

Time Series Analysis

Teaching Session II: ARIMA models-3 Seasonal models

Tohid Ardeshiri

Linköping University
Division of Statistics and Machine Learning

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Seasonal ARMA

- Seasonal patterns
 - ▶ Yearly (ocean temperature)
 - ▶ Daily, weekly (Server workload)
- Strong correlation of x_t and x_{t+s}
 - ▶ $s = 12, 24, \dots$
- Applications
 - ▶ Physics, biology, economics, computer science

Seasonal ARMA

- Pure seasonal $ARMA(P, Q)_s$

$$\Phi_p(B^s)x_t = \theta_Q(B^s)w_t$$

- Seasonal autoregressive operator

$$\Phi_p(B^s) = 1 - \Phi_1(B^{1 \cdot s}) - \dots - \Phi_p B^{P \cdot s}$$

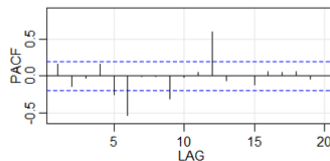
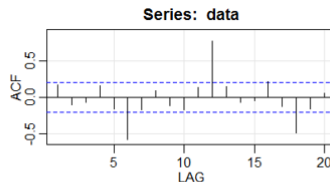
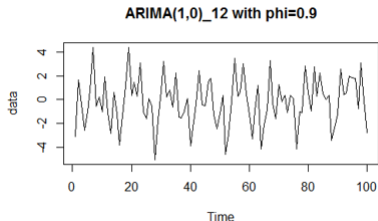
- Seasonal moving average operator

$$\Theta_Q(B^s) = 1 + \Theta_1(B^{1 \cdot s}) - \dots - \Theta_Q B^{Q \cdot s}$$

- Same principles for causality and invertibility
- **Example:** $ARMA(1, 0)_{12}$ and $ARMA(0, 1)_{12}$
 - ▶ Autocovariance

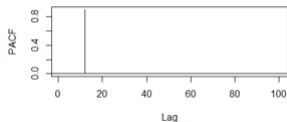
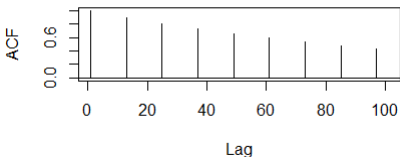
Seasonal ARMA

- **Example:** Simulated $ARMA(1,0)_{12}$, $\Phi = 0.9$



Seasonal ARMA

- **Example:** Simulated $ARMA(1, 0)_{12}$, $\Phi = 0.9$
 - ▶ Theoretical ones



Seasonal ARMA

	$AR(P)_s$	$MA(Q)_s$	$ARMA(P, Q)_s$
ACF*	Tails off at lags ks , $k = 1, 2, \dots$,	Cuts off after lag Qs	Tails off at lags ks
PACF*	Cuts off after lag P_s	Tails off at lags ks $k = 1, 2, \dots$,	Tails off at lags ks

*The values at nonseasonal lags $h \neq ks$, for $k = 1, 2, \dots$, are zero.

Multiplicative seasonal ARMA

- **Problem:** in real data, it is hard to assume x_t is dependent on x_{t-kh} only...
 - ▶ Combinational seasonal and nonseasonal!

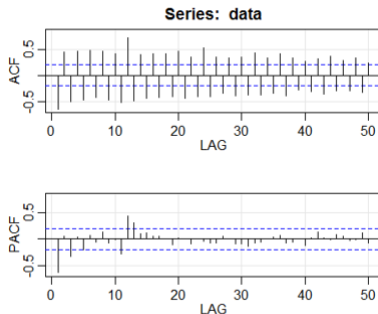
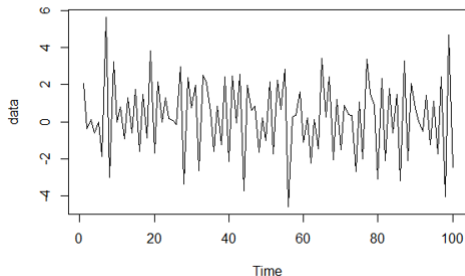
- Multiplicative Seasonal $ARMA(p, q) \times (P, Q)_s$

$$\Phi_p(B^s)\phi(B)x_t = \Theta_Q(B^s)\theta(B)w_t$$

- **Example** Expression for $ARMA(p, q) \times (P, Q)_s$

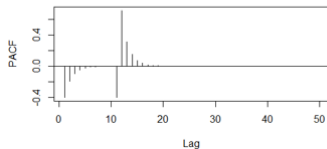
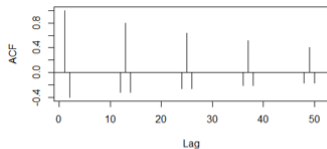
Multiplicative seasonal ARMA

