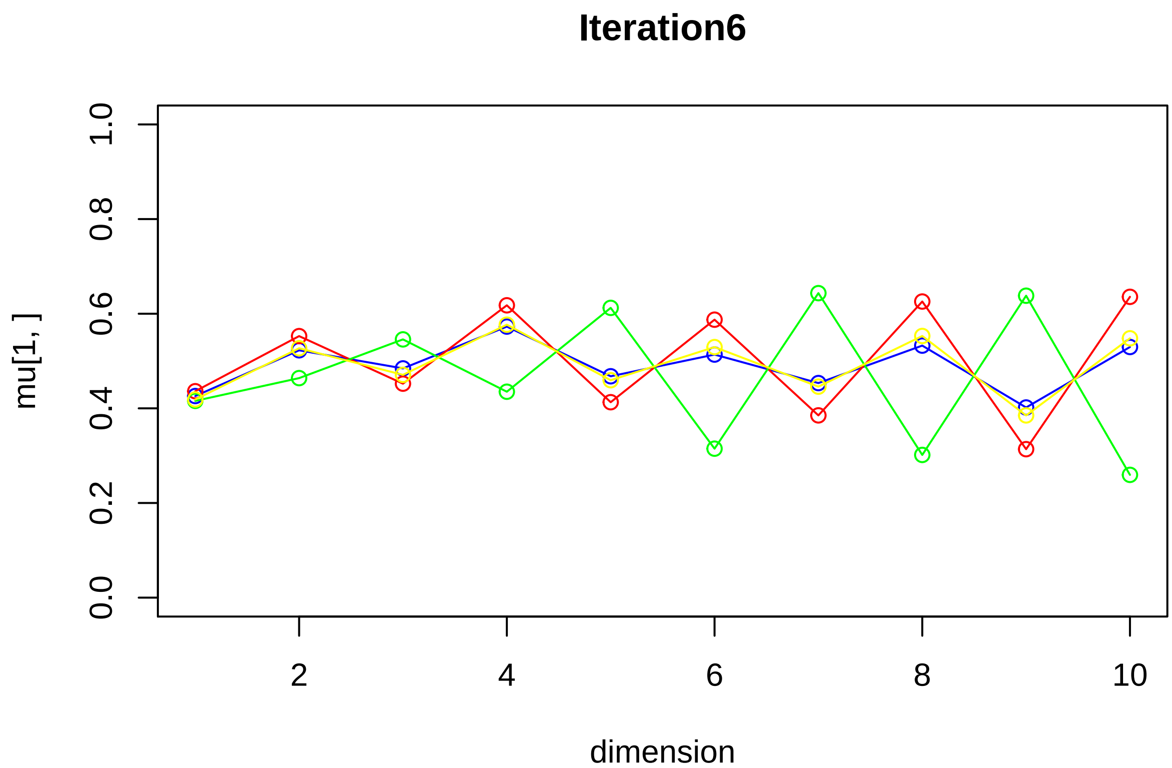

```
## iteration:  3 log likelihood:  -8286.966
```



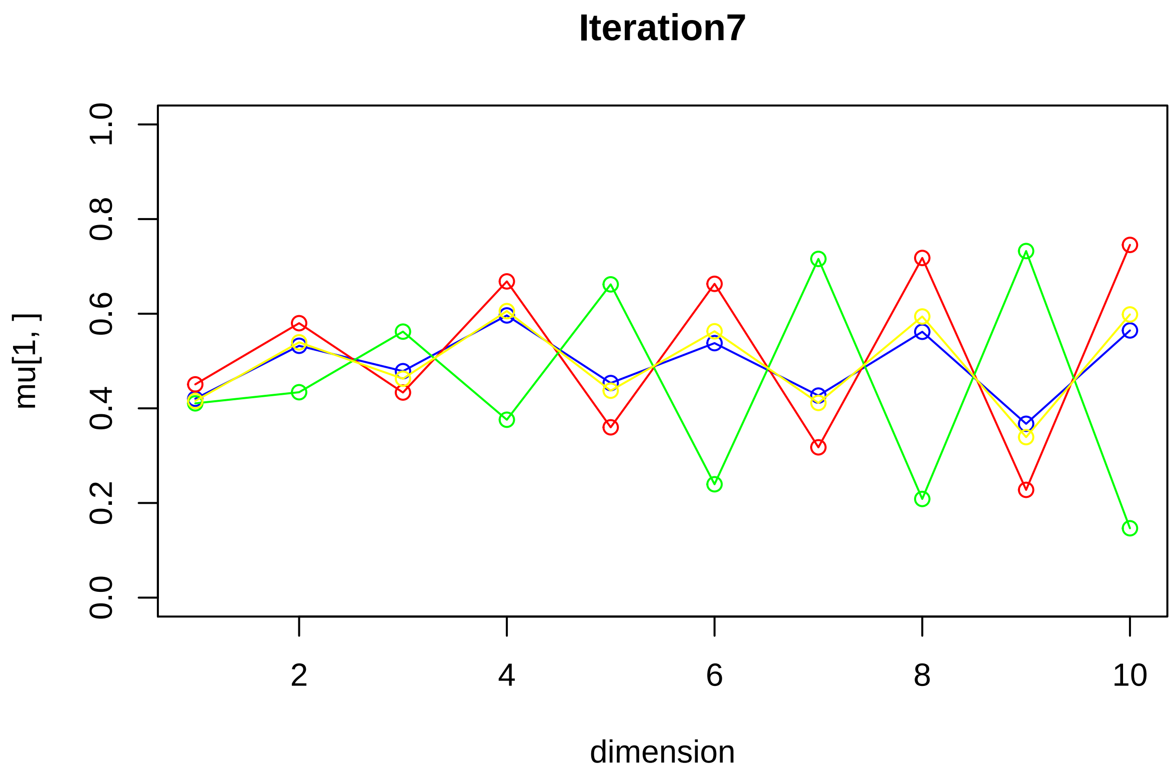

```
## iteration:  4 log likelihood:  -8264.806
```



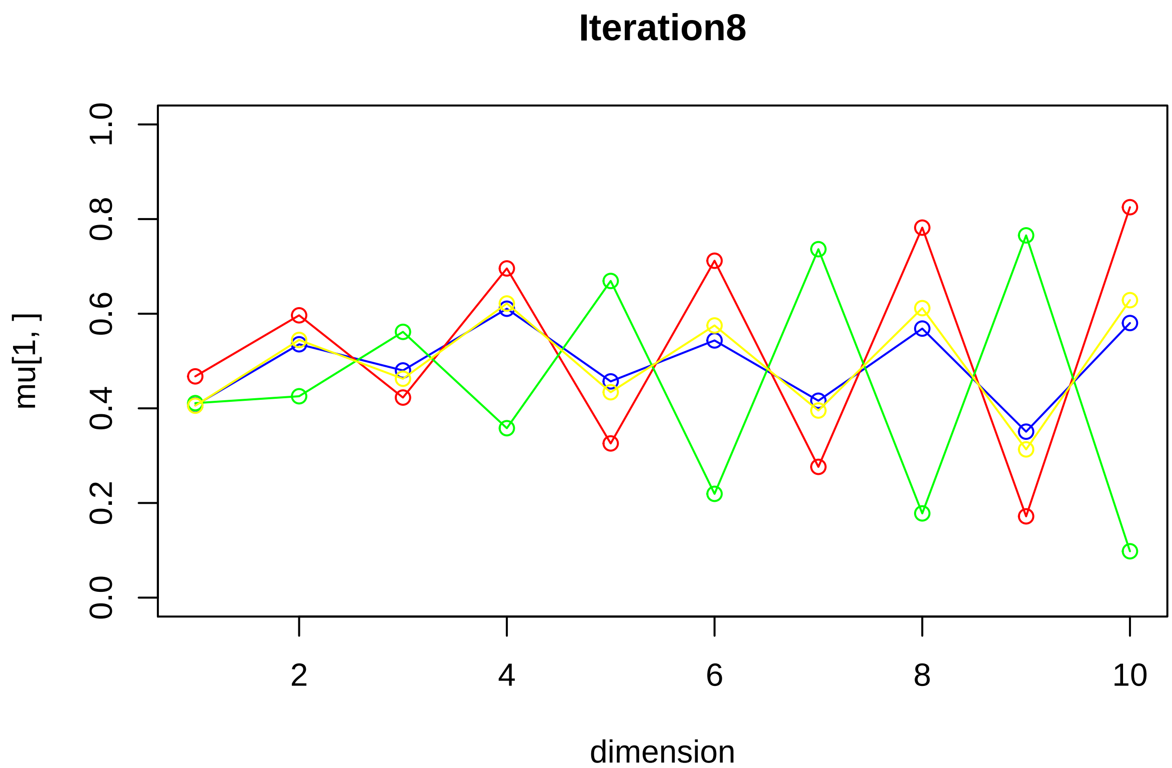

```
## iteration:  5 log likelihood:  -8161.19
```



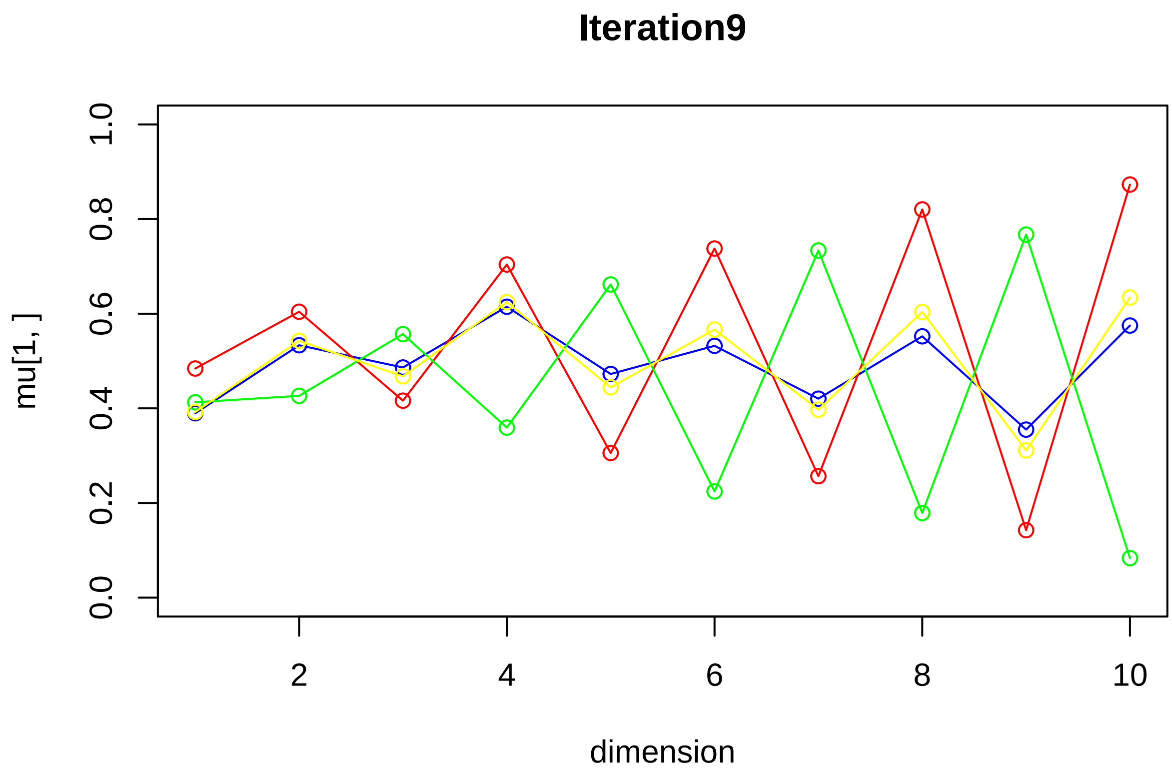

```
## iteration:  6 log likelihood:  -7868.89
```



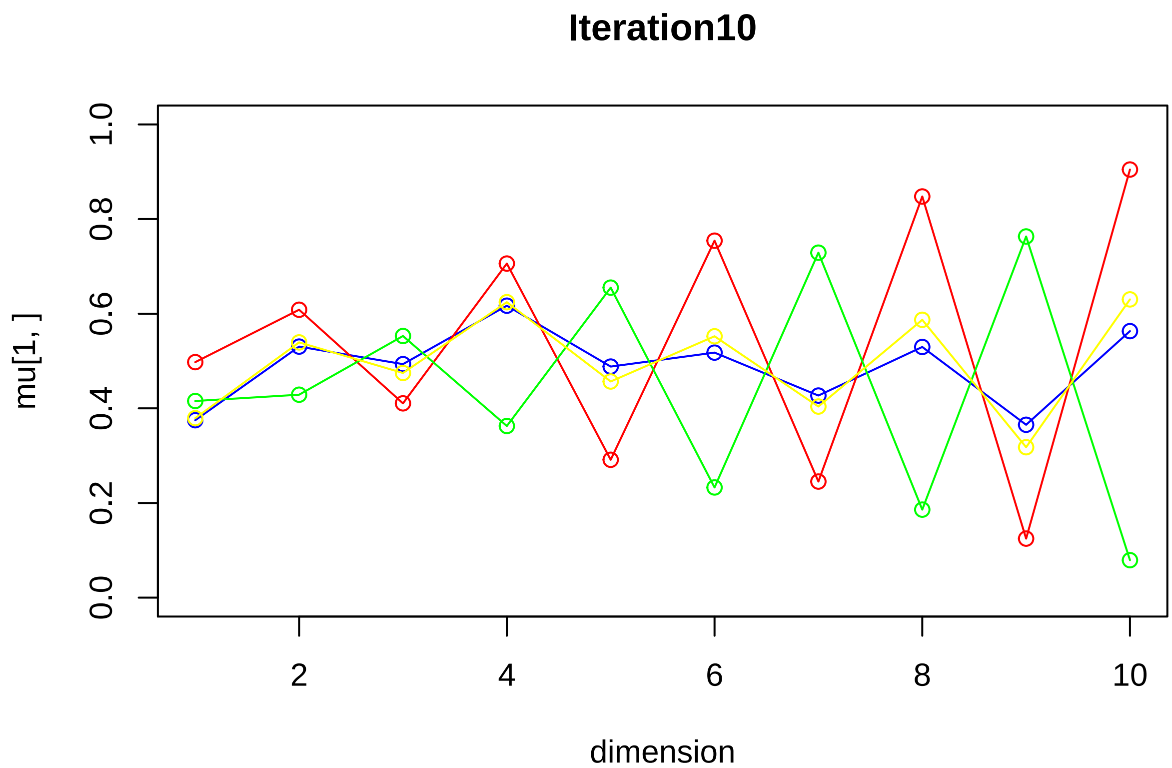

```
## iteration:  7 log likelihood:  -7570.873
```



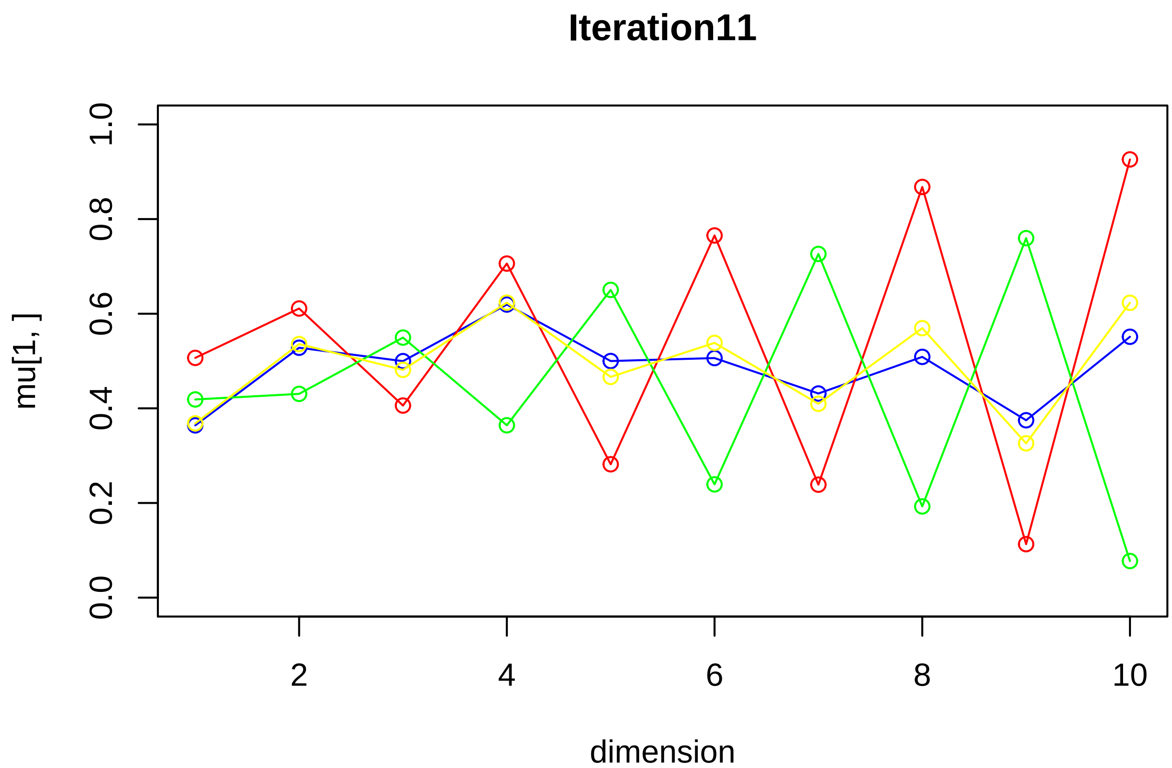

```
## iteration:  8 log likelihood:  -7445.719
```



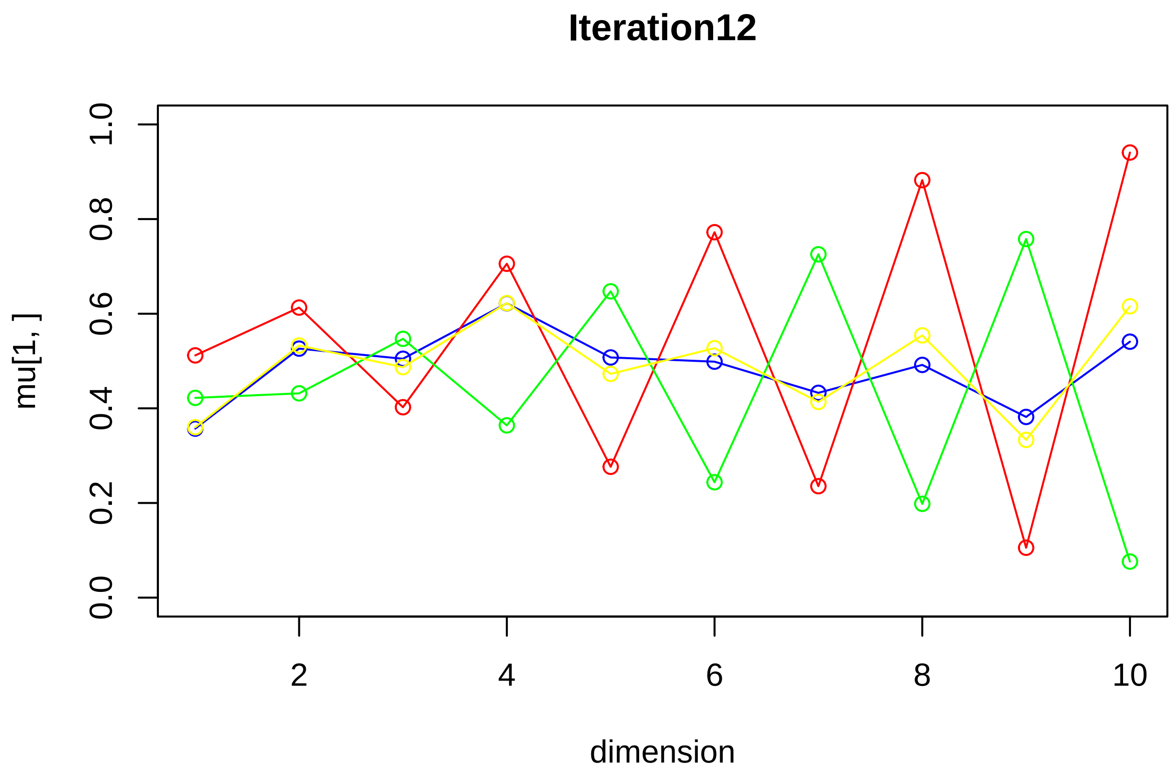

```
## iteration:  9 log likelihood:  -7389.741
```



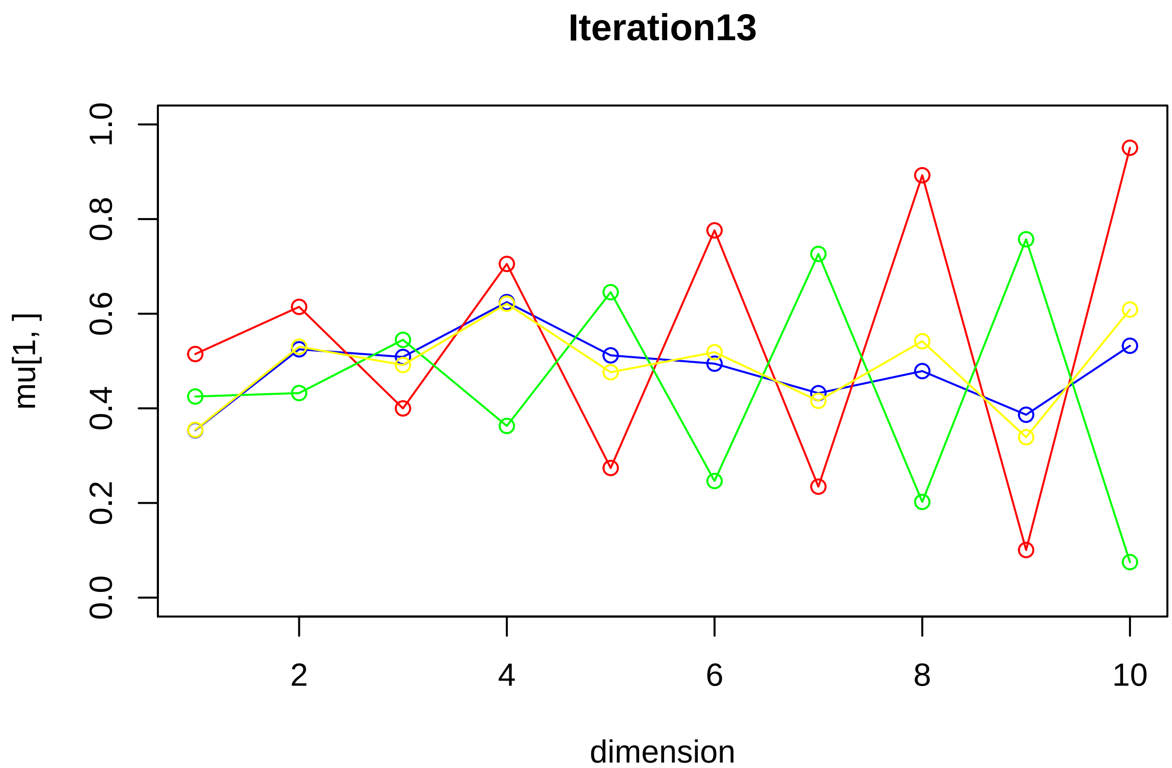

```
## iteration:  10 log likelihood:  -7356.803
```



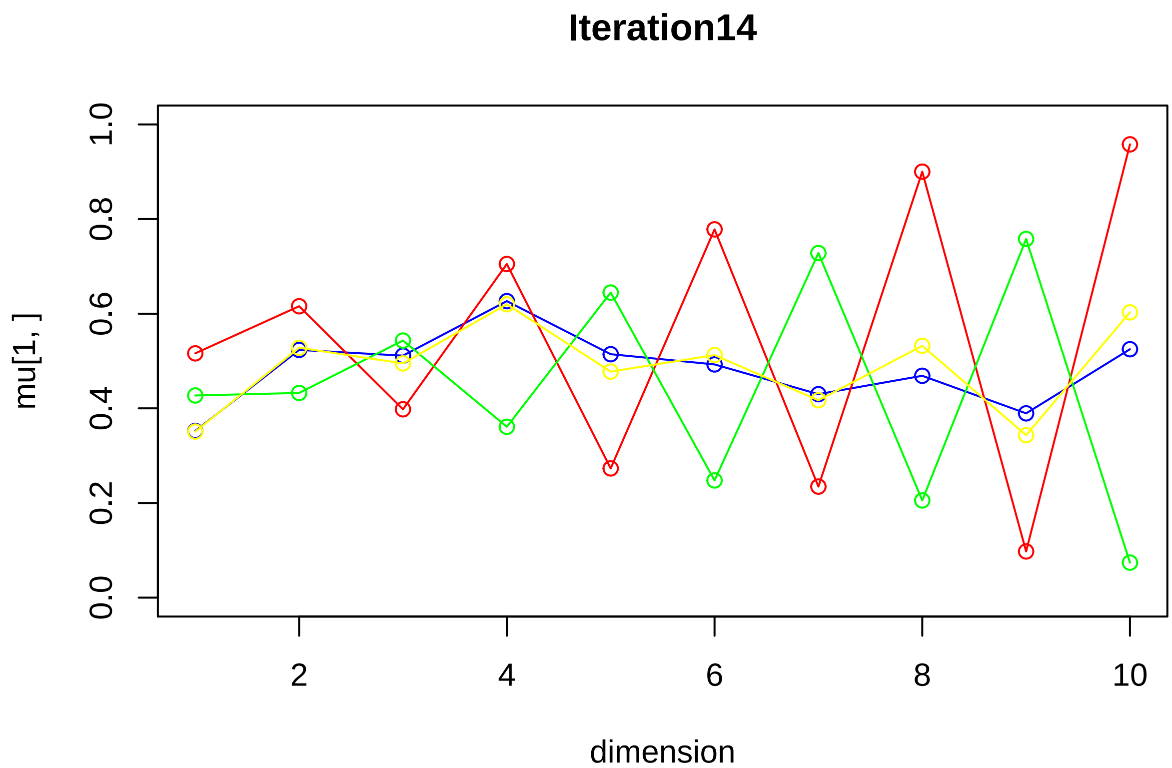

```
## iteration:  11 log likelihood:  -7337.208
```



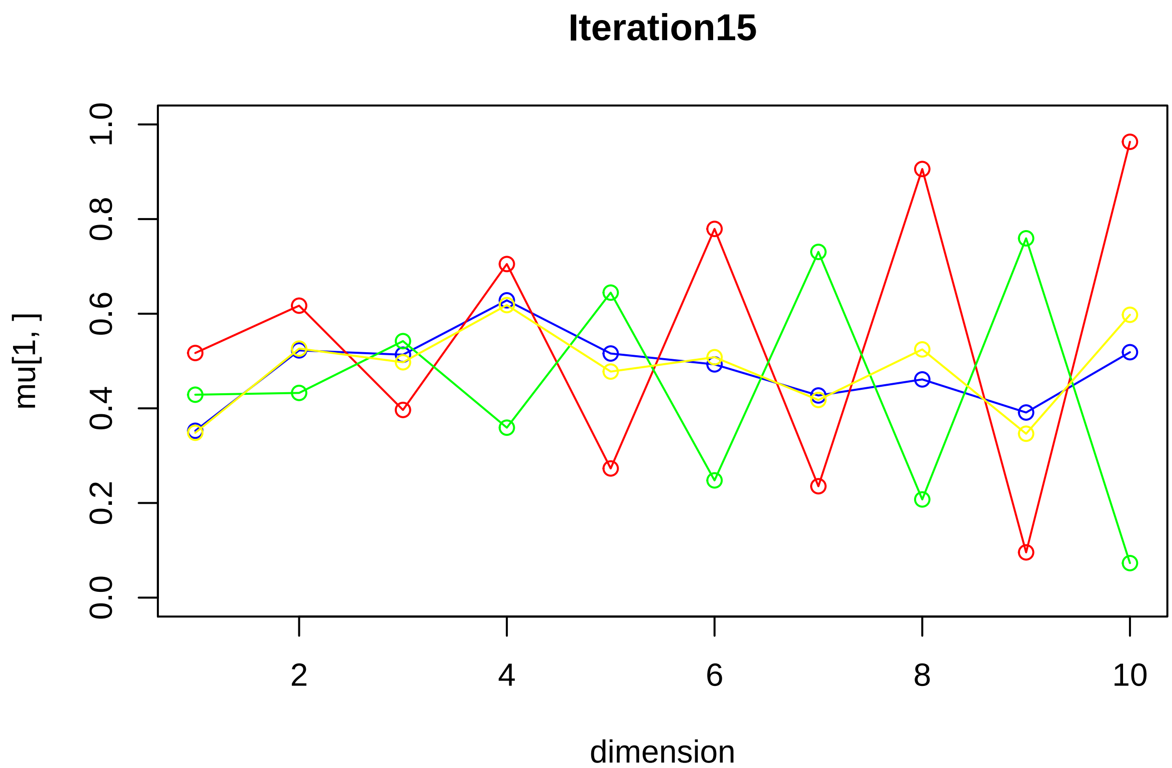

```
## iteration:  12 log likelihood:  -7326.118
```



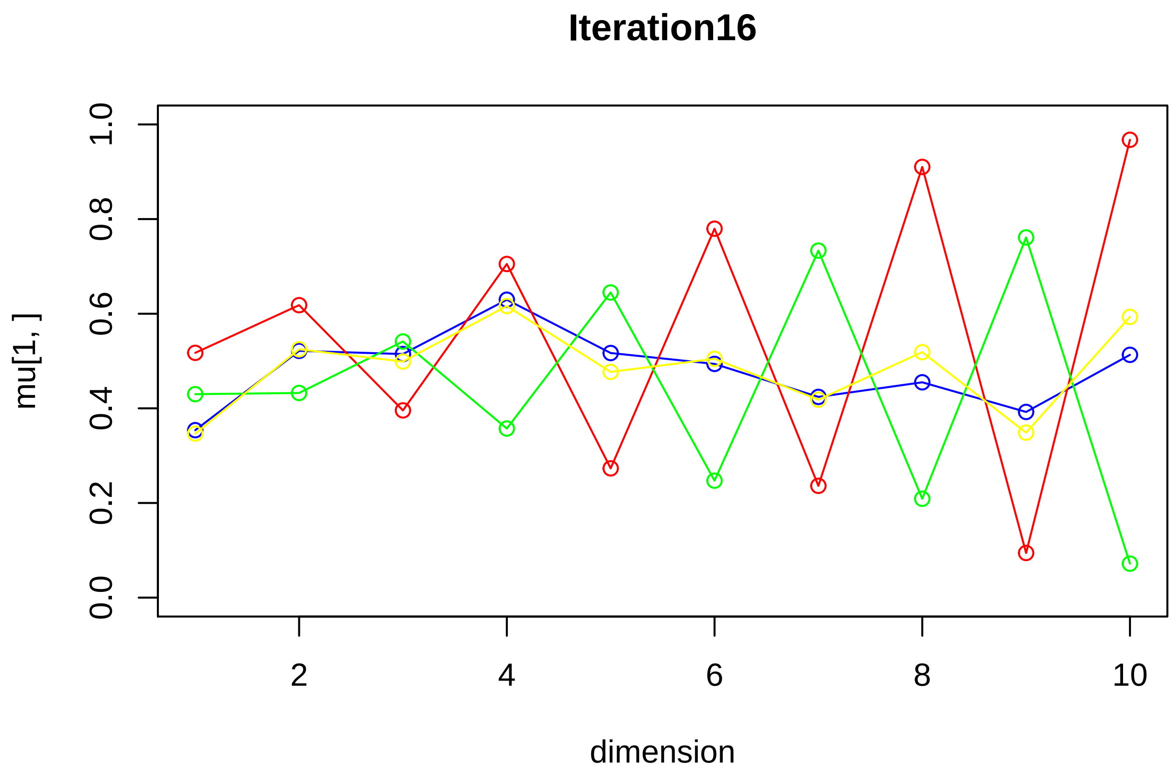

```
## iteration:  13 log likelihood:  -7319.998
```



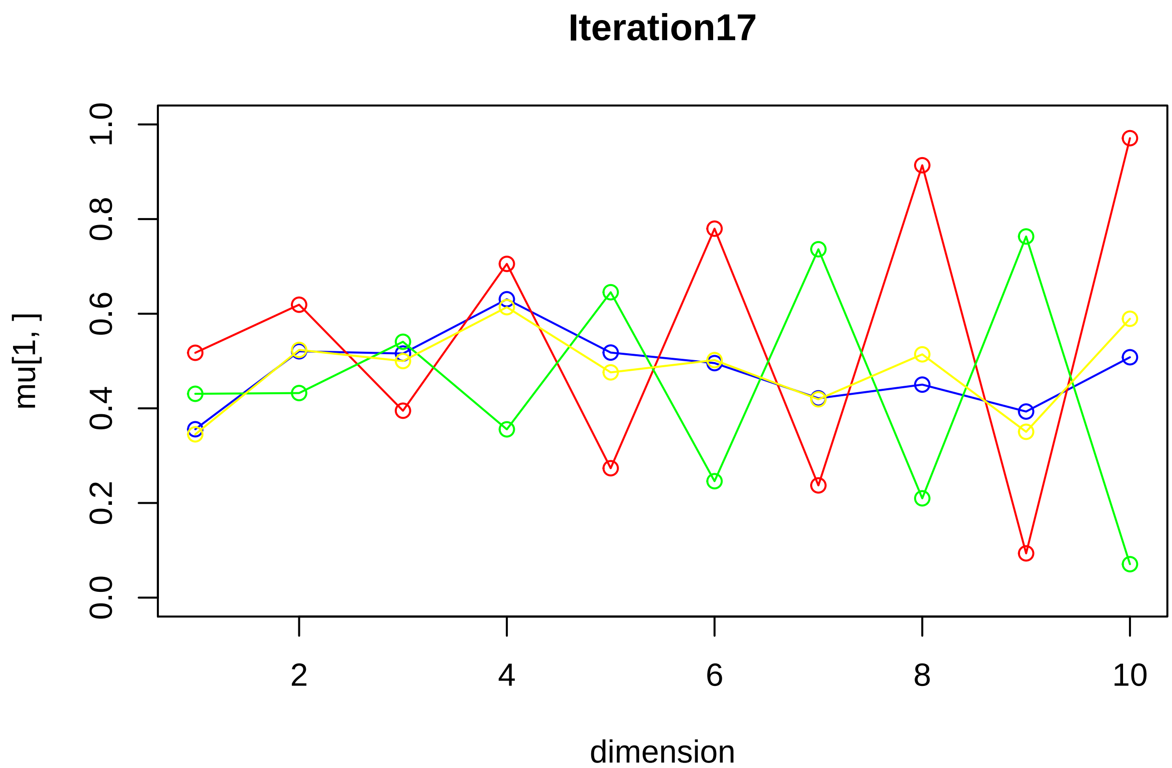

```
## iteration:  14 log likelihood:  -7316.6
```



