machine learning(732A99) lab1 Block2

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Contents

Contributions

During the lab, Lennart and Thijs focused on assignment 2 using loops, Anubhav focused on assignment 1. All codes and analysis was indepedently done and is also reflected in the individual reports.

1. Ensemble Methods

Code

Loading Input files

```
spam_data <- read.csv(file = "spambase.data", header = FALSE)
colnames(spam_data)[58] <- "Spam"
spam_data$Spam <- factor(spam_data$Spam, levels = c(0,1), labels = c("0", "1"))
```

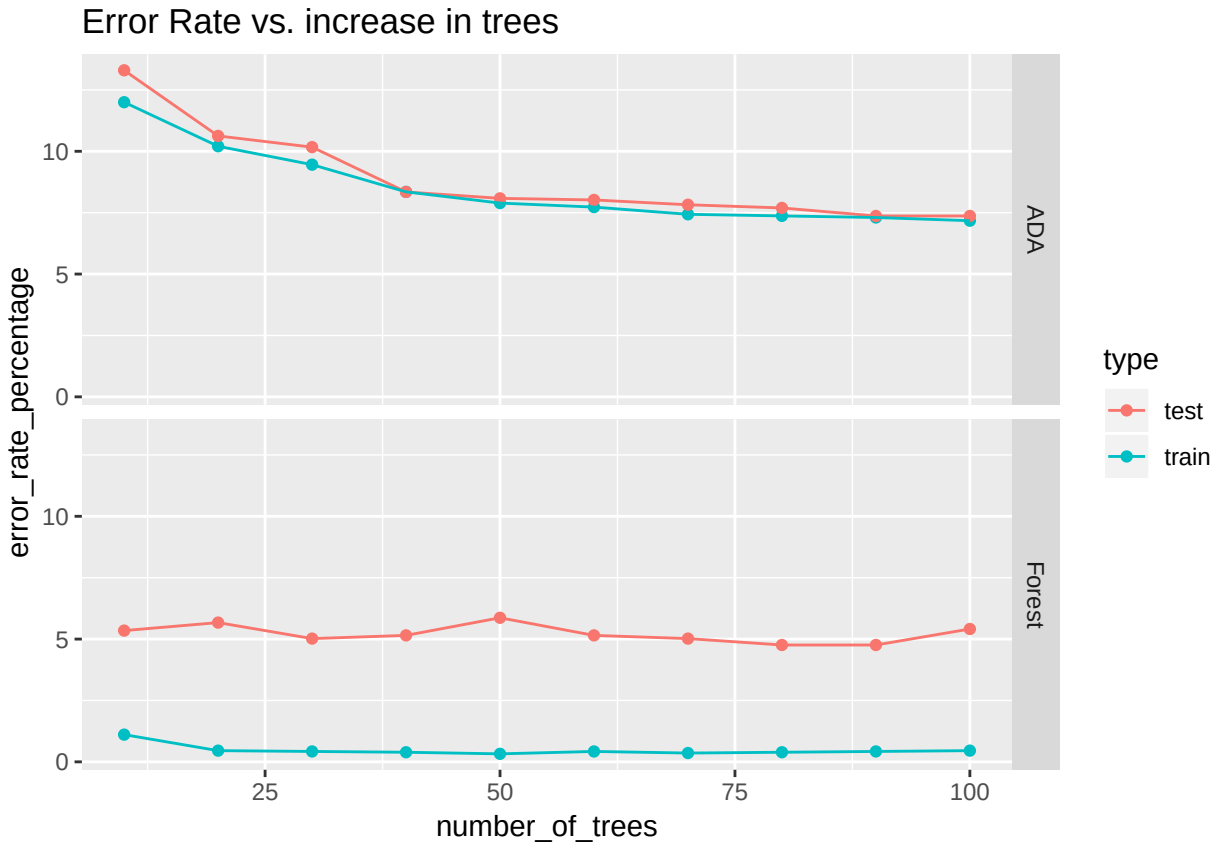
Splitting into Train and Test with 66% and 33% ratio.

```
set.seed(12345)
n =  NROW(spam_data)
id = sample(1:n, floor(n*(2/3)))
train = spam_data[id,]
test = spam_data[-id,]
```

Trainning the Model

```
final_result <- NULL
for(i in seq(from = 10, to = 100, by = 10)){

ada_model <- mboost::blackboost(Spam~.,
                                data = train,
                                family = AdaExp(),
                               control=boost_control(mstop=i))

forest_model <- randomForest(Spam~., data = train, ntree = i)


prediction_function <- function(model, data){
  predicted <- predict(model, newdata = data, type = c("class"))
  predict_correct <- ifelse(data$Spam == predicted, 1, 0)
  score <- sum(predict_correct)/NROW(data)
  return(score)
}


train_ada_model_predict <- predict(ada_model, newdata = train, type = c("class"))
test_ada_model_predict <- predict(ada_model, newdata = test, type = c("class"))
train_forest_model_predict <- predict(forest_model, newdata = train, type = c("class"))
test_forest_model_predict <- predict(forest_model, newdata = test, type = c("class"))

test_predict_correct <- ifelse(test$Spam == test_forest_model_predict, 1, 0)
train_predict_correct <- ifelse(train$Spam == train_forest_model_predict, 1, 0)


train_ada_score <-  prediction_function(ada_model, train)
test_ada_score <-  prediction_function(ada_model, test)
train_forest_score <-  prediction_function(forest_model, train)
test_forest_score <-  prediction_function(forest_model, test)
```

```
iteration_result <- data.frame(number_of_trees = i,
                               accuracy = c(train_ada_score,
                                            test_ada_score,
                                            train_forest_score,
                                            test_forest_score),
                               type  = c("train", "test", "train", "test"),
                               model = c("ADA", "ADA",  "Forest", "Forest"))


final_result <- rbind(iteration_result, final_result)
}

final_result$error_rate_percentage <- 100*(1 - final_result$accuracy)
ggplot(data = final_result, aes(x = number_of_trees,
                                y = error_rate_percentage,
                                group = type, color = type)) +
  geom_point() +
  geom_line() +
  ggtitle("Error Rate vs. increase in trees") + facet_grid(rows = vars(model))
```



Analysis

From the plots we can clearly see that ADA boosted methods uses more trees(~50) to reduce the test error, while randomforest achieves saturation in short number of trees(~10). We also see that random forest achieves less error than ADA tree for both tree and test cases.

2. Mixture Models

Description of the EM algorithm

EM is an iterative expectation maximumation technique. The way this works is for a given mixed distribution we guess the components of the data. This is done by first guessing the number of components and then randomly initializing the parameters of the said distribution (Mean, Varience).

Sometimes the data do not follow any known probability distribution but a mixture of known distributions such as:

$$p(x) = \sum_{k=1}^{K} p(k).p(x|k)$$

where p(x|k) are called mixture components and p(k) are called mixing coefficients: where p(k) is denoted by

$$\pi_k$$

With the following conditions

$$0 \leq \pi_k \leq 1$$

and

$$\sum_k \pi_k = 1$$

We are also given that the mixture model follows a Bernoulli distribution, for bernoulli we know that

$$Bern(x|\mu_k) = \prod_i \mu_{ki}^{x_i}.(1-\mu_{ki})^{(1-x_i)}$$

The EM algorithm for an Bernoulli mixed model is:

Set pi and mu to some initial values Repeat until pi and mu do not change E-step: Compute p(z|x) for all k and n M-step: Set piˆk to piˆk(ML) from likehood estimate, do the same to mu

M step:

$$p(z_{nk}|x_n, \mu, \pi) = Z = \frac{\pi_k p(x_n|\mu_k)}{\sum_k p(x_n|\mu_k)}$$

E step:

$$\pi_k^{ML} = \frac{\sum_N p(z_{nk}|x_n, \mu, \pi)}{N}$$

$$\mu_{ki}^{ML} = \frac{\sum_n x_{ni} p(z_{nk}|x_n, \mu, \pi)}{\sum_n p(z_{nk}|x_n, \mu, \pi)}$$

The maximum likehood of E step is:

$$\log_e p(X|\mu, \pi) = \sum_{n=1}^{N} \log_e \sum_{k=1}^{K} .\pi_k.p(x_n|\mu_k)$$

Code

To compare the results for K = 2,3,4, the em_loop-function provides a graphical analysis for every iteration. The function includes comments which explain what I did at which step to create the EM algorithm. The function will be finally run with K = 2,3,4.

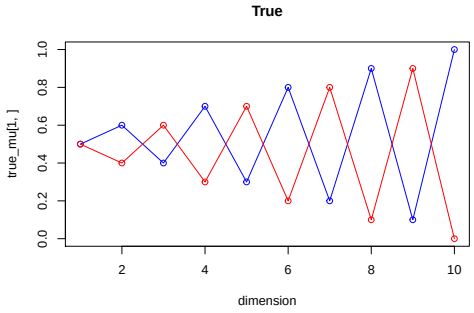
```
em_loop = function(K) {
# Initializing data
set.seed(1234567890)
max_it = 100 # max number of EM iterations
min_change = 0.1 # min change in log likelihood between two consecutive EM iterations
N = 1000 # number of training points
D = 10 # number of dimensions
x = matrix(nrow=N, ncol = D) # training data
true_pi = vector(length = K) # true mixing coefficients
true_mu = matrix(nrow = K, ncol = D) # true conditional distributions
true_pi = c(rep(1/K, K))
if (K == 2) {
true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue",
ylim = c(0,1), main = "True")
points(true_mu[2,], type="o", xlab = "dimension", col = "red",
main = "True")
} else if (K == 3) {
true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
true_mu[3,] = c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue", ylim=c(0,1),
main = "True")
points(true_mu[2,], type = "o", xlab = "dimension", col = "red",
main = "True")
points(true_mu[3,], type = "o", xlab = "dimension", col = "green",
main = "True")
} else {
true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
true_mu[3,] = c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
true_mu[4,] = c(0.3,0.5,0.5,0.7,0.5,0.5,0.5,0.5,0.4,0.5)
plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue",
ylim = c(0,1), main = "True")
points(true_mu[2,], type = "o", xlab = "dimension", col = "red",
main = "True")
points(true_mu[3,], type = "o", xlab = "dimension", col = "green",
main = "True")
points(true_mu[4,], type = "o", xlab = "dimension", col = "yellow",
main = "True")
}
z = matrix(nrow = N, ncol = K) # fractional component assignments
pi = vector(length = K) # mixing coefficients
mu = matrix(nrow = K, ncol = D) # conditional distributions
llik = vector(length = max_it) # log likelihood of the EM iterations
# Producing the training data
for(n in 1:N) {
k = sample(1:K, 1, prob=true_pi)
```

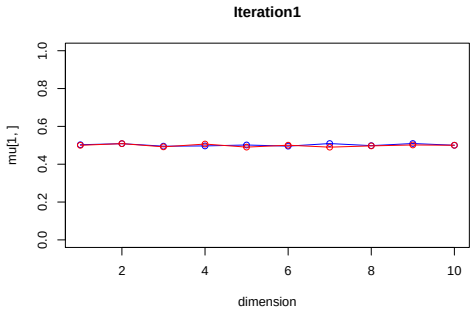
```
for(d in 1:D) {
x[n,d] = rbinom(1, 1, true_mu[k,d])
}
}
# Random initialization of the paramters
pi = runif(K, 0.49, 0.51)
pi = pi / sum(pi)
for(k in 1:K) {
mu[k,] = runif(D, 0.49, 0.51)
}
#EM algorithm
for(it in 1:max_it) {
# Plotting mu
# Defining plot title
title = paste0("Iteration", it)
if (K == 2) {
plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
} else if (K == 3) {
plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
points(mu[3,], type = "o", xlab = "dimension", col = "green", main = title)
} else {
plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
points(mu[3,], type = "o", xlab = "dimension", col = "green", main = title)
points(mu[4,], type = "o", xlab = "dimension", col = "yellow", main = title)
}
Sys.sleep(0.5)
# E-step: Computation of the fractional component assignments
for (n in 1:N) {
# Creating empty matrix (column 1:K = p_x_given_k; column K+1 = p(x|all k)
p_x = matrix(data = c(rep(1,K), 0), nrow = 1, ncol = K+1)
# Calculating p(x|k) and p(x|all k)
for (k in 1:K) {
# Calculating p(x|k)
for (d in 1:D) {
p_x[1,k] = p_x[1,k] * (mu[k,d]^x[n,d]) * (1-mu[k,d])^(1-x[n,d])
}
p_x[1,k] = p_x[1,k] * pi[k] # weighting with pi[k]
# Calculating p(x|all k) (denominator)
p_x[1,K+1] = p_x[1,K+1] + p_x[1,k]
}
# Calculating z for n and all k
for (k in 1:K) {
z[n,k] = p_x[1,k] / p_x[1,K+1]
}
}
# Log likelihood computation
for (n in 1:N) {
for (k in 1:K) {
log_term = 0
for (d in 1:D) {
```

```
log_term = log_term + x[n,d] * log(mu[k,d]) + (1-x[n,d]) * log(1-mu[k,d])
}
llik[it] = llik[it] + z[n,k] * (log(pi[k]) + log_term)
}
}
cat("iteration: ", it, "log likelihood: ", llik[it], "\n")
flush.console()
# Stop if the log likelihood has not changed significantly
if (it != 1) {
if (abs(llik[it] - llik[it-1]) < min_change) {
break
}
}
# M-step: ML parameter estimation from the data and fractional component assignments
# Updating pi
for (k in 1:K) {
pi[k] = sum(z[,k])/N
}
# Updating mu
for (k in 1:K) {
mu[k,] = 0
for (n in 1:N) {
    mu[k,] = mu[k,] + x[n,] * z[n,k]
}
mu[k,] = mu[k,] / sum(z[,k])
}
}
# Printing pi, mu and development of log likelihood at the end
return(list(
pi = pi,
mu = mu,
logLikelihoodDevelopment = plot(llik[1:it],
type = "o",
main = "Development of the log likelihood",
xlab = "iteration",
ylab = "log likelihood")
))
}
```

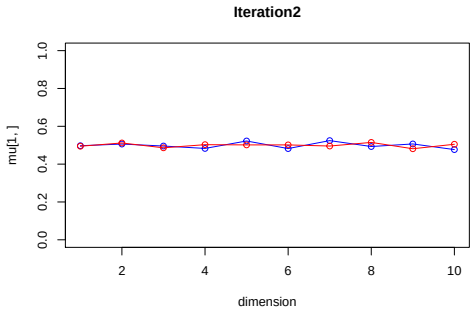
K=2

```
em_loop(2)
```

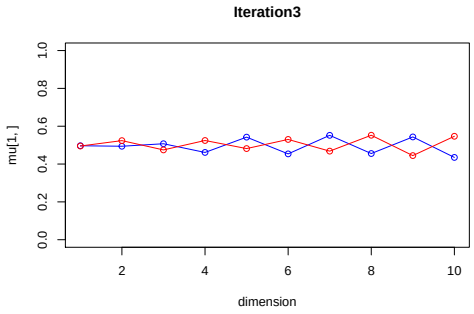




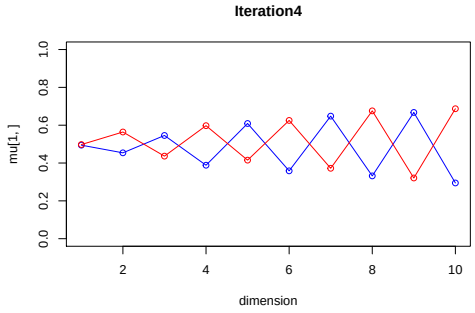

```
## iteration:  1 log likelihood:  -7623.897
```



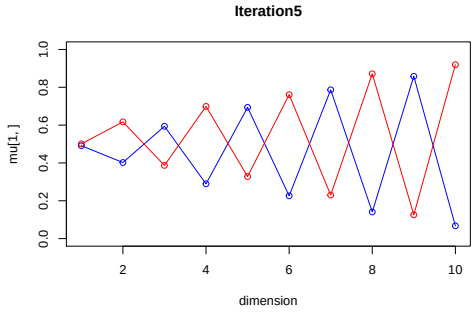

```
## iteration:  2 log likelihood:  -7610.745
```



