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# $\underline{\text{II}^{nd} \text{ Hand-in}}$

Given ARIMA $(p, d, q) * (P\ D\ Q)_s$, write the state space.

That is

$$\Phi^P(B^s)\ \phi^p(B)\ (1-B^s)^D (1-B)^d X_t = \Theta^Q(B^s)\ \theta^q(B)\ W_t$$

$$\rightarrow \Phi^P(B^s)\, \phi^p(B)\, (1-B^s)^D (1-B)^d X_t \left[\Theta^Q(B^s)\ \theta^q(B)\right] = W_t.$$

Let $Z_t = \left[\Theta^Q(B^s)\ \theta^q(B)\right]^{-1} X_t.$

$$\rightarrow \boxed{\begin{array}{l} X_t = \Theta^Q(B^s)\ \theta^q(B)\ Z_t. \ (AR) \\ W_t = \Phi^P(B^s)\ \phi^p(B)(1-B^s)(1-B)^d Z_t. \ (MA) \end{array}} \rightarrow (A).$$

Now the space model is given by:-

$$\boxed{\begin{array}{l} Z_t = A Z_{t-1} + e_t \\ X_t = C Z_t + v_t \end{array}} \rightarrow (1). \quad \begin{array}{l} e_t \sim N(0, R) \\ v_t \sim N(0, Q) \end{array}$$

Comparing eq (1) & (A) we get that the $1^{st}$ term is the AR & $2^{nd}$ equation is for MA.

Thus we need to find matrix 'A', 'C'. in order to determine the state space equivalence.

We know from ~~Hamilton~~ Harvey representation of ARIMA(p,q).

$$F = \begin{pmatrix} \phi_1 & 1 & 0 & 0 & 0 & \cdots & 0 \\ \phi_2 & 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & & \vdots & & & & \vdots \\ \vdots & & & & & & 1 \\ \phi_d & & & & & & 0 \end{pmatrix}$$

$$\boxed{Z_t = F Z_{t-1} + w_t \begin{pmatrix} 1 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{d_s - 1} \end{pmatrix}}$$

$\rightarrow$ AR

$\rightarrow$ MA.

$\rightarrow (2)$

$d = \max(p, q+1)$

$$\boxed{X_t = (1 \ 0 \ \dots \ 0)^T Z_t}$$

$\rightarrow (3)$

We can rewrite the expression such that.

'C' matrix $= [1 \ \theta_1 \ \theta_2 \ \dots \ \theta_{d_s - 1}]$.

$$\text{'A' matrix} = \begin{bmatrix} \phi_1 & 1 & 0 & 0 & \dots & 0 \\ \phi_2 & 0 & 1 & \dots & & \vdots \\ \vdots & & \ddots & & & \vdots \\ \vdots & & & \ddots & & 1 \\ \phi_d & \dots & & & & 0 \end{bmatrix}$$

The dimension of matrix A will be

$\max(p + sp + d + sD, \ sQ + q)$.

Substituting the values given $p = 3, d = 2, q = 1, P = 2, D = 1, Q = 1, s = 5$

in eq(A) we can get a expression to plug into the matrix & solve.