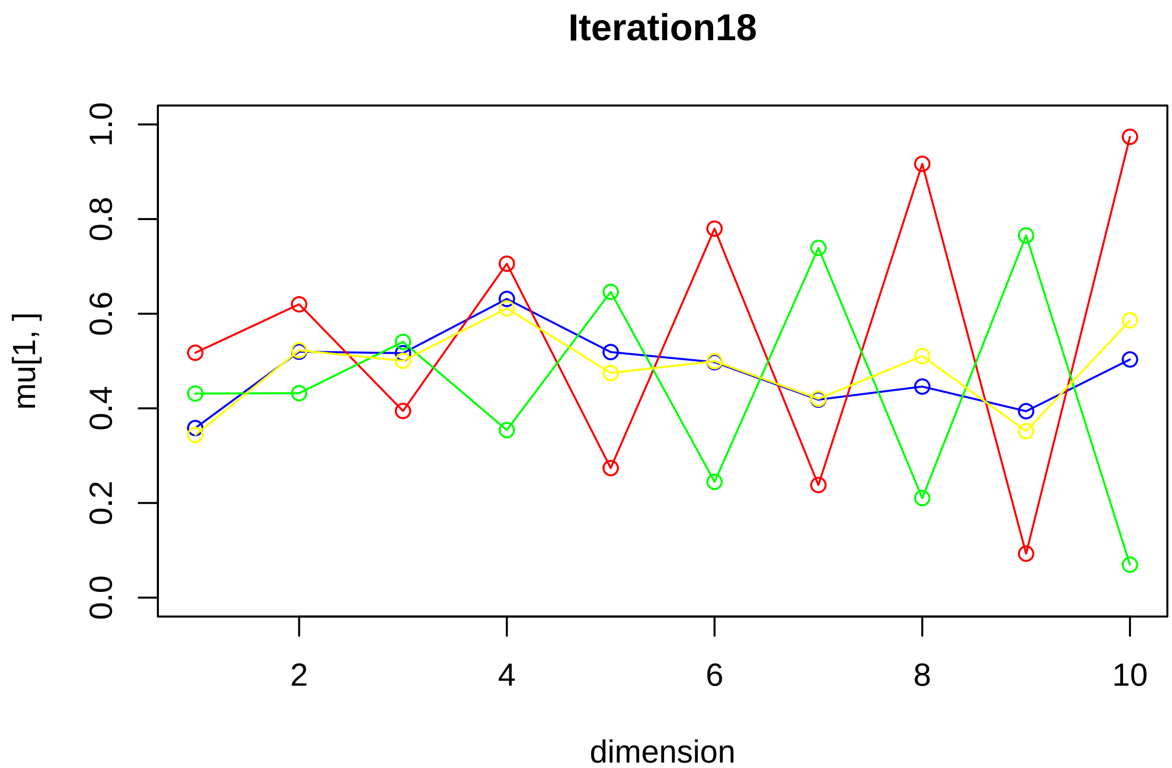

```
## iteration:  15 log likelihood:  -7314.666
```



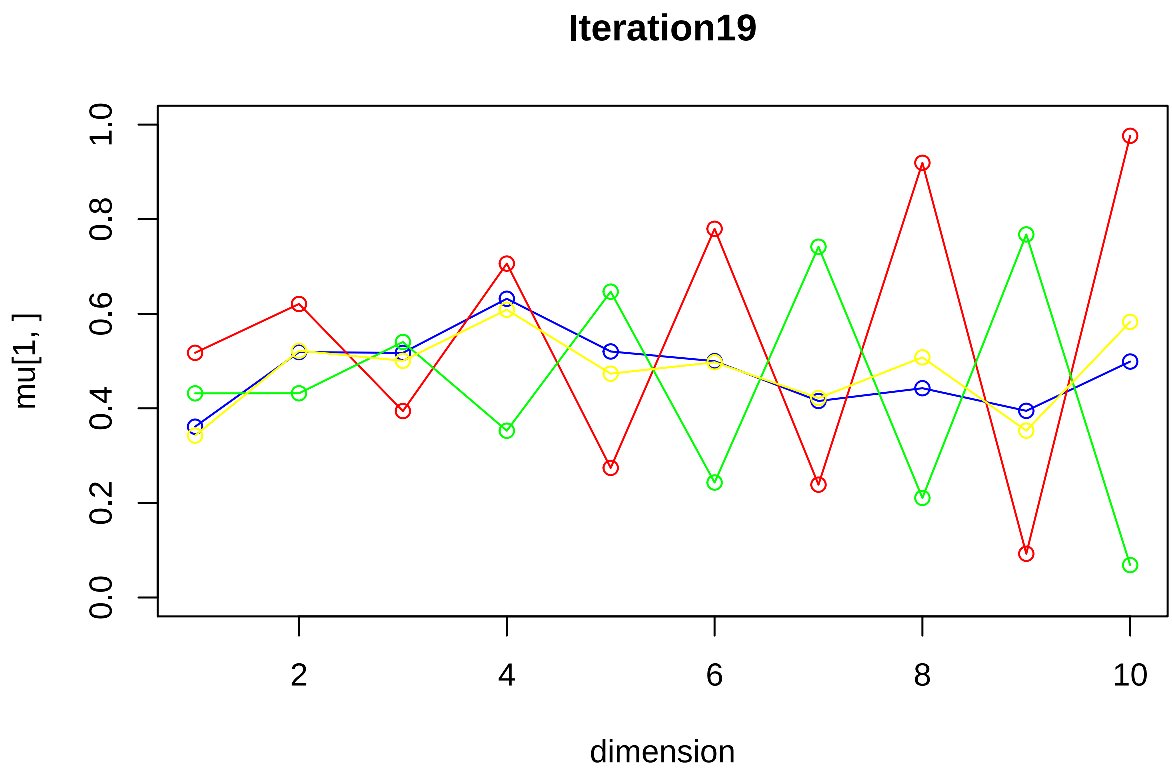

```
## iteration:  16 log likelihood:  -7313.528
```



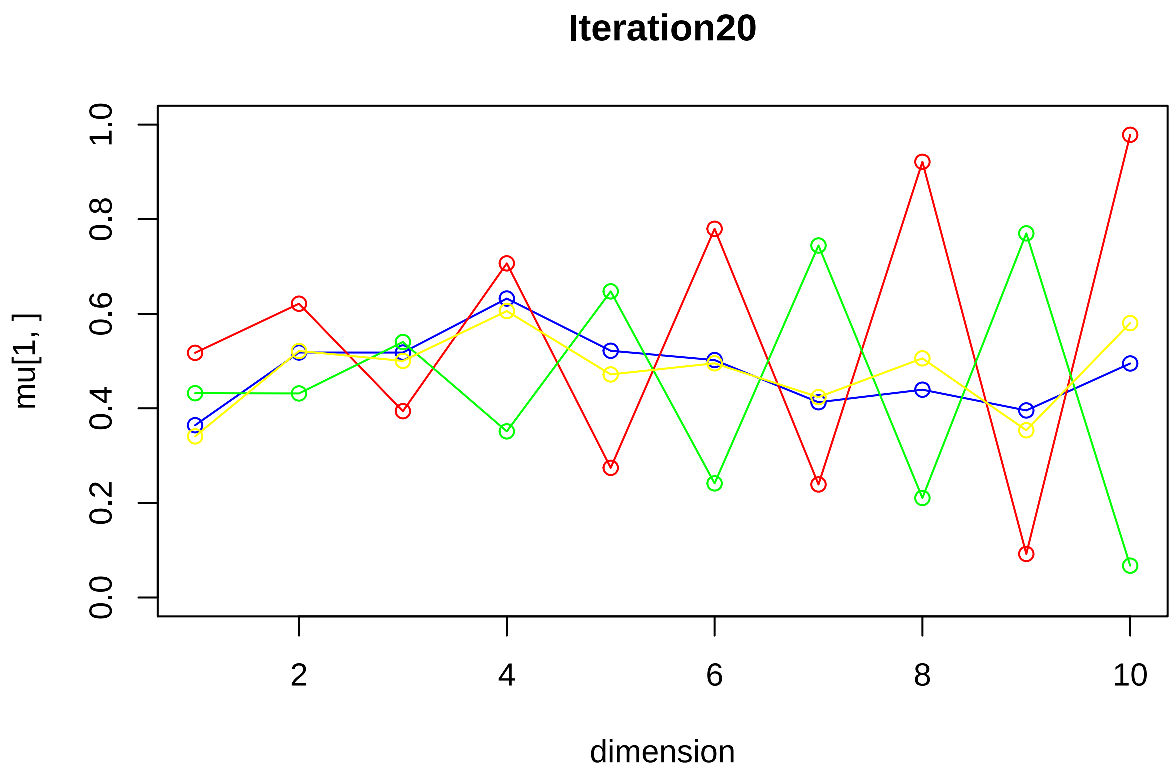

```
## iteration:  17 log likelihood:  -7312.829
```



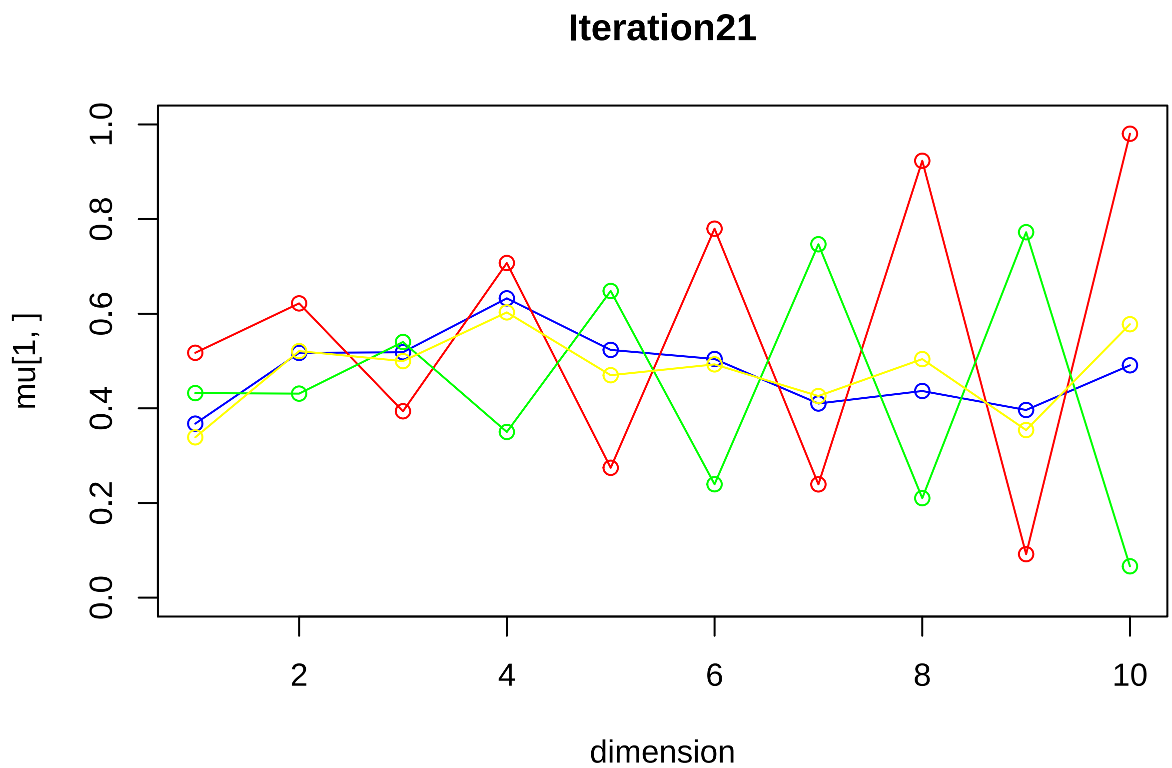

```
## iteration:  18 log likelihood:  -7312.367
```



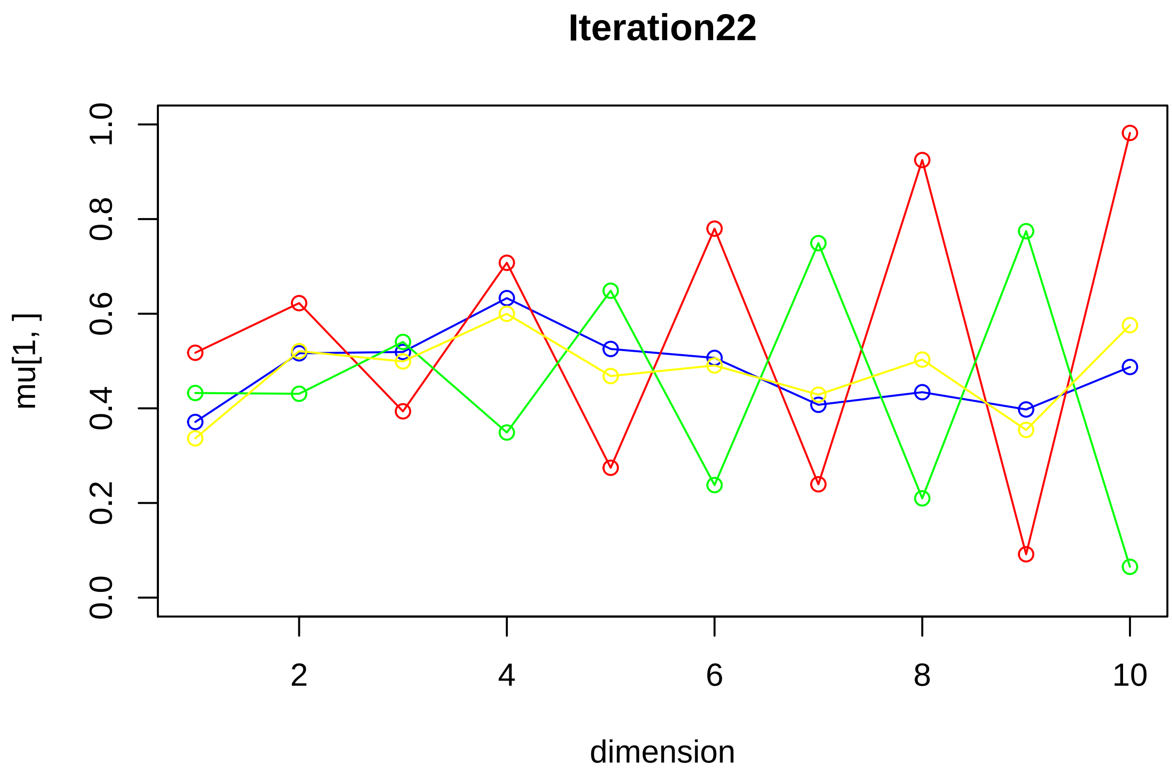

```
## iteration:  19 log likelihood:  -7312.024
```



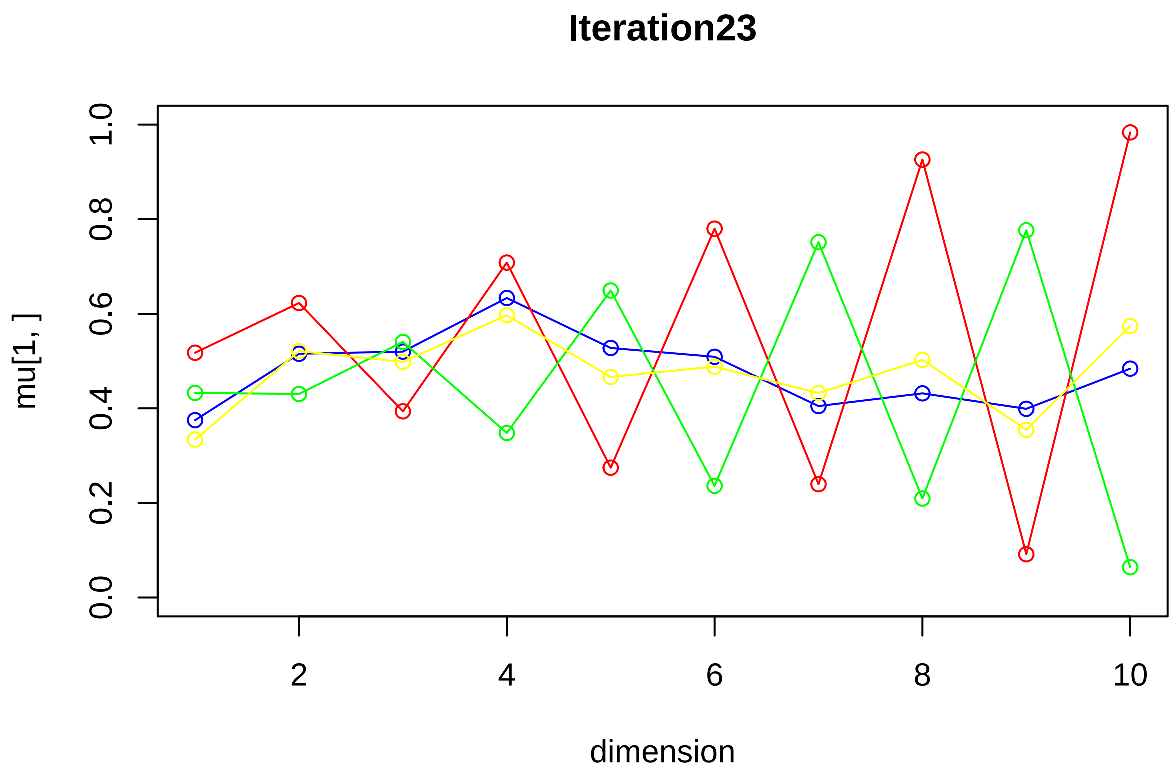

```
## iteration:  20 log likelihood:  -7311.723
```



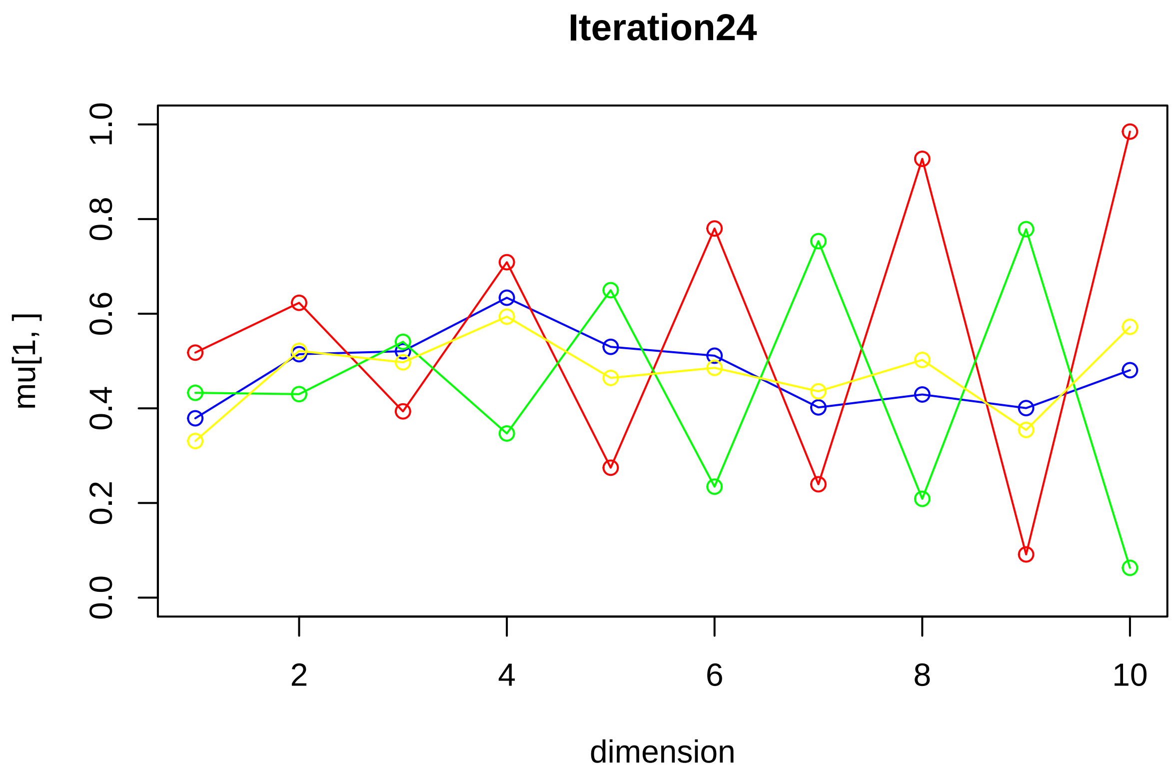

```
## iteration:  21 log likelihood:  -7311.407
```



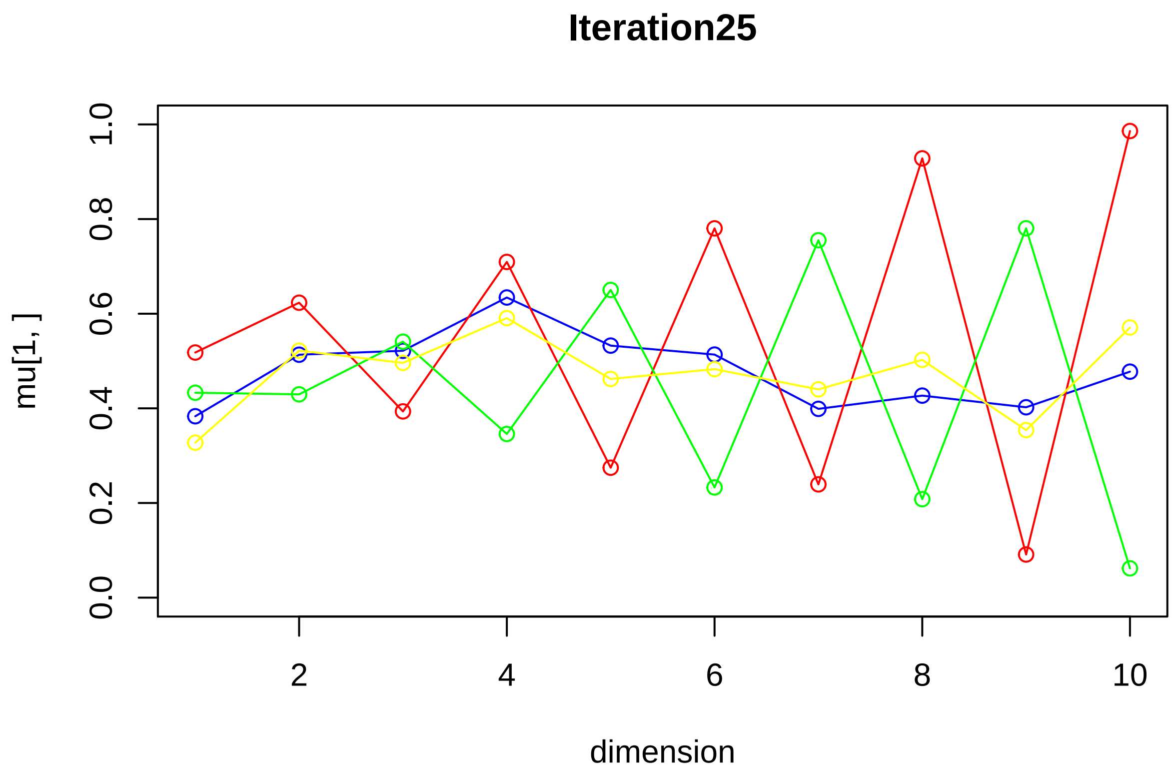

```
## iteration:  22 log likelihood:  -7311.036
```



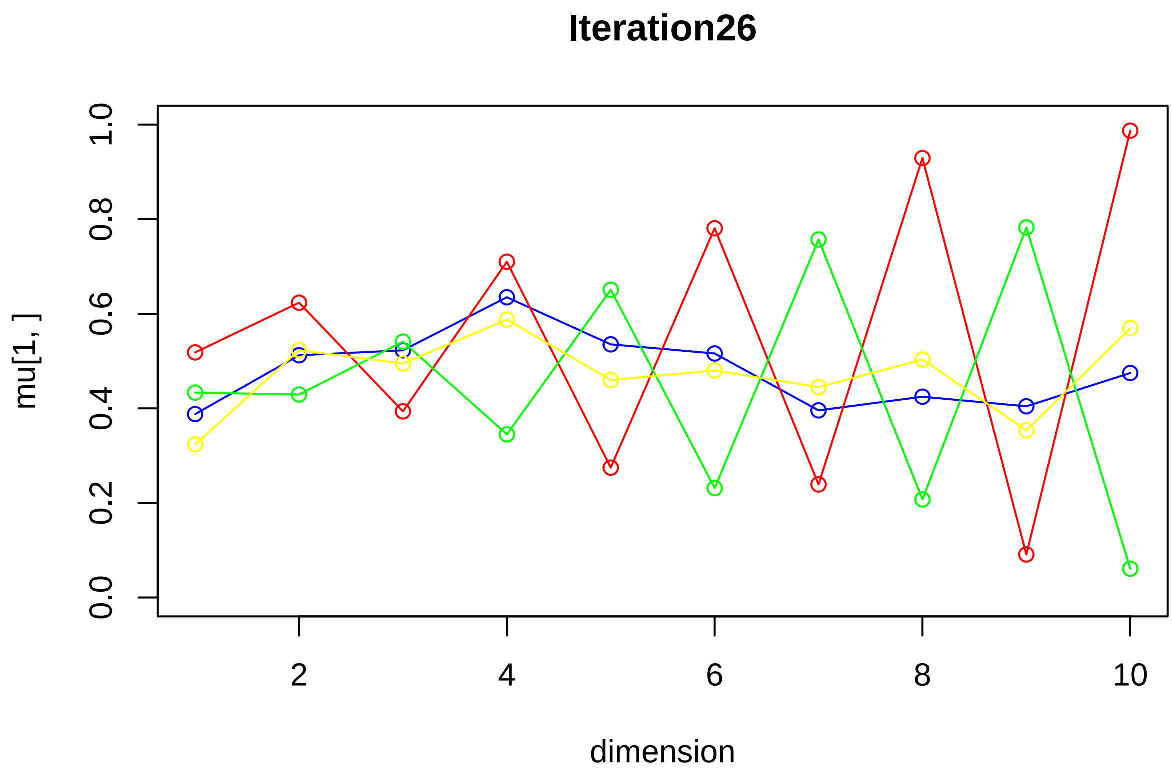

```
## iteration:  23 log likelihood:  -7310.574
```



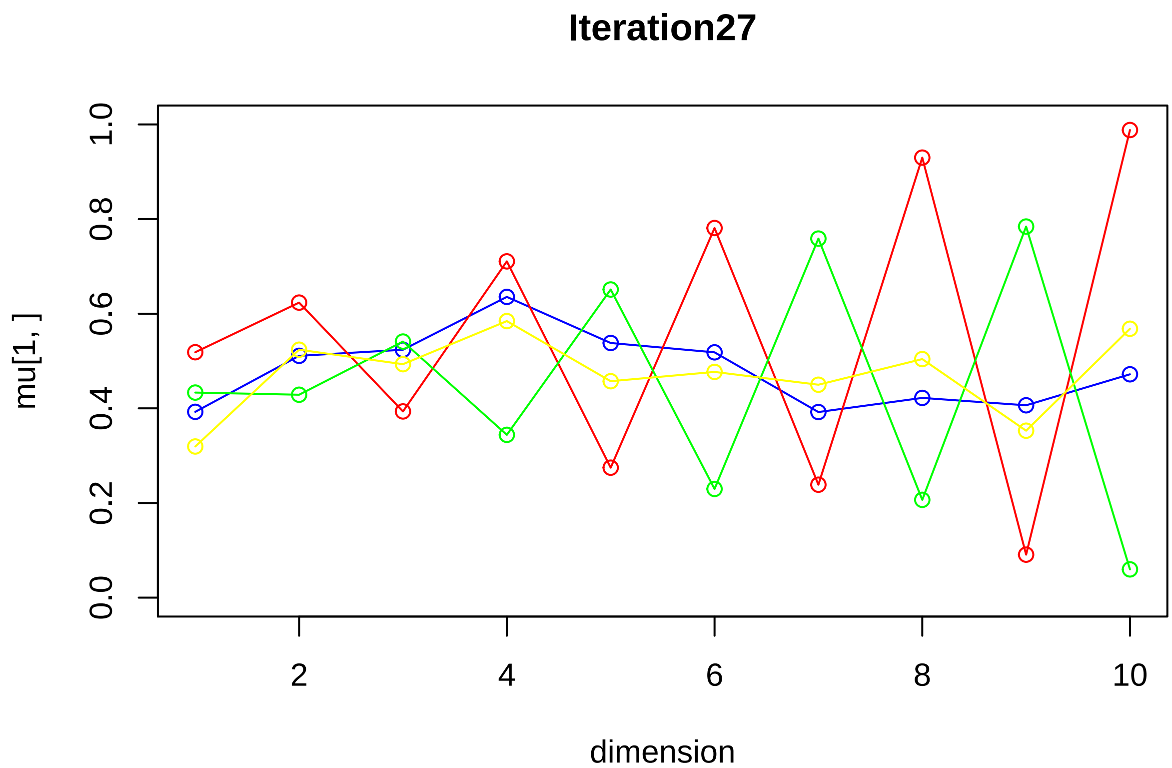

```
## iteration:  24 log likelihood:  -7309.988
```



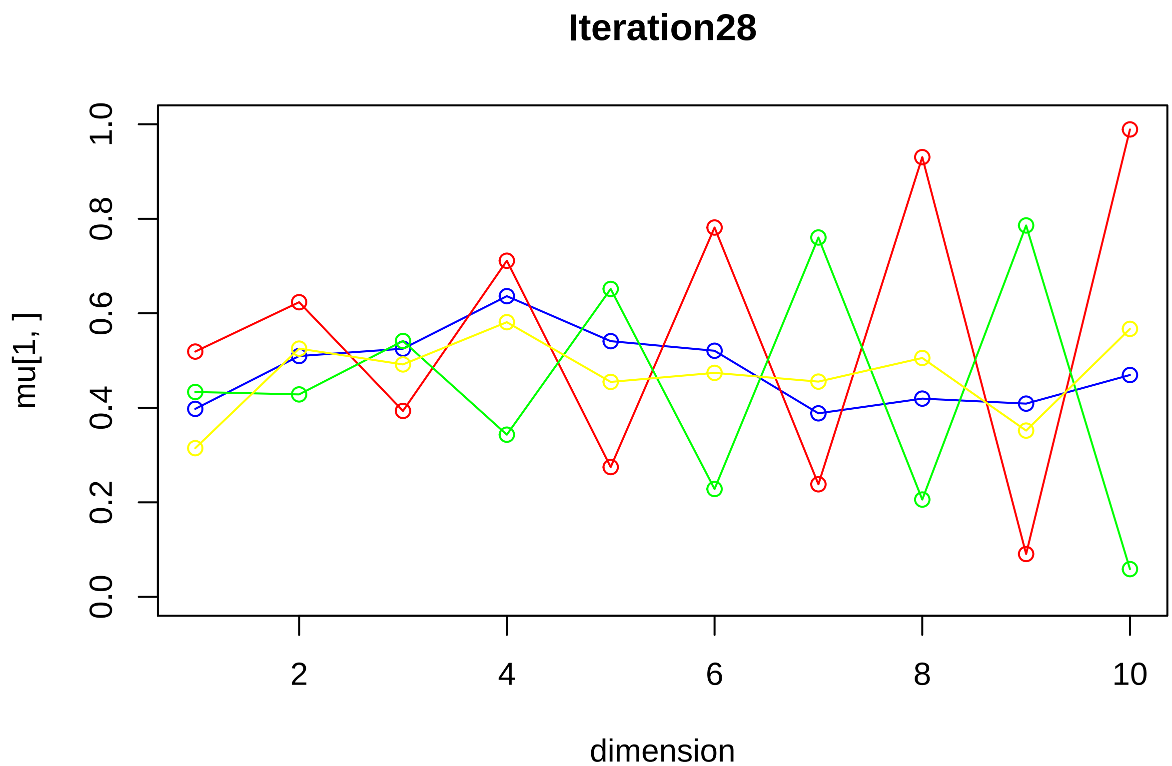

```
## iteration:  25 log likelihood:  -7309.248
```



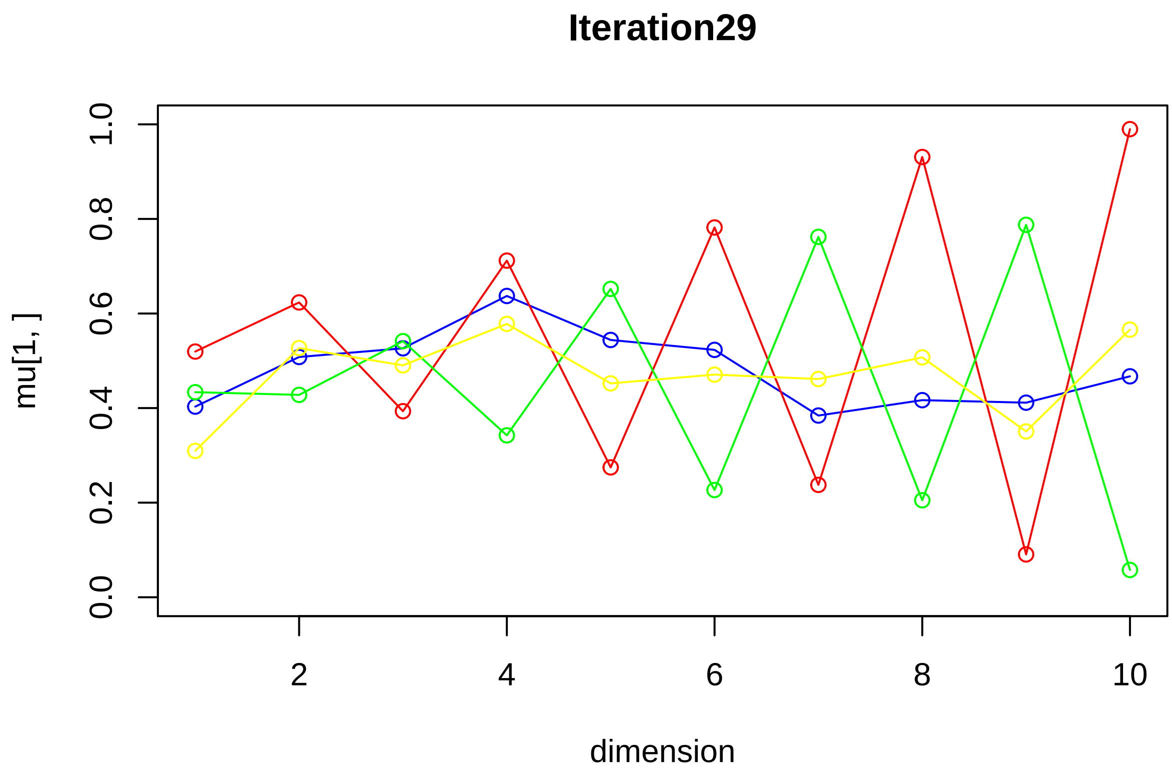

```
## iteration:  26 log likelihood:  -7308.322
```