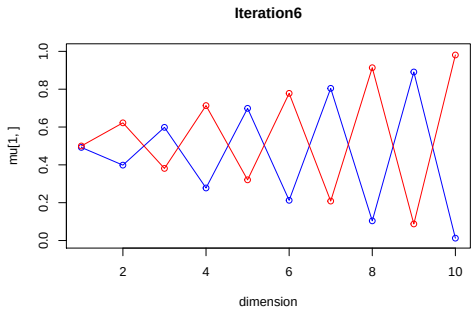

```
## iteration:  3 log likelihood:  -7463.445
```



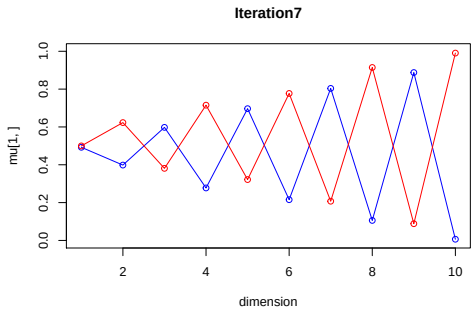

```
## iteration:  4 log likelihood:  -6575.121
```



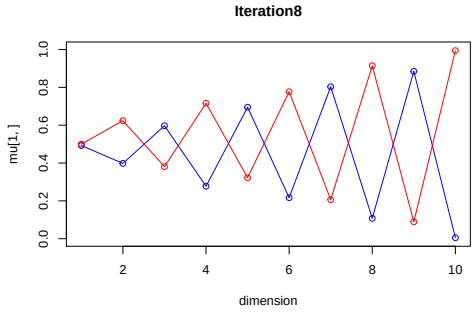

```
## iteration:  5 log likelihood:  -5731.559
```



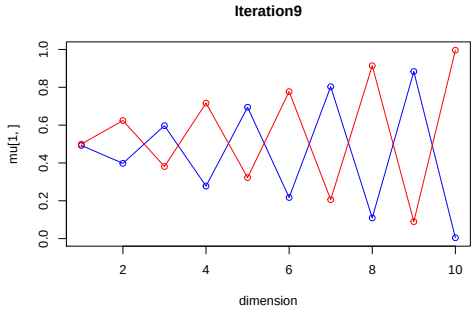

```
## iteration:  6 log likelihood:  -5656.174
```



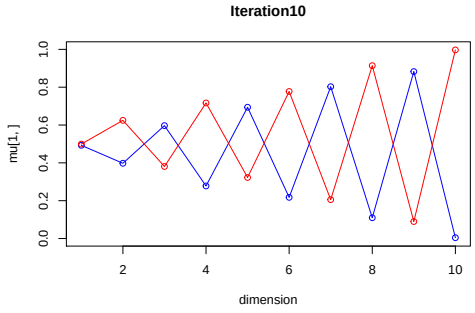

```
## iteration:  7 log likelihood:  -5648.904
```



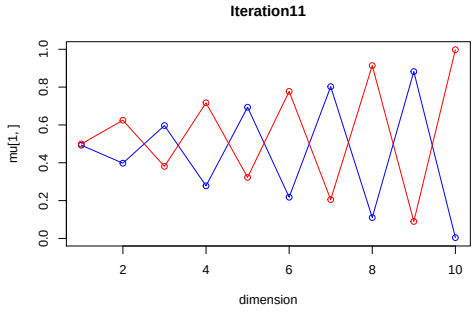

```
## iteration:  8 log likelihood:  -5646.139
```



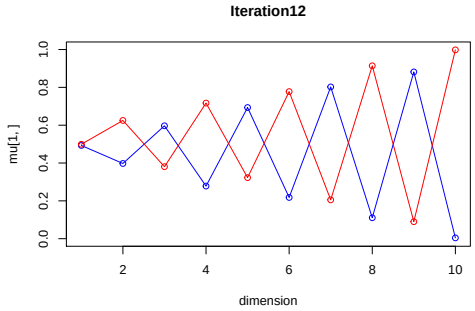

```
## iteration:  9 log likelihood:  -5644.608
```



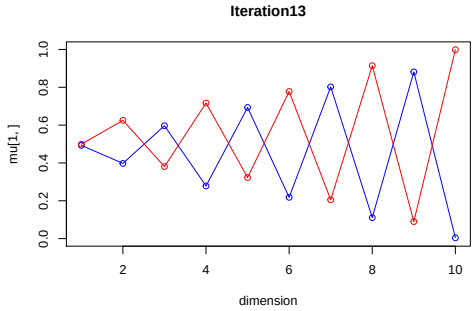

```
## iteration:  10 log likelihood:  -5643.615
```



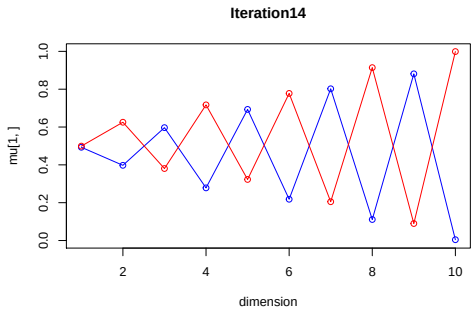

```
## iteration:  11 log likelihood:  -5642.913
```



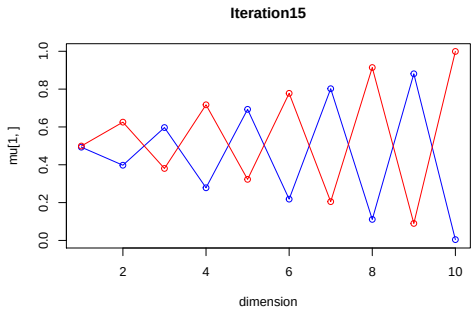

```
## iteration:  12 log likelihood:  -5642.386
```



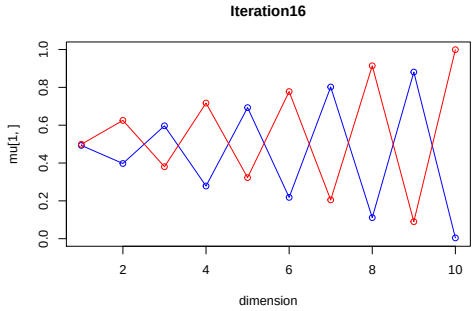

```
## iteration:  13 log likelihood:  -5641.977
```



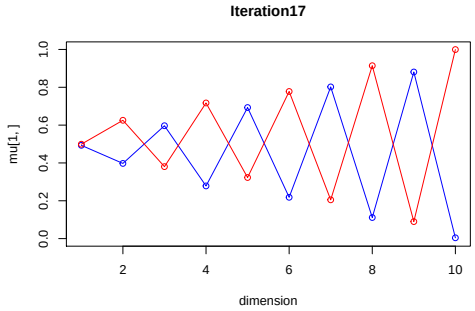

```
## iteration:  14 log likelihood:  -5641.649
```



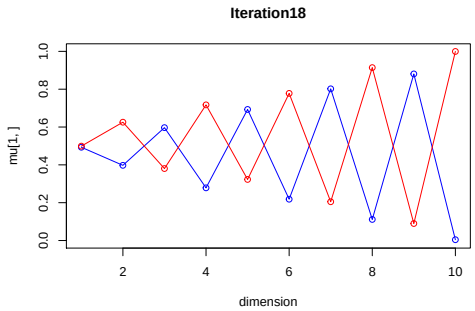

```
## iteration:  15 log likelihood:  -5641.382
```



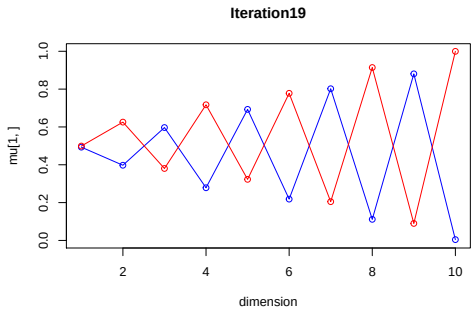

```
## iteration:  16 log likelihood:  -5641.161
```



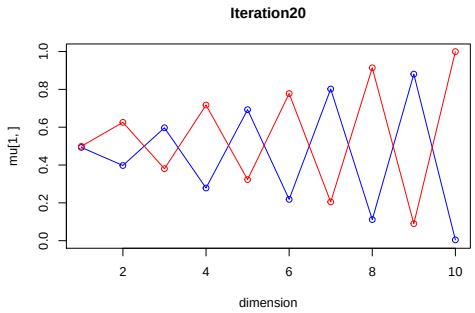

```
## iteration:  17 log likelihood:  -5640.975
```



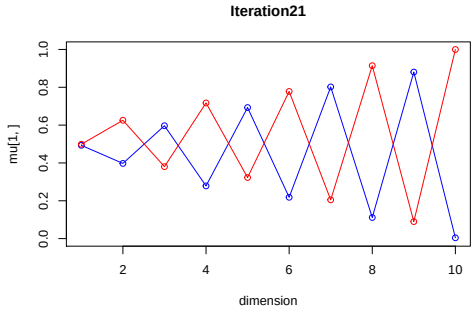

```
## iteration:  18 log likelihood:  -5640.819
```



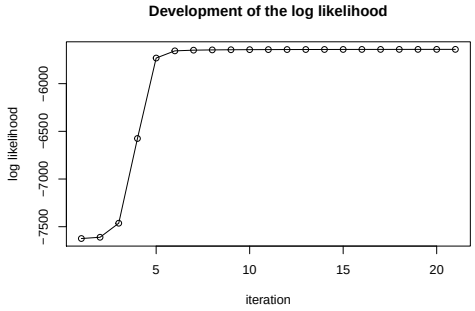

```
## iteration:  19 log likelihood:  -5640.685
```




```
## iteration:  20 log likelihood:  -5640.571
```



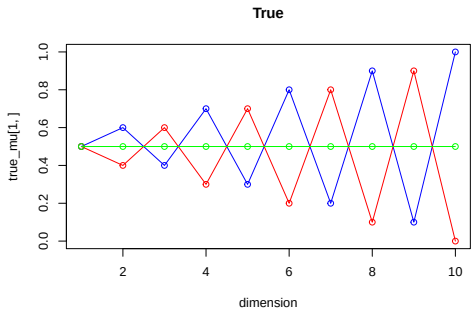

```
## iteration:  21 log likelihood:  -5640.473
```

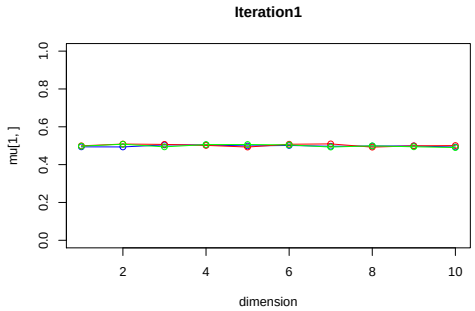



```
## $pi
## [1] 0.5110531 0.4889469
##
## $mu
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4931735 0.3974606 0.5967811 0.2785480 0.6927917 0.2184957 0.8018491
## [2,] 0.4989543 0.6255823 0.3804363 0.7171478 0.3230343 0.7778699 0.2049559
##           [,8]       [,9]       [,10]
## [1,] 0.1116477 0.88054439 0.004290353
## [2,] 0.9140913 0.08997919 0.999714736
##
## $logLikelihoodDevelopment
## NULL
```

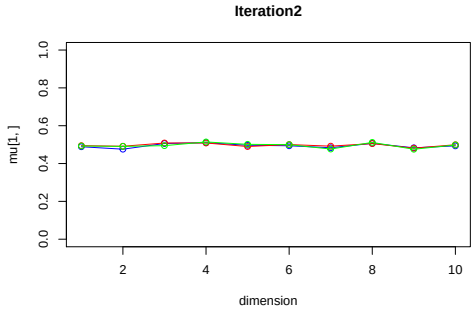
K=3

```
em_loop(3)
```

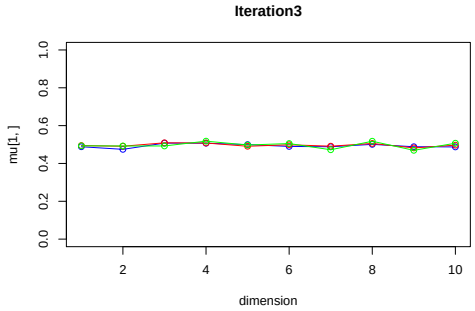




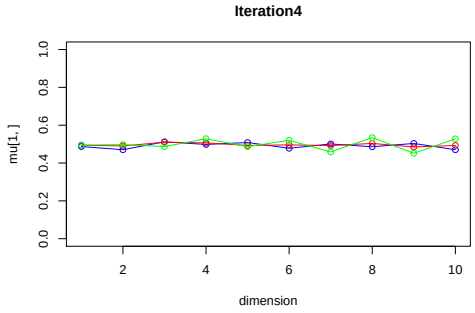

```
## iteration:  1 log likelihood:  -8029.723
```



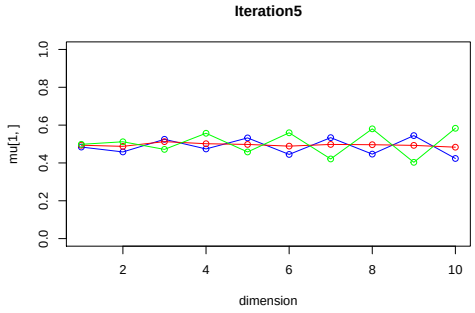

```
## iteration:  2 log likelihood:  -8027.183
```



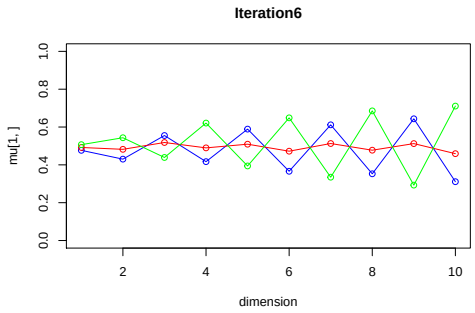

```
## iteration:  3 log likelihood:  -8024.696
```



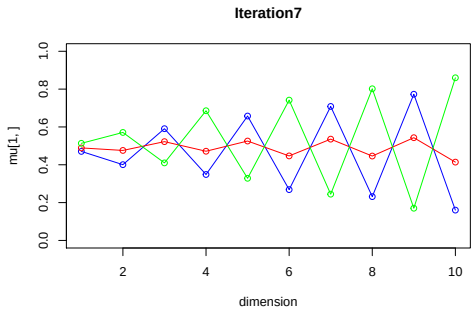

```
## iteration:  4 log likelihood:  -8005.631
```



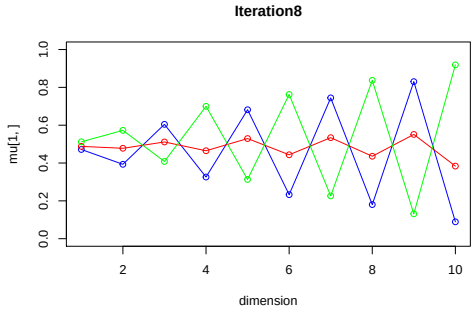

```
## iteration:  5 log likelihood:  -7877.606
```



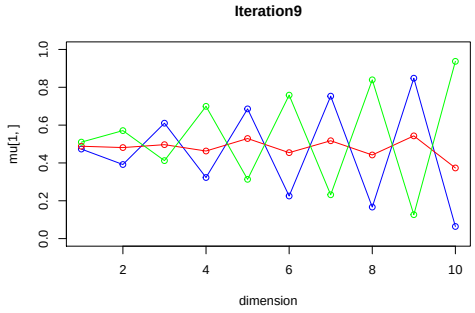

```
## iteration:  6 log likelihood:  -7403.513
```



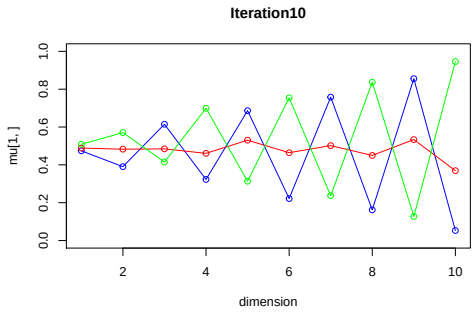

```
## iteration:  7 log likelihood:  -6936.919
```



