Assignment 12&18 (Time series)

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Given $y_t = \mu_t + \sigma_t x_t$.

(a) Find $E(y_t)$ & $cov(y_s, y_p)$.

→ ~~$E(y_t) = E(y_t - E(y_t))$~~

→ $E(y_t) = E(\mu_t) + E(\sigma_t x_t)$

$= E(\mu_t) + 0 \quad \{E(\sigma_t x_t) = E(\sigma_t) E(x_t) \overset{?}{=} 0\}$

Ans $\boxed{E(y_t) = E(\mu_t) = \mu_t}$ Since μ_t is function.

$cov(y_s, y_p) = E((y_s - E(y_s))(y_p - E(y_p)))$

$= E((y_s - \mu_s)(y_p - \mu_p))$ {From above}

$cov(y_s, y_p) = E((\cancel{\mu_s} + \sigma_s x_s - \cancel{\mu_s})(\cancel{\mu_p} + \sigma_p x_p - \cancel{\mu_p}))$

→ $cov(y_s, y_p) = E(\sigma_s \sigma_p x_s x_p)$

→ $cov(y_s, y_p) = \sigma_s \sigma_p E((x_s - 0)(x_p - 0))$ {since σ is function}

Ans $\boxed{cov(y_s, y_p) = \sigma_s \sigma_p \, cov(x_s, x_p)}$

But given autocorrelation of x is f_h.

we know $\boxed{corr(X, Y) = \frac{cov(X, Y)}{\sqrt{Var(X) Var(Y)}}}$

→ $\boxed{cov(y_s, y_p) = \sigma_s \sigma_p f_h}$ {since $var(x_s) = 1$}

Ans

Auto correlation.

(b)

$$\boxed{\rho(s,t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\,\gamma(t,t)}}}$$

From above we get.

Answer

$$\boxed{\rho(s,t) = \frac{\cancel{\sigma_s}\cancel{\sigma_t}\,\rho_h}{\sqrt{\cancel{\sigma_s^2}\cancel{\sigma_t^2}(1)(1)}} = \rho_h}$$

Since $E(y_t) \neq$ constant thus y_t is not stationary.

(c) For a timeseries (y_t) to be stationary there are three conditions.

$\checkmark\ E(y_t) = \text{constant}.$

$\checkmark\ \gamma(s,t) = \gamma(|t+h-t|)$

$\text{var}(y_t) < \infty.$

Given is mean is constant the first condition is met.

We also know from defination that $\text{cov}(y_t, y_{t+h}) = \gamma(y_t, y_{t+h}) = \gamma(h)$ Thus second condition is true

Answer

We are not given any information about $\text{var}(y_t)$ thus if this is possible then y_t can be stationary.

Assignment 18

c) $\underbrace{x_t - 3x_{t-1}}_{AR_p(\phi)} = \underbrace{w_t + 2w_{t-1}}_{MA_q(\theta)} - 8w_{t-2}$.

iff $|z_\phi| > 1$ Then ARMA(p,q) is Causal

$|z_\theta| > 1$ Then ARMA(p,q) is invertible.

Comparing co-efficient we get $p = 1$, $q = 2$.

ARMA(1,2).

roots of θ are $\theta(z) = 1 + 2z - 8z^2$.

gives us roots as. $z_{\theta 1} = 0.5$

$z_{\theta 2} = -0.25$

$\phi(z) = (1 - 3z)$

$\phi_{z_1} = 0.3333$.

Since neither $|z_\phi| > 1$ & $|z_\theta| > 1$ is not met The series is neither Causal or invertible.

$$\overbrace{}^{AR_{\phi}(p)} \qquad \overbrace{}^{MA_{\theta}(q)}$$

d) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1}$.

$\theta_z = \left(1 - \frac{8}{9}z\right) \Rightarrow$

$\theta_{z_1} = \frac{9}{8} = 1.125$

Answer: $\boxed{\text{since } |z_\theta| \geq 1 \text{, we can say that this time series is invertible.}}$

$\phi_z = (1 - 2z + 2z^2)$

$\Rightarrow \phi_z$ roots are $\phi_z = 0.5 + 0.5i$
$0.5 - 0.5i$

Answer: $\boxed{\text{since } |z_\phi| > 1 \text{ is not met, we can say time series cannot be causal.}}$

Answer: $\boxed{p = 2,\ q = 1. \text{ This series is ARMA}(2,1).}$

$$\overbrace{x_t - 4x_{t-2}}^{AR_\phi(p)} = \overbrace{w_t - w_{t-1} + 0.5w_{t-2}}^{MA_\theta(q)}.$$

(e)

$\phi_z = (1 - 4z^2).$ | $\theta_z = (1 - z + 0.5z^2)$

roots are $\phi_z \Rightarrow 0.5, -0.5$ | roots are $\theta_z = 1+i$, $1-i$

Since no common roots exists Thus There is no redundancy.

Answer: $\boxed{\text{Thus } ARMA(p,q) = ARMA(2,2)}$

Answer: Since $|z_\theta| > 1$ is not met The series is not invertible

$|z_\phi| > 1$ is met, Thus series is causal.

(f) $\underbrace{x_t - \frac{9}{4} x_{t-1} - \frac{9}{4} x_{t-2}}_{AR_\phi(p)} = \underbrace{w_t}_{MA_\theta(q)}$

$q = 0$.

Since there is no MA component.

This is purely a AR time series.
Thus no point in checking for invertible.

Answer

$\phi(z) = \left(1 - \frac{9}{4} z - \frac{9}{4} z^2\right)$

$\phi(z)$. roots are $\frac{1}{3}$, $\frac{-4}{3}$.

roots of $\theta(z) = 1$.

Answer ARMA(2,0).

Since $|z_\phi| > 1$ is not met, thus series is not causal.