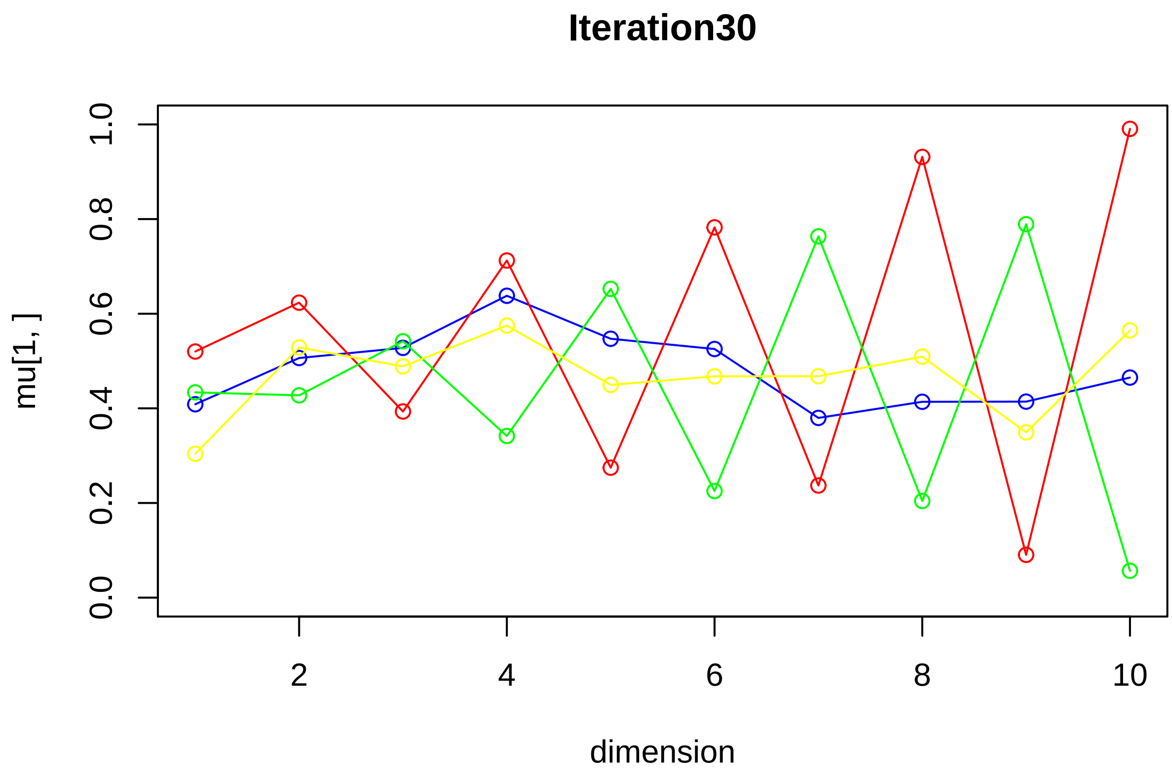

```
## iteration:  27 log likelihood:  -7307.185
```



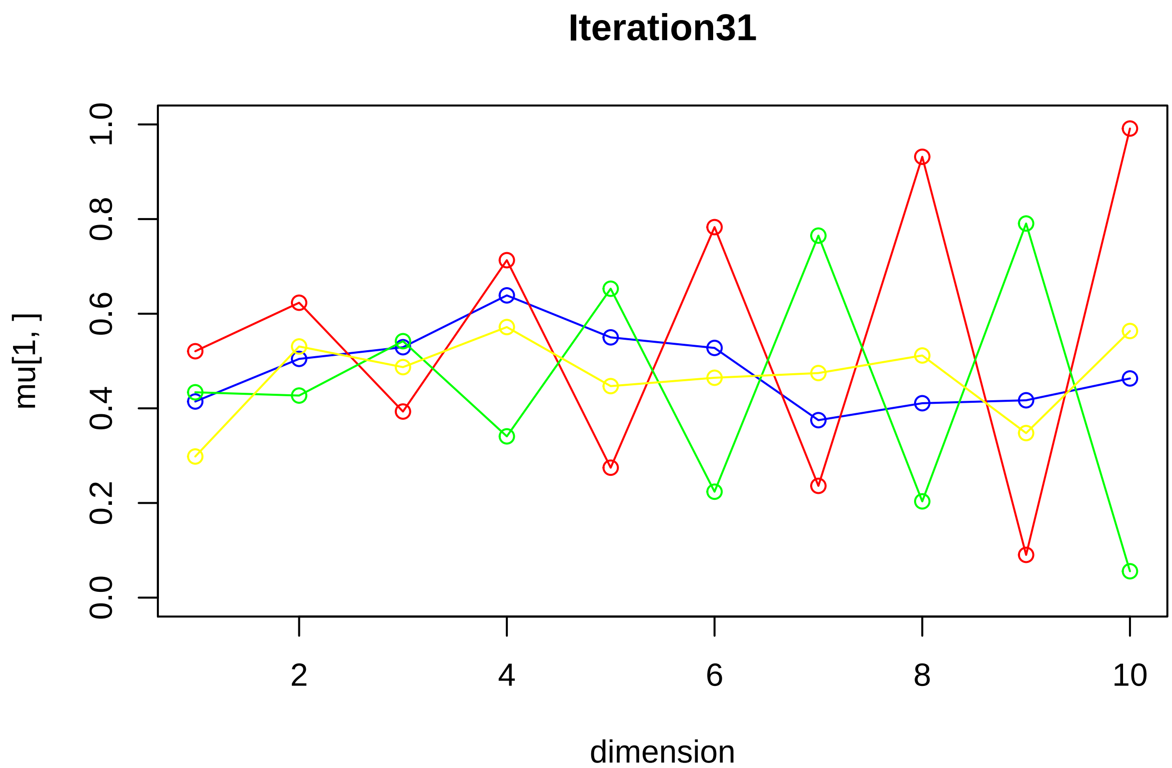

```
## iteration:  28 log likelihood:  -7305.809
```



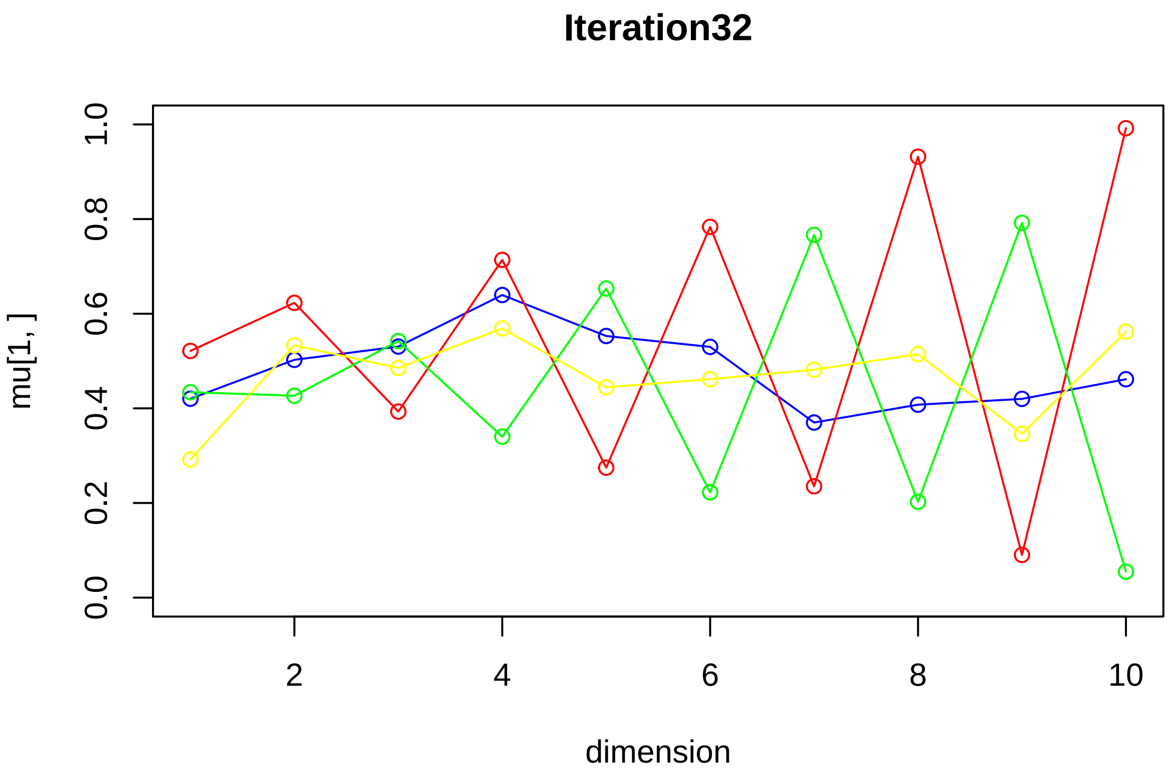

```
## iteration:  29 log likelihood:  -7304.176
```



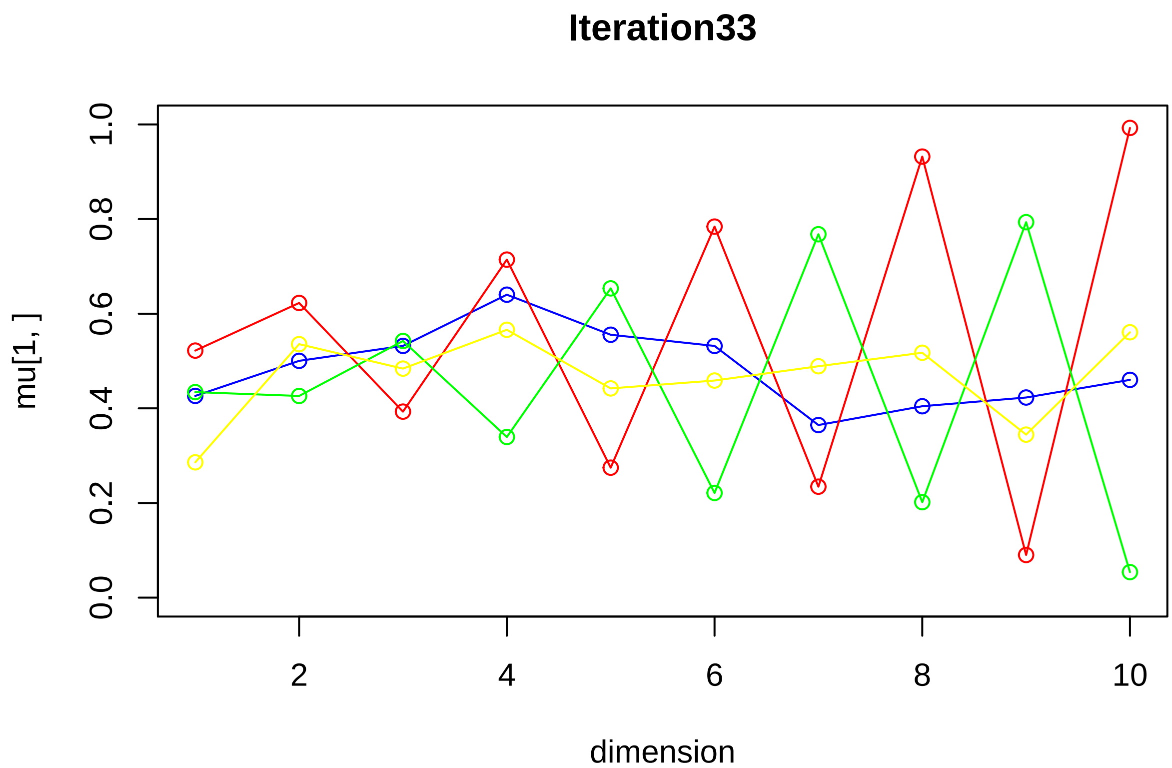

```
## iteration:  30 log likelihood:  -7302.273
```



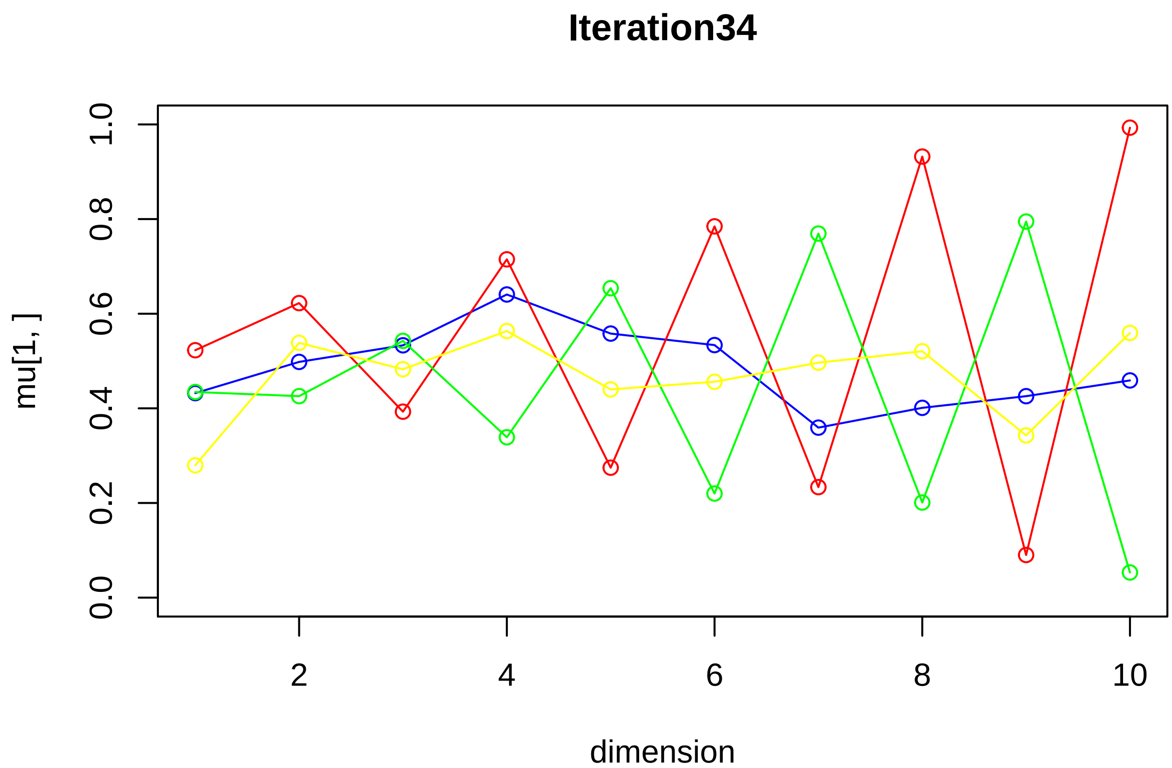

```
## iteration:  31 log likelihood:  -7300.1
```



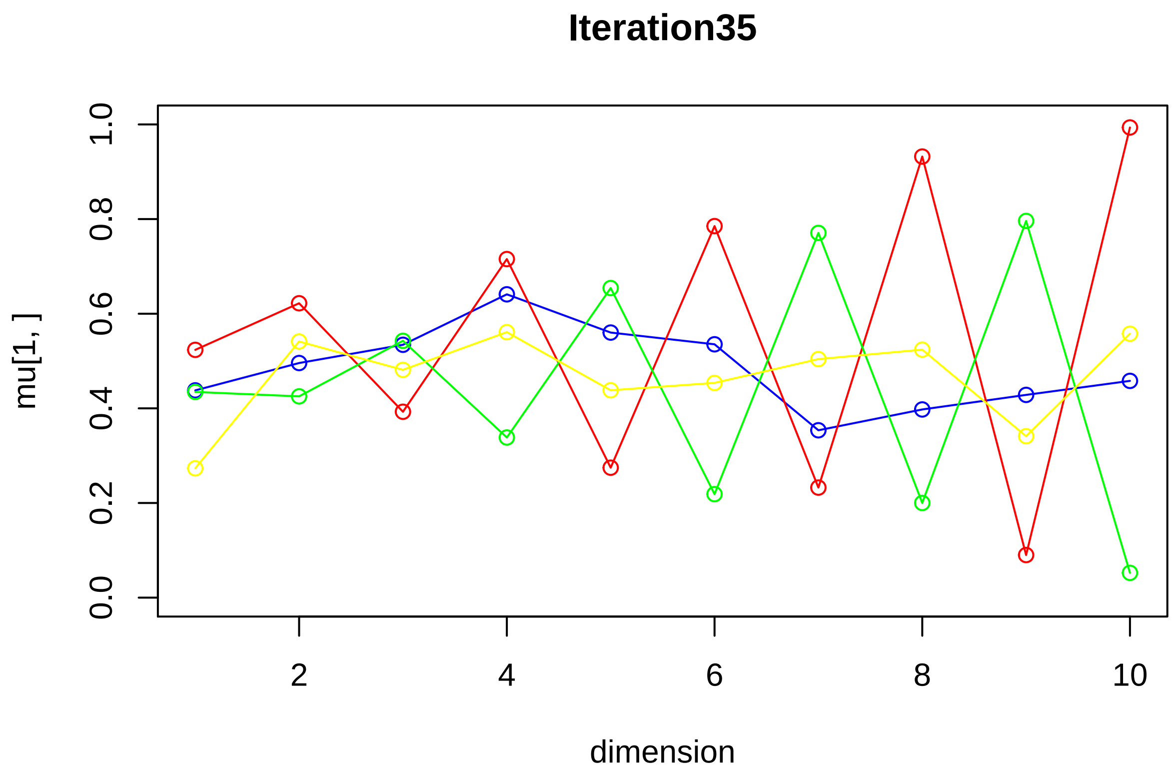

```
## iteration:  32 log likelihood:  -7297.671
```



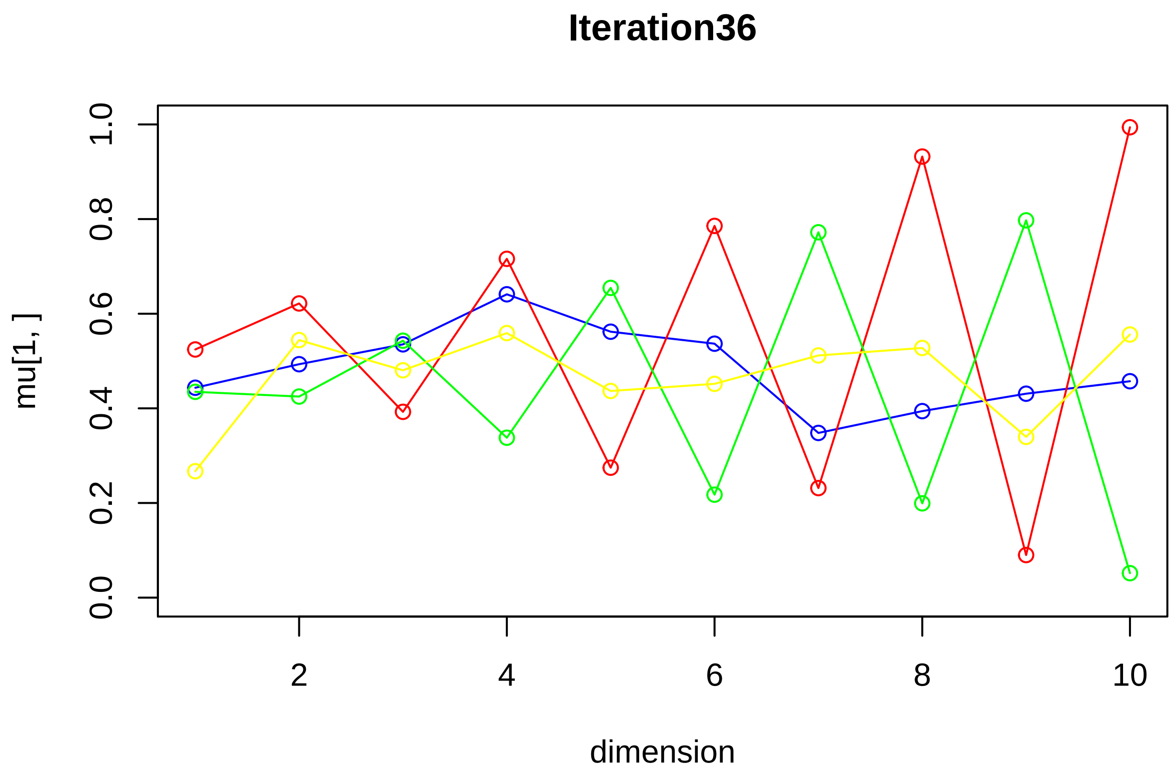

```
## iteration:  33 log likelihood:  -7295.014
```



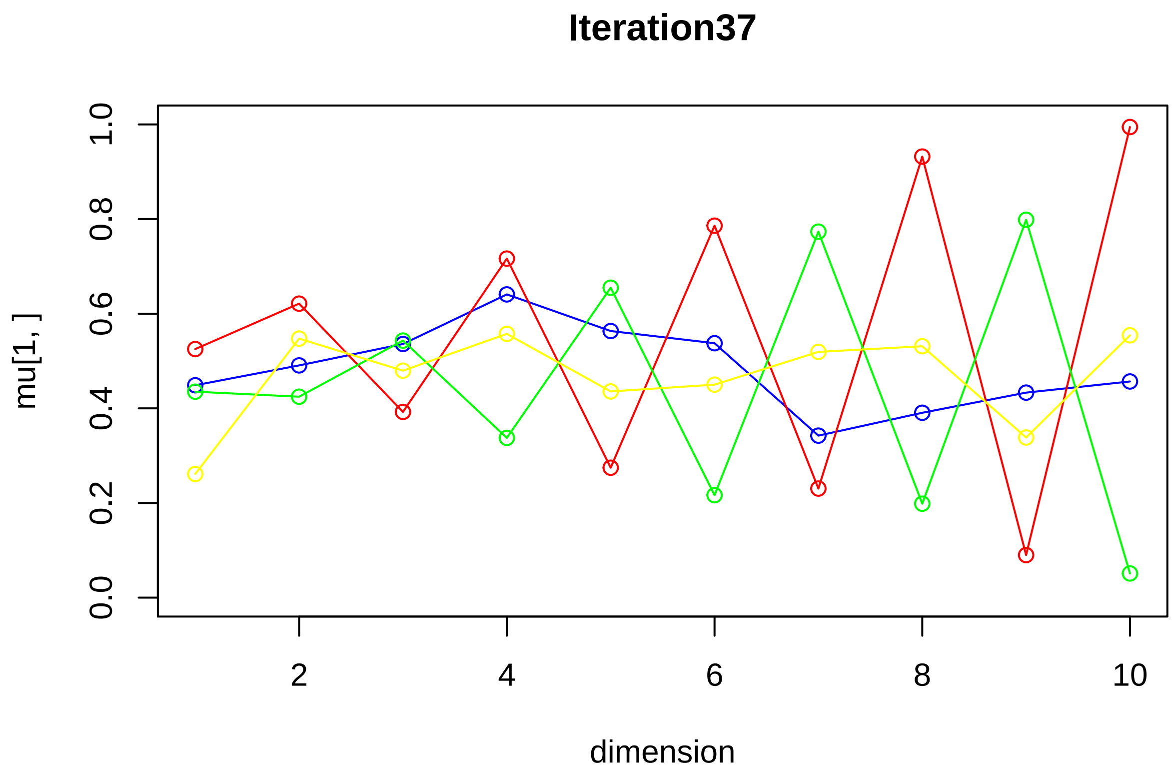

```
## iteration:  34 log likelihood:  -7292.171
```



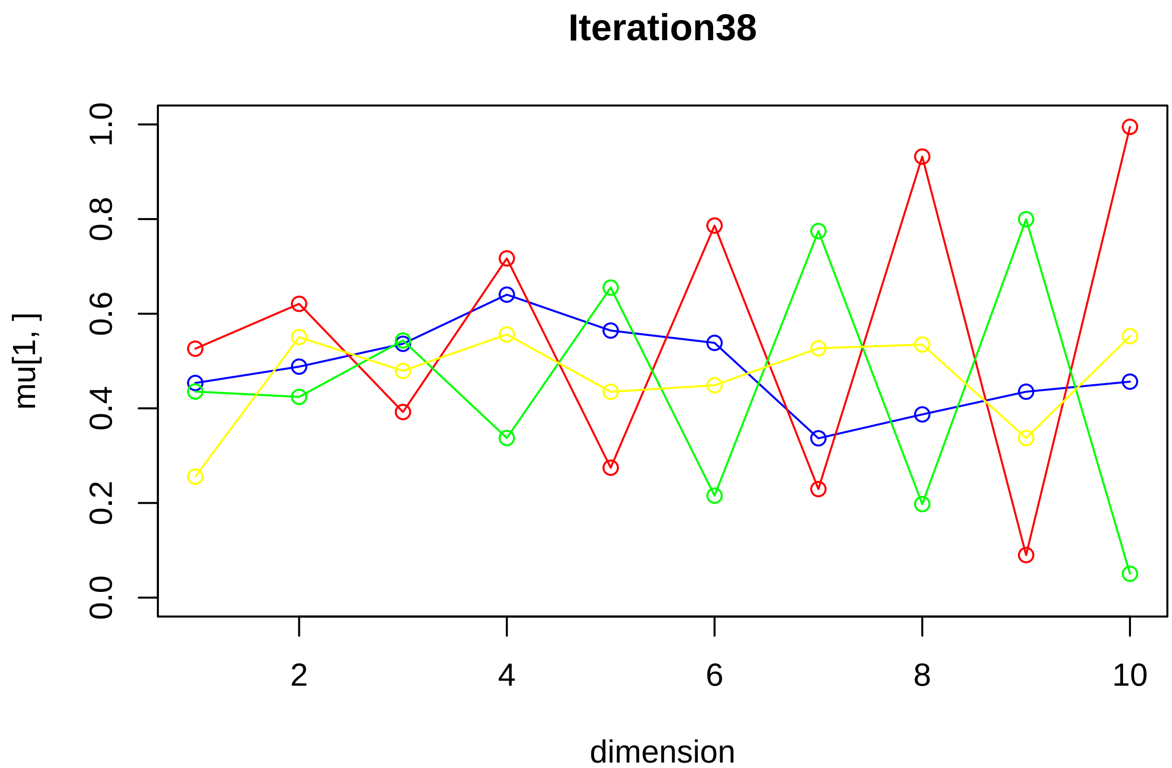

```
## iteration:  35 log likelihood:  -7289.196
```



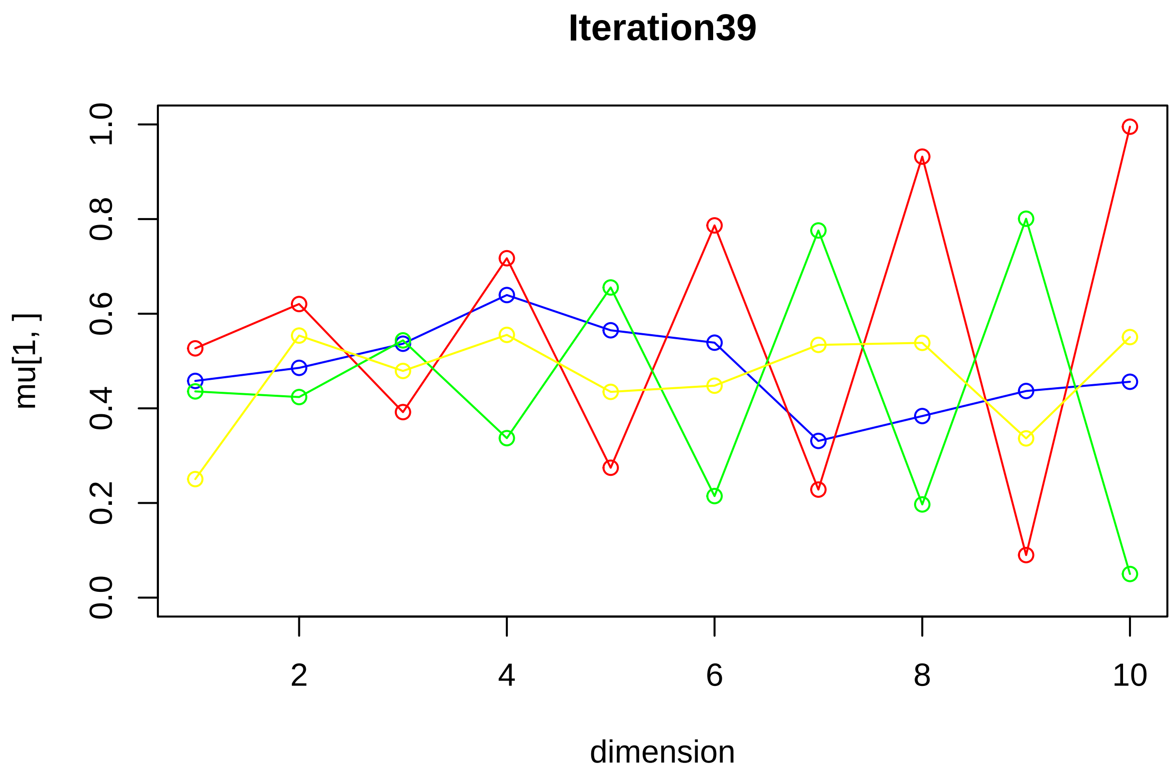

```
## iteration:  36 log likelihood:  -7286.15
```



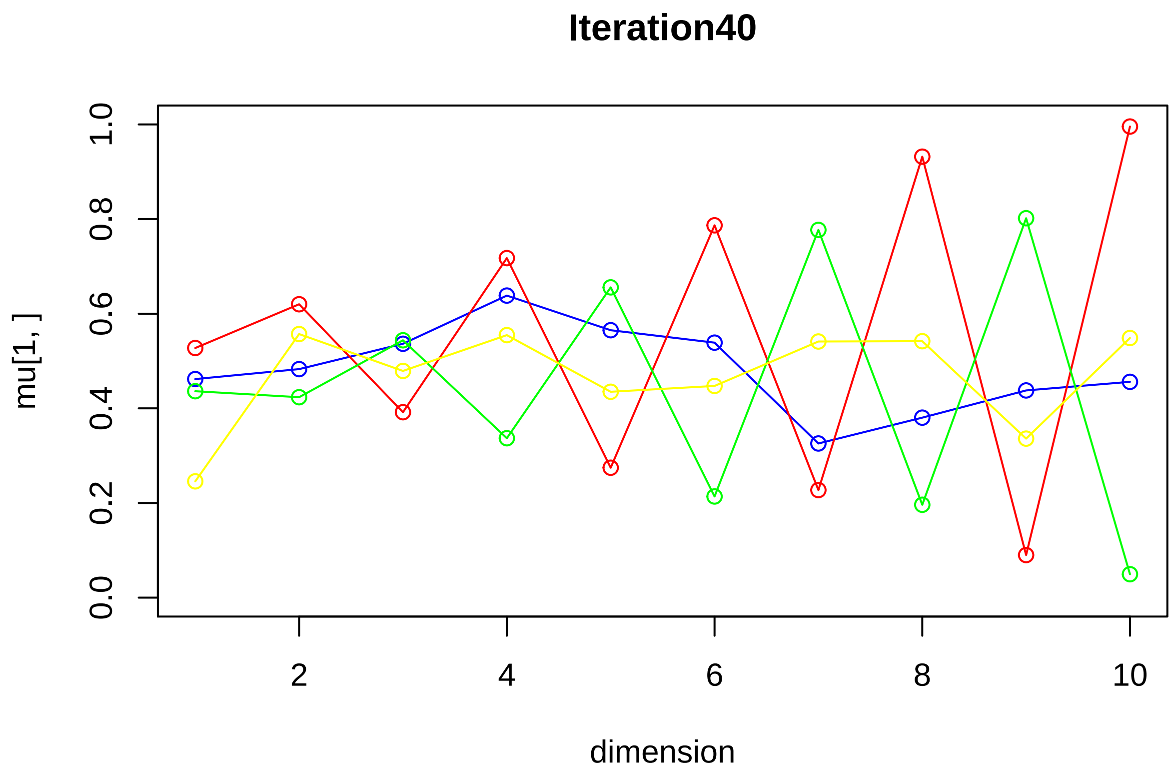

```
## iteration:  37 log likelihood:  -7283.093
```



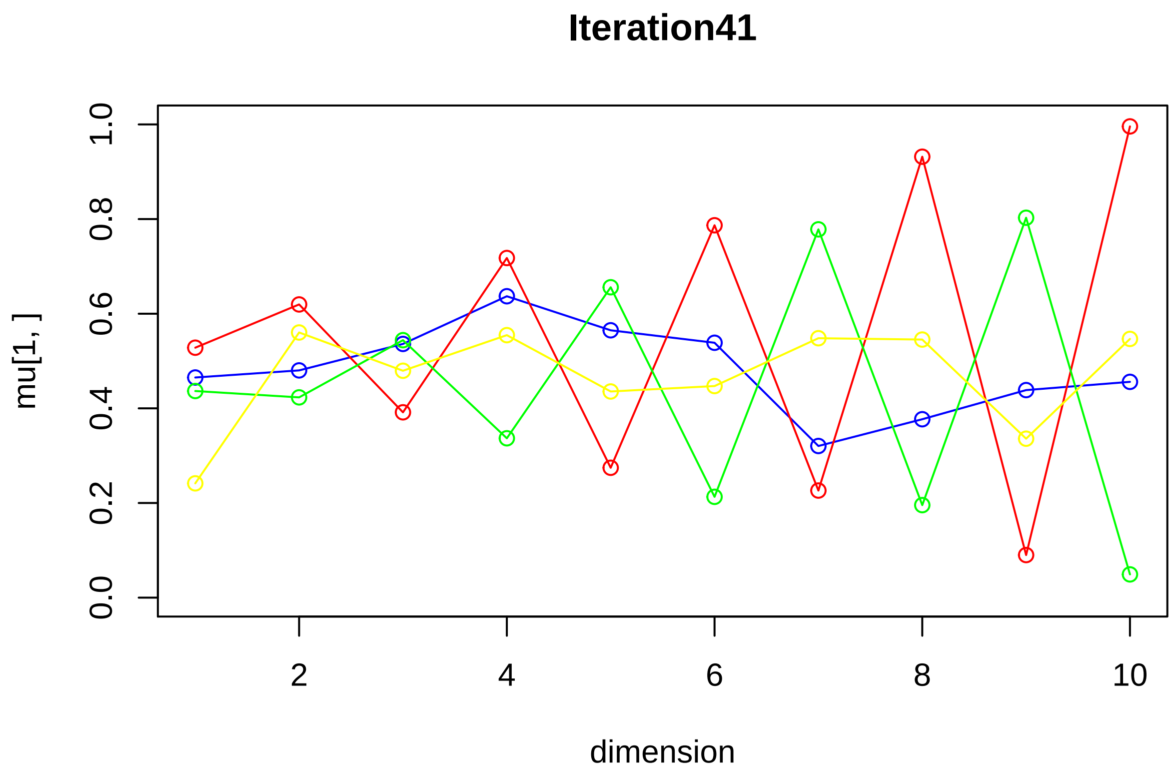

```
## iteration:  38 log likelihood:  -7280.079
```