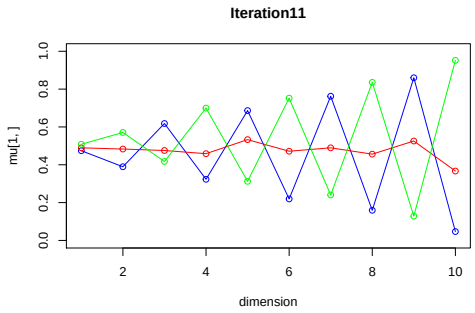

```
## iteration:  8 log likelihood:  -6818.582
```



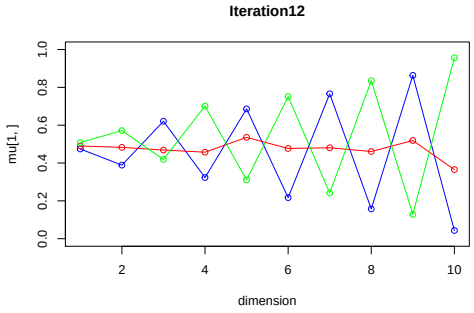

```
## iteration:  9 log likelihood:  -6791.377
```



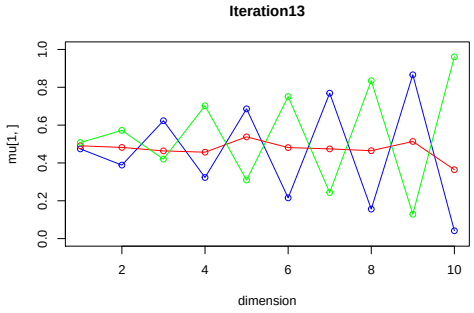

```
## iteration:  10 log likelihood:  -6780.713
```



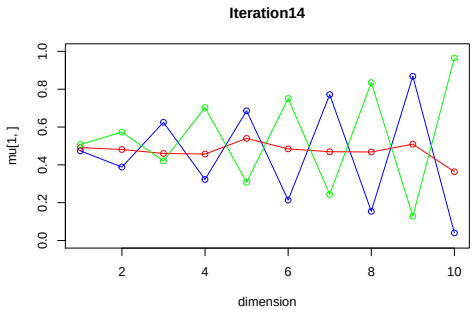

```
## iteration:  11 log likelihood:  -6774.958
```



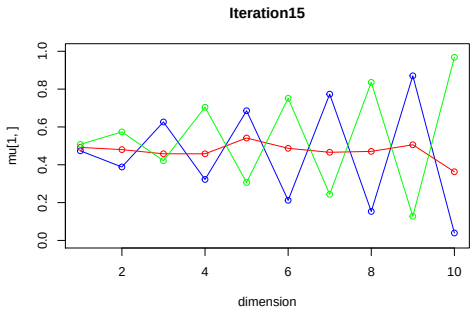

```
## iteration:  12 log likelihood:  -6771.261
```



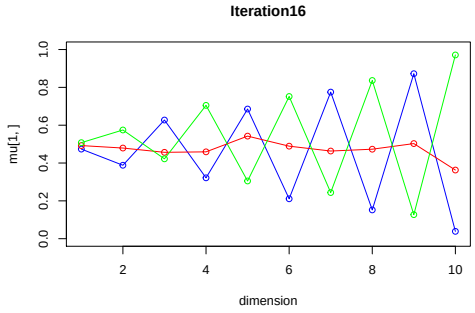

```
## iteration:  13 log likelihood:  -6768.606
```



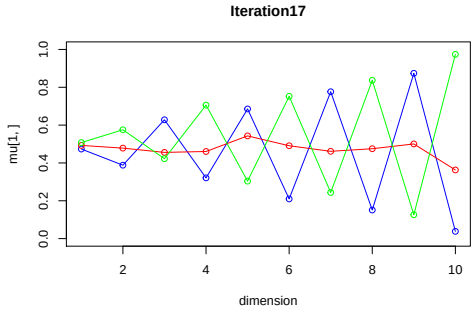

```
## iteration:  14 log likelihood:  -6766.535
```



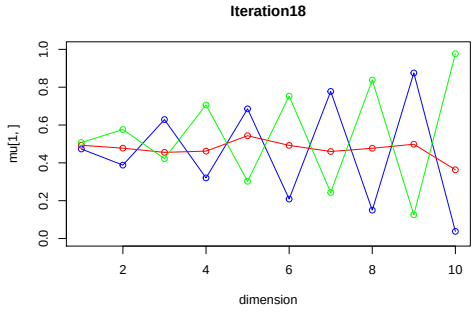

```
## iteration:  15 log likelihood:  -6764.815
```



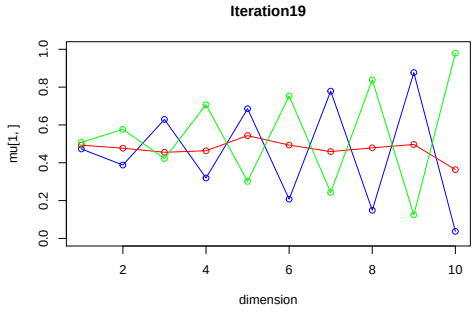

```
## iteration:  16 log likelihood:  -6763.316
```



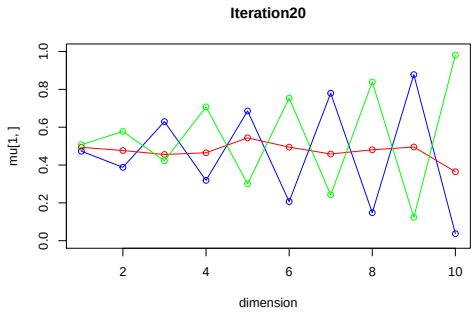

```
## iteration:  17 log likelihood:  -6761.967
```



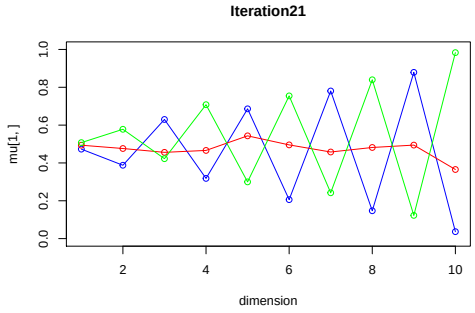

```
## iteration:  18 log likelihood:  -6760.727
```



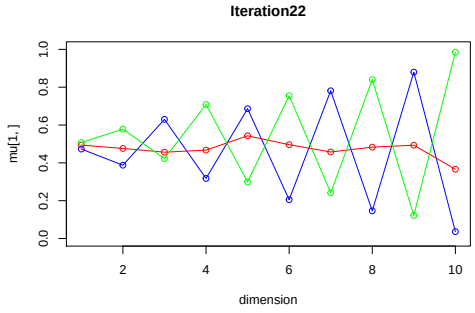

```
## iteration:  19 log likelihood:  -6759.572
```



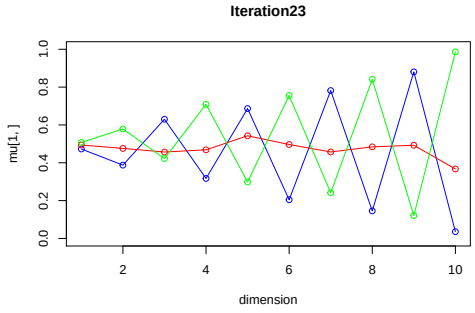

```
## iteration:  20 log likelihood:  -6758.491
```



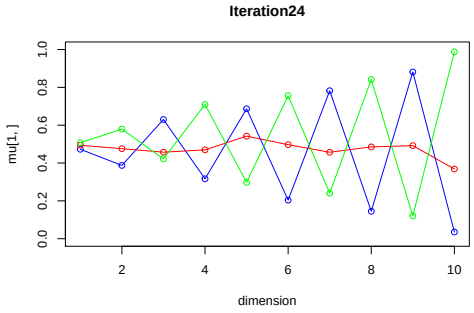

```
## iteration:  21 log likelihood:  -6757.475
```



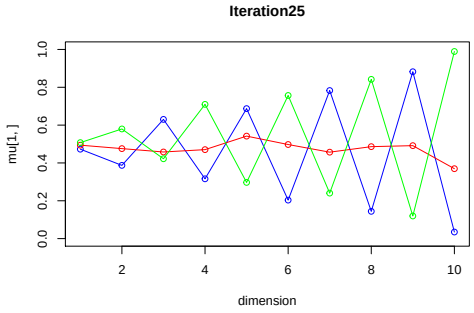

```
## iteration:  22 log likelihood:  -6756.521
```



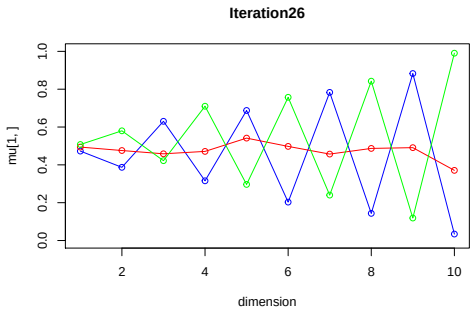

```
## iteration:  23 log likelihood:  -6755.625
```



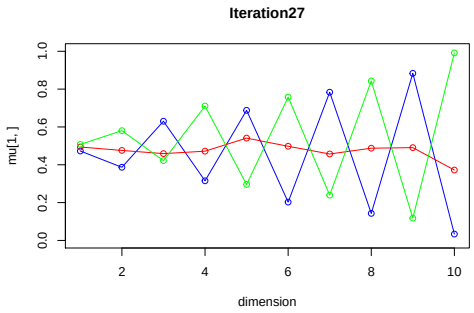

```
## iteration:  24 log likelihood:  -6754.784
```



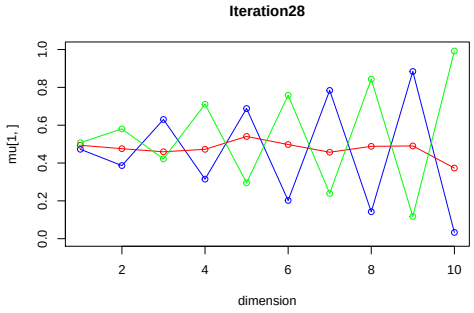

```
## iteration:  25 log likelihood:  -6753.996
```



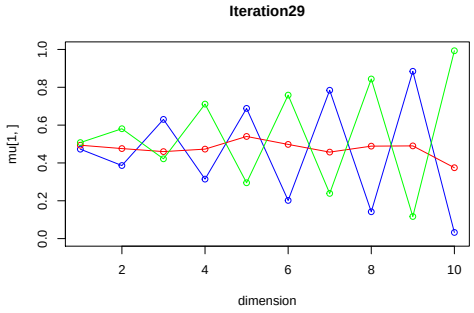

```
## iteration:  26 log likelihood:  -6753.26
```



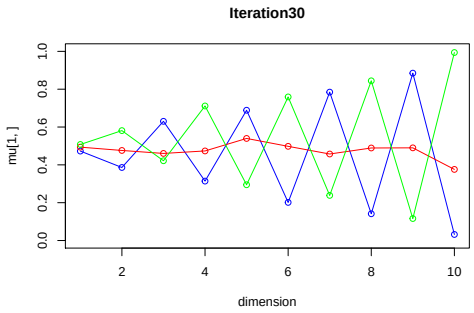

```
## iteration:  27 log likelihood:  -6752.571
```



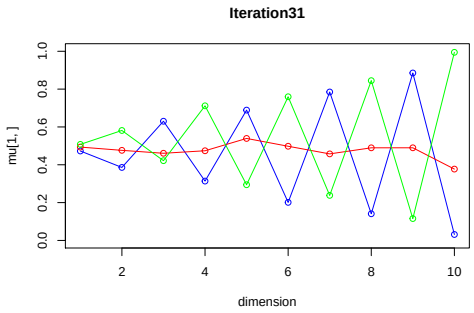

```
## iteration:  28 log likelihood:  -6751.928
```



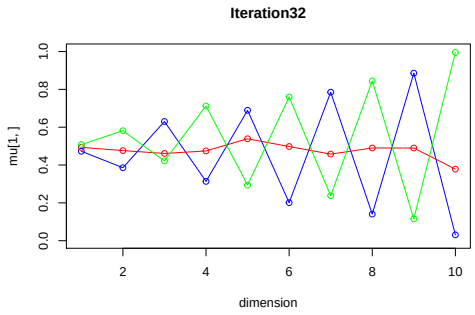

```
## iteration:  29 log likelihood:  -6751.328
```



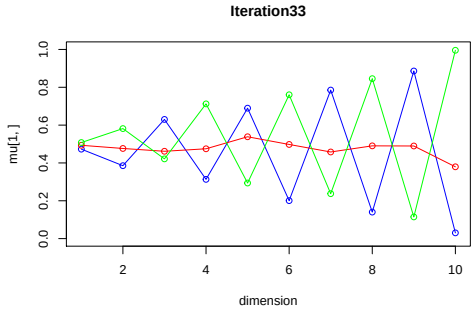

```
## iteration:  30 log likelihood:  -6750.768
```



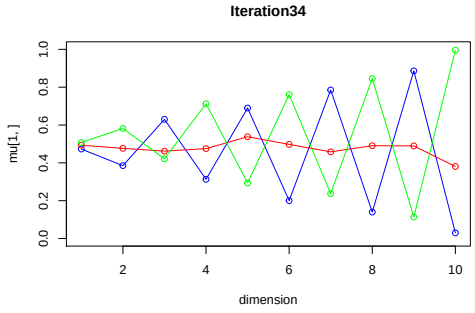

```
## iteration:  31 log likelihood:  -6750.246
```



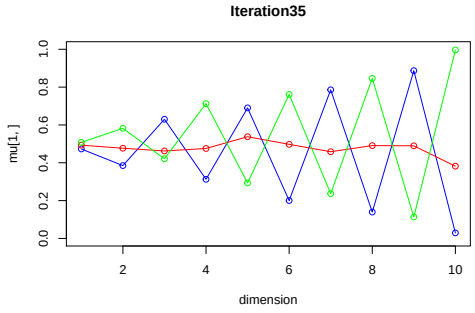

```
## iteration:  32 log likelihood:  -6749.758
```



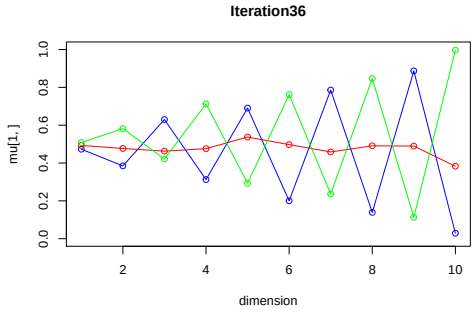

```
## iteration:  33 log likelihood:  -6749.304
```



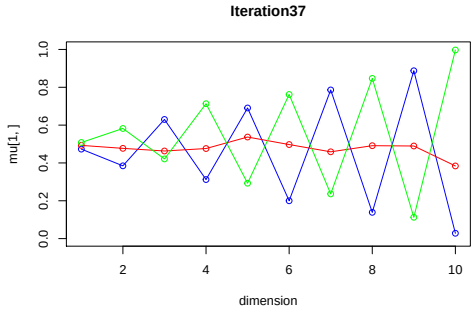

```
## iteration:  34 log likelihood:  -6748.88
```



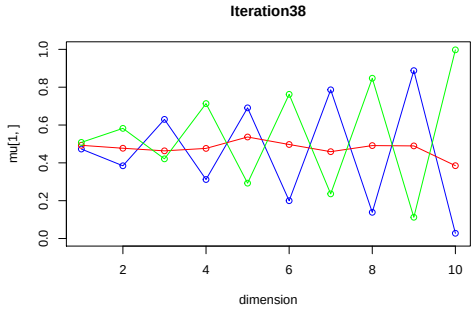

```
## iteration:  35 log likelihood:  -6748.484
```



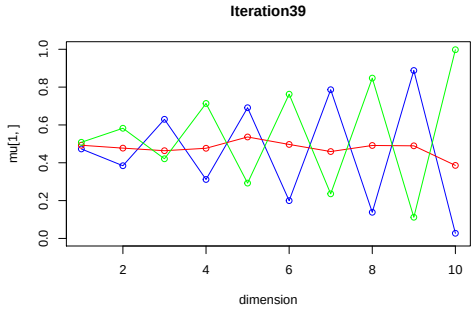

```
## iteration:  36 log likelihood:  -6748.114
```