- **Example** $x_t = 0.8x_{t-12} + w_t - 0.5w_{t-1}$



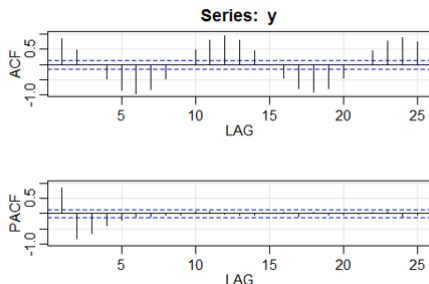
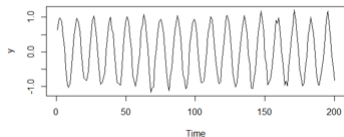
Multiplicative seasonal ARMA

- **Example** $x_t = 0.8x_{t-12} + w_t - 0.5w_{t-1}$
 - ▶ Theoretical



SARIMA

- What if there is a seasonal pattern which differs a little between the series



Note: ACF almost decays very slowly at peaks 12h

SARIMA

- Multiplicative seasonal autoregressive integrated moving average model $ARIMA(p, d, q) \times (P, D, Q)_s$

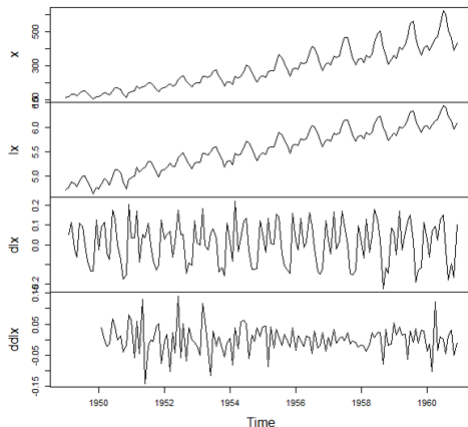
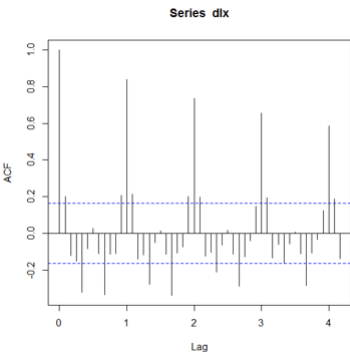
$$\Phi_p(B^s)\phi(B)\nabla_s^D\nabla^d x_t = \delta + \Theta_Q(B^s)\theta(B)w_t$$

$$\nabla_s^D = (1 - B^s)^D$$

- How to identify SARIMA?
 - ① Perform differencing first (trend)
 - ② Investigate ACF \rightarrow slowly decays at peaks
 - ① Yes \rightarrow Additional differencing by ∇_s^D
 - ③ Model non-seasonal part
 - ④ Model seasonal part (check peaks), check ACF and PACF of residuals

SARIMA

- **Example:** Air passengers



SARIMA

- Remove AR term!

Is one model much better the other one?

$$(1, 1, 1) \times (1, 1, 0)_{12}$$

```
> m1$fit  
Call:  
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,  
Q), period = S), include.mean = !no.constant, optim.control = list(trace = trc,  
REPORT = 1, reltol = tol))
```

```
Coefficients:  
      ar1      ma1      sar1  
0.0547 -0.4886 -0.4731  
s.e. 0.2161 0.1933 0.0800
```

```
sigma^2 estimated as 0.001425: log likelihood = 241.73, aic = -475.47
```

```
> m2$fit  
Call:  
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,  
Q), period = S), include.mean = !no.constant, optim.control = list(trace = trc,  
REPORT = 1, reltol = tol))
```

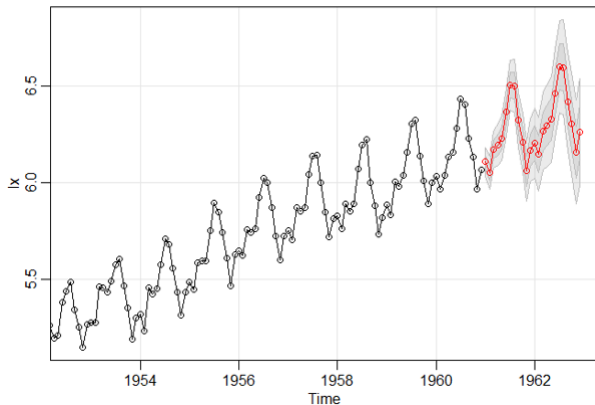
```
Coefficients:  
      ar1      ma1      sma1  
0.1960 -0.5784 -0.5643  
s.e. 0.2475 0.2132 0.0747
```

```
sigma^2 estimated as 0.001341: log likelihood = 244.95, aic = -481.9
```

$$(1, 1, 1) \times (0, 1, 1)_{12}$$

SARIMA

- Forecasting



Read home

- Shumway and Stoffer, section 3.9
- R code: `sarima`, `sarima.for`, `runs`