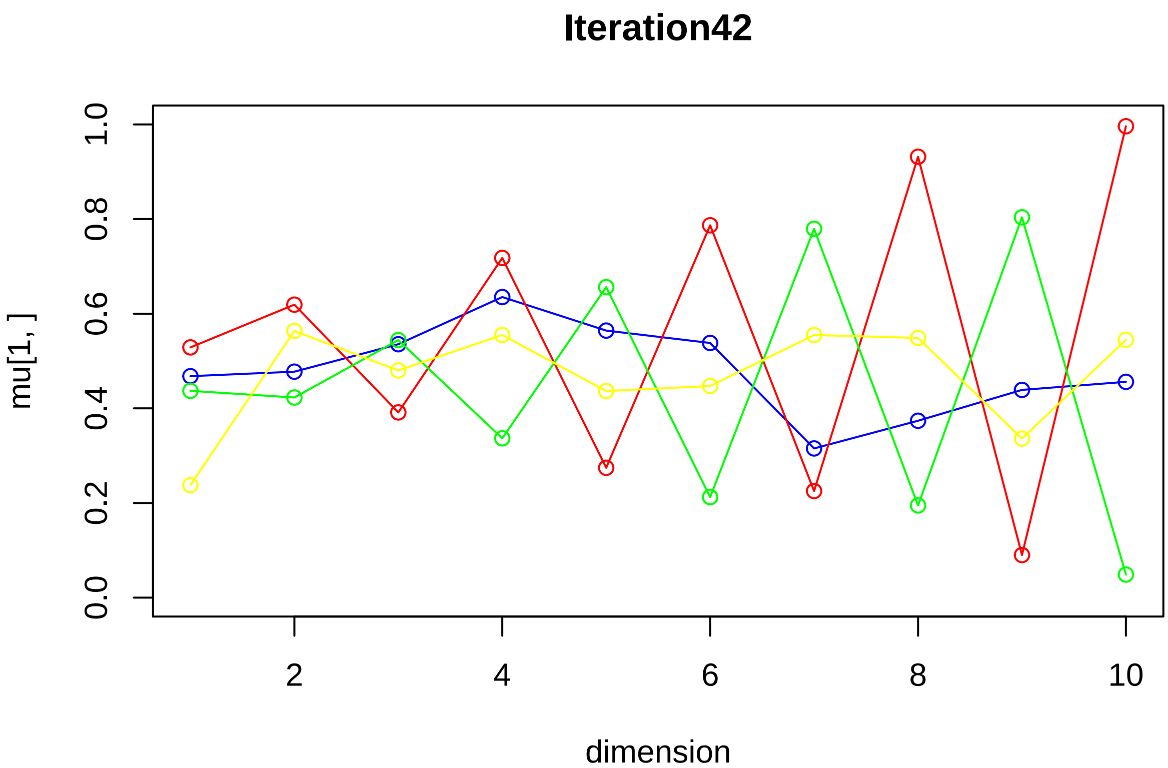

```
## iteration:  39 log likelihood:  -7277.151
```



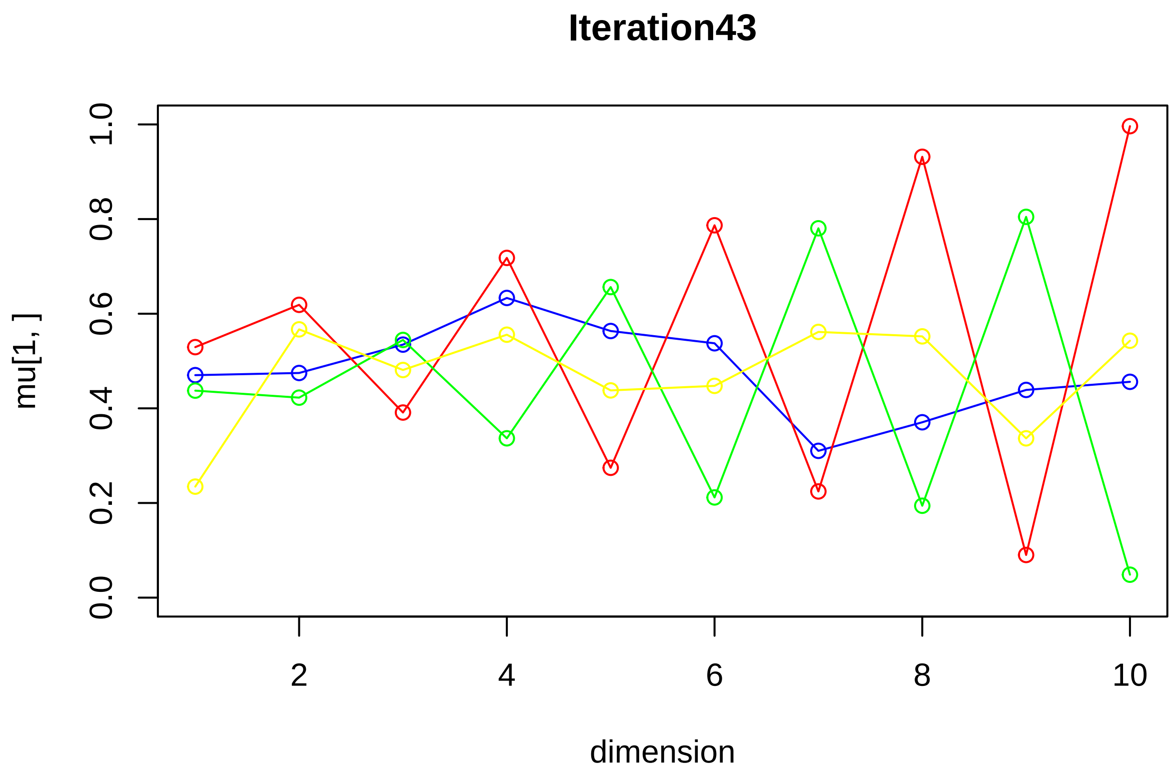

```
## iteration:  40 log likelihood:  -7274.34
```



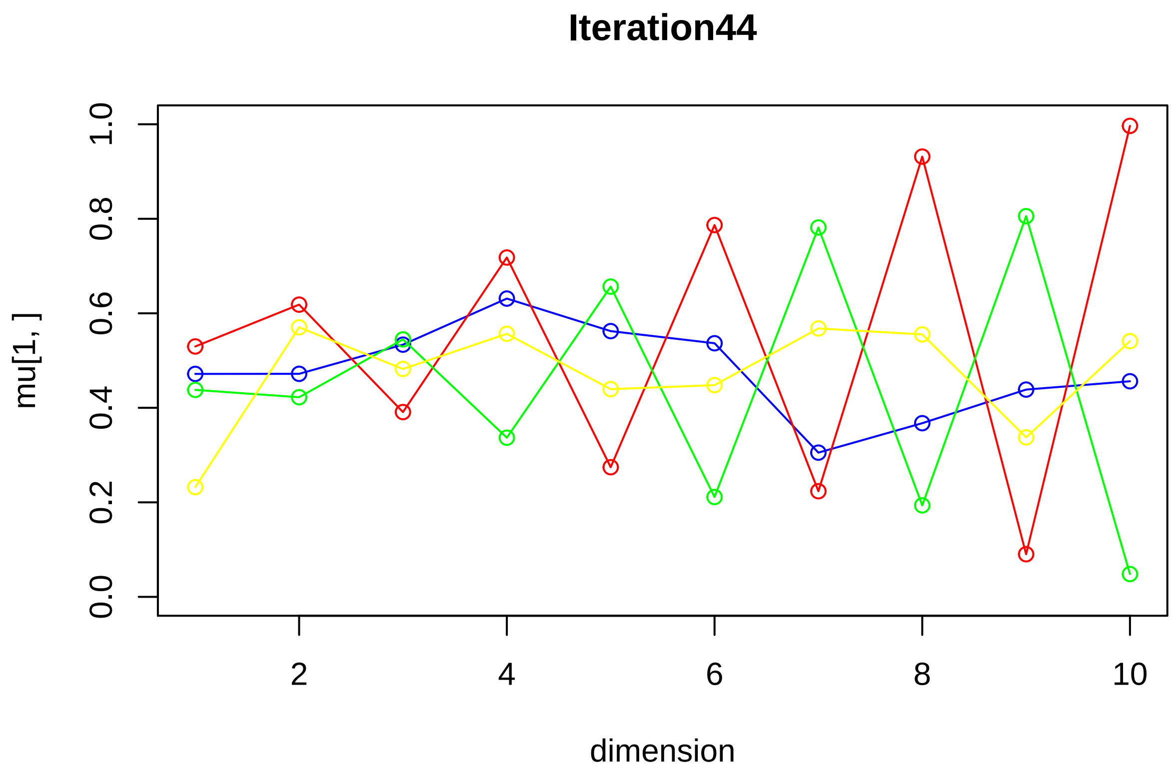

```
## iteration:  41 log likelihood:  -7271.66
```



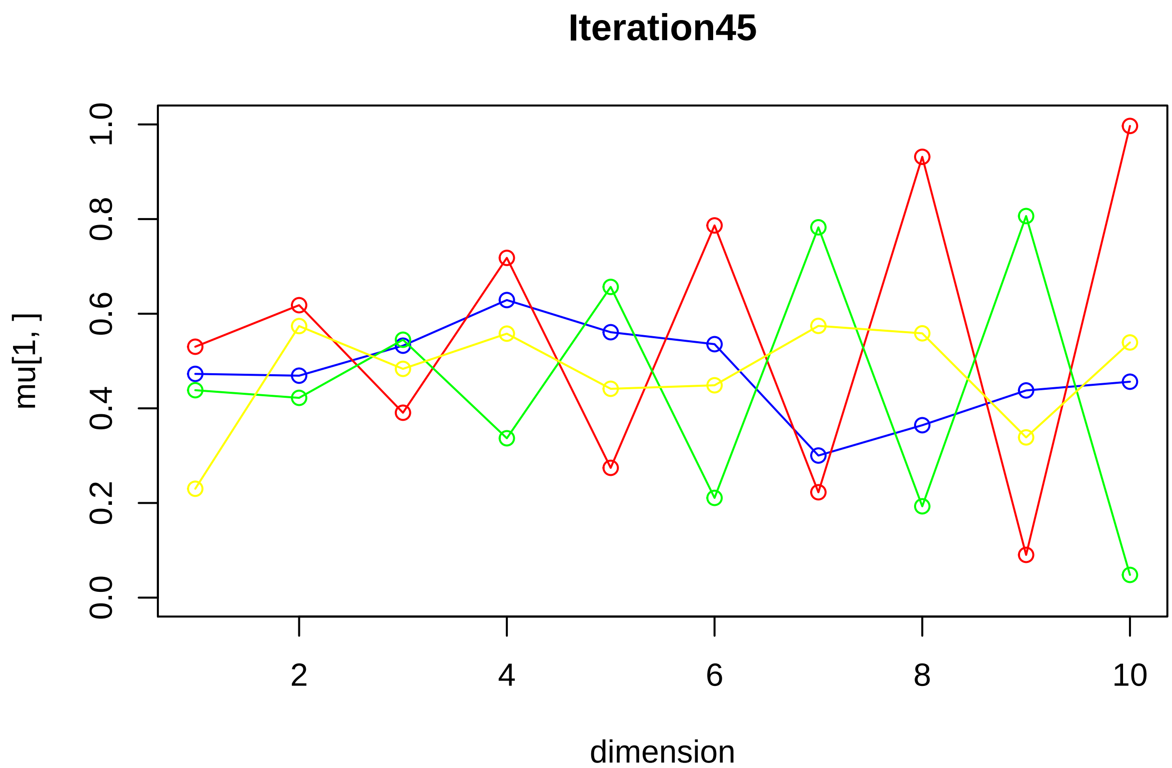

```
## iteration:  42 log likelihood:  -7269.116
```



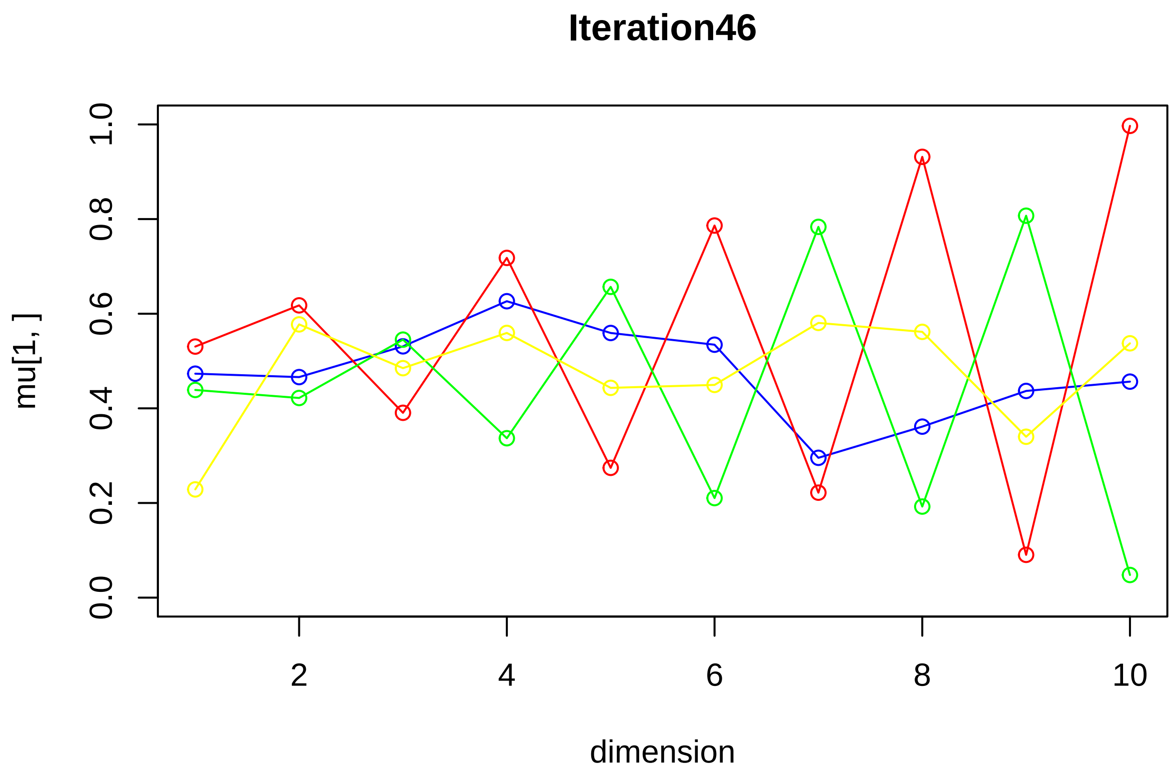

```
## iteration:  43 log likelihood:  -7266.7
```



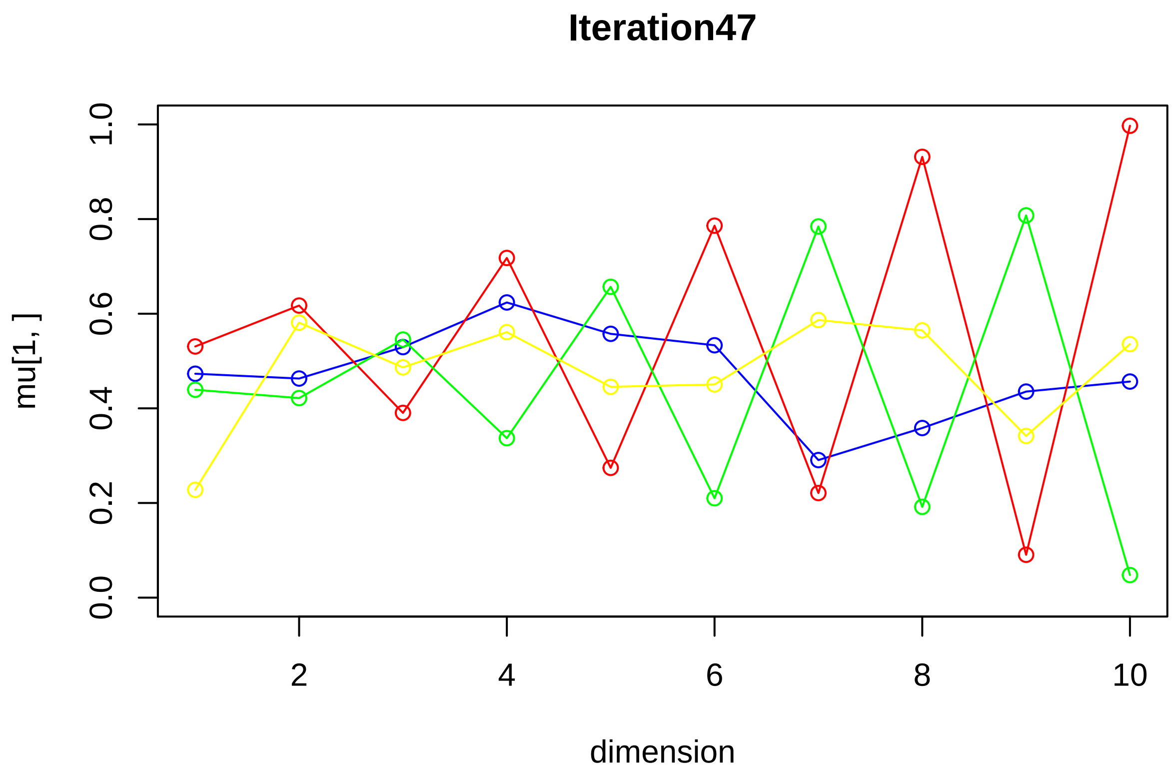

```
## iteration:  44 log likelihood:  -7264.398
```



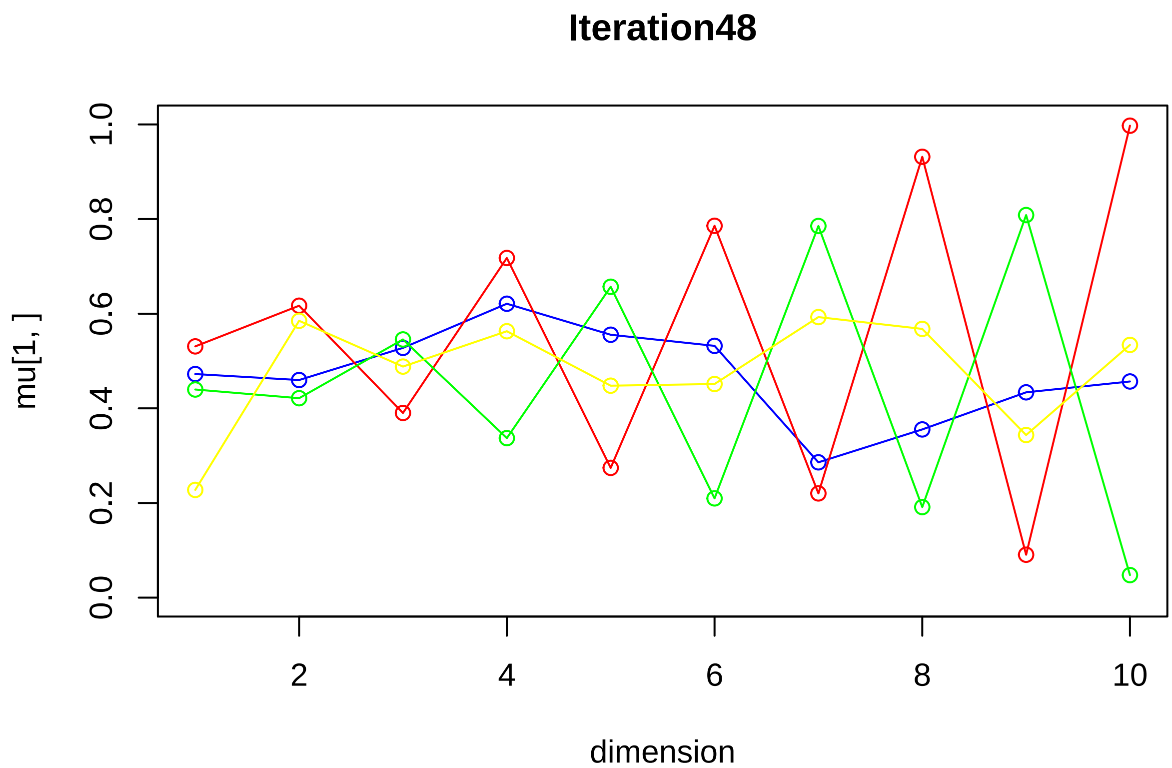

```
## iteration:  45 log likelihood:  -7262.189
```



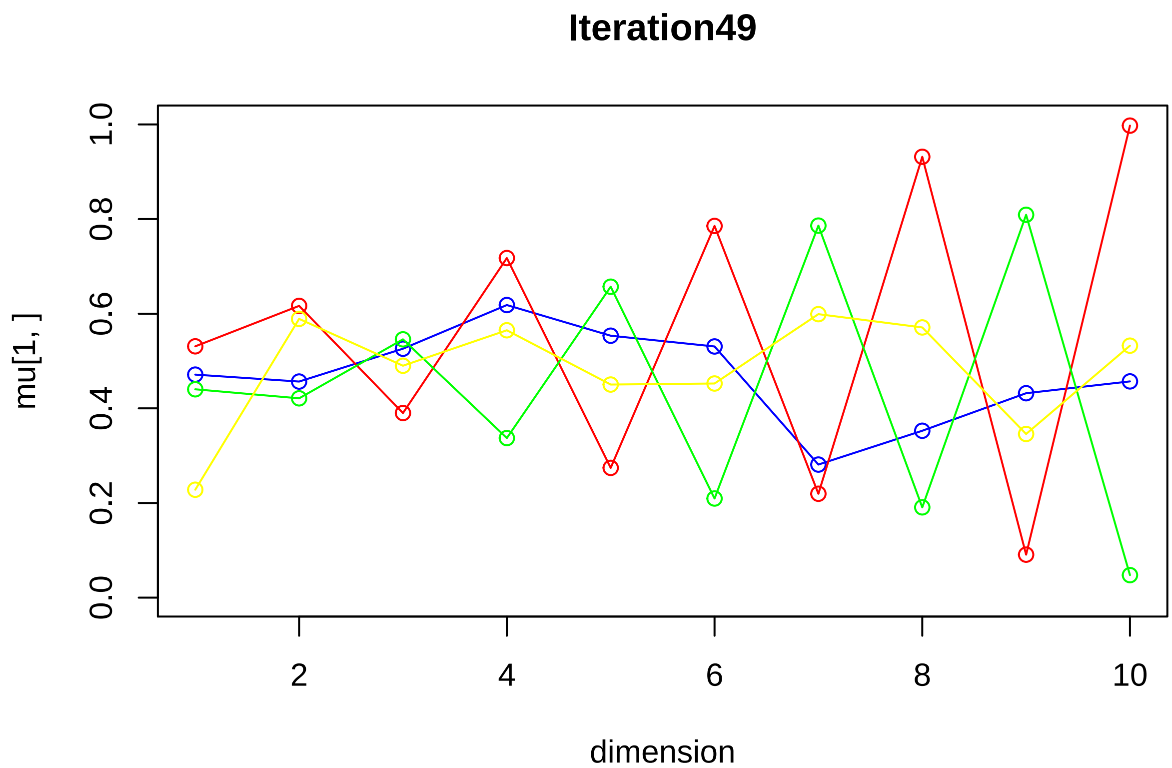

```
## iteration:  46 log likelihood:  -7260.051
```



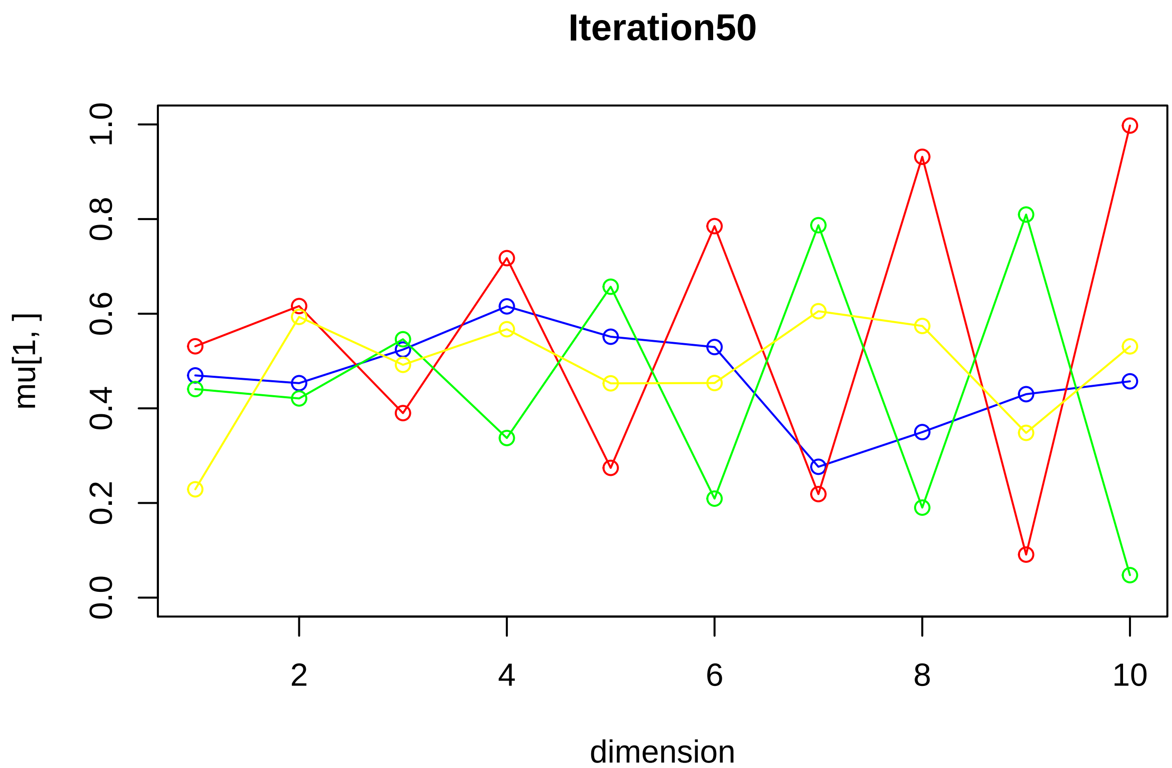

```
## iteration:  47 log likelihood:  -7257.96
```



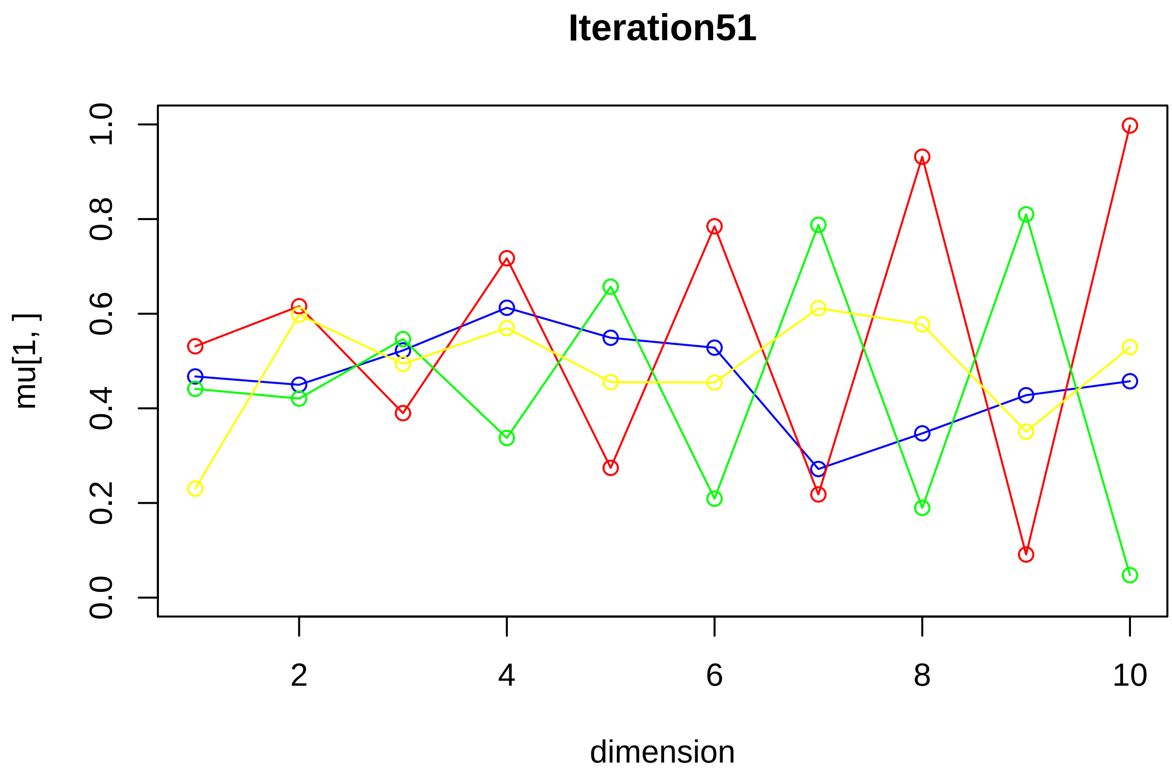

```
## iteration:  48 log likelihood:  -7255.892
```



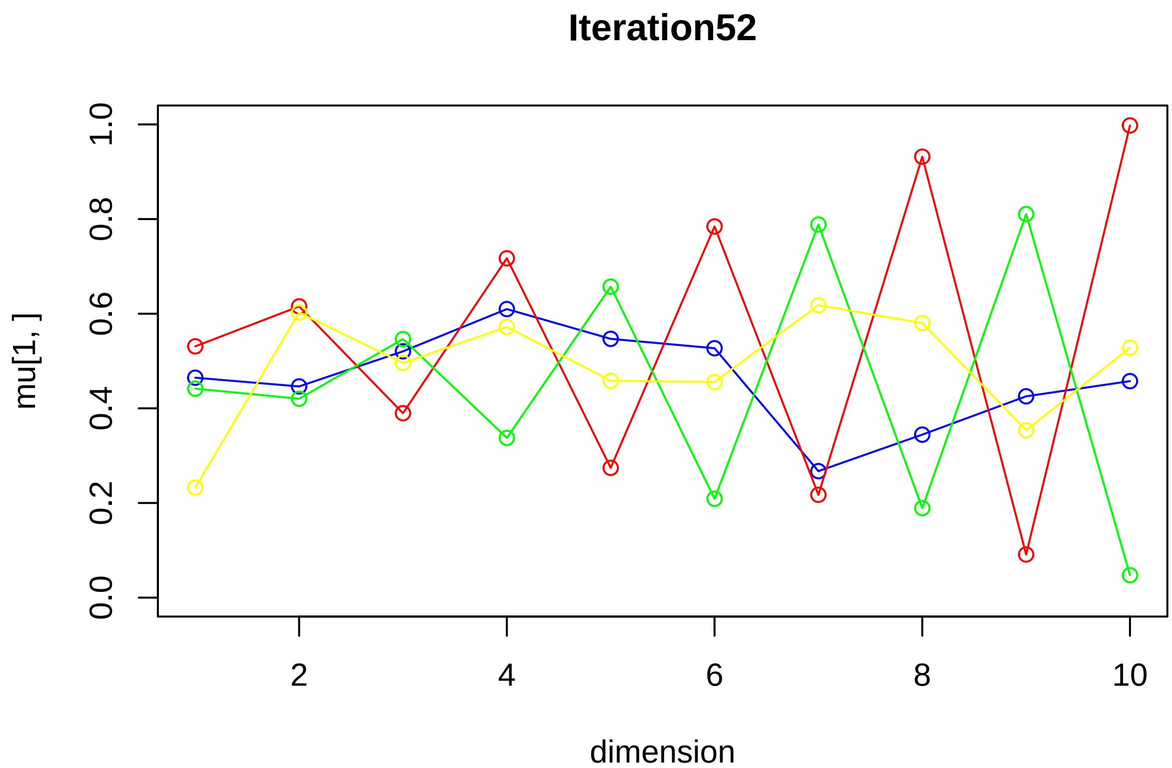

```
## iteration:  49 log likelihood:  -7253.824
```



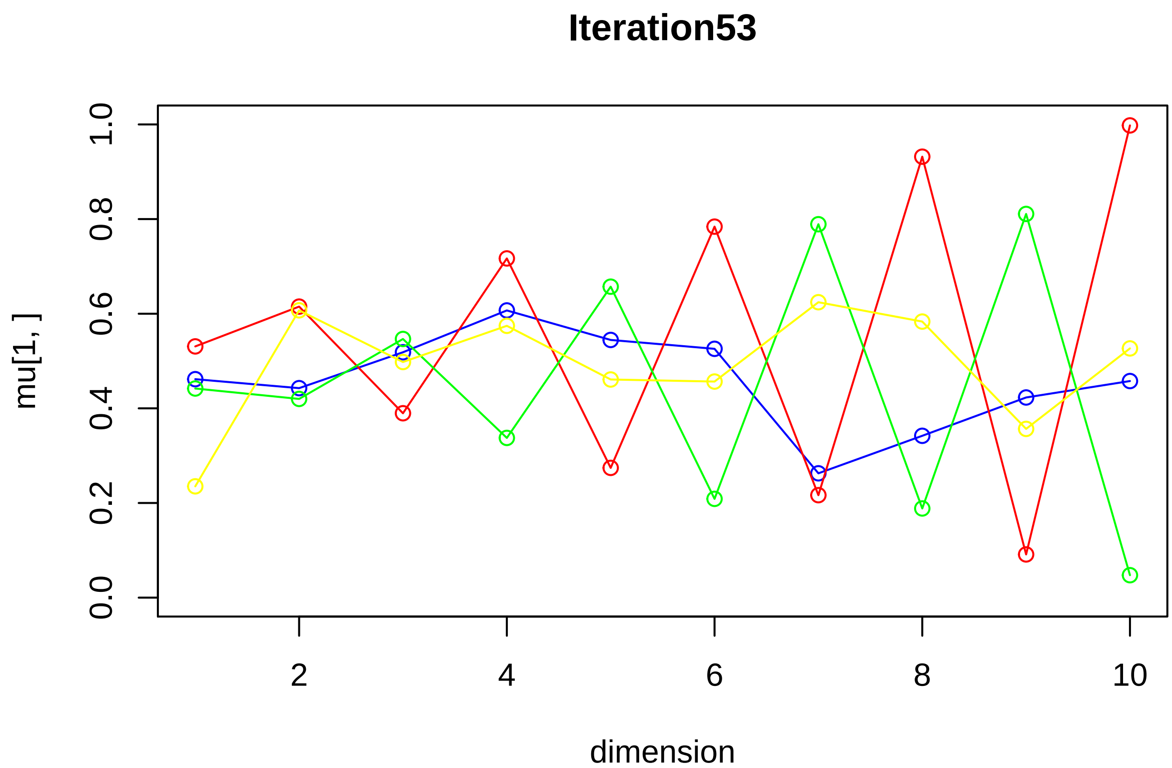

```
## iteration:  50 log likelihood:  -7251.733
```