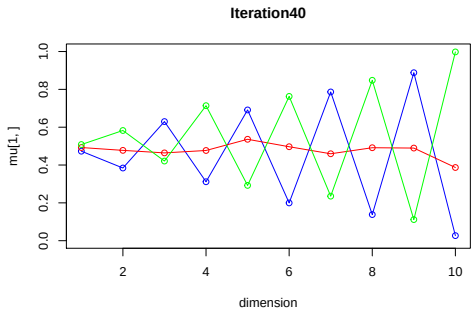

```
## iteration:  37 log likelihood:  -6747.767
```



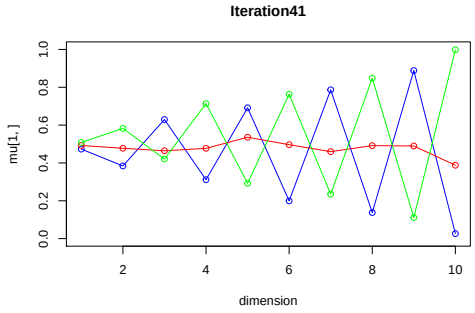

```
## iteration:  38 log likelihood:  -6747.444
```



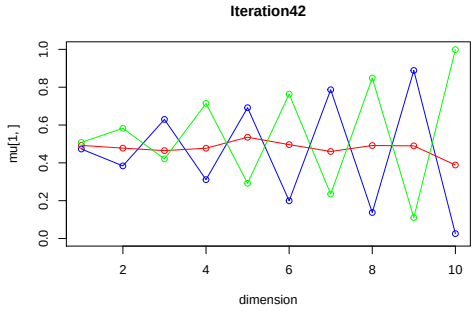

```
## iteration:  39 log likelihood:  -6747.14
```



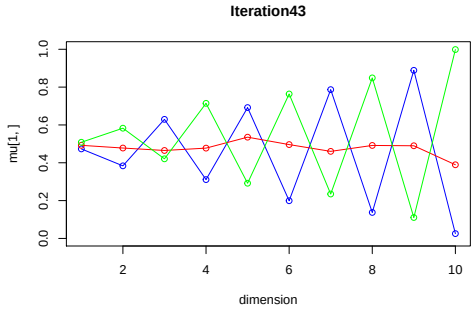

```
## iteration:  40 log likelihood:  -6746.856
```



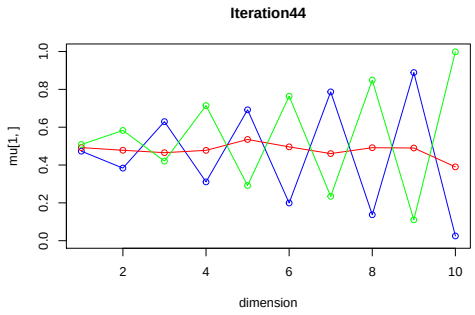

```
## iteration:  41 log likelihood:  -6746.589
```



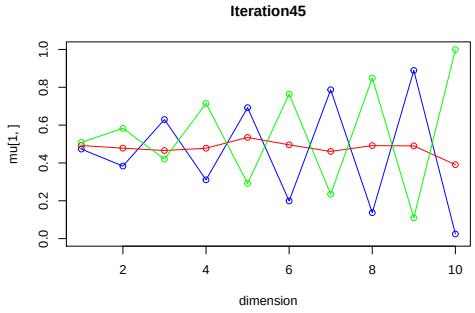

```
## iteration:  42 log likelihood:  -6746.338
```



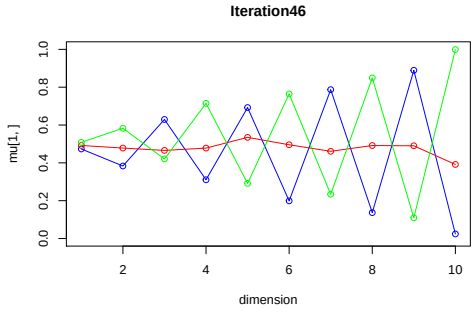

```
## iteration:  43 log likelihood:  -6746.102
```



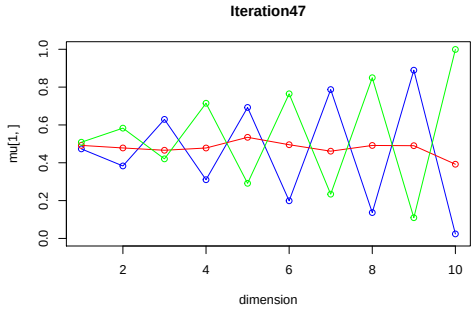

```
## iteration:  44 log likelihood:  -6745.88
```



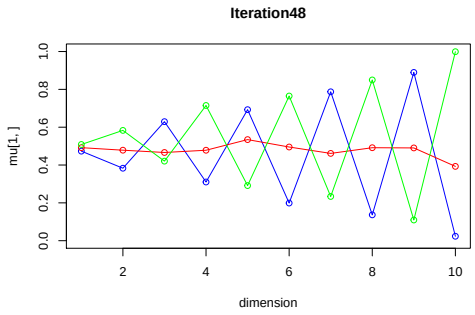

```
## iteration:  45 log likelihood:  -6745.67
```



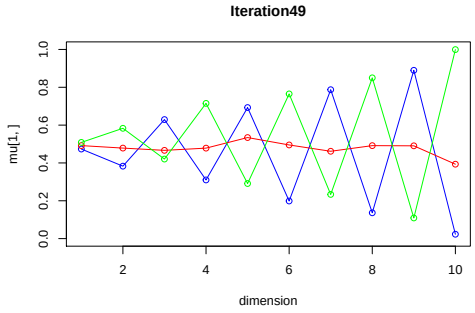

```
## iteration:  46 log likelihood:  -6745.472
```



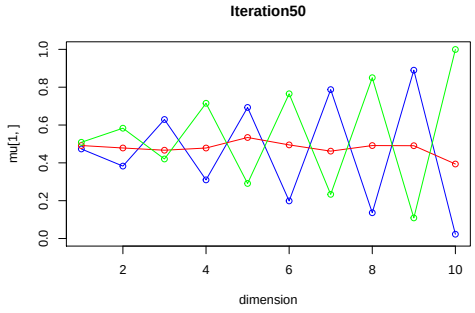

```
## iteration:  47 log likelihood:  -6745.285
```



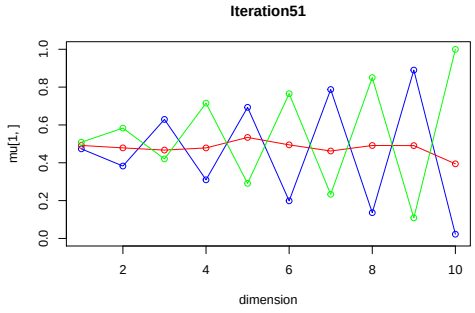

```
## iteration:  48 log likelihood:  -6745.108
```



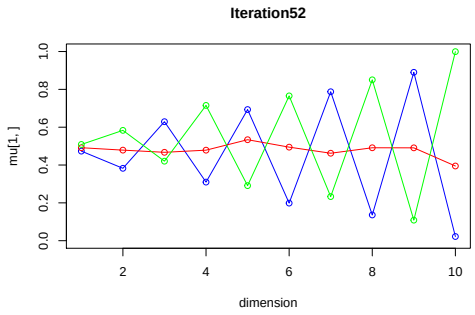

```
## iteration:  49 log likelihood:  -6744.939
```



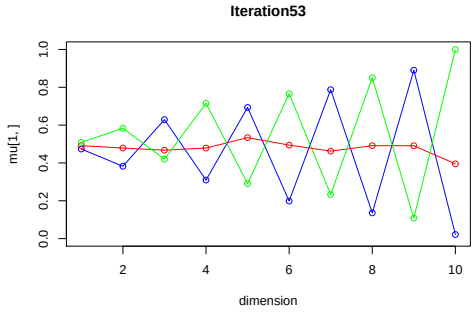

```
## iteration:  50 log likelihood:  -6744.78
```



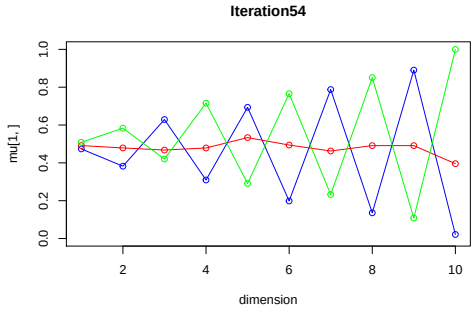

```
## iteration:  51 log likelihood:  -6744.627
```



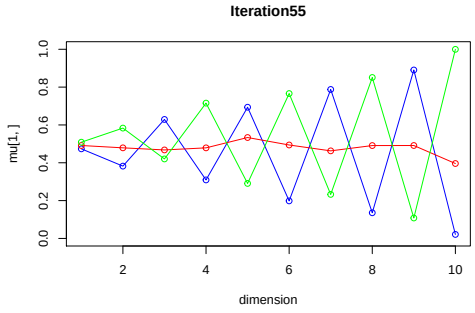

```
## iteration:  52 log likelihood:  -6744.483
```



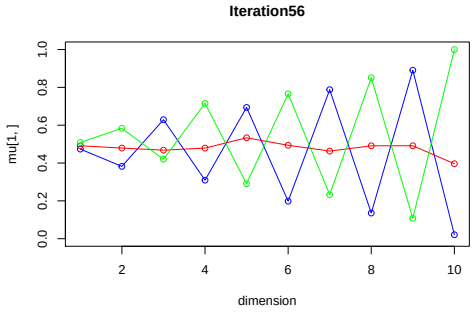

```
## iteration:  53 log likelihood:  -6744.344
```



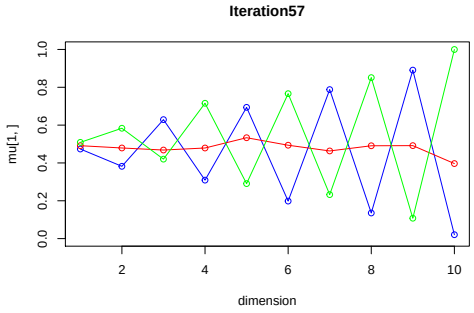

```
## iteration:  54 log likelihood:  -6744.212
```



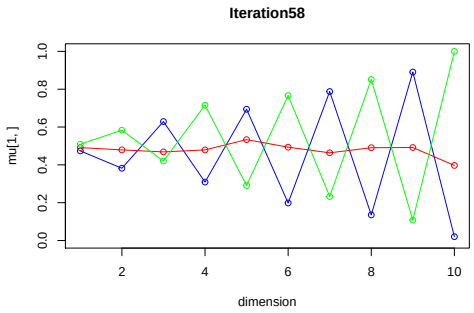

```
## iteration:  55 log likelihood:  -6744.086
```



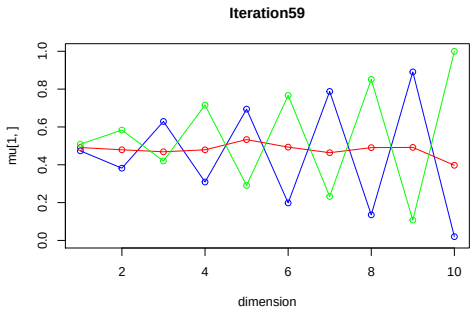

```
## iteration:  56 log likelihood:  -6743.964
```



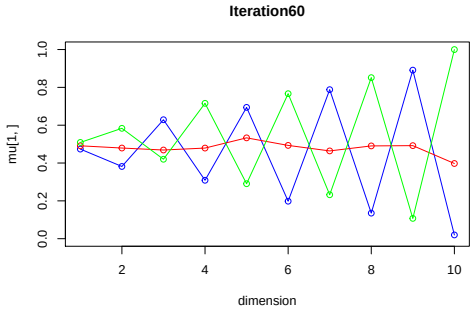

```
## iteration:  57 log likelihood:  -6743.848
```



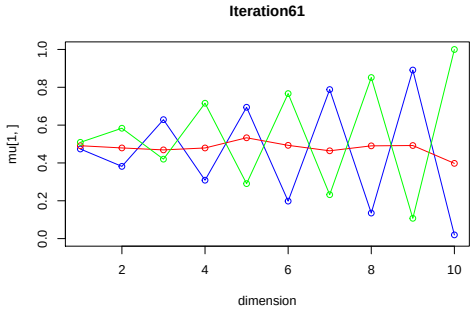

```
## iteration:  58 log likelihood:  -6743.736
```



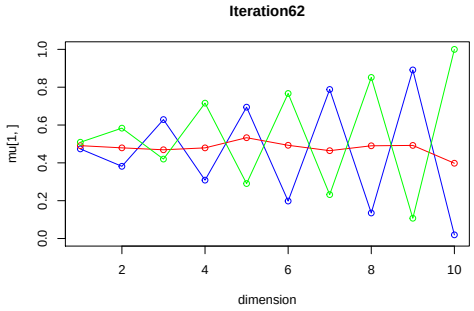

```
## iteration:  59 log likelihood:  -6743.628
```



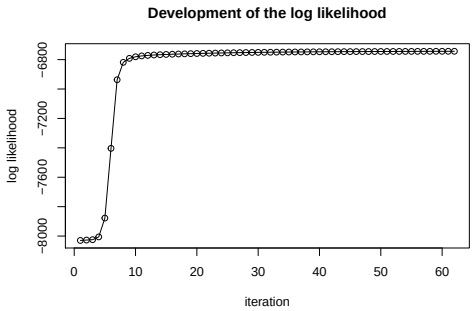

```
## iteration:  60 log likelihood:  -6743.524
```




```
## iteration:  61 log likelihood:  -6743.423
```




```
## iteration:  62 log likelihood:  -6743.326
```

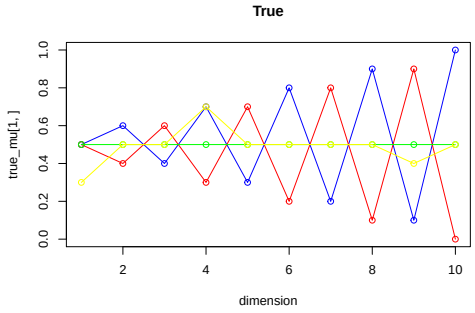


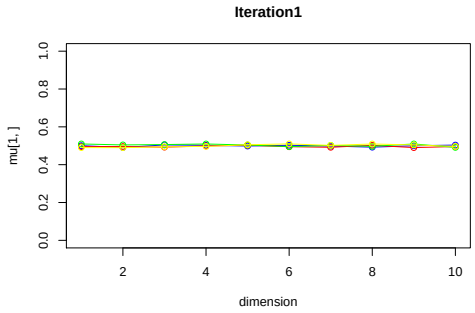

```
## $pi
## [1] 0.3259592 0.3044579 0.3695828
##
```

```
## $mu
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4737193 0.3817120 0.6288021 0.3086143 0.6943731 0.1980896 0.7879447
## [2,] 0.4909874 0.4793213 0.4691560 0.4791793 0.5329895 0.4928830 0.4643990
## [3,] 0.5089571 0.5834802 0.4199272 0.7157107 0.2905703 0.7667258 0.2320784
##           [,8]      [,9]      [,10]
## [1,] 0.1349651 0.8912534 0.01937869
## [2,] 0.4902682 0.4922194 0.39798407
## [3,] 0.8516111 0.1072226 0.99981353
##
## $logLikelihoodDevelopment
## NULL
```

