

# Dirac Library

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## 1 Math used in the library

Definition of a quantum state:

$$|\psi\rangle := \sum \psi_n |n\rangle$$

$$|\psi\rangle := \int_{-\infty}^{\infty} \psi(x) |x\rangle dx$$

Normalization:

$$\langle\psi|\psi\rangle = 1$$

$$\sum \psi_n^* \psi_n = 1$$

$$\int_{-\infty}^{\infty} \psi^*(x) \psi(x) dx = 1$$

Probability density:

$$\psi^*(x) \psi(x) = \rho(x)$$

Operators:

$$\hat{x}|\psi\rangle = x|\psi\rangle$$

$$\hat{p}|\psi\rangle = -i\hbar \frac{\partial}{\partial x} |\psi\rangle$$

Expected value of an operator:

$$\langle Q \rangle = \langle\psi|\hat{Q}|\psi\rangle$$

$$\langle Q \rangle = \sum \psi_n^* \hat{Q} \psi_n$$

$$\langle Q \rangle = \int_{-\infty}^{\infty} \psi^*(x) \hat{Q} \psi(x) dx$$

Schrodinger's Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{U}$$

$$\hat{H} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \hat{U}$$

Schrodinger's Equation:

$$i\hbar \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle$$

Time Evolution of the quantum state:

$$|\psi(t + \Delta t)\rangle = \left[ 1 - \frac{i}{\hbar} \hat{H} \Delta t \right] |\psi(t)\rangle$$

Discrete derivatives:

$$\frac{\partial}{\partial x} \psi_n \approx \frac{\psi_{n+1} - \psi_n}{\Delta x}$$

$$\frac{\partial^2}{\partial x^2} \psi_n \approx \frac{\psi_{n+1} - 2\psi_n + \psi_{n-1}}{\Delta x^2}$$

## 2 Future

$$i\hbar \gamma^\mu \partial_\mu |\psi\rangle - mc |\psi\rangle = 0$$

$$\hat{H} = \alpha \cdot \hat{p} + \beta mc$$

$$K = \int_{-\infty}^{\infty} \mathcal{D}x e^{\frac{i}{\hbar} S}$$

$$\langle x' | \psi(t') \rangle = \int_{-\infty}^{\infty} dx K(x | \psi(t))$$