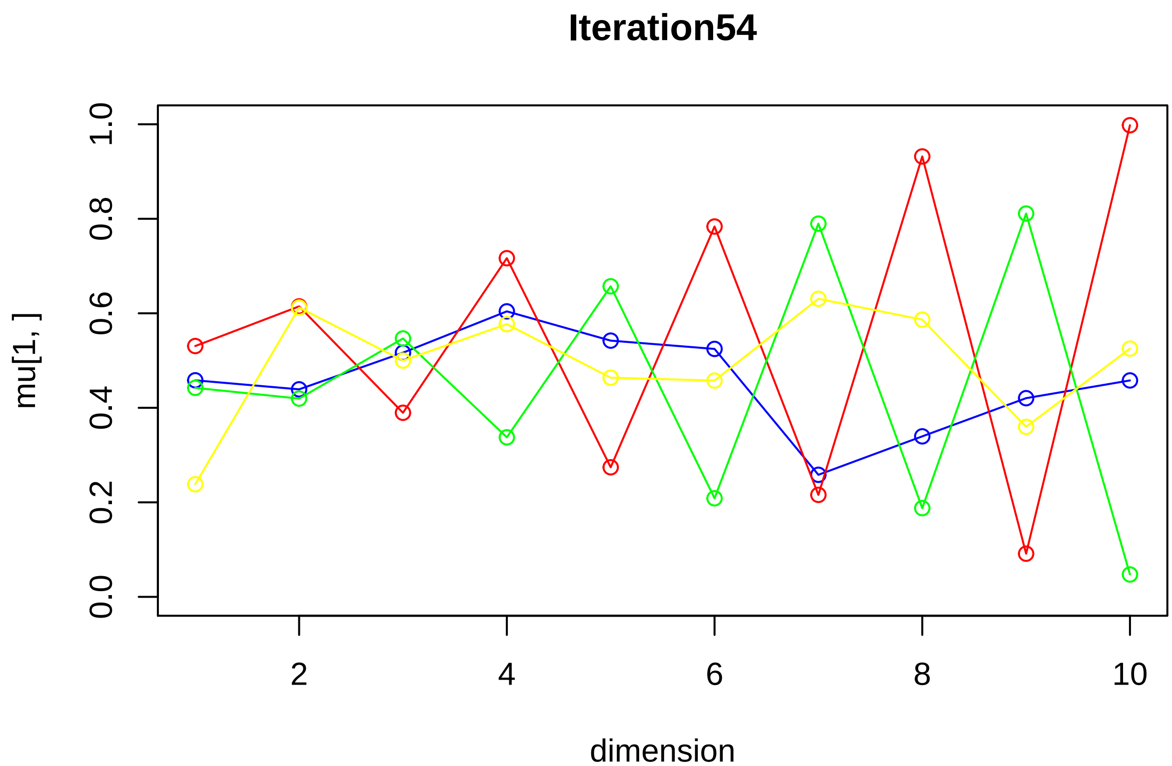

```
## iteration:  51 log likelihood:  -7249.603
```



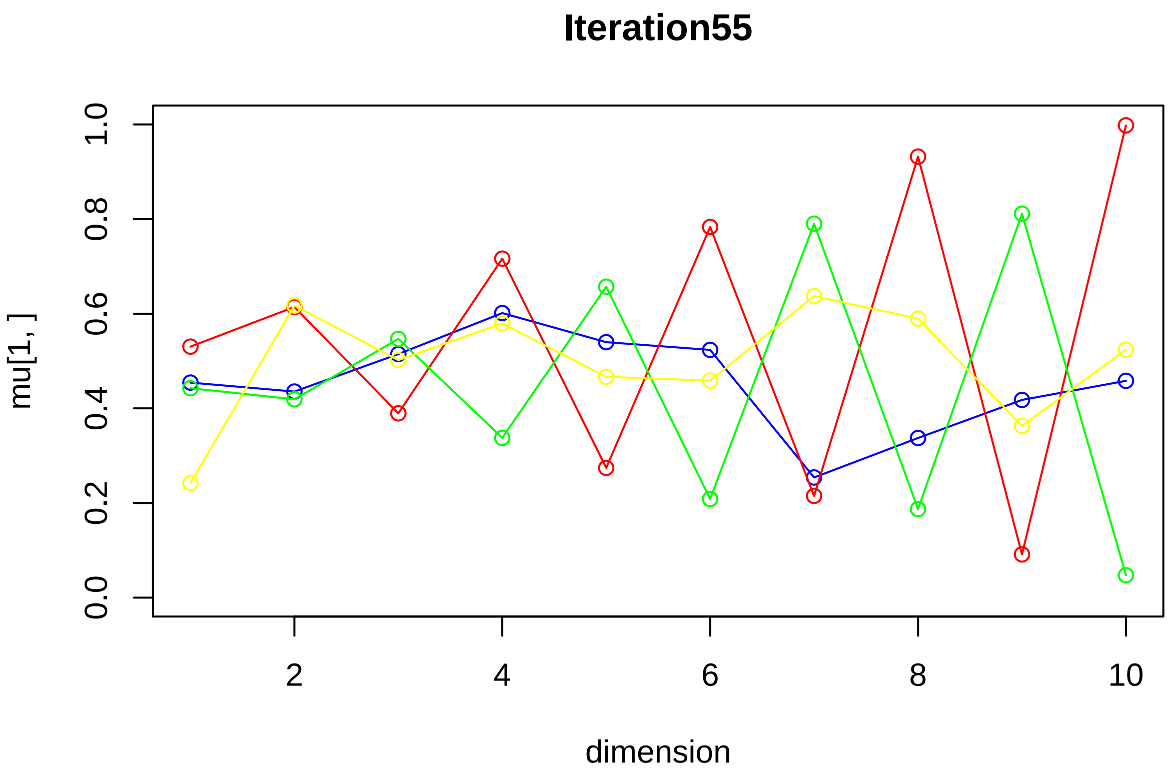

```
## iteration:  52 log likelihood:  -7247.419
```



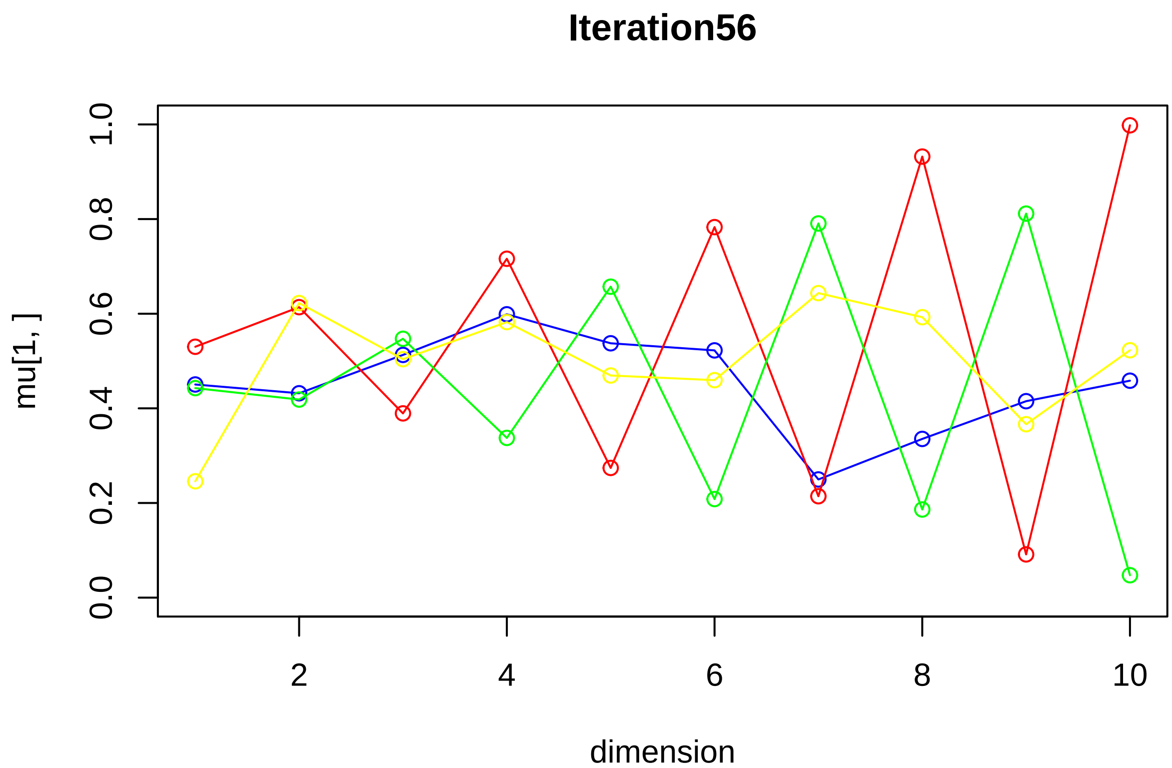

```
## iteration:  53 log likelihood:  -7245.17
```



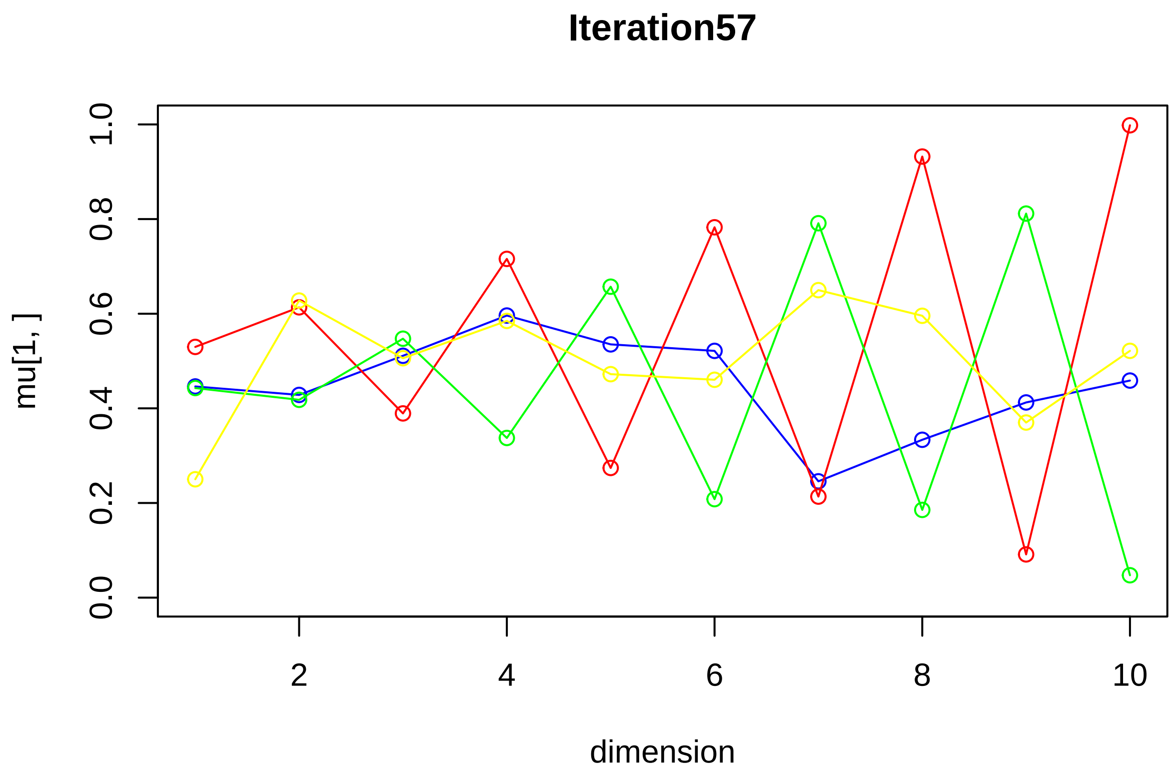

```
## iteration:  54 log likelihood:  -7242.853
```



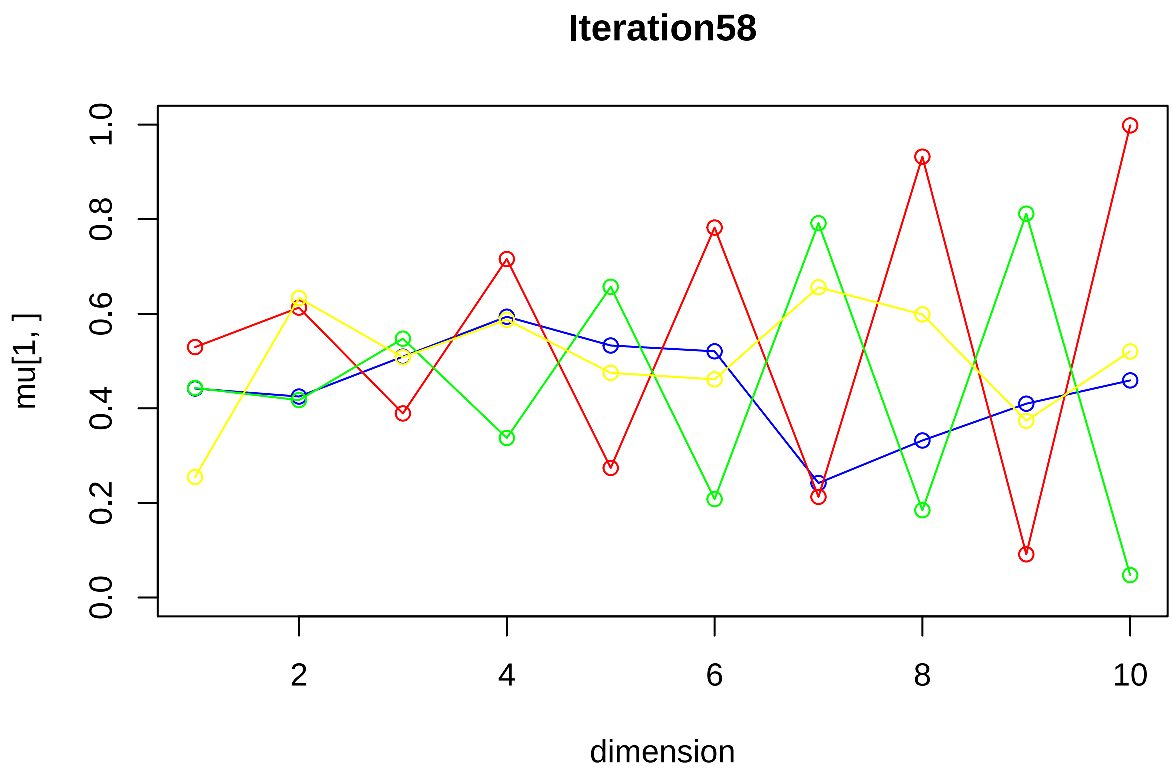

```
## iteration:  55 log likelihood:  -7240.472
```



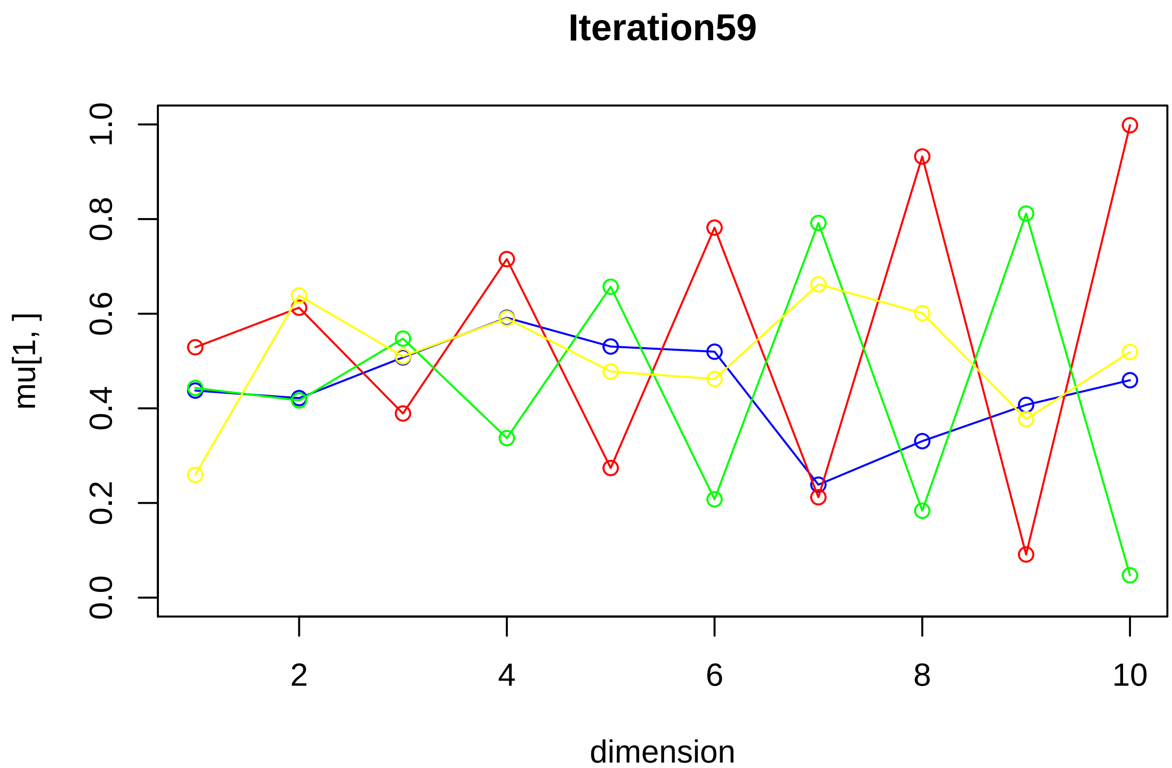

```
## iteration:  56 log likelihood:  -7238.038
```



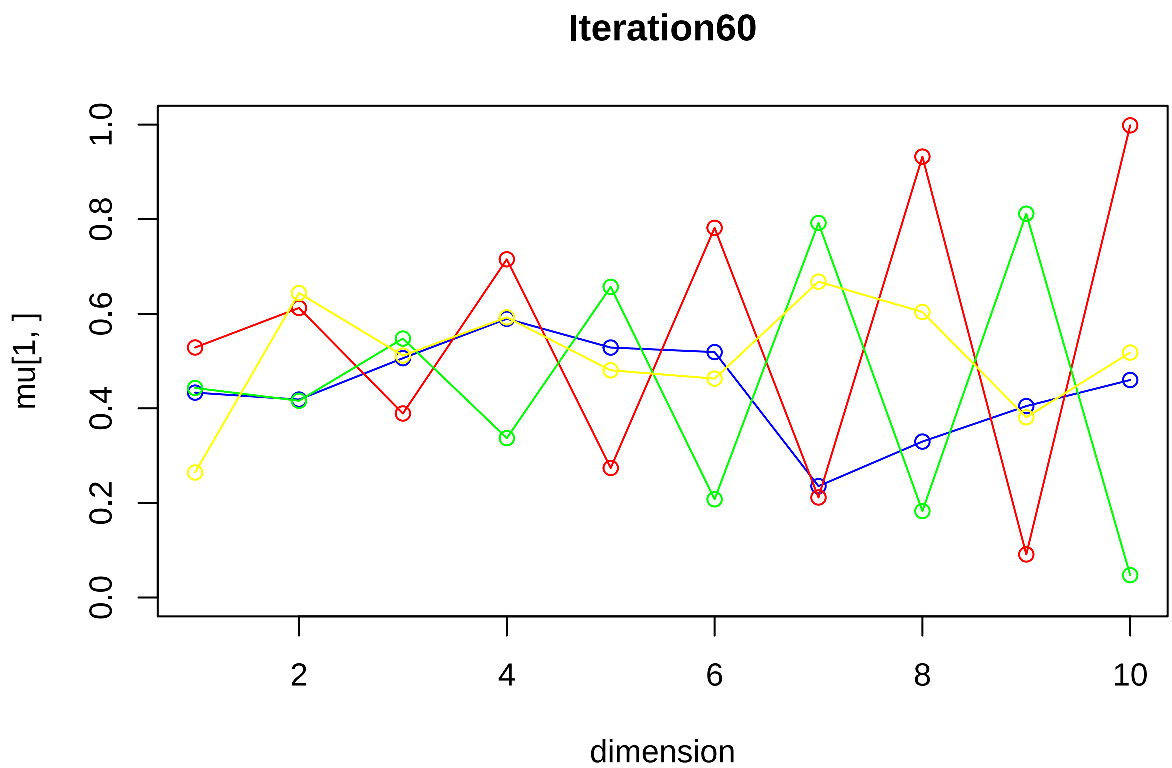

```
## iteration:  57 log likelihood:  -7235.571
```



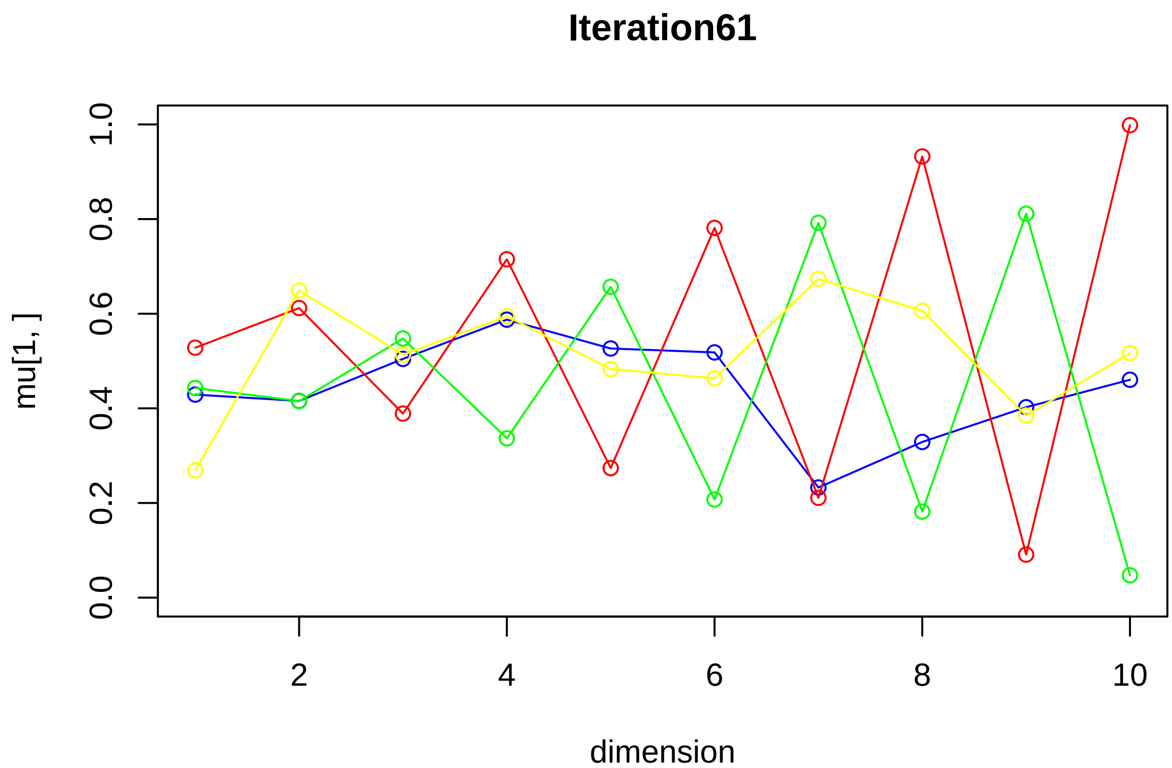

```
## iteration:  58 log likelihood:  -7233.095
```



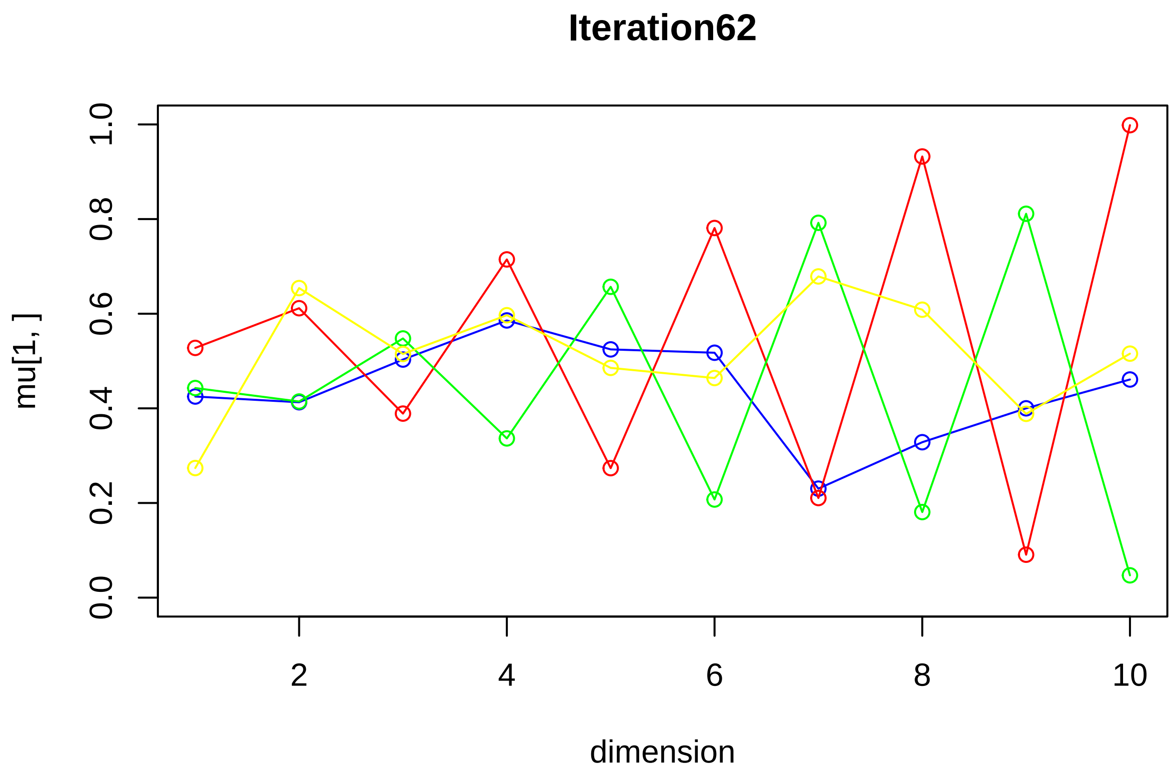

```
## iteration:  59 log likelihood:  -7230.64
```



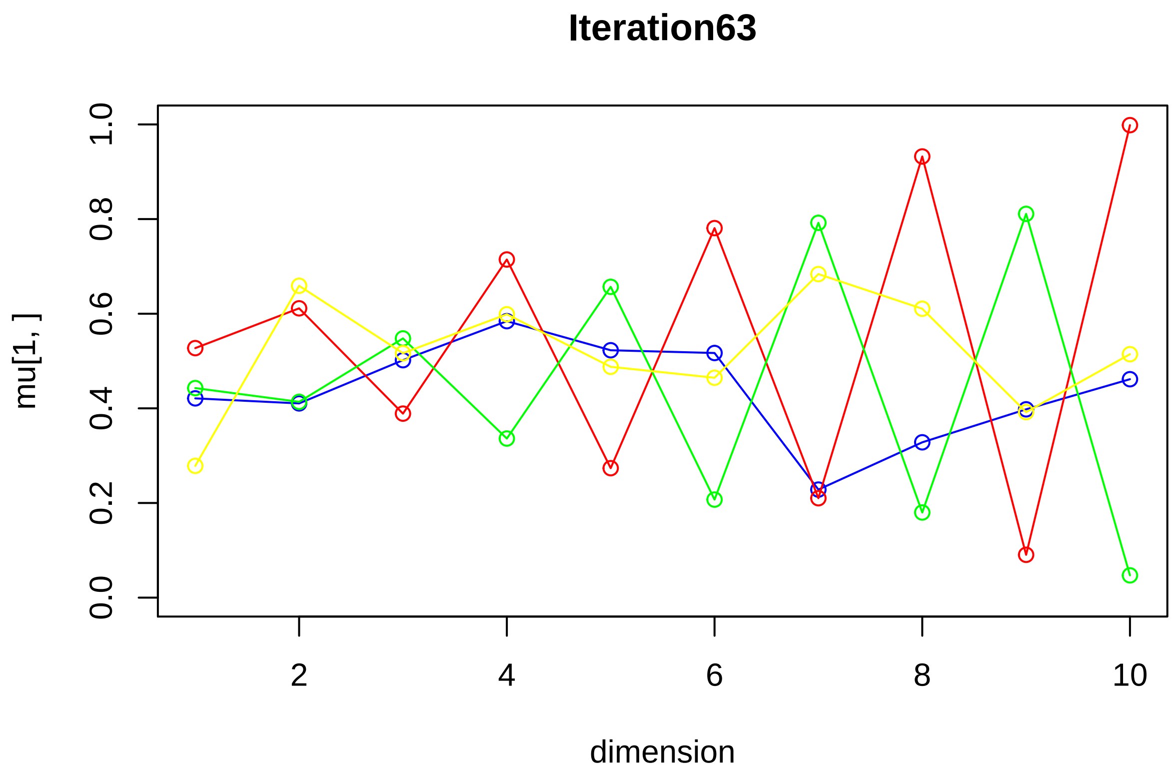

```
## iteration:  60 log likelihood:  -7228.239
```



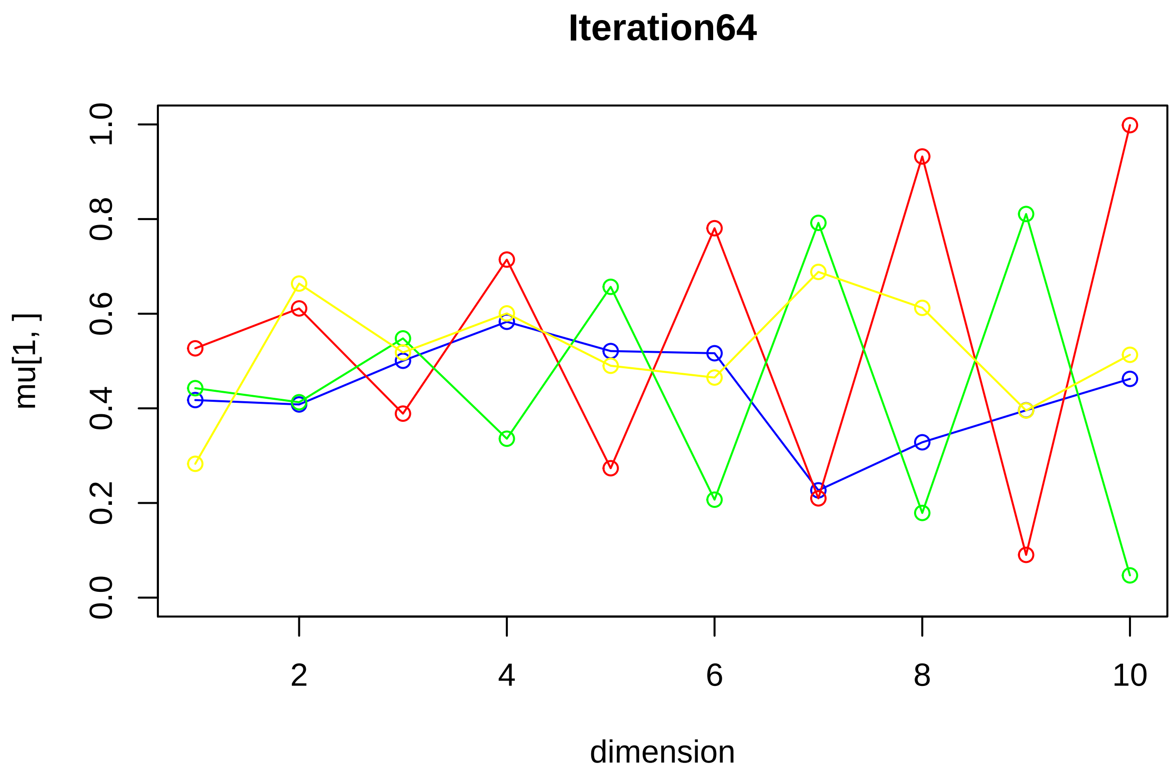

```
## iteration:  61 log likelihood:  -7225.925
```



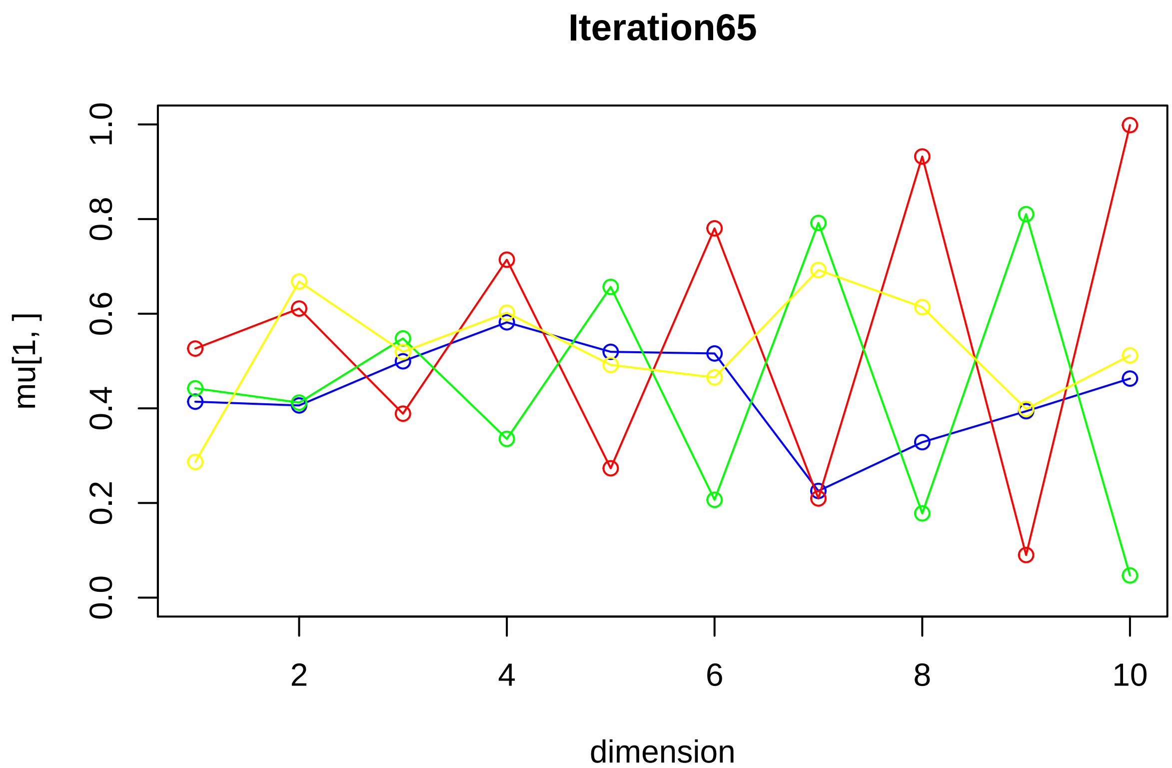

```
## iteration:  62 log likelihood:  -7223.725
```