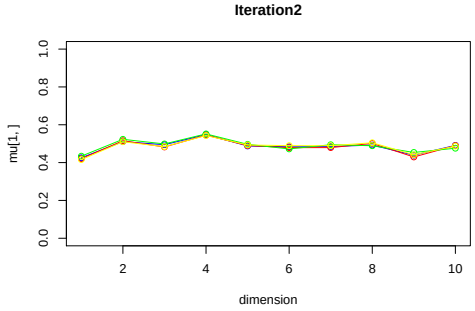
K=4

```
em_loop(4)
```

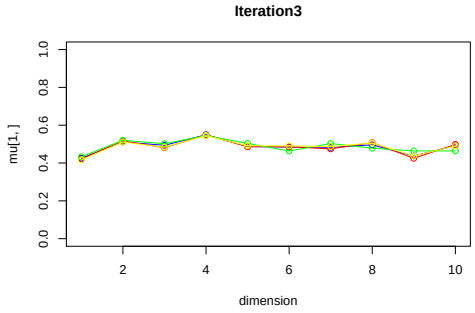




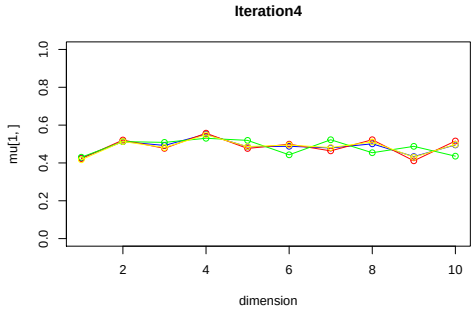

```
## iteration:  1 log likelihood:  -8316.904
```



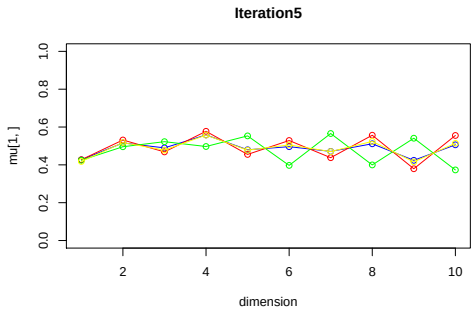

```
## iteration:  2 log likelihood:  -8291.114
```



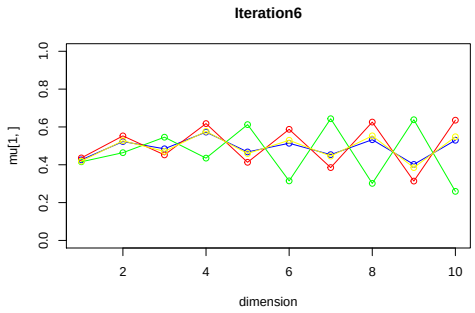

```
## iteration:  3 log likelihood:  -8286.966
```



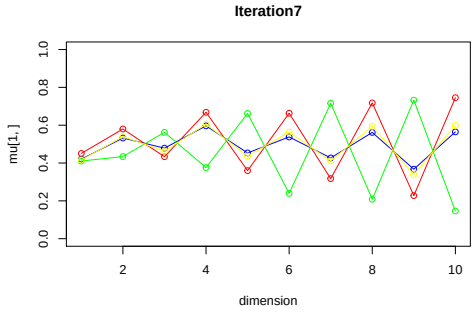

```
## iteration:  4 log likelihood:  -8264.806
```



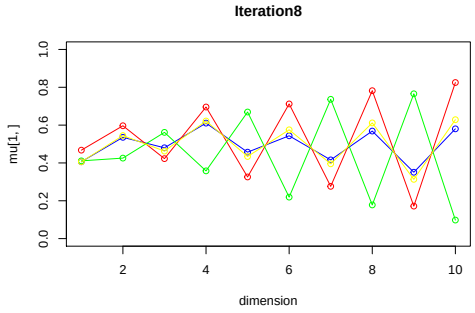

```
## iteration:  5 log likelihood:  -8161.19
```



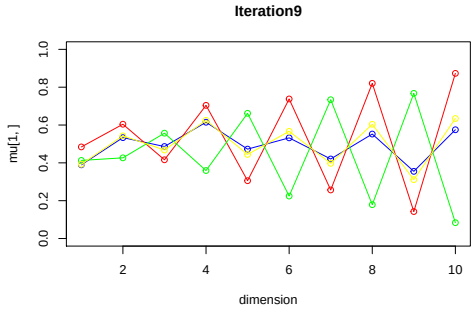

```
## iteration:  6 log likelihood:  -7868.89
```



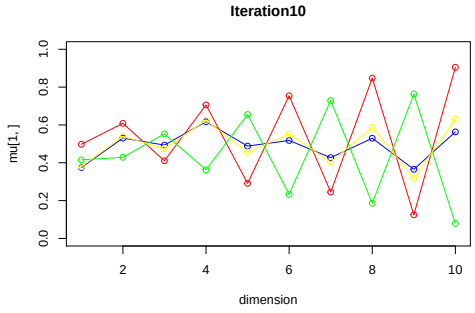

```
## iteration:  7 log likelihood:  -7570.873
```



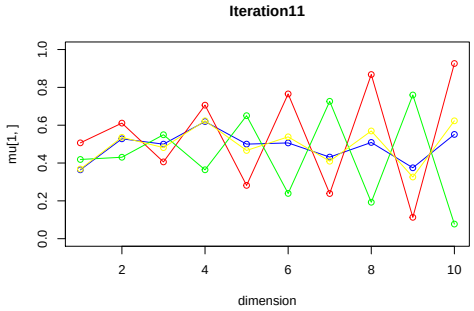

```
## iteration:  8 log likelihood:  -7445.719
```



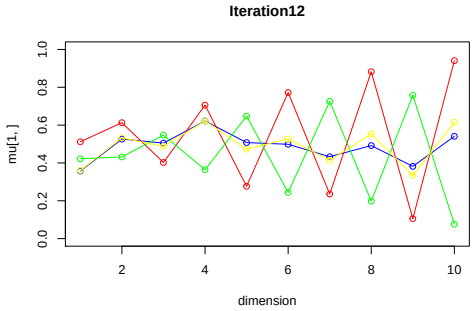

```
## iteration:  9 log likelihood:  -7389.741
```



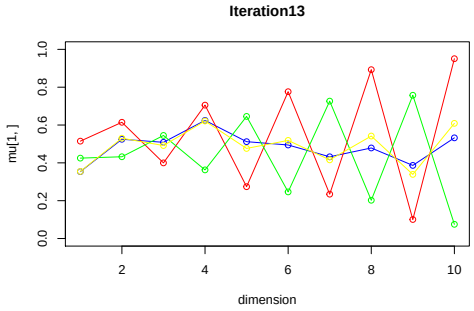

```
## iteration:  10 log likelihood:  -7356.803
```



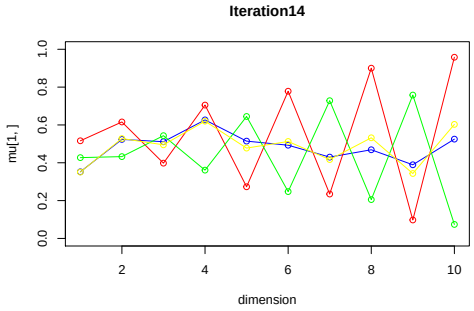

```
## iteration:  11 log likelihood:  -7337.208
```



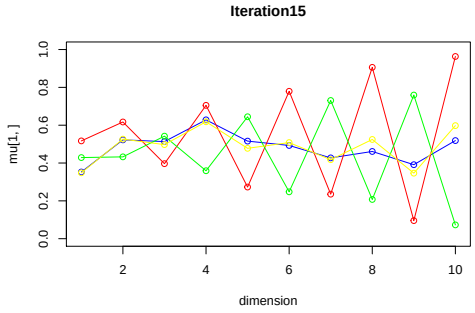

```
## iteration:  12 log likelihood:  -7326.118
```



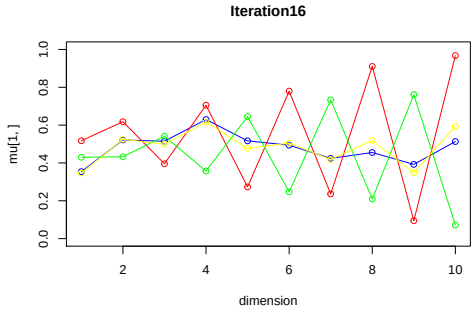

```
## iteration:  13 log likelihood:  -7319.998
```



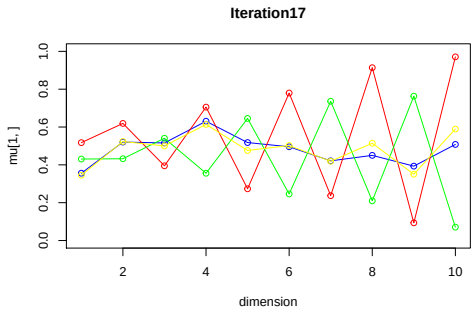

```
## iteration:  14 log likelihood:  -7316.6
```



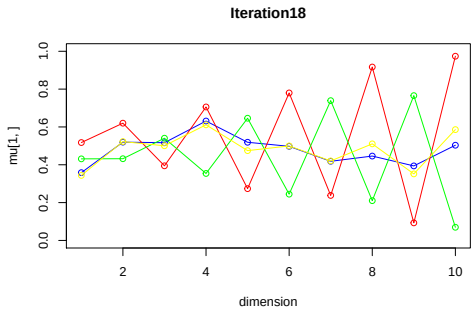

```
## iteration:  15 log likelihood:  -7314.666
```



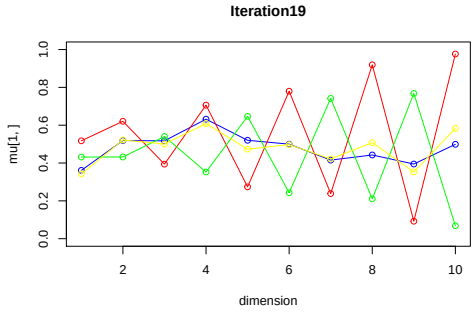

```
## iteration:  16 log likelihood:  -7313.528
```



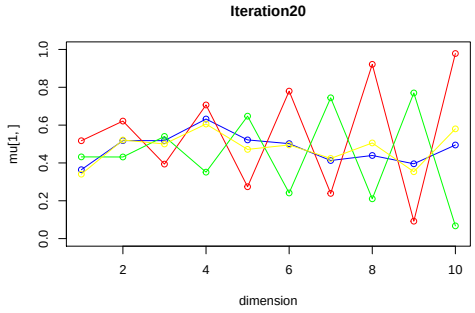

```
## iteration:  17 log likelihood:  -7312.829
```



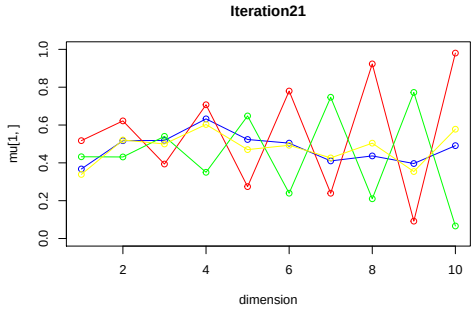

```
## iteration:  18 log likelihood:  -7312.367
```



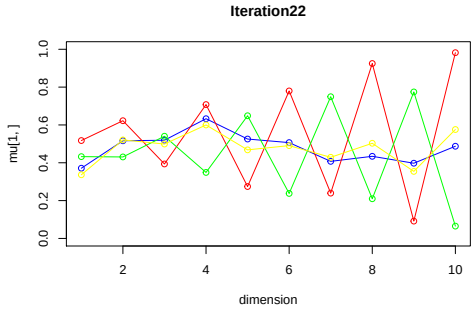

```
## iteration:  19 log likelihood:  -7312.024
```



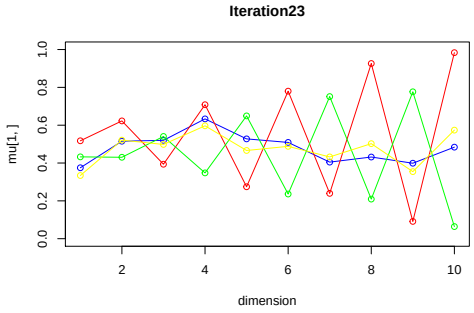

```
## iteration:  20 log likelihood:  -7311.723
```



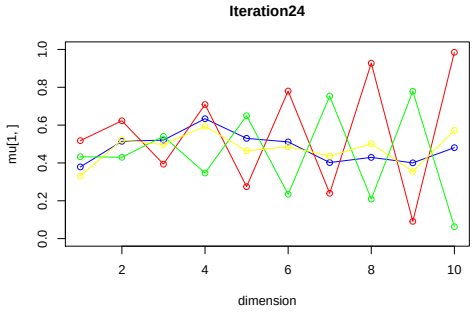

```
## iteration:  21 log likelihood:  -7311.407
```



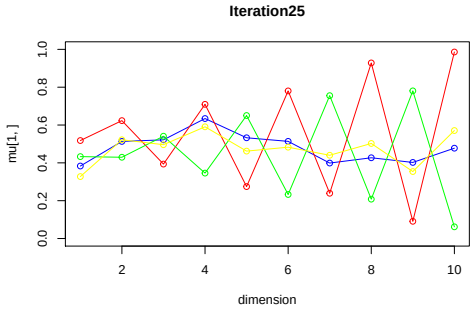

```
## iteration:  22 log likelihood:  -7311.036
```



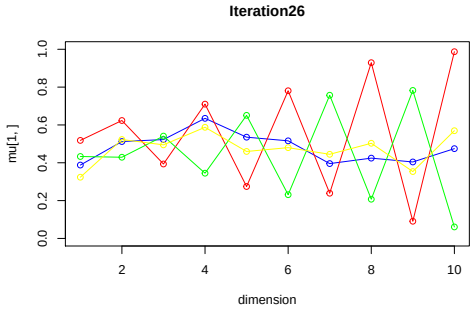

```
## iteration:  23 log likelihood:  -7310.574
```



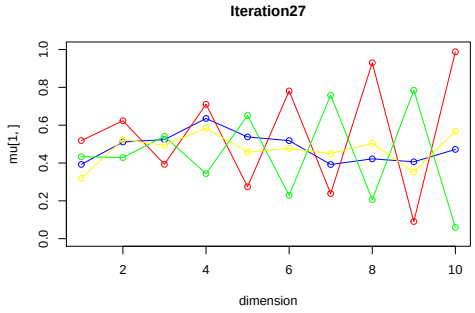

```
## iteration:  24 log likelihood:  -7309.988
```



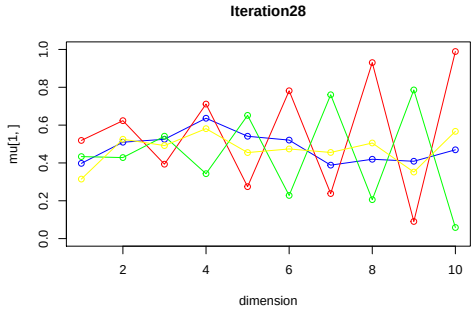

```
## iteration:  25 log likelihood:  -7309.248
```



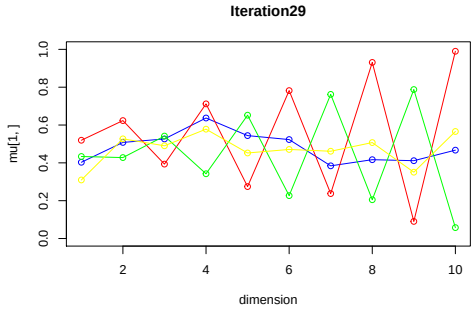

```
## iteration:  26 log likelihood:  -7308.322
```



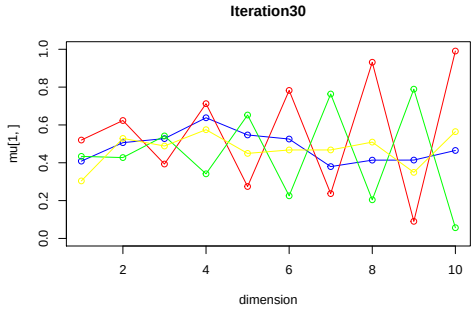

```
## iteration:  27 log likelihood:  -7307.185
```



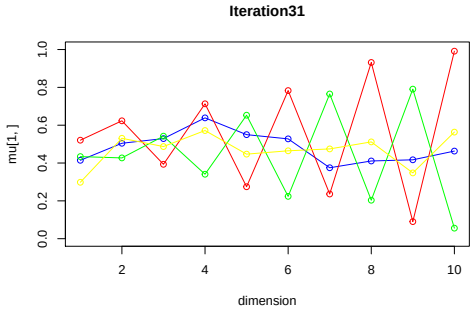

```
## iteration:  28 log likelihood:  -7305.809
```