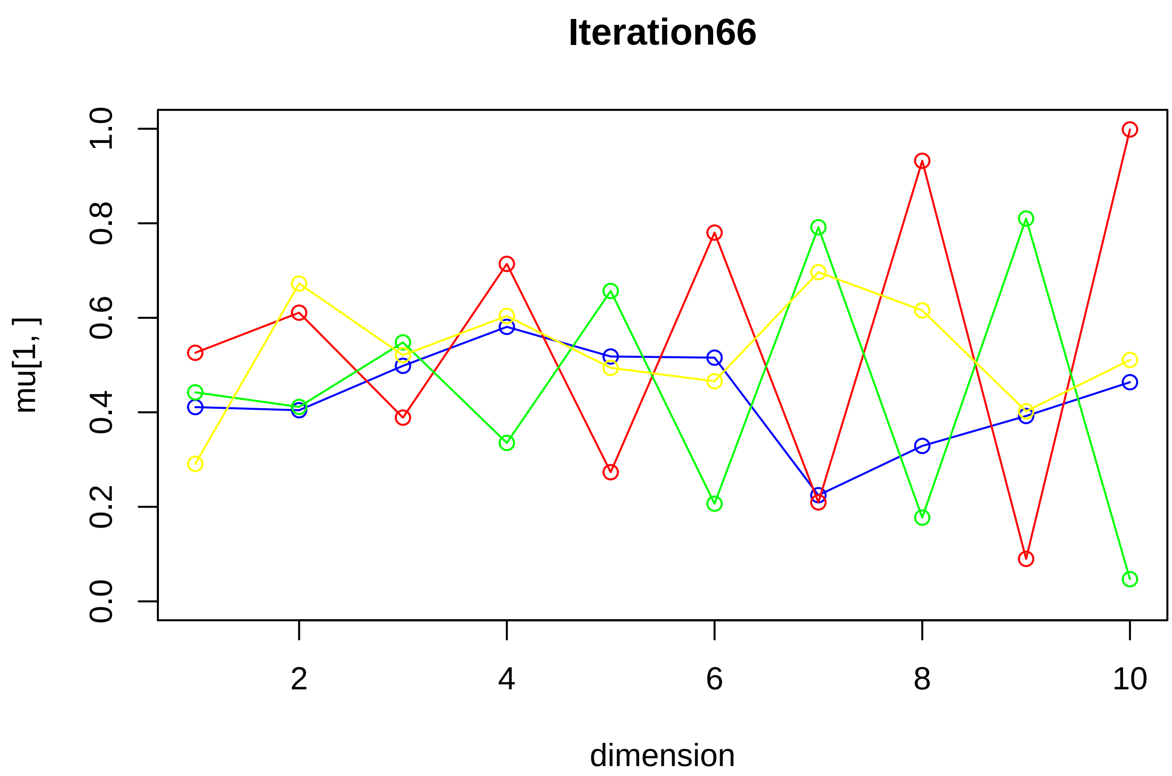

```
## iteration:  63 log likelihood:  -7221.663
```



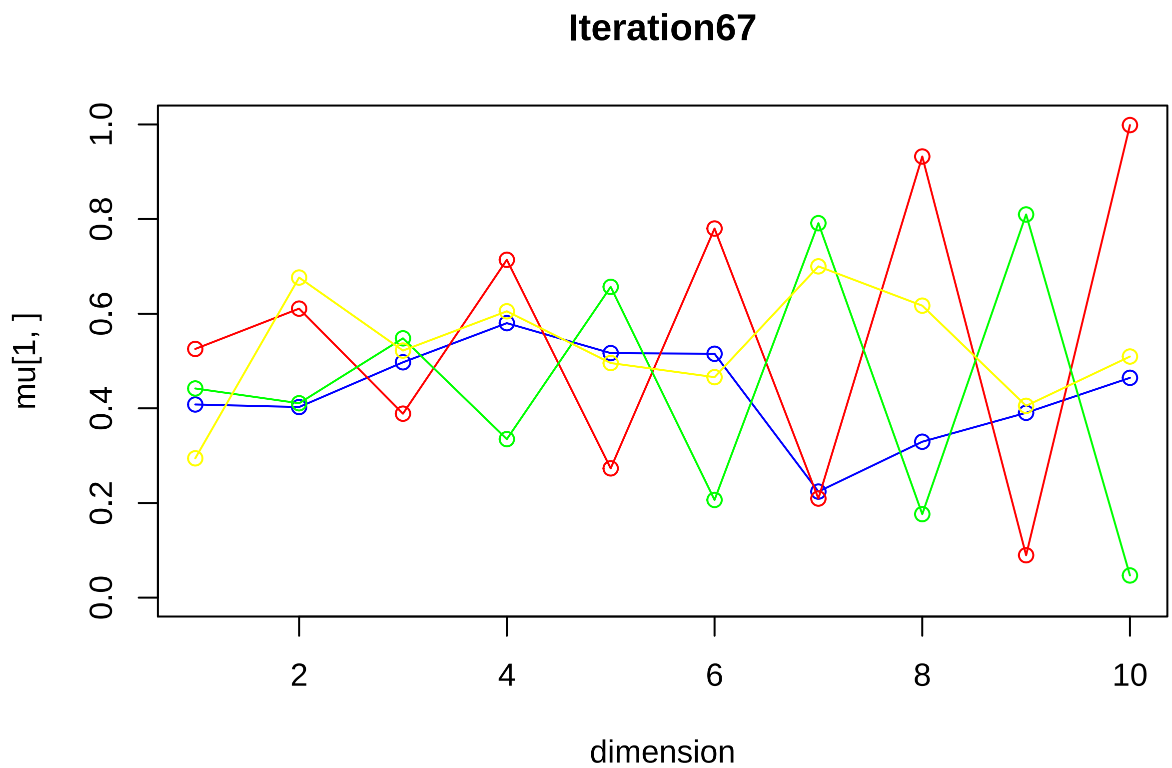

```
## iteration:  64 log likelihood:  -7219.755
```



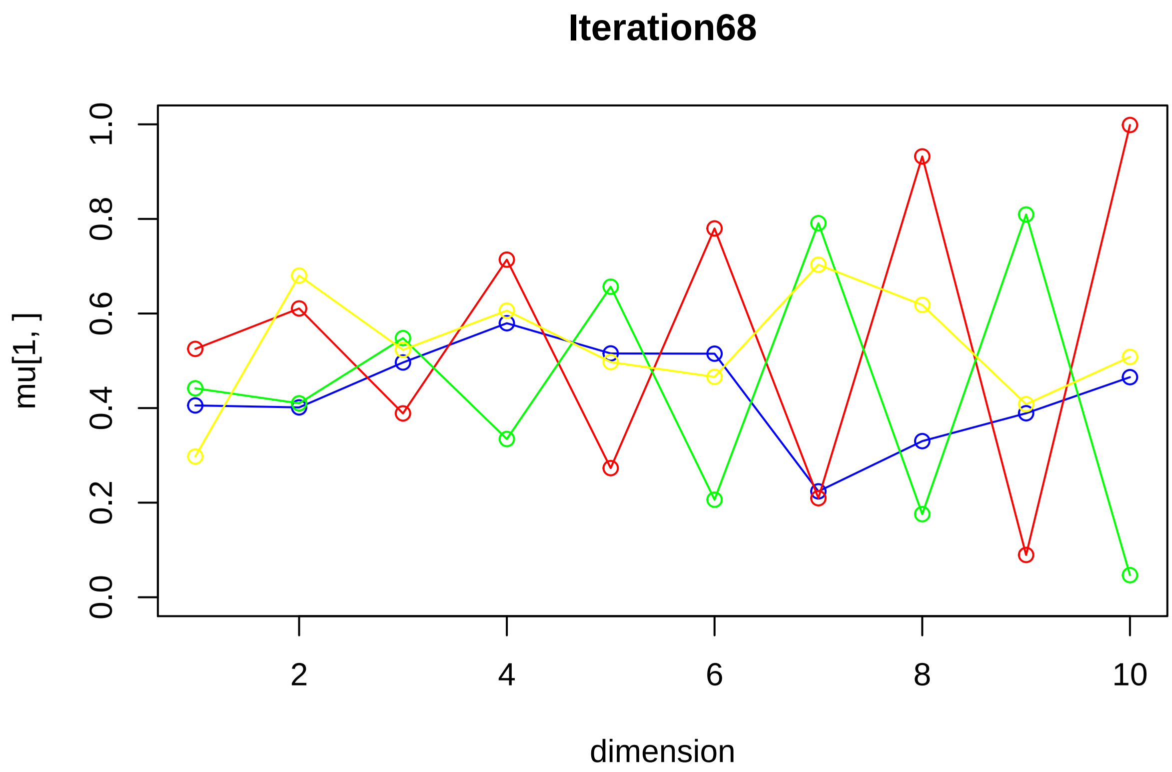

```
## iteration:  65 log likelihood:  -7218.01
```



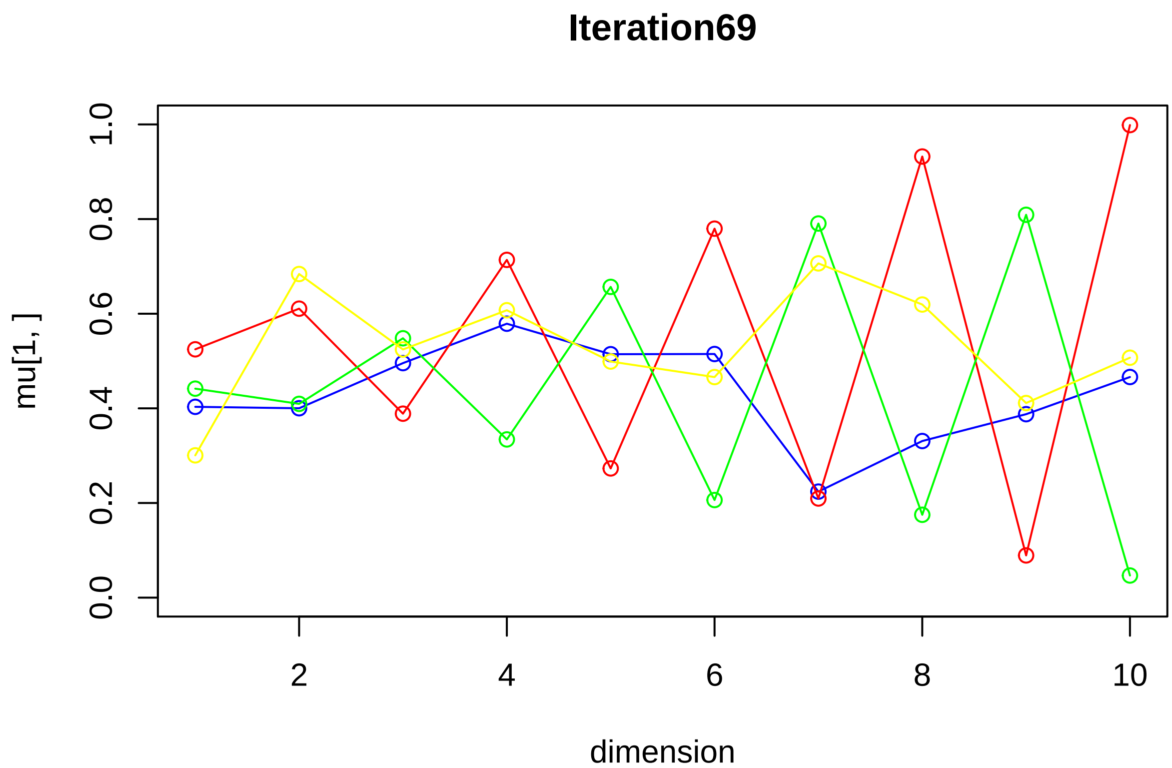

```
## iteration:  66 log likelihood:  -7216.431
```



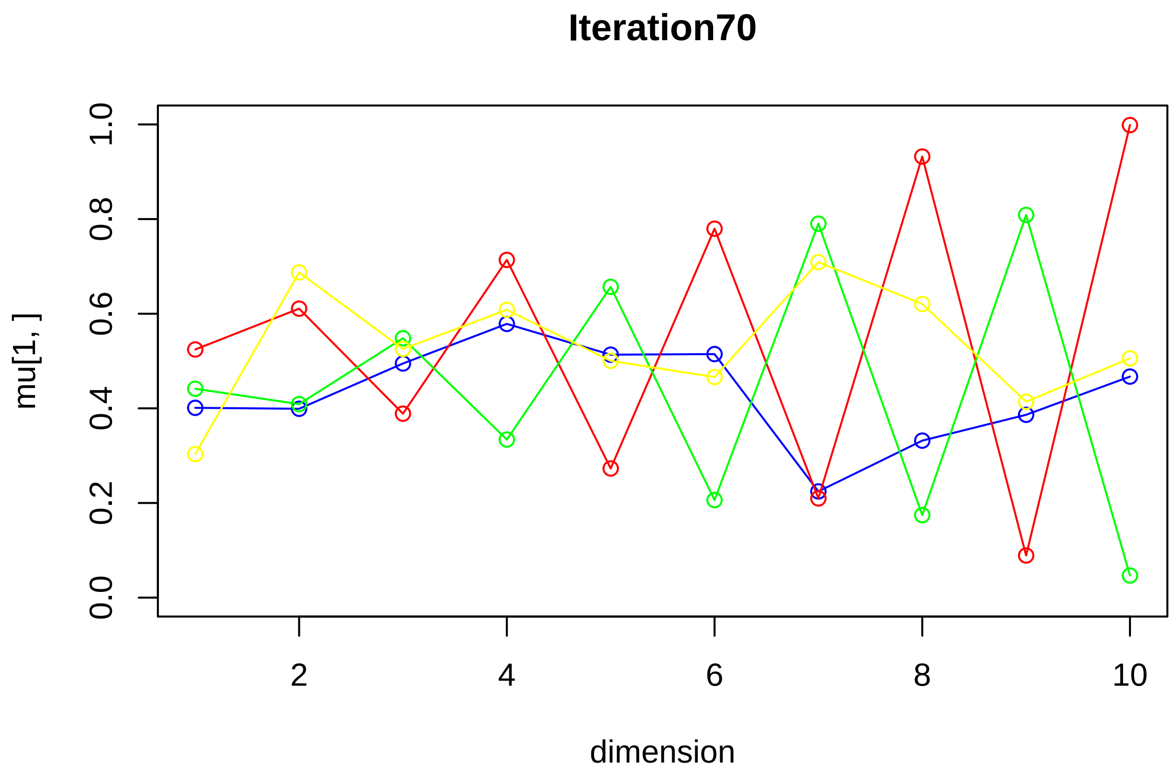

```
## iteration:  67 log likelihood:  -7215.013
```



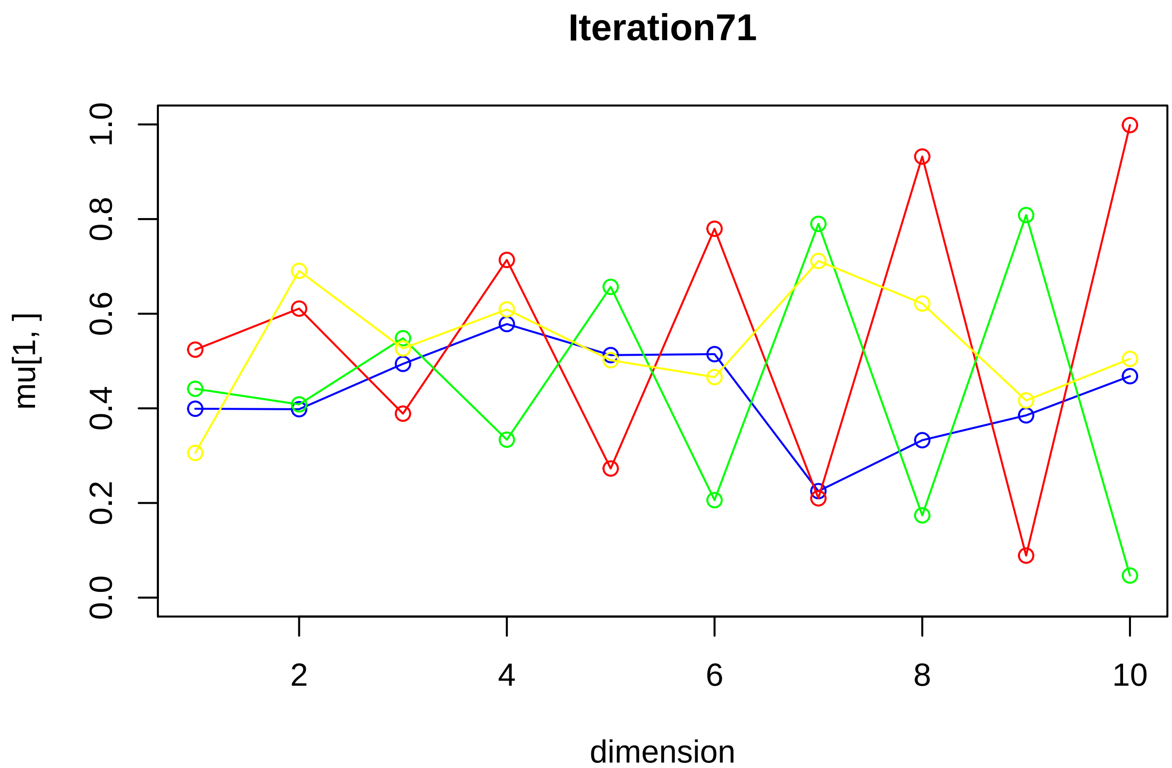

```
## iteration:  68 log likelihood:  -7213.748
```



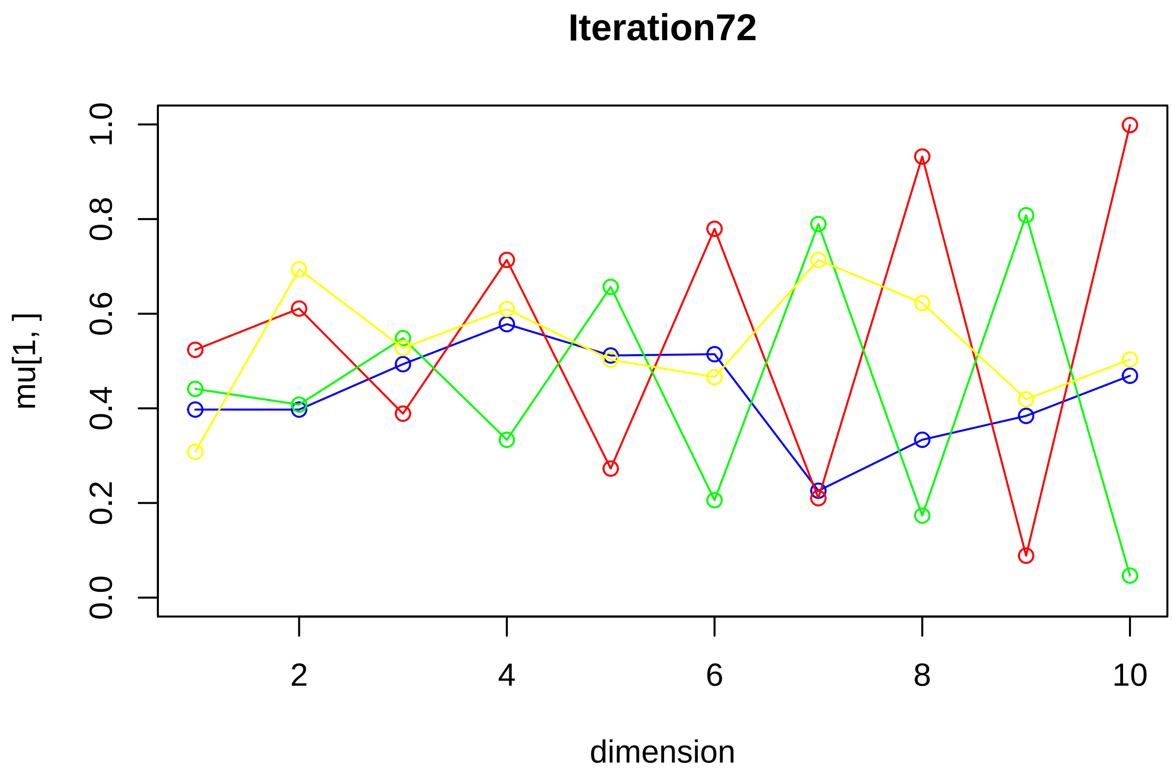

```
## iteration:  69 log likelihood:  -7212.621
```



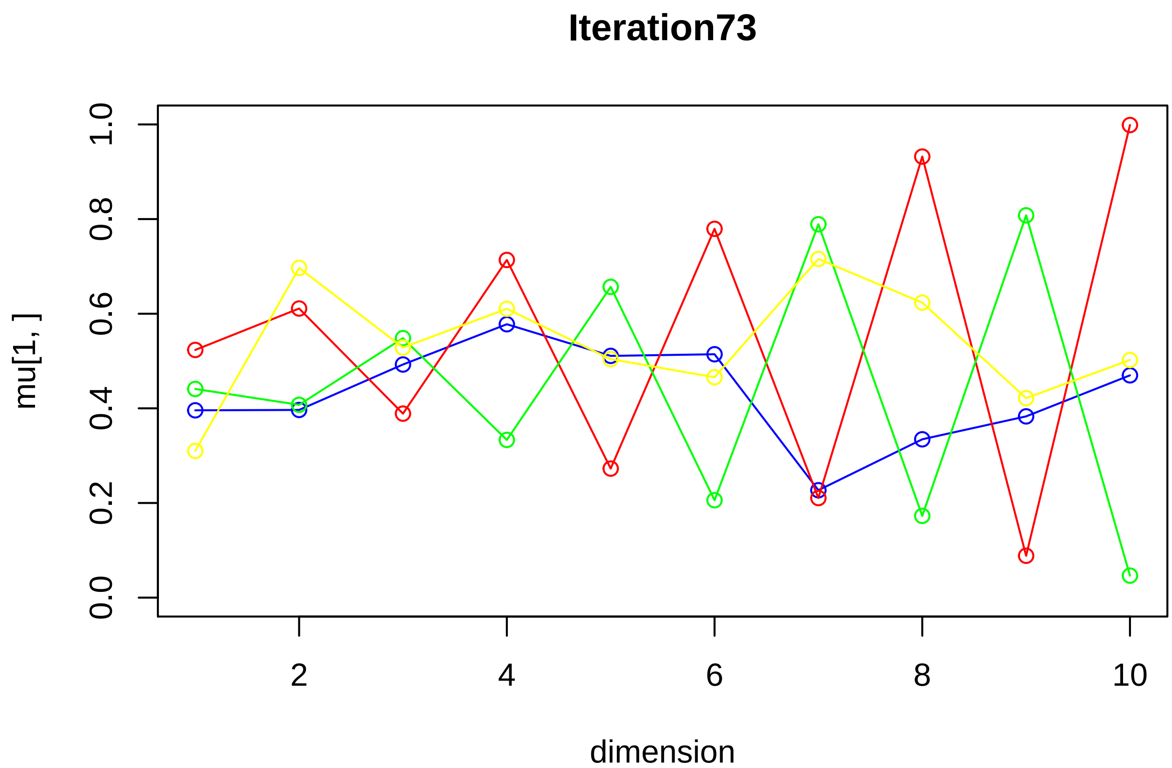

```
## iteration:  70 log likelihood:  -7211.62
```



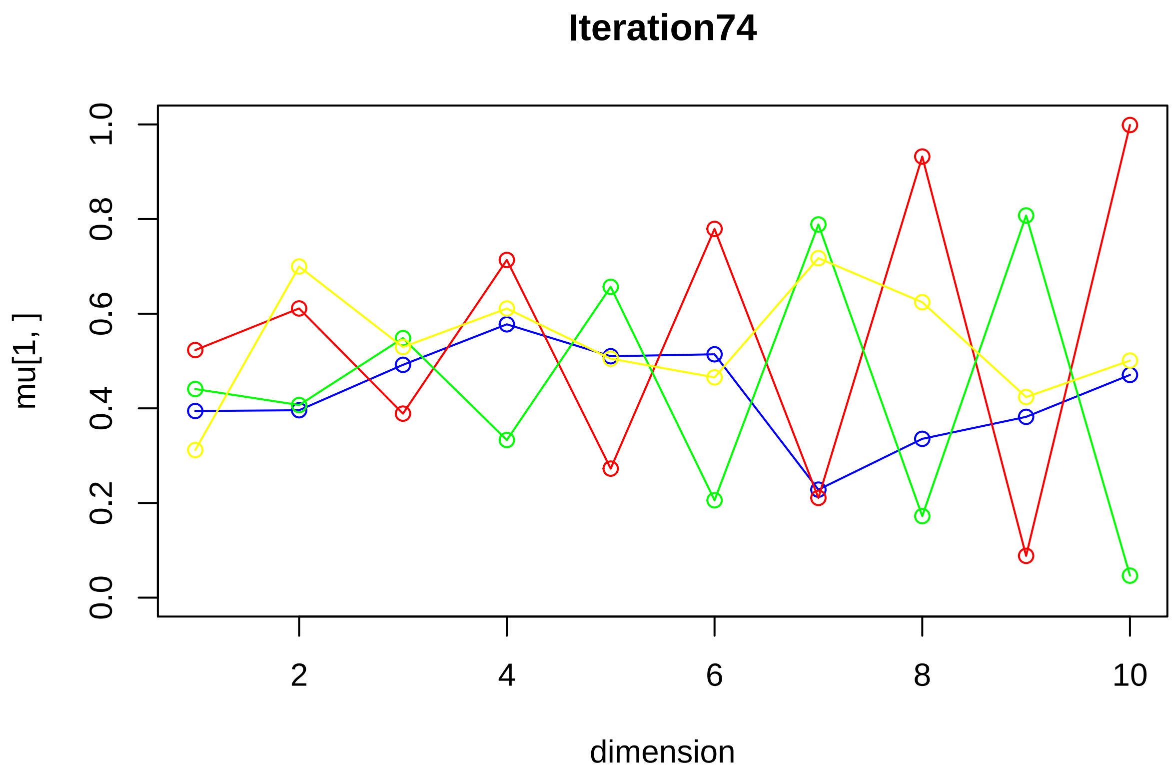

```
## iteration:  71 log likelihood:  -7210.727
```



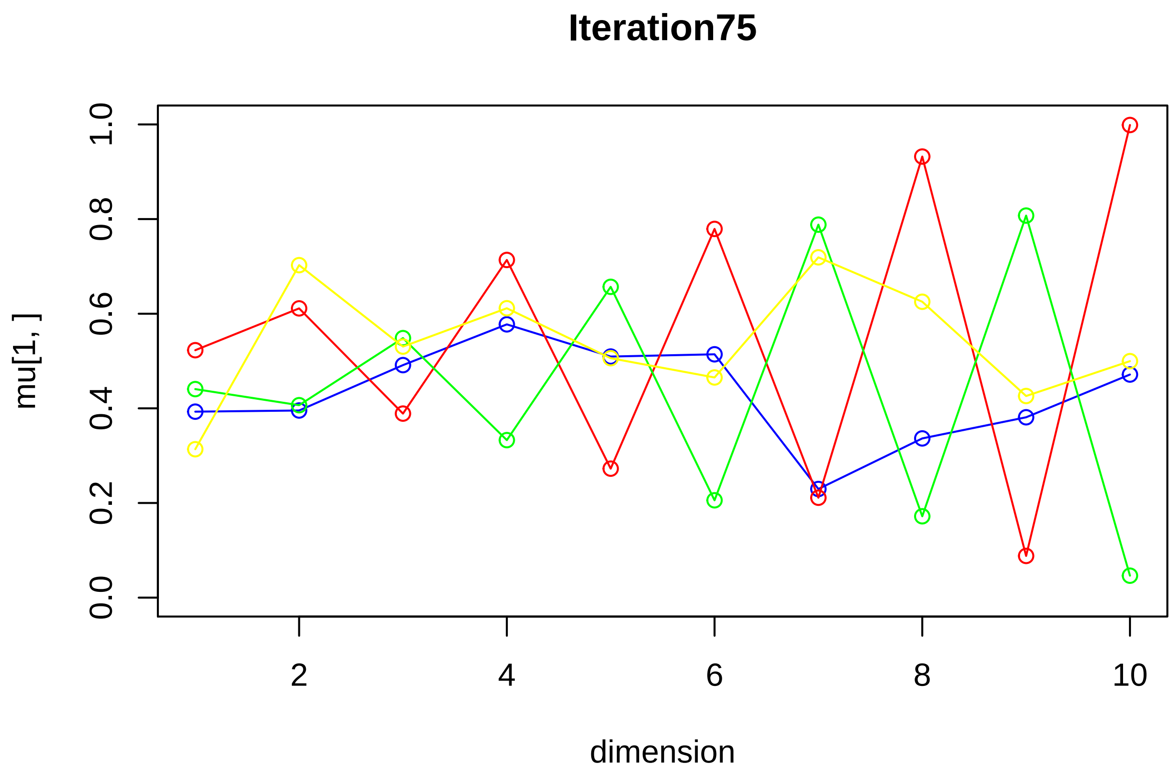

```
## iteration:  72 log likelihood:  -7209.929
```



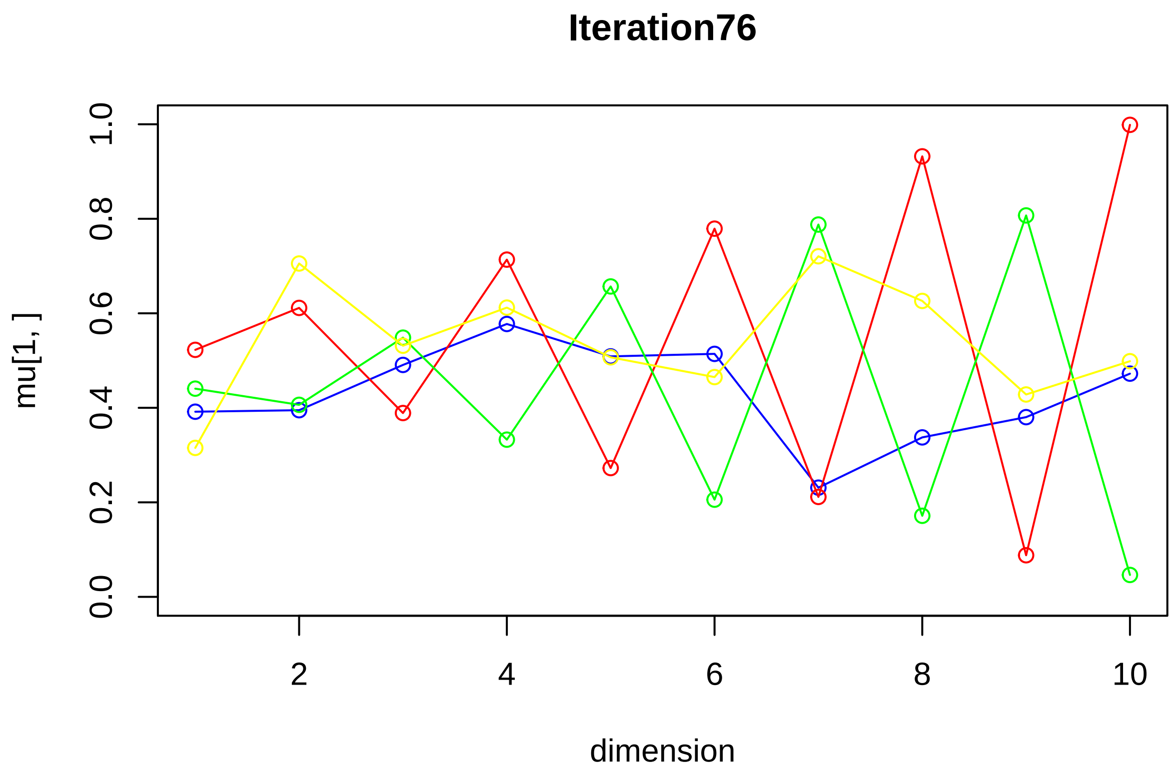

```
## iteration:  73 log likelihood:  -7209.208
```



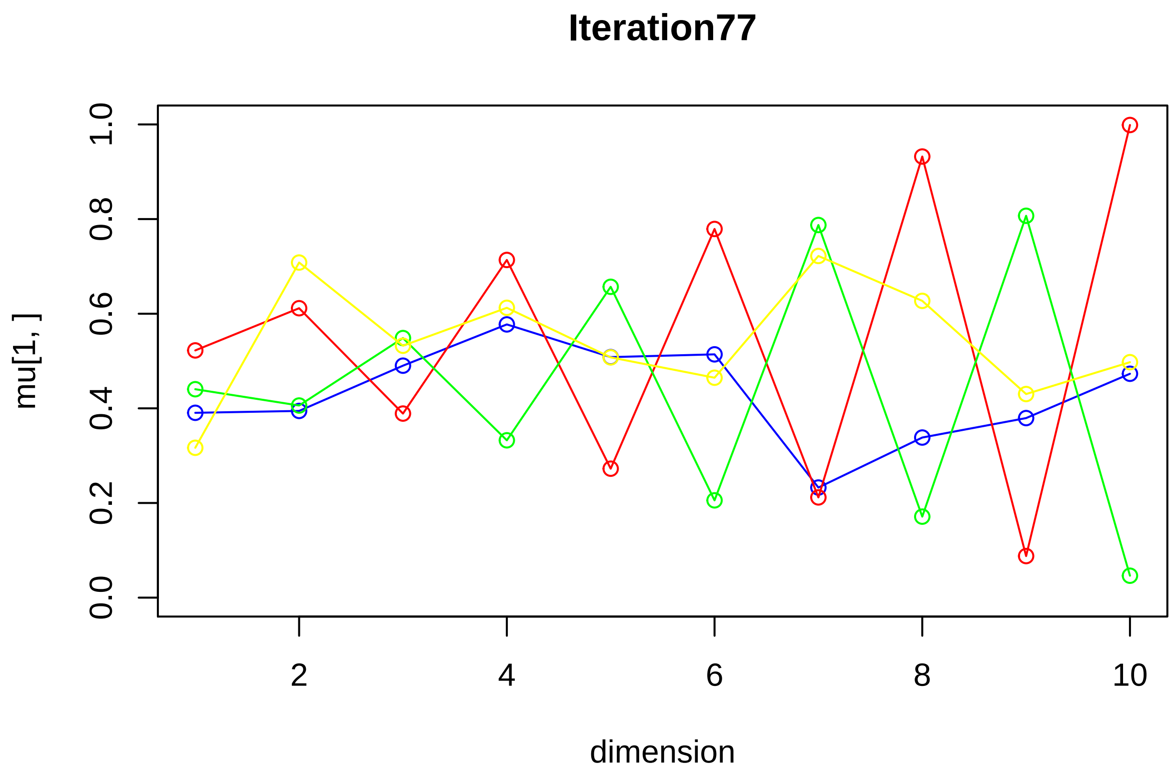

```
## iteration:  74 log likelihood:  -7208.552
```



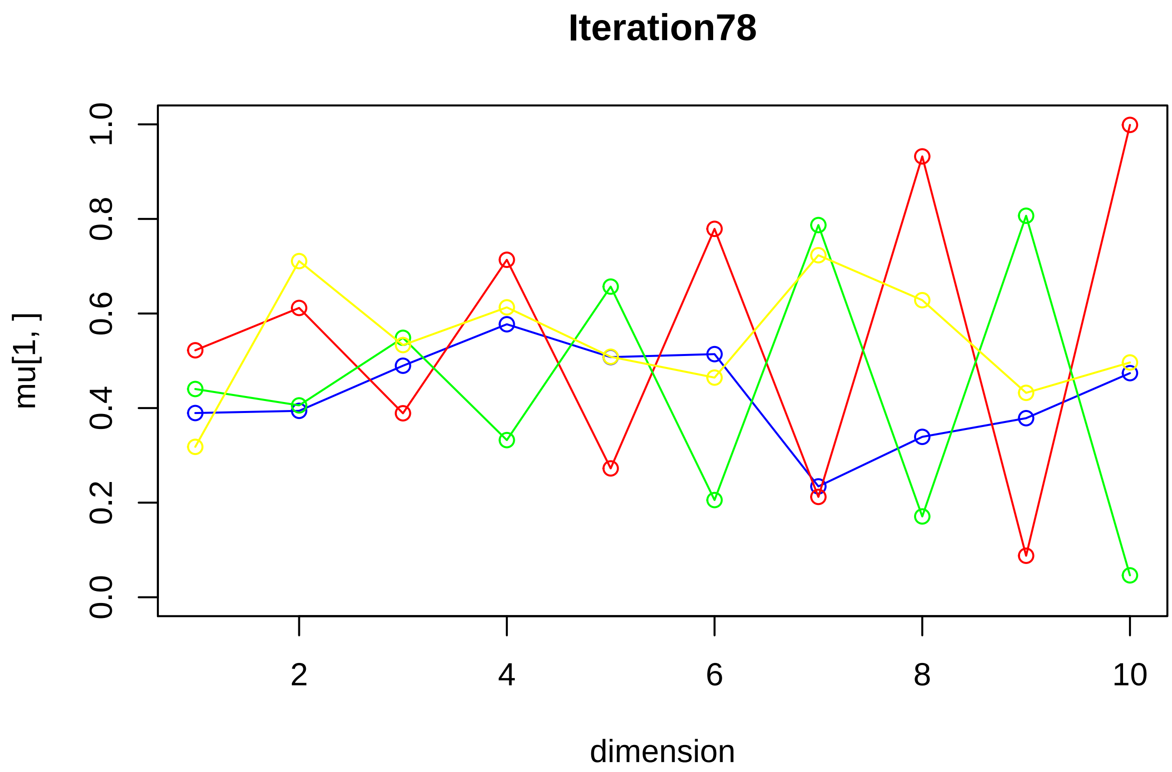

```
## iteration:  75 log likelihood:  -7207.946
```



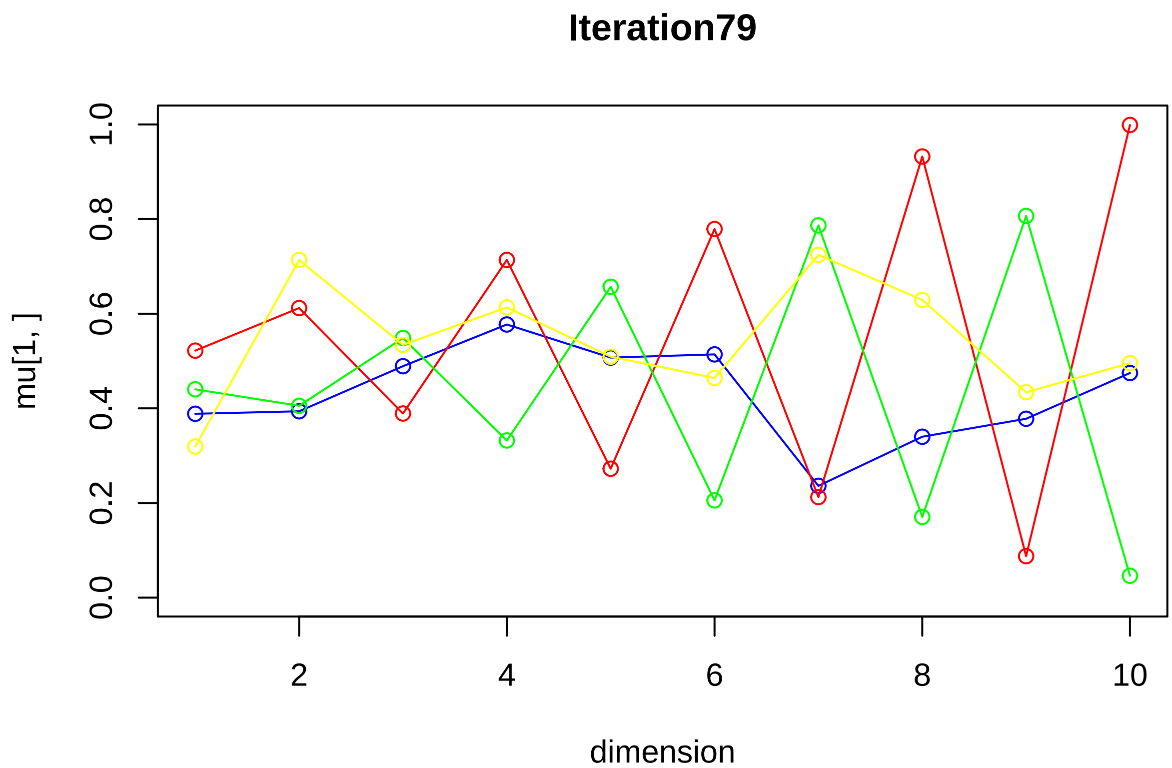

```
## iteration:  76 log likelihood:  -7207.38
```



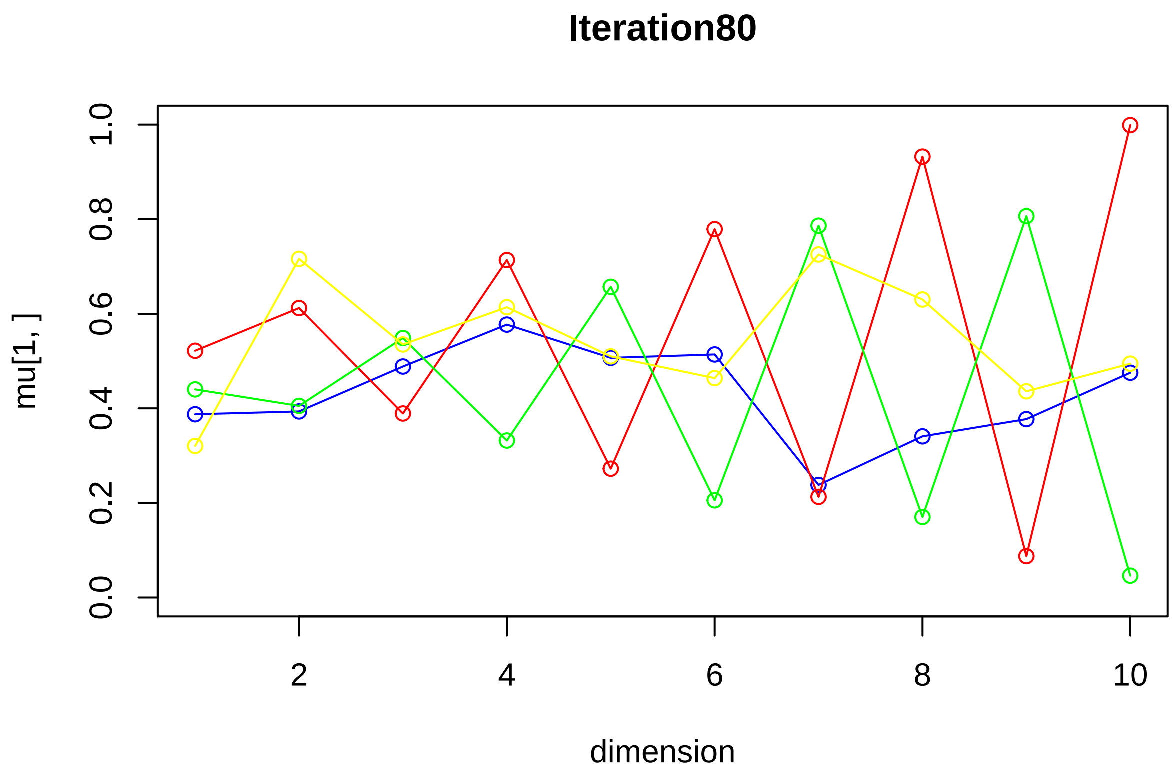

```
## iteration:  77 log likelihood:  -7206.844
```



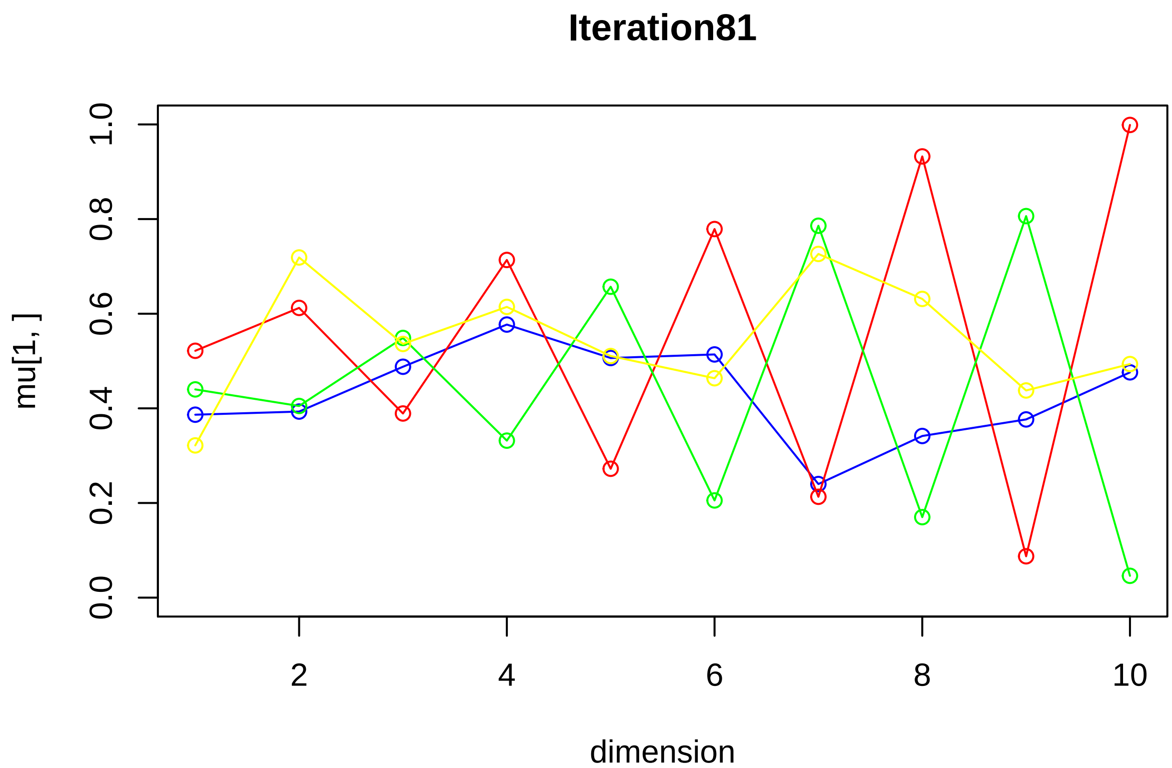

```
## iteration:  78 log likelihood:  -7206.327
```



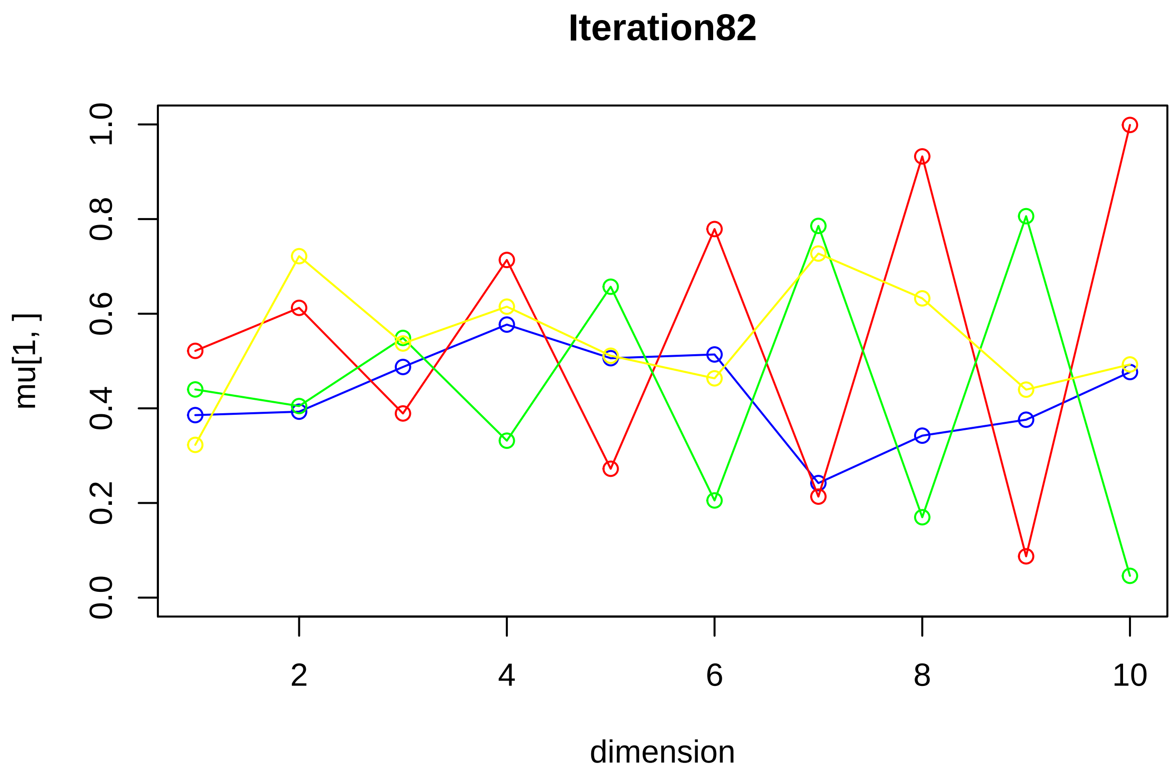

```
## iteration:  79 log likelihood:  -7205.824
```



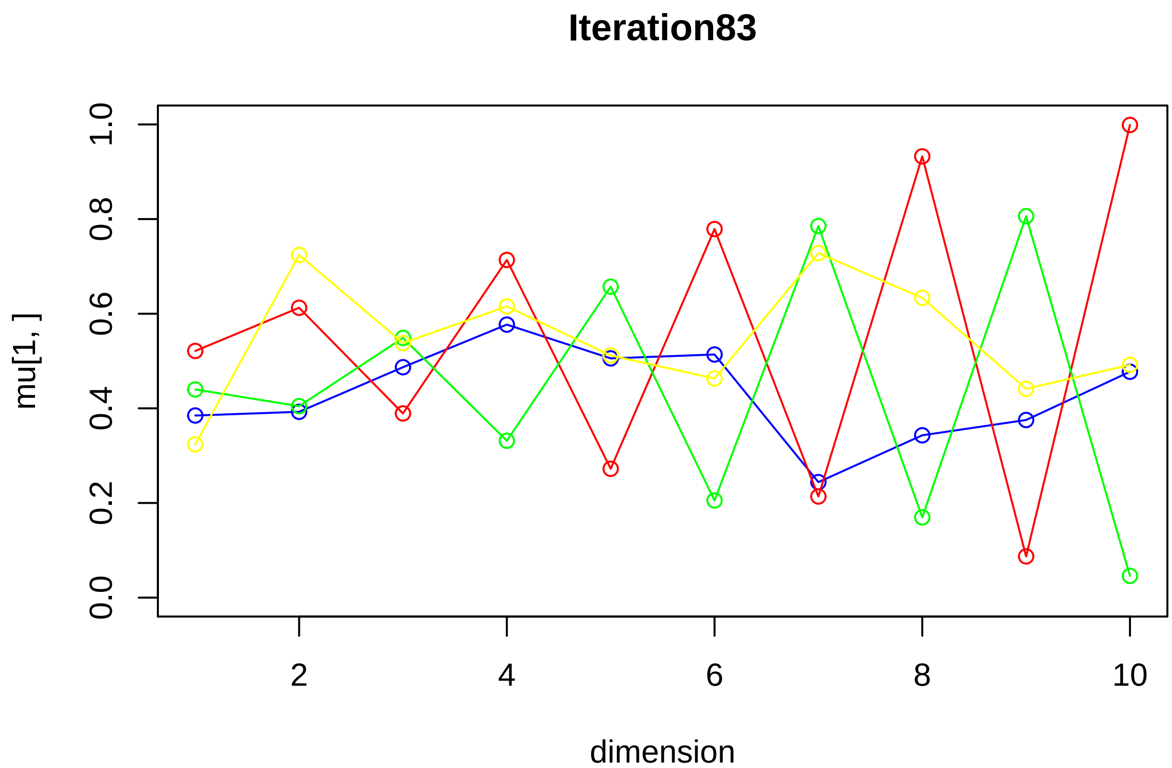

```
## iteration:  80 log likelihood:  -7205.326
```



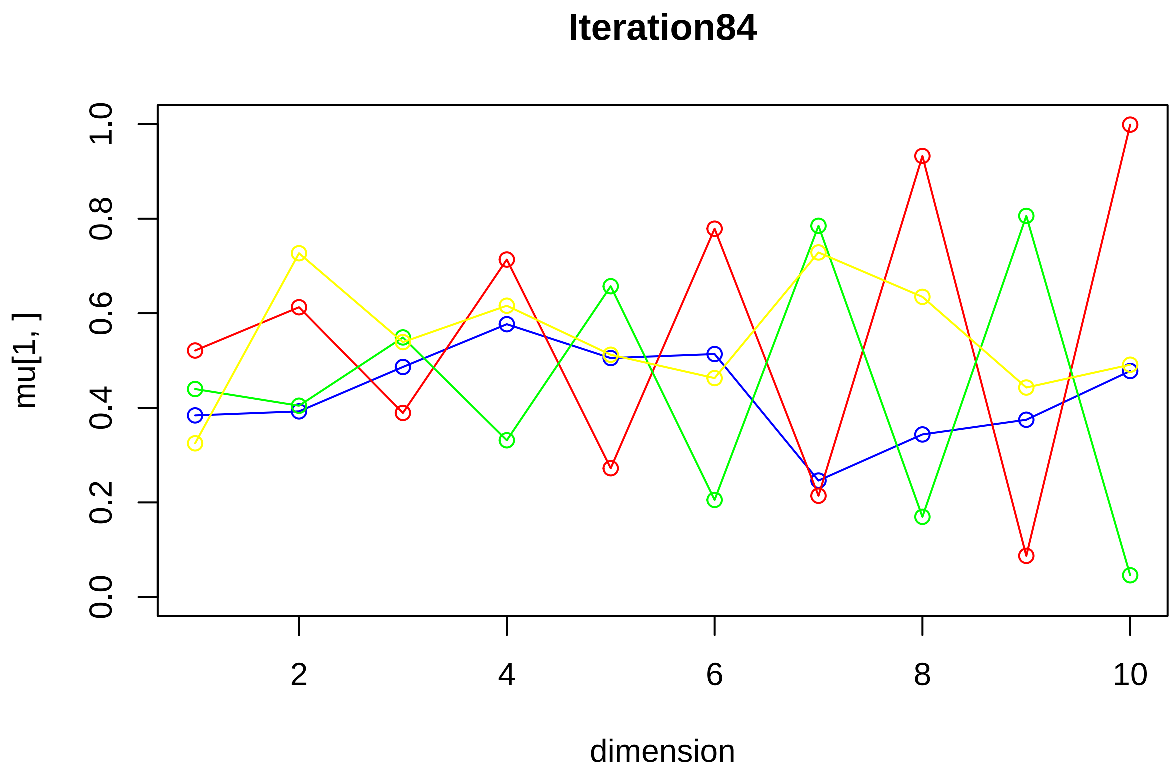

```
## iteration:  81 log likelihood:  -7204.829
```



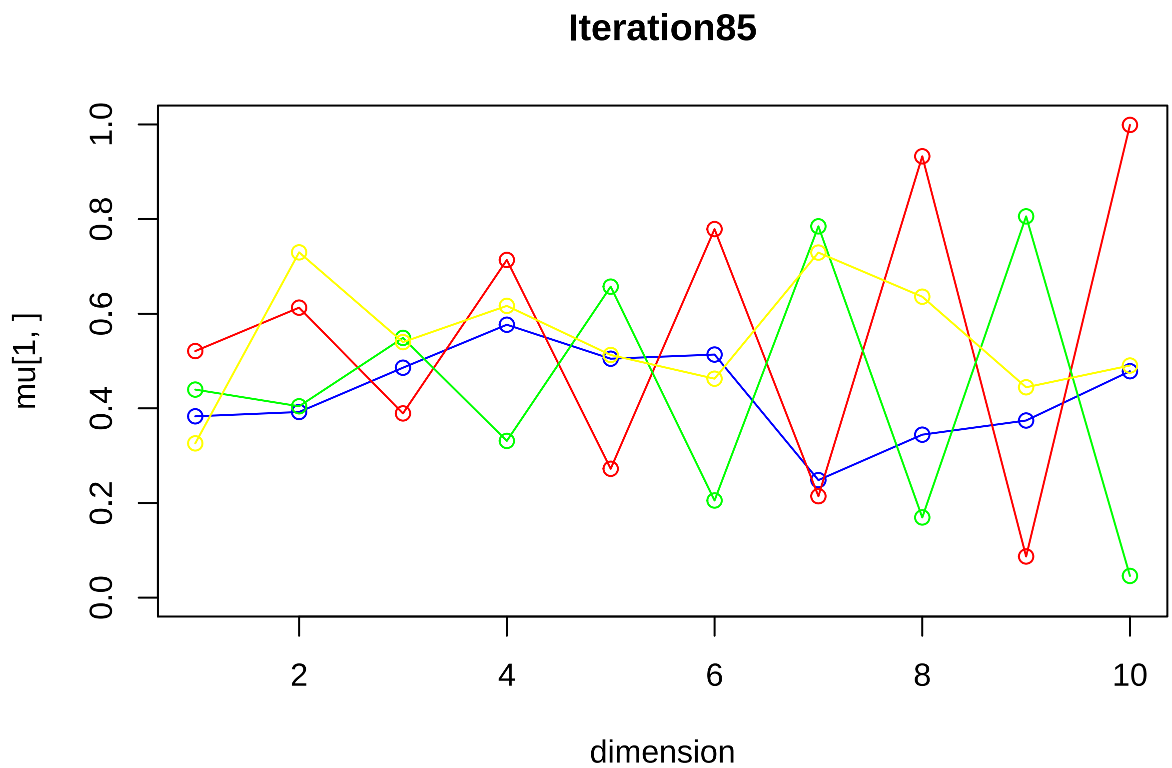

```
## iteration:  82 log likelihood:  -7204.327
```



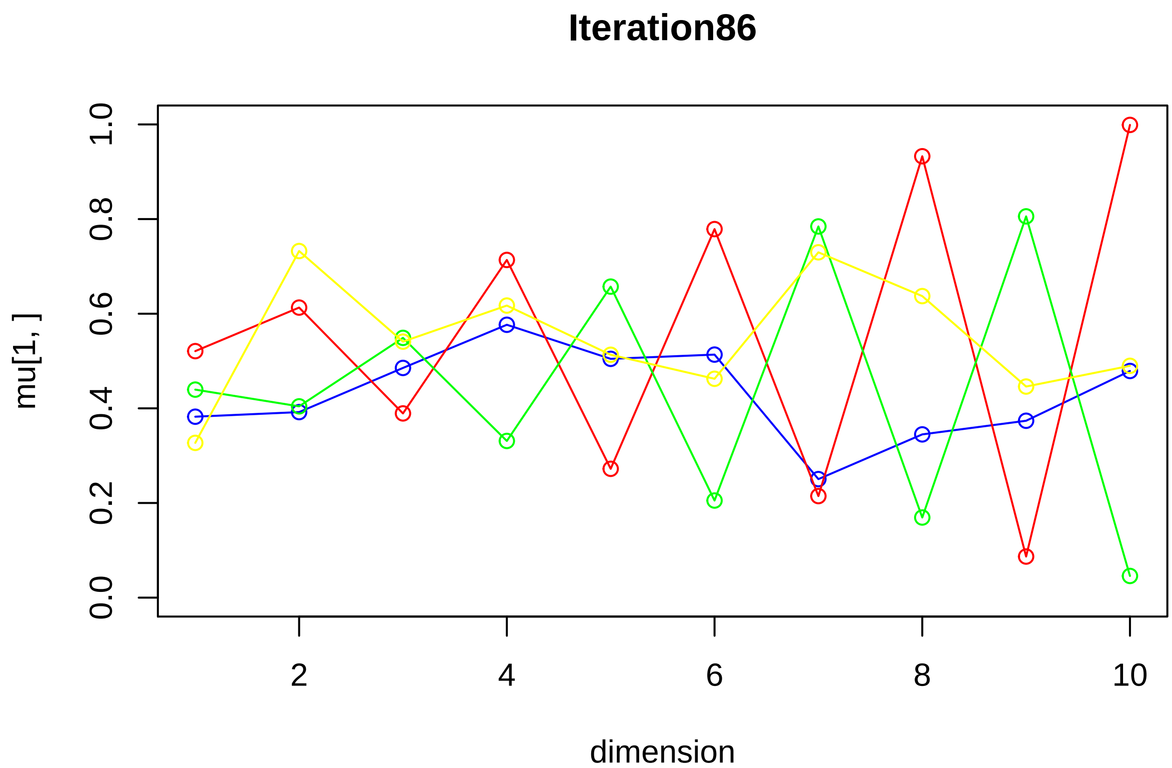

```
## iteration:  83 log likelihood:  -7203.816
```



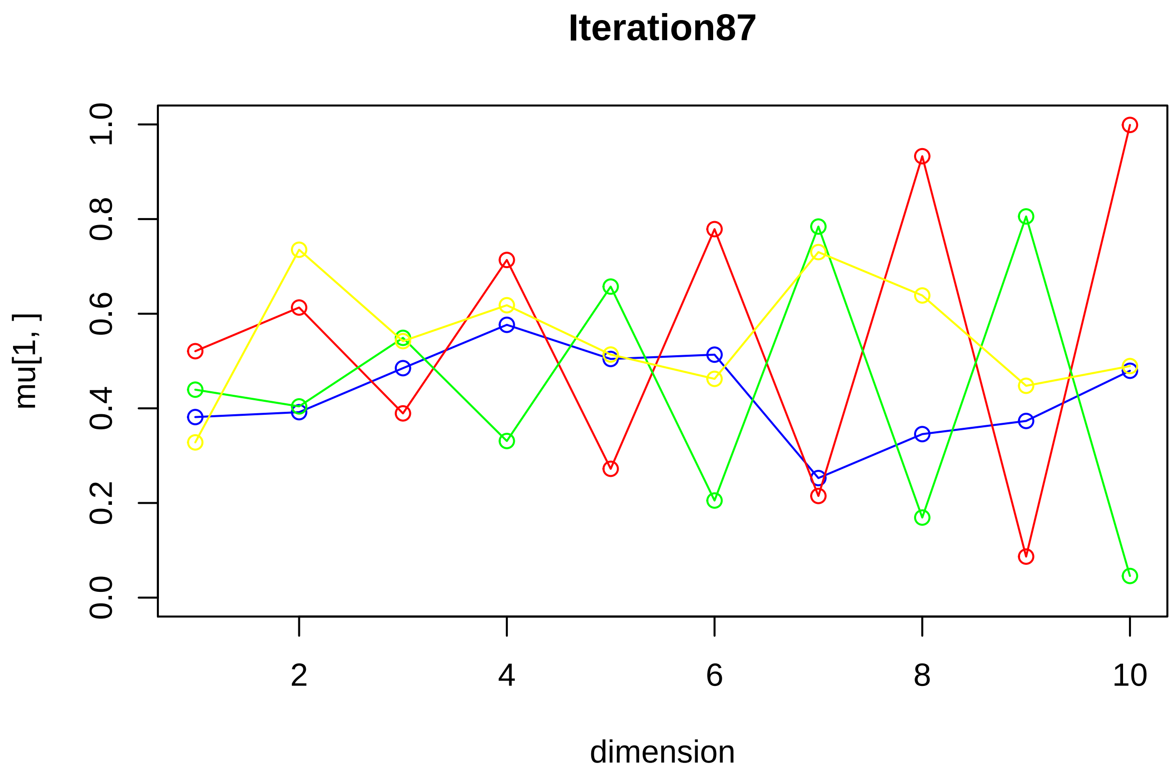

```
## iteration:  84 log likelihood:  -7203.294
```



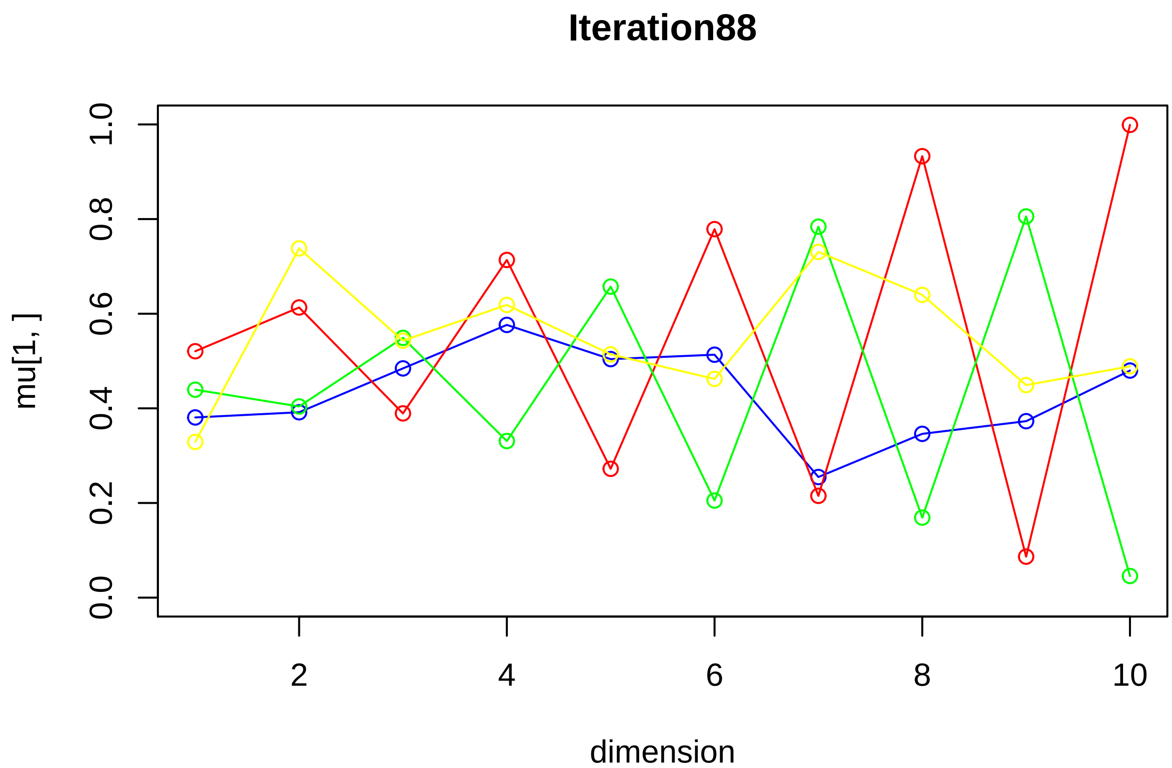

```
## iteration:  85 log likelihood:  -7202.756
```



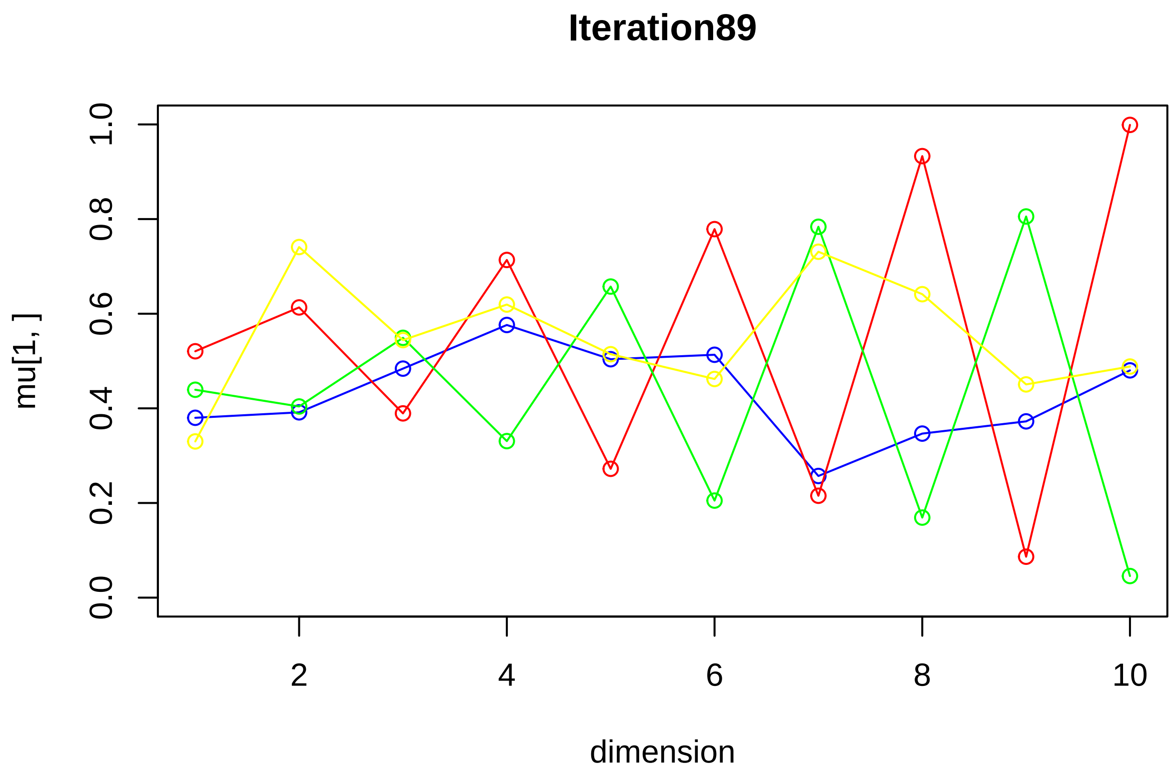

```
## iteration:  86 log likelihood:  -7202.201
```