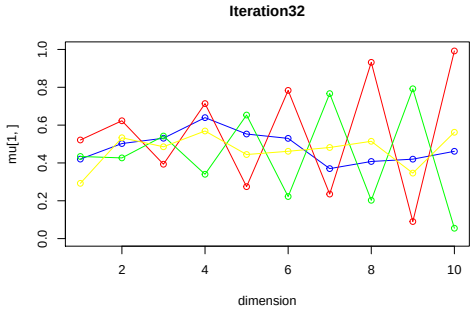

```
## iteration:  29 log likelihood:  -7304.176
```



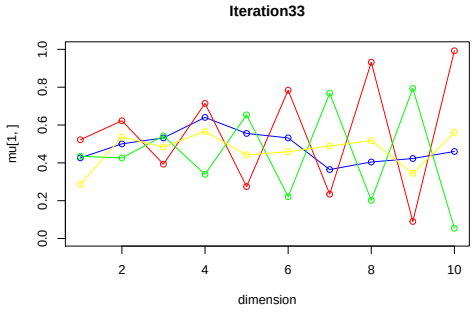

```
## iteration:  30 log likelihood:  -7302.273
```



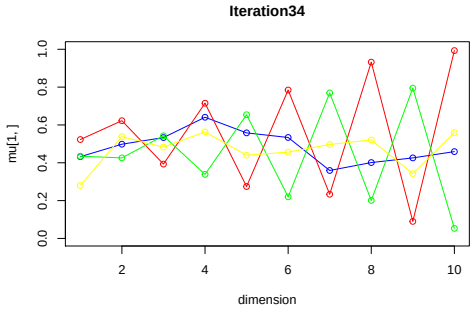

```
## iteration:  31 log likelihood:  -7300.1
```



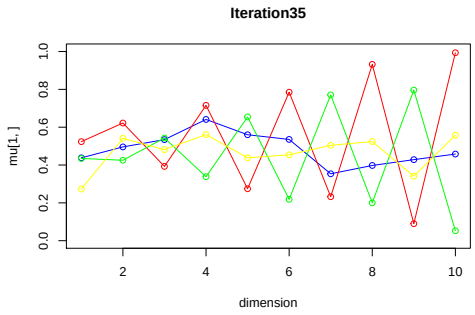

```
## iteration:  32 log likelihood:  -7297.671
```



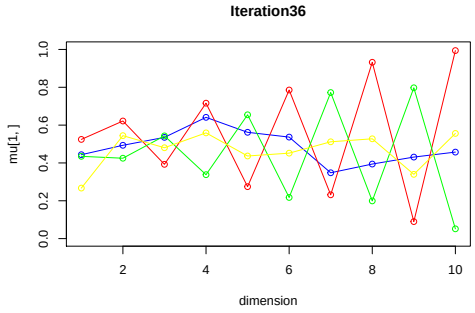

```
## iteration:  33 log likelihood:  -7295.014
```



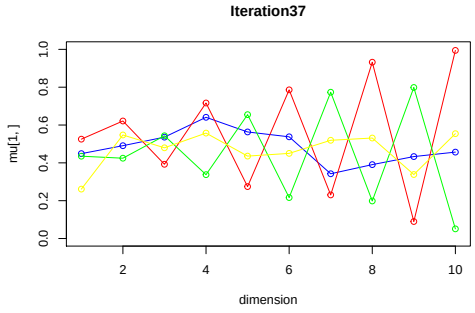

```
## iteration:  34 log likelihood:  -7292.171
```



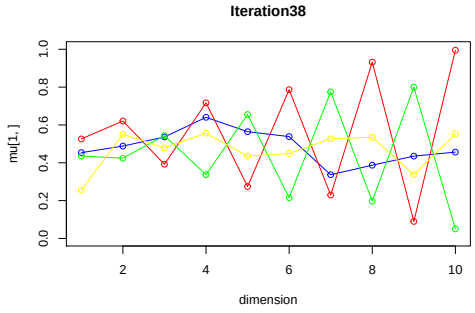

```
## iteration:  35 log likelihood:  -7289.196
```



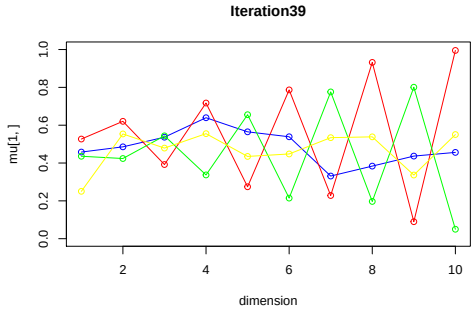

```
## iteration:  36 log likelihood:  -7286.15
```



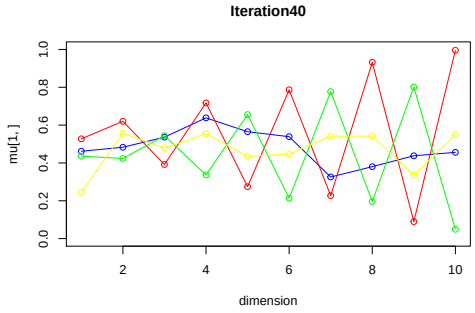

```
## iteration:  37 log likelihood:  -7283.093
```



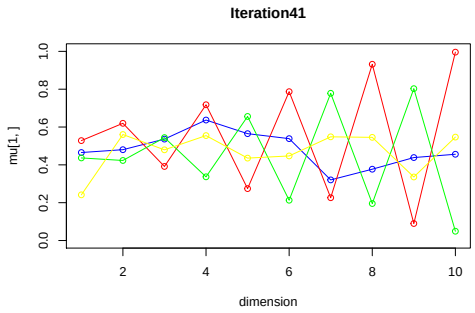

```
## iteration:  38 log likelihood:  -7280.079
```



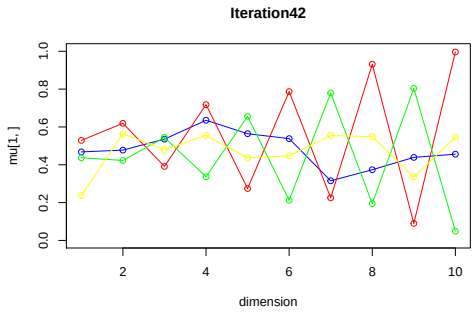

```
## iteration:  39 log likelihood:  -7277.151
```



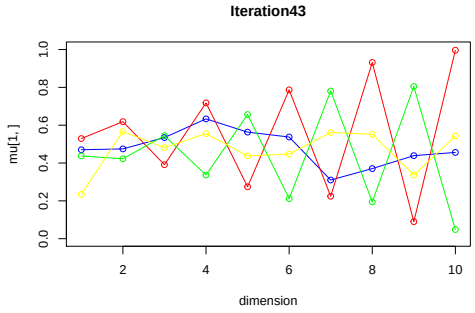

```
## iteration:  40 log likelihood:  -7274.34
```



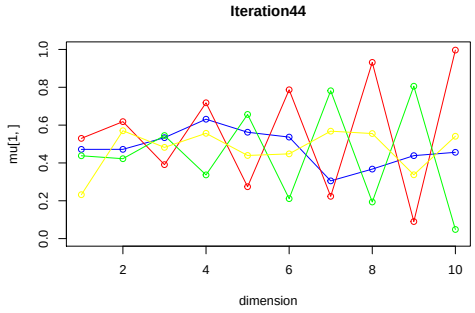

```
## iteration:  41 log likelihood:  -7271.66
```



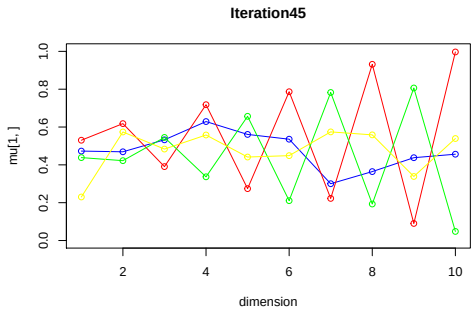

```
## iteration:  42 log likelihood:  -7269.116
```



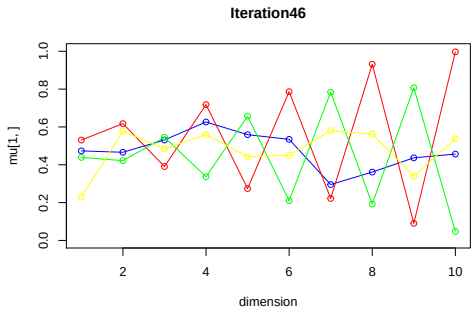

```
## iteration:  43 log likelihood:  -7266.7
```



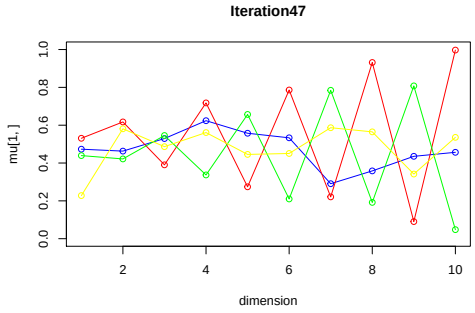

```
## iteration:  44 log likelihood:  -7264.398
```



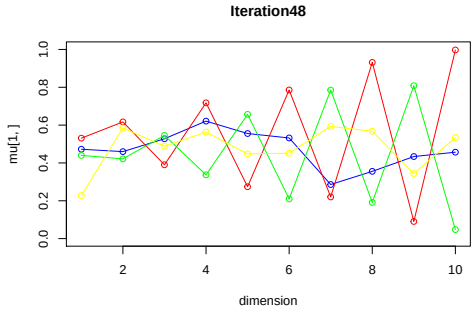

```
## iteration:  45 log likelihood:  -7262.189
```



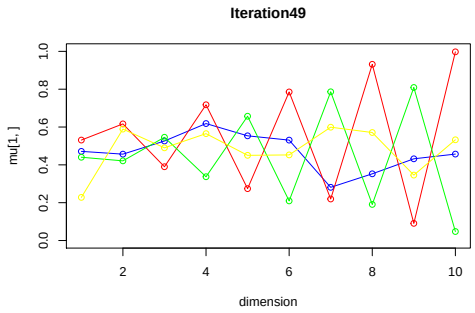

```
## iteration:  46 log likelihood:  -7260.051
```



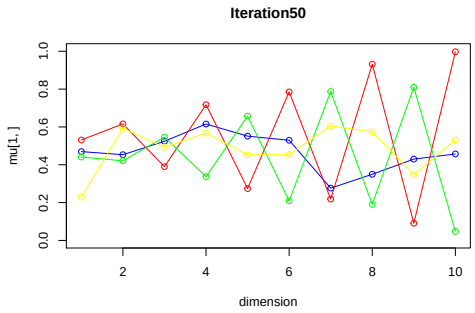

```
## iteration:  47 log likelihood:  -7257.96
```



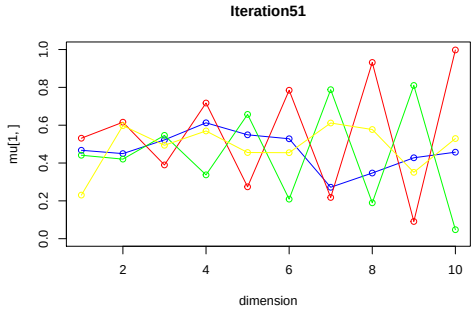

```
## iteration:  48 log likelihood:  -7255.892
```



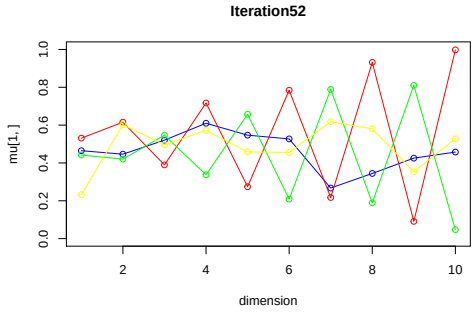

```
## iteration:  49 log likelihood:  -7253.824
```



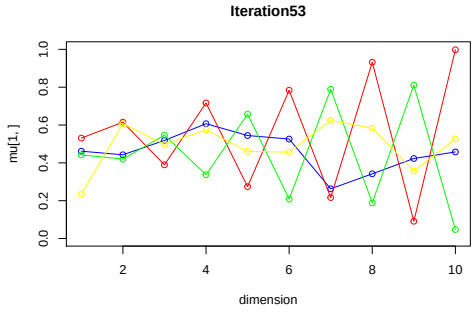

```
## iteration:  50 log likelihood:  -7251.733
```



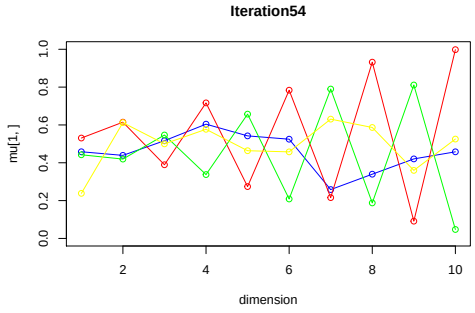

```
## iteration:  51 log likelihood:  -7249.603
```



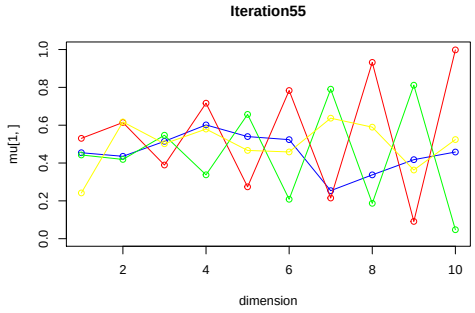

```
## iteration:  52 log likelihood:  -7247.419
```



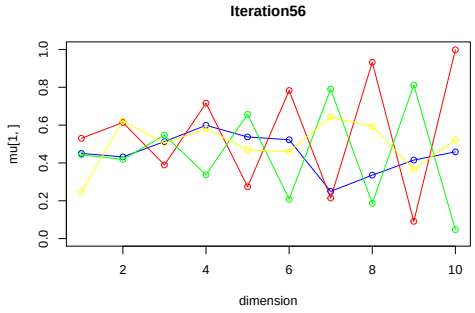

```
## iteration:  53 log likelihood:  -7245.17
```



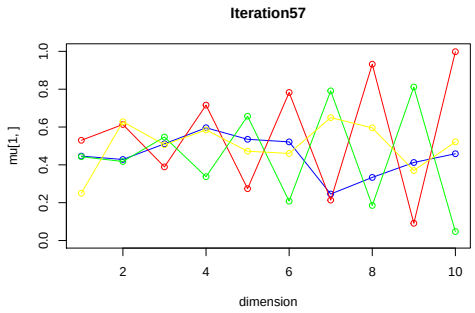

```
## iteration:  54 log likelihood:  -7242.853
```



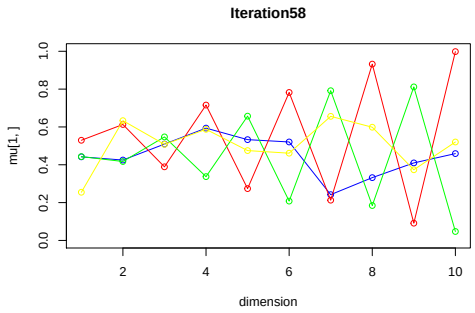

```
## iteration:  55 log likelihood:  -7240.472
```



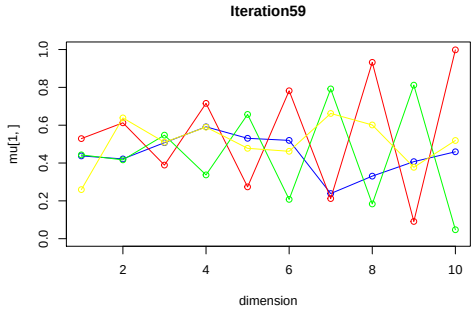

```
## iteration:  56 log likelihood:  -7238.038
```