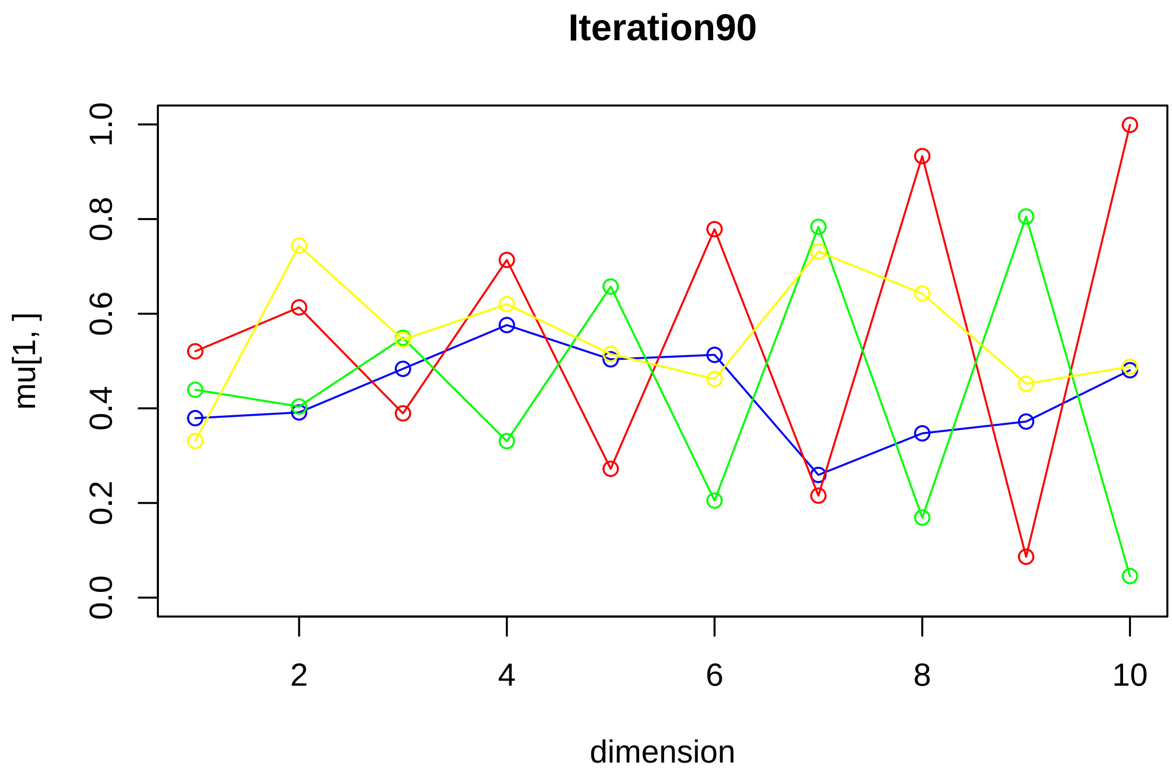

```
## iteration:  87 log likelihood:  -7201.627
```



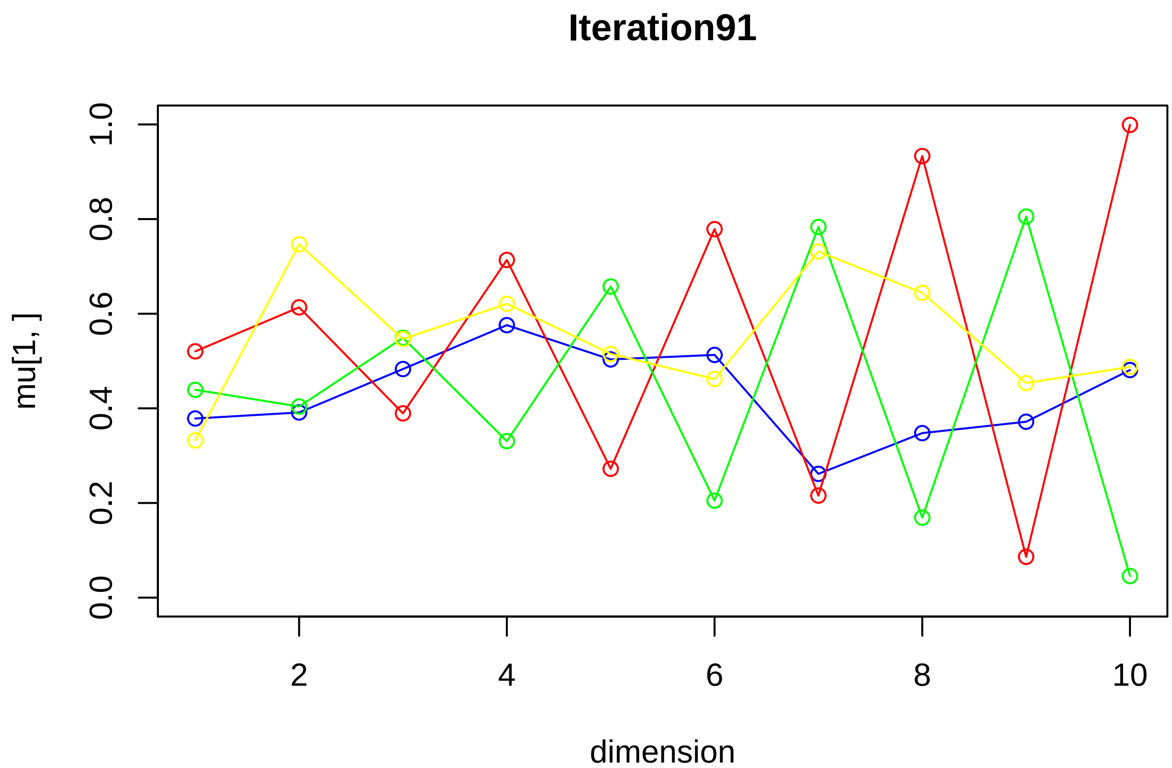

```
## iteration:  88 log likelihood:  -7201.032
```



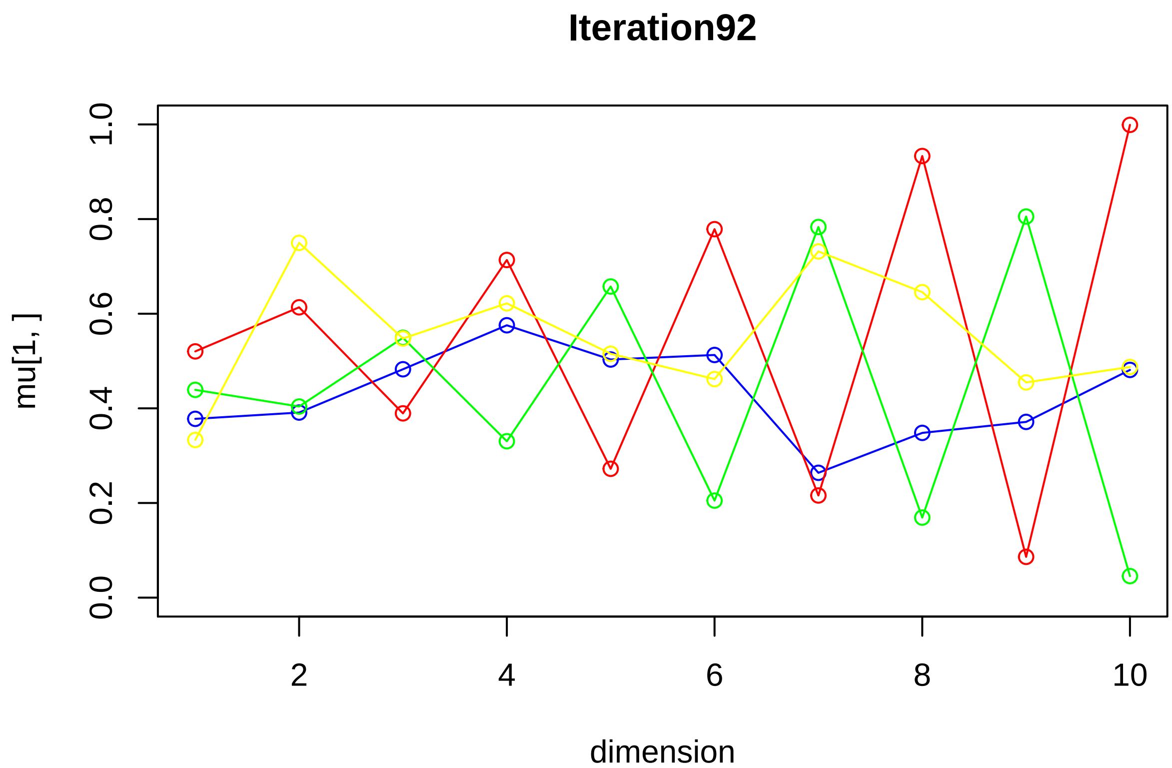

```
## iteration:  89 log likelihood:  -7200.414
```



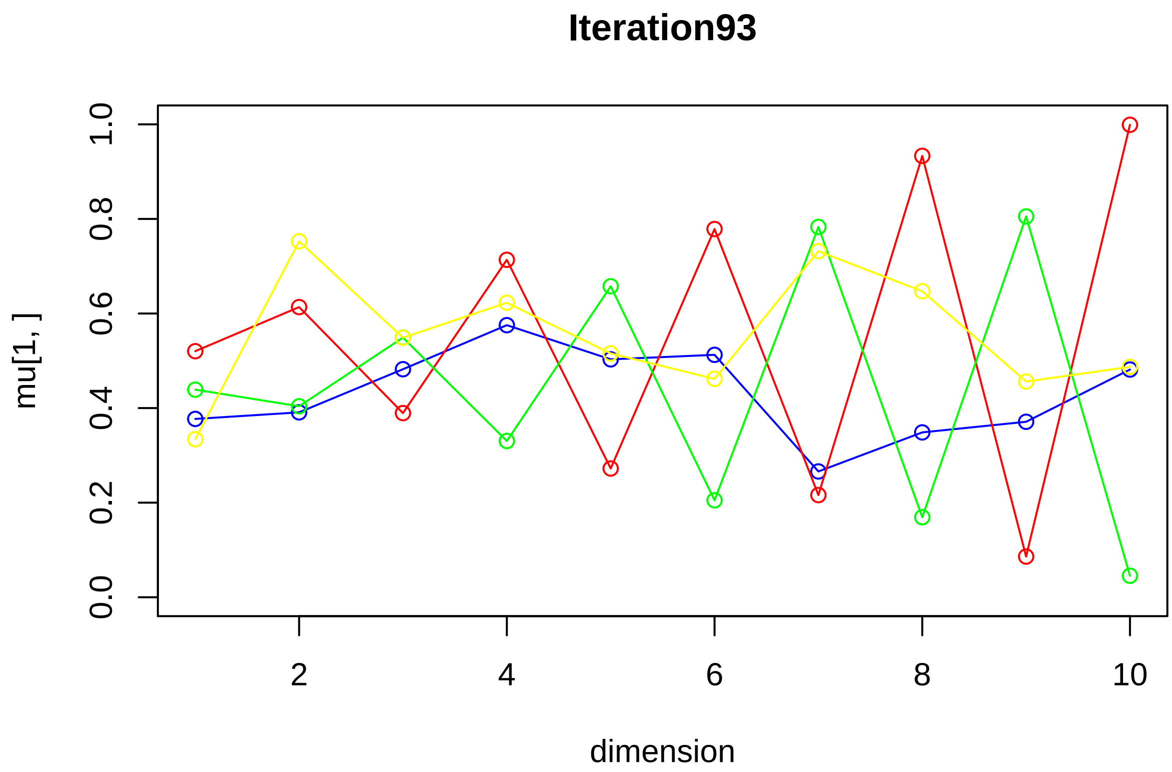

```
## iteration:  90 log likelihood:  -7199.773
```



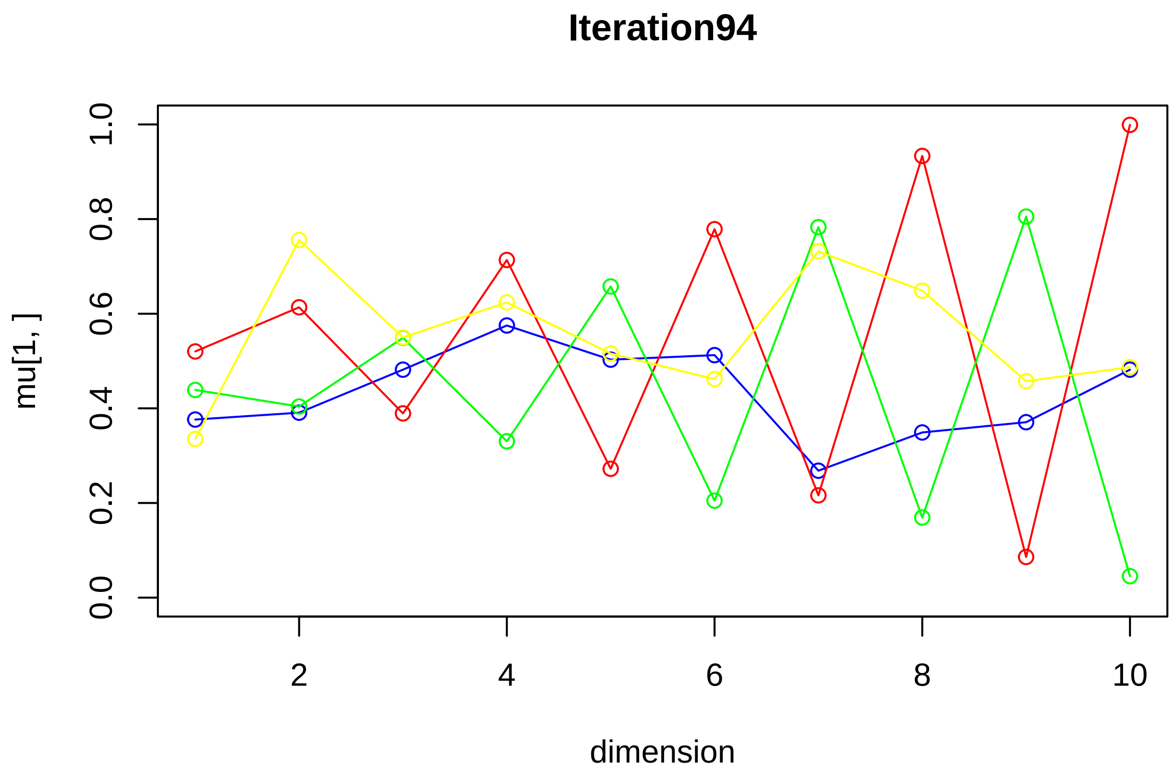

```
## iteration:  91 log likelihood:  -7199.107
```



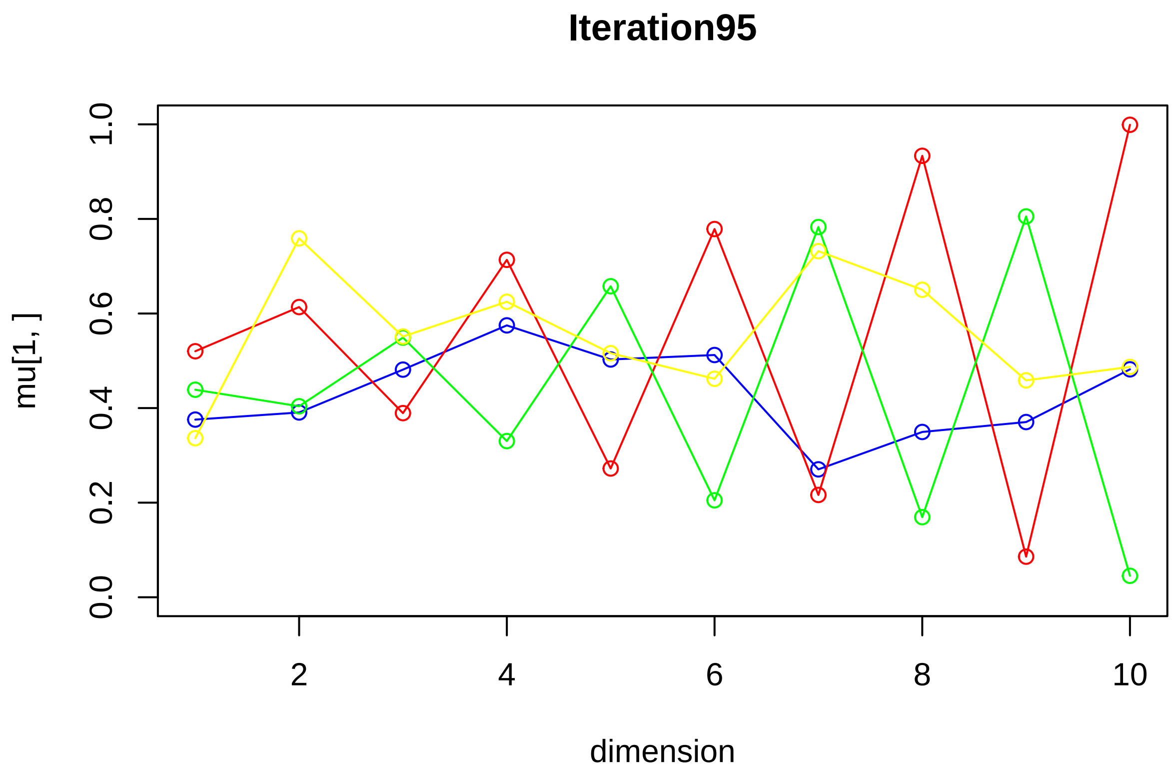

```
## iteration:  92 log likelihood:  -7198.416
```



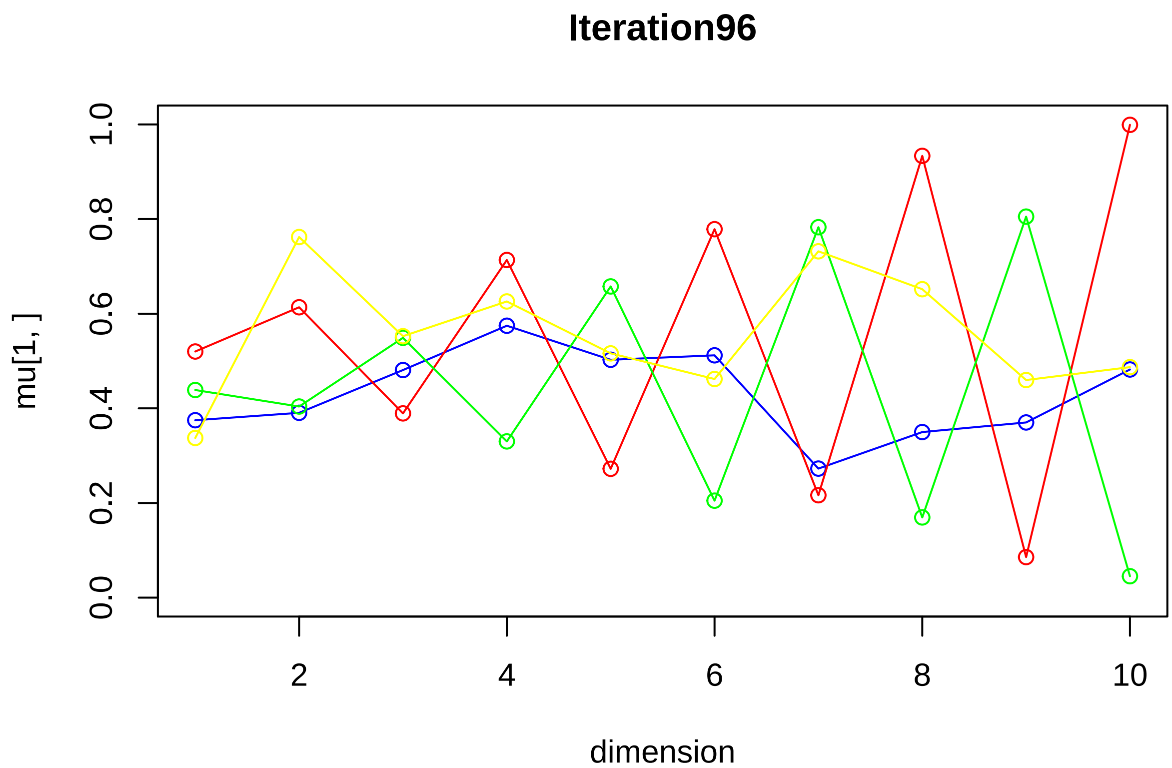

```
## iteration:  93 log likelihood:  -7197.7
```



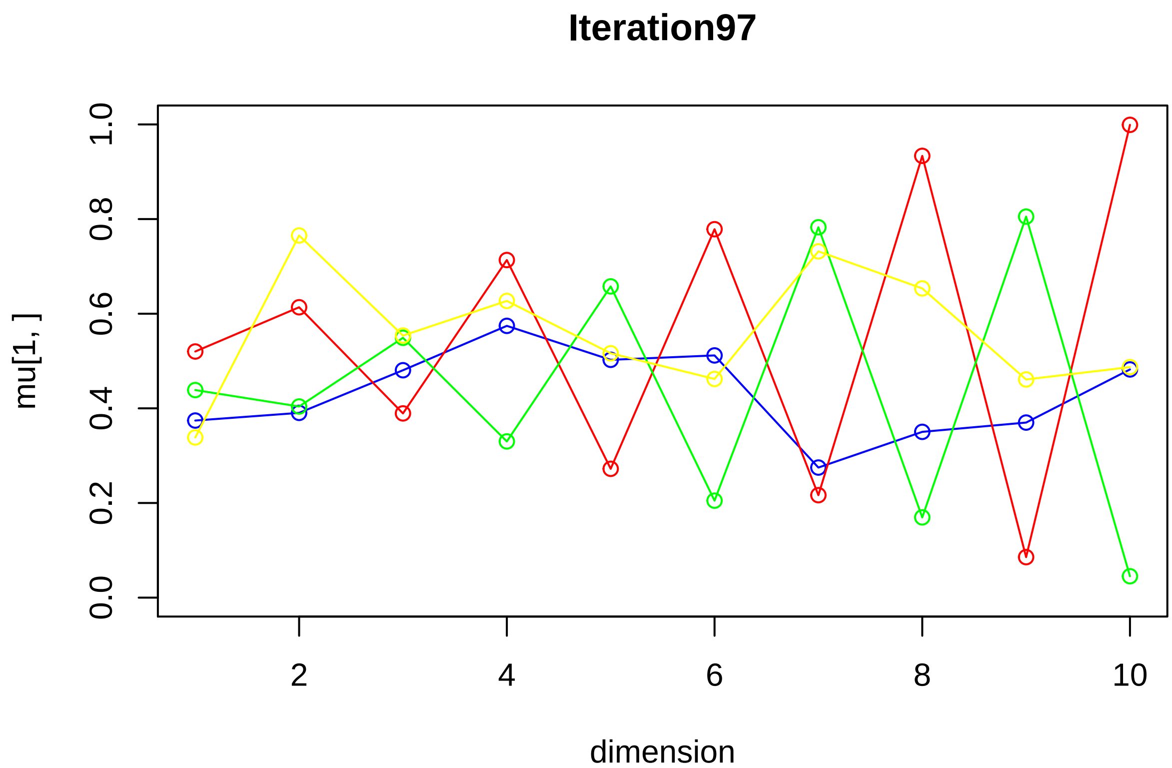

```
## iteration:  94 log likelihood:  -7196.957
```



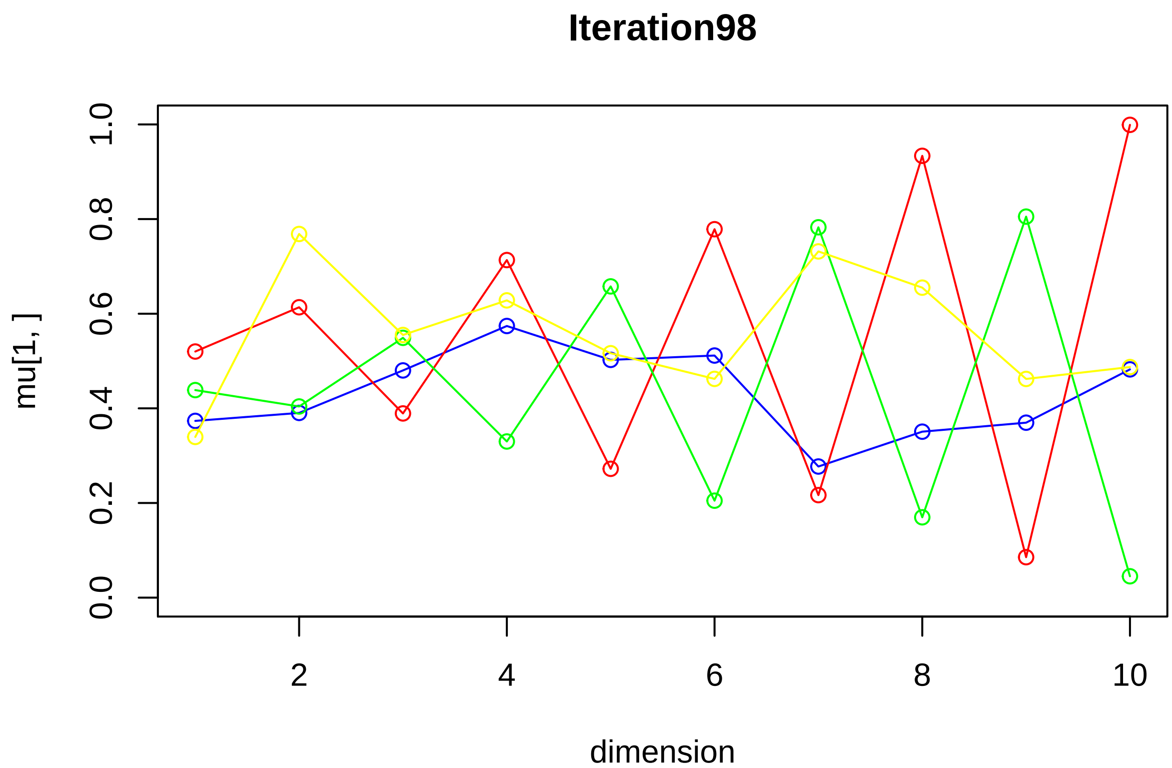

```
## iteration:  95 log likelihood:  -7196.188
```



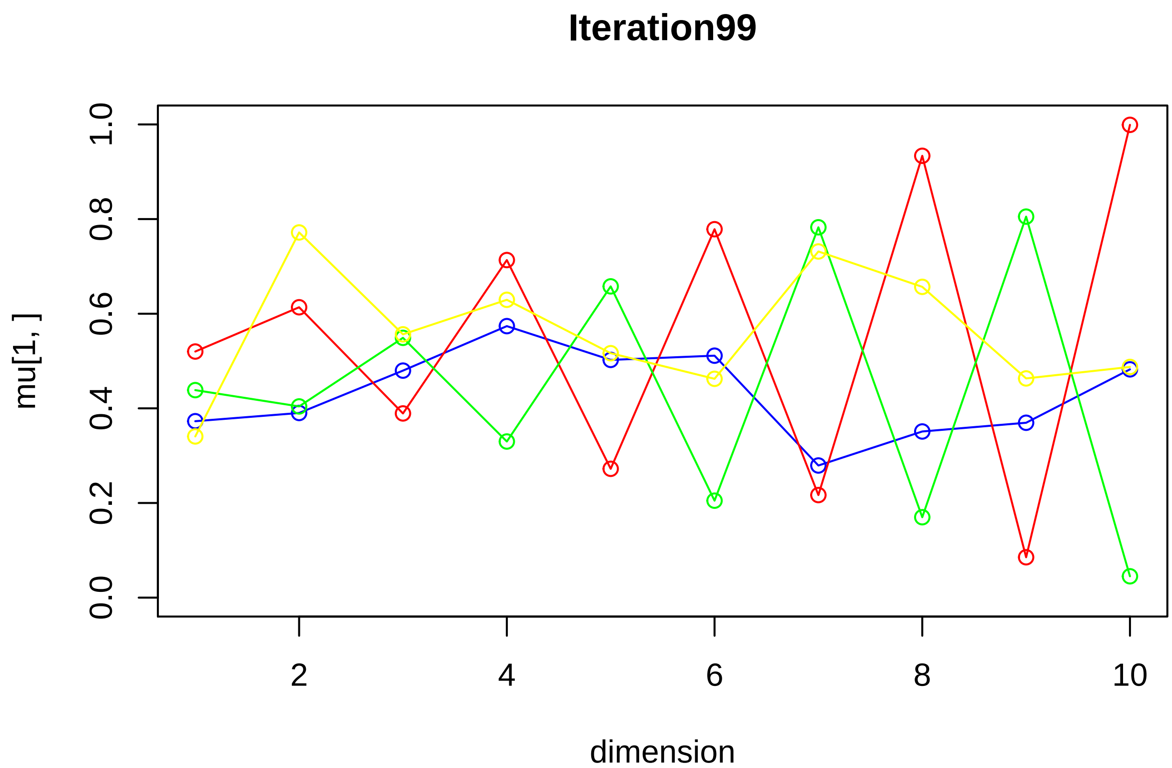

```
## iteration:  96 log likelihood:  -7195.392
```



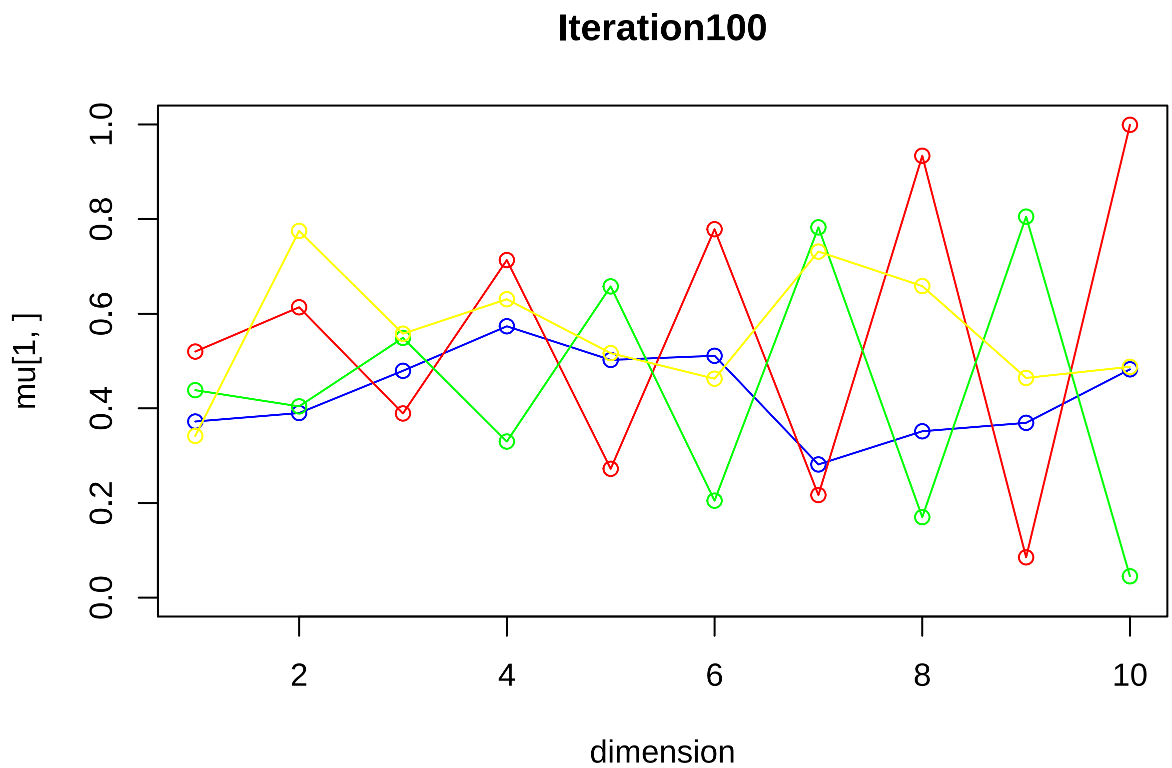

```
## iteration:  97 log likelihood:  -7194.57
```



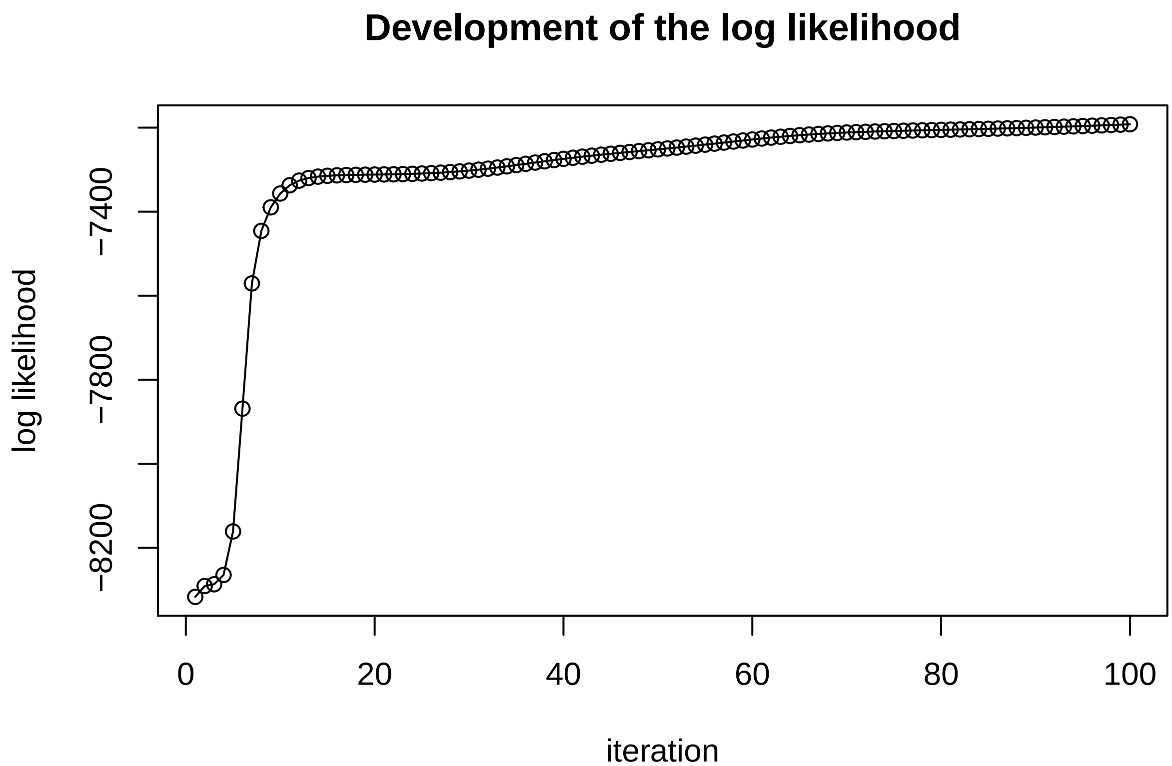

```
## iteration:  98 log likelihood:  -7193.722
```




```
## iteration:  99 log likelihood:  -7192.847
```




```
## iteration:  100 log likelihood:  -7191.946
```




```
## $pi
## [1] 0.2880470 0.2533761 0.2933710 0.1652060
##
## $mu
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.3714855 0.3899958 0.4790260 0.5731886 0.5022651 0.5108478 0.2835691
## [2,] 0.5199997 0.6135841 0.3891214 0.7132736 0.2722448 0.7785461 0.2168891
## [3,] 0.4383456 0.4042497 0.5489526 0.3298363 0.6578057 0.2049012 0.7825505
## [4,] 0.3428531 0.7784238 0.5591637 0.6319621 0.5167044 0.4629058 0.7311279
##           [,8]       [,9]      [,10]
## [1,] 0.3519184 0.36924863 0.48252239
## [2,] 0.9337959 0.08504806 0.99916297
## [3,] 0.1703330 0.80517853 0.04500171
## [4,] 0.6601375 0.46532151 0.48814639
##
## $logLikelihoodDevelopment
## NULL
```