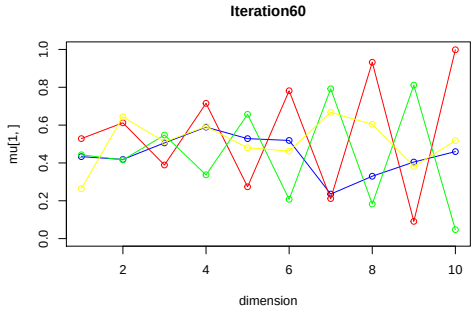

```
## iteration:  57 log likelihood:  -7235.571
```



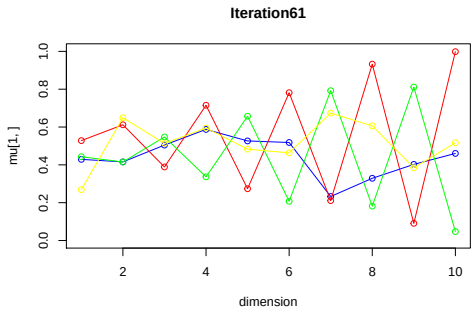

```
## iteration:  58 log likelihood:  -7233.095
```



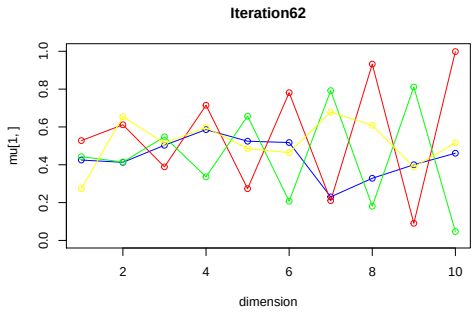

```
## iteration:  59 log likelihood:  -7230.64
```



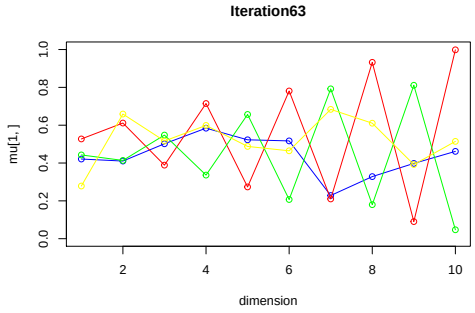

```
## iteration:  60 log likelihood:  -7228.239
```



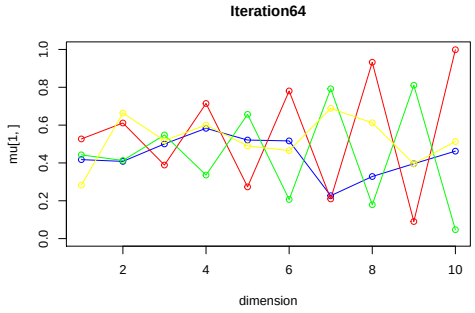

```
## iteration:  61 log likelihood:  -7225.925
```



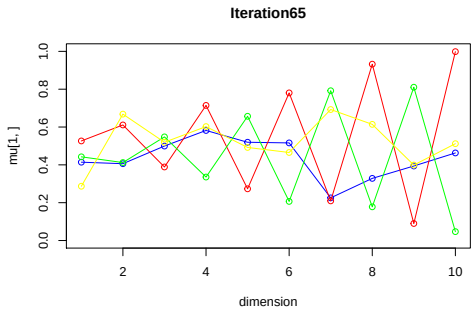

```
## iteration:  62 log likelihood:  -7223.725
```



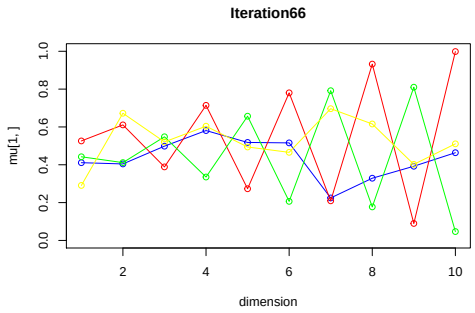

```
## iteration:  63 log likelihood:  -7221.663
```



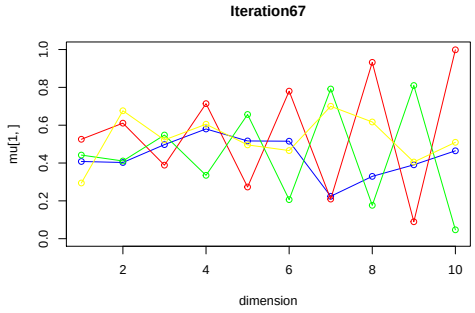

```
## iteration:  64 log likelihood:  -7219.755
```



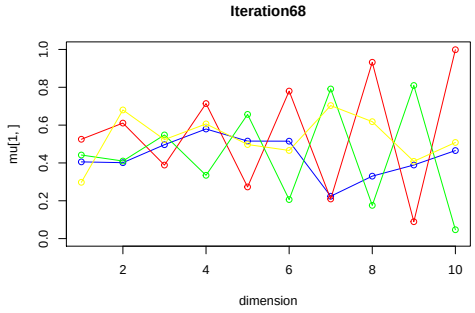

```
## iteration:  65 log likelihood:  -7218.01
```



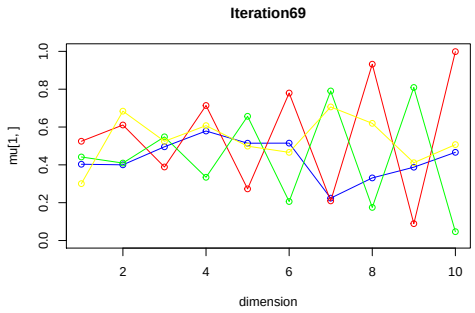

```
## iteration:  66 log likelihood:  -7216.431
```



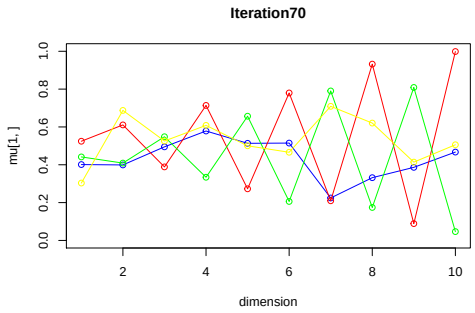

```
## iteration:  67 log likelihood:  -7215.013
```



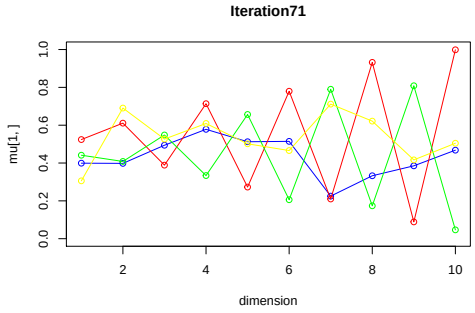

```
## iteration:  68 log likelihood:  -7213.748
```



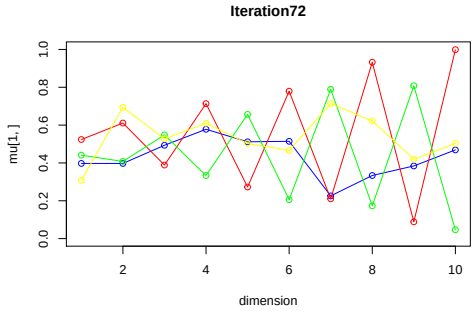

```
## iteration:  69 log likelihood:  -7212.621
```



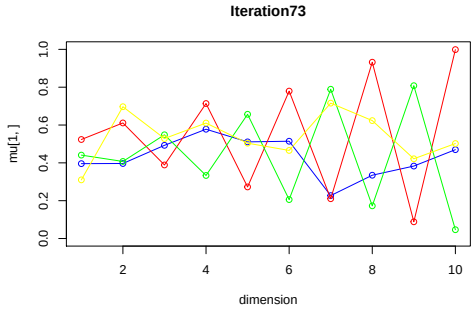

```
## iteration:  70 log likelihood:  -7211.62
```



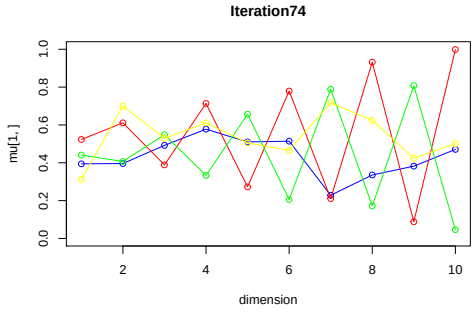

```
## iteration:  71 log likelihood:  -7210.727
```



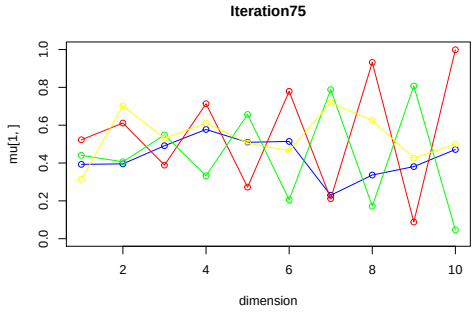

```
## iteration:  72 log likelihood:  -7209.929
```



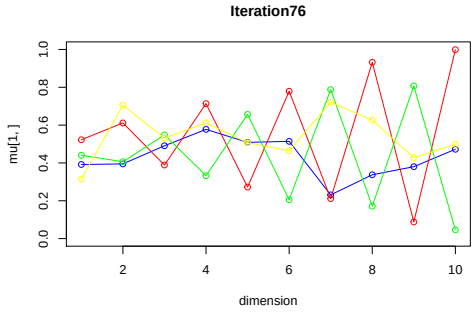

```
## iteration:  73 log likelihood:  -7209.208
```



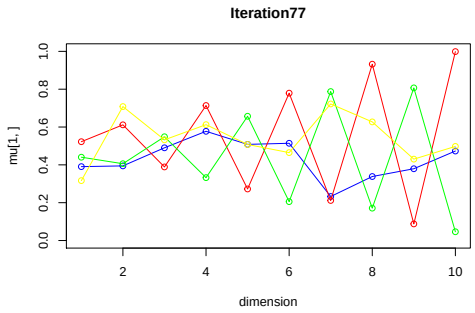

```
## iteration:  74 log likelihood:  -7208.552
```



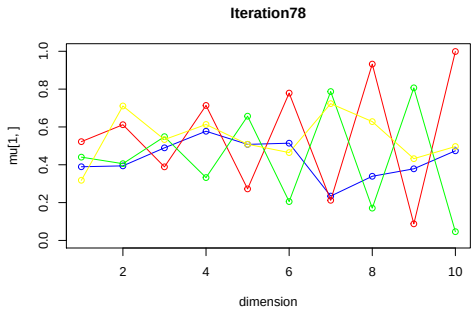

```
## iteration:  75 log likelihood:  -7207.946
```



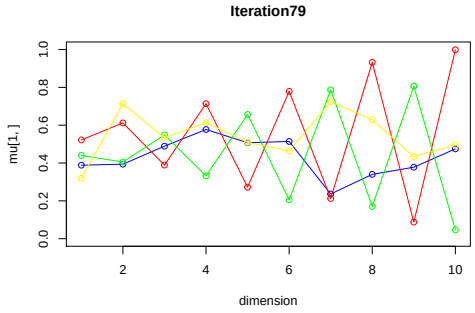

```
## iteration:  76 log likelihood:  -7207.38
```



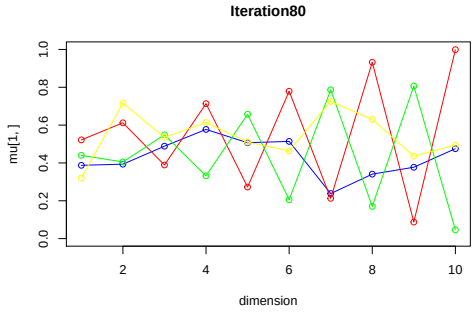

```
## iteration:  77 log likelihood:  -7206.844
```



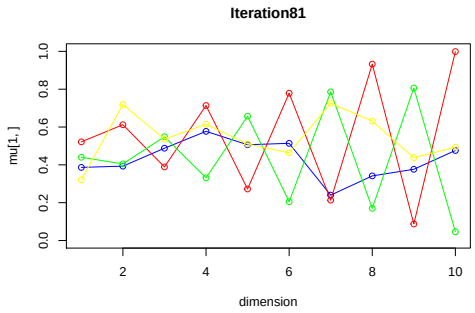

```
## iteration:  78 log likelihood:  -7206.327
```



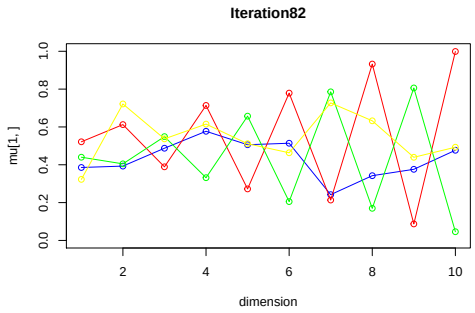

```
## iteration:  79 log likelihood:  -7205.824
```



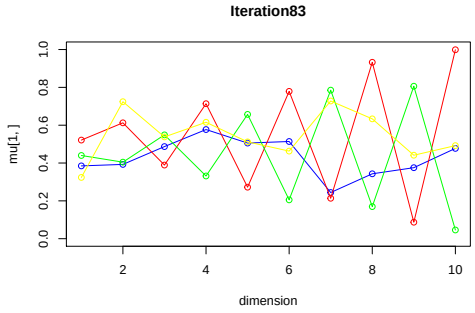

```
## iteration:  80 log likelihood:  -7205.326
```



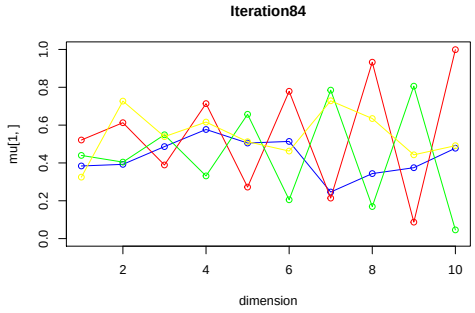

```
## iteration:  81 log likelihood:  -7204.829
```



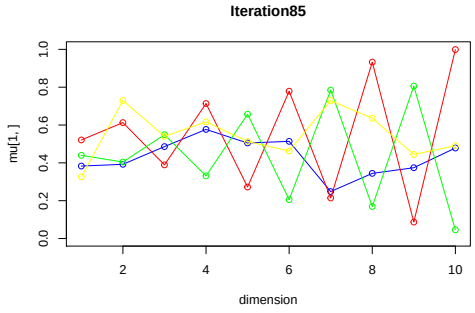

```
## iteration:  82 log likelihood:  -7204.327
```



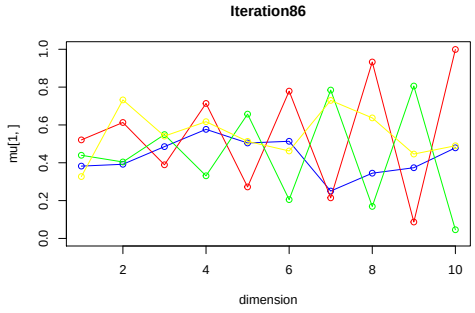

```
## iteration:  83 log likelihood:  -7203.816
```



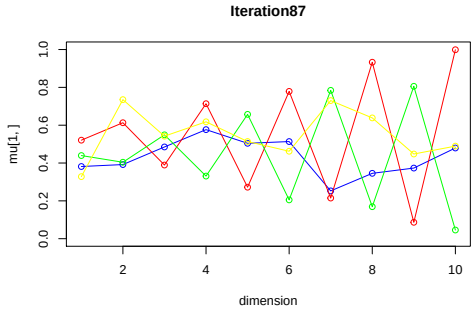

```
## iteration:  84 log likelihood:  -7203.294
```



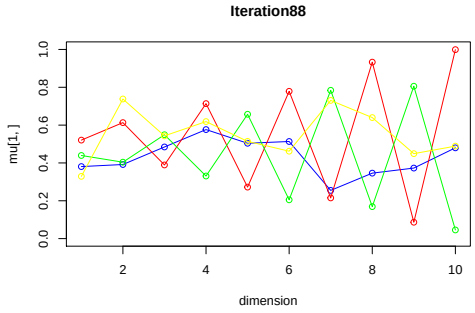

```
## iteration:  85 log likelihood:  -7202.756
```



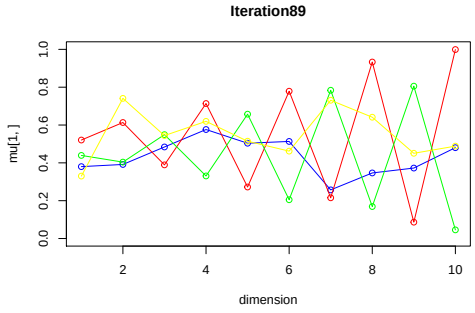

```
## iteration:  86 log likelihood:  -7202.201
```



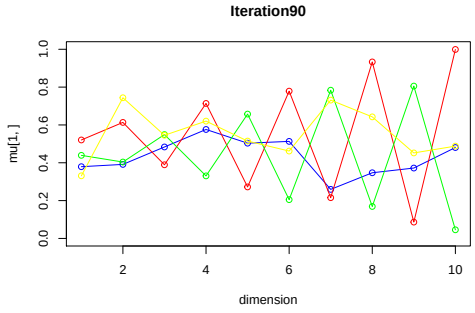

```
## iteration:  87 log likelihood:  -7201.627
```



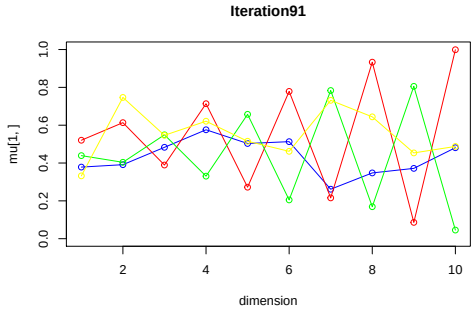

```
## iteration:  88 log likelihood:  -7201.032
```



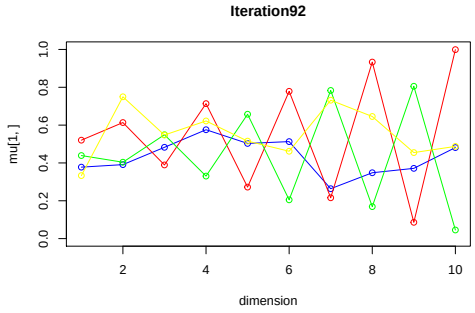

```
## iteration:  89 log likelihood:  -7200.414
```



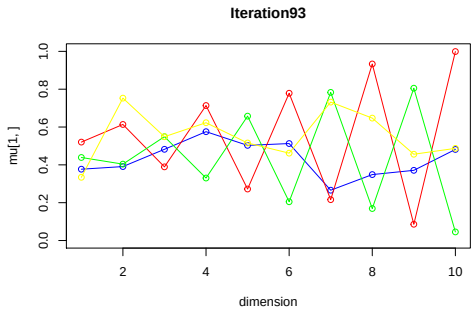

```
## iteration:  90 log likelihood:  -7199.773
```



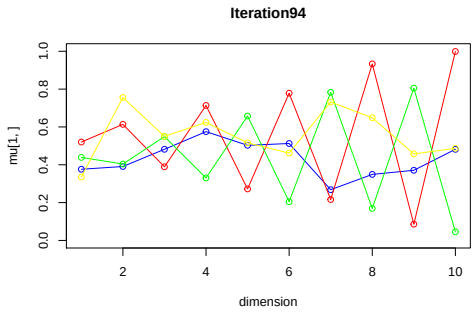

```
## iteration:  91 log likelihood:  -7199.107
```



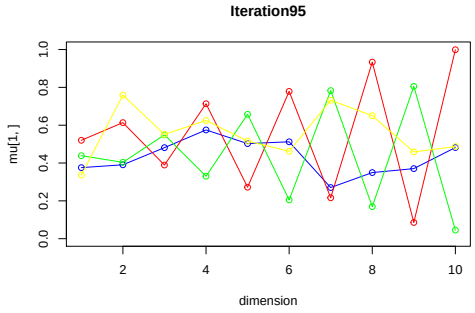

```
## iteration:  92 log likelihood:  -7198.416
```



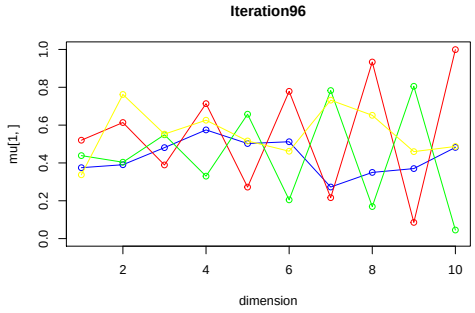

```
## iteration:  93 log likelihood:  -7197.7
```



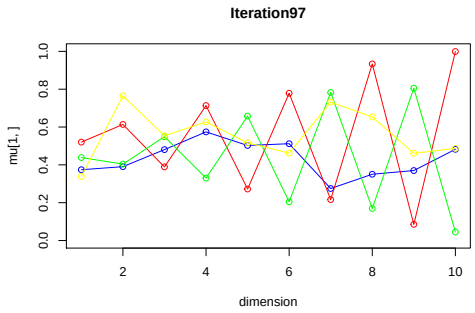

```
## iteration:  94 log likelihood:  -7196.957
```



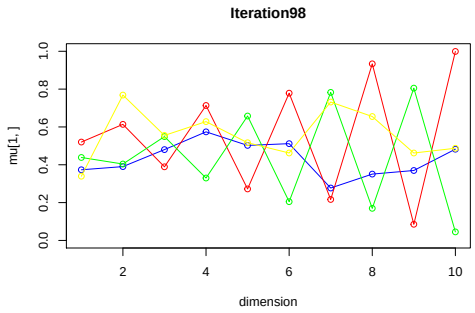

```
## iteration:  95 log likelihood:  -7196.188
```



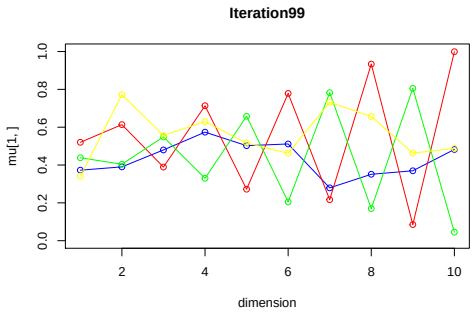

```
## iteration:  96 log likelihood:  -7195.392
```



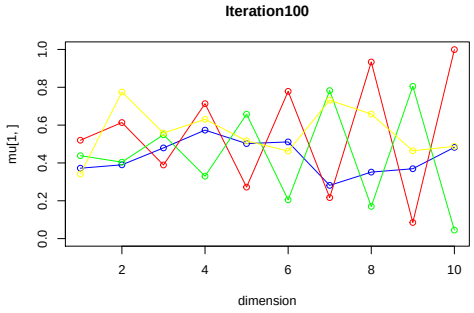

```
## iteration:  97 log likelihood:  -7194.57
```



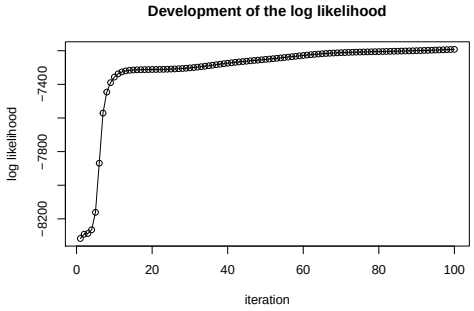

```
## iteration:  98 log likelihood:  -7193.722
```




```
## iteration:  99 log likelihood:  -7192.847
```




```
## iteration:  100 log likelihood:  -7191.946
```




```
## $pi
## [1] 0.2880470 0.2533761 0.2933710 0.1652060
##
## $mu
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.3714855 0.3899958 0.4790260 0.5731886 0.5022651 0.5108478 0.2835691
## [2,] 0.5199997 0.6135841 0.3891214 0.7132736 0.2722448 0.7785461 0.2168891
## [3,] 0.4383456 0.4042497 0.5489526 0.3298363 0.6578057 0.2049012 0.7825505
## [4,] 0.3428531 0.7784238 0.5591637 0.6319621 0.5167044 0.4629058 0.7311279
##           [,8]       [,9]      [,10]
## [1,] 0.3519184 0.36924863 0.48252239
## [2,] 0.9337959 0.08504806 0.99916297
## [3,] 0.1703330 0.80517853 0.04500171
## [4,] 0.6601375 0.46532151 0.48814639
##
## $logLikelihoodDevelopment
```

```
## NULL
```

Analysis

Comparing the final plots for each of the cases, it becomes clear that when the mixture model has more components (K = 4), the EM algorithm does not perform as accurate as for fewer components (K = 2 or K = 3). The segregation between each component gets diluted as the components get higher.

Appendix

```
if (!require("pacman")) install.packages("pacman")
pacman::p_load(mboost, randomForest, ggplot2)

options("jtools-digits" = 2, scipen = 999)

spam_data <- read.csv(file = "spambase.data", header = FALSE)
colnames(spam_data)[58] <- "Spam"
spam_data$Spam <- factor(spam_data$Spam, levels = c(0,1), labels = c("0", "1"))
set.seed(12345)
n =  NROW(spam_data)
id = sample(1:n, floor(n*(2/3)))
train = spam_data[id,]
test = spam_data[-id,]

final_result <- NULL
for(i in seq(from = 10, to = 100, by = 10)){

ada_model <- mboost::blackboost(Spam~.,
                                data = train,
                                family = AdaExp(),
                               control=boost_control(mstop=i))

forest_model <- randomForest(Spam~., data = train, ntree = i)


prediction_function <- function(model, data){
  predicted <- predict(model, newdata = data, type = c("class"))
  predict_correct <- ifelse(data$Spam == predicted, 1, 0)
  score <- sum(predict_correct)/NROW(data)
  return(score)
}


train_ada_model_predict <- predict(ada_model, newdata = train, type = c("class"))
test_ada_model_predict <- predict(ada_model, newdata = test, type = c("class"))
train_forest_model_predict <- predict(forest_model, newdata = train, type = c("class"))
test_forest_model_predict <- predict(forest_model, newdata = test, type = c("class"))

test_predict_correct <- ifelse(test$Spam == test_forest_model_predict, 1, 0)
train_predict_correct <- ifelse(train$Spam == train_forest_model_predict, 1, 0)


train_ada_score <-  prediction_function(ada_model, train)
```

```
test_ada_score <-  prediction_function(ada_model, test)
train_forest_score <-  prediction_function(forest_model, train)
test_forest_score <-  prediction_function(forest_model, test)

iteration_result <- data.frame(number_of_trees = i,
                               accuracy = c(train_ada_score,
                                            test_ada_score,
                                            train_forest_score,
                                            test_forest_score),
                               type  = c("train", "test", "train", "test"),
                               model = c("ADA", "ADA",  "Forest", "Forest"))


final_result <- rbind(iteration_result, final_result)
}

final_result$error_rate_percentage <- 100*(1 - final_result$accuracy)
ggplot(data = final_result, aes(x = number_of_trees,
                                y = error_rate_percentage,
                                group = type, color = type)) +
  geom_point() +
  geom_line() +
  ggtitle("Error Rate vs. increase in trees") + facet_grid(rows = vars(model))

em_loop = function(K) {
# Initializing data
set.seed(1234567890)
max_it = 100 # max number of EM iterations
min_change = 0.1 # min change in log likelihood between two consecutive EM iterations
N = 1000 # number of training points
D = 10 # number of dimensions
x = matrix(nrow=N, ncol = D) # training data
true_pi = vector(length = K) # true mixing coefficients
true_mu = matrix(nrow = K, ncol = D) # true conditional distributions
true_pi = c(rep(1/K, K))
if (K == 2) {
true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue",
ylim = c(0,1), main = "True")
points(true_mu[2,], type="o", xlab = "dimension", col = "red",
main = "True")
} else if (K == 3) {
true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
true_mu[3,] = c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue", ylim=c(0,1),
main = "True")
points(true_mu[2,], type = "o", xlab = "dimension", col = "red",
main = "True")
points(true_mu[3,], type = "o", xlab = "dimension", col = "green",
main = "True")
} else {
```

```
true_mu[1,] = c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,] = c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
true_mu[3,] = c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
true_mu[4,] = c(0.3,0.5,0.5,0.7,0.5,0.5,0.5,0.5,0.4,0.5)
plot(true_mu[1,], type = "o", xlab = "dimension", col = "blue",
ylim = c(0,1), main = "True")
points(true_mu[2,], type = "o", xlab = "dimension", col = "red",
main = "True")
points(true_mu[3,], type = "o", xlab = "dimension", col = "green",
main = "True")
points(true_mu[4,], type = "o", xlab = "dimension", col = "yellow",
main = "True")
}
z = matrix(nrow = N, ncol = K) # fractional component assignments
pi = vector(length = K) # mixing coefficients
mu = matrix(nrow = K, ncol = D) # conditional distributions
llik = vector(length = max_it) # log likelihood of the EM iterations
# Producing the training data
for(n in 1:N) {
k = sample(1:K, 1, prob=true_pi)
for(d in 1:D) {
x[n,d] = rbinom(1, 1, true_mu[k,d])
}
}
# Random initialization of the paramters
pi = runif(K, 0.49, 0.51)
pi = pi / sum(pi)
for(k in 1:K) {
mu[k,] = runif(D, 0.49, 0.51)
}
#EM algorithm
for(it in 1:max_it) {
# Plotting mu
# Defining plot title
title = paste0("Iteration", it)
if (K == 2) {
plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
} else if (K == 3) {
plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
points(mu[3,], type = "o", xlab = "dimension", col = "green", main = title)
} else {
plot(mu[1,], type = "o", xlab = "dimension", col = "blue", ylim = c(0,1), main = title)
points(mu[2,], type = "o", xlab = "dimension", col = "red", main = title)
points(mu[3,], type = "o", xlab = "dimension", col = "green", main = title)
points(mu[4,], type = "o", xlab = "dimension", col = "yellow", main = title)
}
Sys.sleep(0.5)
# E-step: Computation of the fractional component assignments
for (n in 1:N) {
# Creating empty matrix (column 1:K = p_x_given_k; column K+1 = p(x|all k)
p_x = matrix(data = c(rep(1,K), 0), nrow = 1, ncol = K+1)
```

```
# Calculating p(x|k) and p(x|all k)
for (k in 1:K) {
# Calculating p(x|k)
for (d in 1:D) {
p_x[1,k] = p_x[1,k] * (mu[k,d]^x[n,d]) * (1-mu[k,d])^(1-x[n,d])
}
p_x[1,k] = p_x[1,k] * pi[k] # weighting with pi[k]
# Calculating p(x|all k) (denominator)
p_x[1,K+1] = p_x[1,K+1] + p_x[1,k]
}
# Calculating z for n and all k
for (k in 1:K) {
z[n,k] = p_x[1,k] / p_x[1,K+1]
}
}
# Log likelihood computation
for (n in 1:N) {
for (k in 1:K) {
log_term = 0
for (d in 1:D) {
log_term = log_term + x[n,d] * log(mu[k,d]) + (1-x[n,d]) * log(1-mu[k,d])
}
llik[it] = llik[it] + z[n,k] * (log(pi[k]) + log_term)
}
}
cat("iteration: ", it, "log likelihood: ", llik[it], "\n")
flush.console()
# Stop if the log likelihood has not changed significantly
if (it != 1) {
if (abs(llik[it] - llik[it-1]) < min_change) {
break
}
}
# M-step: ML parameter estimation from the data and fractional component assignments
# Updating pi
for (k in 1:K) {
pi[k] = sum(z[,k])/N
}
# Updating mu
for (k in 1:K) {
mu[k,] = 0
for (n in 1:N) {
    mu[k,] = mu[k,] + x[n,] * z[n,k]
}
mu[k,] = mu[k,] / sum(z[,k])
}
}
# Printing pi, mu and development of log likelihood at the end
return(list(
pi = pi,
mu = mu,
logLikelihoodDevelopment = plot(llik[1:it],
type = "o",
```

```
main = "Development of the log likelihood",
xlab = "iteration",
ylab = "log likelihood")
))
}

em_loop(2)
em_loop(3)
em_loop(4)
```